The asymptotic behaviour of parton distributions at small and large x

XXIV International Workshop on Deep-Inelastic Scattering and Related Subjects

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Foreword: parametrising parton distribution functions

An accurate determination of Parton Distribution Functions (PDFs)
is an essential ingredient in the precision physics program at the LHC
but cannot be achieved from Quantum Chromodynamics (QCD) first principles

PDFs for each flavor f_i are determined in a global fit to hard-scattering data
A (general, smooth, flexible) parametrisation of the PDFs at an initial scale Q_0^2 is chosen

$$\boxed{xf_i(x,Q_0^2) = A_{f_i}\, x^{a_{f_i}}\, (1-x)^{b_{f_i}}\, \mathscr{F}(x,\{c_{f_i}\})}$$

small x		large x
$xf_i(x,Q^2) \xrightarrow{x\to 0} x^{a_{f_i}}$	$\mathscr{F}(x,\{c_{f_i}\}) \underset{x\to 1}{\xrightarrow{x\to 0}}$ finite; $\xrightarrow{\text{smooth interpolation in between}}$	$xf_i(x,Q^2) \xrightarrow{x\to 1} (1-x)^{b_{f_i}}$
(Regge theory)		(quark counting rules)
[Nuovo Cim. 14 (1959) 951]		[Phys. Rev. Lett. 31 (1973) 1153]

[R.G. Roberts, The structure of the proton: deep inelastic scattering, Cambridge University Press, 1994]
[R. Devenish and A. Cooper-Sarkar, Deep inelastic scattering, Oxford University Press, 2004]

Is the assumption of the power-law behaviour a source of bias?
How small/large should x be in order for the power-law behaviour to hold?
At which values of Q^2 should the power-law behaviour apply?

Aim

Present a methodology to quantify
the effective asymptotic behaviour of PDFs at small and large x

Quantify for which ranges of x and Q^2 (if any) PDFs exhibit a power-law behaviour

Investigate how the effective exponents determined from global PDF determinations compare to the theoretical predictions provided by Regge theory and quark counting rules

Outline

1. Tools

A robust definition of effective asymptotic exponents; PDF sets

2. Small x: $\alpha_{f_i}(x, Q^2)$

x dependence at fixed Q^2; double asymptotic scaling and Q^2 dependence at fixed x
comparison with Regge theory expectations

3. Large x: $\beta_{f_i}(x, Q^2)$

x dependence at fixed Q^2; cusp anomalous dimension and Q^2 dependence at fixed x
comparison with quark counting rule expectations and models of nucleon structure

4. Conclusions

This presentation is based on a work in collaboration with R. D. Ball and J. Rojo
[arXiv:1604.00024]

1. Tools

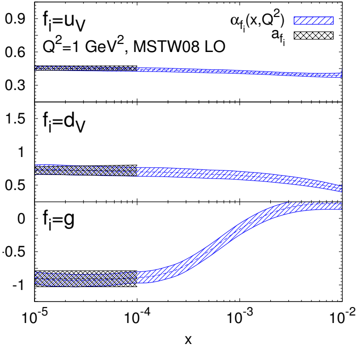
A robust definition of effective asymptotic exponents

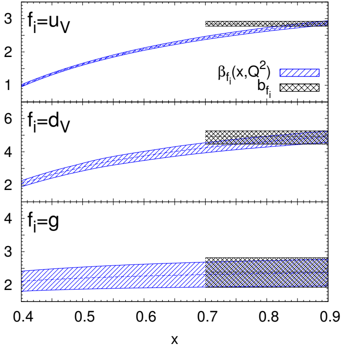
$$\alpha_{f_i}(x,Q^2) \equiv \frac{\partial \ln[x f_i(x,Q^2)]}{\partial \ln x}$$

$$\beta_{f_i}(x,Q^2) \equiv \frac{\partial \ln[x f_i(x,Q^2)]}{\partial \ln(1-x)}$$

$$\alpha_{f_i}(x,Q_0^2) = a_{f_i} + x\left[\frac{d\ln[\mathscr{F}(x,\{c_{f_i}\})]}{dx} - \frac{b_{f_i}}{1-x}\right] \xrightarrow{x\to 0} a_{f_i} + O(x)$$

$$\beta_{f_i}(x,Q_0^2) = b_{f_i} - (1-x)\left[\frac{d\ln[\mathscr{F}(x,\{c_{f_i}\})]}{dx} + \frac{a_{f_i}}{x}\right] \xrightarrow{x\to 1} b_{f_i} + O(1-x)$$





PDF sets

`NNPDF3.0` [JHEP 04 (2015) 040]
PDFs are parametrised in the basis that diagonalises the DGLAP equations
$\mathscr{F}(x,\{c_{f_i}\})$ is a multi-layer feed-forward neural network
a_{f_i} and b_{f_i} are not parameters of the fit

`MMHT14` [Eur. Phys. J. C75 (2016) 204]
Parametrised PDFs: u_V, d_V, S, Δ_S, s^+, s^-, g
$\mathscr{F}(x,\{c_{f_i}\})$ is a linear combination of Chebyschev polynomials
a_{f_i} and b_{f_i} are parameters of the fit ($a_{s^+}=a_S$)

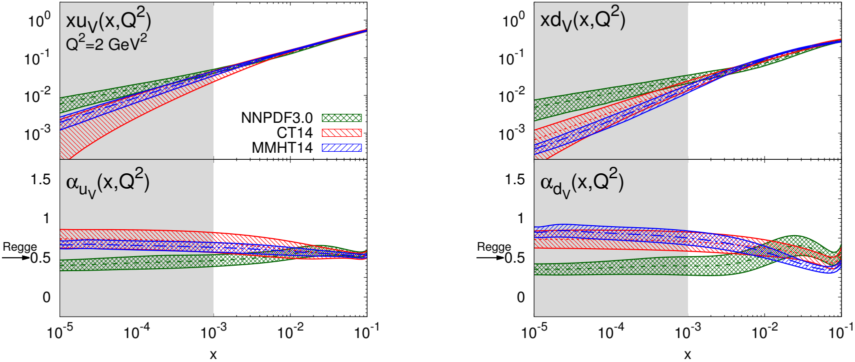
`CT14` [Phys. Rev. D93 (2016) 033006]
Parametrised PDFs: u_V, d_V, $\bar{u}$, $\bar{d}$, s^+ ($s=\bar{s}$), g
$\mathscr{F}(x,\{c_{f_i}\})$ is a linear combination of Bernstein polynomials
a_{f_i} and b_{f_i} are parameters of the fit, but not all of them are independent
($a_{\bar{u}}=a_{\bar{d}}$ so that $\bar{u}/\bar{d}(x,Q_0^2)\xrightarrow{x\to 0}k$ and $b_{u_V}=b_{d_V}$ so that $u_V/d_V(x,Q_0^2)\xrightarrow{x\to 1}k$)

`ABM12` [Phys.Rev. D89 (2014) 054028]
Parametrised PDFs: u_V, d_V, $\bar{u}$, s ($s=\bar{s}$), Δ_S, g
$\mathscr{F}(x,\{c_{f_i}\})$ has the form $x^{P_{f_i}(x)}$; for s, $\mathscr{F}(x,\{c_{f_i}\})=1$
a_{f_i} and b_{f_i} are parameters of the fit (except fixed $a_{\Delta_S}=0.7$)

`CJ15` [arXiv:1602.03154]
Parametrised PDFs: u_V, d_V $\bar{u}+\bar{d}$, $\bar{d}/\bar{u}$, s^+ ($s=\bar{s}$), g
$\mathscr{F}(x,\{c_{f_i}\})=1+c_{f_i}^{(1)}\sqrt{x}+c_{f_i}^{(2)}x$ + built-in assumptions $\bar{d}/\bar{u}\xrightarrow{x\to 1}1$ and $d_V/u_V\xrightarrow{x\to 1}k$
a_{f_i} and b_{f_i} are parameters of the fit

2. Small x: $\alpha_{f_i}(x, Q^2)$

x dependence, fixed (small) Q^2



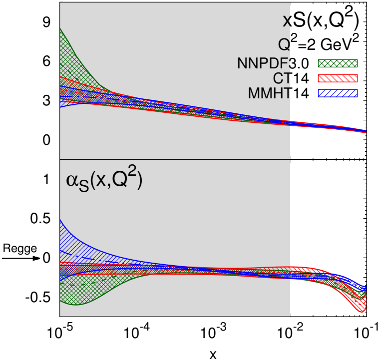
There is an asymptotic region where $\alpha_{f_i}(x, Q^2)$ becomes roughly independent of x

Global PDF sets are consistent among one other at the level of PDFs and $\alpha_{f_i}(x, Q^2)$
(NNPDF becomes compatible with CT and MMHT if 1-σ uncertainty bands are considered)

The onset of the asymptotic regime depends on the PDF flavour and on the PDF set
(this onset takes place at x values close to the data/extrapolation region boundary)

A detailed comparison with Regge theory expectations will be addressed later

x dependence, fixed (small) Q^2



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Interlude: double asymptotic scaling (DAS)

[Sov. Phys. JTEP 46 (1977) 641; Phys. Rev. D10 (1974) 1649; Phys. Lett. B335 (1994) 77; Phys. Lett. B336 (1994) 77]

As $x \to 0$ and $Q^2 \to \infty$ PDFs can be solely determined by DGLAP equations provided that their behaviour is sufficiently soft at the input scale

The singlet sector grows as

$$x\Sigma(x,Q^2) \xrightarrow[Q^2\to\infty]{x\to 0} \mathcal{N}_\Sigma \frac{\gamma}{\rho} \frac{1}{\sqrt{4\pi\gamma\sigma}} e^{2\gamma\sigma - \delta\sigma/\rho}$$

$$xg(x,Q^2) \xrightarrow[Q^2\to\infty]{x\to 0} \mathcal{N}_g \frac{1}{\sqrt{4\pi\gamma\sigma}} e^{2\gamma\sigma - \delta\sigma/\rho}$$

$$\gamma \equiv \left(\frac{12}{\beta_0}\right)^{1/2}, \qquad \delta \equiv \left(11 + \frac{2n_f}{27}\right) \Big/ \beta_0, \qquad \beta_0 = 11 - \frac{2}{3}n_f$$

$$\sigma \equiv \left[\ln\frac{x_0}{x} \ln\frac{\ln\left(Q^2/\Lambda^2\right)}{\ln\left(Q_0^2/\Lambda^2\right)}\right]^{1/2} \qquad \rho \equiv \left[\frac{\ln\left(x_0/x\right)}{\ln\left(\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)\right)}\right]^{1/2}$$

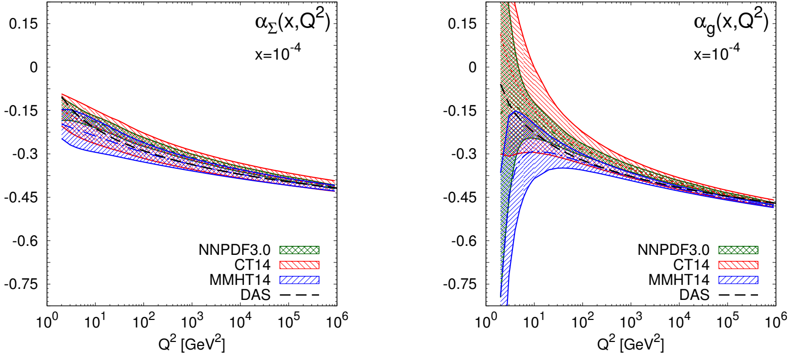
The parameters x_0 and Q_0^2 define the boundaries of the asymptotic region ($\mathcal{N}_\Sigma$ and $\mathcal{N}_g$ are normalisation constants and n_f is the number of active flavours)

Using our definition of $\alpha_{f_i}(x,Q^2)$ we expect (at large σ but fixed ρ)

$$\alpha_\Sigma(x,Q^2) \xrightarrow[Q^2\to\infty]{x\to 0} -\frac{\gamma}{\rho} + \frac{3}{4\sigma\rho}$$

$$\alpha_g(x,Q^2) \xrightarrow[Q^2\to\infty]{x\to 0} -\frac{\gamma}{\rho} + \frac{1}{4\sigma\rho}$$

Q^2 dependence, fixed (small) x



We observe a transition from a low-Q^2 region to a high-Q^2 region
(low-Q^2: non-perturbative dynamics dominates; high-Q^2: perturbative QCD dominates)

As Q^2 increases, $\alpha_{f_i}(x, Q^2)$ becomes less sensitive to Q^2
(this feature is broadly independent of x when x is sufficiently small, $x \lesssim 10^{-3}$)

Both $\alpha_\Sigma(x, Q^2)$ and $\alpha_g(x, Q^2)$ converge asymptotically to the same value $-\gamma/\rho \sim 0.45$
(as expected, since the QCD evolution of g seeds the evolution of Σ)

With our definition, PDF sets reproduce the DAS expectations at $Q^2 \gtrsim 10$ GeV^2
(we have used $x_0 = 0.1$, $Q_0^2 = 1$ GeV^2, $n_f = 5$, $\Lambda^{(n_f=5)} = 0.220$ GeV)

Comparison with Regge theory expectations

f_i	Q^2 [GeV^2]	$\alpha_{f_i}(x_a=10^{-4},Q^2)$ NNPDF3.0	CT14	MMHT14	ABM12	CJ15	a_{f_i}
u_V	2.0	$+0.48\pm0.11$	$+0.72\pm0.12$	$+0.65\pm0.06$	$+0.76\pm0.07$	$+0.61\pm0.01$	$+0.5$
	10.0	$+0.46\pm0.09$	$+0.66\pm0.09$	$+0.61\pm0.04$	$+0.70\pm0.04$	$+0.60\pm0.01$	
d_V	2.0	$+0.41\pm0.11$	$+0.73\pm0.12$	$+0.79\pm0.06$	$+1.39\pm0.10$	$+1.11\pm0.03$	$+0.5$
	10.0	$+0.41\pm0.11$	$+0.66\pm0.07$	$+0.70\pm0.04$	$+0.91\pm0.08$	$+0.95\pm0.05$	
S	2.0	-0.14 ± 0.06	-0.15 ± 0.05	-0.09 ± 0.04	-0.16 ± 0.02	-0.18 ± 0.03	-0.08
	10.0	-0.18 ± 0.04	-0.20 ± 0.05	-0.15 ± 0.04	-0.19 ± 0.01	-0.14 ± 0.02	(-0.2)
g	2.0	-0.16 ± 0.63	$+0.06\pm0.31$	-0.79 ± 0.43	$+0.18\pm0.10$	$+0.08\pm0.03$	-0.08
	10.0	-0.20 ± 0.46	-0.15 ± 0.15	-0.29 ± 0.09	-0.15 ± 0.01	-0.14 ± 0.01	(-0.2)

At small x, Regge theory predicts that $\boxed{xf_i \sim x^{a_{f_i}}}$

(a_{f_i}, Q^2-independent, is related to the intercept of the corresponding Regge trajectory)

valence

$a_{u_V} = a_{d_V} \sim +0.5$: non-singlet Regge trajectory intercept $1-\alpha_R(0)$

gluon

$a_g \simeq -0.08$: close to the singlet (soft) Pomeron trajectory $1-\alpha_P(0)$
($a_g \simeq -0.2$: perturbative solution of the running coupling LLx BFKL equations)

sea

$a_S \simeq a_g$ (large Q^2): dominance of the process $g \to q\bar{q}$ in the evolution of sea quarks

Regge predictions are expected to hold only at low scales

Comparison with Regge theory expectations

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valence

the scale dependence of $\alpha_{u_V}(x,Q^2)$ and $\alpha_{d_V}(x,Q^2)$ is rather weak

the values determined from NNPDF3.0 are in good agreement with expectations

the values determined from other PDF sets are generally a little high

gluon and sea

the scale dependence of $\alpha_g(x,Q^2)$ and $\alpha_S(x,Q^2)$ is rather strong (DAS behaviour)

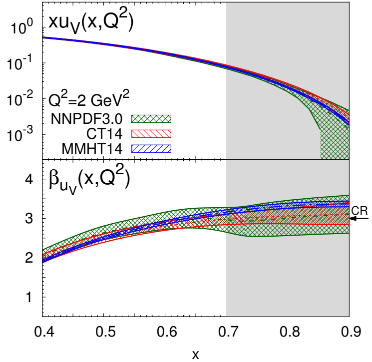
uncertainties for the gluon intercept are inevitably large and the agreement is only qualitative

at $Q^2 = 10$ GeV^2 $\alpha_S(x,Q^2) \sim \alpha_g(x,Q^2)$ (within uncertainties) as predicted

at $Q^2 = 10$ GeV^2 $\alpha_S(x,Q^2)$ is in reasonable agreement with the NLLx perturbative prediction

3. Large x: $\beta_{f_i}(x, Q^2)$

x dependence, fixed (small) Q^2



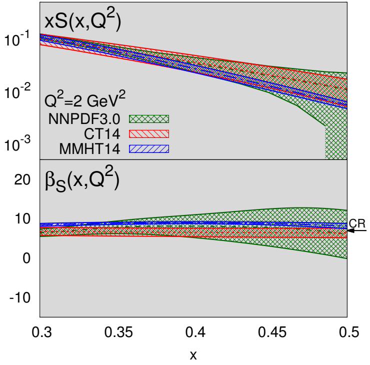
There is an asymptotic region where $\beta_{f_i}(x, Q^2)$ becomes roughly independent of x

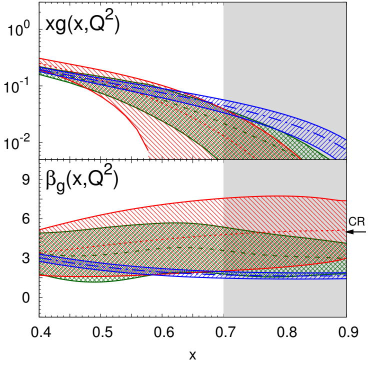
Global PDF sets are consistent among one other at the level of PDFs and $\alpha_{f_i}(x, Q^2)$
(though slightly different behaviours are observed, *e.g.* $\beta_{d_V}(x, Q^2)$ and $\beta_g(x, Q^2)$)

The onset of the asymptotic regime depends on the PDF flavour and on the PDF set
(this onset takes place at x values close to the data/extrapolation region boundary)

A detailed comparison with quark counting rule expectations will be discussed later

x dependence, fixed (small) Q^2





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Interlude: $\overline{\mathrm{MS}}$ cusp quark anomalous dimension

[Mod. Phys. Lett. A4 (1989) 1257; Phys. Lett. B513 (2001) 93]

The quark anomalous dimension in the $\overline{\mathrm{MS}}$ scheme at large N takes the universal form

$$\boxed{\gamma_q(N,\alpha_s(q^2)) \sim -c(\alpha_s(q^2))\ln N + d(\alpha_s(q^2)) + O(1/N)}$$

where $c\left(\alpha_s(q^2)\right)$ and $d\left(\alpha_s(q^2)\right)$ can be computed perturbatively; at NLO:

$$c(\alpha_s(q^2)) = \frac{\alpha_s(q^2)}{2\pi}c_1 + \left(\frac{\alpha_s(q^2)}{2\pi}\right)^2 c_2 + O\left(\alpha_s^3(q^2)\right)$$

$$c_1 = \frac{8}{3} \qquad c_2 = 4\left(\frac{67}{9} - 2\zeta_2\right) - \frac{40}{27}n_f$$

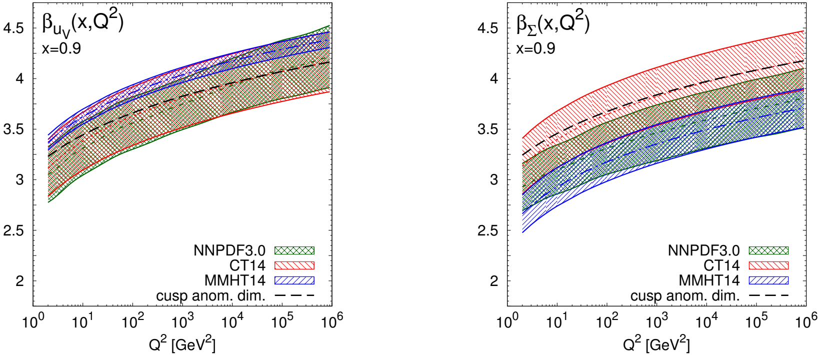
If $xf_i(x,Q_0^2) \sim (1-x)^{b(Q_0^2)}$ as $x \to 1$ at a scale Q_0^2 ($f_i = u_V, d_V, \Sigma$), then this asymptotic behaviour persists at $Q^2 > Q_0^2$ with

$$b(Q^2) = b(Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} c(\alpha_s(q^2))$$

Using our definition of $\beta_{f_i}(x,Q^2)$ we expect (at $x \to 1$)

$$\boxed{\beta_{f_i}(x,Q^2) = \beta_{f_i}(x,Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} c(\alpha_s(q^2))}$$

Q^2 dependence, fixed (large) x



As Q^2 increases, $\beta_{f_i}(x, Q^2)$ becomes less sensitive to Q^2
(this feature is broadly independent of x when x is sufficiently large, $x \gtrsim 0.9$)

The agreement between the cusp. anom. dim expectations and global PDF fits is good
($\beta_{f_i}(x, Q_0^2)$ is fixed to match the central values obtained from `CT14` at $Q^2 = 10^6$ GeV^2)

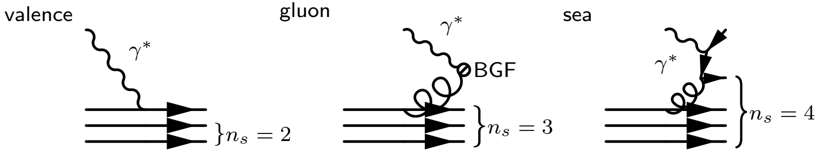
A slight deterioration appears at small values of Q^2 (missing higher order corrections)
(similar conclusions can be derived for other PDF sets when $\beta_{f_i}(x, Q_0^2)$ is chosen consistently)

Comparison with quark counting rules expectations

f_i	Q^2 [GeV^2]	$\beta_{f_i}(x_b = 0.9, Q^2)$ NNPDF3.0	CT14	MMHT14	ABM12	CJ15	b_{f_i}
u_V	2.0	$+2.94 \pm 0.52$	$+3.11 \pm 0.28$	$+3.37 \pm 0.07$	$+3.38 \pm 0.06$	$+3.50 \pm 0.01$	~ 3
	10.0	$+3.30 \pm 0.69$	$+3.38 \pm 0.29$	$+3.62 \pm 0.07$	$+3.61 \pm 0.05$	$+3.78 \pm 0.01$	
d_V	2.0	$+3.03 \pm 1.96$	$+3.27 \pm 0.37$	$+2.05 \pm 0.59$	$+4.72 \pm 0.43$	$+3.42 \pm 0.06$	~ 3
	10.0	$+3.23 \pm 1.88$	$+3.52 \pm 0.36$	$+2.29 \pm 0.59$	$+4.92 \pm 0.42$	$+3.68 \pm 0.05$	
S	2.0	$+6.86 \pm 7.25$	$+6.41 \pm 1.22$	$+8.19 \pm 0.68$	$+8.16 \pm 0.38$	$+7.73 \pm 0.18$	~ 7
	10.0	$+6.76 \pm 6.71$	$+6.91 \pm 1.14$	$+6.83 \pm 0.88$	$+8.51 \pm 0.38$	$+8.15 \pm 0.18$	
g	2.0	$+2.95 \pm 1.25$	$+5.08 \pm 2.18$	$+1.65 \pm 0.23$	$+4.18 \pm 0.06$	$+6.11 \pm 0.33$	~ 5
	10.0	$+3.25 \pm 0.98$	$+5.13 \pm 0.51$	$+2.24 \pm 0.23$	$+4.44 \pm 0.06$	$+4.91 \pm 0.33$	

At large x, quark counting rules predict that $\boxed{x f_i \sim (1-x)^{2n_s-1}}$

(n_s is the minimum number of spectator partons, not struck in a hard scattering process)



It is unclear from the quark model argument at which scale quark counting rules hold

(fortunately the scale dependence of $\beta_{f_i}(x, Q^2)$ is reasonably moderate)

Comparison with quark counting rules expectations

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valence

$\beta_{u_V}(x, Q^2)$ and $\beta_{d_V}(x, Q^2)$ (all PDF sets) are in broad agreement with counting rules
some deviations are observed for the MMHT14 β_{d_V} (low) and ABM12 and CJ15 (high)

gluon and sea

NNPDF3.0 provide too large uncertainties for $\beta_S(x, Q^2)$ to be meaningful, $\beta_g(x, Q^2)$ is low
CT14 provide results in good agreement with counting rules
MMHT14 provide a (far too) low result for the gluon
ABM12 provide a slightly large $\beta_S(x, Q^2)$ and correspondingly a slighlty low $\beta_g(x, Q^2)$
CJ15 provide a slightly large $\beta_S(x, Q^2)$ and a slighlty large $\beta_g(x, Q^2)$

The ratios d_V/u_V and F_2^n/F_2^p

All PDFs vanish at $x=1$, while the ratios d_V/u_V and F_2^n/F_2^p do not necessarily do so

$$\frac{d_V}{u_V} \xrightarrow{x\to 1} (1-x)^{b_{d_V}-b_{u_V}} \xrightarrow[\text{counting rules}]{b_{d_V}=b_{u_V}} k$$

$$\frac{F_2^n}{F_2^p} = \frac{\sum_{i=u,d,d} q_i^2 x f_i}{\sum_{i=u,u,d} q_i^2 x f_i} \xrightarrow{x\to 1} \frac{4(1-x)^{b_{u_V}} + (1-x)^{b_{d_V}}}{(1-x)^{b_{u_V}} + 4(1-x)^{b_{d_V}}} \xrightarrow[\text{counting rules}]{b_{d_V}=b_{u_V}} 1$$

Several non-perturbative models of nucleon structure try to predict the constant k

[Phys. Lett. B377 (1996) 11; Rev. Mod. Phys. 82 (2010) 2991]

SU(6) [Phys. Lett. B43 (1973) 422]
CQM [Phys. Lett. B58 (1975) 345]
pQCD [Phys. Rev. Lett. 35 (1975) 1416]
NJL [Phys. Lett. B621 (2005) 246]
DSE1 [Phys. Lett. B727 (2013) 249]
DSE2 [Phys. Lett. B727 (2013) 249]

One may expect $b_{u_V} \neq b_{d_V}$ because of isospin breaking and electromagnetic effects:

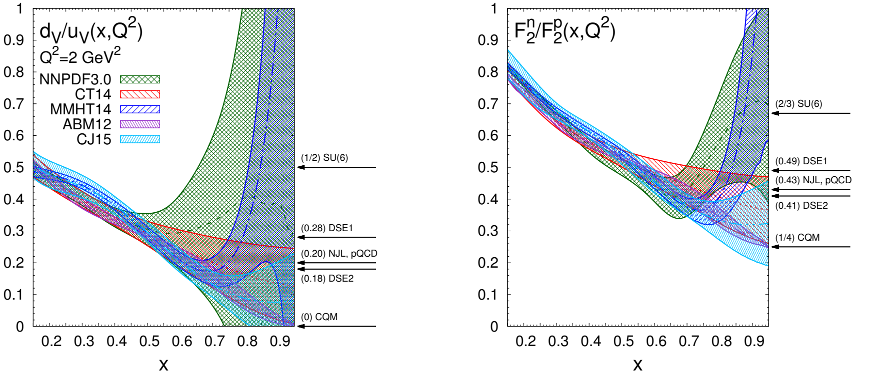
if $b_{u_V} \gg g_{d_V}$ then $\frac{d_V}{u_V} \xrightarrow{x\to 1} \infty$ if $b_{u_V} \ll g_{d_V}$ then $\frac{d_V}{u_V} \xrightarrow{x\to 1} 0$

if $b_{u_V} \gg g_{d_V}$ then $\frac{F_2^n}{F_2^p} \xrightarrow{x\to 1} 4$ if $b_{u_V} \ll g_{d_V}$ then $\frac{F_2^n}{F_2^p} \xrightarrow{x\to 1} \frac{1}{4}$

The latter two limits result in the Nachtmann bound [Nucl. Phys. B38 (1972) 397]

$$\frac{1}{4} \leq \frac{F_2^n}{F_2^p} \leq 4$$

The ratios d_V/u_V and F_2^n/F_2^p



All PDF sets are in good agreement where experimental data are available ($x \lesssim 0.5$)

The mutual consistency among PDF sets deteriorates rapidly at $x \gtrsim 0.5$ (lack of data)

Different behaviours are seen at $x \gtrsim 0.5$ (reflecting different kinds of extrapolation)

no or weak assumptions: no predictive power at all (NNDPF3.0 and MMHT14)

$d_v/u_V \xrightarrow{x\to 1} k$: a value of k is predicted with reduced uncertainties (CT14 and CJ15)

$b_{d_V} \gg b_{u_V}$: $k=0$, with a far too small uncertainty band (ABM12)

A discrimination among different models is prone to the extrapolation assumptions

4. Conclusions

Summary

We introduced a definition of effective exponents, valid for any value of x and Q^2, which allows for a quantitative determination of the asymptotic behaviour of PDFs

We identified the (x, Q^2) ranges in which an asymptotic regime sets in

Our definition reproduces QCD expectations excellently
(double asymptotic scaling and universality of the cusp anomalous dimension in the $\overline{\mathrm{MS}}$ scheme)

We compared predictions based on global PDF determinations with expectations from Regge theory (small x) and quark counting rules (large x)
(broad agreement for valence PDFs and only qualitative agreement for gluon and sea PDFs)

We compared the ratios d_V/u_V and F_2^n/F_2^p among PDF fits and with models
(the comparison with models is inconclusive
because its interpretation is prone to the assumptions built in the PDF parametrisation)

The ancient wisdom of Regge theory and quark counting rules has some degree of truth, but they are no substitute for the empirical PDF determination in a global analysis, and when used as constraints may lead to unrealistically accurate predictions, in kinematic regions where there is no experimental data

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Thank you

5. Extra material

Numerical determination of the effective exponents

The evaluation of the logarithmic derivative of the PDF in the definition of the effective exponents is performed numerically by means of a Savitzky-Golay smoothing filter

[Analytical Chemistry 36 (1964) 1627]

The methodology is validated with `MSTW08` NLO PDFs

$$R^{\alpha_{f_i}}(x,Q^2) = \frac{\alpha_{f_i}^{(\mathrm{num})}(x,Q^2) - \alpha_{f_i}^{(\mathrm{ana})}(x,Q^2)}{\alpha_{f_i}^{(\mathrm{ana})}(x,Q^2)}$$

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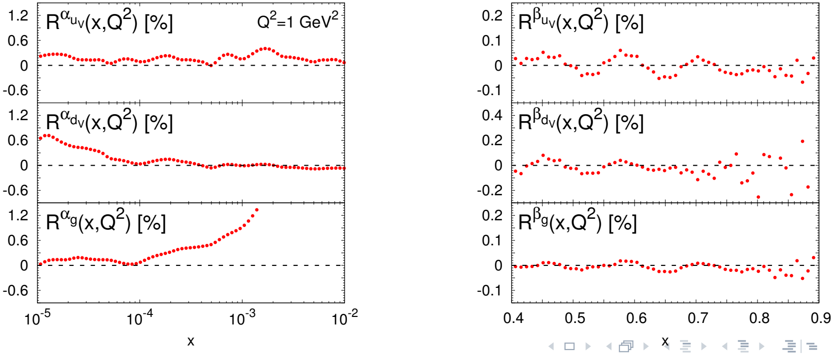
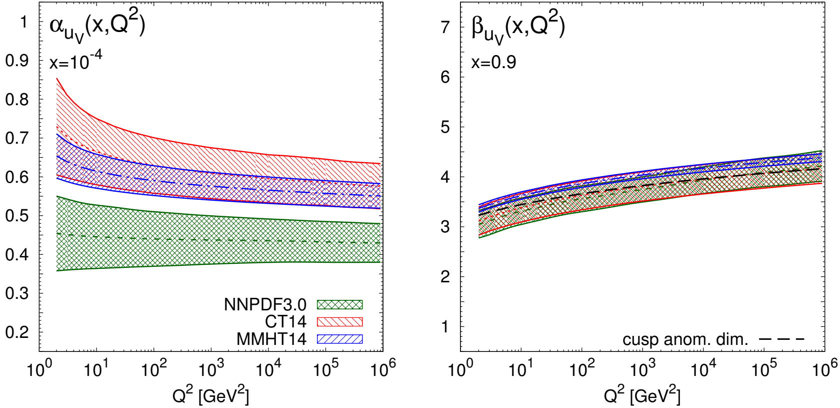


Additional plots



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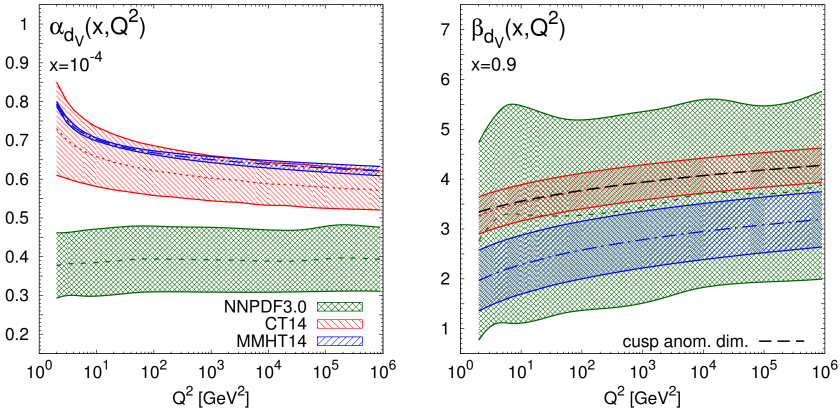

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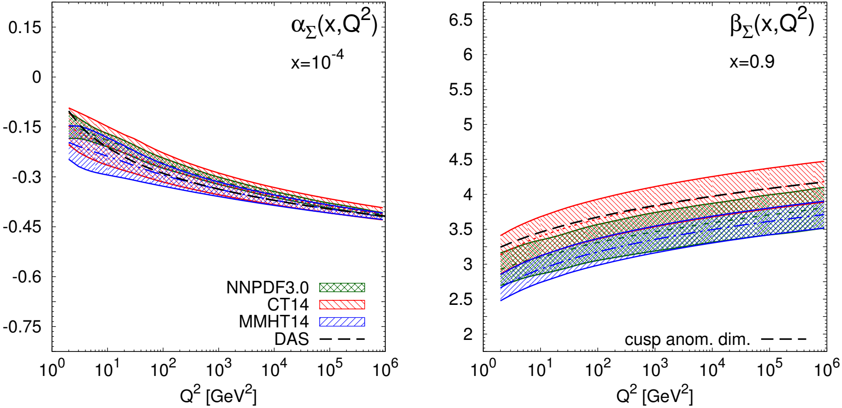

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