Determination of the strong coupling constant from a global PDF fit

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Executive summary

Precision determination of the strong coupling constant within a global PDF analysis

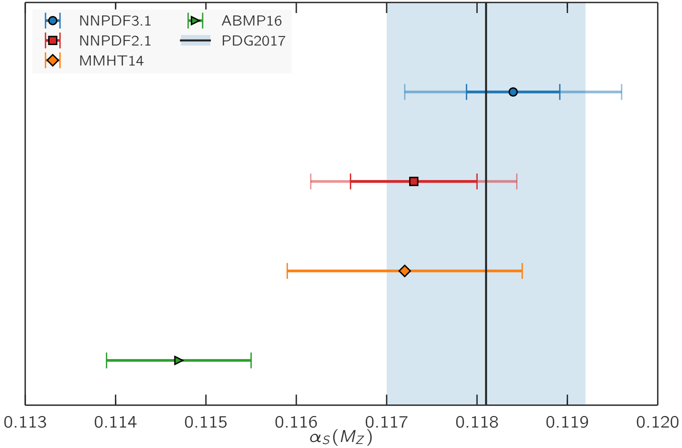
NNPDF Collaboration, arxiv:1802.03398

Result:

$$\alpha_S(M_Z) = 0.1185 \pm 0.0005^{(\mathrm{exp})} \pm 0.0001^{(\mathrm{meth})} (\pm 0.011^{(\mathrm{th})})$$

- Good agreement with the PDG average ($\alpha_{S(\mathrm{PDG})} = 0.1181 \pm 0.001$).
- Experimental uncertainties comparable to most precise determinations.
- MHOU uncertainties hard to quantify (not yet included in any PDF).

Comparison to other PDF-based determinations



α_S from PDFs

- Uses large corpus of data.
- More data $\longrightarrow$ more precision.
- More processes $\longrightarrow$ more accuracy:
  - Possible experimental errors average out.
  - Problems in the theoretical description can average out [Carrazza, Forte, ZK, Rojo, Rottoli, arxiv:1803.07977].
- Many challenges: PDF fitting methodology, MHOU, higher twist, value of the charm mass, EW corrections, nuclear corrections..
- Fit quality of all data taken into account simultaneously.
- Effect of individual dataset hard to quantify. Looking at partial statistics may be misleading.

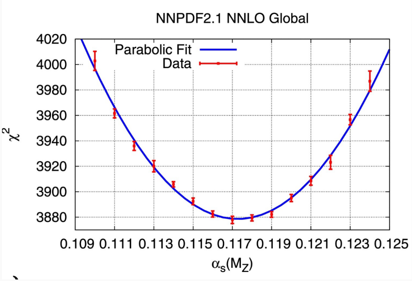
α_S from NNPDF

Many improvements since last determination [arXiv:1103.2369] (where we had $\alpha_{S_{\text{NNPDF2.1}}} = 0.1173 \pm 0.0007$)

- 3979 data points.
- Includes differential top, Z pT and inclusive jets simultaneously for the first time.
- Exact NNLO theory for all included data.
- Improved methodology for α_S extraction.

Old methodology

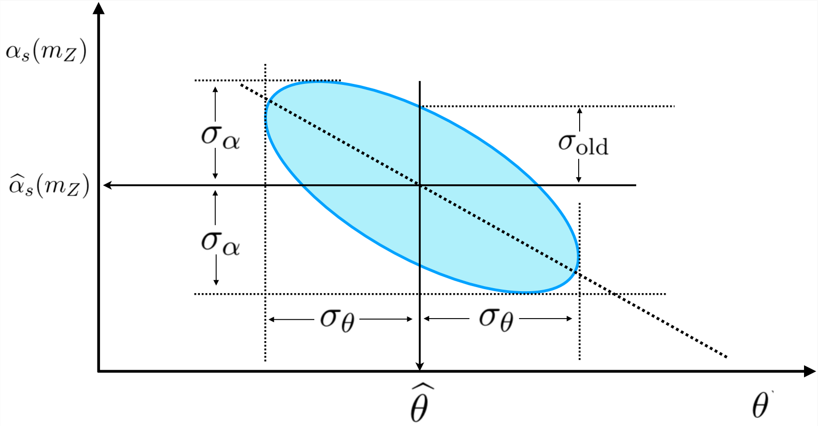
- First find the best fit PDF for a set of values of α_S.
- Then determine χ^2 profile of the best fit PDF and determine α_S as the minimum.



- Uncertainty determined from $\Delta\chi^2 = 1$ in the parabola.

Improvements on the old methodology

- Correlation between data fluctuations and α_S not fully taken into account.
- $\Delta\chi^2 = 1$ not equivalent to the NNPDF replica-based uncertainty propagation.



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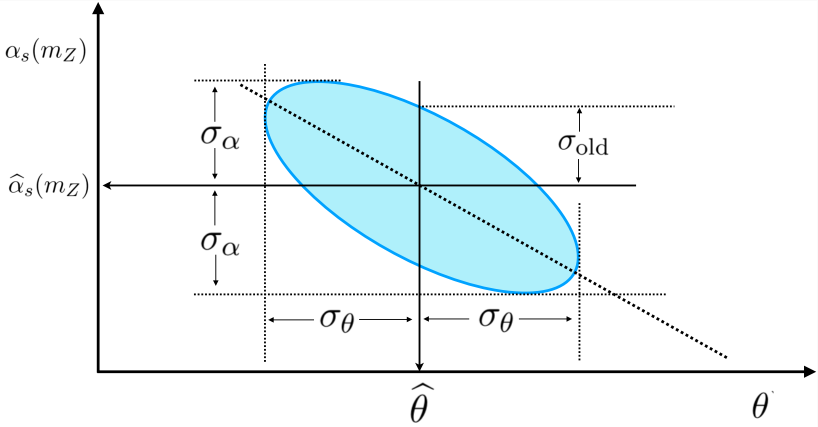
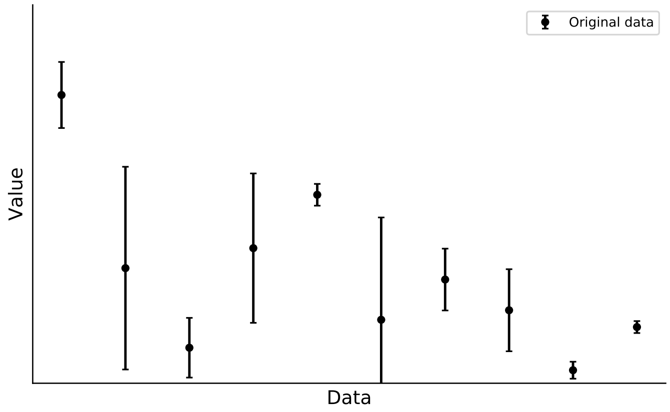


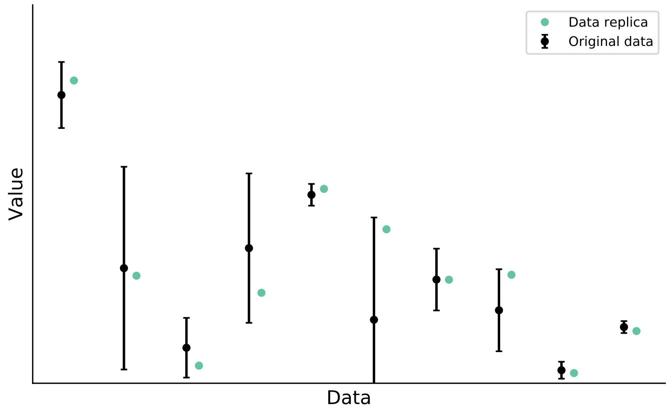
- Ideally, we would have minimization in (α_S, PDF) space.

We try to find the best fit to some dataset



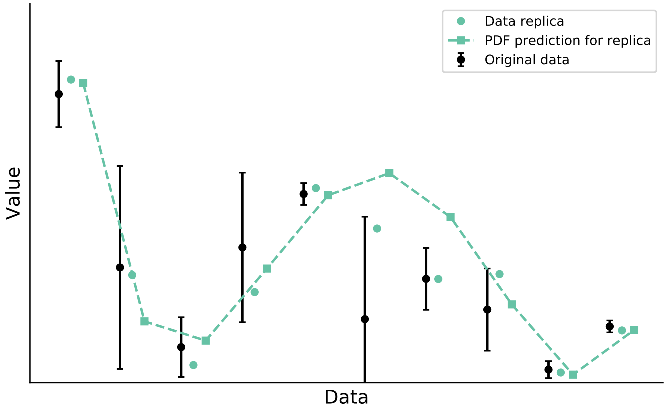
Sample a pseudodata replica

We sample from a probability distribution built from the data uncertainties.

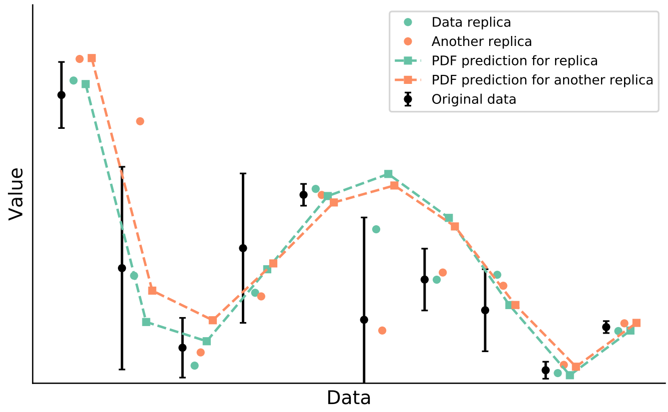


Find the best fit value for that data replica

This is subject to constraints, that in particular depend on the value of α_S.

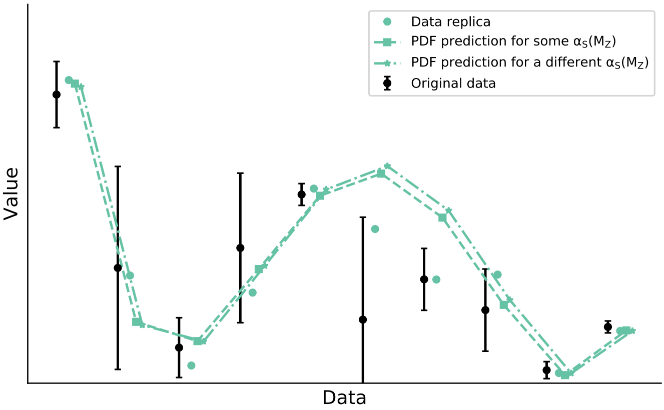


This is the standard PDF fitting procedure.



Make predictions for several values of α_S and a single replica

Obtain different PDF predictions for the same replica. Measure fit quality as a function of α_S.



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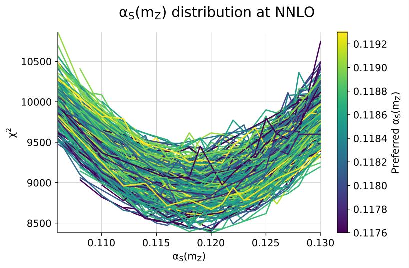

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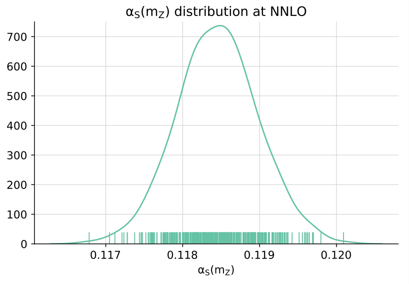
- Each minimum yields one sampled α_S value.



Implementation intricacies

- Need to account for various features of the NNPDF methodology: replicas not converging, cross validation split... See paper for details.
- Need to verify the effect on the α_S value.
- In the end, the actual methodology is equivalent to the idea described above within experimental uncertainties.

Estimates of MHOU

No reliable method known for PDF based quantities. We have:

$$\alpha_S^{\text{NNLO}}(M_Z) = 0.1184 \pm 5 \times 10^{-4}$$

$$\alpha_S^{\text{NLO}}(M_Z) = 0.1206 \pm 6 \times 10^{-4}$$

- Methods based on continuation of perturbative series such as Cacciari-Houdeau [arXiv:1105.5152] hampered by:
  - Poor fit quality at LO and even at NLO.
  - Lack of unique series expansion (many processes involved).
- CH yields a theoretical uncertainty of 4×10^{-4}, smaller that the experimental ones. Likely too optimistic.
- PDF fits with scale variation errors currently under development.

Final MHOU estimate

For lack of better options, make it be crude and conservative.

$$\Delta\alpha_S^{\text{th}} = \frac{1}{2}\left|\alpha_S^{\text{NLO}} - \alpha_S^{\text{NNLO}}\right| = 0.0011(0.9\%)$$

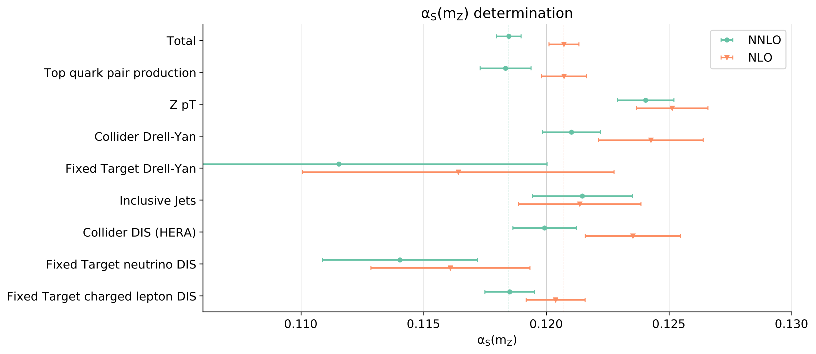
Uncertainty likely overblown by the poor fit quality at NLO. E.g.

- ATLAS $Zp_T\ y$ dist: best fit $\chi^2 = 1.78$ at NLO and 0.94 at NNLO.
- For ATLAS $t\bar{t}$ total cross sections, $\chi^2 = 1.96$ at NLO and 0.85 at NNLO.

Other theoretical uncertainties likely subdominant.

Determinations based on a partial dataset

Consider:



- Can we say that e.g. the Zp_T data “pulls the most” (e.g. talk by R.Thorne)?
- Pulls necessarily depend on th rest of the data. Not an intrinsic property of each dataset.
- Best fit PDF not very meaningful if the fit is bad in the first place! [Z.K, arxiv:1802.05236]

Definition: global χ^2

The global χ^2 minimizes to fit (α_S, PDF) looks like

$$\chi^2\left[\{\theta\}, \alpha_S, \mathcal{D}\right] = \sum_{I,J=1}^{N_{\mathcal{D}}} \left(T_I[\{\theta\}, \alpha_S] - D_I\right) C_{IJ}^{-1} \left(T_J[\{\theta\}, \alpha_S] - D_J\right)$$

where

- $\{\theta\}$ are the parameters of the PDF.
- $\mathcal{D}$ is the set of $N_{\mathcal{D}}$ data points entering the fit.
- T_I is the theoretical prediction for the data point indexed by I.
- D_I is the experimentally measured value.
- C_{IJ} measures the experimental covariance.

The profile of a c-replica is

$$\chi^{2(r)}(\alpha_s) = \min_{\{\theta\}} \chi^2\left[\{\theta\}, \alpha_S, \mathcal{D}^{(r)}\right]$$

Partial χ^2

The partial χ^2 is:

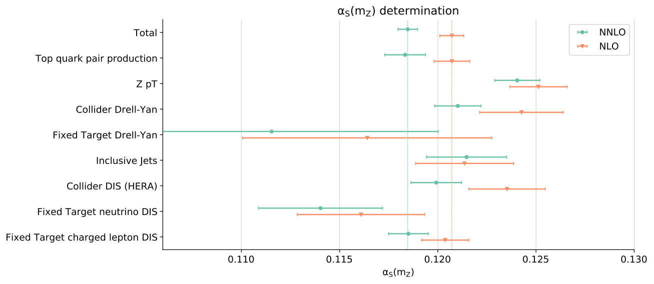
$$\chi_p^2\left[\{\theta\}, \alpha_S, \mathcal{P}\right] = \sum_{I,J=1}^{N_{\mathcal{P}}} \left(T_I[\{\theta\}, \alpha_S] - D_I\right) C_{IJ}^{-1} \left(T_J[\{\theta\}, \alpha_S] - D_J\right)$$

where we have replaced $\mathcal{D}$ with $\mathcal{P}$, a subset of $\mathcal{D}$ with $N_{\mathcal{P}}$ points, and neglected the correlations between the points in $\mathcal{P}$ and $\mathcal{D}$.

Up to missing correlations, for a set of processes that cover all $\mathcal{D}$

$$\chi^2(\alpha_s) = \sum_p \chi_p^2(\alpha_s)$$

Results from the partial χ^2



Define

$$\alpha_S^p = \arg\min_{\alpha_S} \chi_p^2(\alpha_S)$$

Collider determinations of α_S are essentially based on finding a minimum to th partial χ^2 profile.

> *By* collider determinations *I mean determinations of* α_S *based on cross sections measured at hadron–hadron and hadron–lepton colliders that are used to constrain the strong coupling independently of a PDF fit.*
> *G.Salam [arxiv:1712.05165]*

Collider determinations depend on the PDF in two ways:

- Best fit PDF changes strongly with α_S, i.e. $\{\theta\}(\alpha_S) = \arg\min_{\{\theta\}} \chi^2(\alpha_s, \{\theta\}, \mathcal{D})$.
- Global fit quality $\chi^2(\alpha_S)$ changes strongly with α_S.

Consistency of collider determinations of α_S

Claim:

Collider determinations of α_S that optimize the partial χ^2, but however do not take into account the total χ^2 in the input PDF set are potentially unreliable.

Will show by producing an example that exposes the contradictions.

- Take ZpT, where the apparent preferred value from the partial χ^2 minimization is $\alpha_S^{\mathcal{P}=Zp_T} \sim 0.124$.
- Show that under some assumptions, we can find a *better* preferred value.

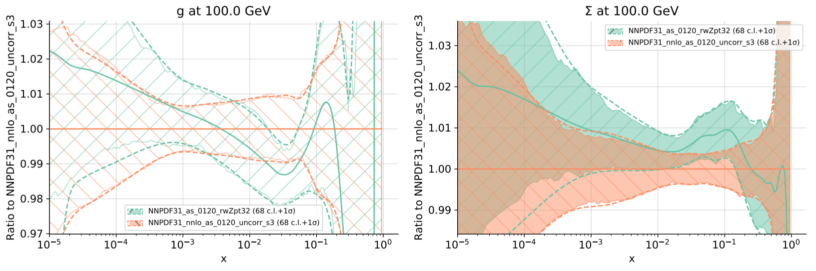
Assumptions

1. All things being equal, a PDF fit that has lower total χ^2 to data is *better*.
2. All things equal, a collider determination of α_S that has lower partial χ^2 to data is *better*.
3. All things equal, an hadronic determination that uses a *better* PDF (in the sense of 1.) is *better*.

Experiment set up

- Easier to find contradictions in the data with biggest apparent discrepancies: Zp_T.
- Obvious way to improve partial χ^2: Give more weight to the ZpT data in the (PDF + α_S) fit. $\chi^2_w \approx \chi^2 + (w-1)\chi^2_p$
- Optimizing χ^2_w means higher χ^2 because we are not optimizing for it anymore.
- But this can still be a better total χ^2 at a *different* value of α_S

- Take $w = 32$ (~same weight to Zp_T and the rest of the data) and
- $\alpha_S = 0.120$, a value between the total best fit result and the minimum of the partial χ^2.



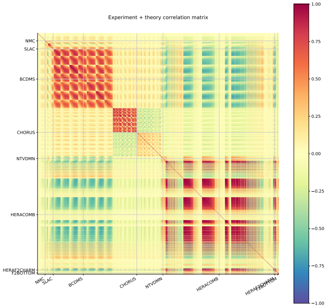
Results: χ^2 of reweighted fit

χ^2/d.o.f..	$\alpha_S = 0.120$ weighted Zp_T	$\alpha_S = 0.124$ default
Total	1.226	1.281
Zp_T	0.94	1.11

- With the definitions above, we have found that $\alpha_S = 0.120$ is a *better* value for α_S from Zp_T.
- $\alpha_S = 0.120$ is not the answer either. But the experiment shows that the partial χ^2 definition is inconsistent.

Conclusions

- We have produced a determination of α_S that is highly competitive within the PDG average.
- MHOU estimation is currently the main limitation. Work underway.



- Global fit are advantageous in that they correctly account for the effect of all data in both α_S and PDFs.

Thank you!

Backup: Theoretical uncertainties intro

- Theoretical effects necessarily have to affect the PDFs in order to affect our determination. Need to be sizable compared to experimental uncertainties.
- But not sufficient: Dependence on $\alpha_S(M_Z)$ required.

Higher twist found to be small compared to experimental uncertainties [arxiv:1303.1189].

Charm mass Improved by parameterizing the charm PDF [arXiv:1605.06515].

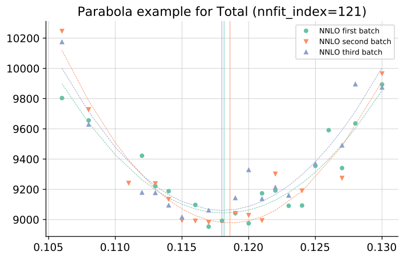
Electroweak corrections Kept under control with suitable cuts [arxiv:1706.00428].

Nuclear corrections Studied in [arxiv:1706.00428] and found to be small compared to experimental uncertainties (but subject to limitations of the models).

In conclusion MHOU highly likely to be the dominant theoretical uncertainty.

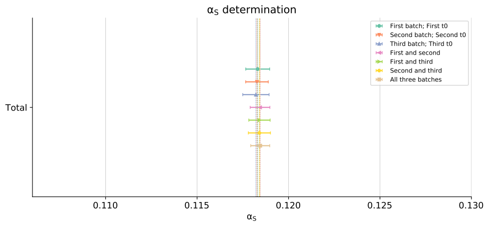
Backup: Improvement by batch minimization

- The NNPDF methodology discards replicas that do not meet certain convergence criteria (25-30%).
- $\chi^2(\alpha_S)$ noisy for each c-replica.
- Solution: Repeat each fit three times, and take the minimum (requiring at least 2 successful replicas).



Backup: Effect of batch minimization

- Small on central value $\sim O(0.4) \times \Delta^{(\text{exp})}$.
- Significative on uncertainty. Reduction to up to 27%.
- Convergence already good with two batches.



Backup: properties of weighted fits

- Cross validation used in the fits forces that χ^2 is not smaller than ~ 1.
- $w \to \infty$ is not the same as a fit with the data subset only,
    - Because e.g. Zp_T does not determine all the parameters of PDF+α_S
- A high weight makes the discrepancies with the weighted data only statistical, i.e. no bias due to disagreement with other data, as long as the weighed data is self consistent.

Backup: Reweighted profiles