The PDF-Lattice Study:
Connecting PDFs from phenomenology and lattice QCD

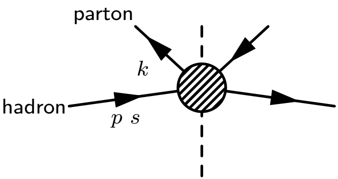
Smowmass EF06 meeting

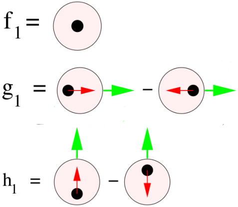
Emanuele R. Nocera – Nikhef

July 1, 2020

MARIE CURIE ACTIONS

Foreword: (collinear) leading twist PDF map





$$\phi_{ij}(k;p,s) = 2\pi \sum_X \int \frac{d^3\mathbf{P}_X}{2E_X} \delta^4(p-k-P_X)\langle p,s|\bar{\psi}_j(0)|X\rangle\langle X|\psi_i(0)|p,s\rangle$$

$$\boxed{\phi(x,s) = \frac{1}{2}\left[\mathbf{f_1}(x)\not{n}_+ + s_L g_1(x)\gamma^5\not{n}_+ + h_1 i\sigma_{\mu\nu}\gamma^5 n_+^\mu s_T^\nu\right]}$$

In this talk $\mathbf{f_1} \to f$, $g_1 \to \Delta f$ and $h_1 \to \delta f$

$$f(x) = \frac{1}{4\pi}\int dy^- e^{-ixp^+y^-}\langle p,s|\bar{\psi}_f(0,0,\mathbf{0}_\perp)\gamma^+\mathcal{P}\psi_f(0,y^-,\mathbf{0}_\perp)|p,s\rangle$$

$$\Delta f(x) = \frac{1}{4\pi}\int dy^- e^{-ixp^+y^-}\langle p,s|\bar{\psi}_f(0,0,\mathbf{0}_\perp)\gamma^+\gamma^5\mathcal{P}\psi_f(0,y^-,\mathbf{0}_\perp)|p,s\rangle$$

$$\delta f(x) = \frac{1}{4\pi}\int dy^- e^{-ixp^+y^-}\langle p,s|\bar{\psi}_f(0,0,\mathbf{0}_\perp)i\sigma^{1+}\gamma^5\mathcal{P}\psi_f(0,y^-,\mathbf{0}_\perp)|p,s\rangle$$

Strategy to fit PDFs from data

1. Collinear, leading-twist factorisation of physical observables

$$\mathcal{O}_I = \sum_{f=q,\bar{q},g} C_{If}(y,\alpha_s(\mu^2)) \otimes f(y,\mu^2) + \text{p.s. corrections} \qquad f\otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$

2. Parametrisation: general, smooth, flexible at an initial scale Q_0^2

$$x f_i(x,Q_0^2) = A_{f_i}\, x^{a_{f_i}}\, (1-x)^{b_{f_i}}\, \mathscr{F}(x,\{c_{f_i}\})$$

$$x f_i(x,Q^2) \xrightarrow{x\to 0} x^{a_{f_i}} \qquad \xrightarrow[\text{smooth interpolation in between}]{\mathscr{F}(x,\{c_{f_i}\}) \xrightarrow[x\to 1]{x\to 0} \text{finite}} \qquad x f_i(x,Q^2) \xrightarrow{x\to 1} (1-x)^{b_{f_i}}$$

3. A prescription to determine/compute expectation values and uncertainties

$$\chi^2 = \sum_{i,j}^{N_{\rm dat}} [T_i[\{\vec{a}\}] - D_i](\text{cov}^{-1})_{ij}[T_j[\{\vec{a}\}] - D_j]$$

$$E[\mathcal{O}] = \int \mathcal{D}f \mathcal{P}(f|data)\mathcal{O}(f) \qquad V[\mathcal{O}] = \int \mathcal{D}f \mathcal{P}(f|data)[\mathcal{O}(f) - E[\mathcal{O}]]^2$$

Monte Carlo: $\mathcal{P}(f|data) \longrightarrow \{f_k\}$

$$E[\mathcal{O}] \approx \tfrac{1}{N}\sum_k \mathcal{O}(f_k)$$

$$V[\mathcal{O}] \approx \tfrac{1}{N}\sum_k [\mathcal{O}(f_k) - E[\mathcal{O}]]^2$$

Maximum likelihood: $\mathcal{P}(f|data) \longrightarrow f_0$

$$E[\mathcal{O}] \approx \mathcal{O}(f_0)$$

$$V[\mathcal{O}] \approx \text{Hessian}, \Delta\chi^2\text{envelope}, \ldots$$

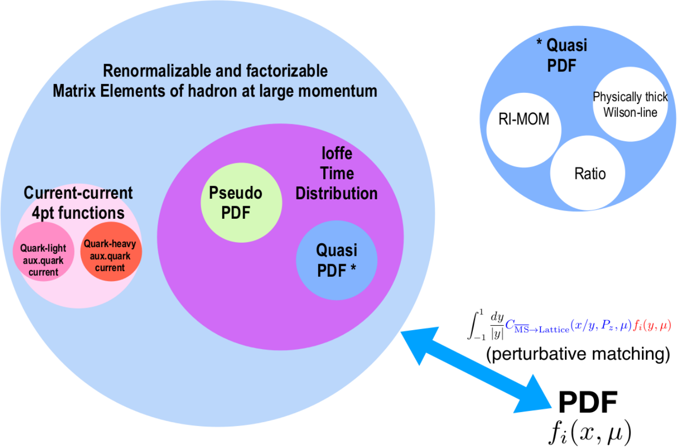
Experimental, theoretical and procedural uncertainties

[More details in Ann.Rev.Nucl.Part.Sci. (2020) 70]

Strategies to reconstruct PDFs from lattice QCD

Hadronic tensor [PRL 72 (1994) 1790]
Auxiliary scalar quarks [PLB 441 (1998) 371]
Ficticious heavy quark [PRD 73 (2006) 014501]
Higher moments [PRD 86 (2012) 054505]

Quasi-PDFs (LaMET) [PRL 110 (2013) 262002]
Good Cross Sections [PRL 120 (2018) 022003]
Compton Amplitudes [PRL 118 (2017) 242001]
Pseudo-PDFs [PRD 96 (2017) 034025]



[Figure by Nikhil Karthik, PDFLattice2019]

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$$h(z,p_z) = \frac{1}{4p_\alpha}\sum_{s=1}^{2}\langle p,s|\,\bar{\psi}(z)\gamma_\alpha e^{ig\int_0^z A_z(z')dz'}\psi(0)\,|p,s\rangle$$

Quasi-PDFs

$$\widetilde{q}(x,\Lambda,p_z) = \int \frac{dz}{2\pi} e^{-ixzp_z} p_z h(z,p_z)$$

$$\widetilde{q}(x,\Lambda,p_z) = \int_{-1}^{1}\frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{p_z},\frac{\Lambda}{p_z}\right)_{\mu^2=Q^2} q(y,Q^2) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{p_z^2},\frac{M^2}{p_z^2}\right)$$

Pseudo-PDFs

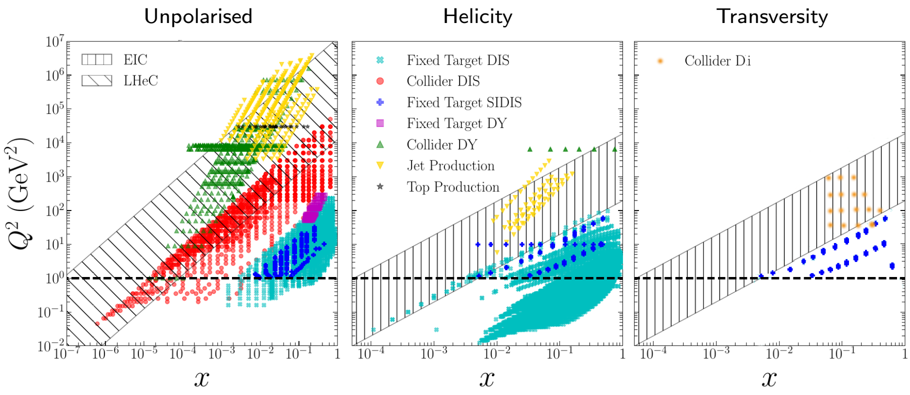
$$\mathcal{P}(x,z^2) = \int \frac{d\nu}{2\pi} e^{-ix\nu}\overline{h}(\nu,z^2) \qquad \overline{h}(\nu,z^2) \equiv h(z,p_z) \qquad \mathcal{M}(\nu,z^2) = \frac{\overline{h}(\nu,z^2)}{\overline{h}(0,z^2)}$$

$$q(x,\mu^2) = \int \frac{d\nu}{2\pi} e^{-ix\nu}\mathcal{M}(\nu,z^2) + \mathcal{O}(z^2)$$

Each step is associated to systematic uncertainties and theoretical challenges

[More details in Adv.High Energy Phys. 2019 (2019) 3036904]

Kinematic coverage



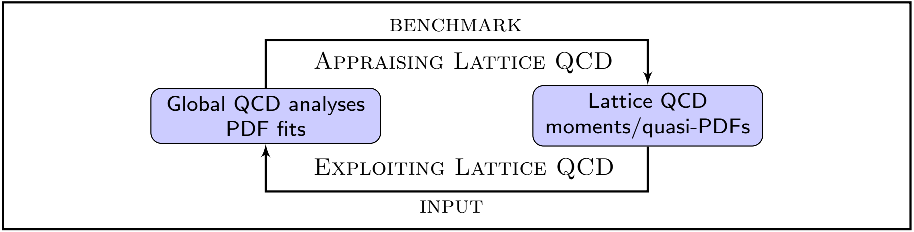
Fits of f
from **thousands** of data
CT, MMHT, NNPDF, ...

Fits of Δf
from hundreds of data
DSSV, JAM, NNPDF, ...

Fits of δf
from tens of data
Kang; Anselmino; Bacchetta

Obvious spread in the distribution of measurements across PDF species

Connecting two faces of the same world



Define a mutually agreed conventional notation
for relevant PDF-related quantities, such as PDF moments.

Assess the sources of systematic uncertainties in lattice-QCD calculations.

Identify a best-set of quantities
to benchmark lattice-QCD calculations against global-fit determinations.

Set precision targets for lattice-QCD calculations
with respect to global-fit determinations.

Assess the impact of lattice-QCD calculations
on global-fit determinations within their current/projected precision.

PDFLattice2017, Balliol College, Oxford, 22-24 March 2017

PDFLattice2019, Kellogg Biological Station, Hickory Corners, 25-27 September 2019

[Prog.Part.Nucl.Phys. 100 (2018) 107; arXiv:2006.08636]

1. Appraising lattice QCD

Define a quantitative benchmark for PDF moments

Benchmark quantities

f $\langle x\rangle_{u^+-d^+} = \int_0^1 dx\, x\left[u^+(x,Q^2) - d^+(x,Q^2)\right]$

$\langle x\rangle_{q^+} = \int_0^1 dx\, xq^+(x,Q^2),\ q=u,d,s \qquad \langle x\rangle_g = \int_0^1 dx\, xg(x,Q^2)$

Δf $g_A = \langle 1\rangle_{\Delta u^+-\Delta d^+} = \int_0^1 dx\left[\Delta u^+(x,Q^2) - \Delta d^+(x,Q^2)\right]$

$\langle 1\rangle_{\Delta q^+} = \int_0^1 dx\,\Delta q^+(x,Q^2),\ q=u,d,s \qquad \langle x\rangle_{\Delta u^- -\Delta d^-} = \int_0^1 x dx\left[\Delta u^-(x,Q^2) - \Delta d^-(x,Q^2)\right]$

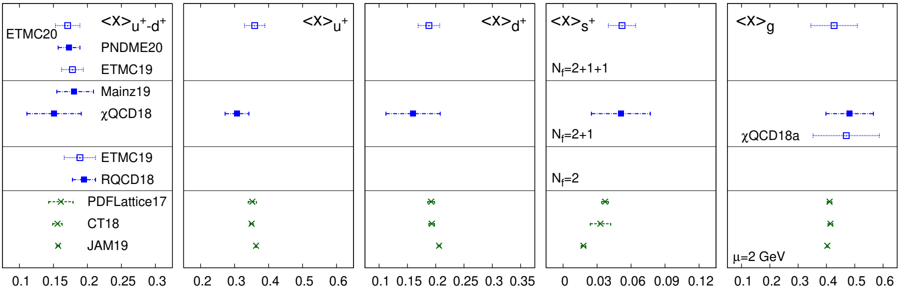
δf $g_T = \langle 1\rangle_{\delta u^- -\delta d^-} = \int_0^1 dx\left[h_1^{u^-}(x,Q^2) - h_1^{d^-}(x,Q^2)\right]$

$g_T^q = \langle 1\rangle_{\delta q^-} = \int_0^1 dx\, h_1^{q^-}(x,Q^2),\ q=u,d,s$

Benchmark criteria

	★	○	■
discretisation	$\{a_1,\dots,a_i,\dots\}\quad i\geq 3$ $a_l, a_m < 0.1$ fm $\left(\frac{a_{\max}}{a_{\min}}\right)^2 \geq 2$	$\{a_1,\dots,a_i,\dots\}\quad i\geq 2$ $a_l < 0.1$ fm $\left(\frac{a_{\max}}{a_{\min}}\right)^2 \geq 1.4$	otherwise
chiral extrapolation	$m_{\pi,i},\ i\geq 3$ $m_{\pi,1,2} < 250$ MeV $m_{\pi,3} < 200$ MeV	$m_{\pi,i},\ i\geq 3$ $m_{\pi,1,2} < 300$ MeV	otherwise
finite volume	$m_{\pi,\min}L \geq 4$ $L_1 \neq L_2 \neq L_3 > 2.5$ fm	$m_{\pi,\min}L \geq 3.4$ $L_1 \neq L_2 > 2.5$ fm	otherwise
renormalisation	non-perturbative (RI-MOM)	perturbative (one-loop or ohigher)	otherwise
excited states	(source-sink)$_i$ $i\geq 3\ \forall\ m_\pi, L$	(source-sink)$_i$ $i\geq 2\ \forall\ m_\pi, L$	otherwise

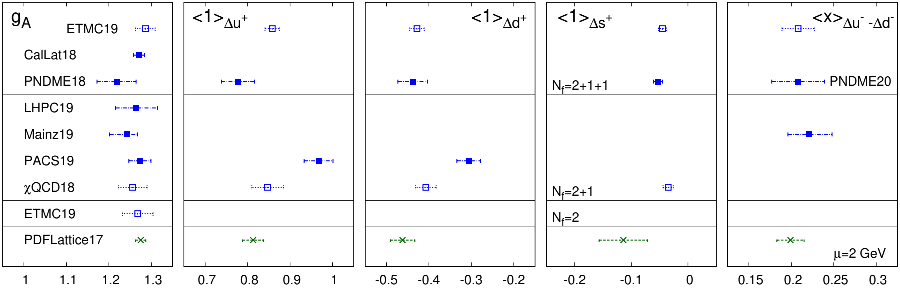
Moments of f



Moment	Lattice QCD			Global Fit
$\langle x \rangle_{u^+-d^+}$	0.153 — 0.194	0.111 — 0.209	0.166 — 0.212	0.161(18)
$\langle x \rangle_{u^+}$	0.359(30)†	0.307(35)†	—	0.353(12)
$\langle x \rangle_{d^+}$	0.188(19)†	0.160(48)†	—	0.192(6)
$\langle x \rangle_{s^+}$	0.052(12)†	0.051(26)†	—	0.037(3)
$\langle x \rangle_{g}$	0.427(92)†	0.353 — 0.587	—	0.411(8)

† Single lattice result

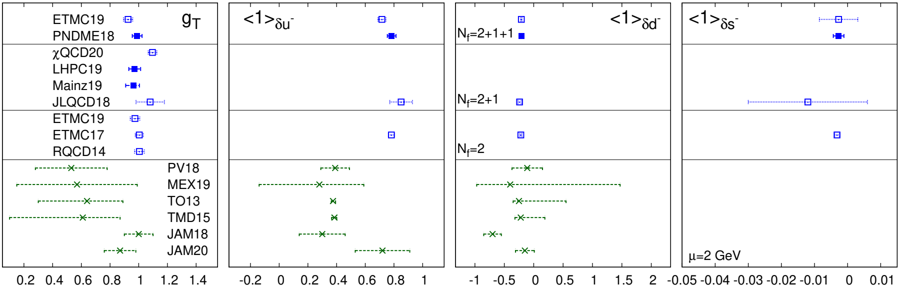
Moments of Δf



Moment	Lattice QCD			Global Fit
g_A	1.179 — 1.309	1.202 — 1.314	1.268(36)†	1.258(28)
$\langle 1\rangle_{\Delta u+}$	0.738 — 0.875	0.810 — 1.001	—	0.813(25)
$\langle 1\rangle_{\Delta d+}$	-0.473 — -0.403	-0.431 — -0.278	—	-0.462(29)
$\langle 1\rangle_{\Delta s+}$	-0.0538 — -0.0379	-0.0035(9)†	—	-0.114(43)
$\langle x\rangle_{\Delta u^- - \Delta d^-}$	0.174 — 0.239	$0.221(^{+27}_{-25})$†	—	0.199(16)

† Single lattice result

Moments of δf

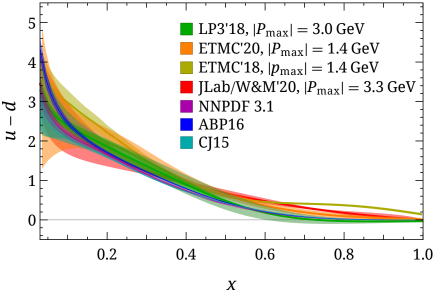


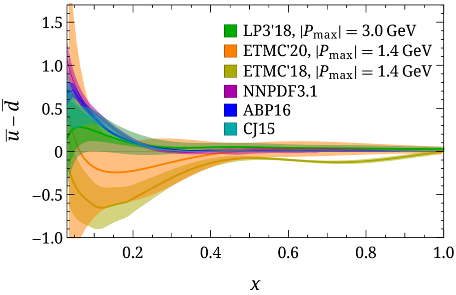
Moment	Lattice QCD			Global Fit
g_T	0.894 — 1.023	0.909 — 1.175	0.941 — 1.039	0.10 — 1.1
$\langle 1 \rangle_{\delta u^-}$	0.688 — 0.814	0.85(8)	0.782(21)	-0.14 — 0.91
$\langle 1 \rangle_{\delta d^-}$	-0.221 — -0.189	-0.24(3)†	-0.219(17)†	-0.97 — 0.47
$\langle 1 \rangle_{\delta s^-}$	-0.0085 — 0.0031	-0.012(18)†	-0.00319(72)†	—

† Single lattice result

Qualitative comparison of lattice QCD and global PDF fits

Unpolarised PDFs



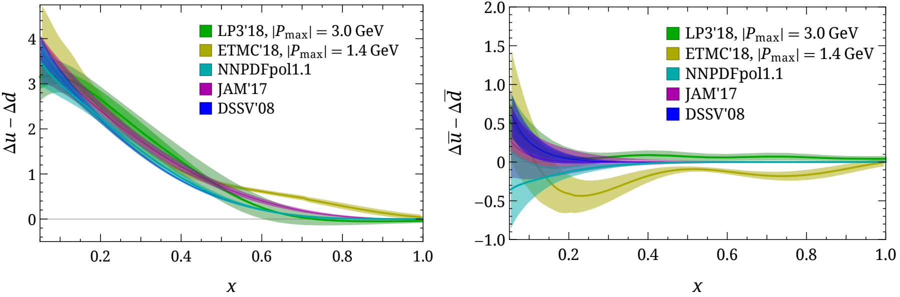


Ref.	Sea quarks	Valence quarks	$N_{\Delta t}$	method	P_{max} (GeV)	a (fm)	M_π (MeV)	$M_\pi L$
ETMC'20	2f twisted mass	twisted mass	4	pseudo-PDF	1.38	0.09	130	3.0
JLab/W&M	2+1 clover	clover	n/a	pseudo-PDF	3.29	0.09	172–358	5.08–5.47
ETMC'18	2f twisted mass	twisted mass	4	quasi-PDF	1.38	0.09	130	3.0
LP3'18	2+1+1f HISQ	clover	4	quasi-PDF	3	0.09	135	4.0
LP3'17	2+1+1f HISQ	clover	2	quasi-PDF	1.3	0.09	135	4.0

ETMC and LP3 determinations are both at the physical pion mass
Lattice determinations are qualitatively similar (among them) and similar to global fits
Nucleon momentum is limited by lattice spacing
Different procedures lead to slightly different behaviour in x

Qualitative comparison of lattice QCD and global PDF fits

Helicity PDFs



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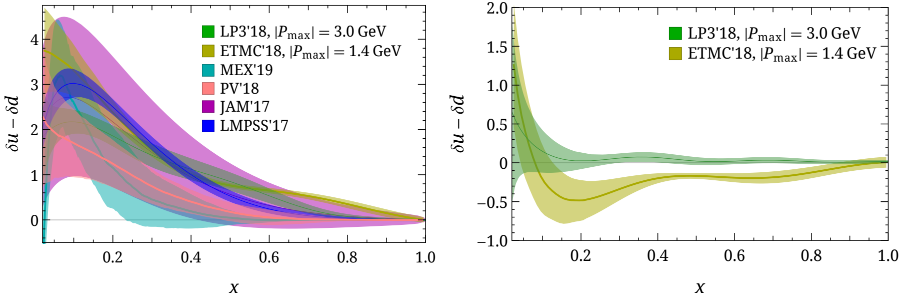
Qualitative comparison of lattice QCD and global PDF fits

Transversity PDFs



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2. Exploiting lattice QCD

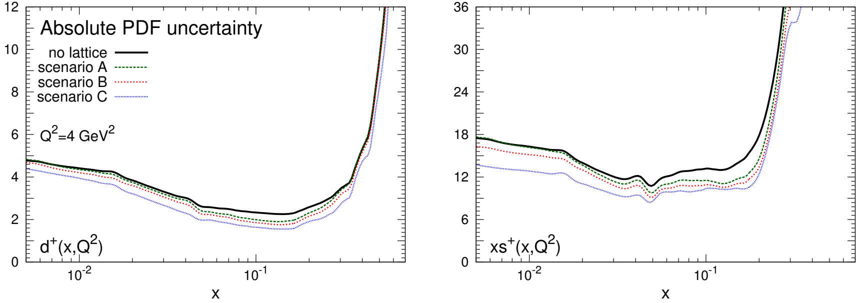
Impact of lattice QCD moments on f

Generate lattice QCD pseudodata assuming NNPDF3.1 central values for
$\langle x \rangle_{u^+}$, $\langle x \rangle_{d^+}$, $\langle x \rangle_{s^+}$, $\langle x \rangle_{g}$, $\langle x \rangle_{u^+-d^+}$

Assume percentage uncertainties according to three scenarios

scenario	$\langle x \rangle_{u^+}$	$\langle x \rangle_{d^+}$	$\langle x \rangle_{s^+}$	$\langle x \rangle_{g}$	$\langle x \rangle_{u^+-d^+}$
A	3%	3%	5%	3%	5%
B	2%	2%	4%	2%	4%
C	1%	1%	3%	1%	3%
current	17%	30%	45%	13%	60%

Reweight NNPDF3.1 with lattice pseudodata and look at the impact



Impact of lattice QCD moments on Δf

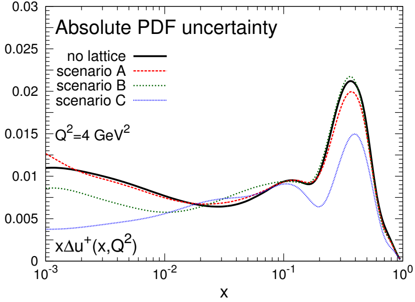
Generate lattice QCD pseudodata assuming NNPDFpol1.1 central values for

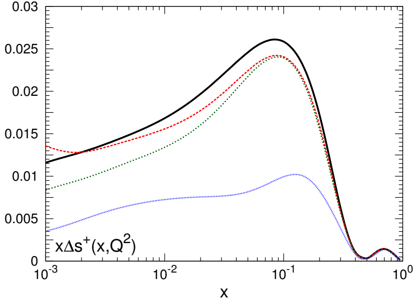
$$g_A \equiv \langle 1 \rangle_{\Delta u^+ - \Delta d^+}, \langle 1 \rangle_{\Delta u^+}, \langle 1 \rangle_{\Delta d^+}, \langle 1 \rangle_{\Delta s^+}, \langle x \rangle_{\Delta u^- - \Delta d^-}$$

Assume percentage uncertainties according to three scenarios

scenario	g_A	$\langle 1 \rangle_{\Delta u^+}$	$\langle 1 \rangle_{\Delta d^+}$	$\langle 1 \rangle_{\Delta s^+}$	$\langle x \rangle_{\Delta u^- - \Delta d^-}$
A	5%	5%	10%	100%	70%
B	3%	3%	5%	50%	30%
C	1%	1%	2%	20%	15%
current	3%	3%	5%	70%	65%

Reweight NNPDFpol1.1 with lattice pseudodata and look at the impact





Impact of lattice QCD moments on δf

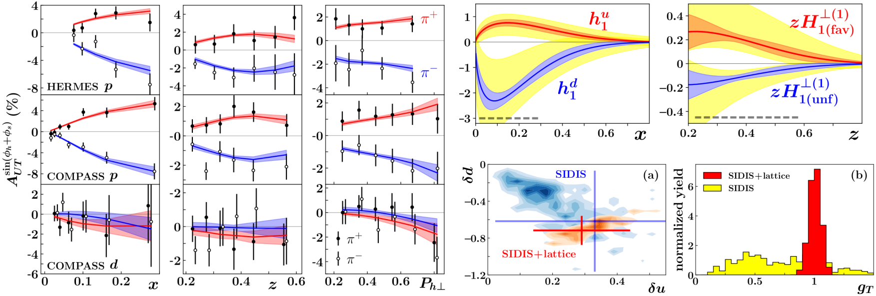
Simultaneous fit to the Collins asymmetry data from HERMES and COMPASS of

$$f_1^q(x, k_\perp^2) \qquad h_1^q(x, k_\perp^2) \qquad D_1^{h/q}(z, p_\perp^2) \qquad H_1^{\perp h/q}(z, p_\perp)$$

and to three lattice *data sets* with an estimate of systematic uncertainties

PDNME [Bhattacharya et al. (2016)] RQCD [Bali et al. (2015)] LHPC [Green et al. (2012)]

using Monte Carlo techniques for the representation of uncertainties



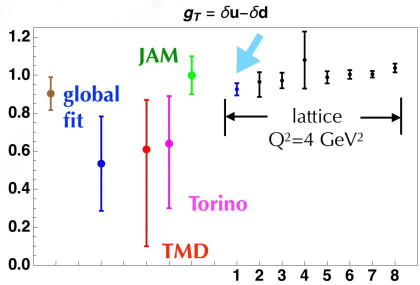
[PRL 120 (2018) 152502]

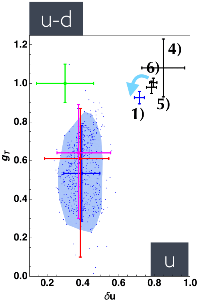
Excellent description of the data with and without lattice results ($\chi^2/N_{\rm dat} = 0.65$)

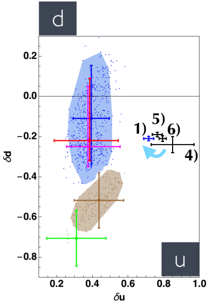
Lattice results seem compatible with measured asymmetries

Lattice results are able to reduce the uncertainty on h_1 and $H_1^\perp$ significantly

Impact of lattice QCD moments on δf [Courtesy of M. Radici]



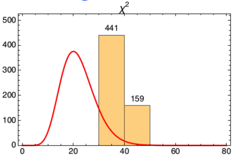




[1] ETMC 19; [2] Mainz 19; [3] LHPC 19; [4] JLQCD 18; [5] PNDME 18; [6] ETMC 17; [7] RQCD 14; [8] LHPC 12

global fit [Radici, in progress]; **JAM** [PRL 120 (2018) 152502]; **TMD** [PRD 93 (2016) 014009]; **Torino** [PRD 92 (2015) 114023]

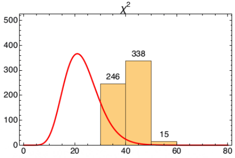
global fit



χ^2/dof = 1.76 ± 0.11

22 d.o.f.

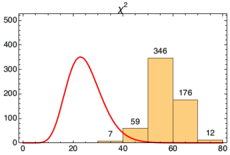
global fit + g_T



χ^2/dof = 1.82 ± 0.25

23 d.o.f.

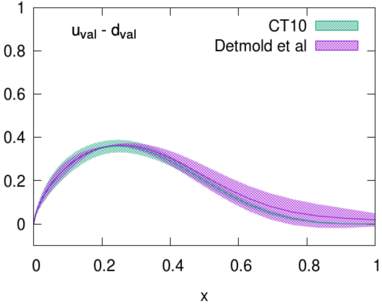
global fit + g_T + δu + δd

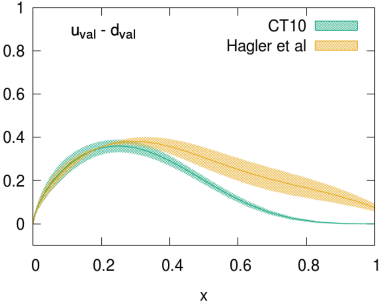


χ^2/dof = 2.29 ± 0.25

25 d.o.f.

Reconstructing PDFs from lattice moments





Detmold *et al.* [EPJ direct 3 (2001) 13]

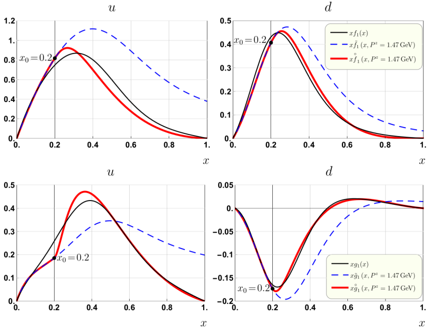
$u - d$ from the lowest few lattice moments, ensure the correct behavior in the chiral and heavy quark limits

Haegler *et al.* [PRD 77 (2008) 094502]

non-perturbative renormalization factor for the axial vector current, only connected diagrams are included

Bacchetta *et al.* [PRD 95 (2017) 014036]

supplement lattice moments with quasi-PDFs (using results of a diquark spectator model) matched at a fixed point x_0

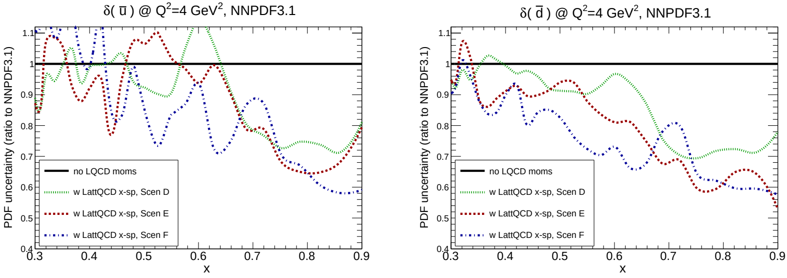


Impact of lattice calculations of x-space PDFs

Apply Bayesian reweighting to the isotriplet PDF combinations

$$\begin{array}{lll} f \quad \text{NNPDF3.1} & u(x_i,Q^2)-d(x_i,Q^2) & \bar{u}(x_i,Q^2)-\bar{d}(x_i,Q^2) \\ \Delta f \quad \text{NNPDFpol1.1} & \Delta u(x_i,Q^2)-\Delta d(x_i,Q^2) & \Delta\bar{u}(x_i,Q^2)-\Delta\bar{d}(x_i,Q^2) \end{array} \quad i=1,\dots,N_x$$

Consider uncorrelated lattice pseudodata $Q^2 = 4$ GeV^2 and $x_i = 0.70, 0.75, 0.80, 0.85, 0.90$ for three scenarios: (D) $\delta_L^{(i)} = 12\%$; (E) $\delta_L^{(i)} = 6\%$; (F) $\delta_L^{(i)} = 3\%$



No large differences among the three scenarios (PDF variations are correlated)

Moderate precision required for lattice QCD to make an impact on antiquarks at large x

Caveat: rough assumptions ($\{x_i\}$, $\delta_L^{(i)}$); can we do something better?

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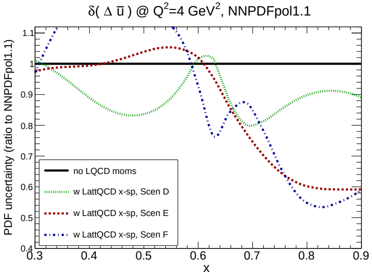

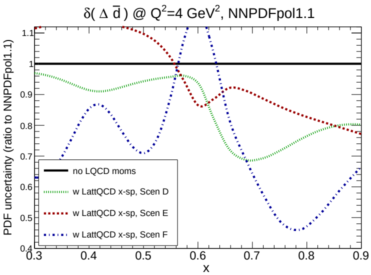

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Impact of lattice calculations of x-space PDFs [JHEP 1910, (2019) 137]

Quasi-PDFs defined as momentum-dependent nonlocal static matrix elements for nucleon states at finite momentum, with an ultraviolet cut-off scale $\Lambda \sim 1/a$

$$\widetilde{q}(x,\Lambda,p_z) = \int \frac{dz}{4\pi} e^{-ixzp_z} \frac{1}{2} \sum_{s=1}^{2} \langle p,s|\, \bar{\psi}(z)\gamma_\alpha e^{ig\int_0^z A_z(z')dz'} \psi(0)\, |p,s\rangle$$

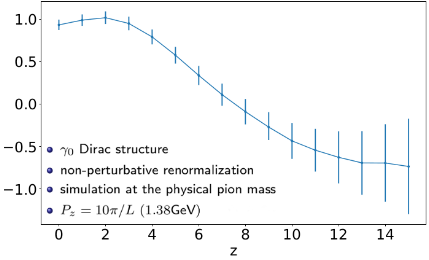
Must be related to the corresponding light-front PDF, usually within LaMET

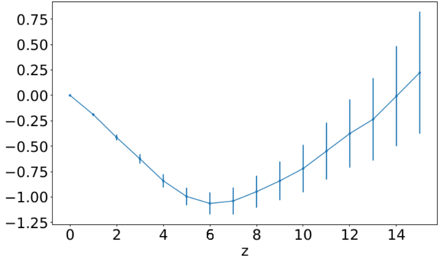
$$\widetilde{q}(x,\Lambda,p_z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{p_z}, \frac{\Lambda}{p_z}\right)_{\mu^2=Q^2} q(y,Q^2) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{p_z^2}, \frac{m^2}{p_z^2}\right)$$

Restrict to the isotriplet distributions and consider ETMC lattice data

$$V_3 = u - \bar{u} - [d - \bar{d}] \qquad\qquad T_3 = u + \bar{u} - [d + \bar{d}]$$

$$\mathcal{O}^{\text{Re}}_{\gamma^0}(z,\mu) \equiv \text{Re}[h_{\gamma^0,3}(zp_z, z^2, \mu^2)] = \mathcal{C}^{\text{Re}}_3 \circledast V_3 \qquad \mathcal{O}^{\text{Im}}_{\gamma^0}(z,\mu) \equiv \text{Im}[h_{\gamma^0,3}(zp_z, z^2, \mu^2)] = \mathcal{C}^{\text{Im}}_3 \circledast T_3$$



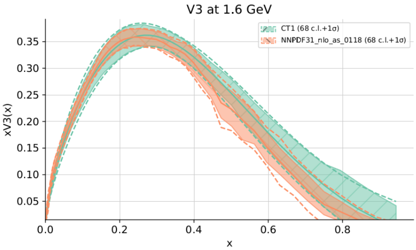


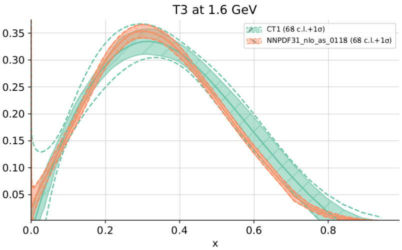
Impact of lattice calculations of x-space PDFs [JHEP 1910, (2019) 137]

Consider various scenarios for systematic uncertainties

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

Percentage values for scenarios S1-S3 should be understood as a given fraction of the central value of the matrix element
Absolute values for scenarios S4-S6 are shifts independent from the matrix element





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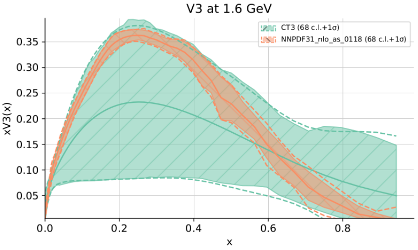

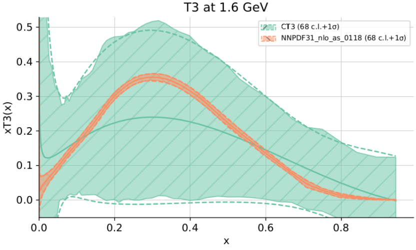

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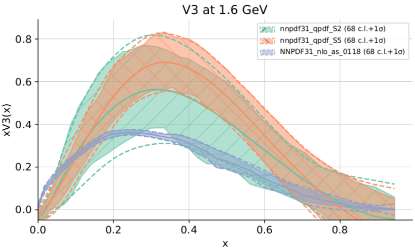
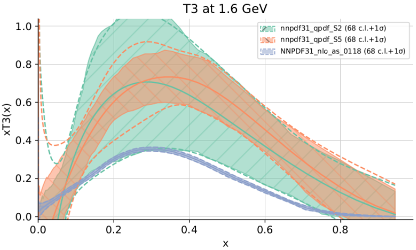
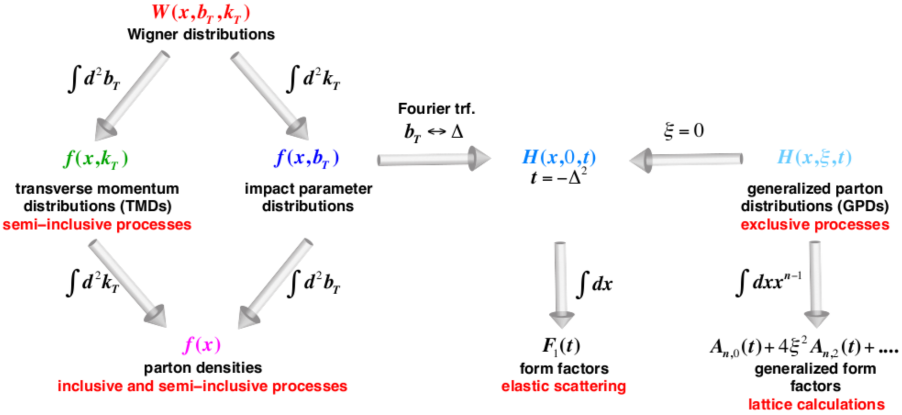




3.Outlook and conclusions

Towards 3D structure

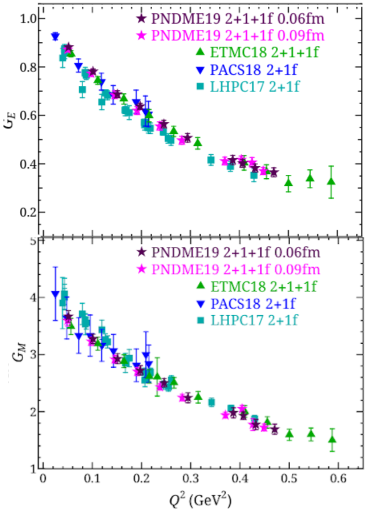


Spectacular theoretical effort on both the lattice and the phenomenological sides

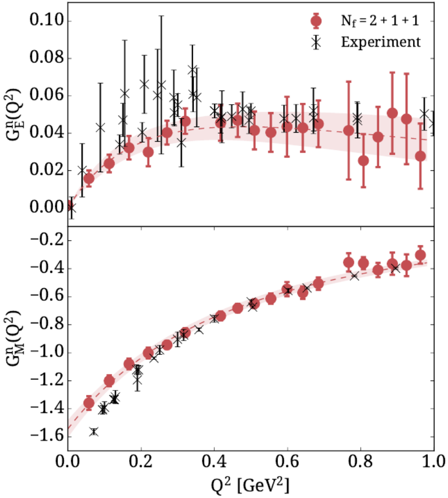
Case for the physics of new facilities and/or upgrades (JLab-12, RHIC, after@LHC, EIC)

Towards 3D structure - Form Factors

Isovector Electric and Magnetic



Neutron Electric and Magnetic



PNDME19 [arXiv:1901.00060]

ETMC18 [PRD 96 (2017) 034503]

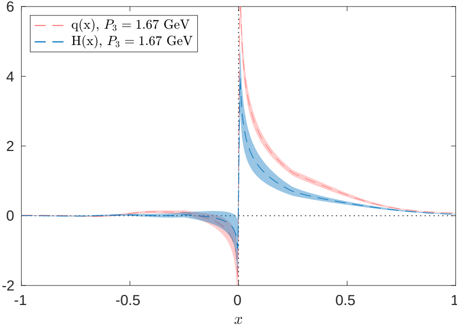
PACS18 [PRD 99 (2019) 014510]

LHPC17 [PRD 97 (2018) 034504]

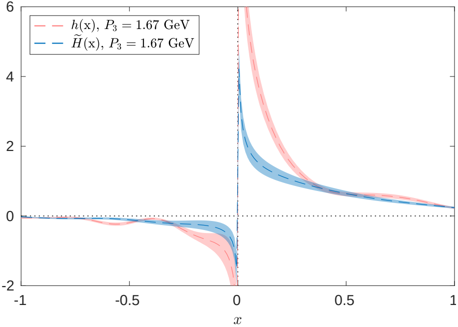
Towards 3D structure - GPDs

GPDs from quasi-PDFs [arXiv:1910.13229]

Unpolarised GPD H



Helicity GPD $\tilde{H}$



Determining GPDs is generally more challenging than determining PDFs

More variables: Wilson line z, hadron momentum P_z, momentum transfer t, skewness ξ

Perturbative matching depends on skewness, but not on momentum transfer

Phenomenological fits complicate because of a general lack of data
(but significant progress ongoing, see *e.g.* PARTONS framework [EPJ C78 (2018) 478])

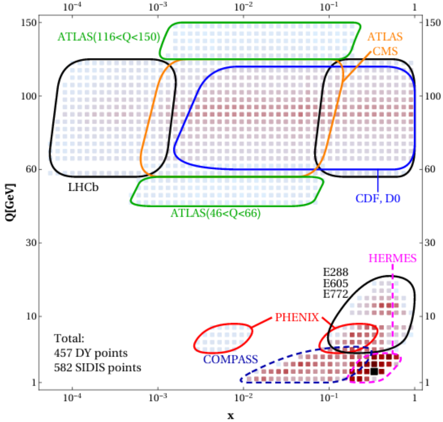
Towards 3D structure - TMDs

quark polarization

nucleon polarization

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

	Framework	HERMES	COMPASS	DY	Z production	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL′	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	NNLL′	✔	✔	✔	✔	1039	1.06
Pavia 2019 arXiv:1912.07550	N³LL	✘	✘	✔	✔	353	1.02



Phenomenology: various challenges (data, theoretical framework)

Lattice: Sivers and Boer-Mulders shifts; Collins-Soper kernel

Summary

There has been an undeniable progress in the determination of PDFs
from both the global fit to data and the lattice QCD sides

Such a progress cannot be ignored

Opportunity to gain further knowledge
by improving cross-talk between the two sides

Attempt to realise such an opportunity within the PDFLattice joint effort
benchmark + impact studies

Some substantial effort is ongoing
the definition and renormalisation of the non-local operators involved in the lattice simulation

the proof of the factorization theorem between PDFs and quasi-PDFs

the computation of the matching coefficients
relating lattice-computable quantities to PDFs in different renormalization schemes

the implementation of efficient methods
to incorporate lattice QCD information into global PDF fit determinations (and *viceversa*)

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Thank you