



# PAST PRESENT AND FUTURE CHALLENGES IN THE DETERMINATION OF THE STRUCTURE OF THE PROTON

## LECTURE IV

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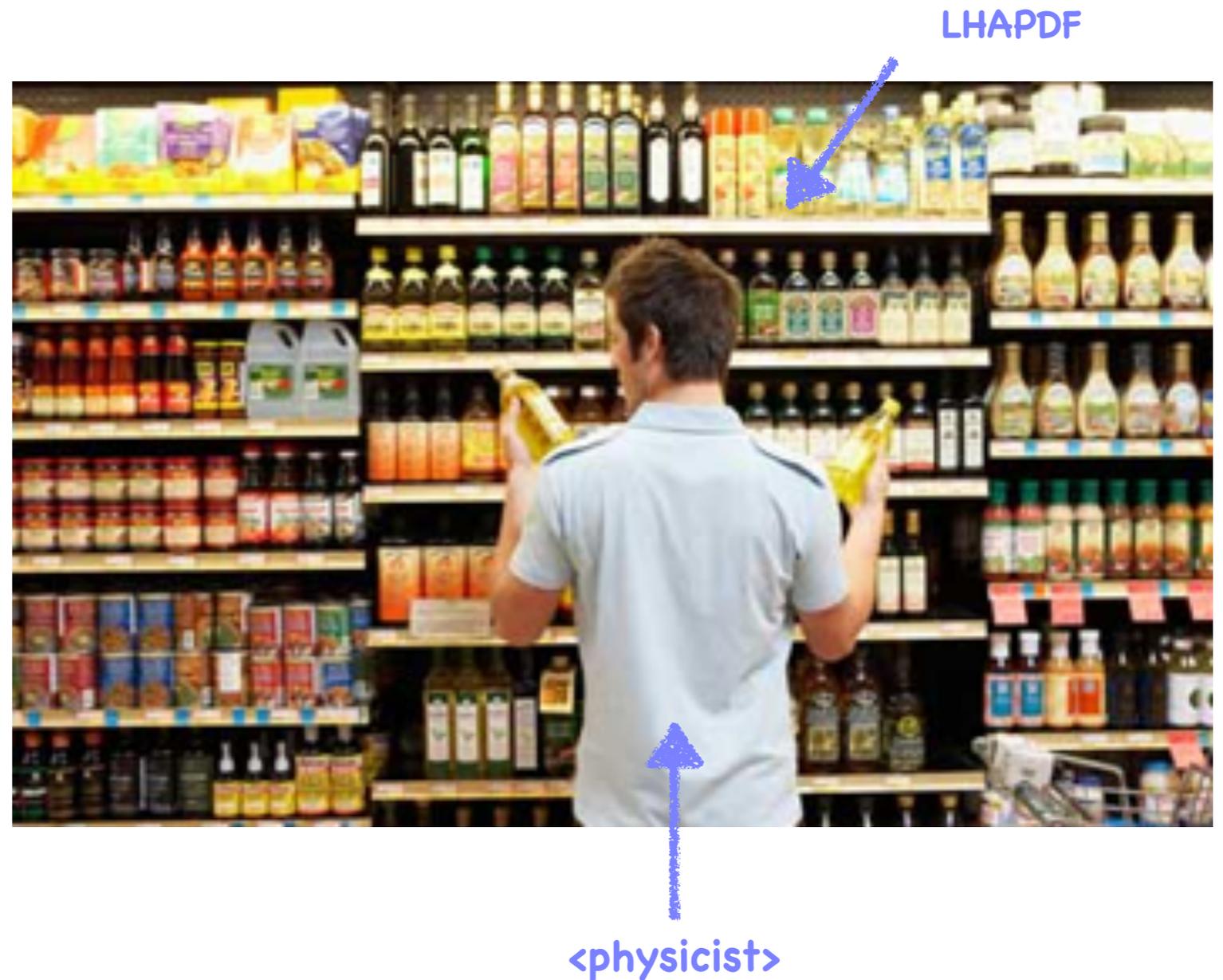
# Outline

- First lecture (Monday)
  - Motivation: the big picture
  - Parton Model and QCD
  - Collinear Factorisation and definition of PDFs
- Second lecture (Tuesday)
  - Experimental Data
  - Disentangling proton's components
- Third lecture (Wednesday)
  - Photon and EW corrections
  - Beyond DGLAP
  - Statistics and Methodology
- **Fourth lecture (today)**
  - State-of-the art PDFs
  - New frontiers and challenges

# State-of-the-art PDFs

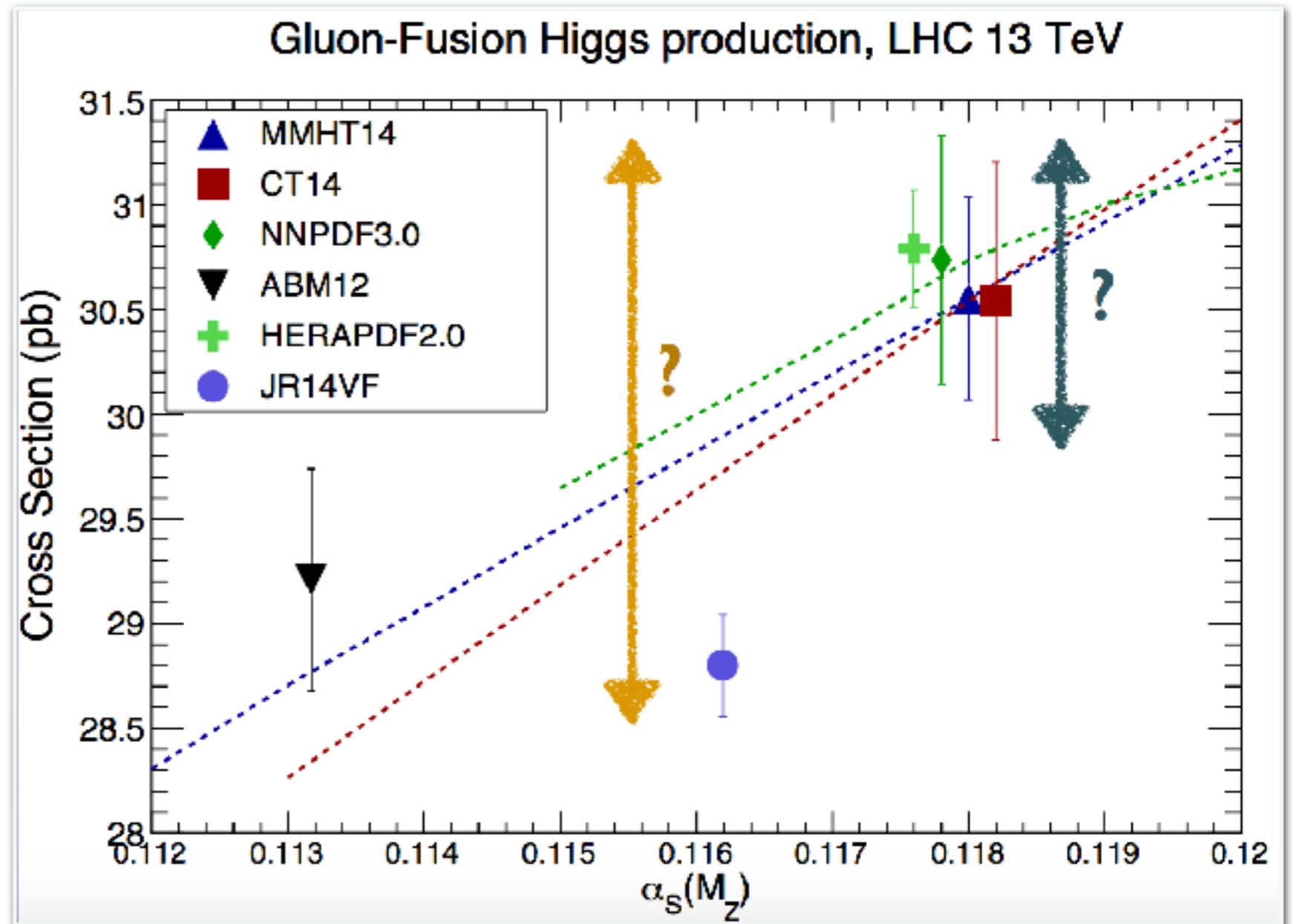
# The choice of PDFs matters

- What does PDF uncertainty include?  
How reliable it is?
- How do we interpret the difference predictions using different PDF sets?
- Shall we just pick a set out of the PDFs  
“supermarket” shelf or  
take the envelope of ALL predictions?



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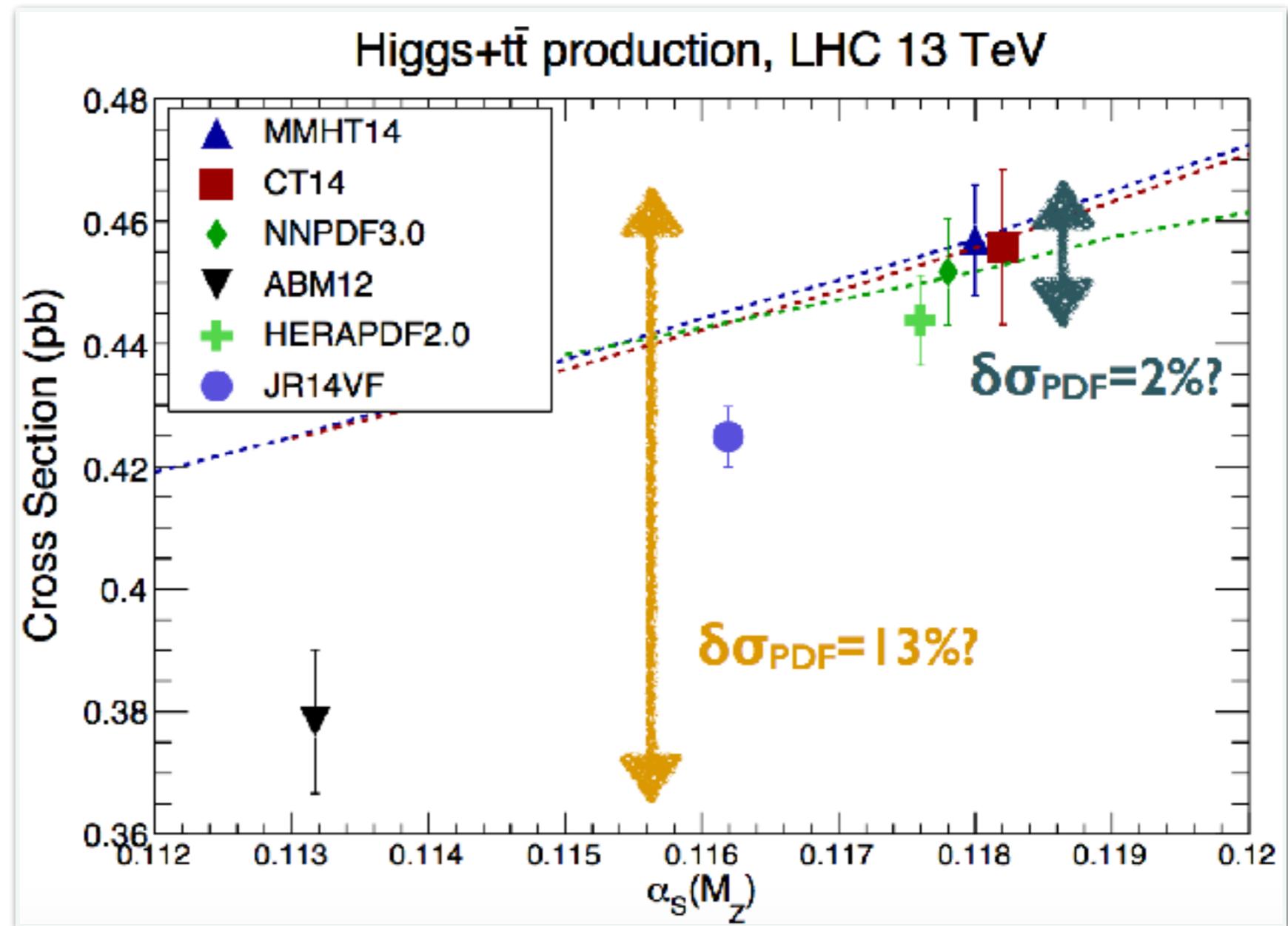


$$\delta\sigma_{PDF} = 2\%$$

$$\delta\sigma_{PDF} = 5\%$$

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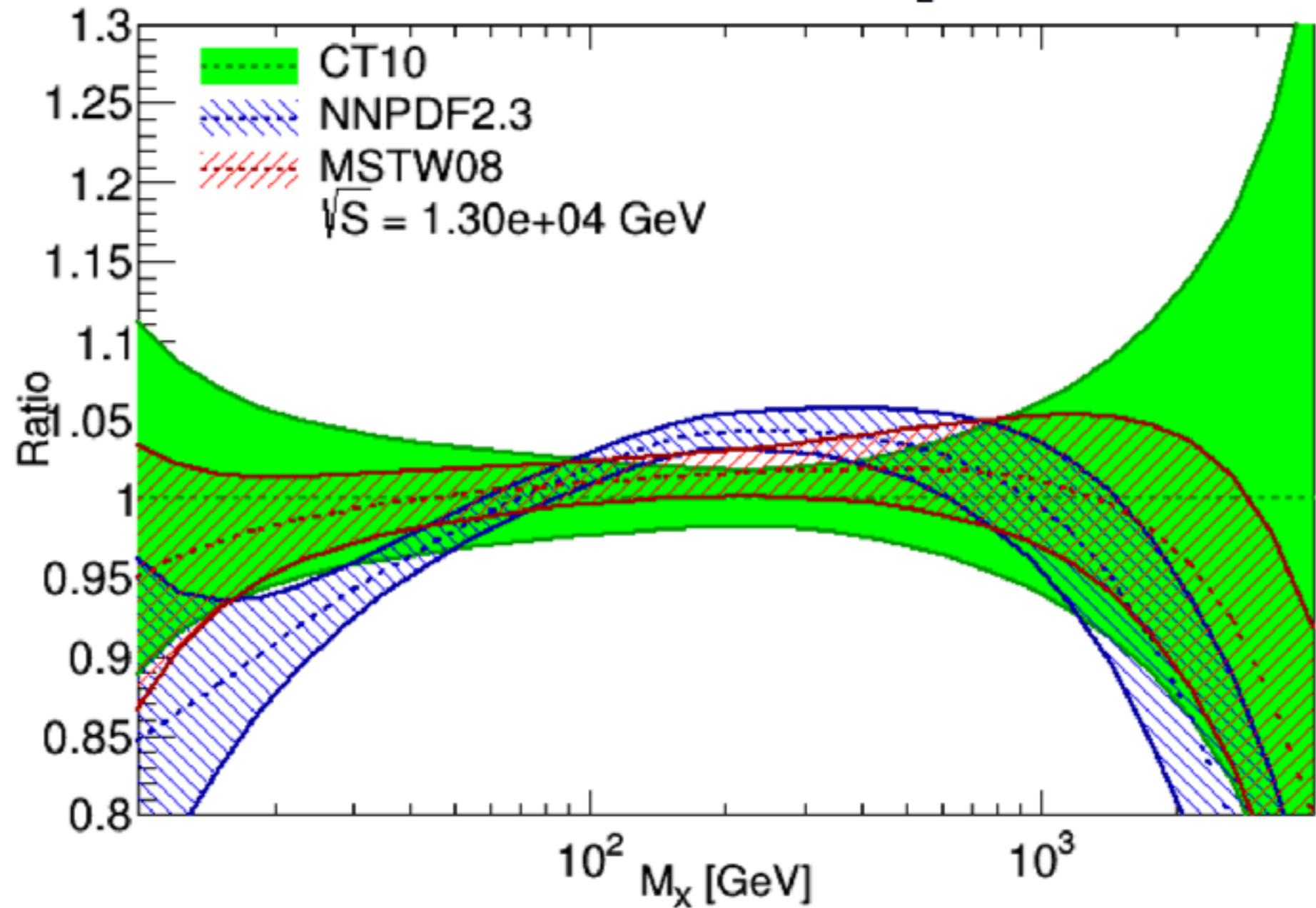
# The players

April 2019	NNPDF3.1	MMHT2014	CT14	ABMP16
Fixed Target DIS	✓	✓	✓	✓
HERA I+II	✓	✓	✓	✓
HERA jets	✗	✓	✗	✗
Fixed Target DY	✓	✓	✓	✓
Tevatron W,Z	✓	✓	✓	✓
Tevatron jets	✓	✓	✓	✗
LHC jets	✓	✓	✓	✗
LHC vector boson	✓	✓	✓	✓
LHC top	✓	✗	✗	✓
Stat. treatment	Monte Carlo	Hessian $\Delta\chi^2$ dynamical	Hessian $\Delta\chi^2$ dynamical	Hessian $\Delta\chi^2=1$
Parametrization	Neural Networks (259 pars)	Chebyshev (37 pars)	Bernstein (30-35 pars)	Polynomial (15 pars)
HQ scheme	FONLL	TR'	ACOT- $\chi$	FFN (+BMST)
Order	NLO/NNLO	NLO/NNLO	NLO/NNLO	NLO/NNLO

# Gluon luminosity

NNPDF2.3 / CT10 / MSTW2008

LHC 13 TeV, NNLO,  $\alpha_s(M_Z)=0.118$



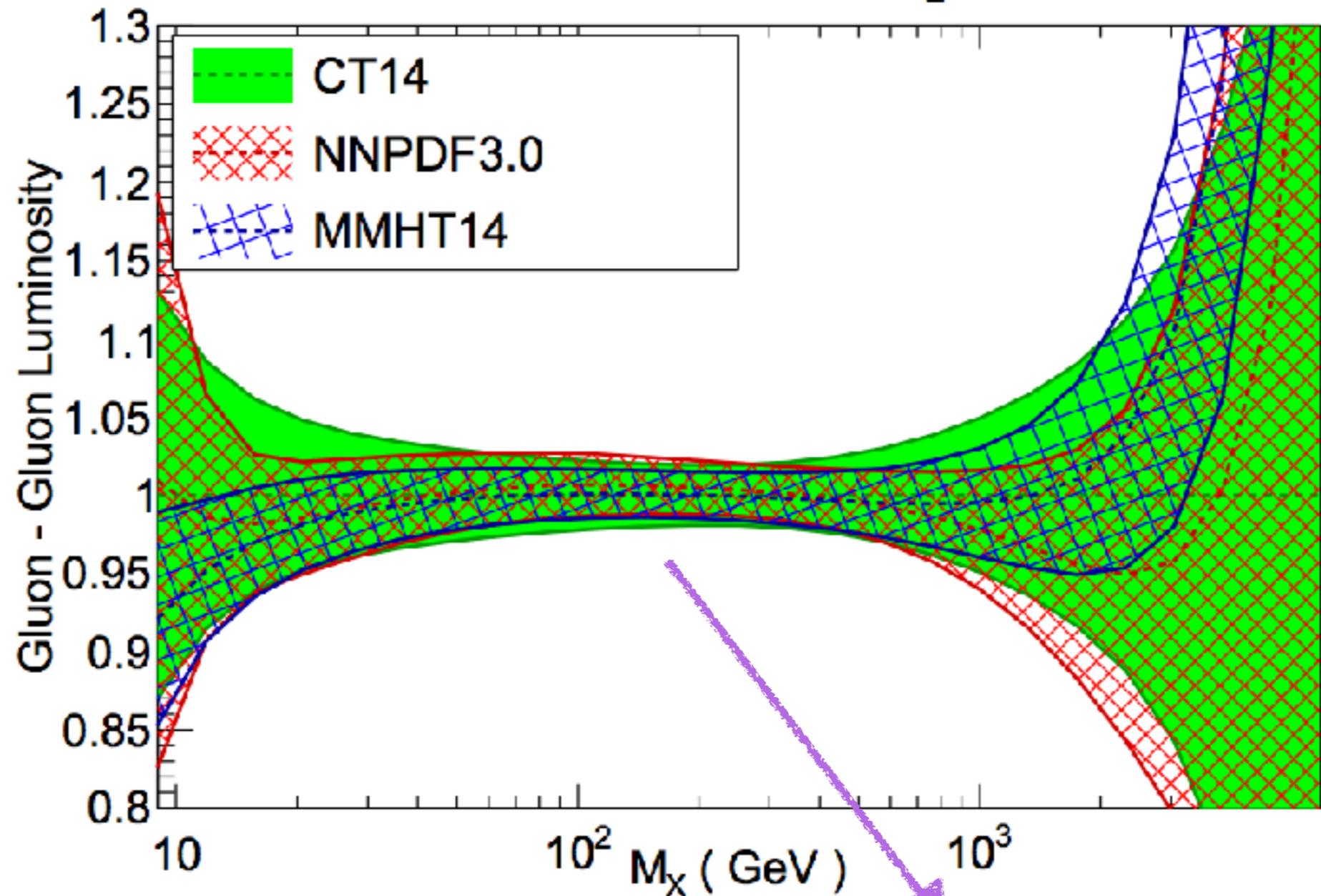
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(2014)

# Gluon luminosity

NNPDF3.0 / CT14 / MMHT14

LHC 13 TeV, NNLO,  $\alpha_s(M_Z)=0.118$

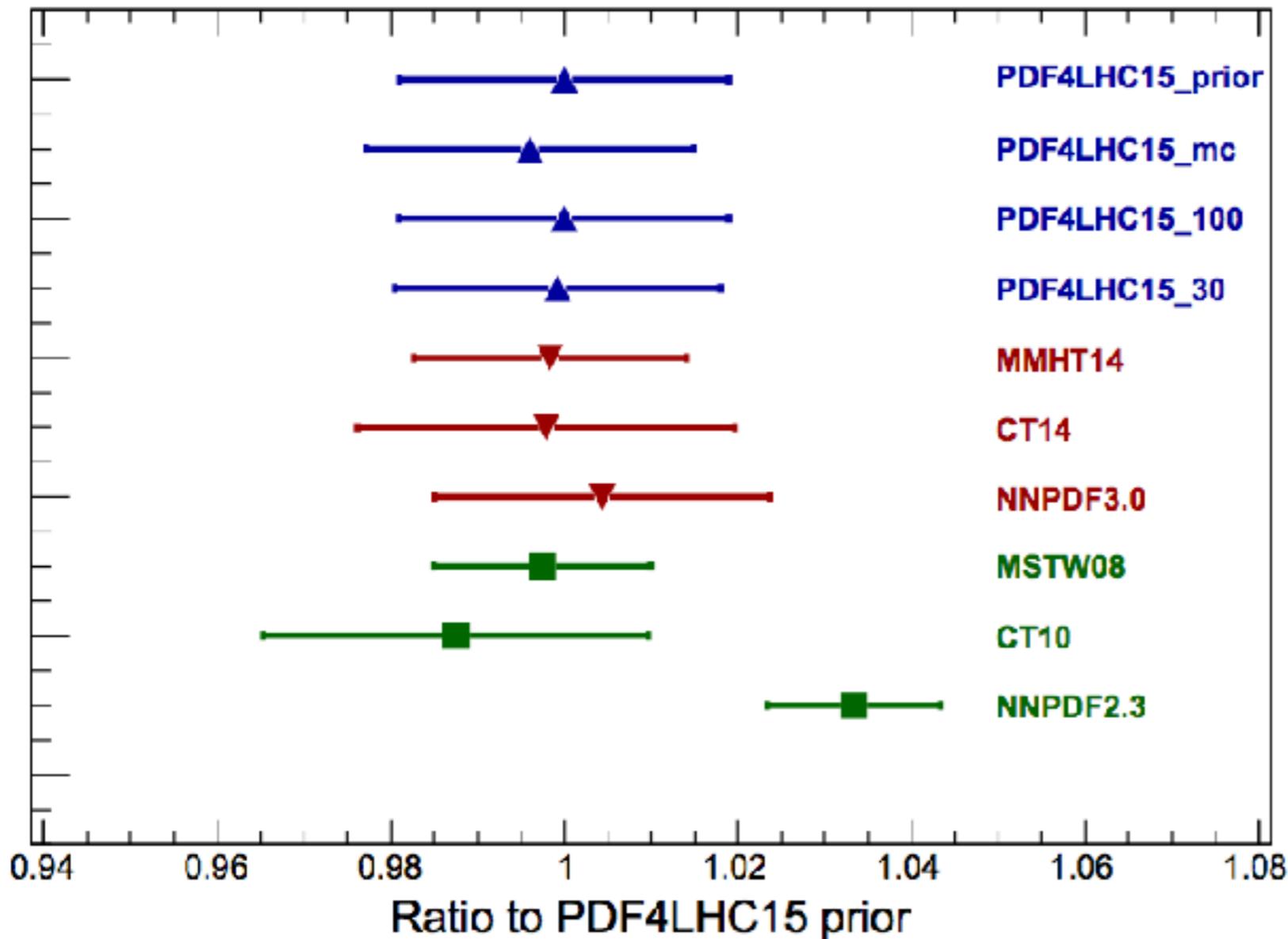


(2016)

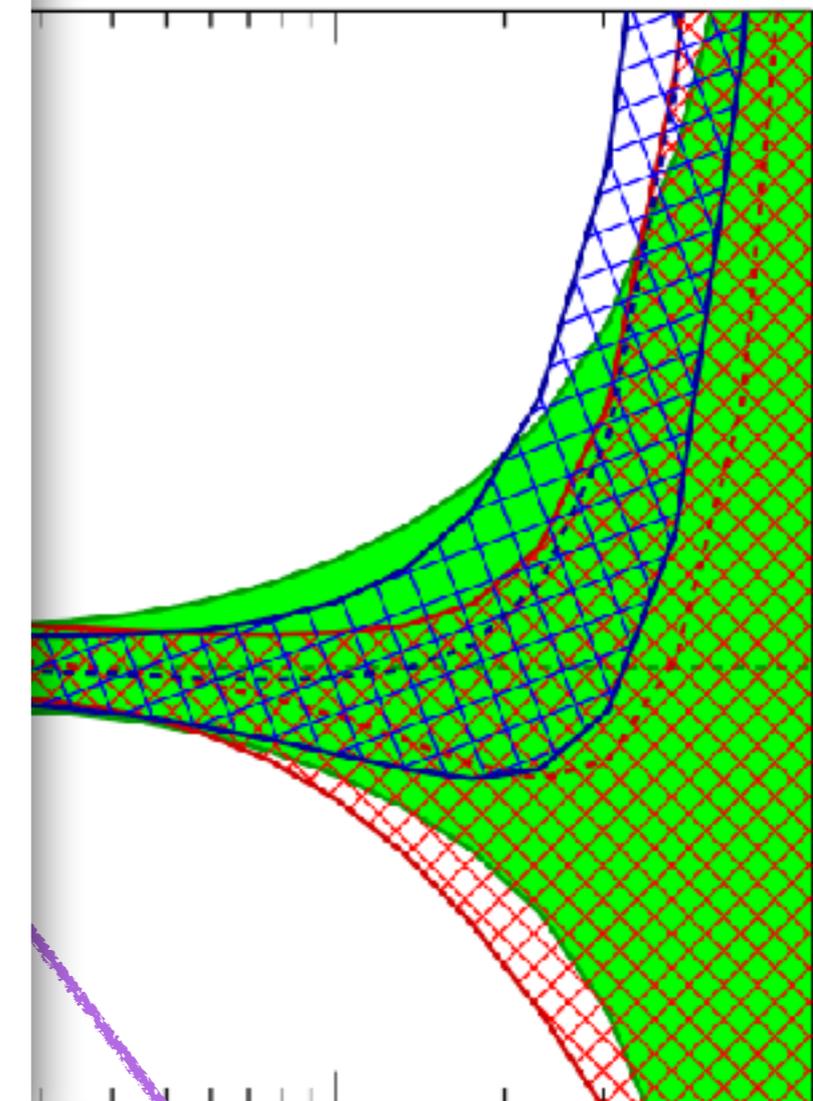
Impact on Higgs physics

# Consequence: Higgs physics

Gluon-Fusion Higgs production, LHC 13 TeV



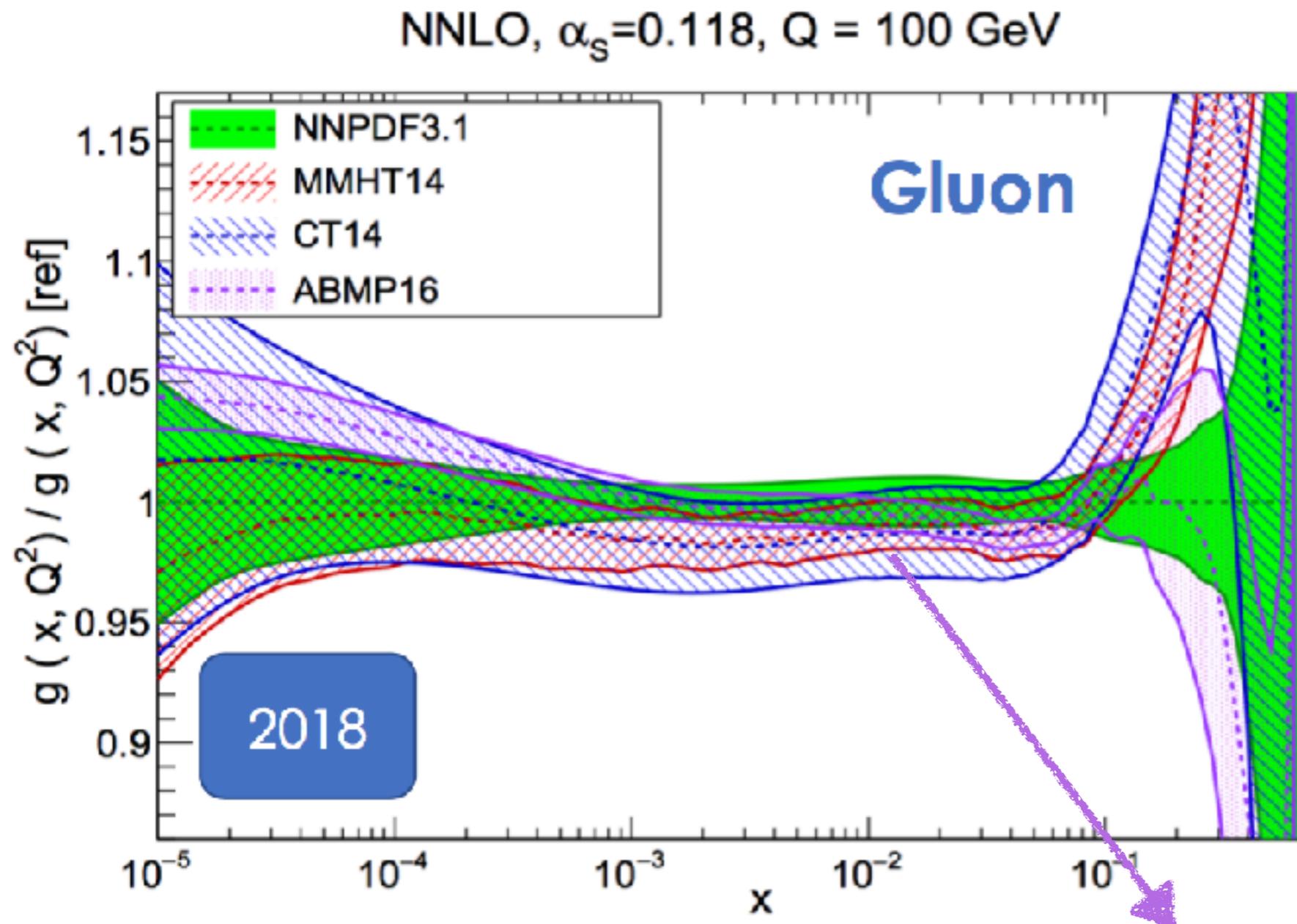
$\alpha_s(M_Z)=0.118$



Impact on Higgs physics

# Gluon luminosity

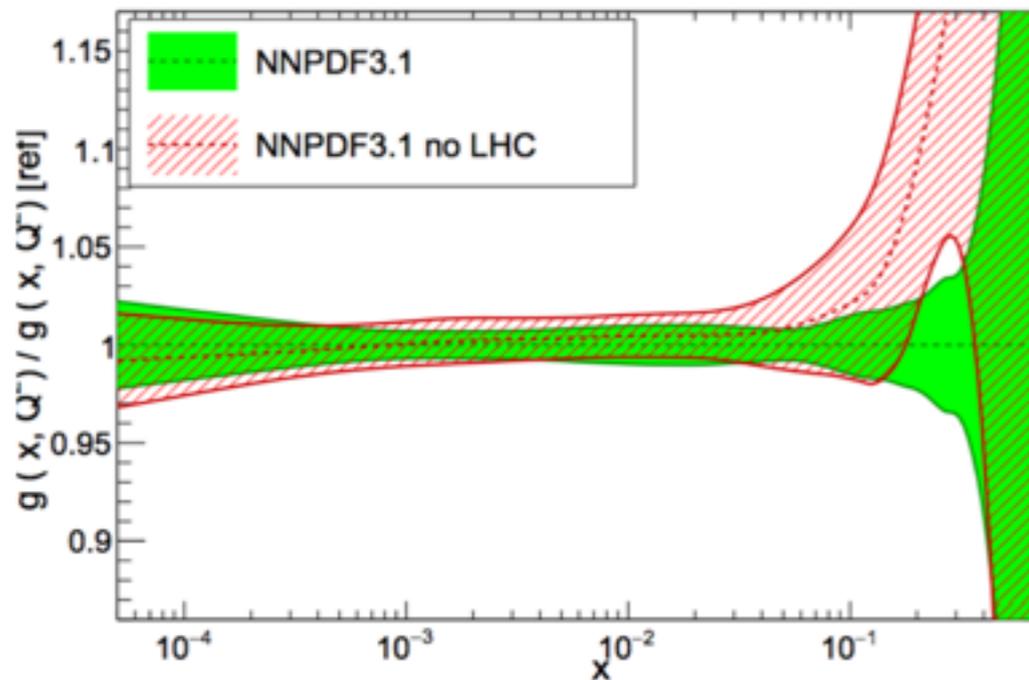
NNPDF3.1 / CT14 / MMHT14/ABMP16



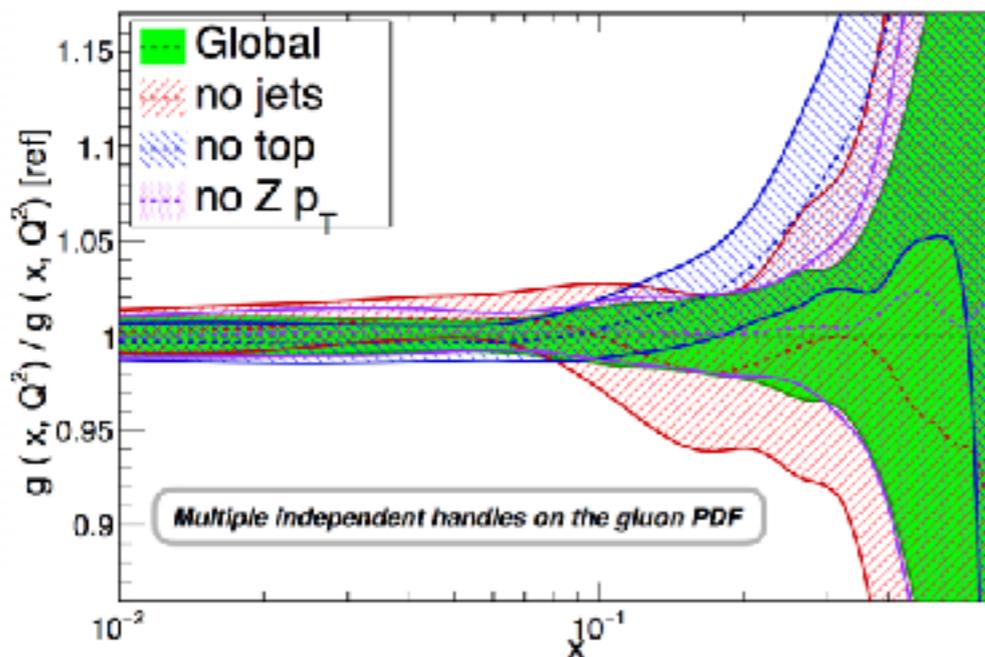
New LHC data?

# Gluon luminosity

NNPDF3.1 NNLO,  $Q = 100$  GeV

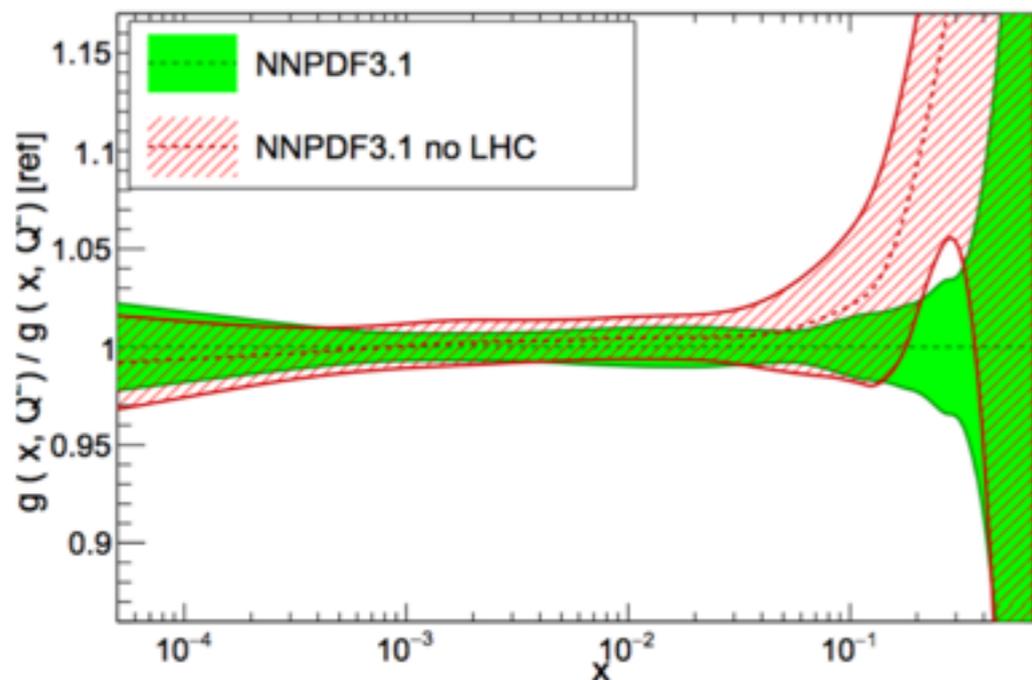


NNPDF3.1 NNLO,  $Q = 100$  GeV

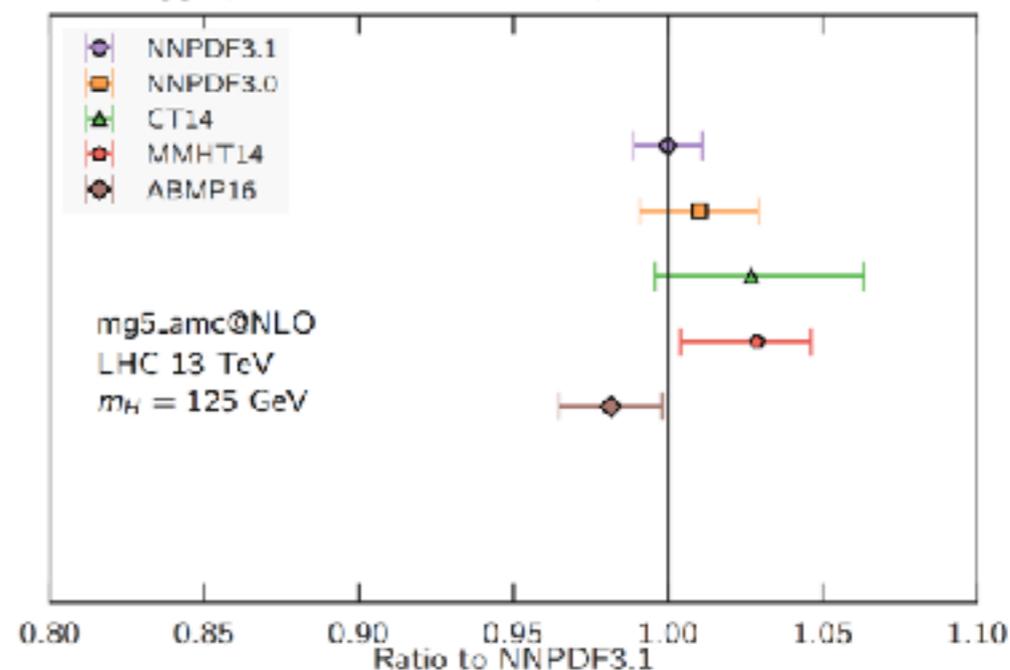


# Gluon luminosity

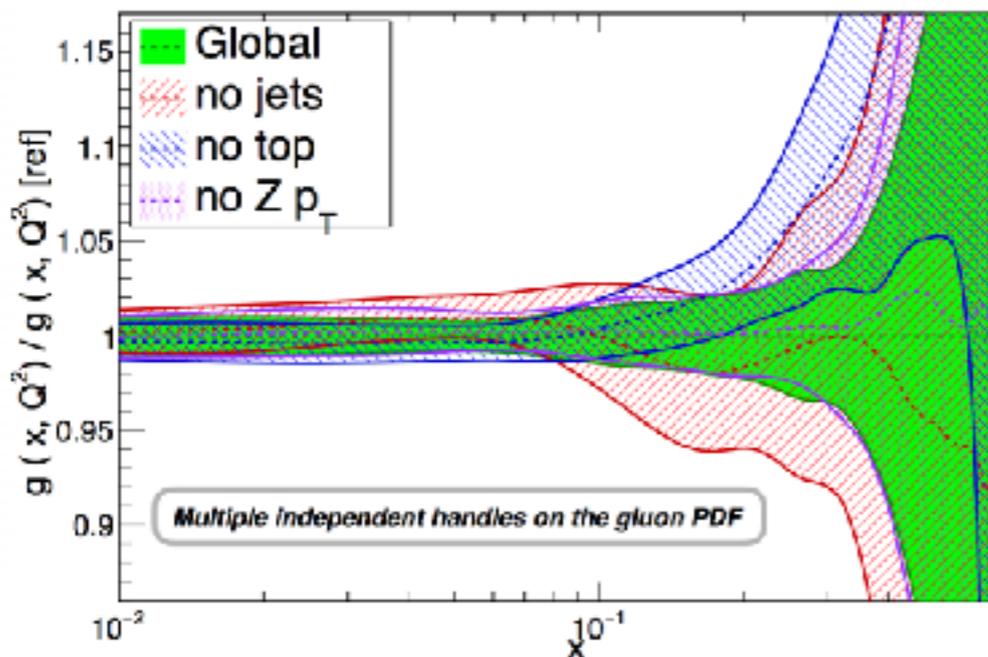
NNPDF3.1 NNLO,  $Q = 100$  GeV



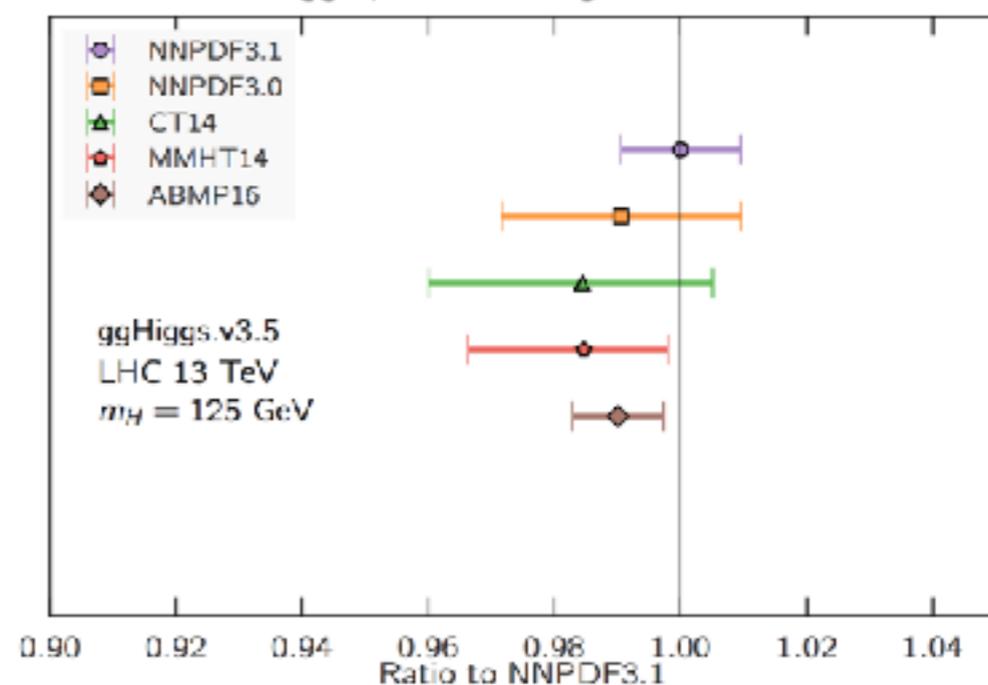
Higgs production: associate production with  $t\bar{t}$



NNPDF3.1 NNLO,  $Q = 100$  GeV



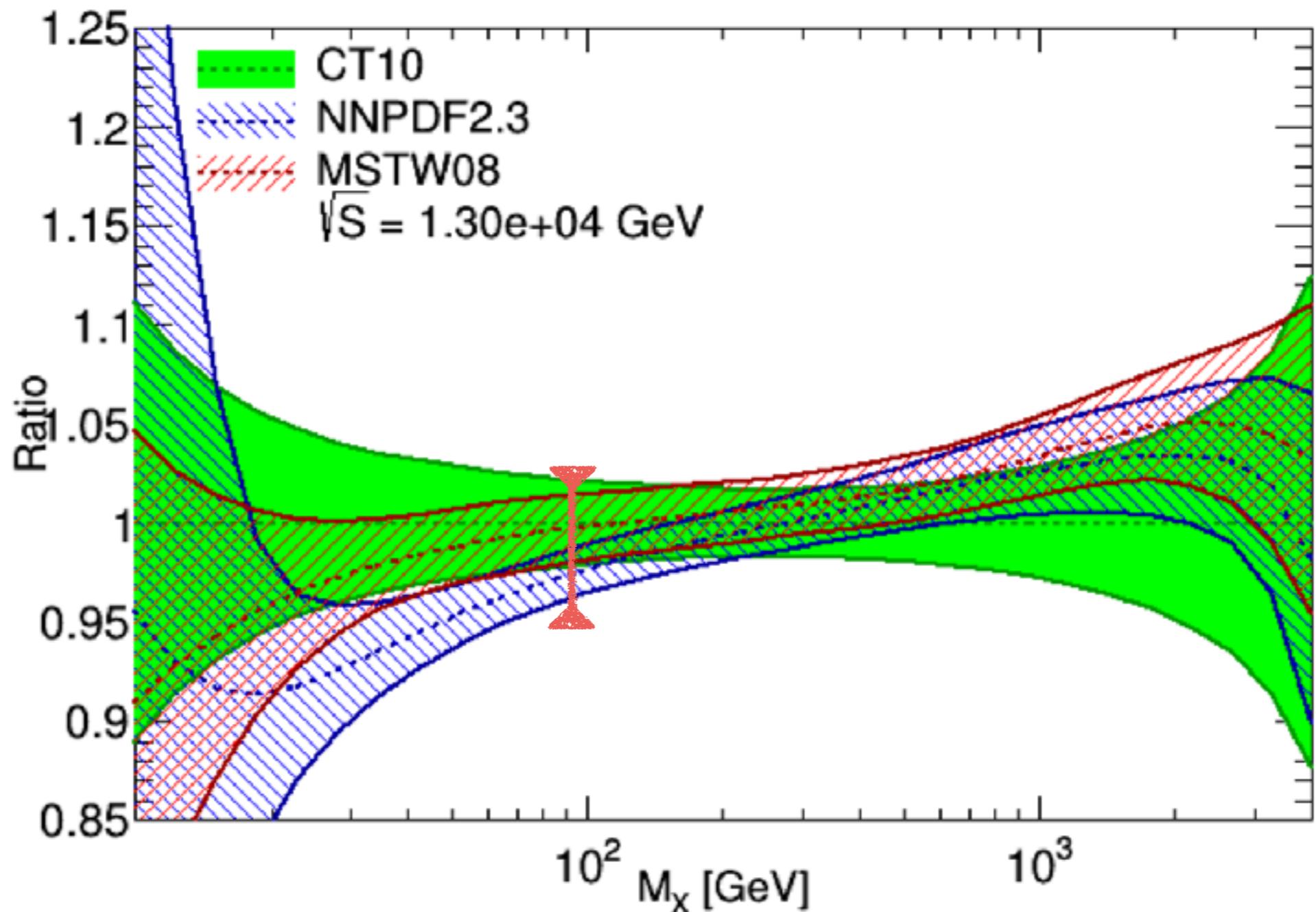
Higgs production: gluon fusion



# Quark-Antiquark luminosity

NNPDF2.3 / CT10 / MSTW2008

LHC 13 TeV, NNLO,  $\alpha_s(M_Z)=0.118$



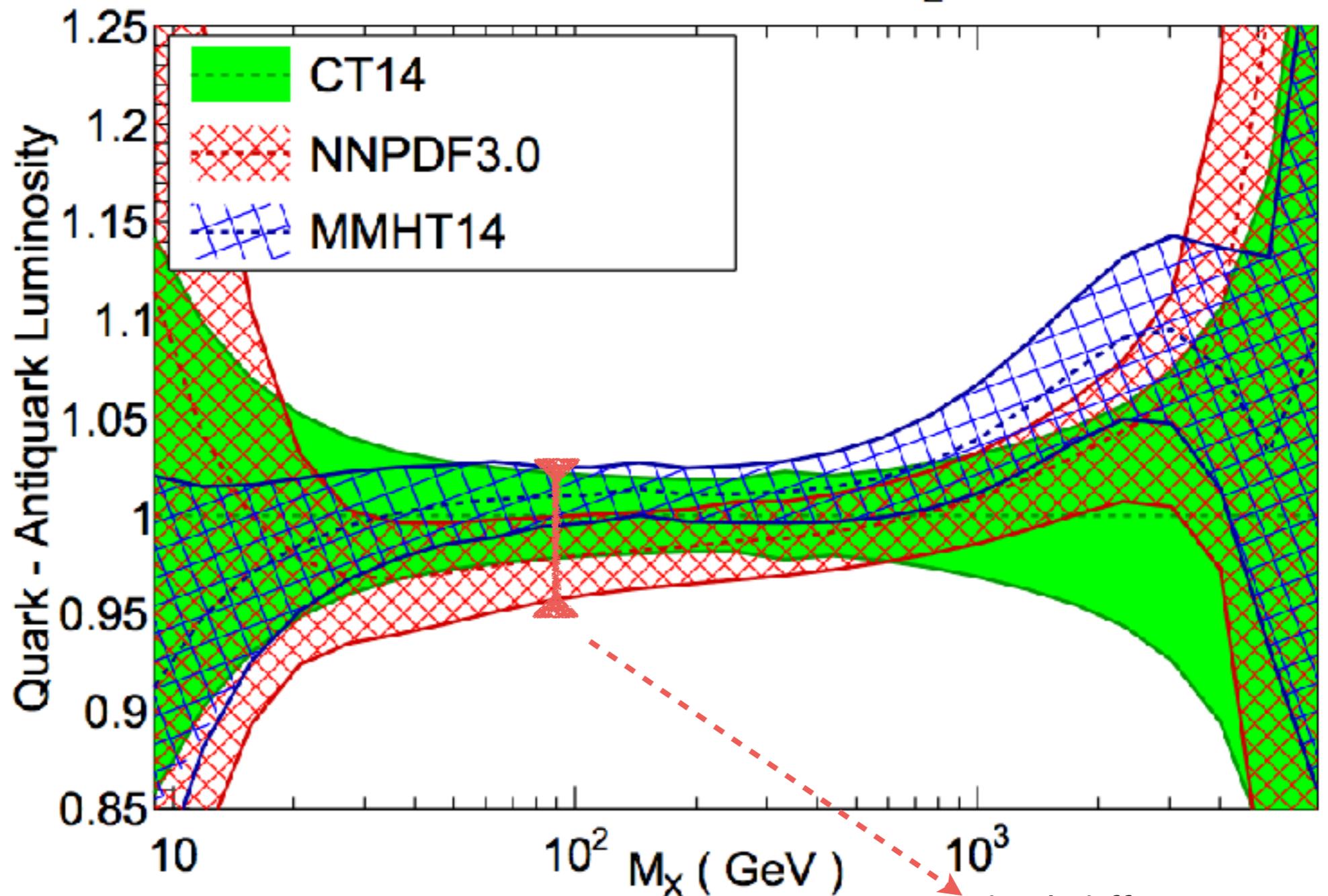
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(2014)

# Quark-Antiquark luminosity

NNPDF3.0 / CT14 / MMHT14

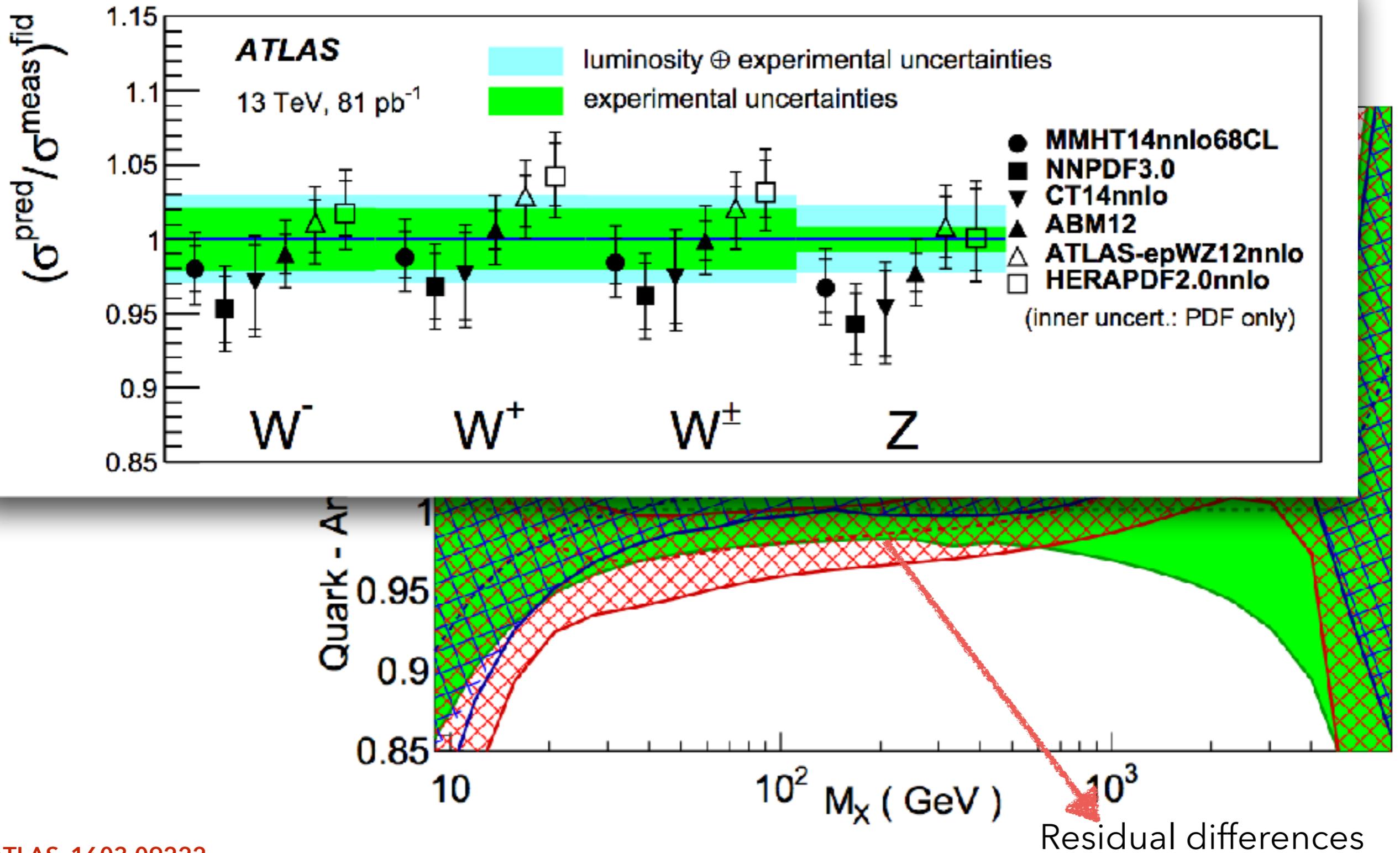
LHC 13 TeV, NNLO,  $\alpha_s(M_Z)=0.118$



(2016)

Residual differences

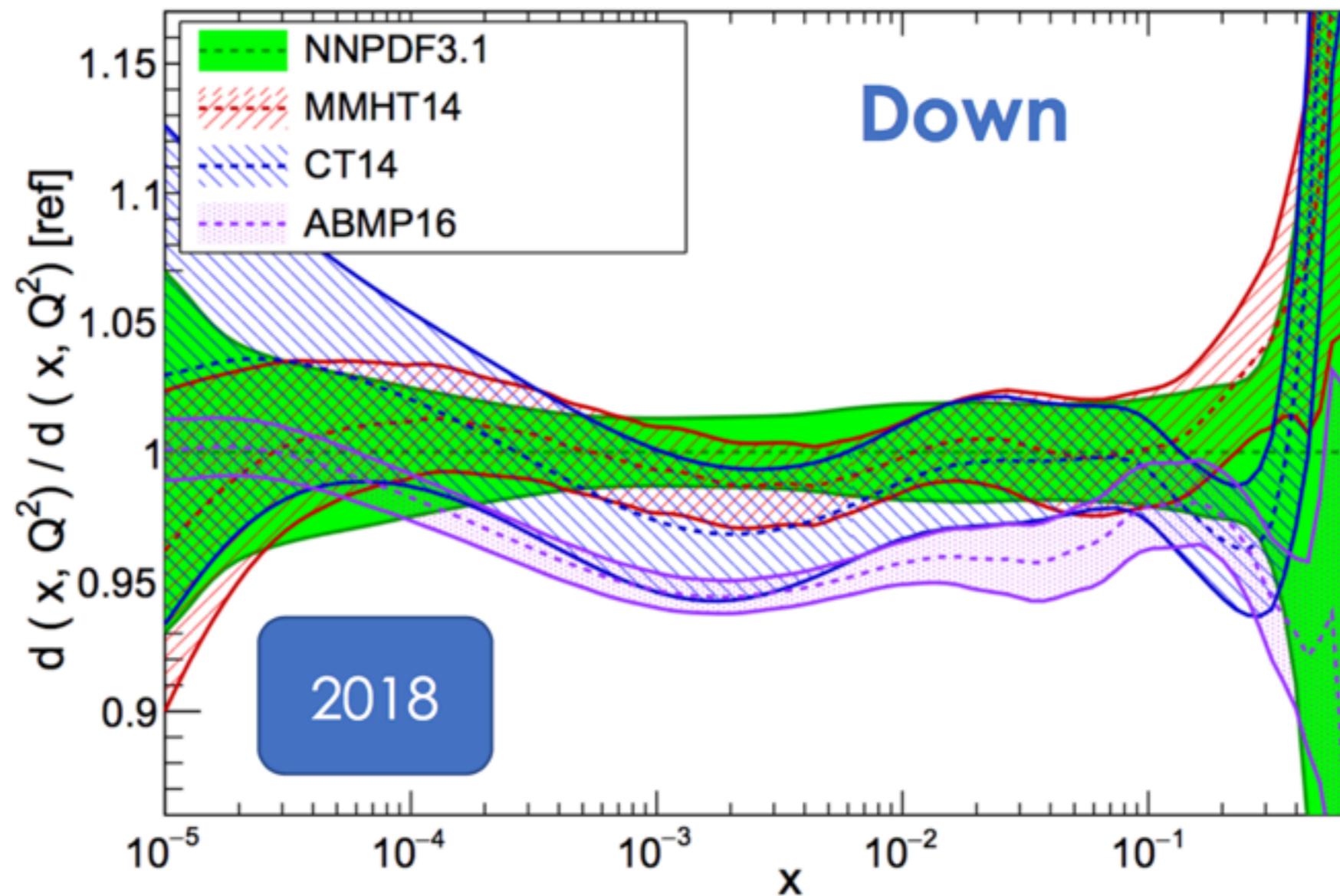
# Quark-Antiquark luminosity



# Quark-Antiquark luminosity

NNPDF3.1 / CT14 / MMHT/ABMP16

NNLO,  $\alpha_s=0.118$ ,  $Q = 100$  GeV



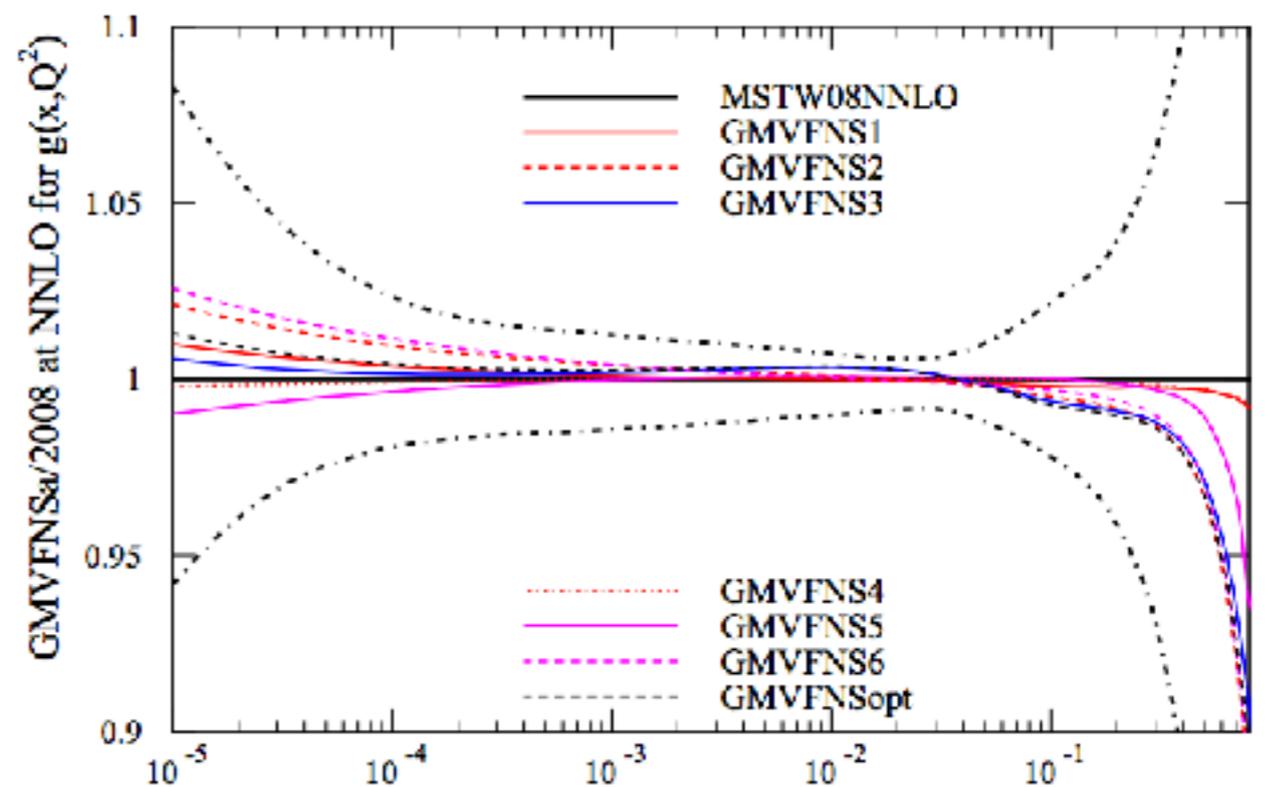
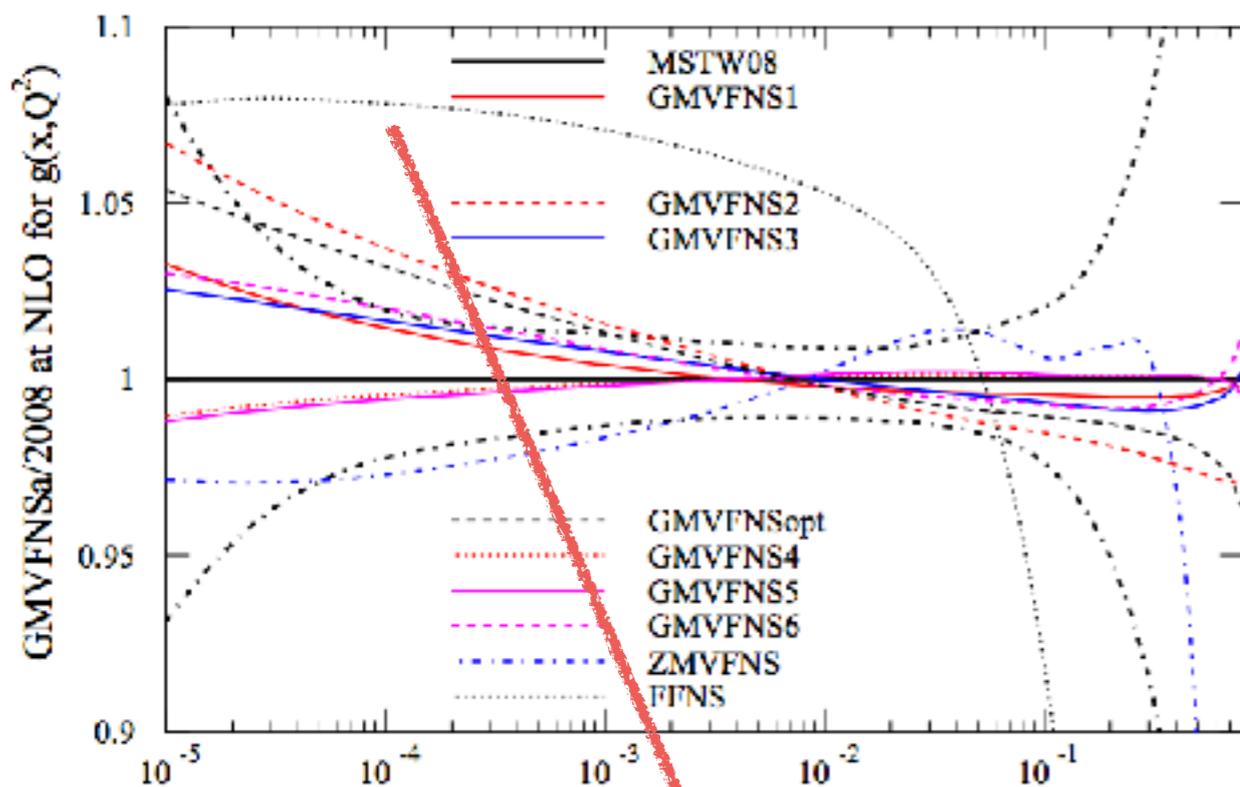
# Data convergence

- Increasingly wide dataset used in PDF analyses: from DIS structure functions only to global analyses including jets, top, W/Z, HQ observables
- HERA PDFs based on maximally consistent set of data, others have to deal with inconsistencies

SET MONTH	2008		2009		2010		2011	2012		2013		2014		2015
	CT6.6 (02)	NN1.0 (08)	MSTW (01)	ABKM09 (08)	NN2.0 (02)	CT10(N) (07)	NN2.1(NN) (07)	ABM11 (02)	NN2.3 (07)	CT10(NN) (02)	ABM12 (10)	NN3.0 (10)	MMHT (12)	CT14 (06)
F. T. DIS	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
ZEUS+H1-HI	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
COMB. HI	✗	✗	✗	✗	✓	✗	✓	✗	✓	✗	✓	✓	✗	✗
ZEUS+H1-HII	✗	✗	✗	✗	✗	✗	some	✗	✗	some	✗	✓	✗	✗
HERA JETS	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗
F. T. DY	✓	✗	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
TEV. W+Z	✓	✗	✓	✗	✓	✓	✓	✗	✓	✓		✓	✓	✓
TEV. JETS	✓	✗	✓	✗	✓	✓	✗	✓	✓	✓	✗	✓	✓	✓
LHC W+Z	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗	some	✓	✓	✓
LHC JETS	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗	✗	✓	✓	✓
TOP	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✓	✗	✗
W+c	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗	✗
W p <sub>T</sub>	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗	✗

# Theory convergence

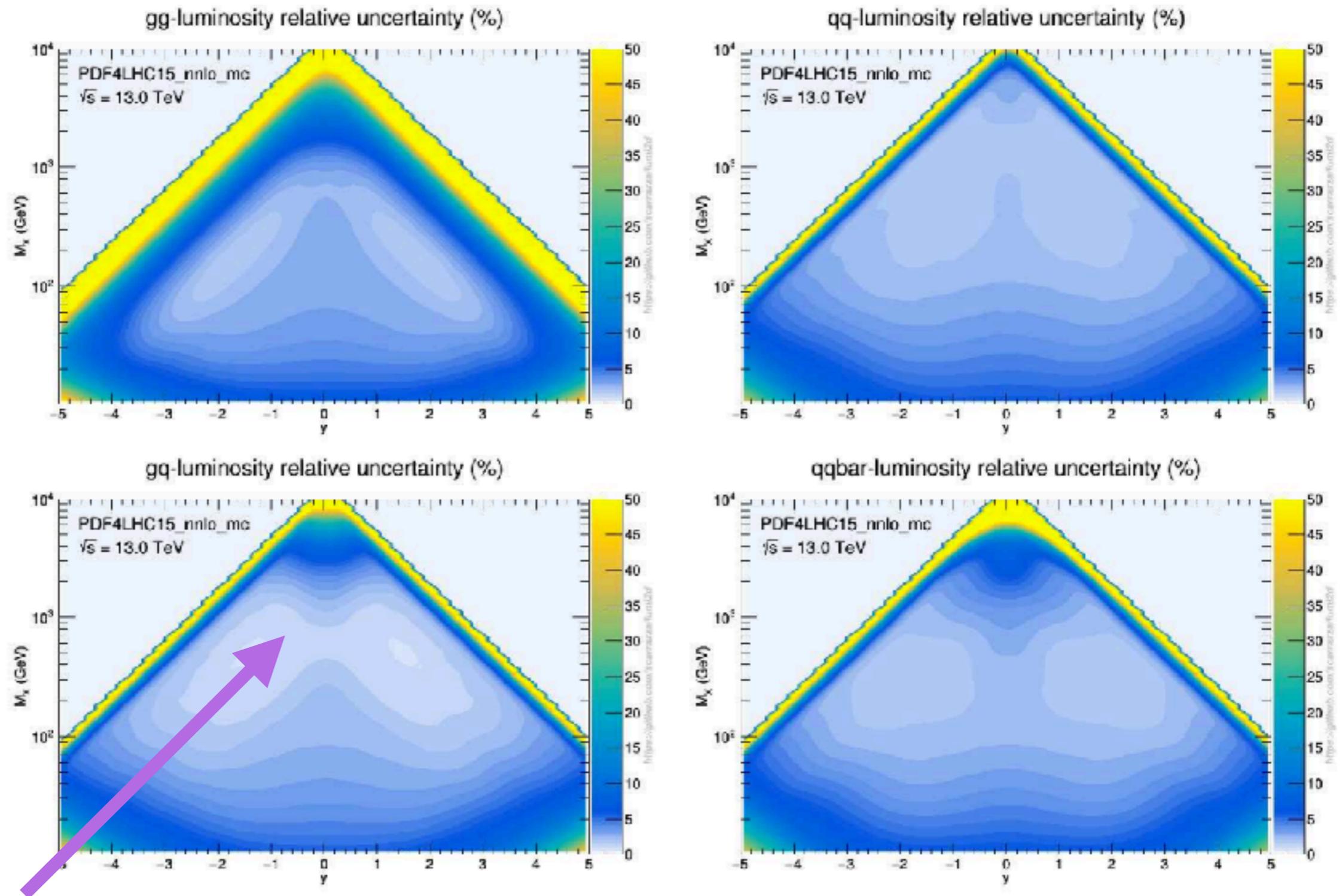
- Comparable GM-VFN schemes for inclusion of HQ masses (sub-leading differences less important at NNLO)
- Common  $\alpha_s(M_Z) = 0.118$  (external parameter)
- NNLO (although with some caveat - especially concerning jets data)
- Extensive benchmarking



Compensate by lower  $\alpha_s(M_Z)$  in structure function scaling

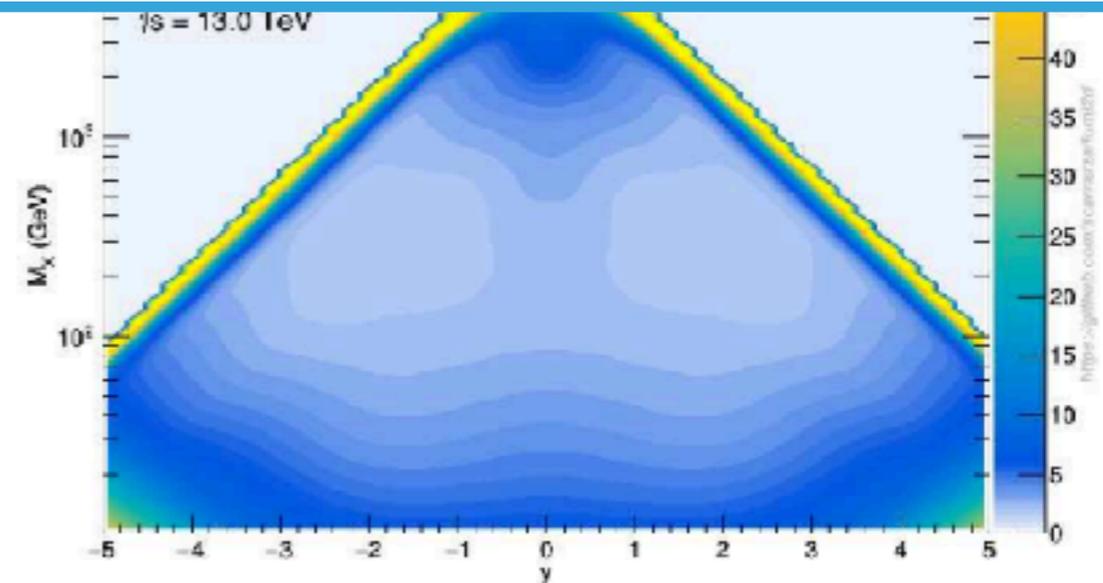
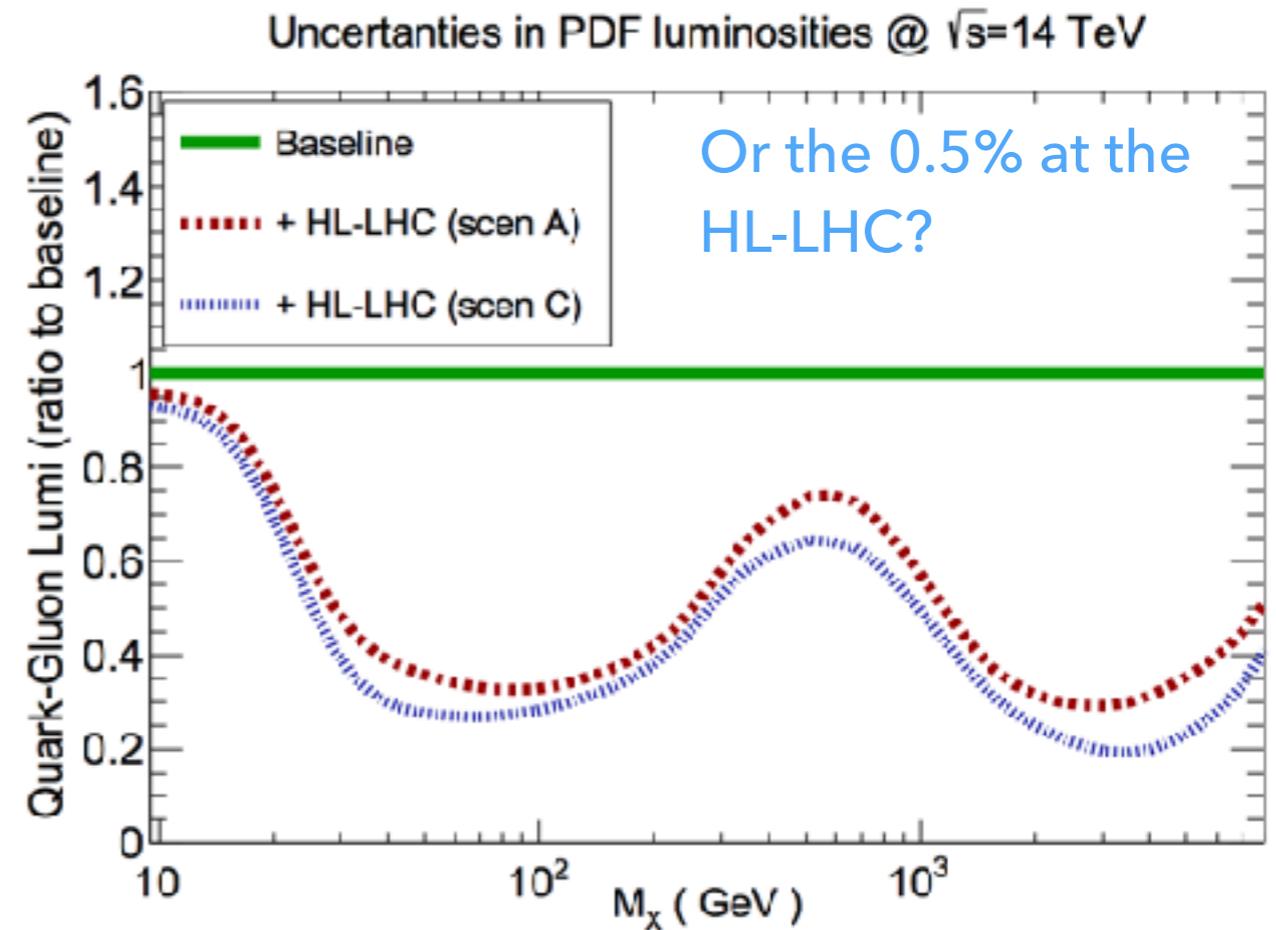
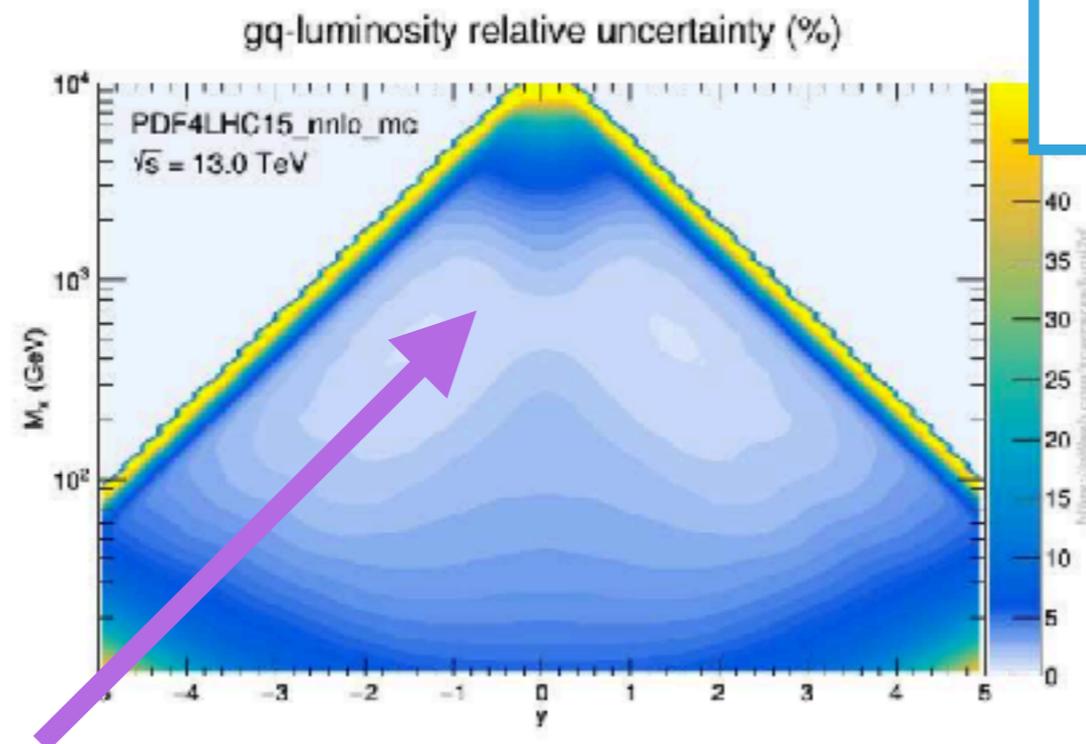
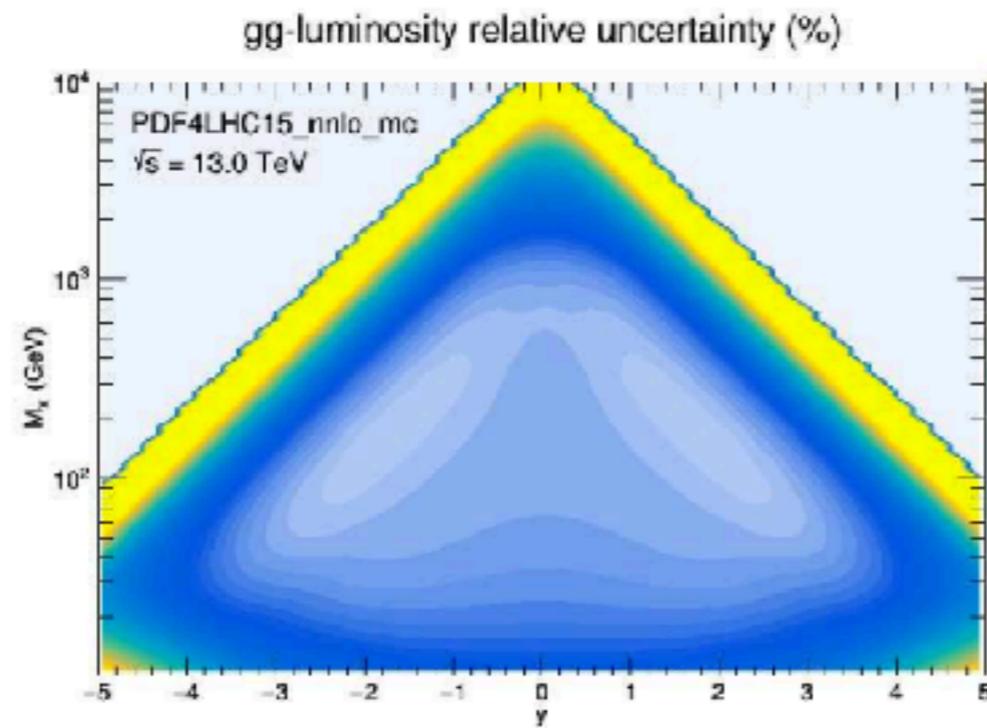
Frontiers #1: missing higher  
order uncertainties

# The precision frontier



Can we trust 1% accuracy?

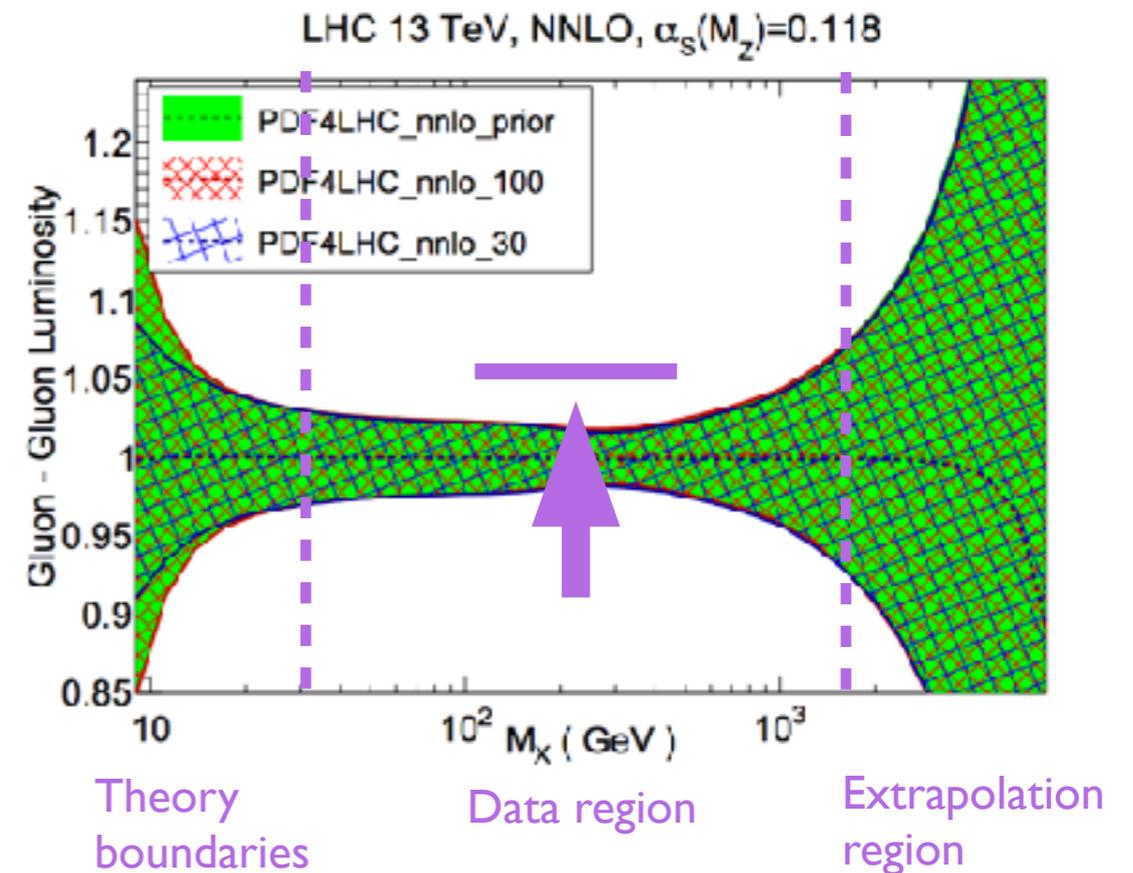
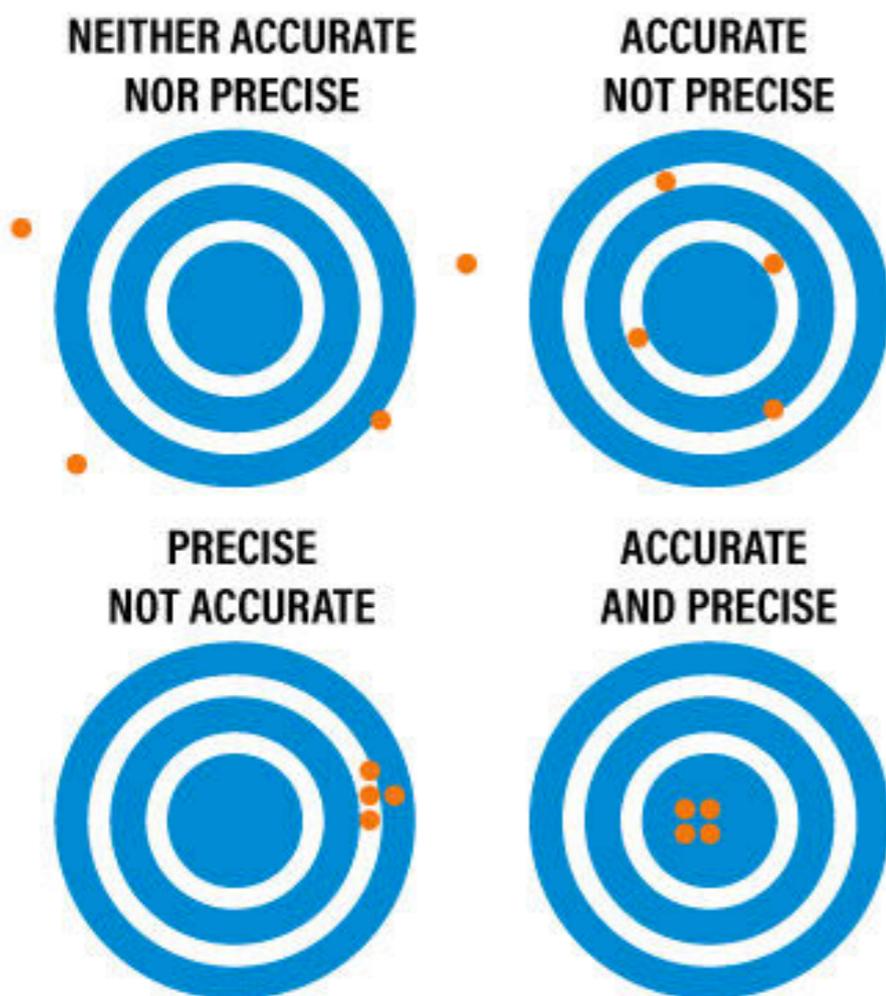
# The precision frontier



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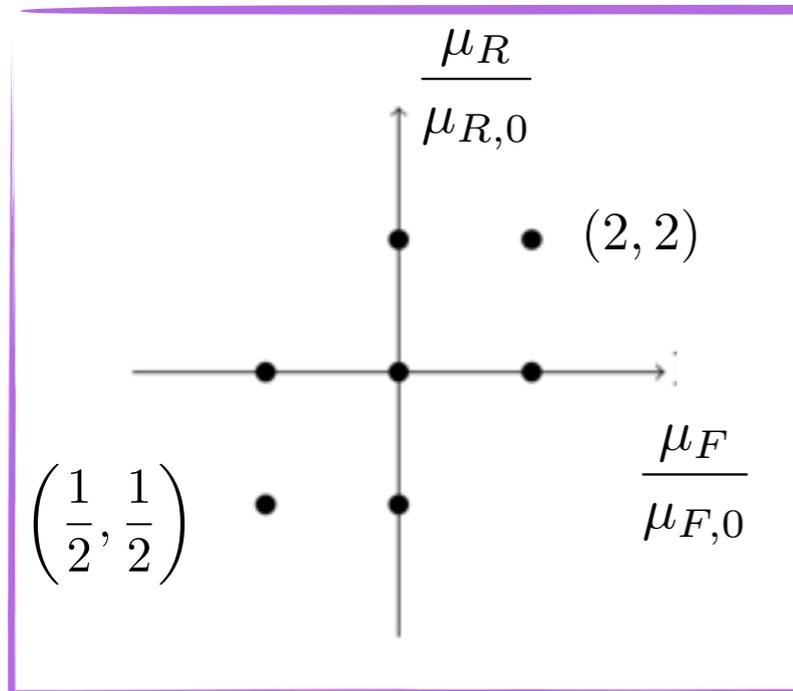
# Theory uncertainties

In updated PDF analysis, shift between old and new set may be larger than PDF uncertainties

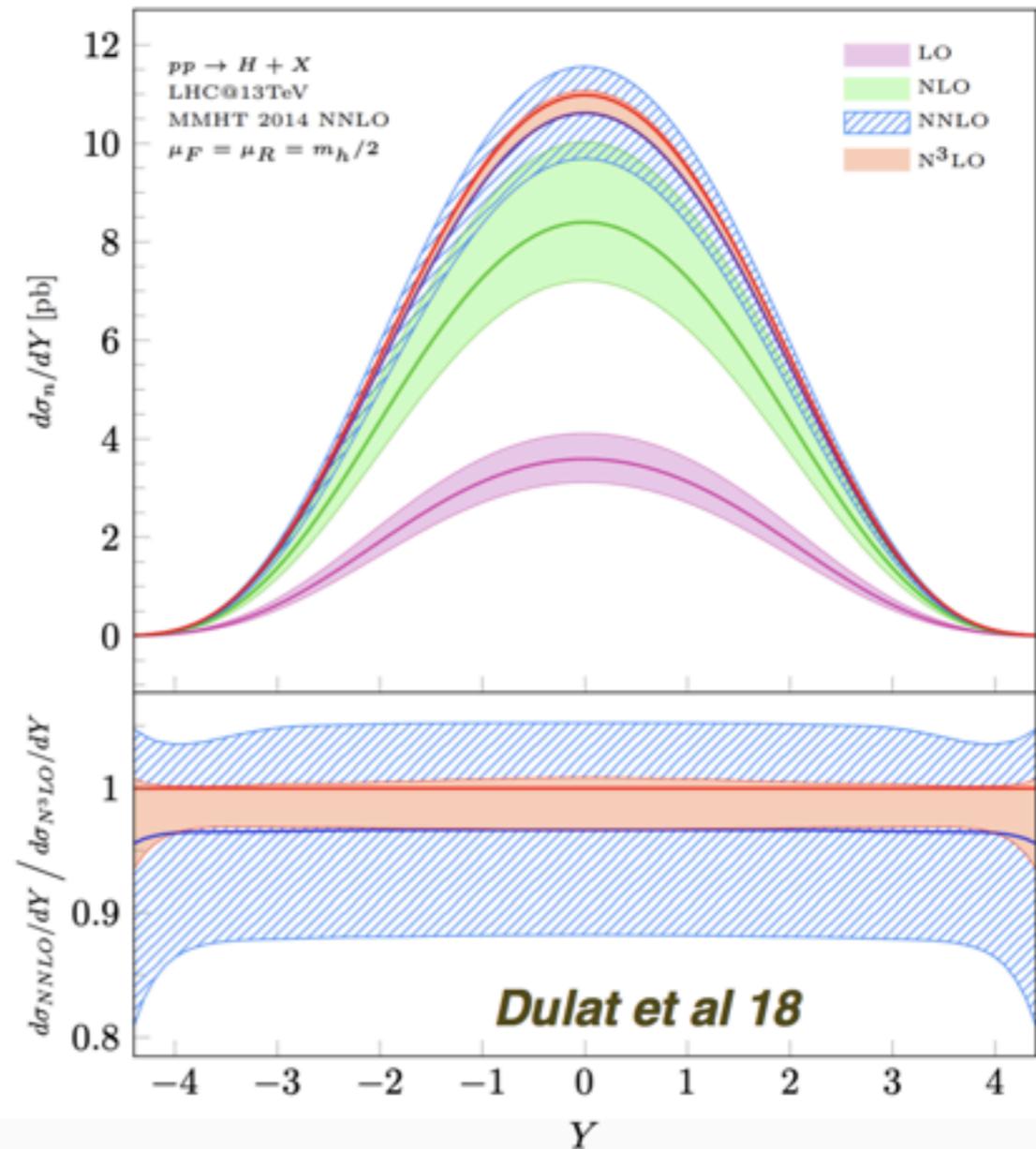


- Inconsistent data → Tolerance/Statistical estimators
- Updated parametrization
- Differences in fitting methodology/minimisation? → Closure Test
- **Changes in theory?**

# MHOU in theoretical predictions

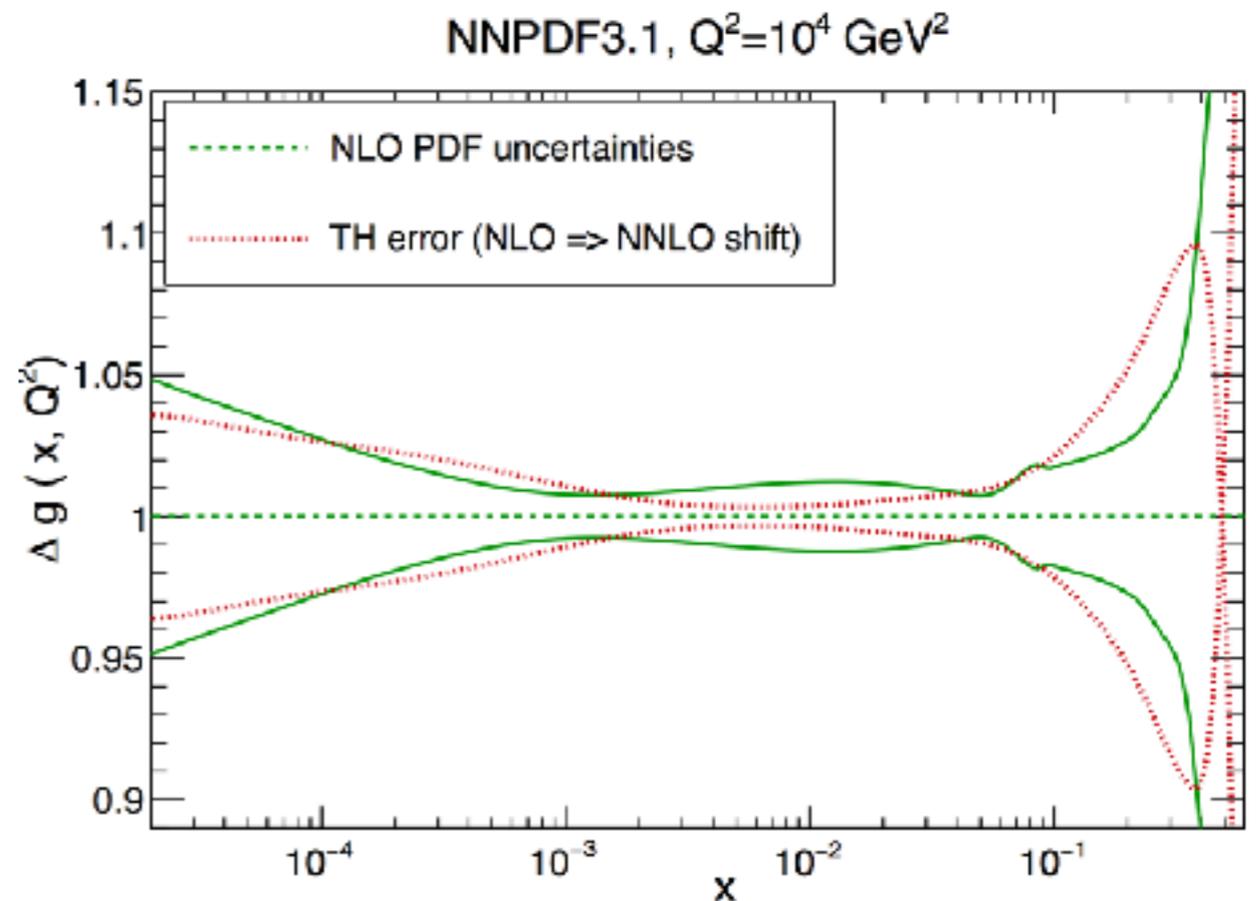
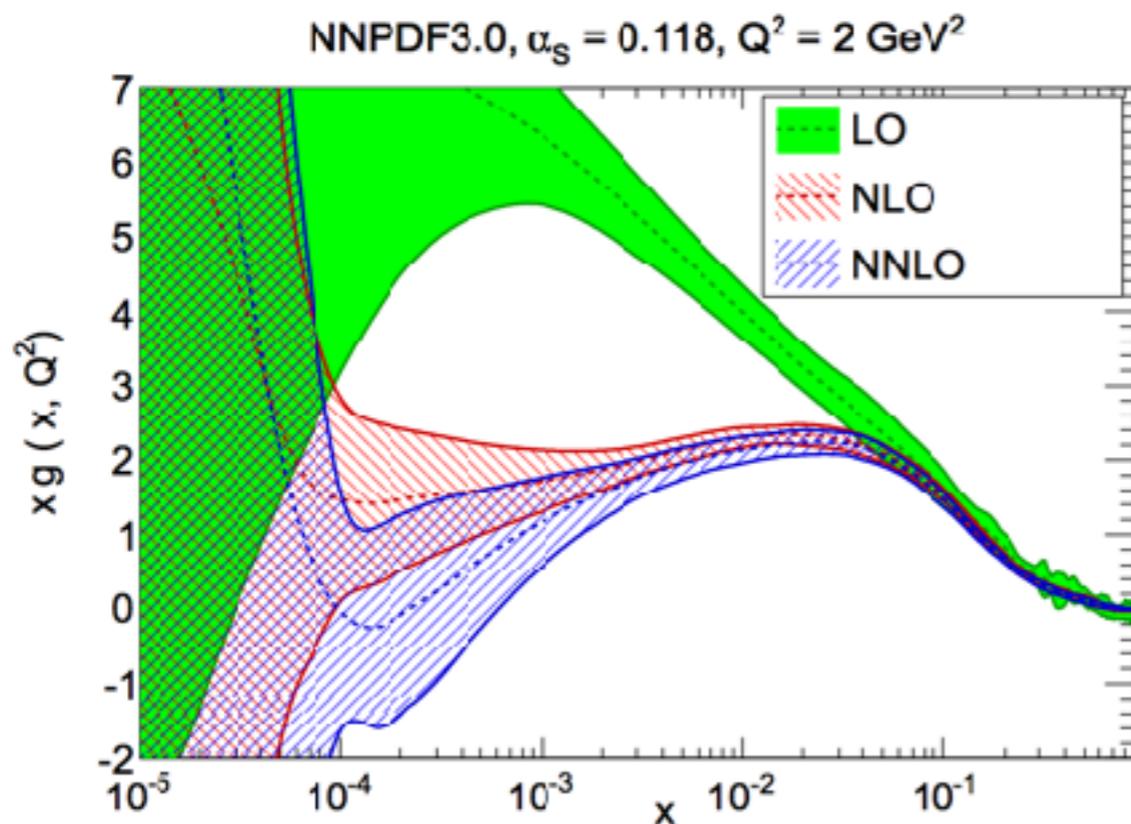


Increasing order in perturbation theory reduced “scale” uncertainty (or MHOU) in theoretical predictions



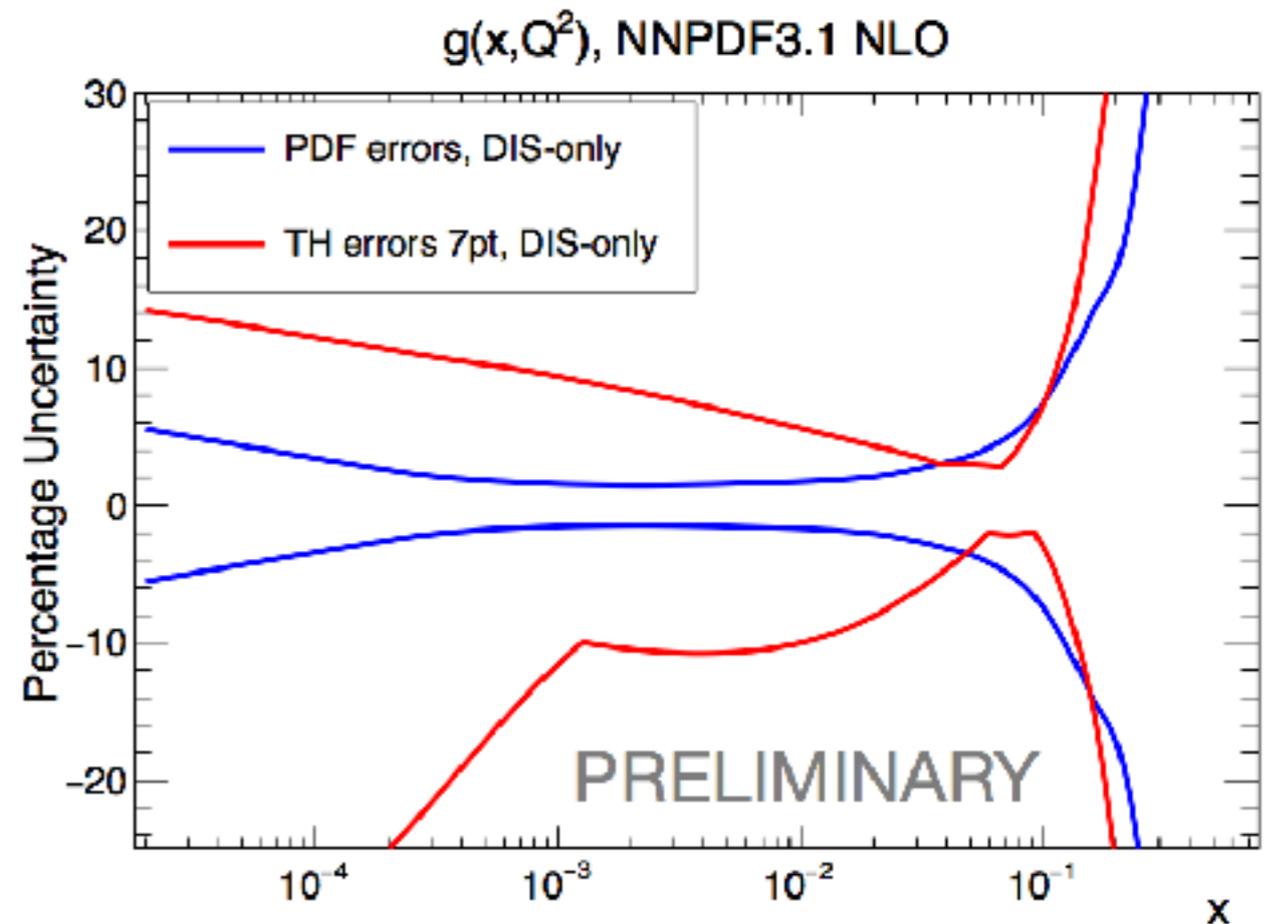
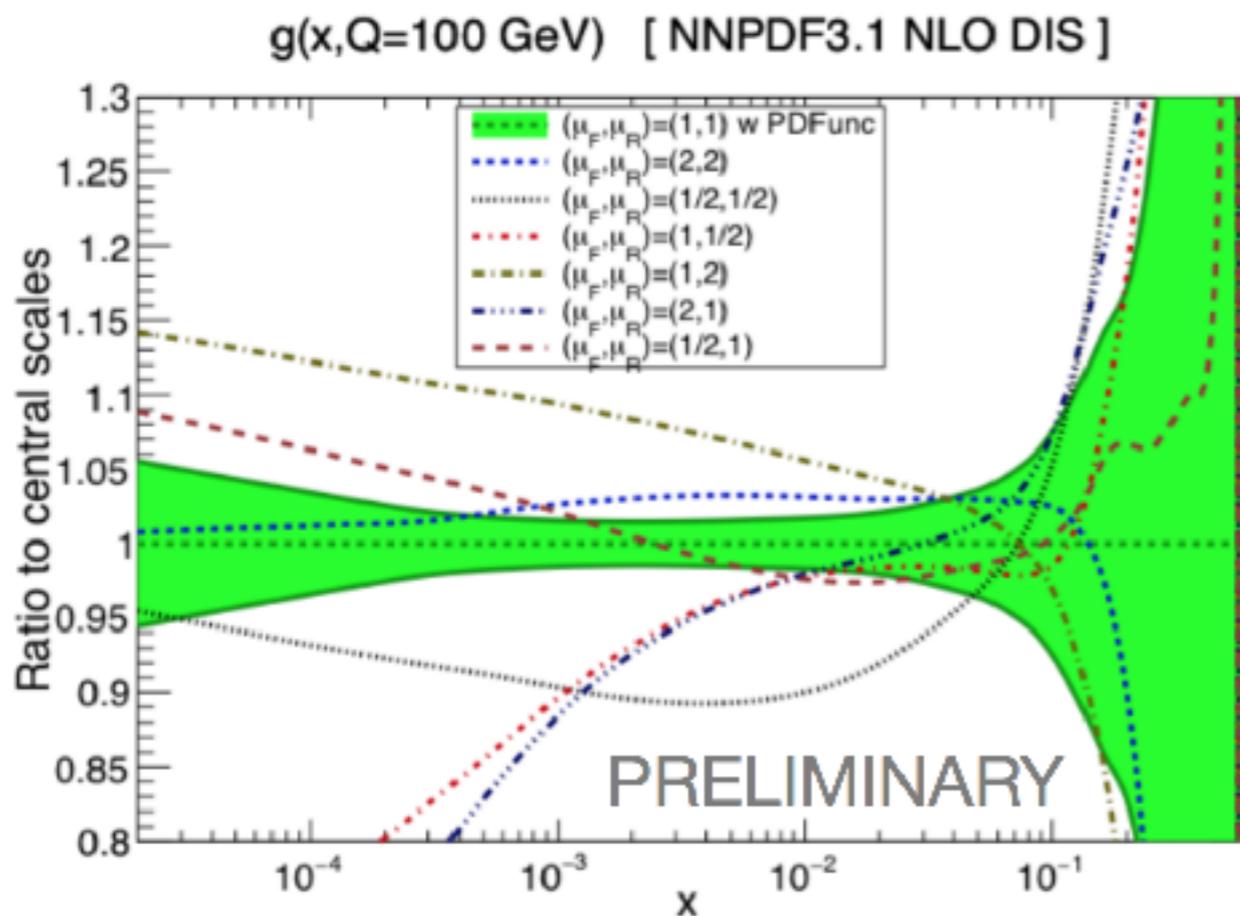
# MHOU in PDF fits

- PDF fits performed at given perturbative order
- PDF uncertainties only reflect lack of information from data
- Theoretical uncertainties (dominated by MHOU) ignored so far
- At NLO PDF uncertainties and MHOU comparable
- Near future: NNLO PDF uncertainties will go down to level of MHOU
- Inclusion of theory uncertainties is the next frontier



# MHOU in PDF fits

- How to estimate MHOU in PDF fits?
- Compare fits with varied scales
- Useful to have indication on the size of MHOU in PDFs
- A posteriori combination?
- How to include them in the fitting methodology along with other sources of theoretical uncertainty?



# Covariance matrix

- Theory is perturbative expansion to some order :  $t_p = \sum_{m=0}^p c_m$

- Standard case:  $P(d|t_p) \propto \exp\left(-\frac{1}{2}(d - t_p)^T \text{cov}_{\text{exp}}^{-1}(d - t_p)\right)$

$\chi_{\text{exp}}^2$

- Bayes' theorem:  $P(t_p|d) = \frac{P(d|t_p)P(t_p)}{P(d)} \propto P(d|t_p)P(t_p)$

- Assume Gaussian theory prior:

$$P(t_p) = \prod_{m=0}^p P(c_m) \quad \text{where} \quad P(c_m) \propto \exp\left(-\frac{1}{2}c_m^T \text{cov}_{\text{th},m}^{-1}c_m\right)$$

$\chi_{\text{th}}^2$

- Assume MHOUs due to  $\mathcal{O}(\alpha^{p+1})$  terms only  $\rightarrow$  marginalise these terms:

$$P(t_p|d) \propto \int dc_{p+1} P(d|c_{p+1}) P(t_{p+1}) \\ \propto \exp\left(-\frac{1}{2}(d - t_p)^T (\text{cov}_{\text{exp}} + \text{cov}_{\text{th}})^{-1}(d - t_p)\right)$$

$\chi_{\text{tot}}^2$

- Include higher order terms by induction

# Covariance matrix

$$\chi^2 = \sum_{m,n=1}^N (d_m - t_m) (\text{COV}_{\text{exp}} + \text{COV}_{\text{th}})^{-1}_{mn} (d_n - t_n)$$

→ How to build correlations between different points?

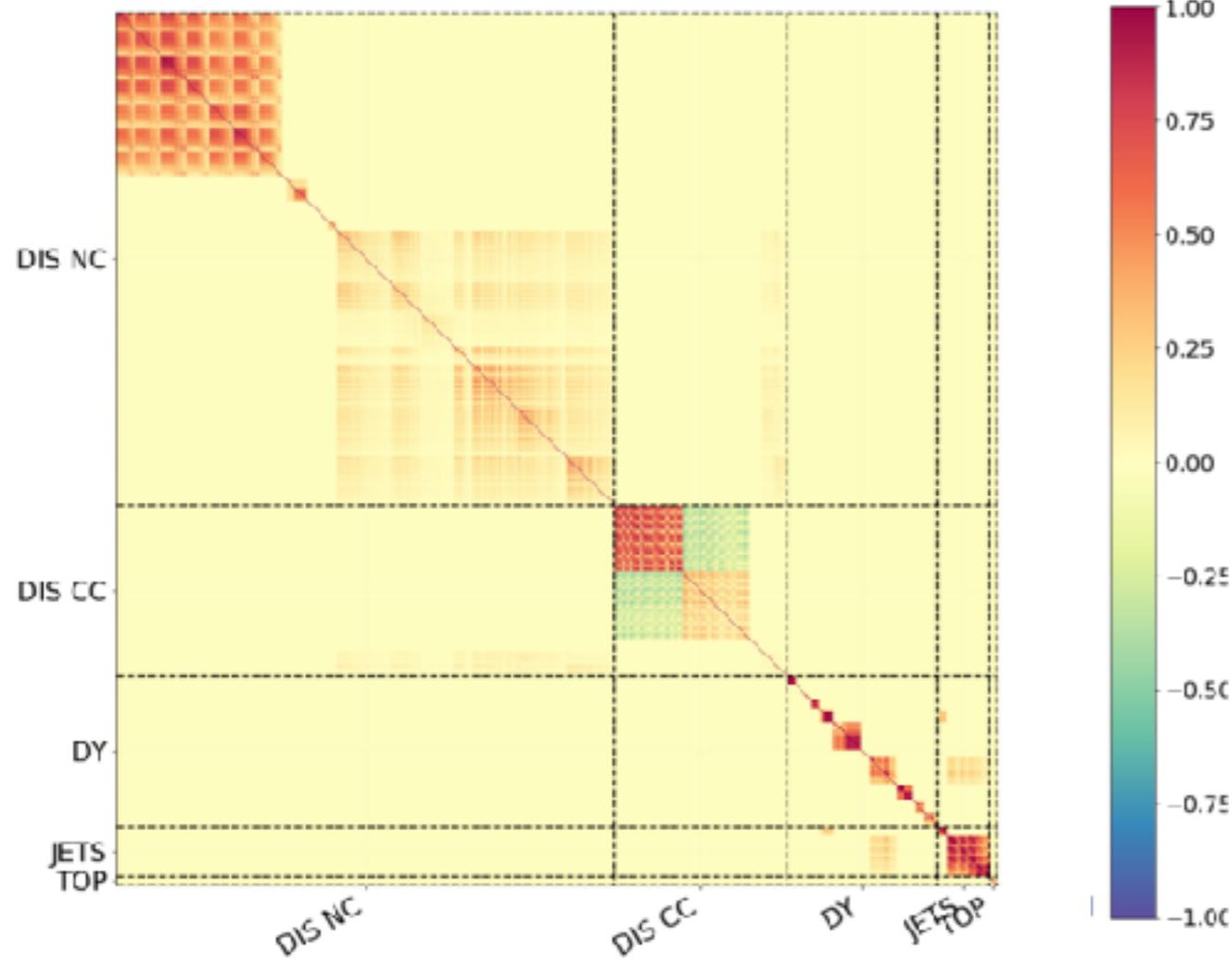
$$(\text{COV}_{\text{th}})_{mn} = \langle (t_p(\mu_R, \mu_F) - t_p(\mu_R^0, \mu_F^0))_m (t_p(\mu_R, \mu_F) - t_p(\mu_R^0, \mu_F^0))_n \rangle$$

- ▶  $\mu_F$  variations correlated across all processes by PDF evolution
- ▶  $\mu_R$  variation correlated by process (hard cross section)

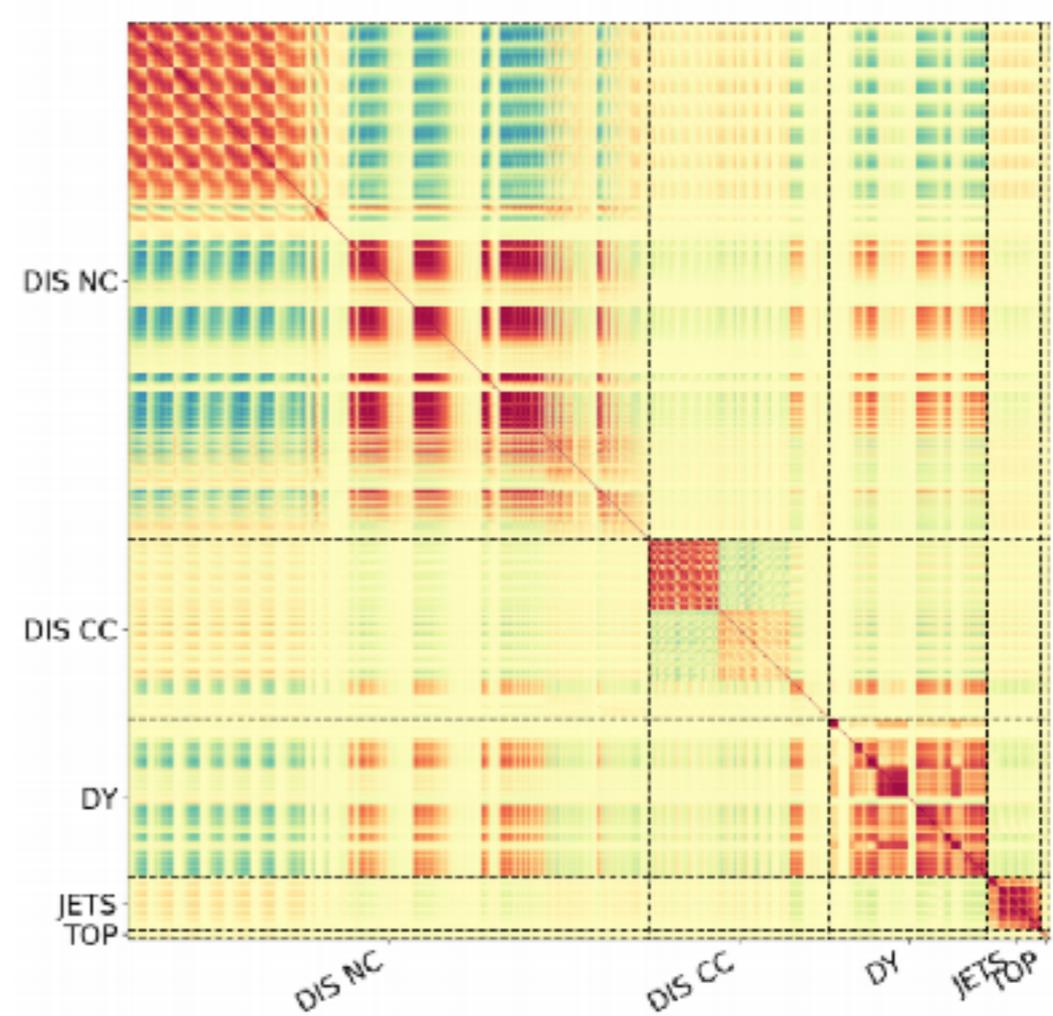
- Several recipes possible (3-points prescriptions, 7-points...)
- Details of correlations are also important
- A lot to be investigated

# Covariance matrix

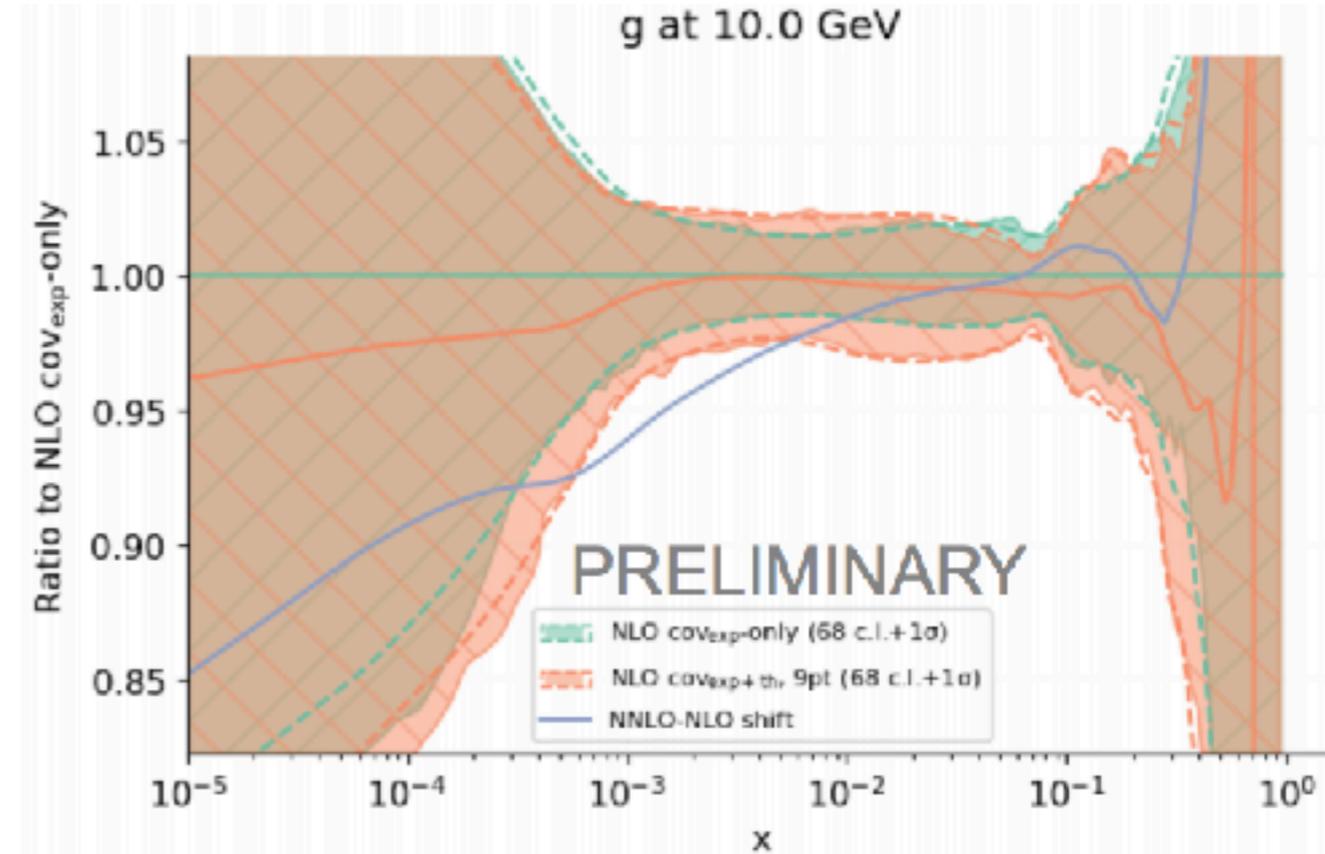
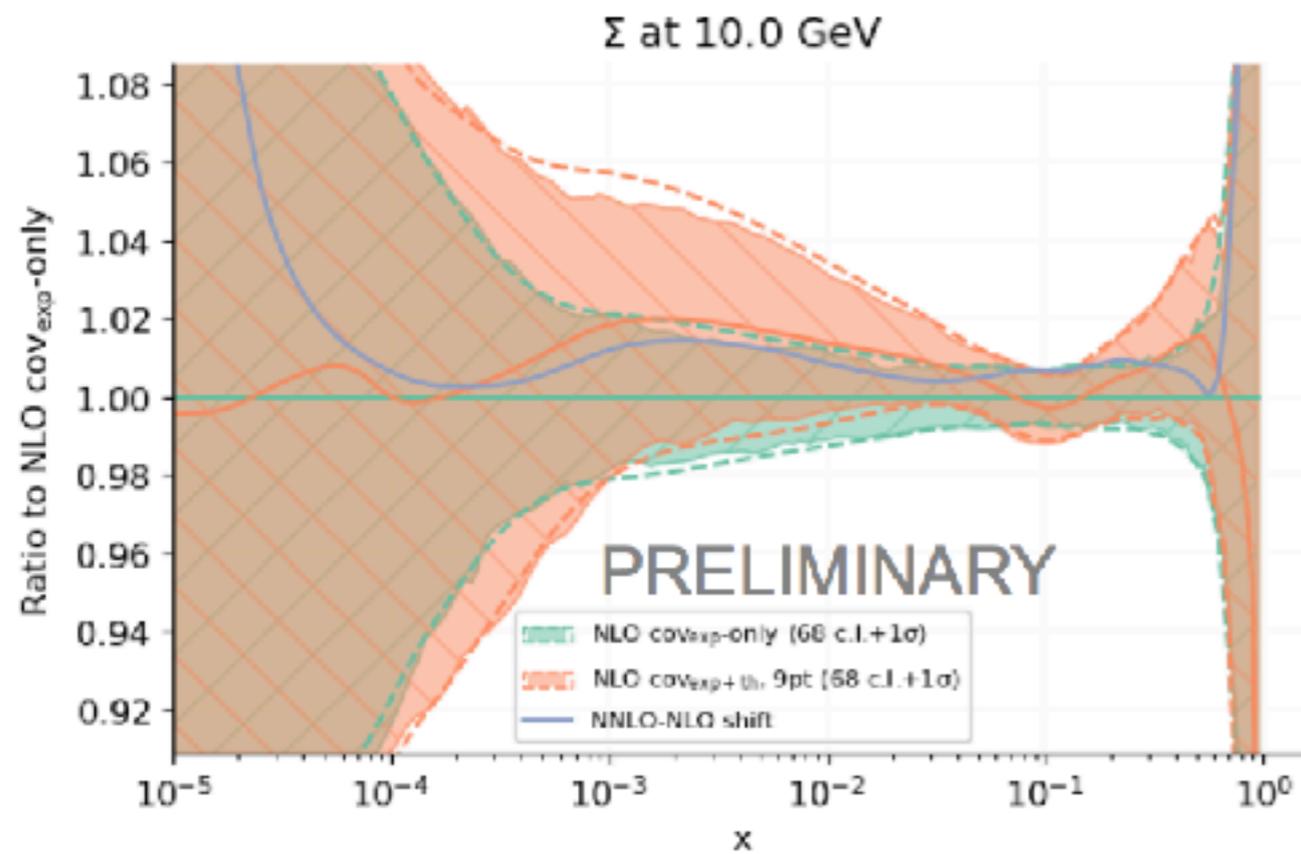
Experiment correlation matrix



Experiment + theory correlation matrix for 9 points



# More reliable uncertainties?



# Frontiers #2: beyond fixed order

# Beyond fixed order

- Multi-scale processes:  $\log(Q_i/Q_j) = L$  arise, which may spoil perturbative expansion
- If  $(\alpha_s * L) \sim O(1)$  fixed order perturbative QCD is no longer justified
- Resummation effectively rearranges perturbative series

fixed order

$$\begin{aligned} \frac{\sigma}{\sigma_0} &= 1 && \text{LO} \\ &+ c_1 \alpha && \text{NLO} \\ &+ c_2 \alpha^2 && \text{NNLO} \\ &+ \dots \end{aligned}$$

all order (L = some large logarithm)

$$\begin{aligned} \ln \frac{\sigma}{\sigma_0} &= \alpha^n L^{n+1} && \text{LL} \\ &+ \alpha^n L^n && \text{NLL} \\ &+ \alpha^n L^{n-1} && \text{NNLL} \\ &+ \dots \end{aligned}$$

- Various kinds of logs:

$L = \log(1-x)$  threshold (soft-gluon) resummation ← **Ball et al, JHEP09(2015)091**  
 $L = \log(1/x)$  high-energy (small-x) resummation ← **BFKL**  
 $L = \log(p_T/M)$  transverse momentum resummation

# Threshold resummation

- Threshold resummation: initial energy just enough to produce final state with mass  $M$ , so emissions forced to be soft and logs at each order in PT are enhanced

$$x = \frac{M^2}{\hat{s}} \quad \text{NLO : } M^2 = z\hat{s} \quad \left[ \frac{\log^k(1-z)}{(1-z)} \right]_+$$

- Transform factorised cross section into Mellin space

$$\sigma(x, Q^2) = x \sum_{a,b} \int_x^1 \frac{dz}{z} \mathcal{L}_{ab} \left( \frac{x}{z}, \mu_F^2 \right) \frac{1}{z} \hat{\sigma}_{ab} \left( z, Q^2, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2} \right)$$

$$\sigma(N, Q^2) = \int_0^1 dx x^{N-2} \sigma(x, Q^2) = \sum_{a,b} \mathcal{L}_{ab}(N, Q^2) \hat{\sigma}_{ab}(N, Q^2, \alpha_s)$$

- In the MSbar scheme PDF evolution does not contain large-x logs and the effect of resummation can be included in resummed coefficient functions

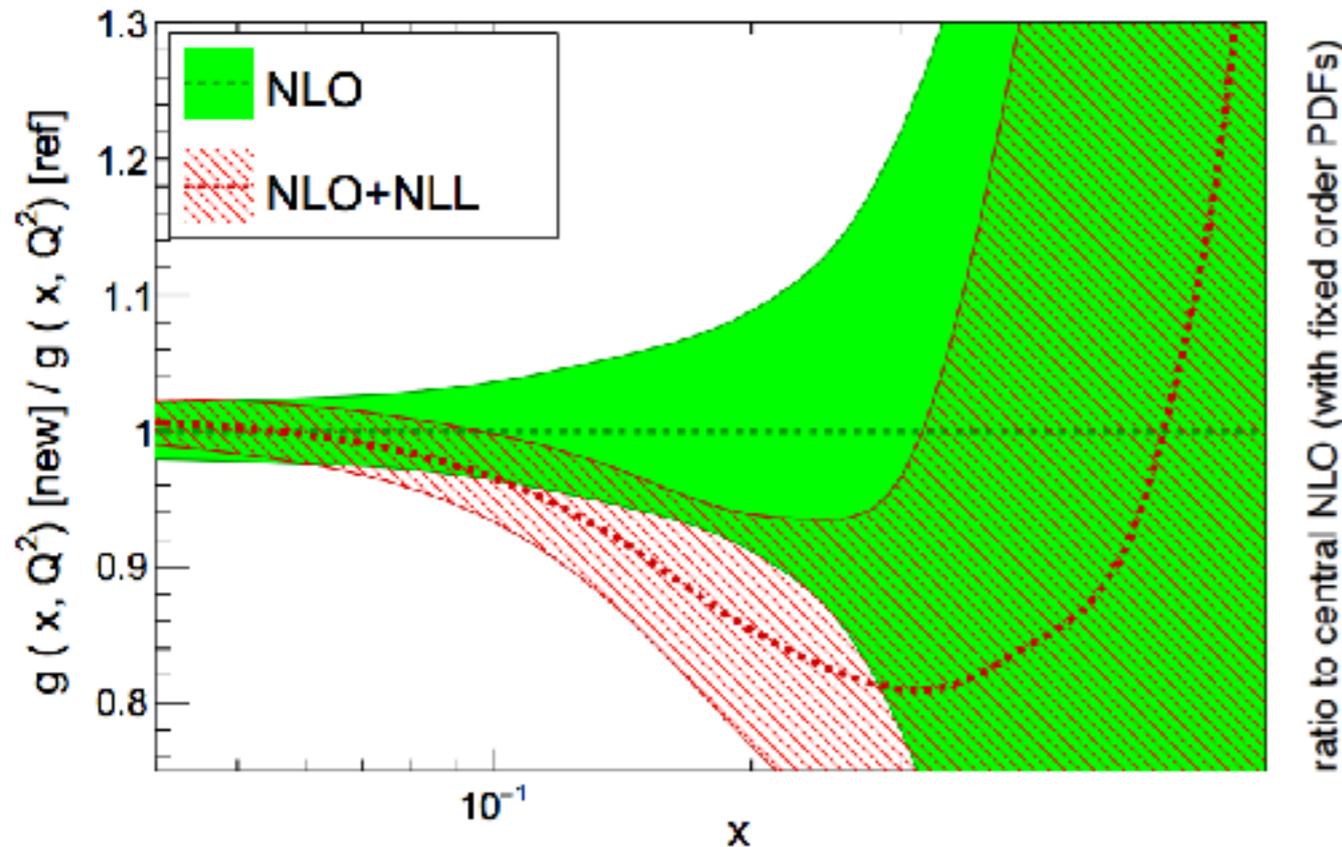
$$\hat{\sigma}_{ab}^{(\text{res})}(N, Q^2; \alpha_s) = \sigma_{ab}^{(\text{born})}(N, Q^2; \alpha_s) C_{ab}^{(\text{res})}(N, \alpha_s)$$

$$C^{(N\text{-soft})}(N, \alpha_s) = g_0(\alpha_s) \exp \mathcal{S}(\ln N, \alpha_s),$$

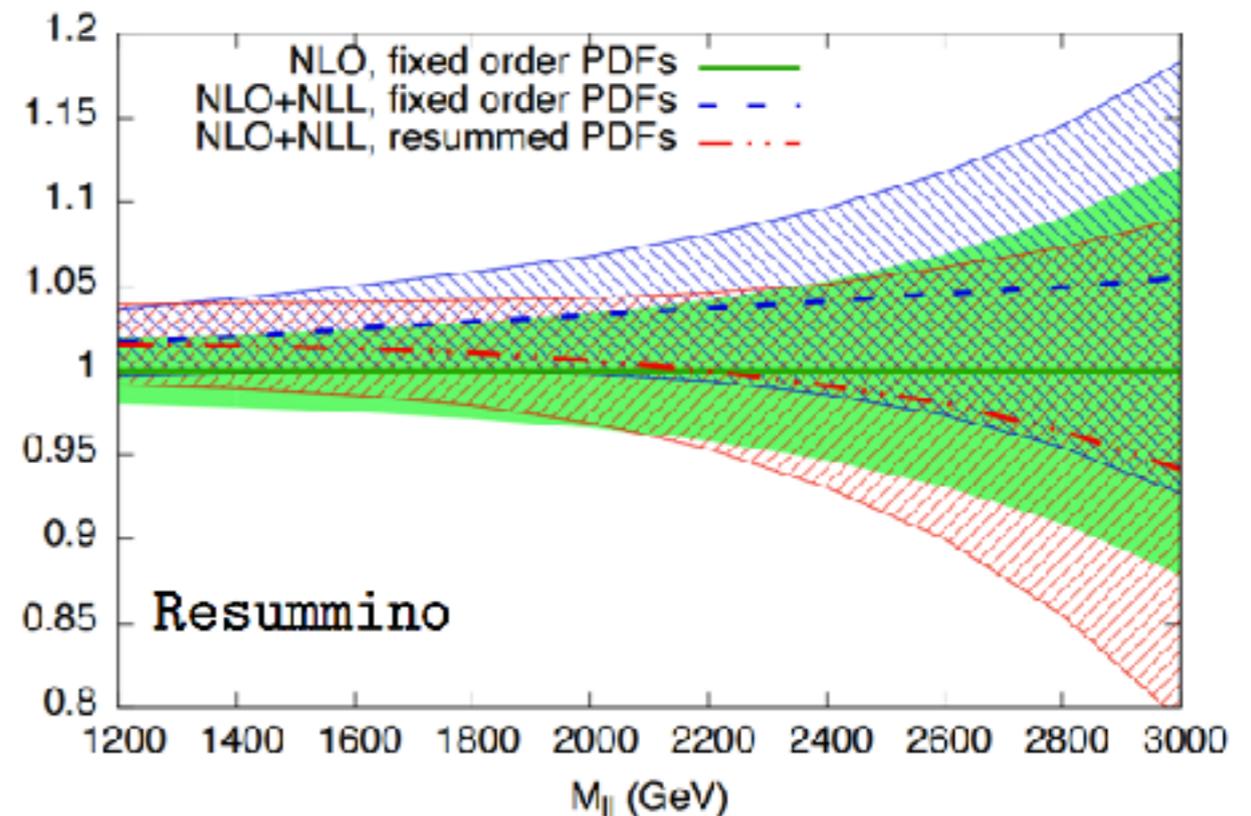
$$\mathcal{S}(\ln N, \alpha_s) = \left[ \frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \dots \right]$$

# Threshold resummation

NNPDF3.0 DIS+DY+Top,  $Q^2=10^4 \text{ GeV}^2$



Slepton pair invariant mass, pp @ 13 TeV,  $m_l = 564 \text{ GeV}$ .



Bonvini et al, JHEP 1509 (2015) 191

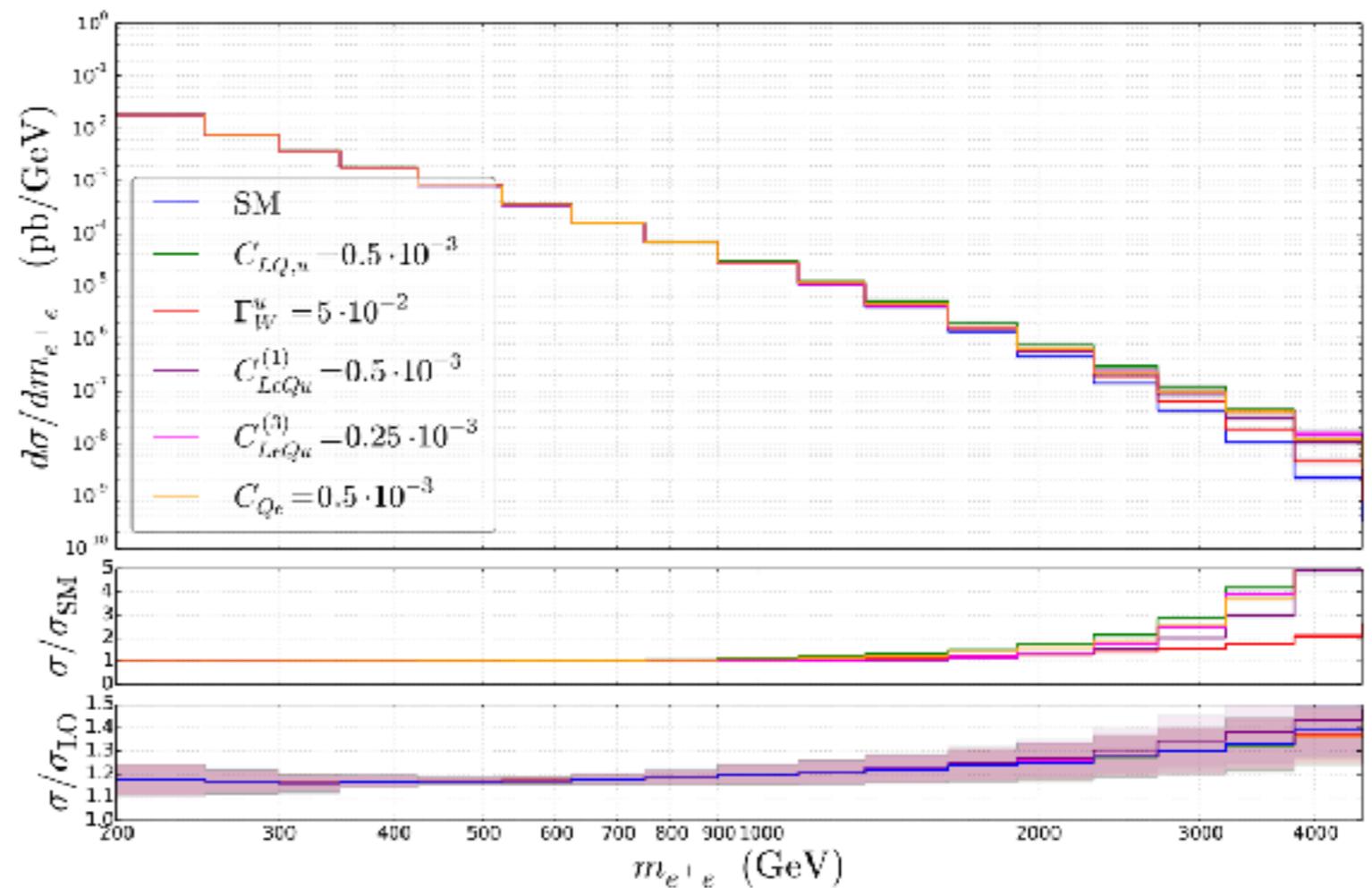
- Threshold-resummed PDFs will be suppressed as compared to fixed-order PDFs
- Mostly due to enhancement of NLO+NLL xsecs used in the fit of DIS structure functions and DY distributions
- This suppression partially or totally compensates enhancements in partonic cross sections
- Phenomenologically relevant for new physics processes [Beenakker et al. EPJC76 (2016)2, 53]

# Frontiers #3: PDFs and new physics

# New Physics and PDFs

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \dots, \quad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4$$

- Many studies analyse effect of higher-dimensional operators on observables measured at the LHC
- Extract constraints on dim-6 operators that contribute to NC and CC Drell-Yan production at the LHC, H+V production and VFB

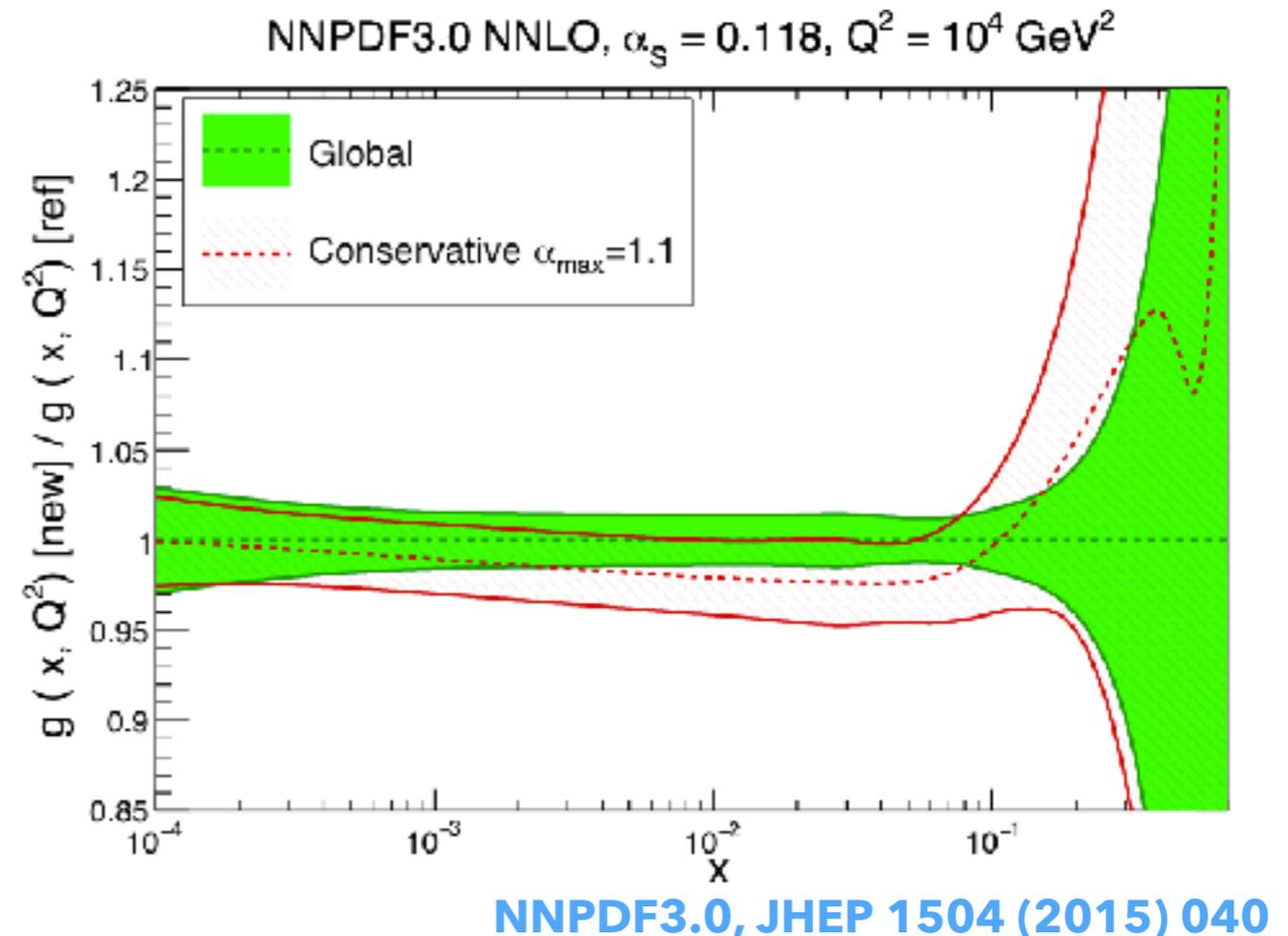


Alioli et al, 1804.07407

**PDFs use some of these data and are determined within SM Framework**

# PDFs and New Physics

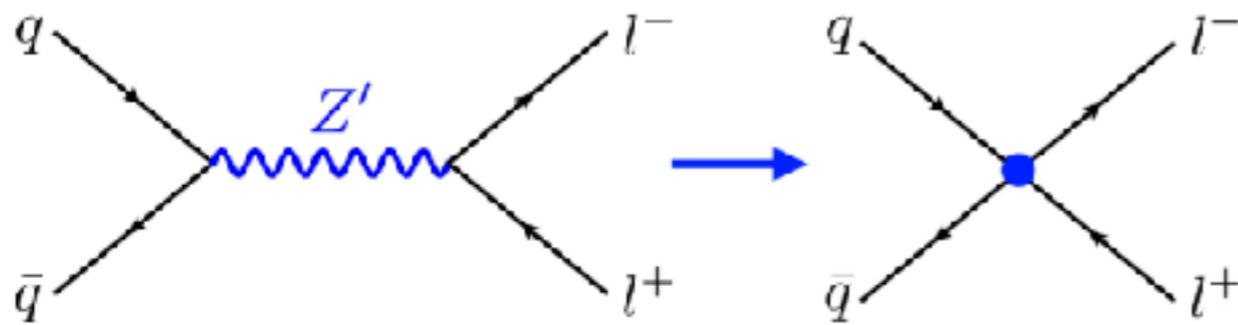
- As more data at higher energy will be released, how can we make sure that new physics effects are not absorbed in the PDFs?
- If effects were big we would have bad and signs of inconsistency but probably would show up as mild inconsistencies
- Inconsistency of any individual dataset with the bulk of global fit may suggest its understanding is incomplete but might be due to many factors



**Are conservative partons the answer?**  
**- Not really: simultaneous fits of EFT coefficients and PDFs is the new frontier**

# A proof of concept

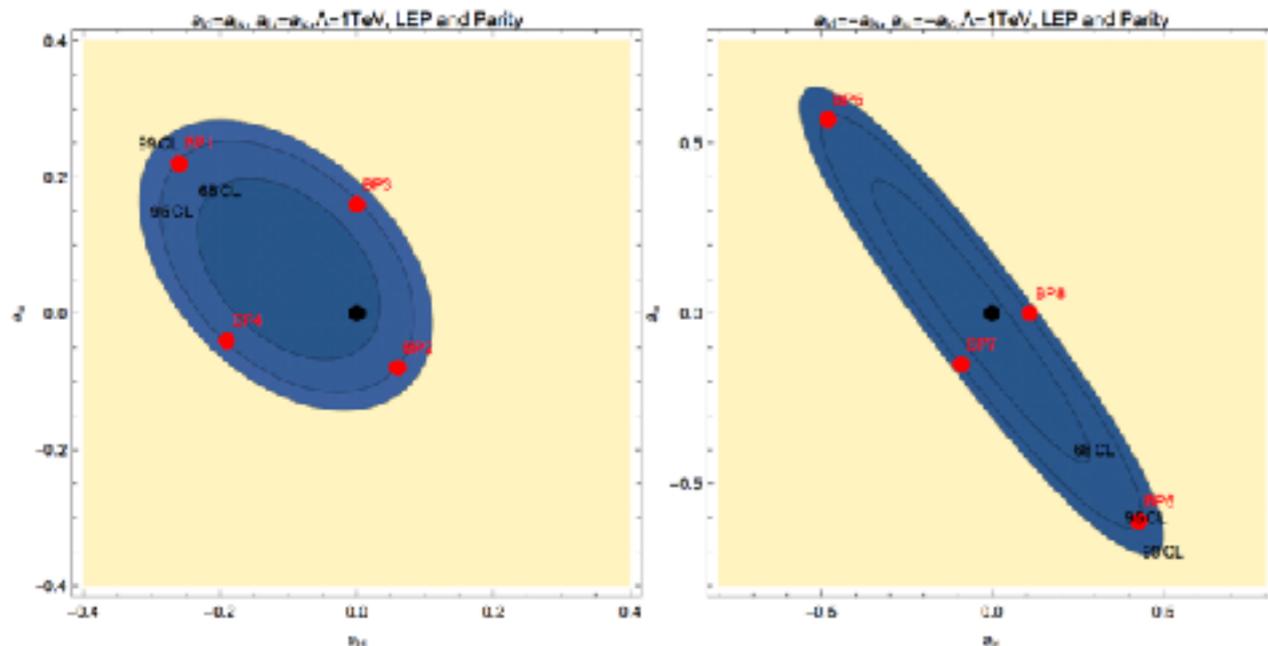
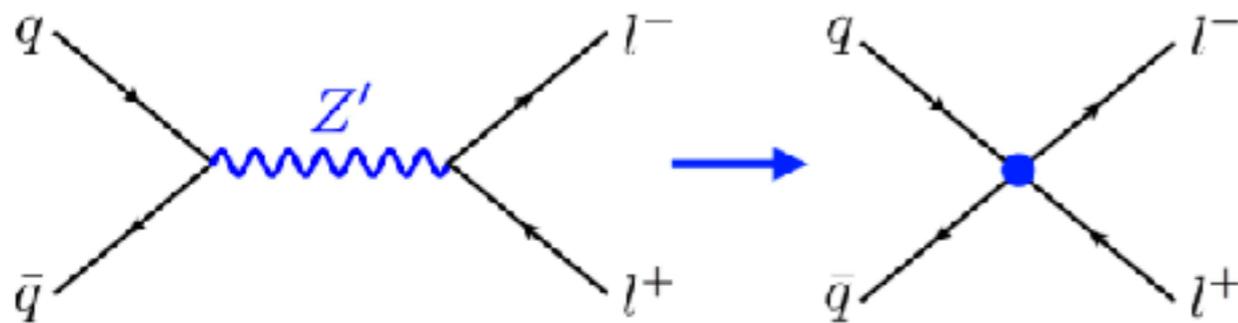
$$\mathcal{O}_{lu} = (\bar{l}_R \gamma^\mu l_R) (\bar{u}_R \gamma_\mu u_R) \quad , \quad \mathcal{O}_{ld} = (\bar{l}_R \gamma^\mu l_R) (\bar{d}_R \gamma_\mu d_R)$$
$$\mathcal{O}_{lc} = (\bar{l}_R \gamma^\mu l_R) (\bar{c}_R \gamma_\mu c_R) \quad , \quad \mathcal{O}_{ls} = (\bar{l}_R \gamma^\mu l_R) (\bar{s}_R \gamma_\mu s_R)$$



# A proof of concept

$$\mathcal{O}_{lu} = (\bar{l}_R \gamma^\mu l_R) (\bar{u}_R \gamma_\mu u_R) \quad , \quad \mathcal{O}_{ld} = (\bar{l}_R \gamma^\mu l_R) (\bar{d}_R \gamma_\mu d_R)$$

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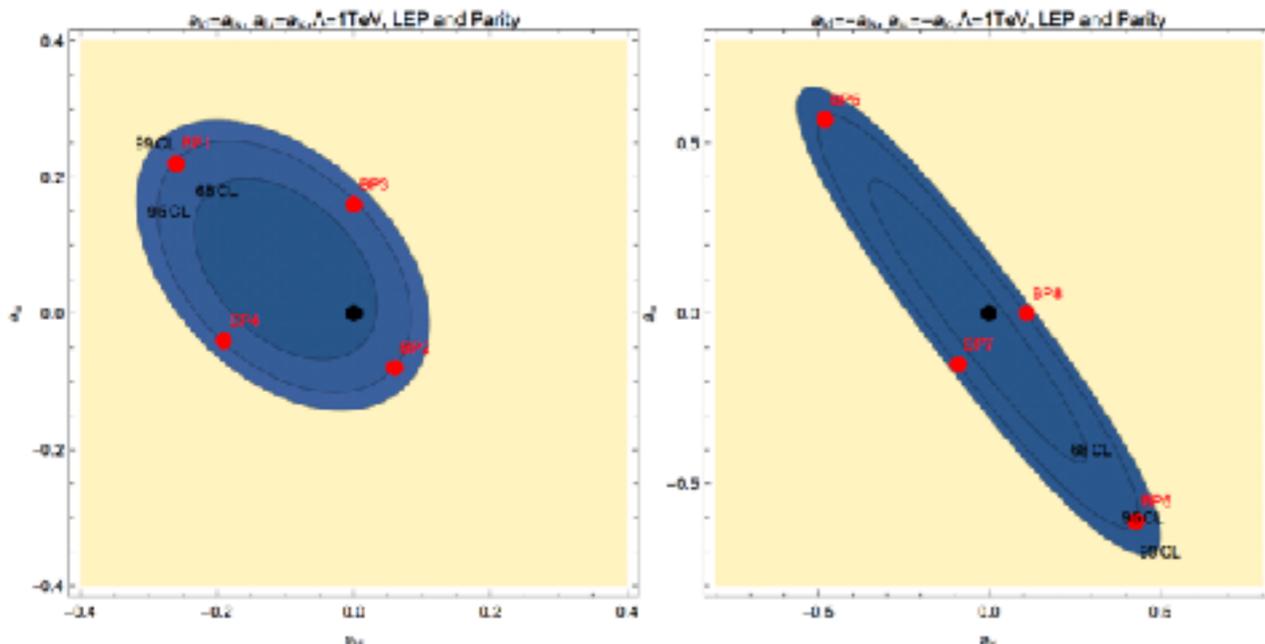
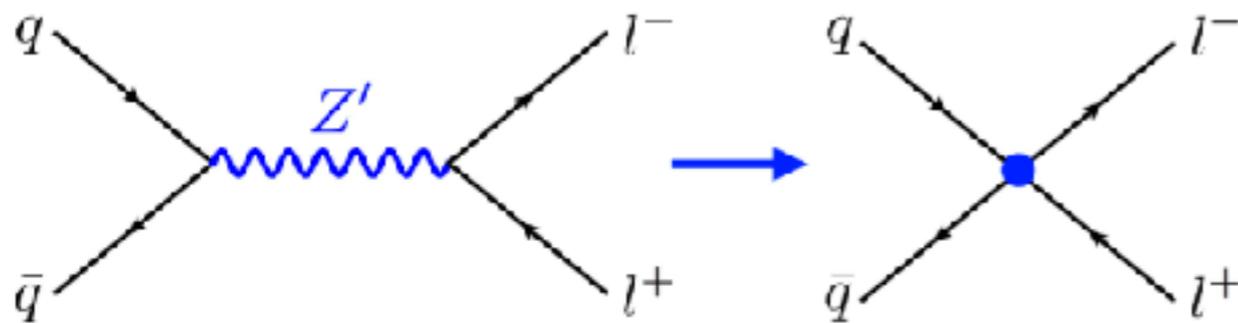


- Constrained by LEP and other experiments

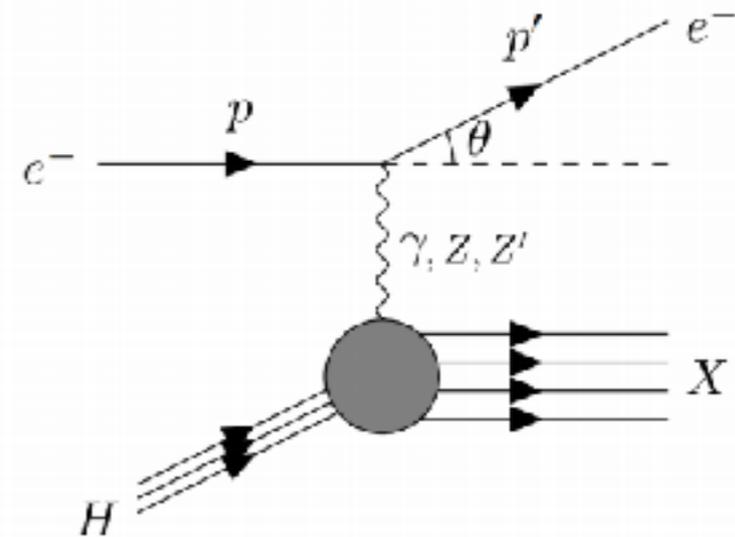
# A proof of concept

$$\mathcal{O}_{lu} = (\bar{l}_R \gamma^\mu l_R) (\bar{u}_R \gamma_\mu u_R) \quad , \quad \mathcal{O}_{ld} = (\bar{l}_R \gamma^\mu l_R) (\bar{d}_R \gamma_\mu d_R)$$

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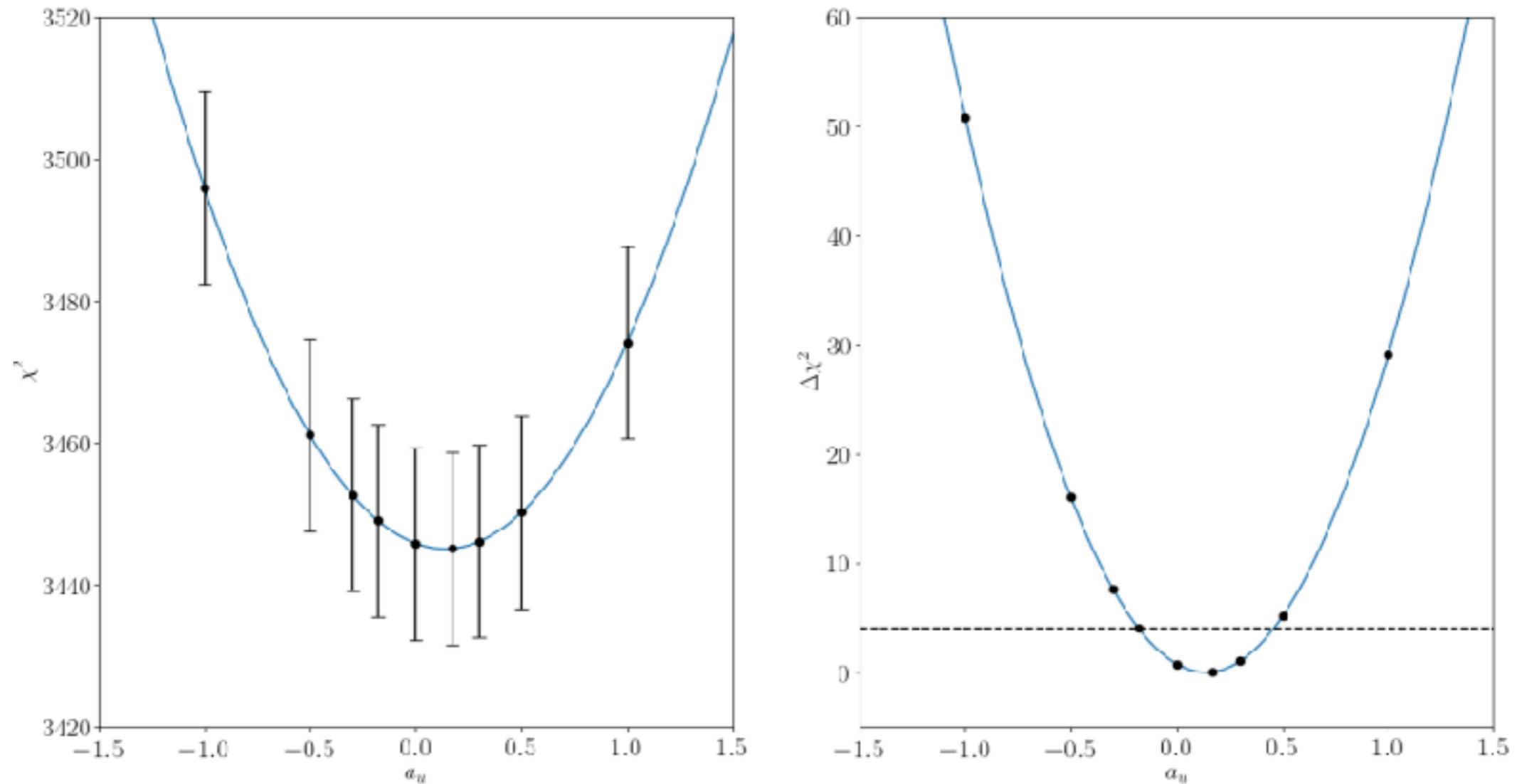
- What happens in a PDF fit if we include in the hadronic tensor the effect of the  $Z'$ ?



- What bounds do I get?
- Would they change if PDFs were fitted assuming new physics in the theory?

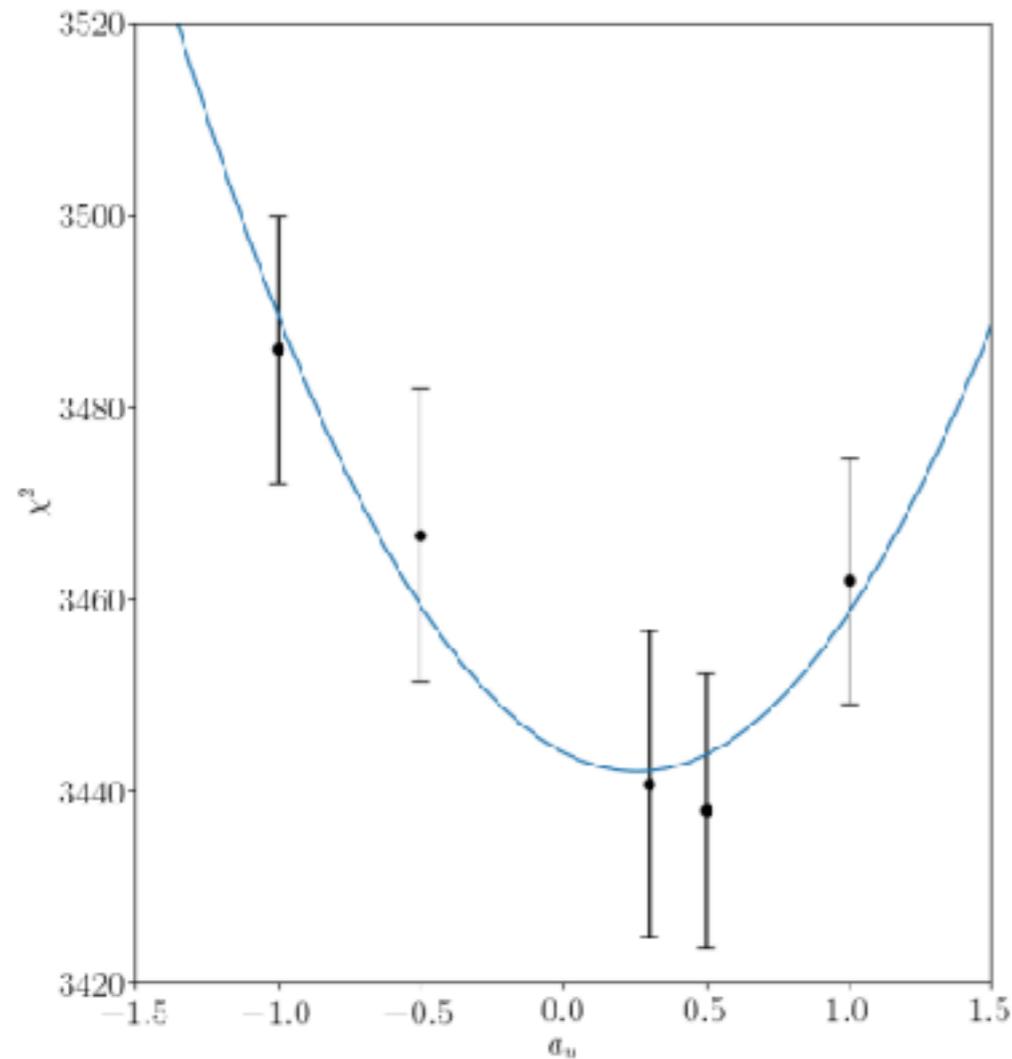
- Constrained by LEP and other experiments

# A proof of concept

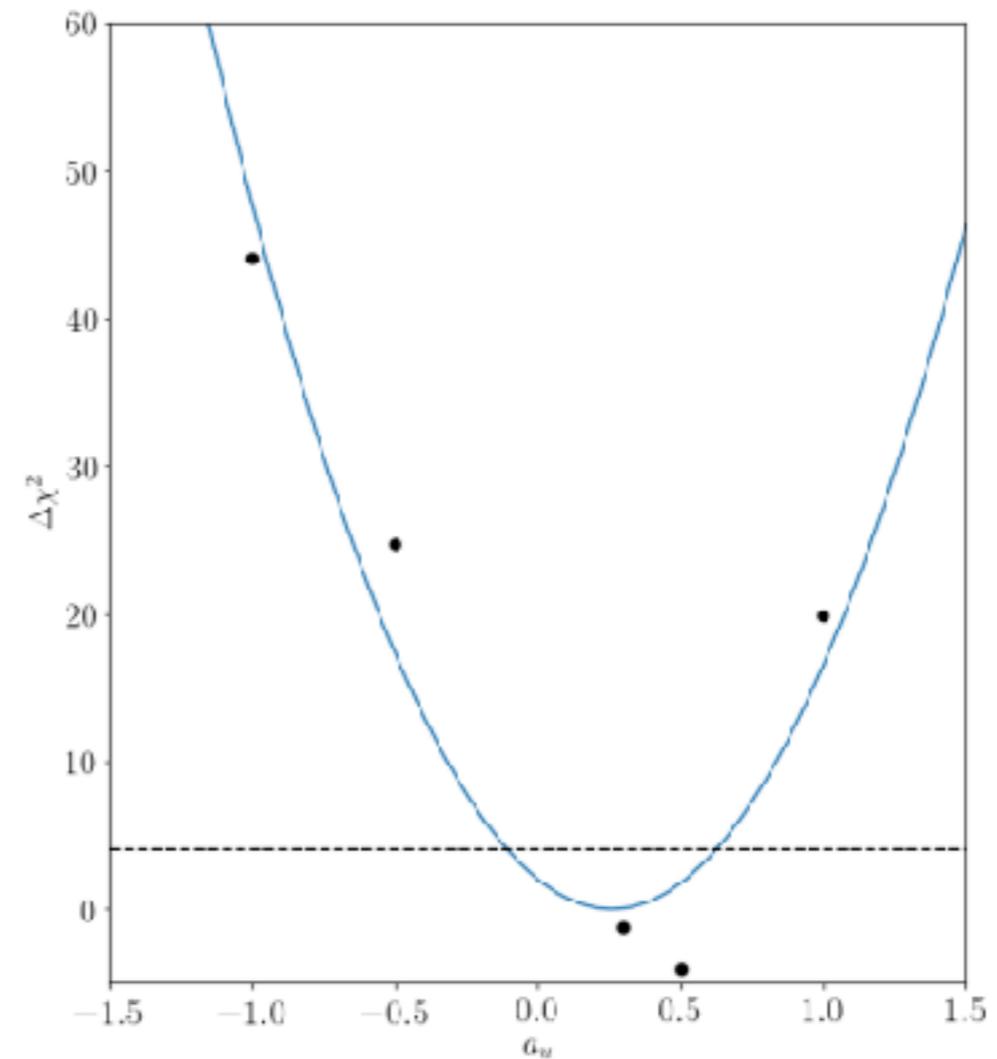


Error bars due to finite number of replicas. 90% CI:  $-0.18 < a_u < 0.46$

# A proof of concept

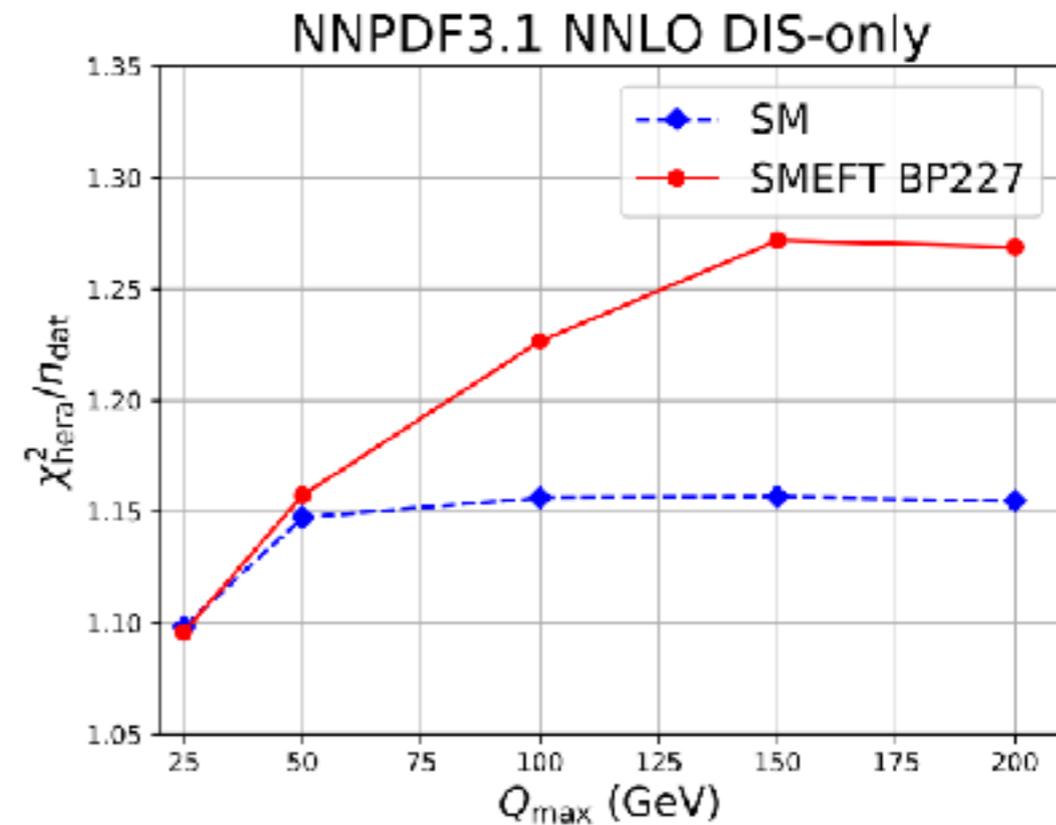
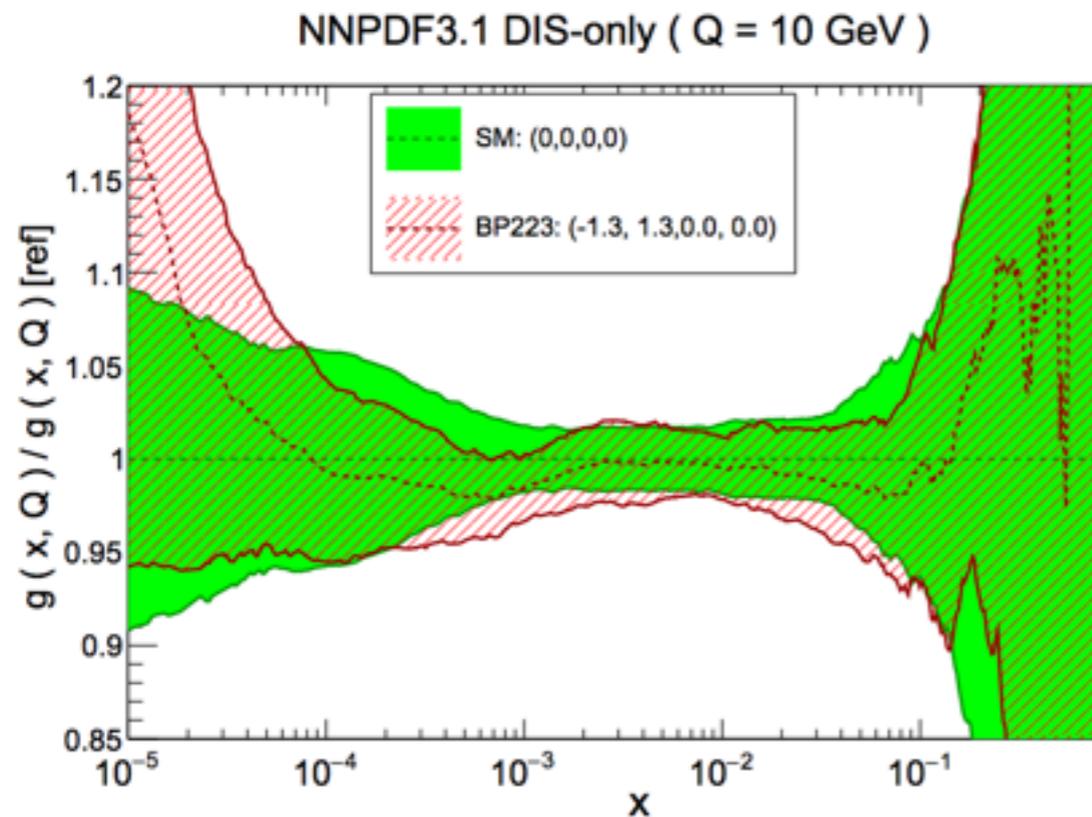


90% CI:  $-0.11 < a_u < 0.62$



- PDFs do not change much
- Larger bounds if new physics effects included in the fit

# Can PDFs absorb New Physics?



- Take a point within region allowed by LEP and other low-energy experiments
- Gluon changes in significant way
- But  $\chi^2$  of PDF fit within this new physics scenario gets worse, as data at higher  $Q$  are included
- Bottomline: PDFs cannot absorb new physics - in this proof-of-concept case

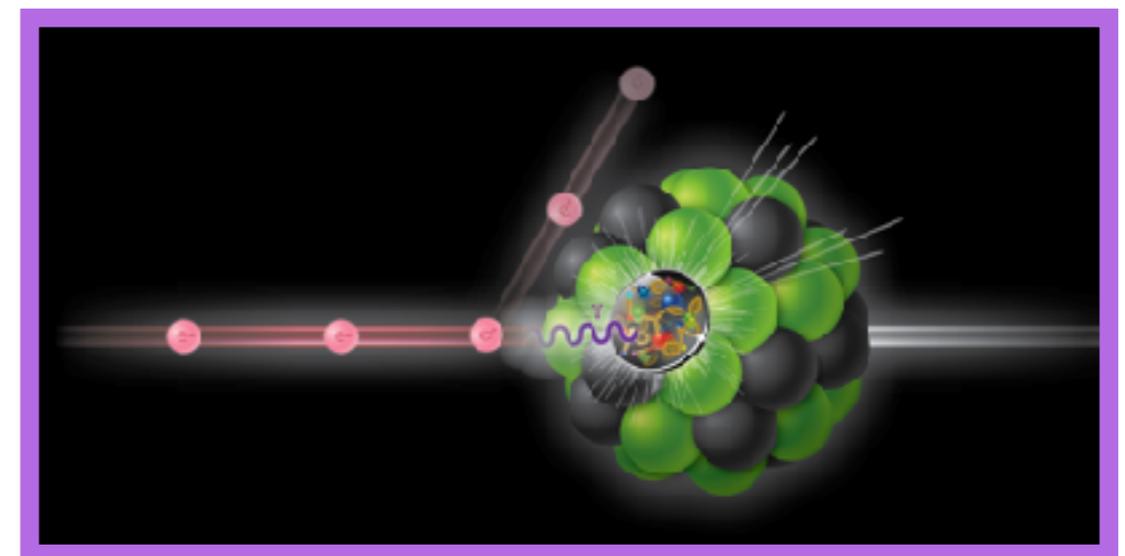
# Frontiers #4: nuclear PDFs

# Nuclear PDFs

## Collinear Factorisation Theorem:

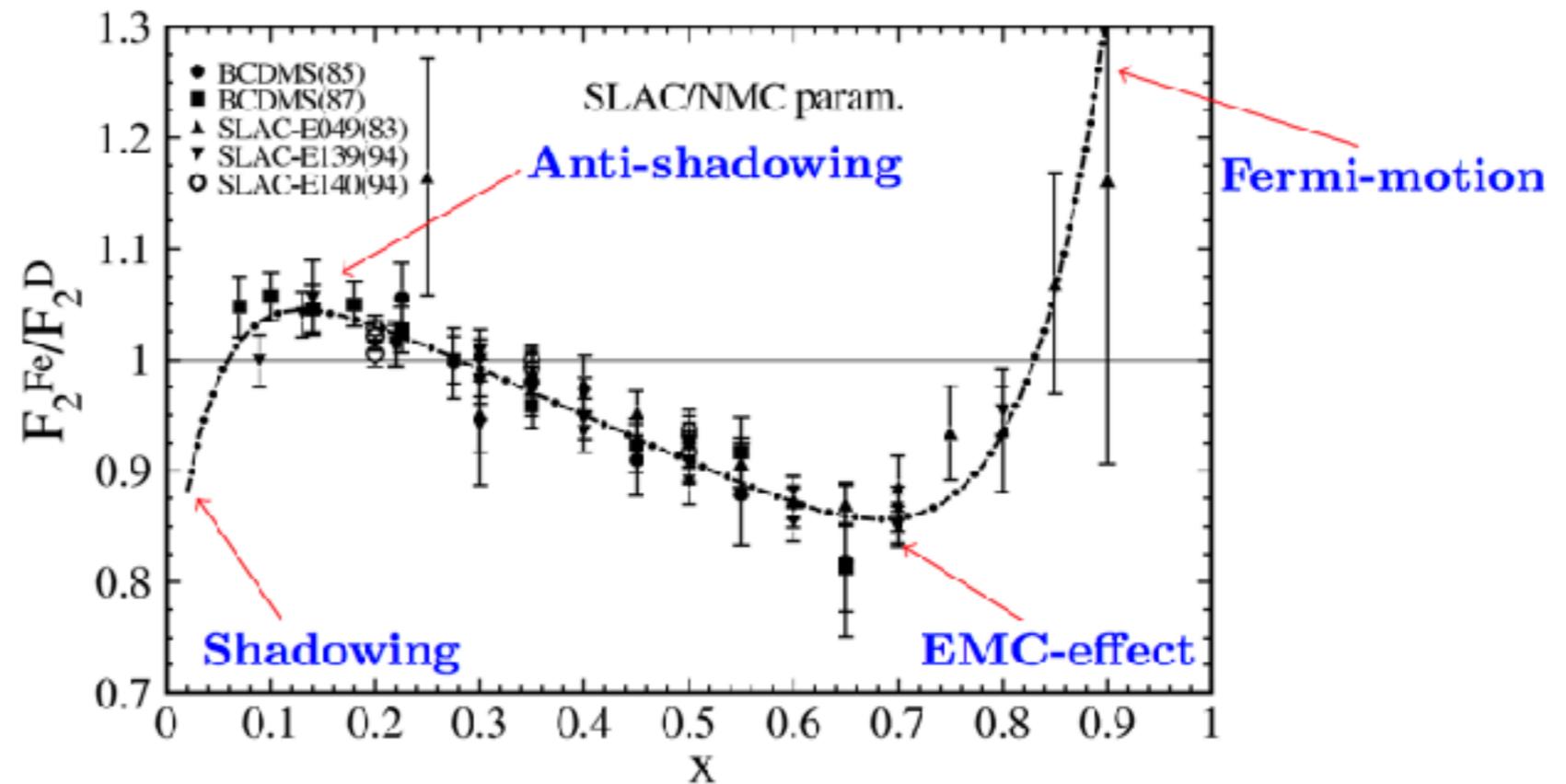
- Provide theoretical definition of universal PDFs
- Make the formalism predictive
- Make a statement about the error of the factorisation formula

- ➔ For pp ( $pp\sim$ ) and ep collisions we have rigorous factorisation proofs
- ➔ For eA factorisation works quite well (although need nuclear corrections)
- ➔ For pA and AA factorisation is a working hypothesis to be tested phenomenologically
- ➔ There might be breaking of QCD factorisation, from DGLAP evolutions or other nuclear effects to be included



# Nuclear PDFs

- EMC effect = a shift in the quark momentum distributions towards lower  $x$  when nucleons are bound
- Elastic ep maxima smeared around  $x = 1$  since nucleons are confined in a nucleus of radius  $\sim 1$  fm. Thus a Fermi momentum

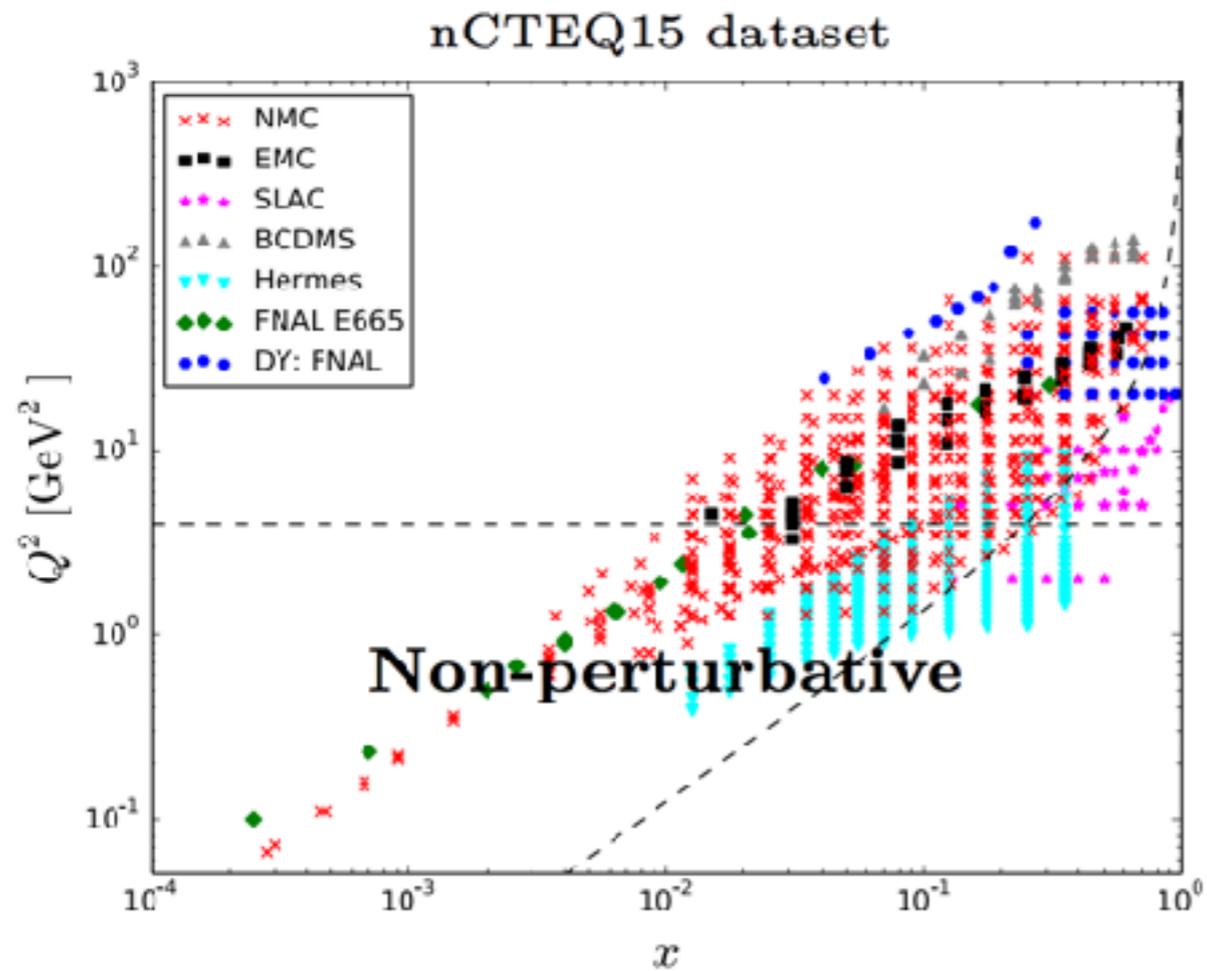


$$f_i^{p/A}(x_N, \mu_0) = R_i(x_N, \mu_0, A) f_i^{\text{free proton}}(x_N, \mu_0)$$

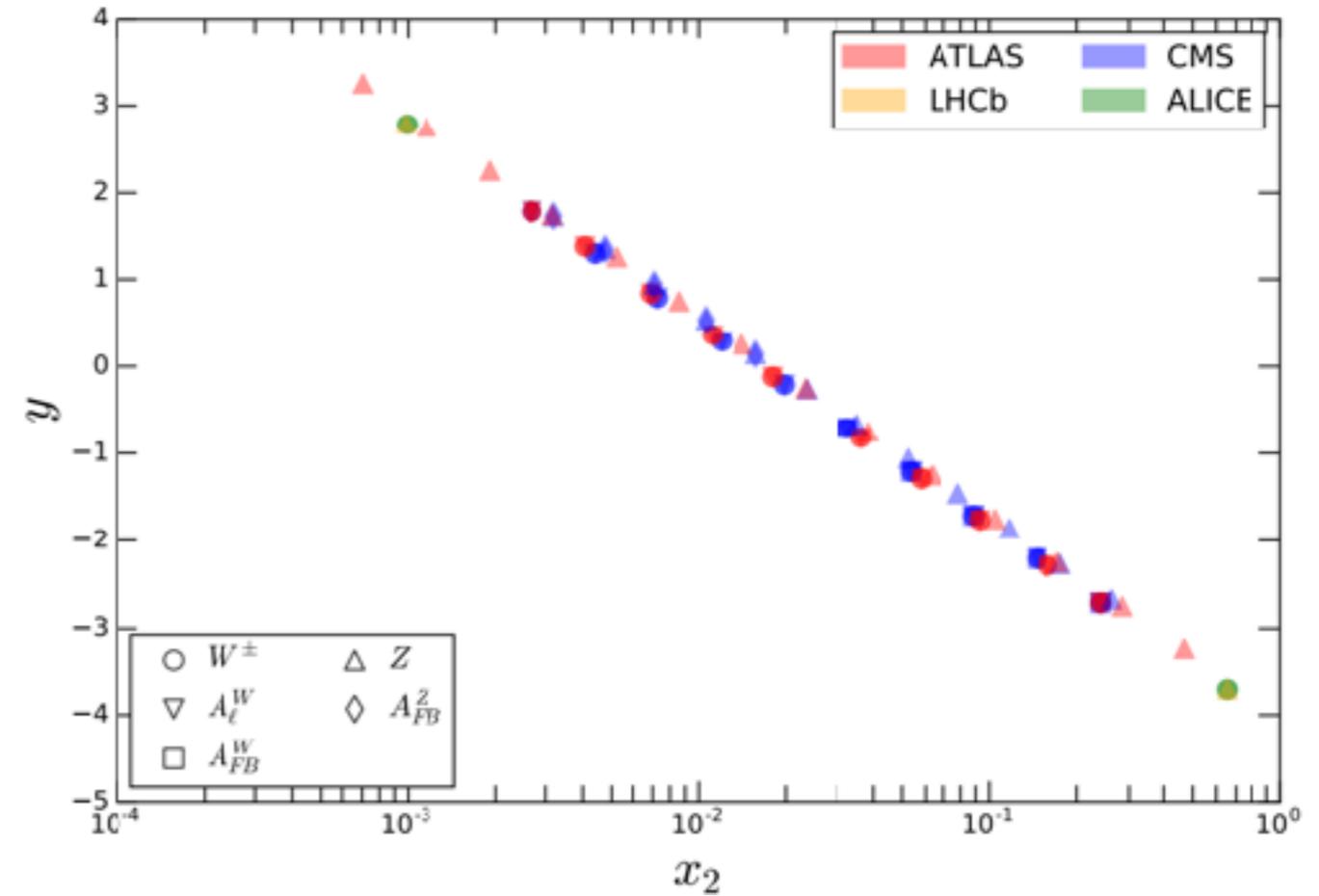
$$f_i^{p/A}(x_N, \mu_0) = f_i(x_N, A, \mu_0)$$

$$f_i(x_N, A = 1, \mu_0) \equiv f_i^{\text{free proton}}(x_N, \mu_0)$$

# Nuclear PDFs



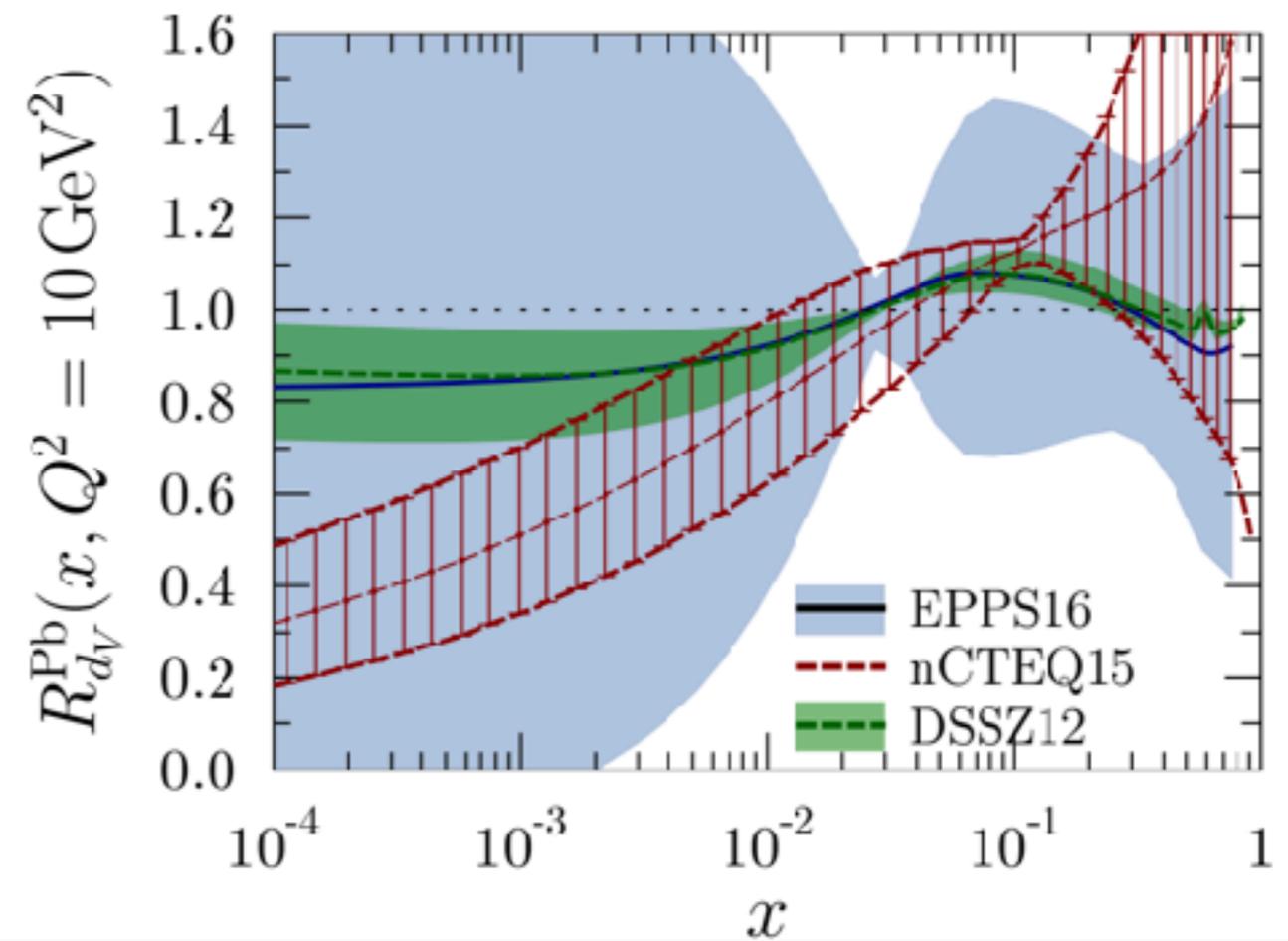
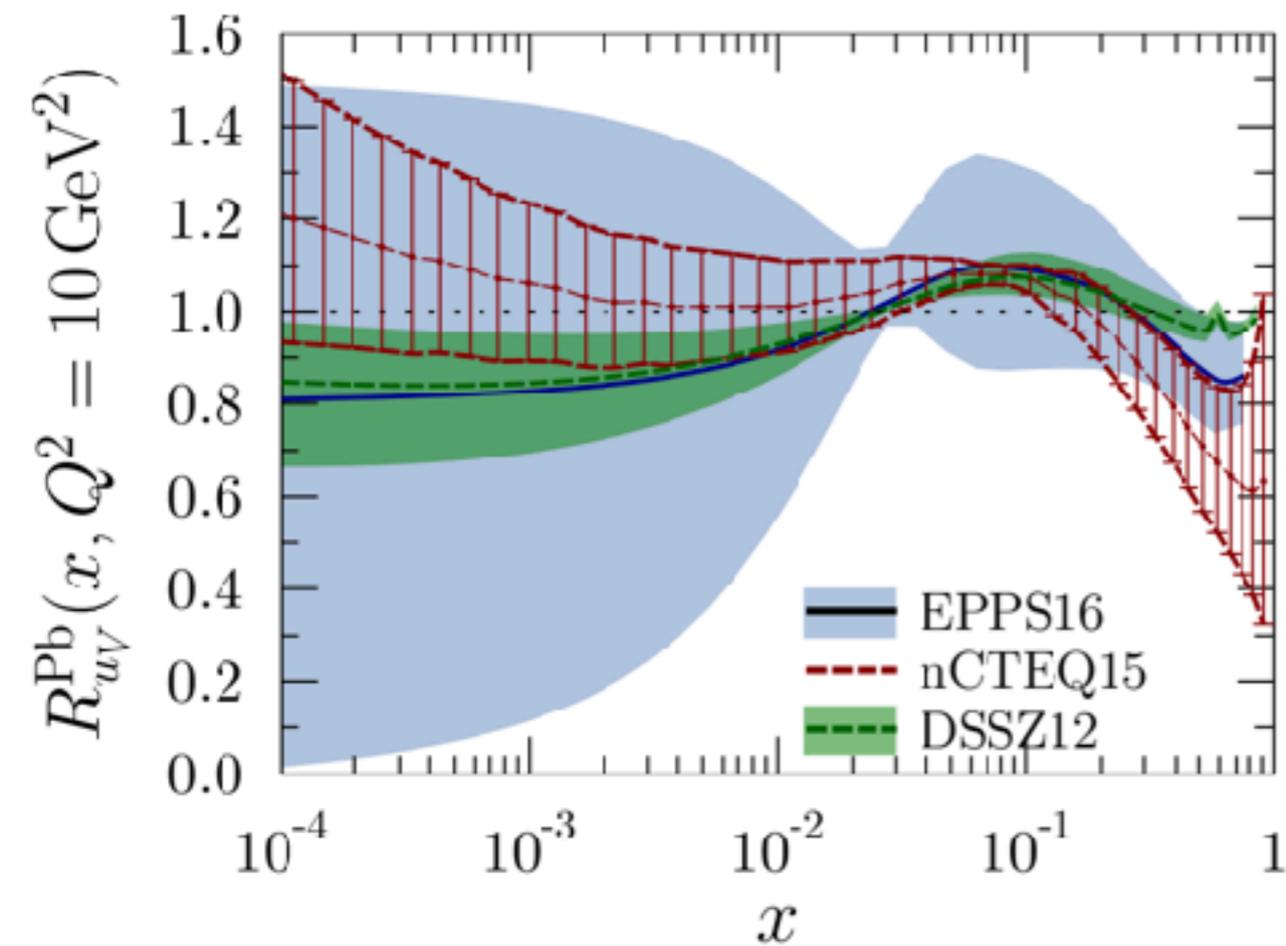
## P - Pb collisions



# Nuclear PDFs

$$R_i^A(x, Q^2) \equiv f_i^{\text{proton in nucleus } A}(x, Q^2) / f_i^{\text{free proton}}(x, Q^2)$$

90% CL



Frontiers #4:  
neutrino telescopes

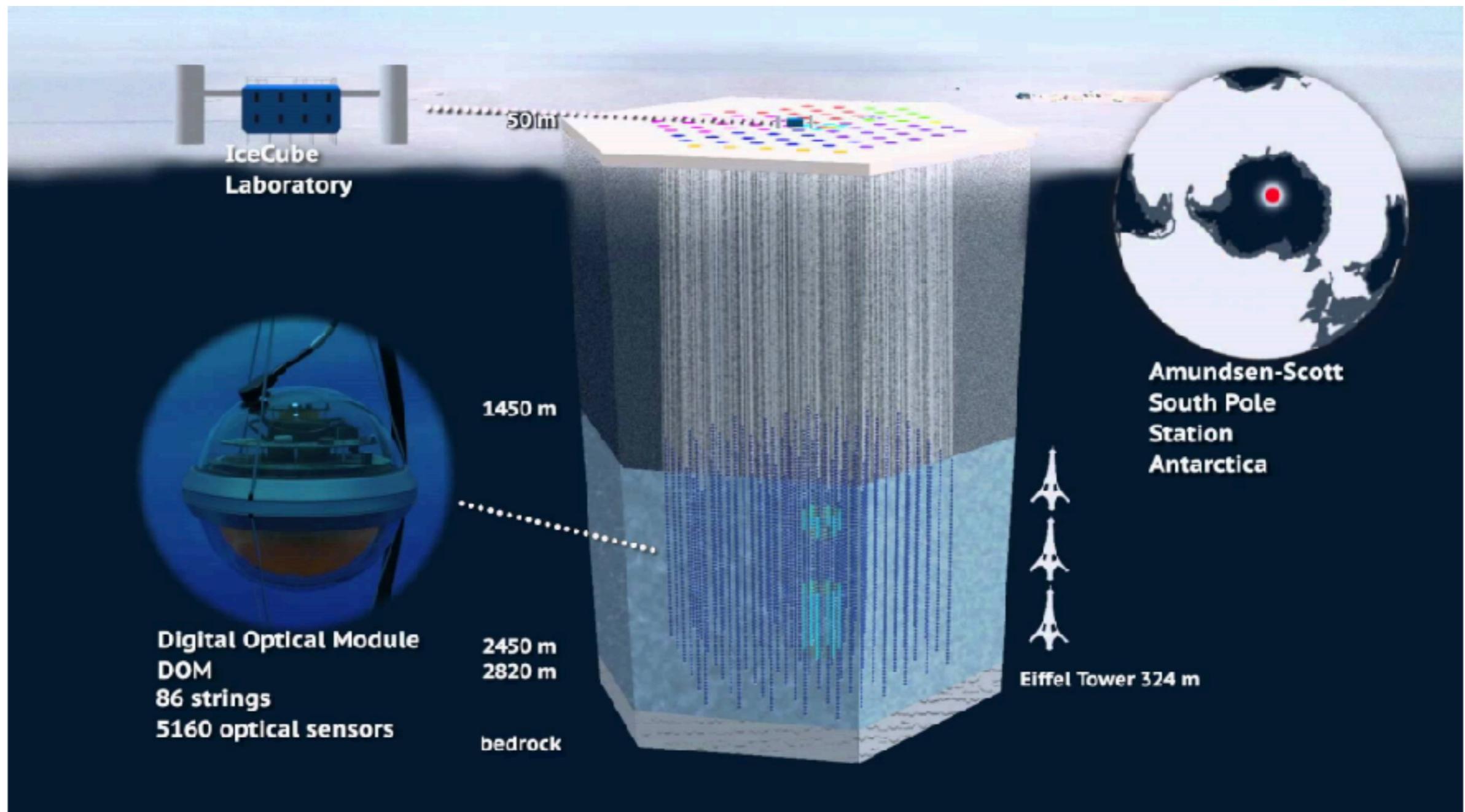
# Neutrino telescopes

- Ultra-high energy (UHE) neutrinos: novel window to the extreme Universe



# Neutrino telescopes

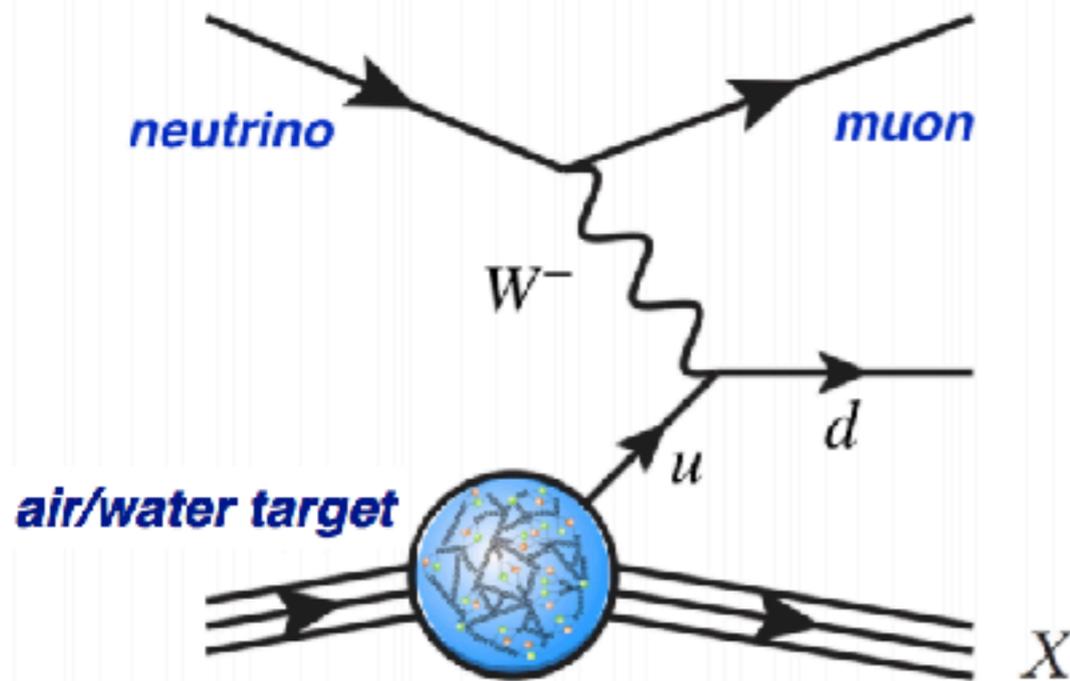
- Ultra-high energy (UHE) neutrinos: novel window to the extreme Universe



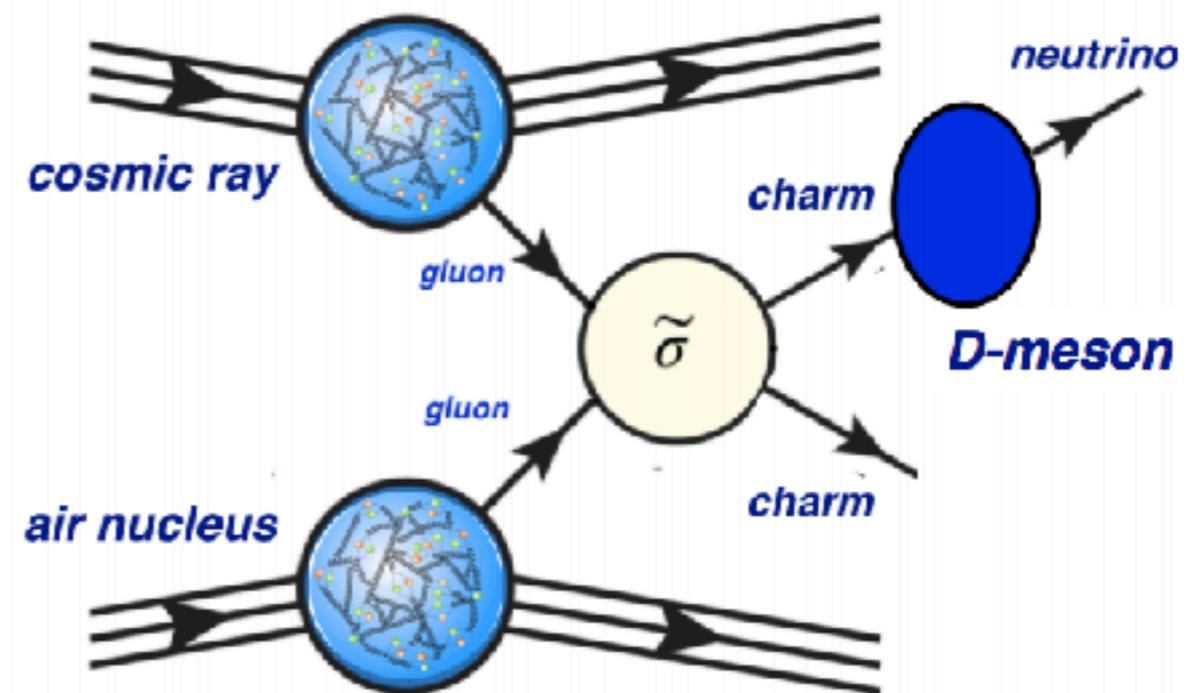
# Neutrino telescopes

**signal:** cosmic neutrino - nucleus scattering

**background:** prompt charm production



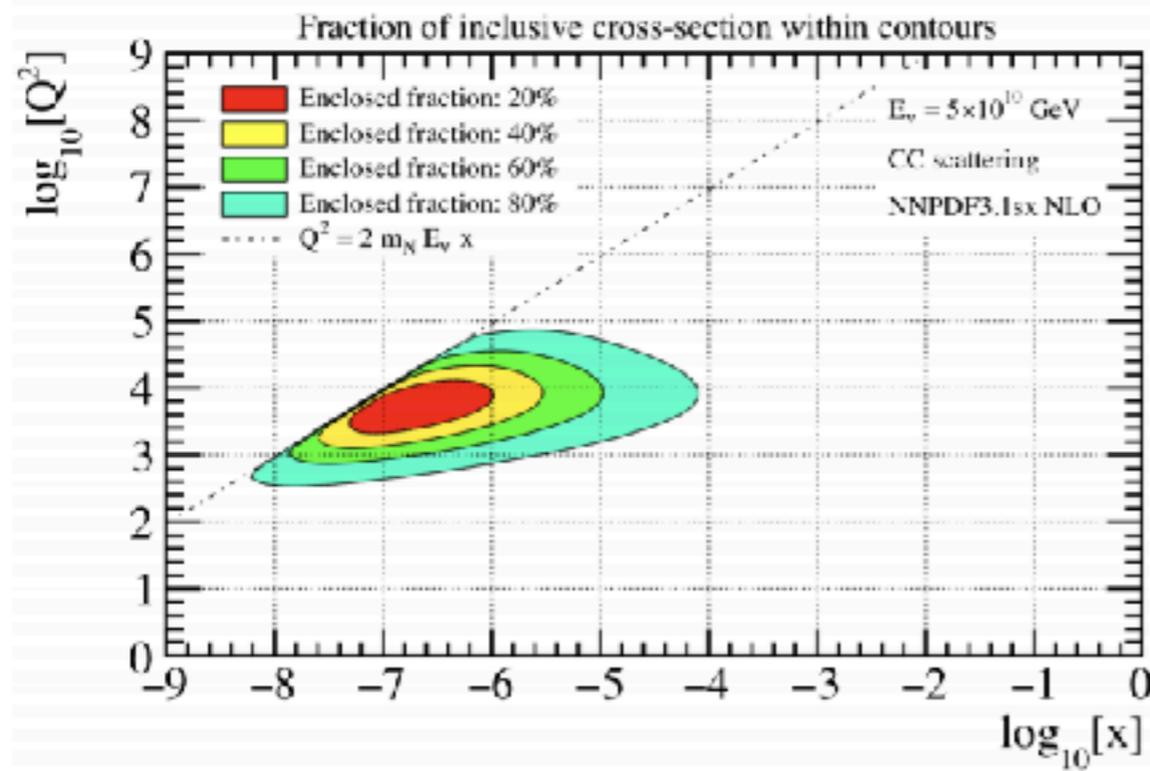
Sensitive to **small- $x$  quarks** (and thus gluons via evolution) down to  $x = 10^{-8}$  and  $Q = M_W$



Sensitive to **small- $x$  gluons** down to  $x = 10^{-6}$  and  $Q = M_{\text{charm}}$  in the **centre-of-mass frame**

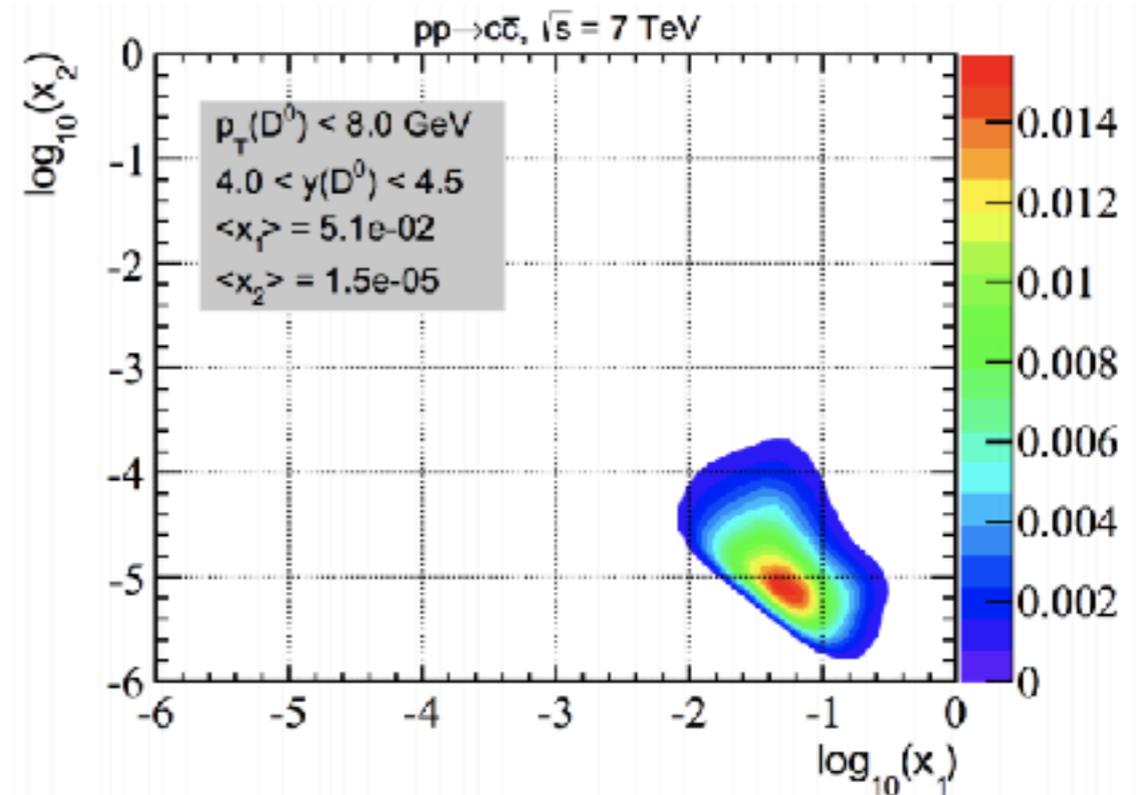
# Neutrino telescopes

**signal:** cosmic neutrino - nucleus scattering



Sensitive to **small- $x$  quarks** (and thus gluons via evolution) down to  **$x = 10^{-8}$**  and  **$Q = M_w$**

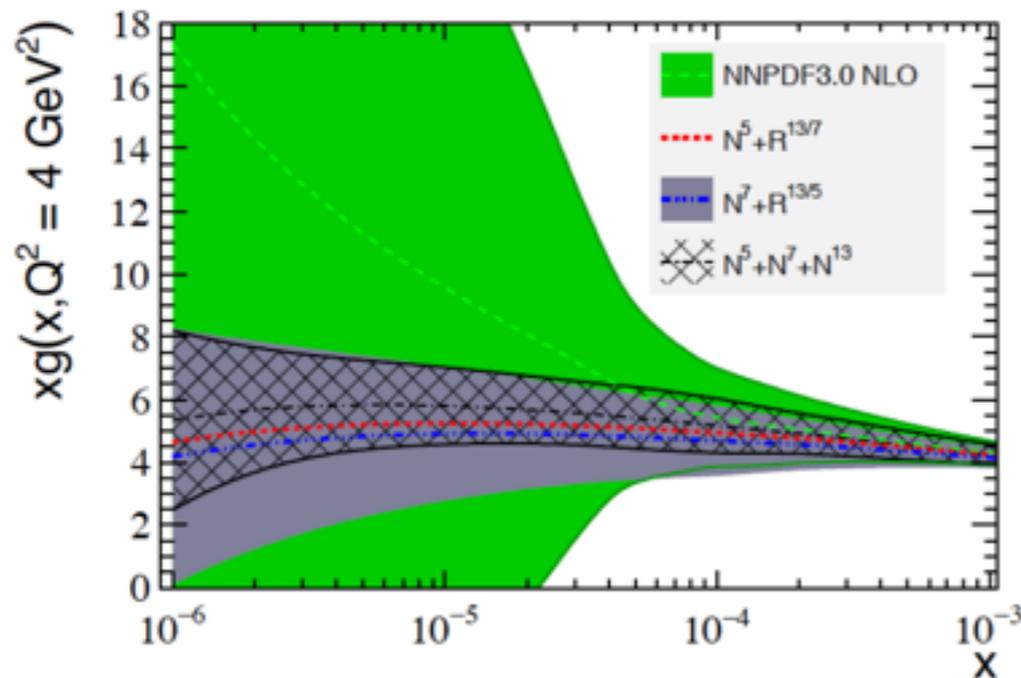
**background:** prompt charm production



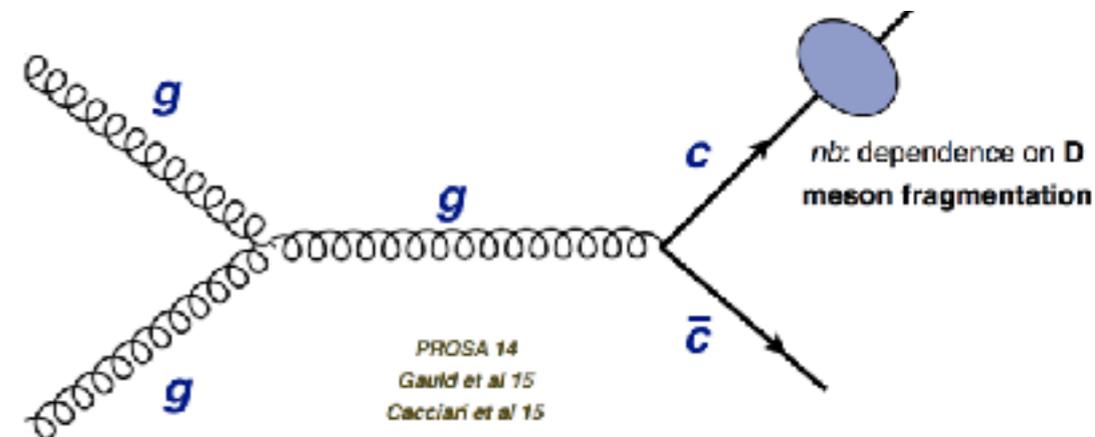
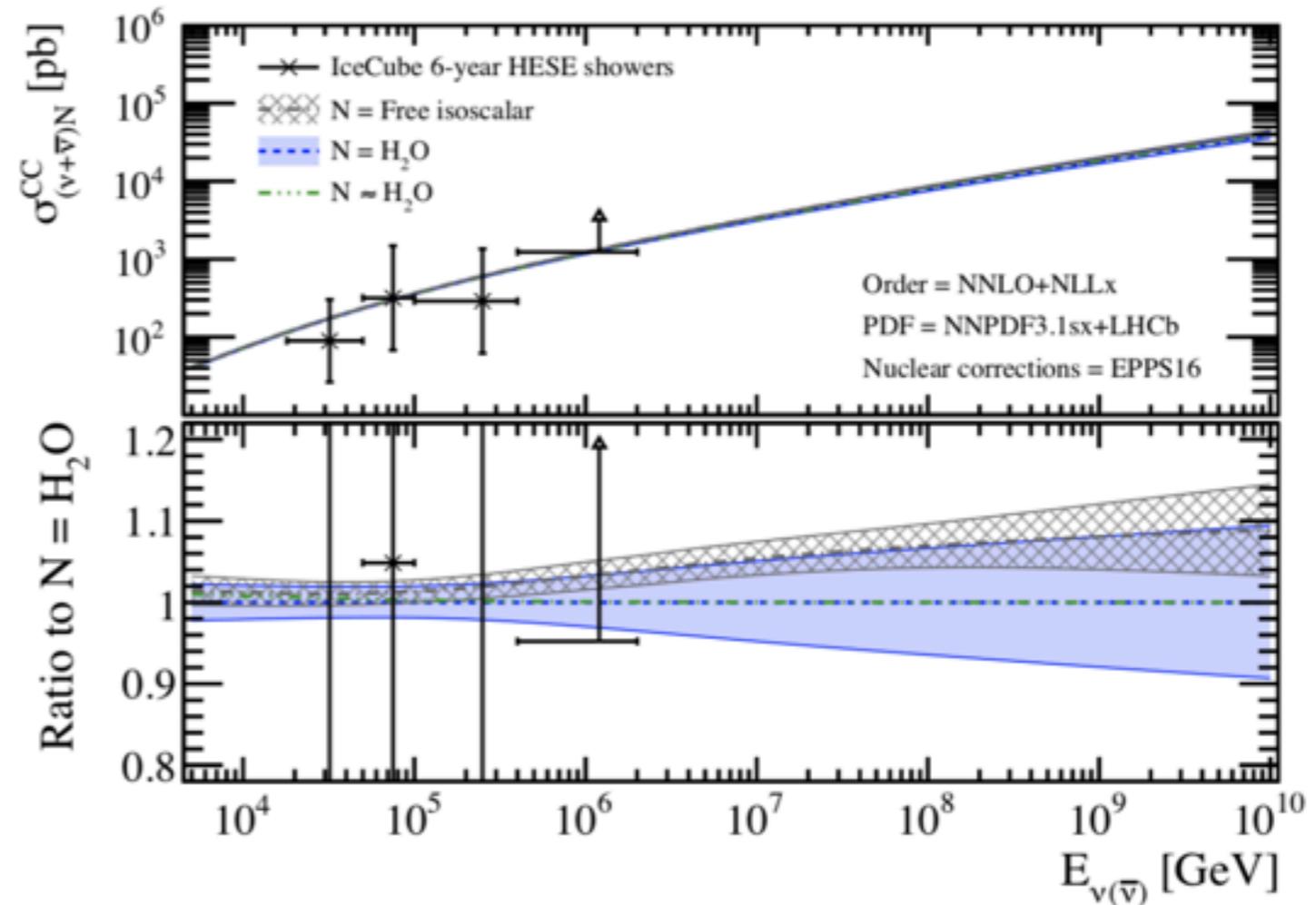
Sensitive to **small- $x$  gluons** down to  **$x = 10^{-6}$**  and  **$Q = M_{\text{charm}}$**  in the **centre-of-mass frame**

# Neutrino telescopes

- The inclusion of LHCb prompt charm data reduces uncertainty of gluon at small  $x$
- Using that gluon to predict prompt charm production background improves agreement between data and theoretical predictions



## Comparison with *IceCube* data

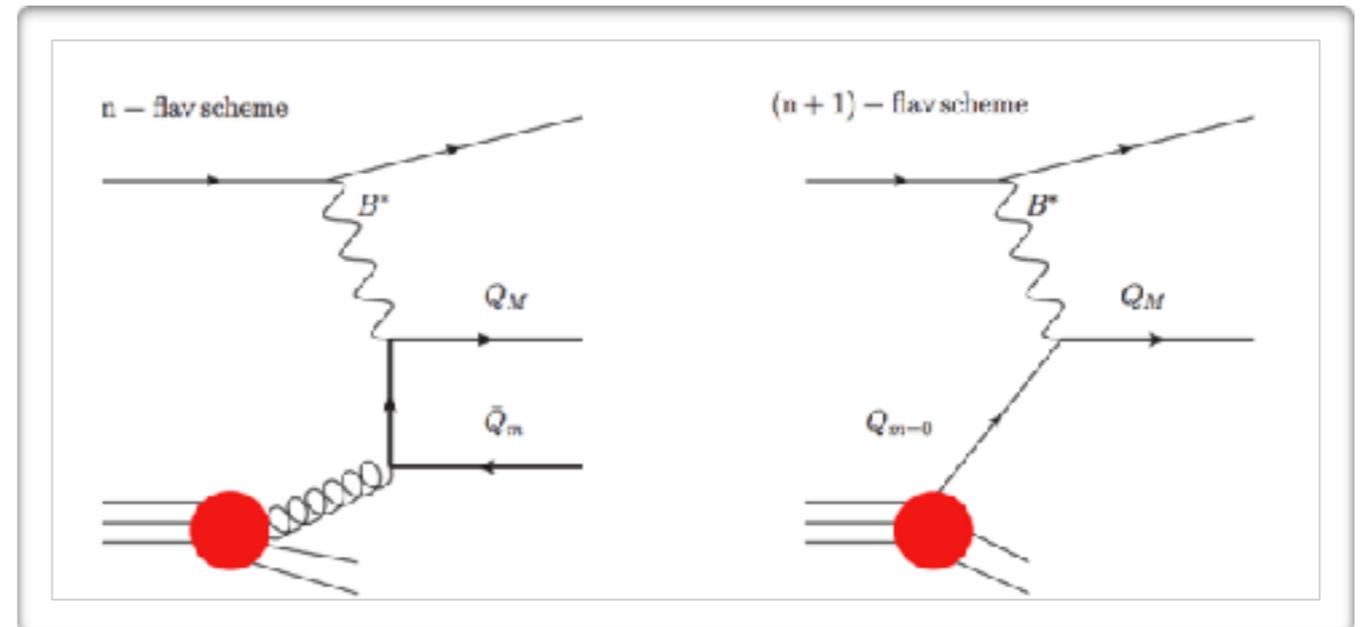


Frontiers #6: heavy quark  
phenomenology

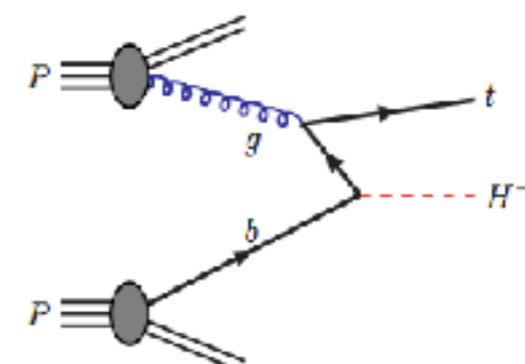
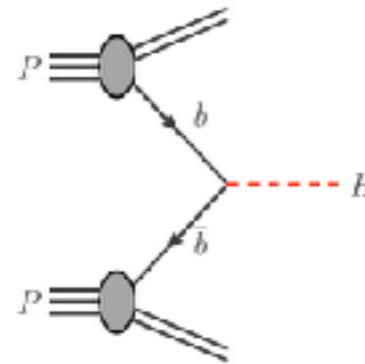
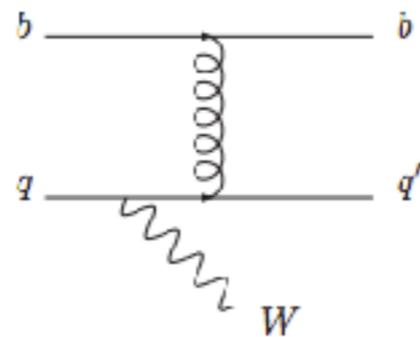
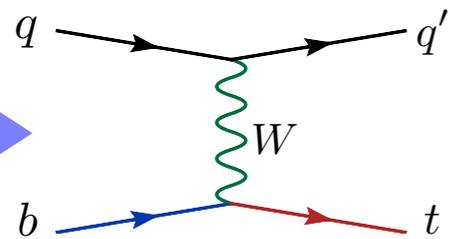
# A rich phenomenology

$$\frac{d\sigma_H^{pp \rightarrow AB}}{dX} = \sum_{i,j=1}^{N_f} \boxed{f_i(x_1, \mu_F) f_j(x_2, \mu_F)} \boxed{\frac{d\hat{\sigma}^{ij \rightarrow ab}}{dX}(x_1 x_2 S_{\text{had}}, \alpha_S(\mu_R), \mu_R, \mu_F, m)} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^{2n}}{S_{\text{had}}^n}\right)$$

DIS

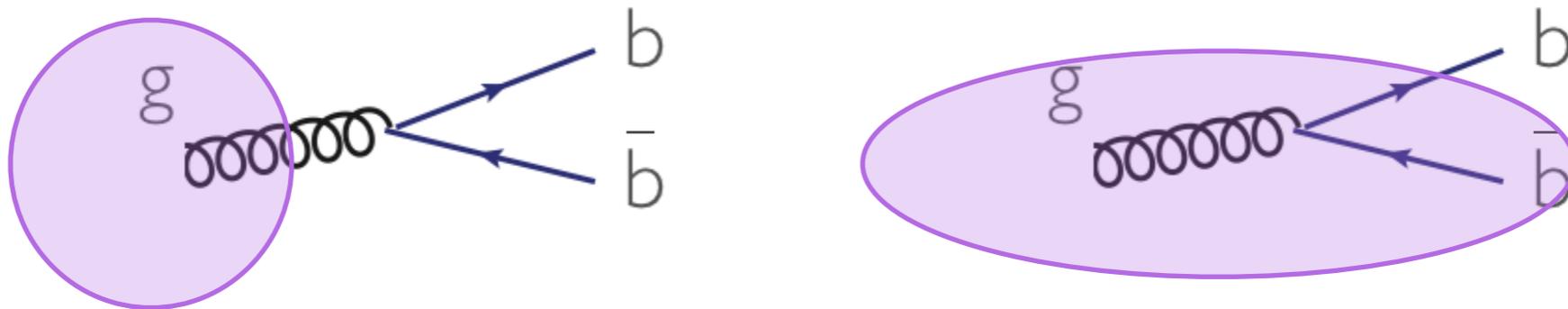


## Hadron colliders



# 4F and 5F schemes

- For all processes that feature bottom quarks at the hard-process level there are two ways of performing computations: **4F** and **5F** schemes



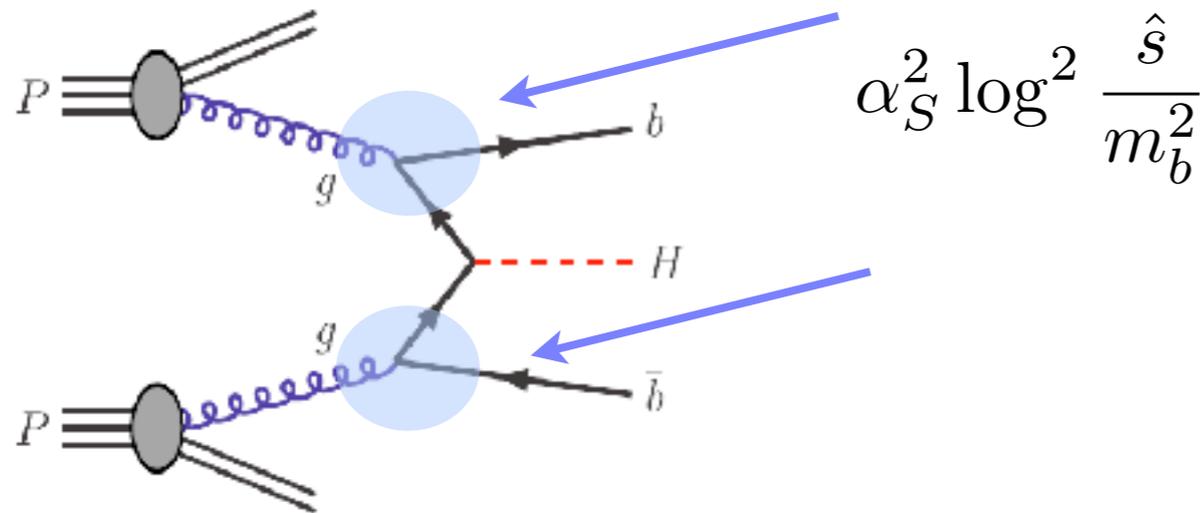
- Each supports the issues that arise in different kinematical regimes

If  $m_b \sim Q$

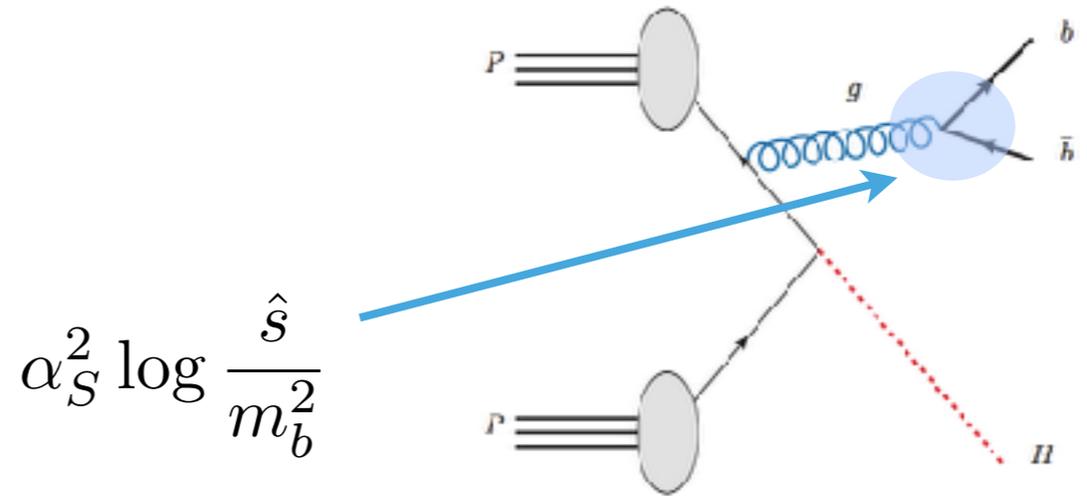
**4F scheme**

- ◆ b quark treated as massive object at the level of short-distance xsec
- ◆ b quark never appears in the initial state
- ◆ In the short-distance xsec logarithms arise

# 4F and 5F schemes



t-channel kinematics  
Initial state



s-channel kinematics  
Final state

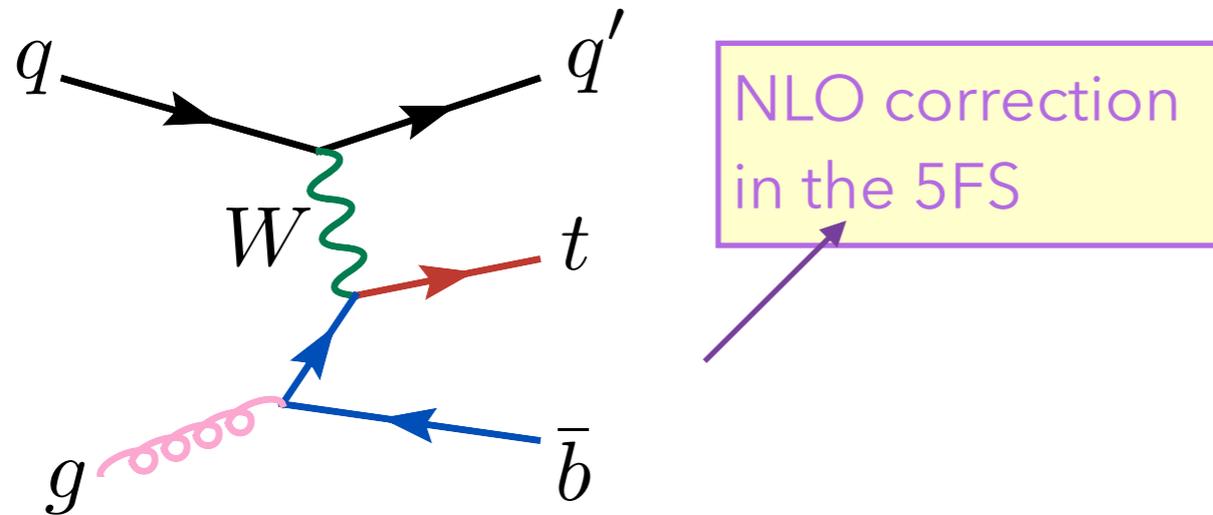
These logs for  $m_b \ll s$ , might be large, possibly spoiling perturbation theory!!

If logs dominate

**5F scheme**

- ◆ b quark treated as a light parton generated at threshold  $\mu_b \sim m_b$  from DGLAP evolution
- ◆ Set  $m_b = 0$  in the short-distance xsec
- ◆ Resummation of the collinear logs achieved through DGLAP evolution equations for bottom PDFs

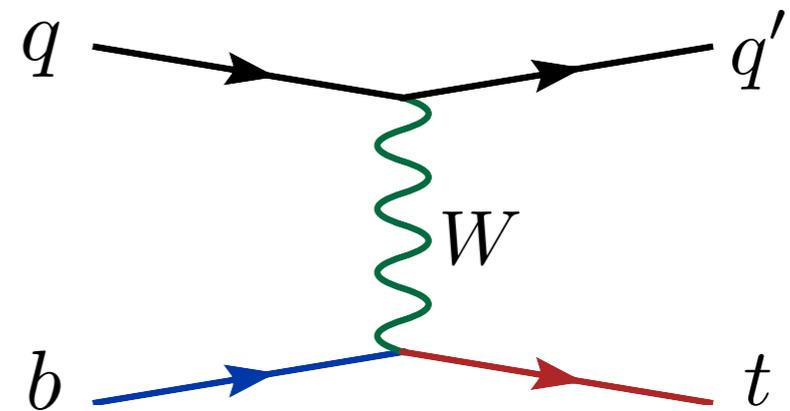
# 4F and 5F schemes



4F scheme

- ✗ It does not resum possibly large logs, yet it has them explicitly
- ✗ Computing higher orders is more difficult
- ✓ Mass effects are there at any order
- ✓ Straightforward implementation in MC event generators at LO and NLO

Decoupling or massive scheme

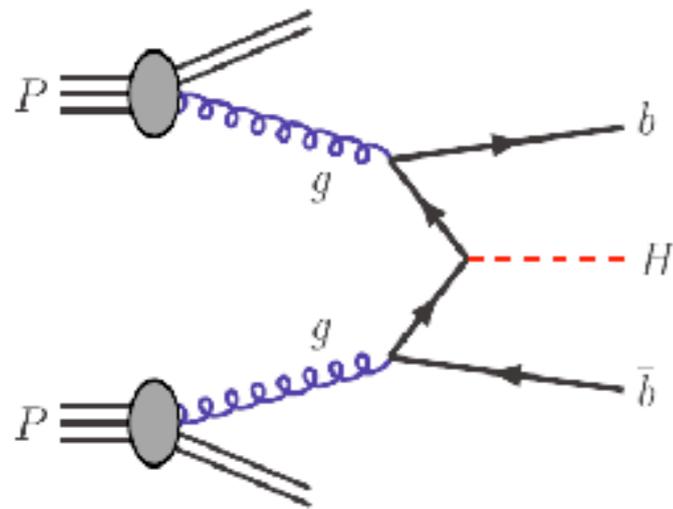


5F scheme

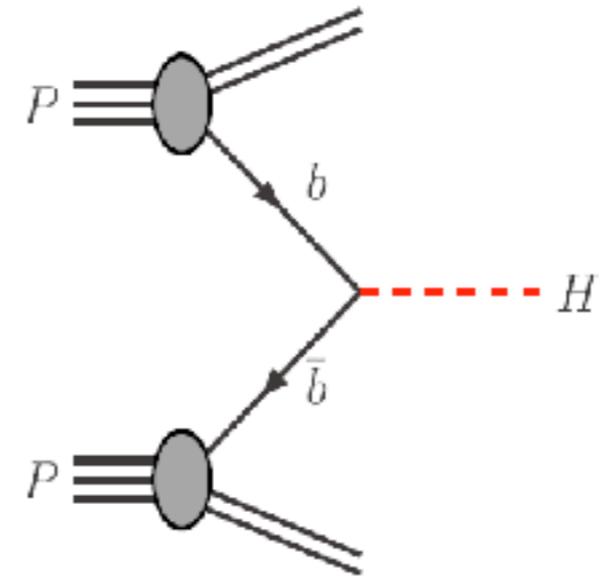
- ✓ It resums initial state large logs into b-PDFs leading to more stable predictions
- ✓ Computing higher orders is easier
- ✗  $p_T$  of bottom enters at higher orders
- ✗ Implementation in MC depends on the gluon splitting model in the PS

Massless scheme

# 4F and 5F schemes



NNLO correction  
in the 5FS



4F scheme

5F scheme

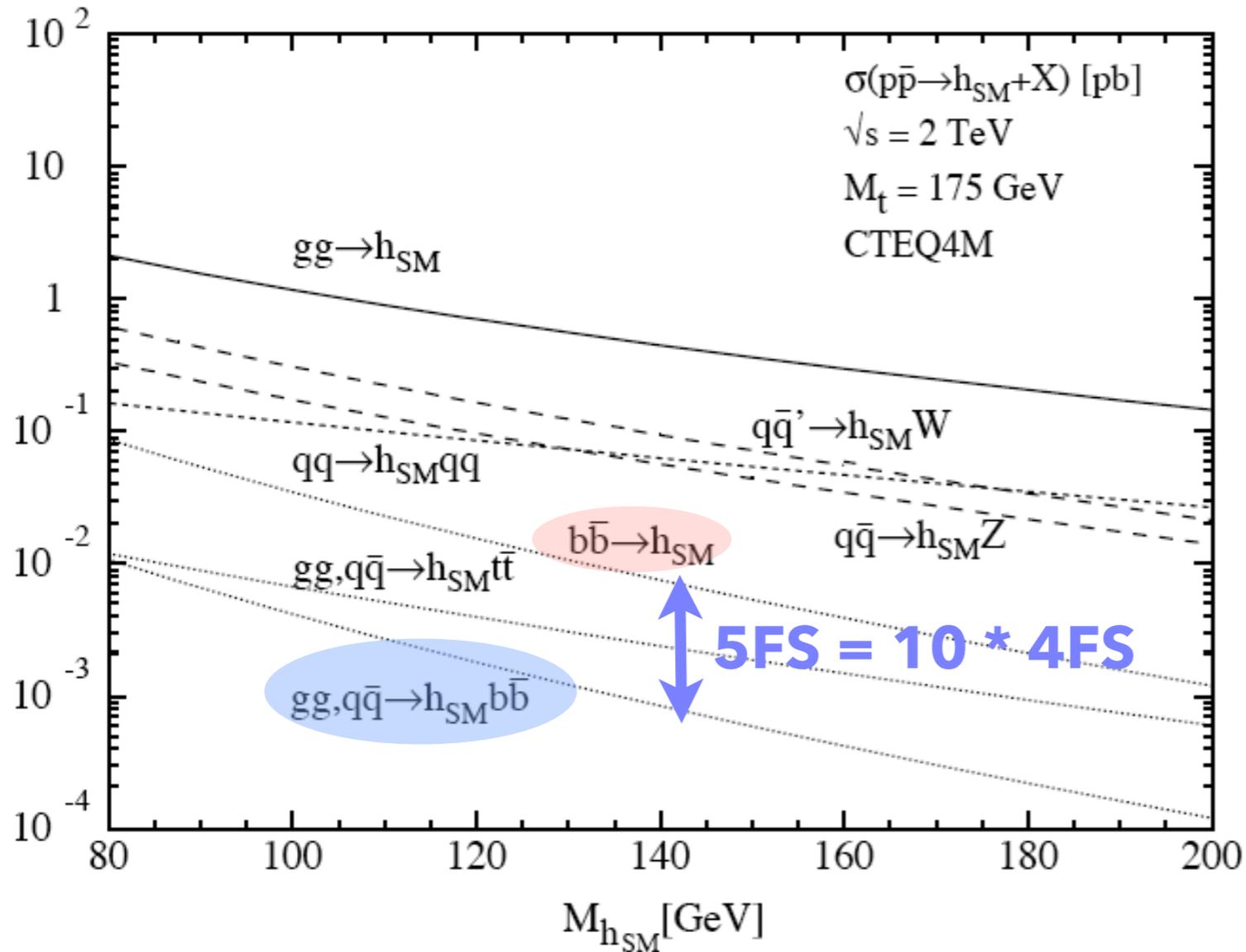
- ✗ It does not resum possibly large logs, yet it has them explicitly
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- ✗ Implementation in MC depends on the gluon splitting model in the PS

Decoupling or massive scheme

Massless scheme

# A lot of (open) questions



- ➔ Why do the two schemes often lead to very different results?
- ➔ Why differences become smaller is a softer scale is used?
- ➔ For exclusive/differential observables: how to proceed?
- ➔ For inclusive observables: how to combine/match the two schemes to maximise the pros?

# Combining the 4F and 5F schemes

- There are cases when both **mass** terms and **resummation** of collinear logs must be included, as they both play a role in getting accurate predictions (e.g. DIS)
- What about predictions for partonic cross sections at the LHC?

4F scheme		5F scheme	
$pp \rightarrow bb$	Nason et al (1989), Mangano et al (1992)		
$pp \rightarrow bbbb$	Greiner et al (2011)		
$pp \rightarrow ttbb$	Bevilacqua et al (2009), Bredenstein et al (2010)	$pp \rightarrow tW$	Campbell et al (2005), Frixione et al (2008)
$pp \rightarrow tbj$	Campbell et al (2009)	$pp \rightarrow tj$	Harris et al (2002), Campbell et al (2005)
$pp \rightarrow tbH^\pm$	Dittmaier et al (2009), Degrande et al (2015)	$pp \rightarrow tH^\pm$	Plehn et al (2003), Weydert et al (2010)
$pp \rightarrow \Phi bb$	Dawson et al (2005), Dittmaier et al (2004)	$pp \rightarrow \Phi(bb), \Phi b(b)$	Campbell et al (2003), Harlander et al (2003)
$pp \rightarrow Vbb$	Ellis et al (1999,2000), Reina et al (2008,2009), Badger et al (2011), Frederix et al (2011)	$pp \rightarrow Z(bb), Vbj, Vb$	Campbell et al (2004,2006,2007,2009), Maltoni et al (2005)

# Combining the 4F and 5F schemes

- Independently of the size of the mass effects and of collinear resummation effects, a prediction that combines the best available 4F and 5F scheme predictions based on standard QCD factorisation is the best one could get
- For inclusive cross sections a “phenomenological approach” is often adopted (HXS WG). Not too harmful is predictions do not differ much, but not theoretically sound!

## **Santander matching:**

Weighted average between the 4F and the 5F scheme predictions

$$\sigma = \frac{\sigma^{(4F)} + w \sigma^{(5F)}}{1 + w}$$

$$w = \log \left( \frac{M}{m_b} \right) - 2$$

# Combining the 4F and 5F schemes

- Independently of the size of the mass effects and of collinear resummation effects, a prediction that combines the best available 4F and 5F scheme predictions based on standard QCD factorisation is the best one could get
- Can we do better than that?

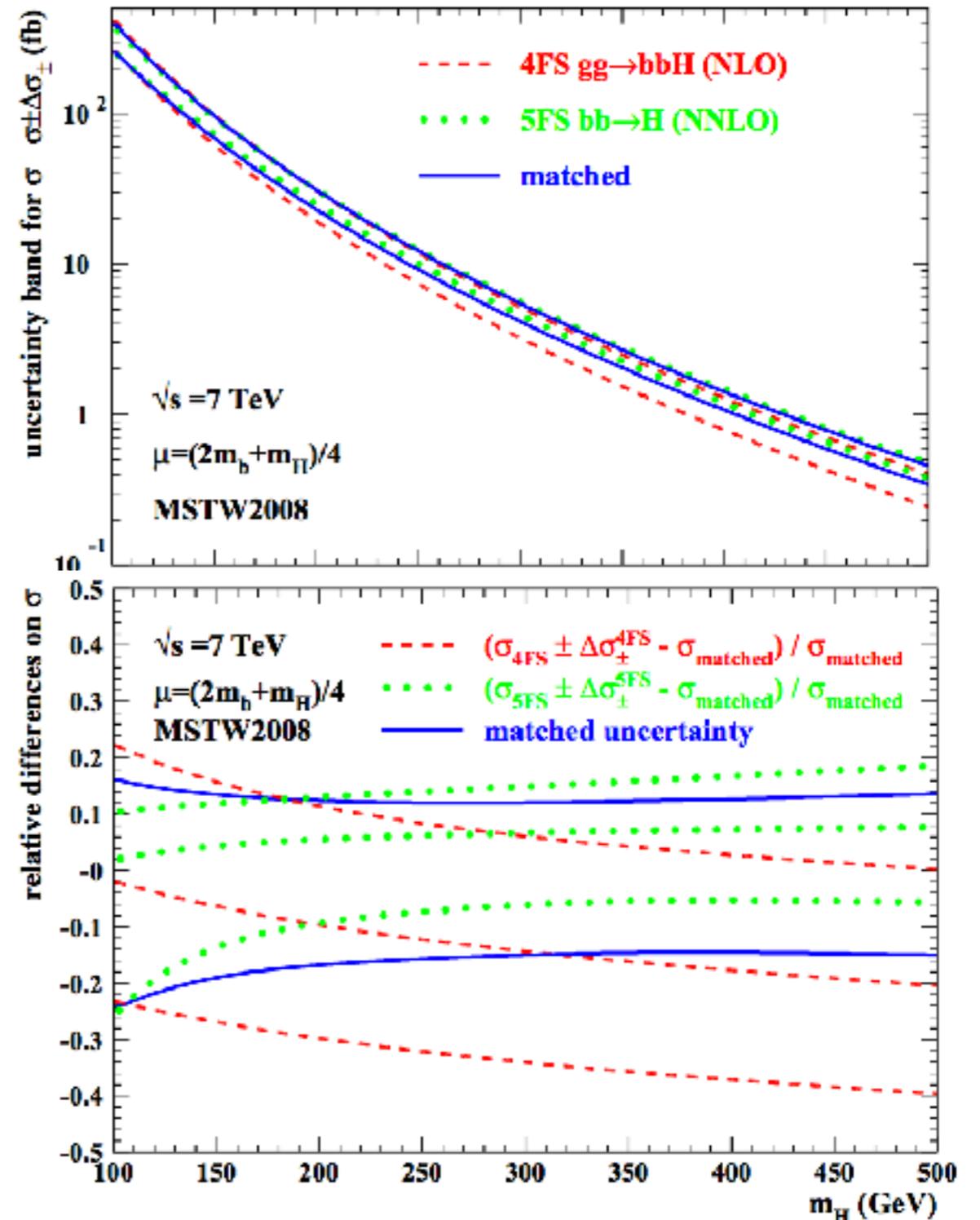
- ▶ DIS [ACOT (1993), TR(2002), FONLL(2010)]
- ▶ b hadro-production [Cacciari et al (1998)]
- ▶ single top t -channel [MCFM, Campbell et al (2002,2009)]
- ▶ W+Q, Z+Q [MCFM, Campbell et al (2004)]
- ▶ ttH' [Han et al (2015)]
- ▶ bbH [Forte et al (2015), Bonvini et al (2015)]

# The bbH case

- Bottom-fusion initiated H production relevant in models in which bH coupling is enhanced (e.g. 2HDM with large tanB)
- 5F known up to NNLO (diff.)
  - Dicus et al (1999)
  - Ballasz et al (1999)
  - Harlander et al (2003)
  - Busheler et al (2012)
- 4FS known up to NLO
  - Dittmaier et al (2004)
  - Dawson et al (2004)
  - Wiesemann et al (2015)

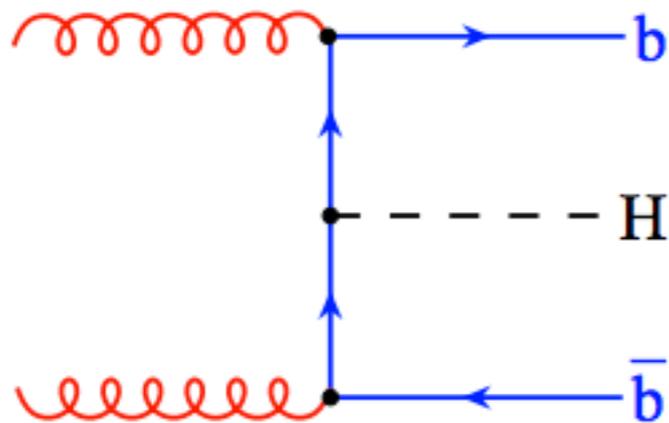
MSTW08 PDFs

Scale + PDF +  $a_s$  uncertainties,  
 $m_b^{\text{pole}} = 4.75 \text{ GeV}$ ,  $y_b$  evolved at  $\mu_R$  at  $n+1$  loops

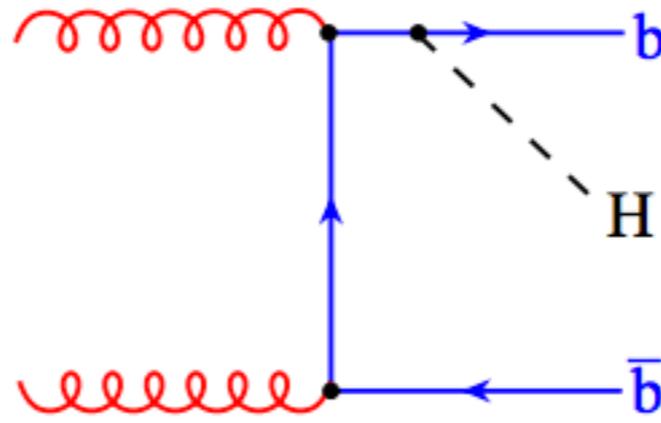


# The $bbH$ case

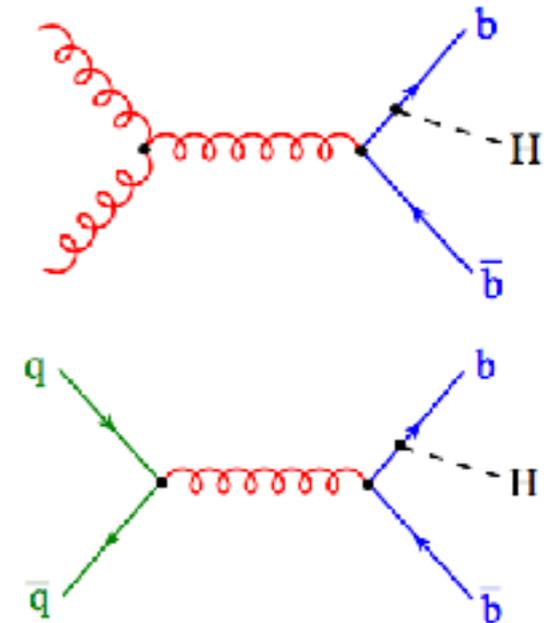
- Anatomy of bottom-fusion initiated Higgs production.
- For simplicity take 4FS **LO** diagrams (exclude cross diagrams and gluon emission from  $b$ )



$$\mathcal{O}(\alpha^2 L^2) + \mathcal{O}(\alpha^2 L^1) + \mathcal{O}(\alpha^2 L^0)$$



$$\mathcal{O}(\alpha^2 L^1) + \mathcal{O}(\alpha^2 L^0)$$

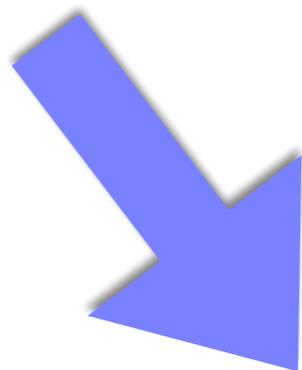
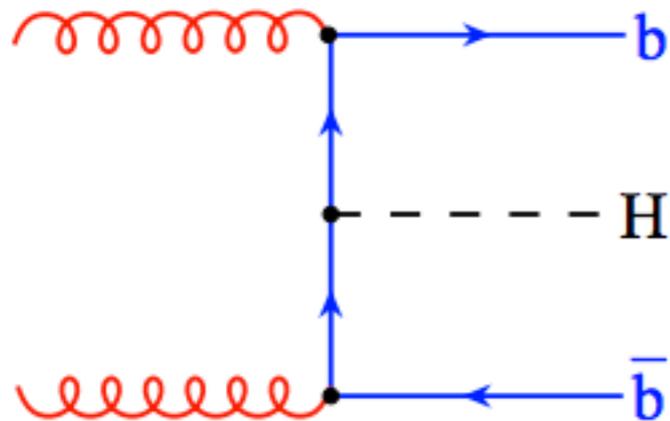


$$\mathcal{O}(\alpha^2 L^0)$$

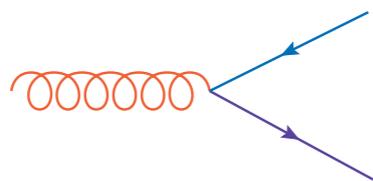
$$L = \log \frac{Q^2}{m_b^2}$$

# The $bbH$ case

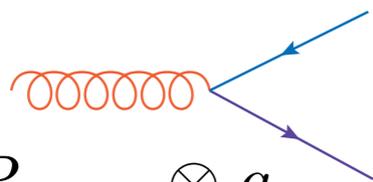
- In the massless/collinear limit, this diagram factorises into



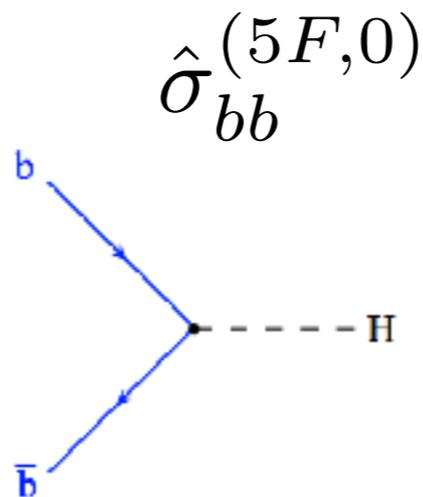
$$\tilde{f}_b^{(1)} = \frac{\alpha_s}{2\pi} L P_{g \rightarrow qq} \otimes g$$



$$\tilde{f}_b^{(1)} = \frac{\alpha_s}{2\pi} L P_{g \rightarrow qq} \otimes g$$

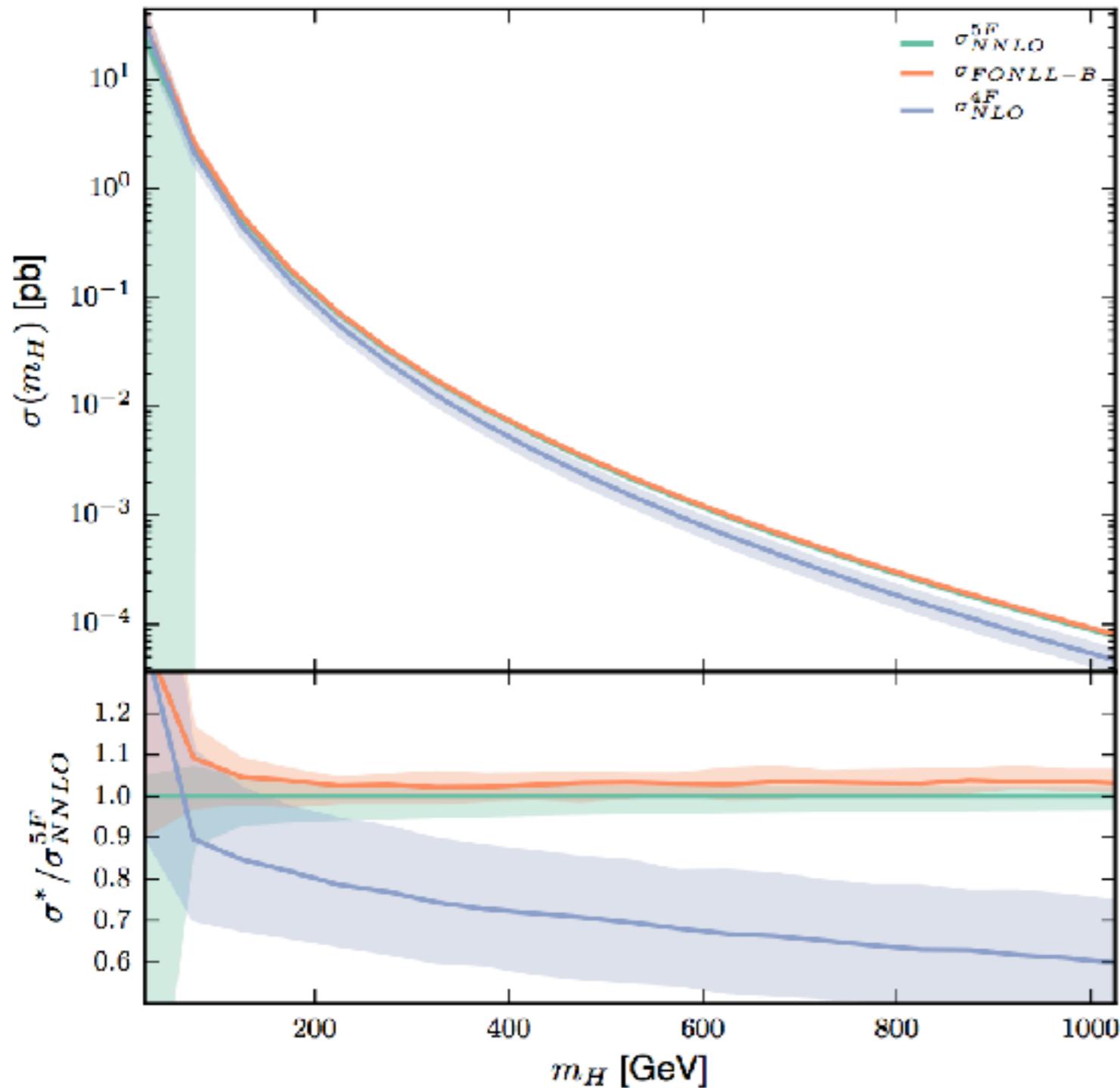


$\otimes$



These logs are double-counted in the 4FS and the 5FS. In the 4FS only the first one (two) log of the tower of logs resummed in the  $b$  PDFs are explicitly present and must be subtracted in the matching procedure

# The bbH case



- Bottomline: a matched cross section can be obtained for total cross section (normalisation)
- For differential cross section (distributions) one has to choose which scheme is more adequate depending on the process

Thank you for listening and  
for your questions!