Theory developments in the NNPDF collaboration

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What are we working on

Some **new features** we are developing:

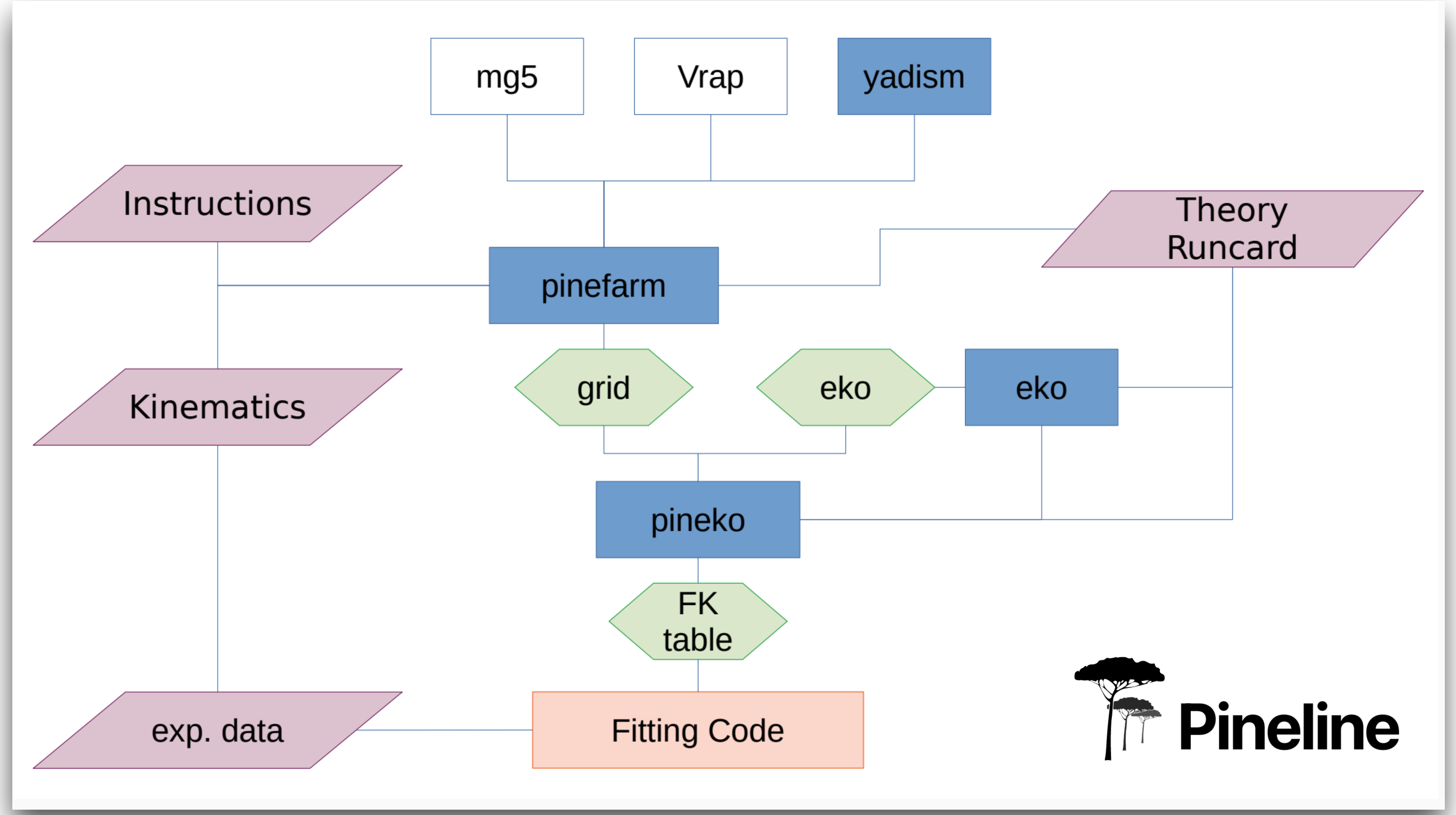
- **Theory uncertainties @ NNLO**
- **aN3LO PDFs fits**
- Full calculation of hadronic processes @ NNLO
- $QCD \otimes QED$ evolution

✓This has required the complete restructure of the theory pipeline used to calculate observables

The new *NNPDF theory pipeline*:

One program, one job.

Easier to maintain. Mainly python written. Open-source



For more documentation: https://nnpdf.github.io/pineline/
https://docs.nnpdf.science/
[arxiv:2211.10447]

- **Towards N3LO PDFs fits.**
- Estimation of MHOU from scale variations.
- Evidence for the IC in the proton.

Towards N3LO PDFs

- ➡ **N3LO DGLAP evolution: splitting functions approximation**
- ➡ N3LO DIS light coefficients *(+ approximation for the massive contributions)*
- ➡ Hadronic observables @ NNLO *(+ K-factors)*
- ➡ Inclusion of theory uncertainties both from scale variations and N3LO accuracy.

N3LO singlet sector

Analytical calculations of the complete N3LO spitting functions are not available yet. Restricting to the singlet sector the known limits are:

large-n_f

- Davies, Vogt, Ruijl, Ueda, and Vermaseren. Large-n_f contributions to the four-loop splitting functions in QCD. [arXiv:1610.07477]

small-x

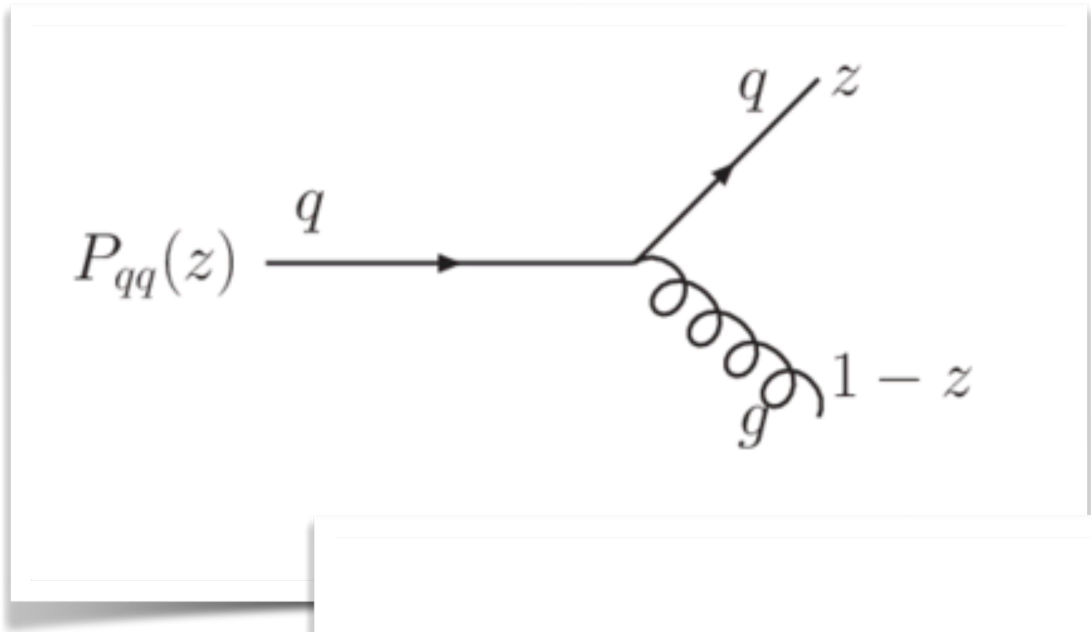
- Bonvini and Marzani. Four-loop splitting functions at small-x. [arXiv:1805.06460]
- Davies, Kom, Moch, and Vogt. Resummation of small-x double logarithms in QCD: inclusive deep-inelastic scattering. 2 2022. arXiv:2202.10362.

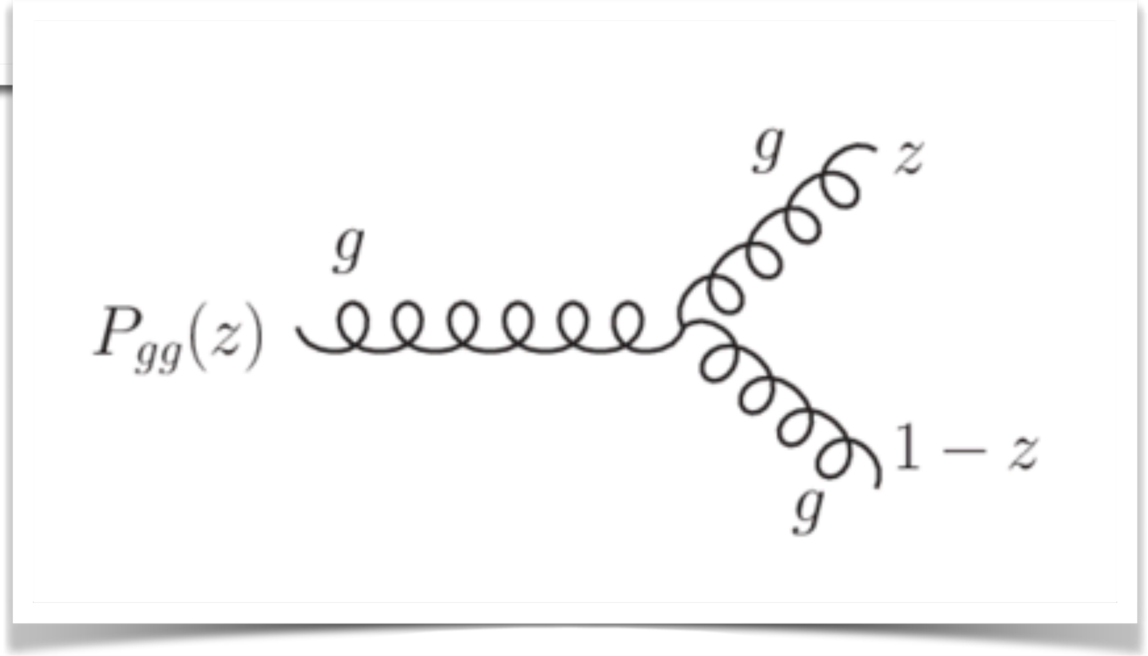
large-x

- Duhr, Mistlberger, and Vita. Soft integrals and soft anomalous dimensions at N3LO and beyond. [arXiv:2205.04493].
- Henn, Korchemsky, and Mistlberger. The full four-loop cusp anomalous dimension in $\mathcal{N} = 4$ super Yang-Mills and QCD. [arXiv:1911.10174].
- Soar, Moch, Vermaseren, and Vogt. On Higgs-exchange DIS, physical evolution kernels and fourth-order splitting functions at large x. [arXiv:0912.0369].

Moments

- Moch, Ruijl, Ueda, Vermaseren, and Vogt. Low moments of the four-loop splitting functions in QCD. [arXiv:2111.15561].





- Theoretical inputs are not enough to determine the full expressions analytically.
- Need to parametrise the unknown part with sub-leading contributions.
- Uncertainties from this determination has to be taken into account during the fit.

Singlet

	n_f^0	n_f^1	n_f^2	n_f^3
$\gamma_{gg}^{(3)}$	✓	✓	✓	✓
$\gamma_{gq}^{(3)}$	✓	✓	✓	✓
$\gamma_{qg}^{(3)}$		✓	✓	✓
$\gamma_{qq,ps}^{(3)}$		✓	✓	✓

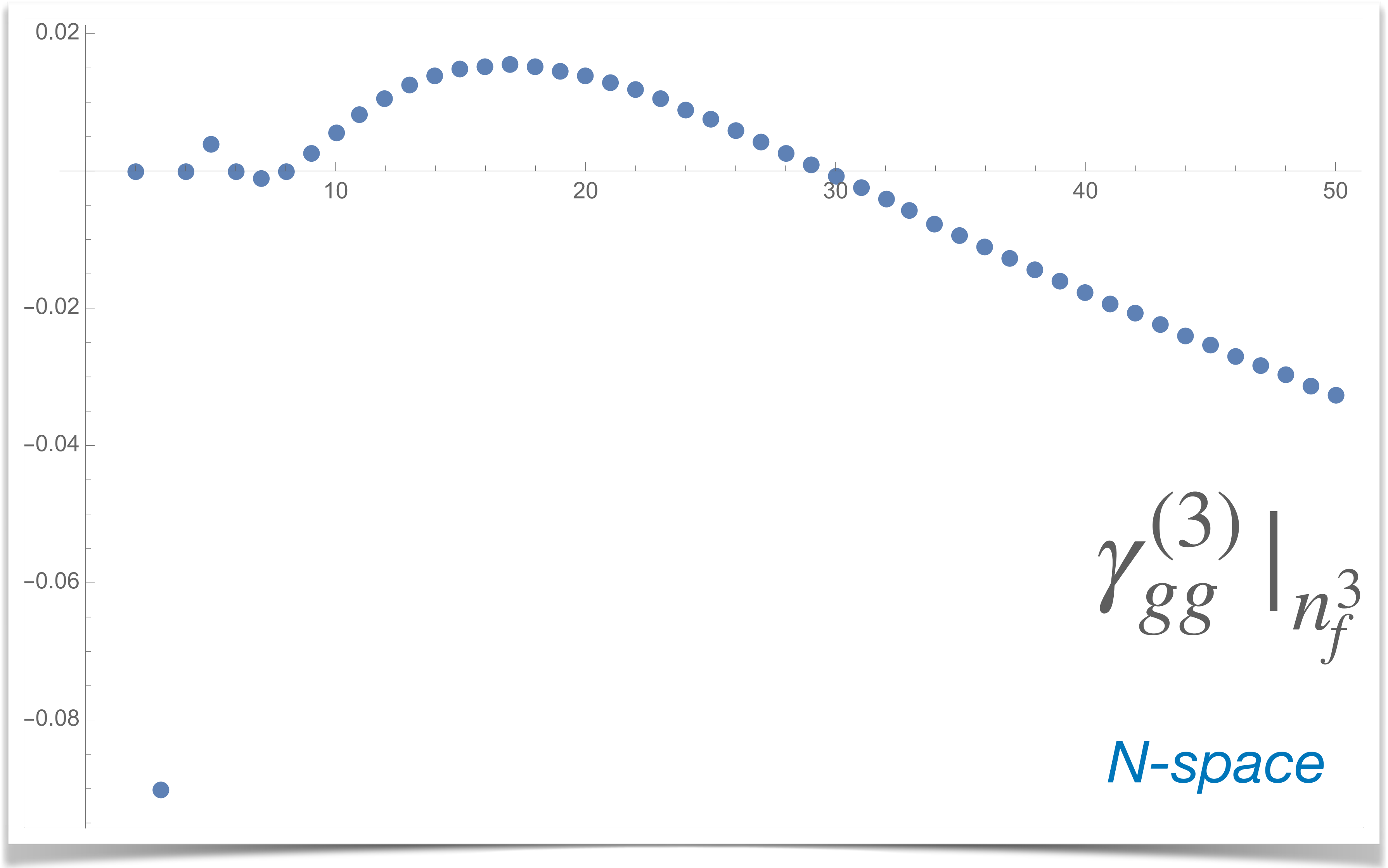
For results about the Non Singlet sector see slides 25-26

Approximation of $P_{gg}(x)$

$$\tilde{f}(N) = \int_0^1 x^{N-1} f(x) dx$$

Rule of thumb:
small-N → small-x,
large-N → large-x

Comparison w.r.t. known analytical part (%)



The approximation procedure is performed in Mellin space for each n_f part independently:

1. Parametrise the difference between the *4* known moments and known limits with *4* functions $f_i(N)$.
2. Varying the sub-leading unknown $f_i(N)$ to produce a large set of parameterisation candidates (≈ 70).
3. Reduce the number of samples discarding too wiggly parameterisations and looking at the most representative cases.

In $P_{gg}(x)$:

Theoretical constrain include:

- large-N:

$$\gamma_{gg}^{(3)}(N \to \infty) \approx \Gamma_A S_1(N) + B_{gg} + \mathcal{O}(\frac{\ln(N)}{N})$$

- small-N pole at $N = 0$, and $N = 1$ *(leading contribution)*:

$$\gamma_{gg}^{(3)}(N \to 1) \approx C_4 \frac{1}{(N-1)^4} + C_3 \frac{1}{(N-1)^3} + \mathcal{O}((N-1)^{-2})$$

- 4 lowest moments $N = \{2,4,6,8\}$

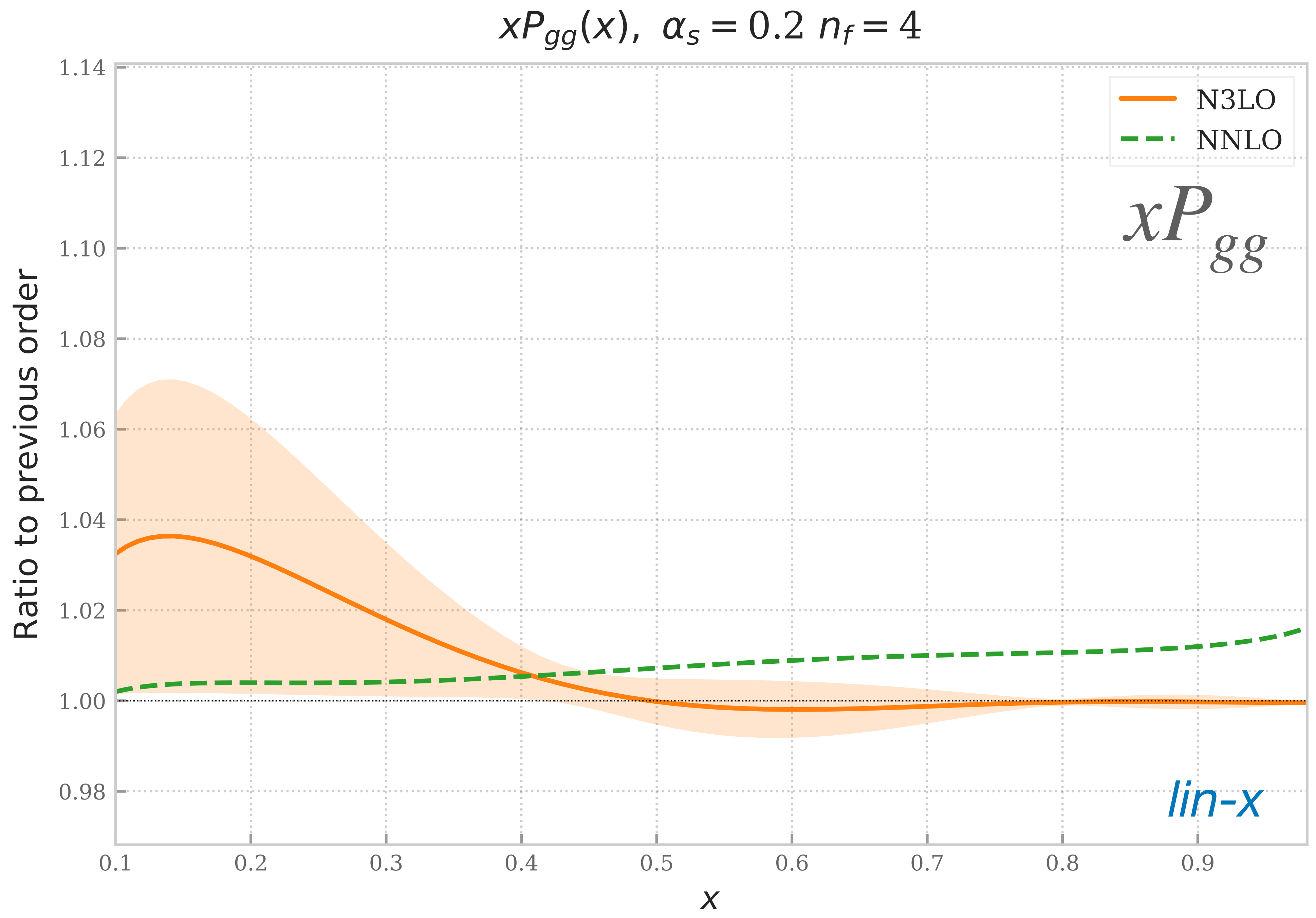
Solve the constrain given by the 4 known Mellin moments with many different candidates $\{f_1, f_2, f_3, f_4\}$:

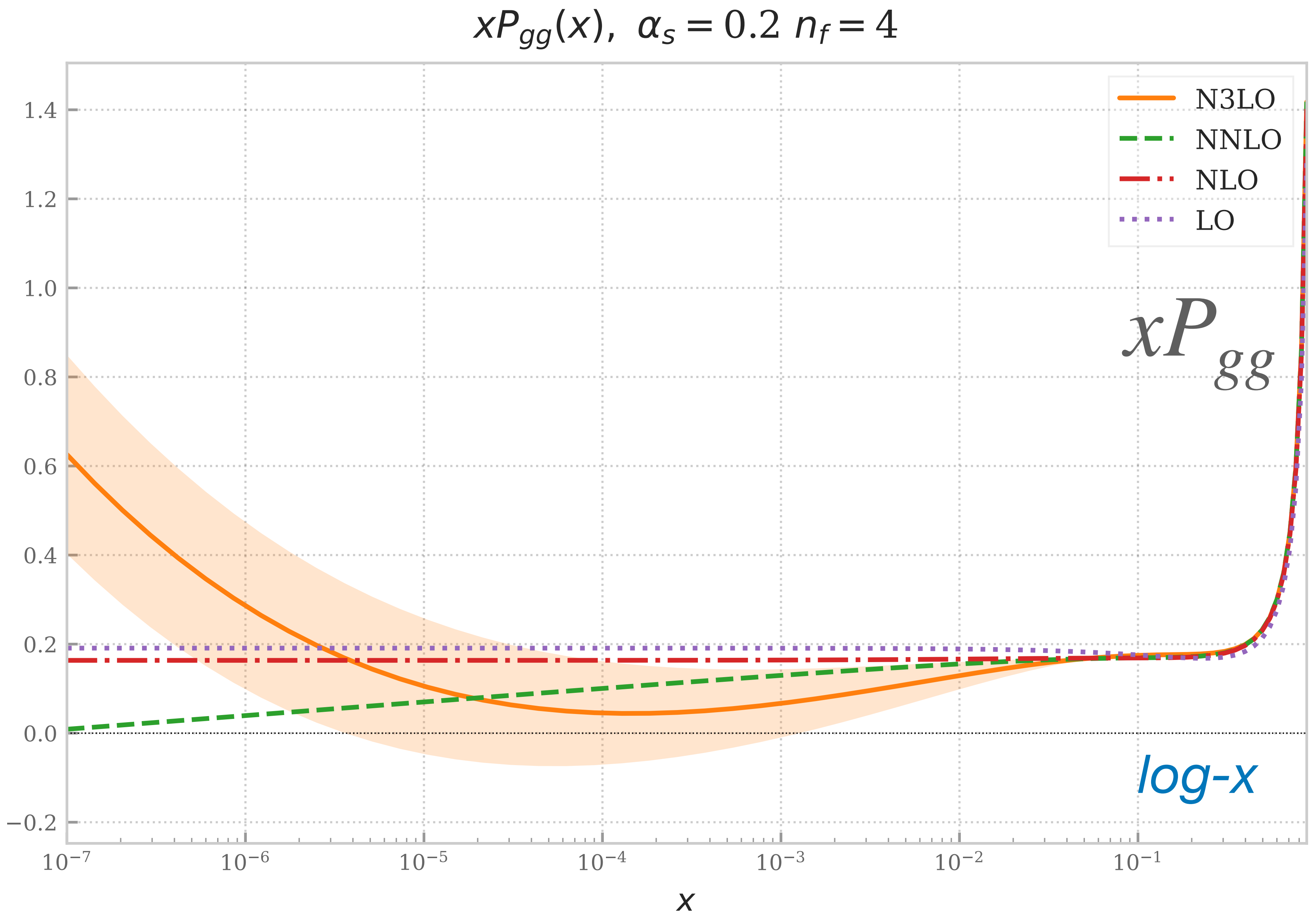
$$f_1 = \frac{S_1(N)}{N}, \; f_2 = \frac{1}{(N-1)^2}$$

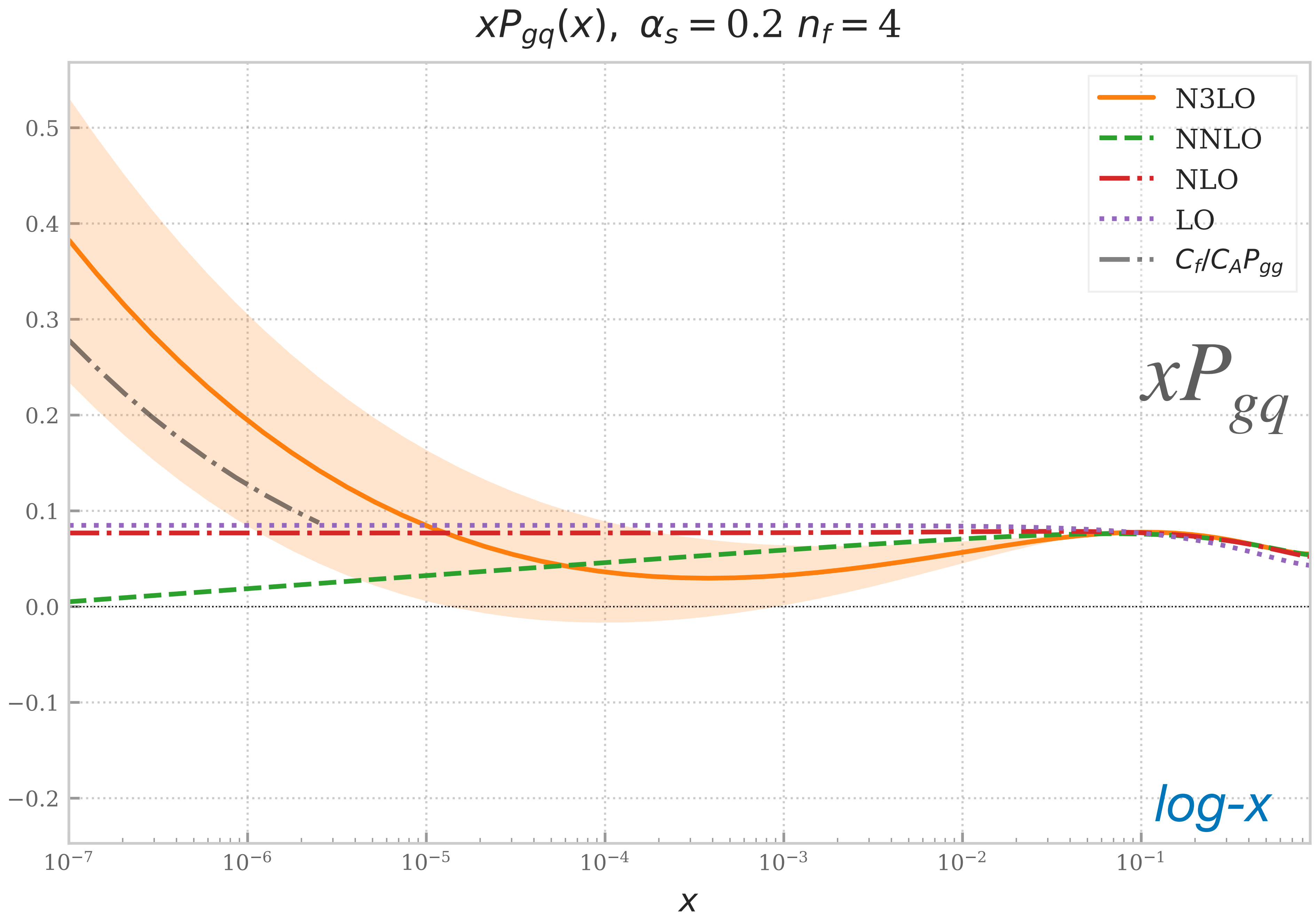
$$f_3, f_4 = \{ \frac{1}{(N-1)}, \frac{1}{N^4}, \frac{1}{N^3}, \frac{1}{N^2}, \frac{1}{N}, \frac{1}{(N+1)^3}, \frac{1}{(N+1)^2}, \frac{1}{N+1}, \frac{1}{N+2}, \mathcal{M}[\ln(1-x)], \mathcal{M}[(1-x)\ln(1-x)], \frac{S_1(N)}{N^2} \}$$

N3LO singlet sector

PRELIMINARY RESULTS



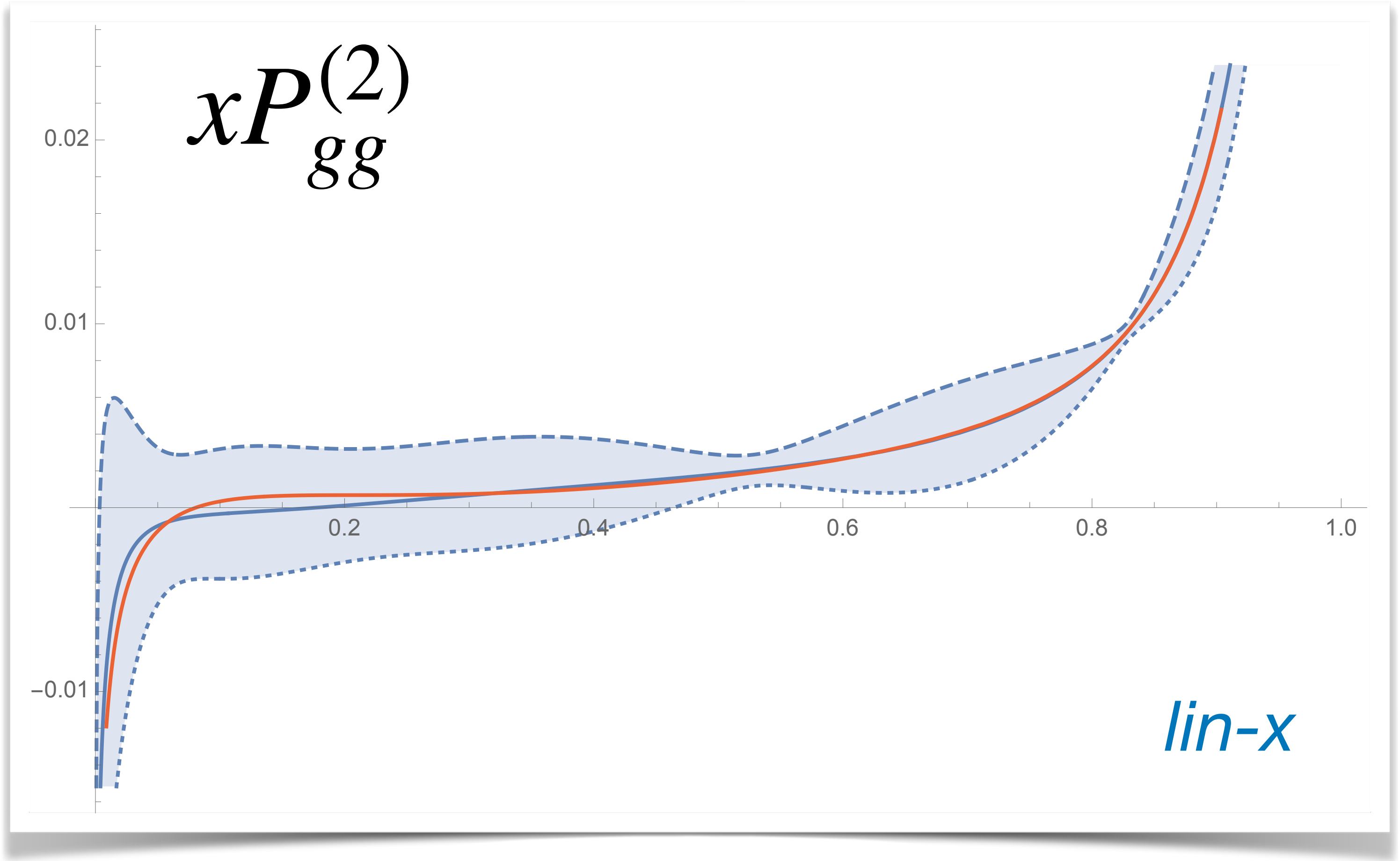


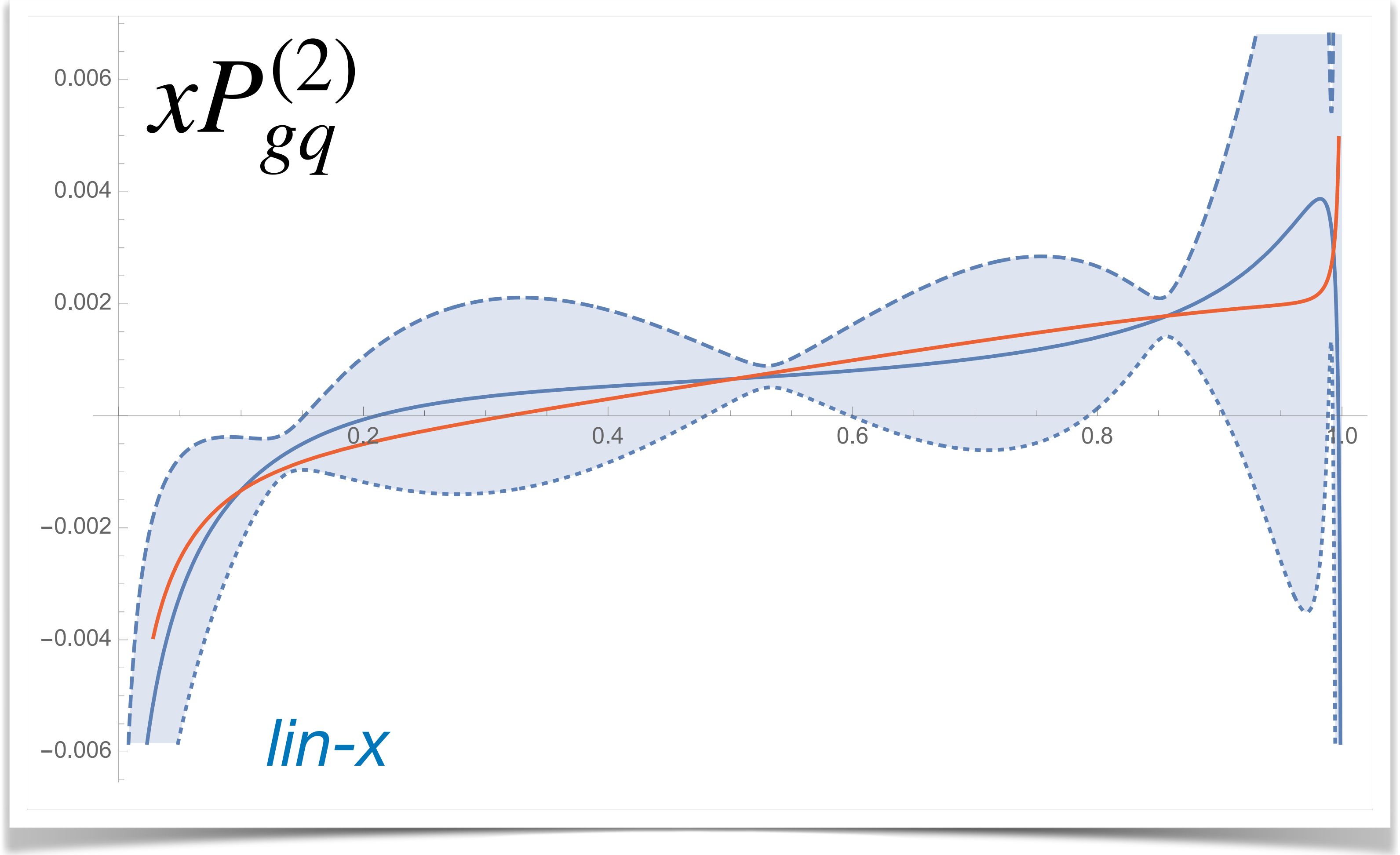


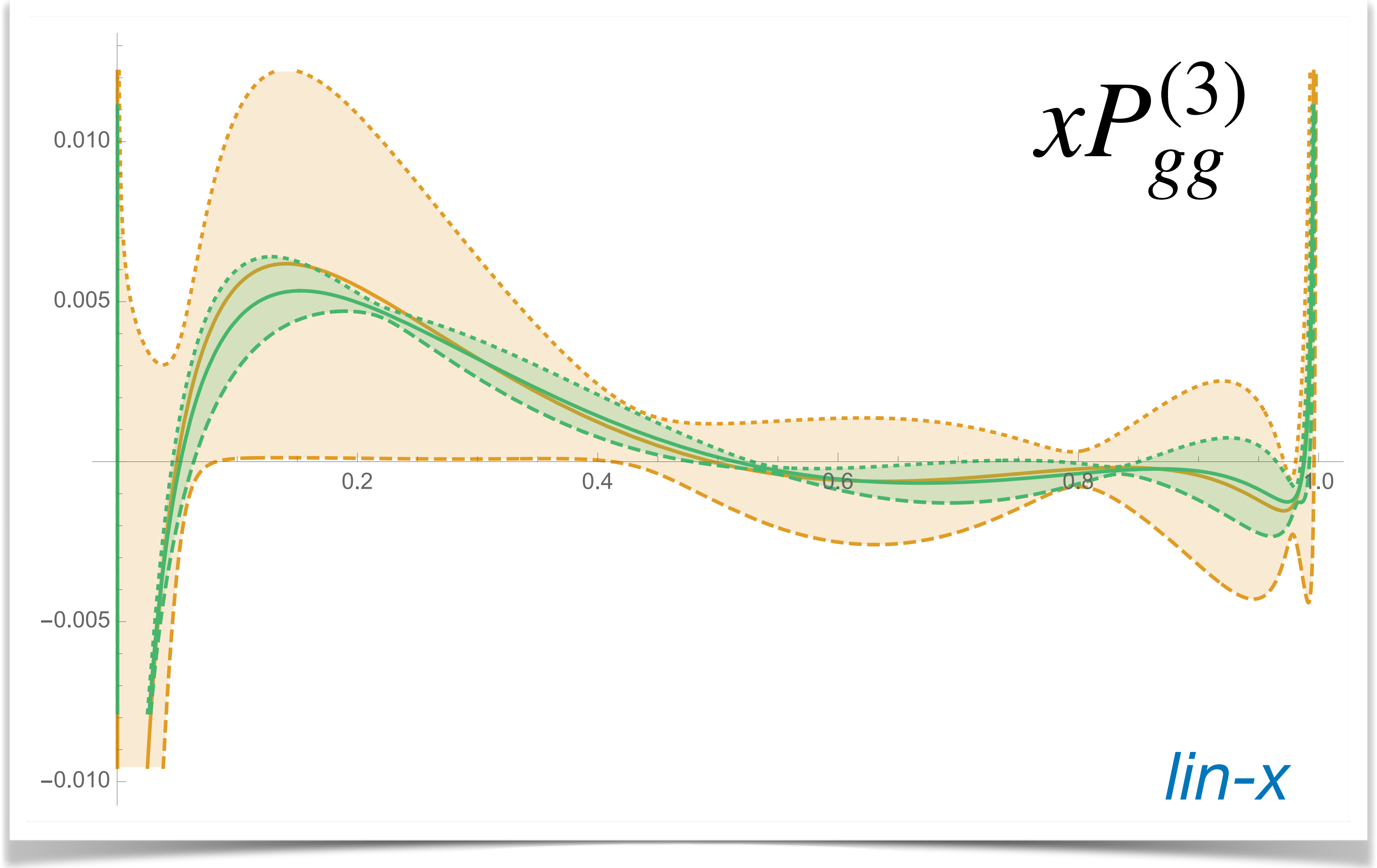
- Singlet approximated spitting functions are less constrained by the known limits. The coefficients of $1/x \ln^2(x),\ 1/x \ln(x)$ play a crucial role in the small-x region.
- Uncertainty arising from the approximation is not negligible.
- Off diagonal terms $P_{qg},\ P_{gq}$ are more difficult to estimate (large-N goes to 0).
- Only theoretical inputs are considered.
- All the implemented approximations respect momentum sum rules.

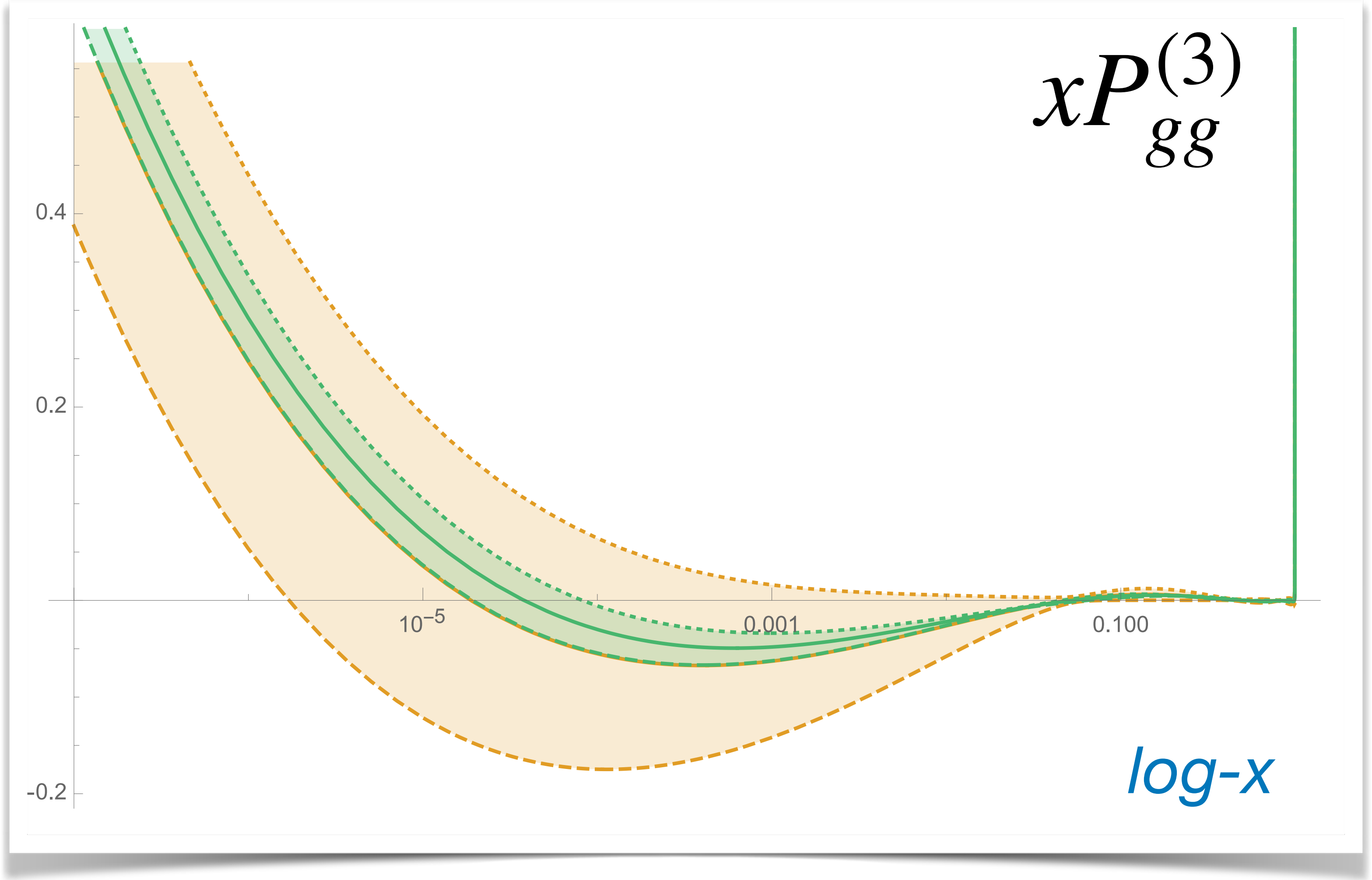
Approximation checks

1. A possible way to validate the procedure is to **reproduce the known NNLO** singlet splitting functions using the very similar constrain that we have right now on the N3LO ones.







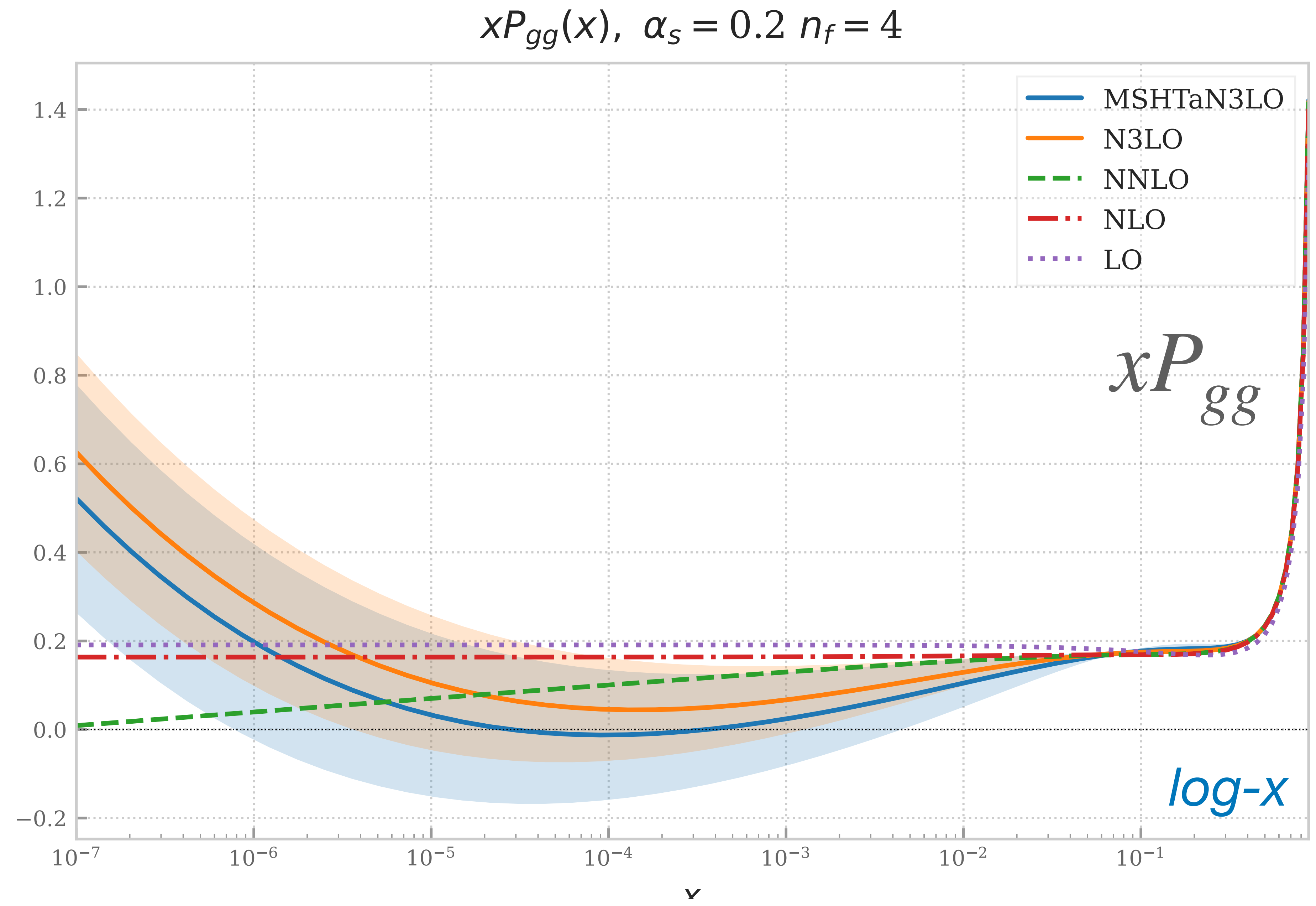


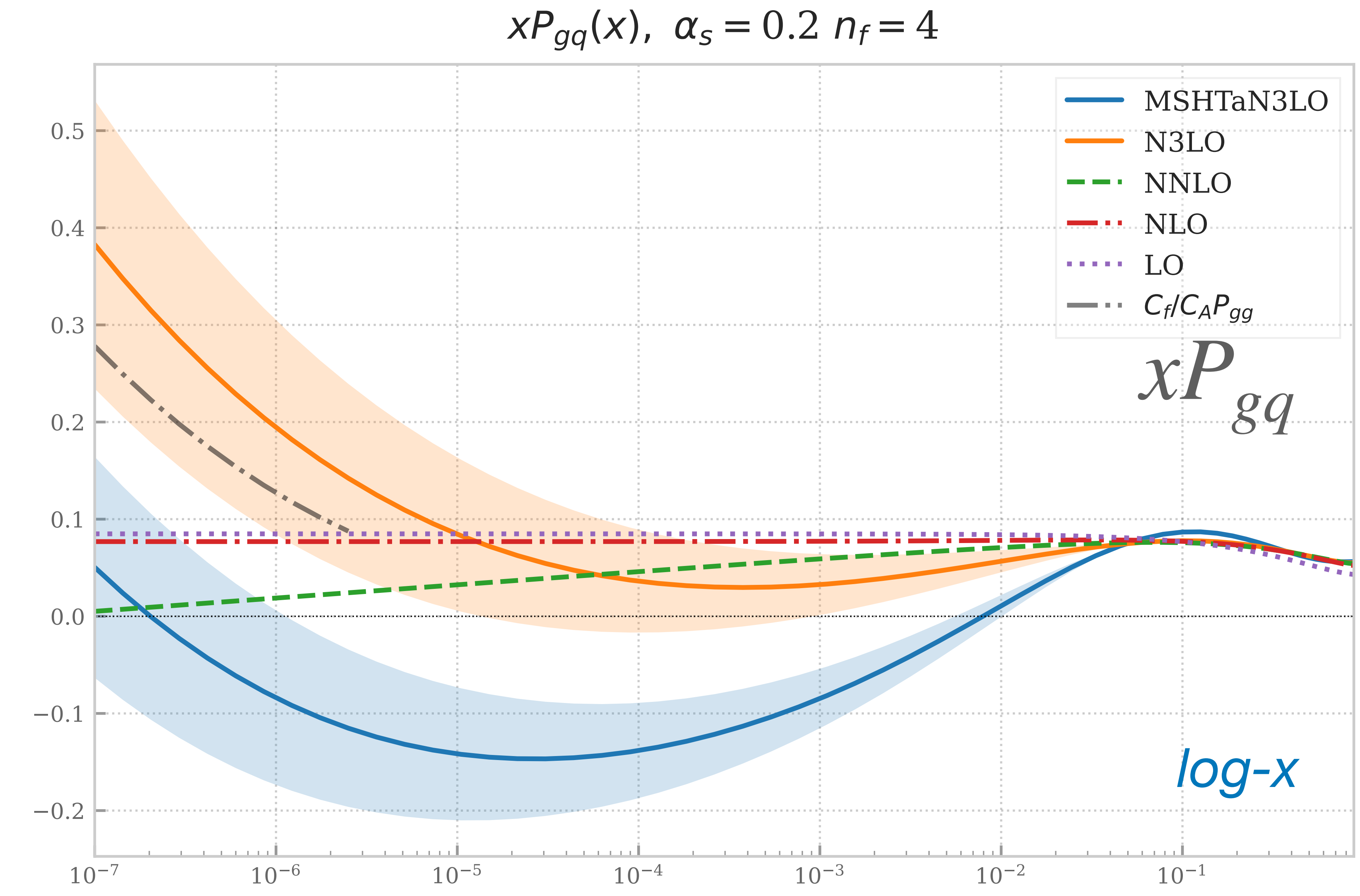
2. Another way to validate the results is to **interpolate the known moments**, and construct a more constrained parametrisation now including 5/6 moments.
If the procedure is working (the samples are varied enough) the uncertainty band obtained in this way should be small than the default one.

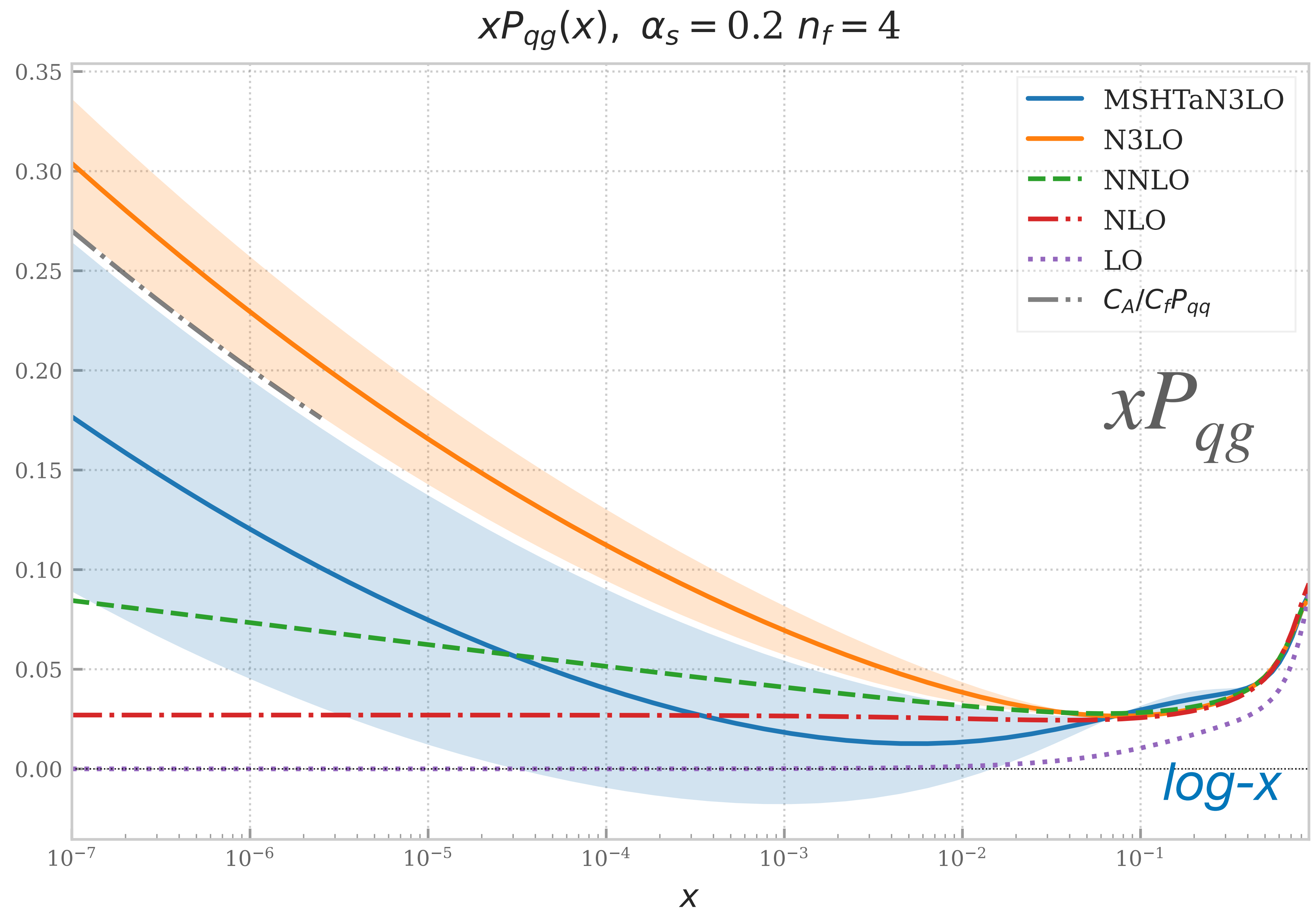
Comparison with MSHT small-x

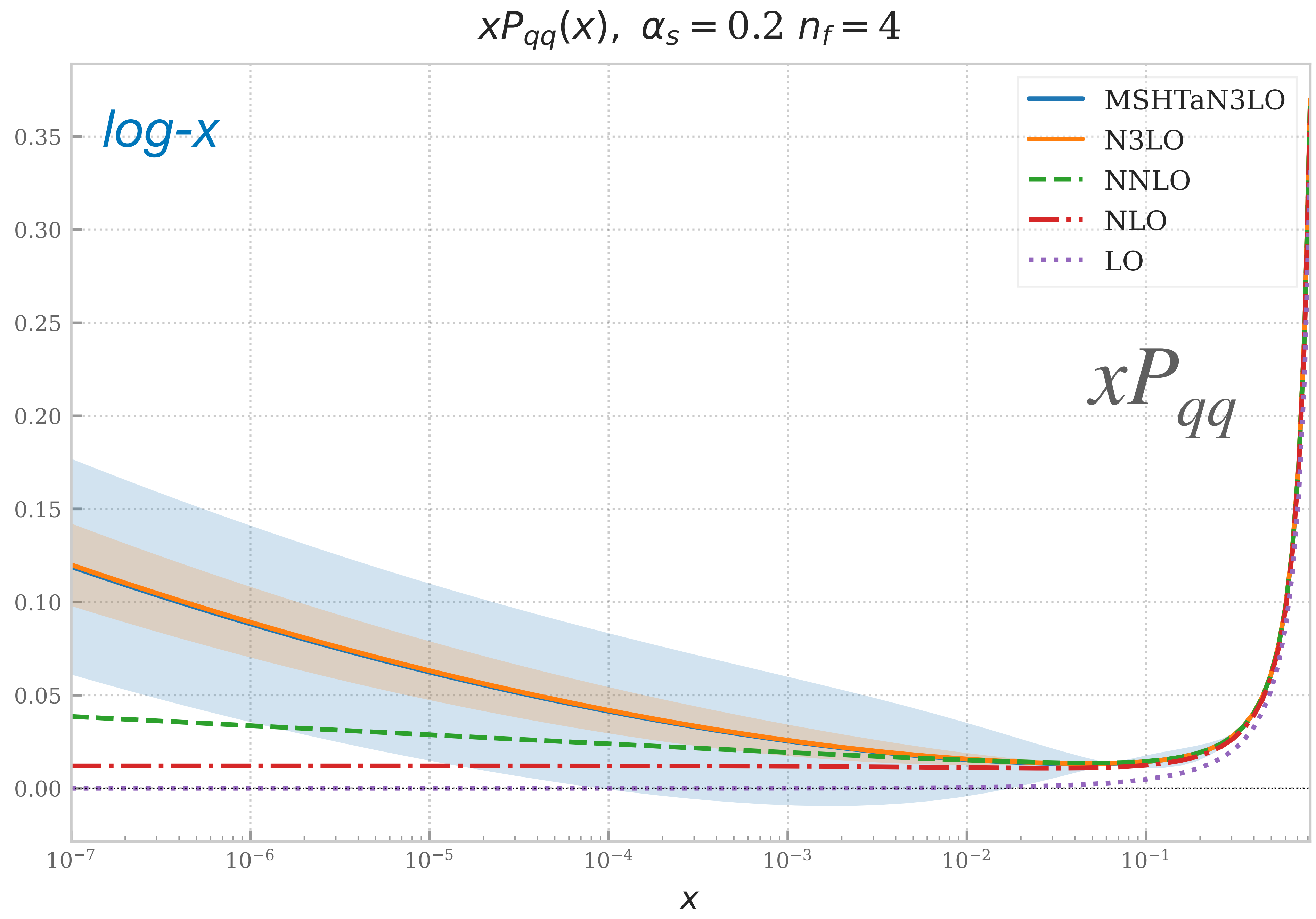
MSHTaN3LO: [arxiv:2207.04739]

PRELIMINARY RESULTS





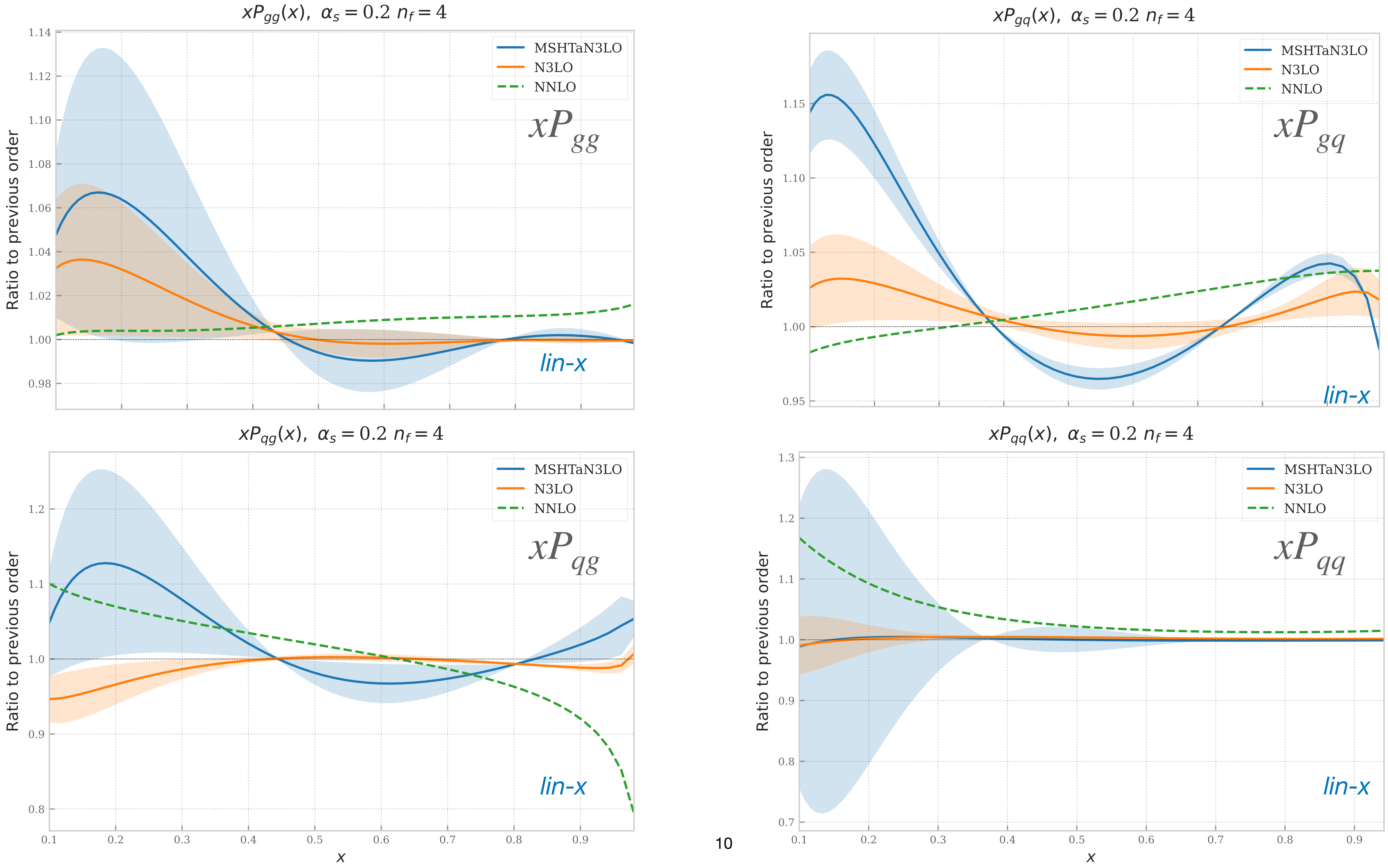




Comparison with MSHT large-x

MSHTaN3LO: [arxiv:2207.04739]

PRELIMINARY RESULTS



Impact on the covariance matrix

What can be the effect of the uncertainties coming from the N3LO splitting functions?

- Construct a theory covariance matrix by varying one single candidate (during the DGLAP evolution) at the time:

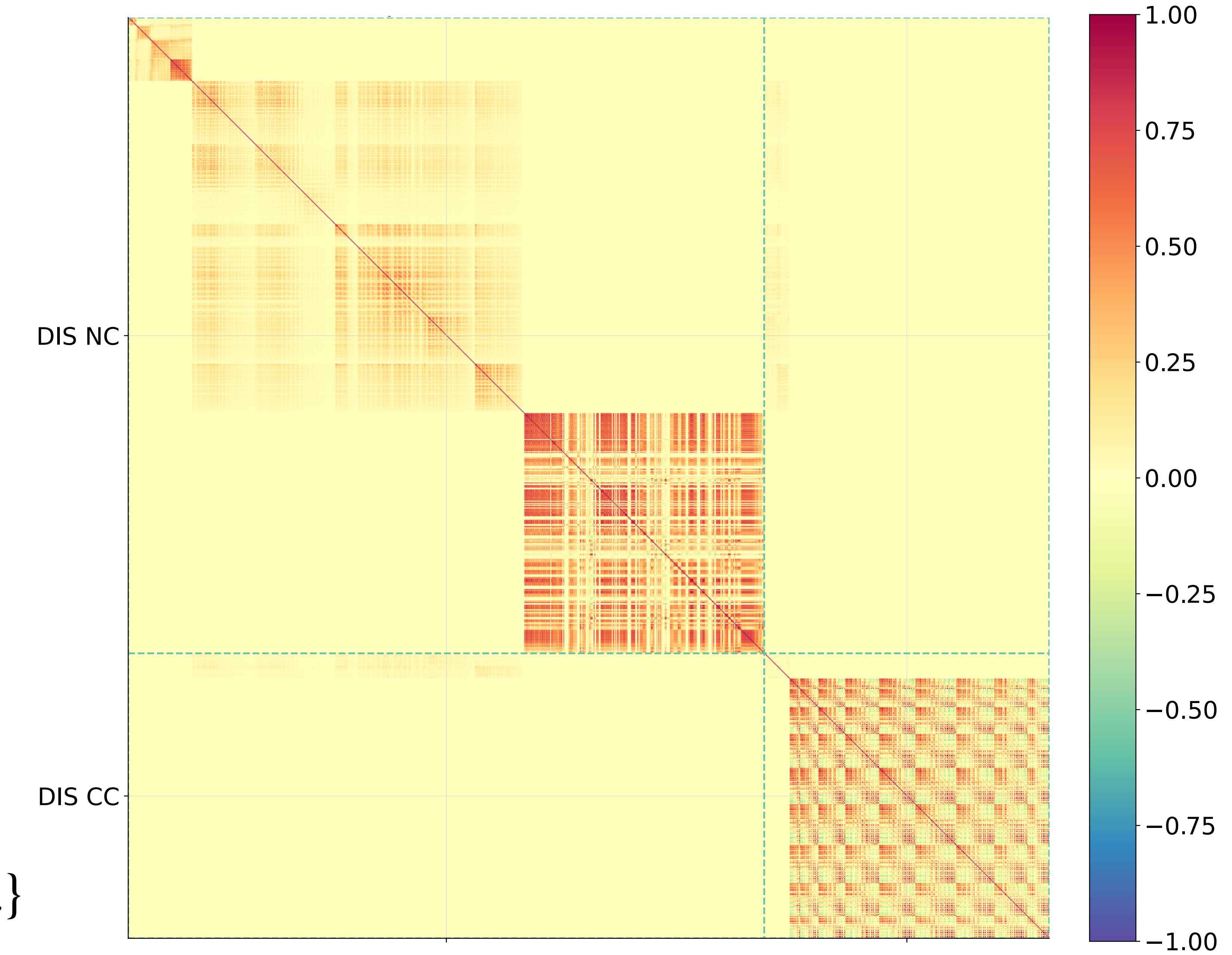
$$\delta^2_{aN3LO_{mn}} = \frac{1}{N_{dat}}(\sigma_{i,n} - \sigma_{0,n})(\sigma_{i,m} - \sigma_{0,m}), \quad n = \{0,\ldots,N_{dat}\}\ i = \{0,\ldots,N_{var}\}$$

- This will produce an ≈ 80 point prescription theory covmat assuming that each variation is not correlated to the others.

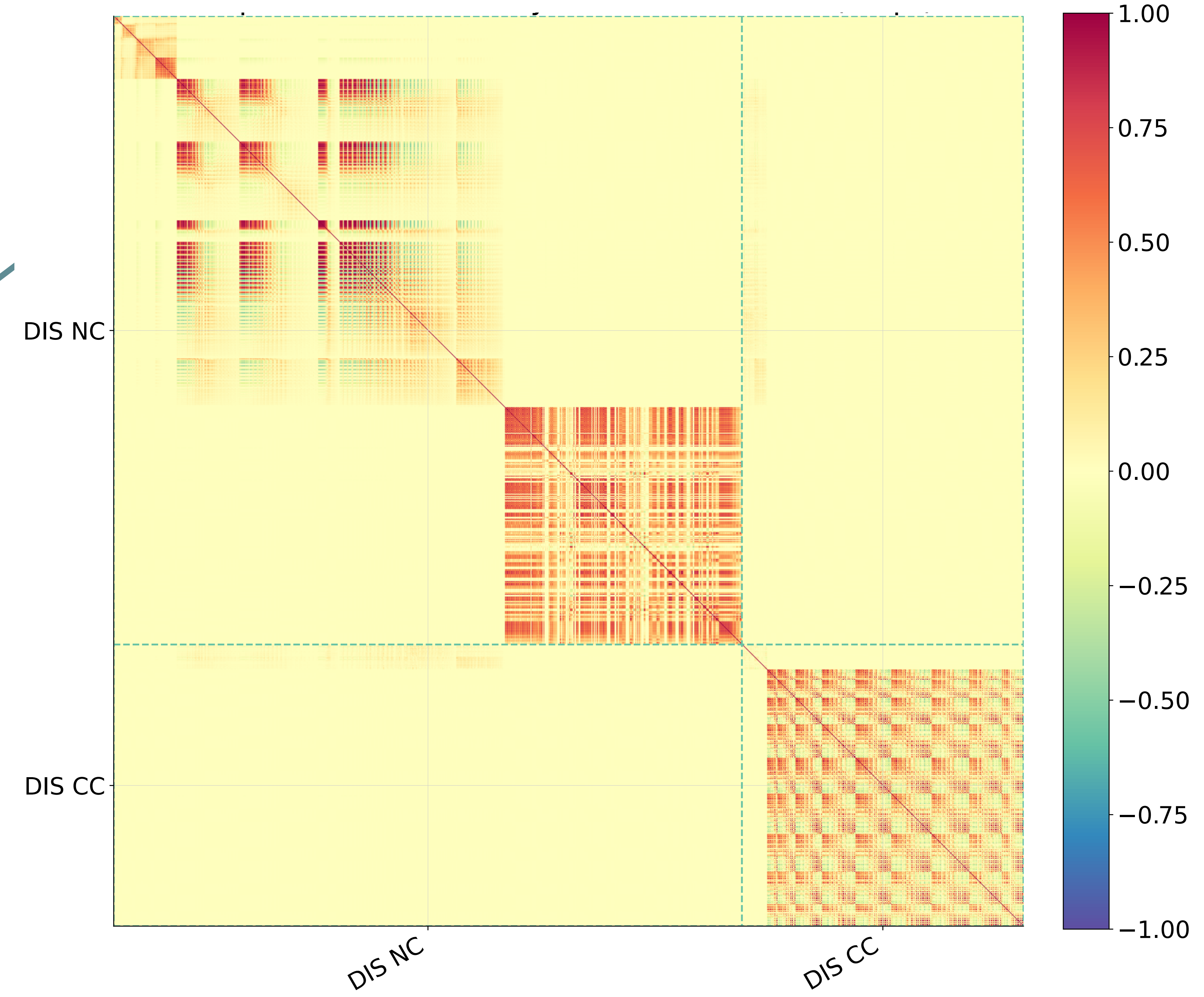
- This source of uncertainty can added to the "standard" theory covariance mat obtained with scale variations:

$$\delta^2_{th} = \delta^2_{MHOU} + \delta^2_{aN3LO}$$

Exp correlations



Exp +Theory correlations **aN3LO**



Larger effect on the small-x HERA data

- Towards N3LO PDFs fits.
- **Estimation of MHOU from scale variations.**
- Evidence for the IC in the proton.

MHOU from scale variations

General formalism how to introduce theory uncertainties in PDFs have been addressed in various studies:
MSTH [arxiv:1811.08434], NNPDF [arxiv:1906.10698], [arxiv:2105.05114]

Scale variation advantages:

- Justified by RGE invariance.
- Valid for every process.

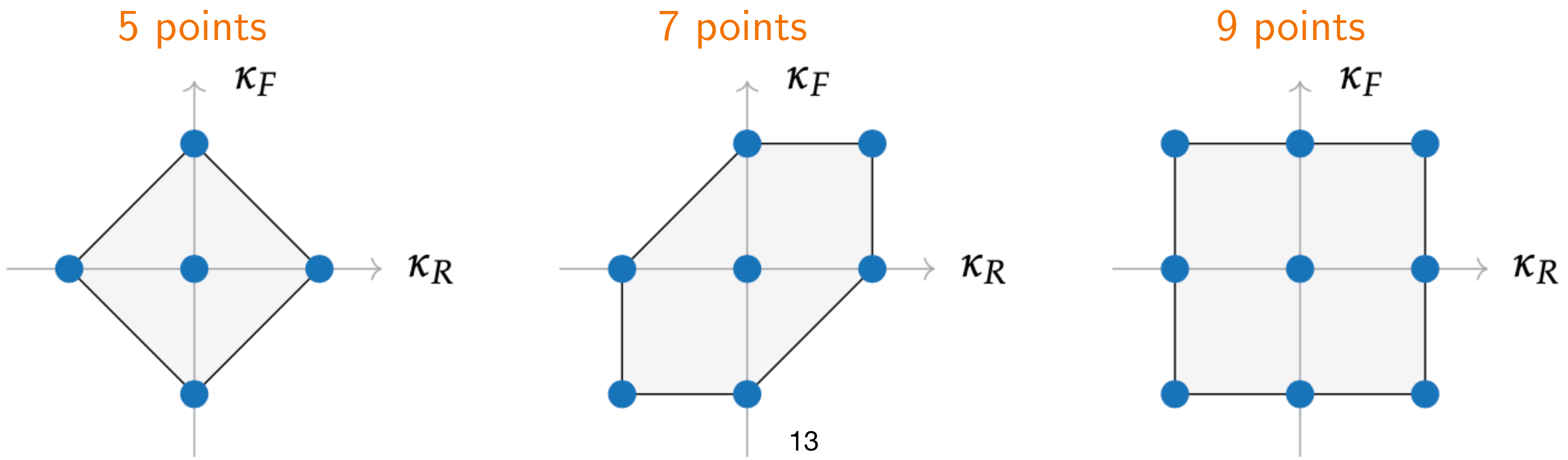
- Not a unique procedure. There are at least 3 different schemes that can be used to compute MHOU. Differences are always higher orders.
- Factorisation scale variations are introduced during the DGLAP evolution. Renormalization scale variations are retained inside the coefficient functions.
- The way in which μ_f, μ_r are varied simultaneously define a so called point prescription.

Evolution

$$FK_{ik,n}(Q, \mu_f, \mu_r) = E_{ij,n}(Q, \alpha_s, \mu_f) \otimes C_{jk,n}(Q/\mu_r, \alpha_s)$$

$$n = \{1 \ldots N_{dat}\}$$

Partonic coeff

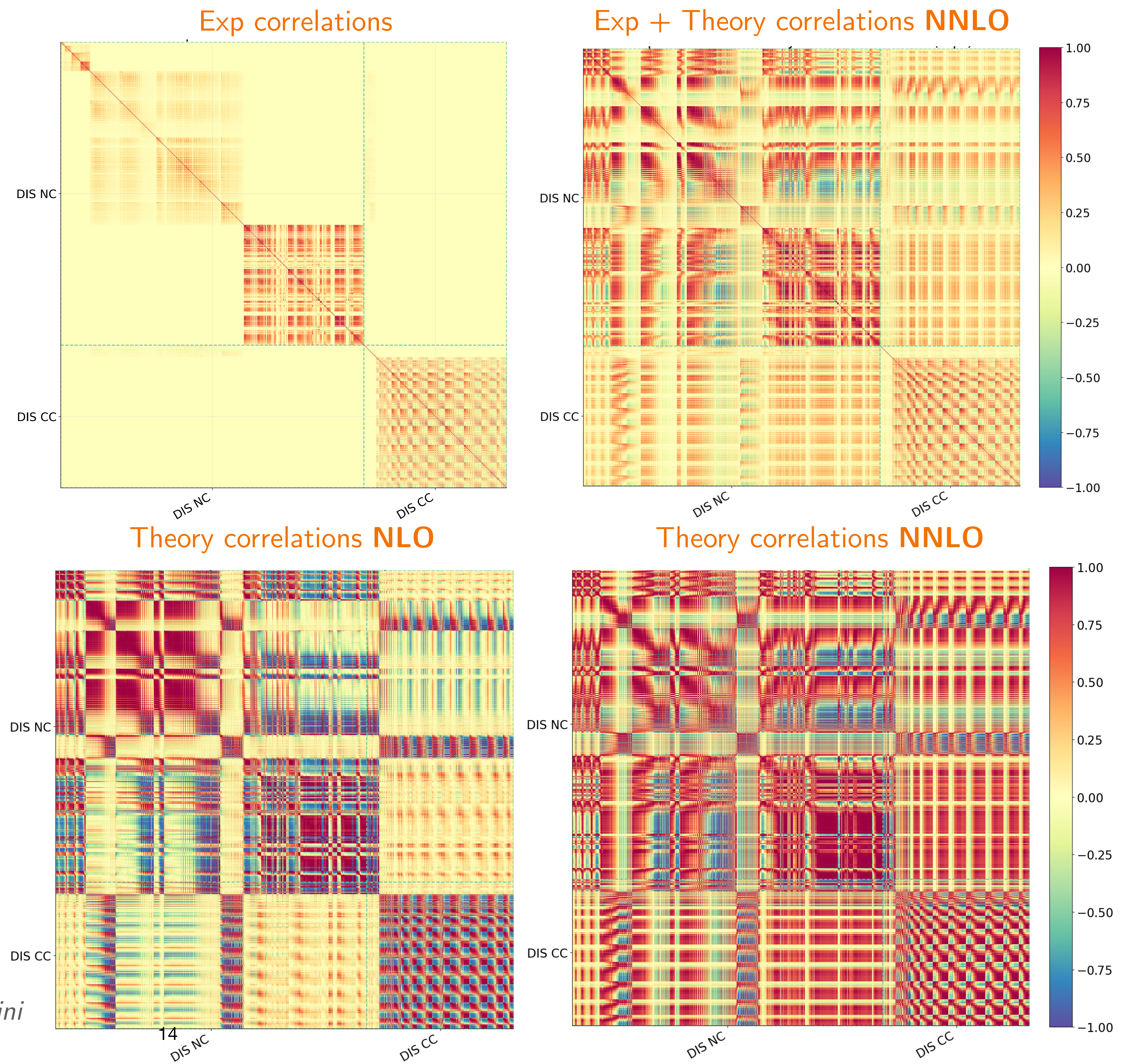


MHOU from scale variations

Impact of the theory uncertainties

- μ_f/Q, μ_r/Q are varied in the range $[0.5,\ 1,\ 2]$
- Theory uncertainties add correlations between datasets, which are not taken into account in the experimental covariance mat.
- Effects on the fit are not trivial.
- Result of [arxiv:1906.10698] at NLO have been fully benchmarked.
- DIS datasets are ready, hadronic process are on the way.

Main goal:
include MHOU at NNLO for all the NNPDF4.0 datasets**.**



Plots from Andrea Barontini

- Towards N3LO PDFs fits.
- Estimation of MHOU from scale variations.
- **Evidence for the IC in the proton.**

Intrinsic charm in the proton

- *Do heavy quarks contribute to the proton PDFs at low scales?*
- *If yes, how can they be determined?*
- *If yes, do they have any impact on LHC processes?*

- Focus on the **charm**, as the *natural candite* to answer this question ($m_c = 1.51\ GeV$).

- Results based on:

[arxiv:2208.08372]
Nature 608 (2022) 7923, 483-487
Richard D. Ball , Alessandro Candido , Juan Cruz Martininez, Stefano Forte, Tommaso Giani, Felix Hekhorn, Kirill Kudashkin, GM and Juan Rojo.

Original idea from 1980

THE INTRINSIC CHARM OF THE PROTON

S.J. BRODSKY [1]
Stanford Linear Accelerator Center, Stanford, California 94305, USA

and

P. HOYER, C. PETERSON and N. SAKAI [2]
NORDITA, Copenhagen, Denmark

Received 22 April 1980

Recent data give unexpectedly large cross-sections for charmed particle production at high x_F in hadron collisions. This may imply that the proton has a non-negligible $uudc\bar{c}$ Fock component. The interesting consequences of such a hypothesis are explored.

Fitted vs perturbative only charm

In VFNS there are 2 options of treating the charm pdf.

In a **purely perturbative** scenario the charm PDF is determined as:

$$c^{(3)}(x) = 0 \;\rightarrow\; c^{(4)}(x, m_c) = \sum_{i=g,q} A_{c,i} f_i^{(3)}(x, m_c) \approx \mathcal{O}(a_s^2)$$

$c^{(4)}(x, Q)$ functional form is fully determined by the DGLAP evolution and the initial boundary conditions.

Allowing for **Intrinsic Charm (IC)** means**:**

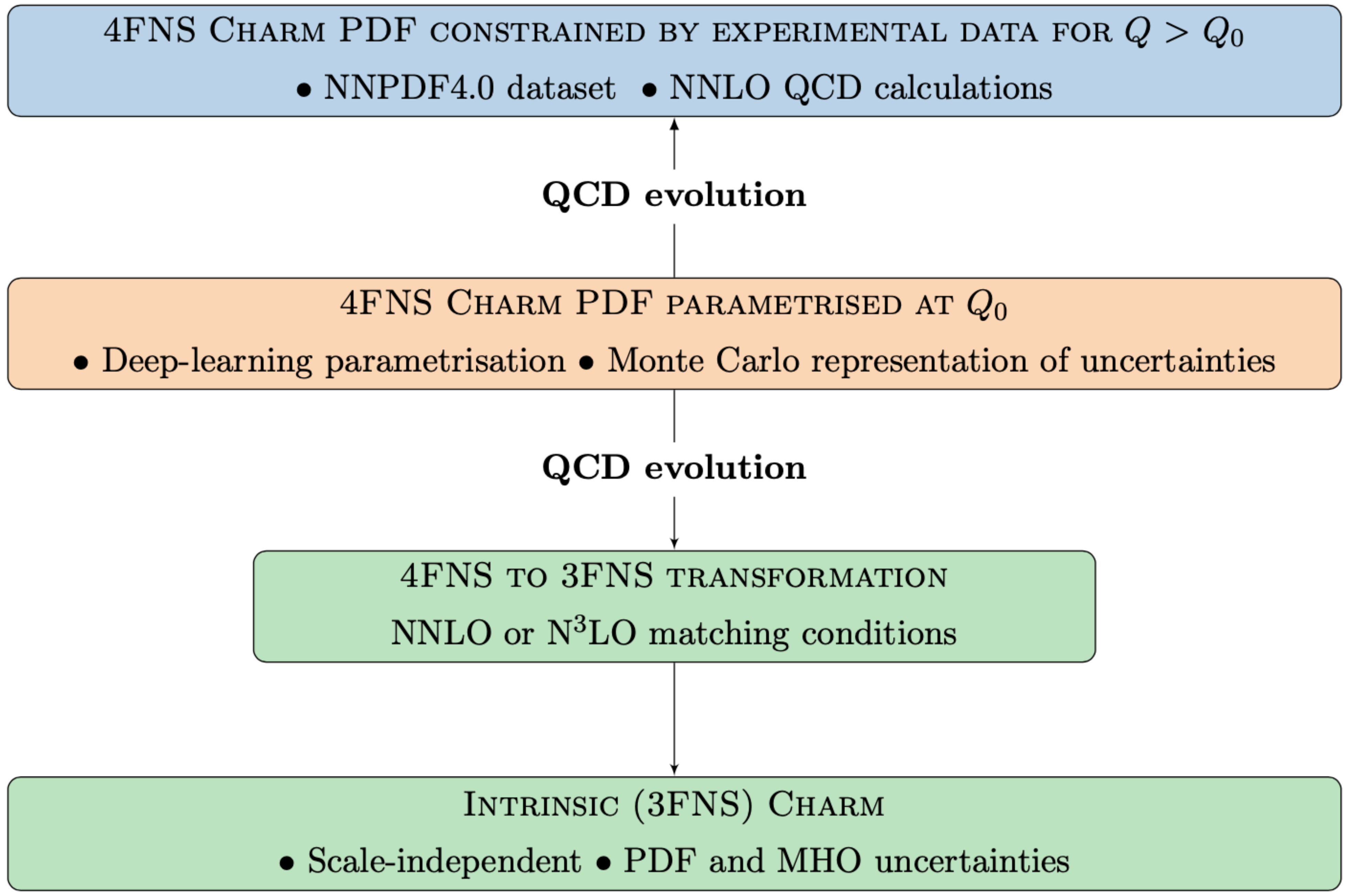
$$c^{(3)}(x) \neq 0 \;\rightarrow\; c^{(4)}(x, m_c) = \sum_{i=g,q} A_{c,i} f_i^{(3)}(x, m_c) + A_{cc} c^{(3)}(x) \approx \mathcal{O}(a_s)$$

$c^{(4)}(x, Q)$ has to treated as the other light flavor and **fitted to the data.** It will consist of two components one perturbative (as before) an one intrinsic which has a NP origin.

From 4FNS to 3FNS

In NNPDF we parametrise initial PDFs in 4FNS at $Q_0 = 1.65\ GeV$.

To extract a possible IC component need to determine 3FNS PDFs.



When crossing the heavy flavor scale $Q^2 = \mu_h^2\ (= m_h^2)$ we need to match the two different flavor schemes using:

$$\begin{pmatrix} V^{(n_f)} \\ h^- \end{pmatrix}^{n_f+1} (\mu_h^2) = \mathbf{A}^{(n_f)}_{NS,h^-}(\mu_h^2) \begin{pmatrix} V^{(n_f)} \\ h^- \end{pmatrix}^{n_f} (\mu_h^2)$$

$$\begin{pmatrix} g \\ \Sigma^{(n_f)} \\ h^+ \end{pmatrix}^{n_f+1} (\mu_h^2) = \mathbf{A}^{(n_f)}_{S,h^+}(\mu_h^2) \begin{pmatrix} g \\ \Sigma^{(n_f)} \\ h^+ \end{pmatrix}^{n_f} (\mu_h^2)$$

$$\{g,\ \Sigma = \sum_i^{n_l} f_i^+,\ V = \sum_i^{n_l} f_i^-, \dots\}$$

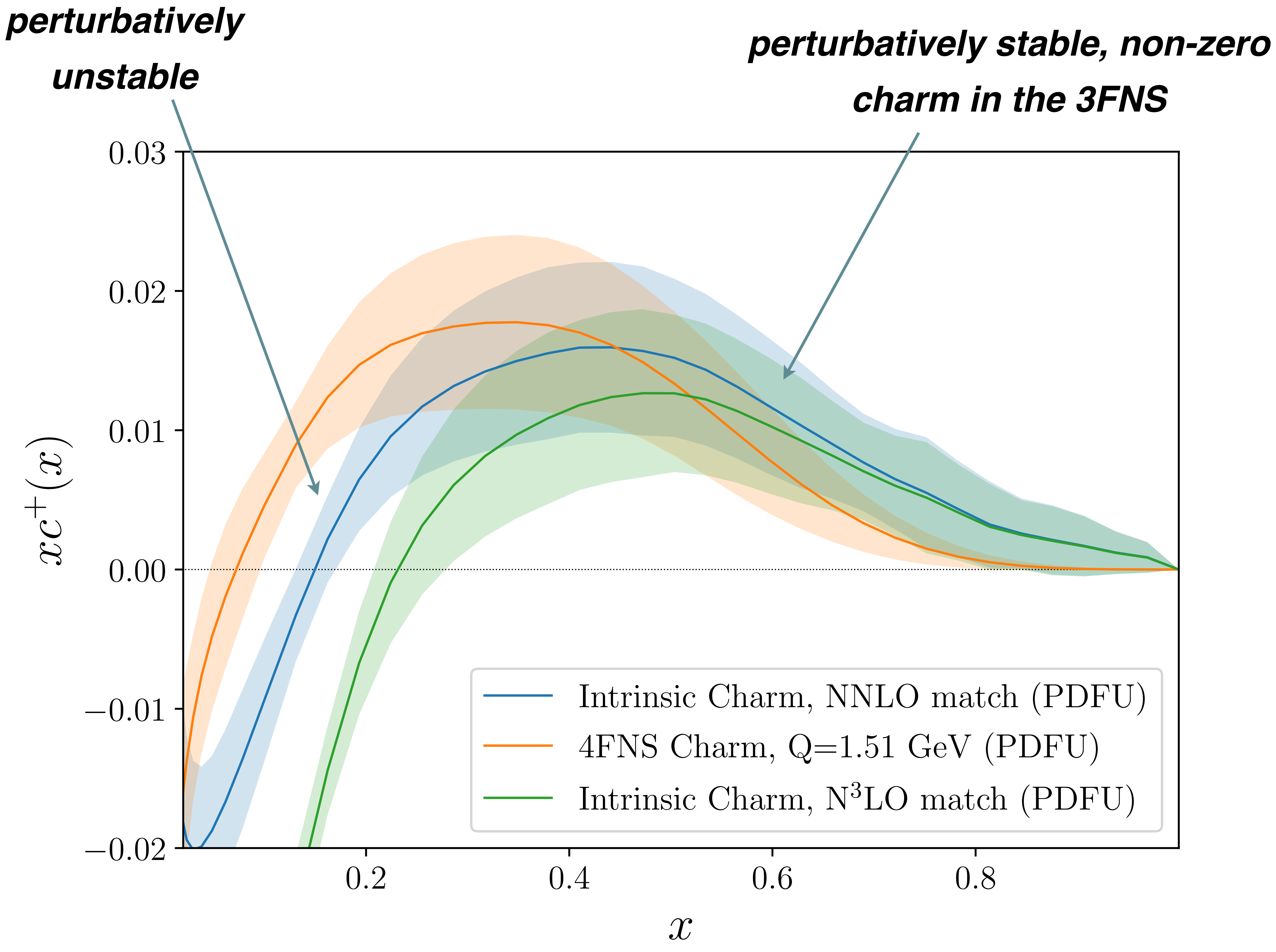
Where $\mathbf{A}_{ij} = \mathbf{A}_{ij}(\alpha_s, \log(\frac{\mu_h^2}{m_h^2}))$ are the **Operator Matrix Elements** available up to aN3LO

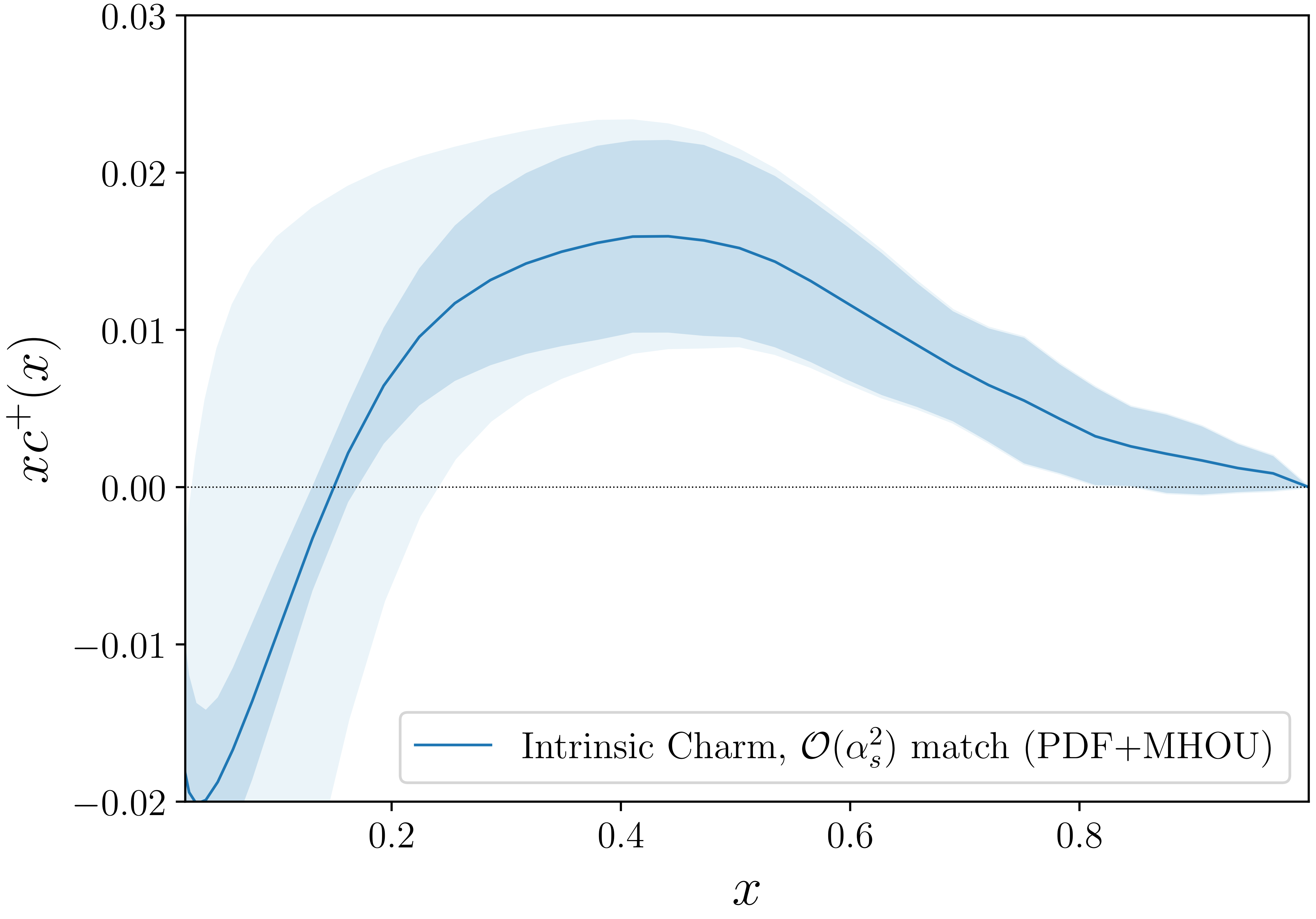
Matching conditions references:
[Eur.Phys.J.C 1 (1998) 301-320],
[arxiv:1510.00009],
[arxiv:1711.06717] et al.,
See slide 34 for complete bibliography

Intrinsic charm

In **3FNS:**

- Charm PDF exhibits a *valence-like* peak.
- For $x \leq 0.2$ the perturbative uncertainties are quite large.
- The carried momentum fraction is within 1%.
- Charm PDF is now scale independent (we are in 3FNS).





Comparison with models

- **BHPS** model: [Phy. Letter B (1980) 451-455]

$$p \to uudc\bar{c}$$

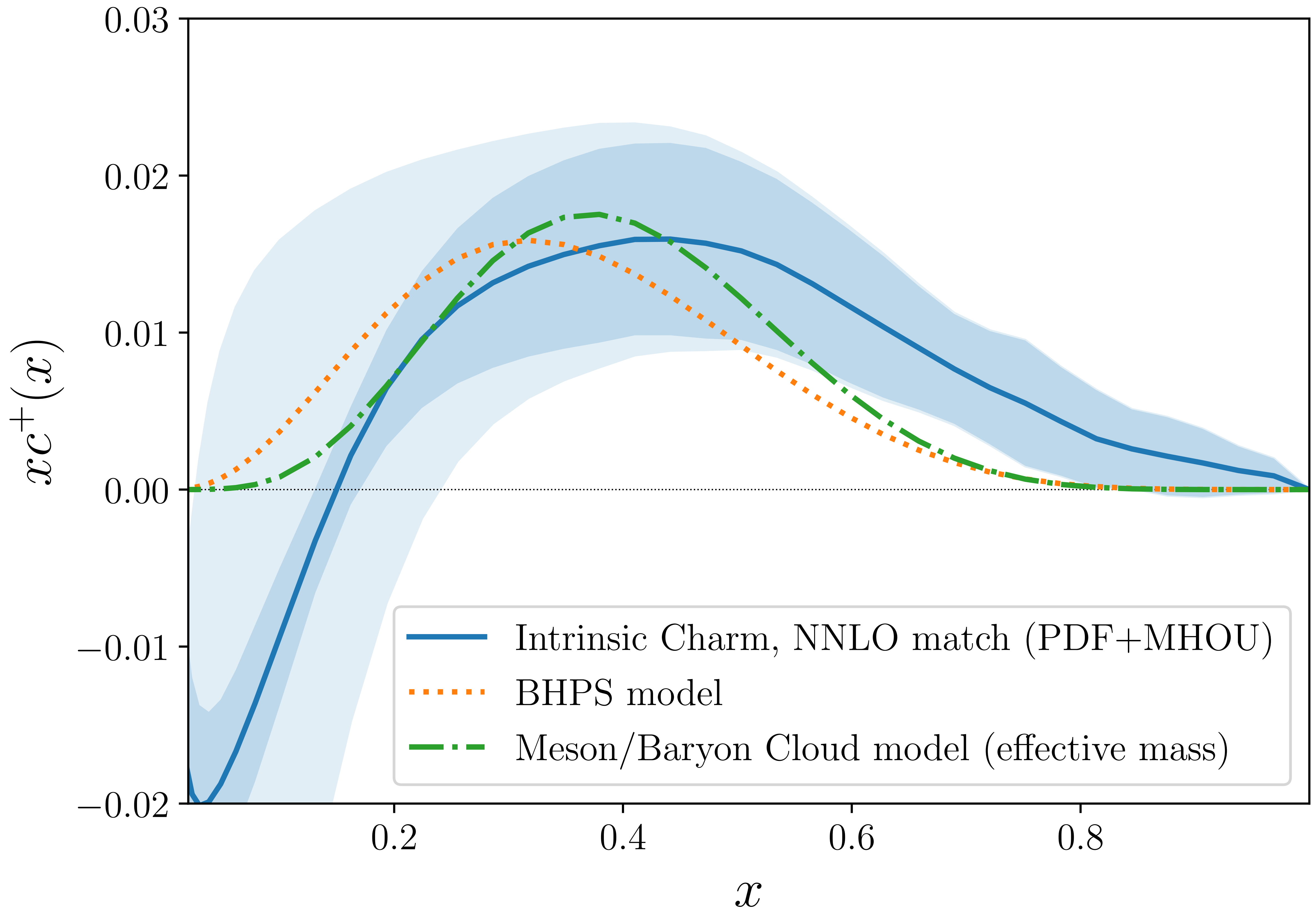
$$xc^+ = \frac{1}{2}Nx^3[\frac{1}{3}(1-x)(1+10x+x^2) + 2x(1+x^2)\ln(x)]$$

- **Meson Baryon** model: [arxiv:1311.1578]

$$p \to \Lambda_c^+ + \bar{D}_0$$

$$xc^+ = \frac{N}{B(\alpha+2,\beta+1)} x^{(1+\alpha)}(1-x)^\beta$$

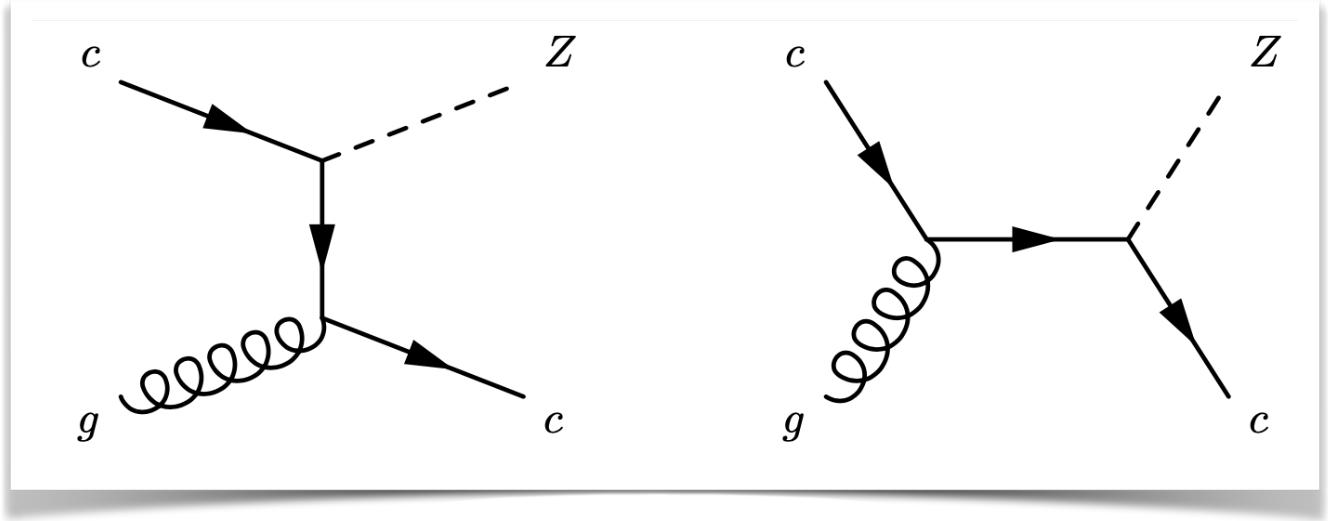
- $\bar{c} = c$ by assumption in BHPS, not true in M/B models.
- A conclusive analysis of what is the origin of IC is beyond of the scope of our study.



See also recents studies:
CTEQ: [arxiv:2211.01387],
LFHQD model: [arxiv:2209.00403]

Impact on LHC observables

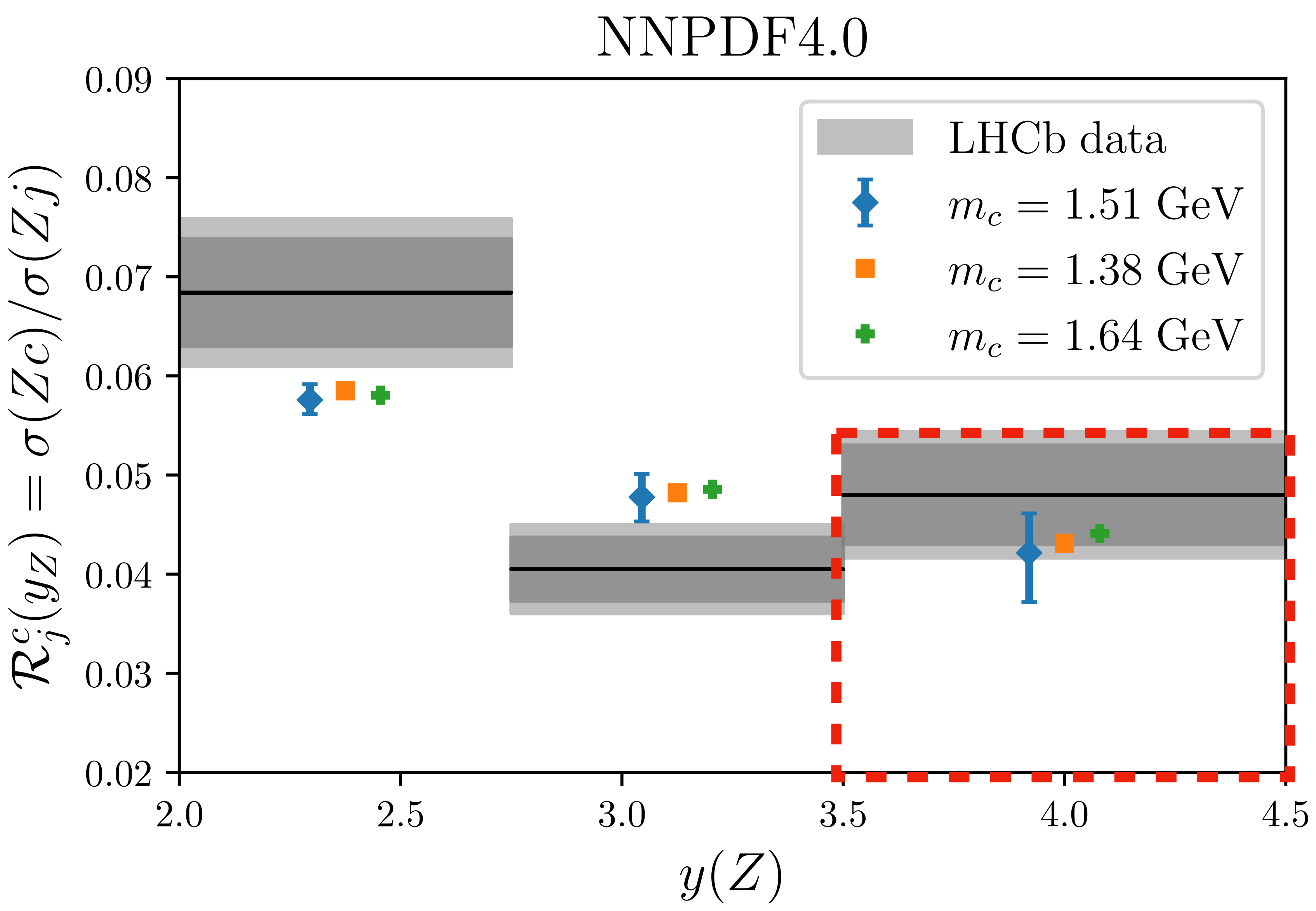
Z+charm production @ LHCb

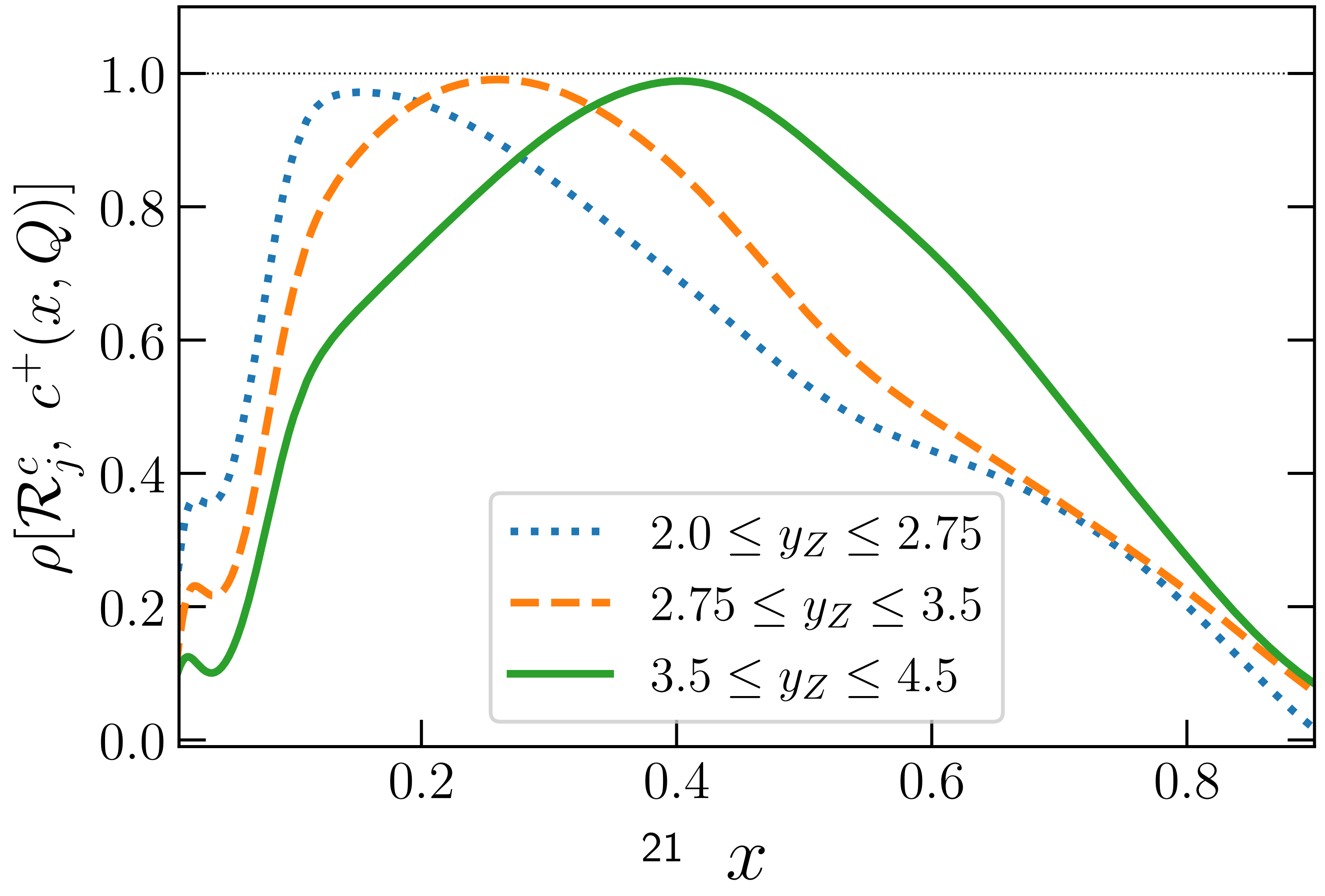


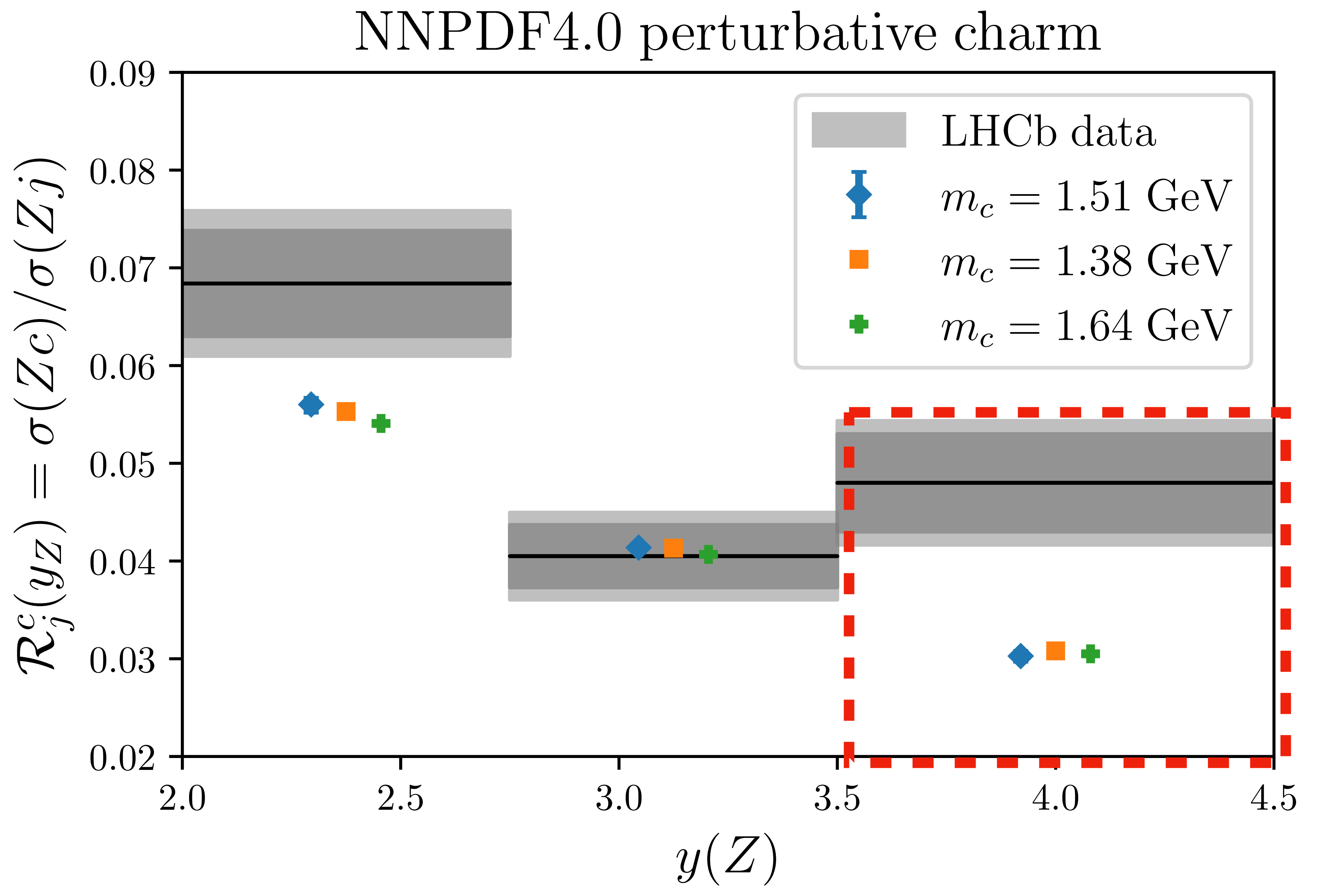
We validate our observation of Intrinsic charm evaluating the prediction for:
$Z + c$ production at LHCb [arxiv:2109.08084]

Compare data to *POWHEG* @ **NLO+PS** [arxiv:1009.5594]

- Better agreement is found with the NNPDF4.0 baseline especially in the **forward region.**
- Predictions are also stable upon charm mass variation.
- NNLO corrections not taken into account yet.
- High correlation with the charm PDF and LHCb observable.







Summary and outlook

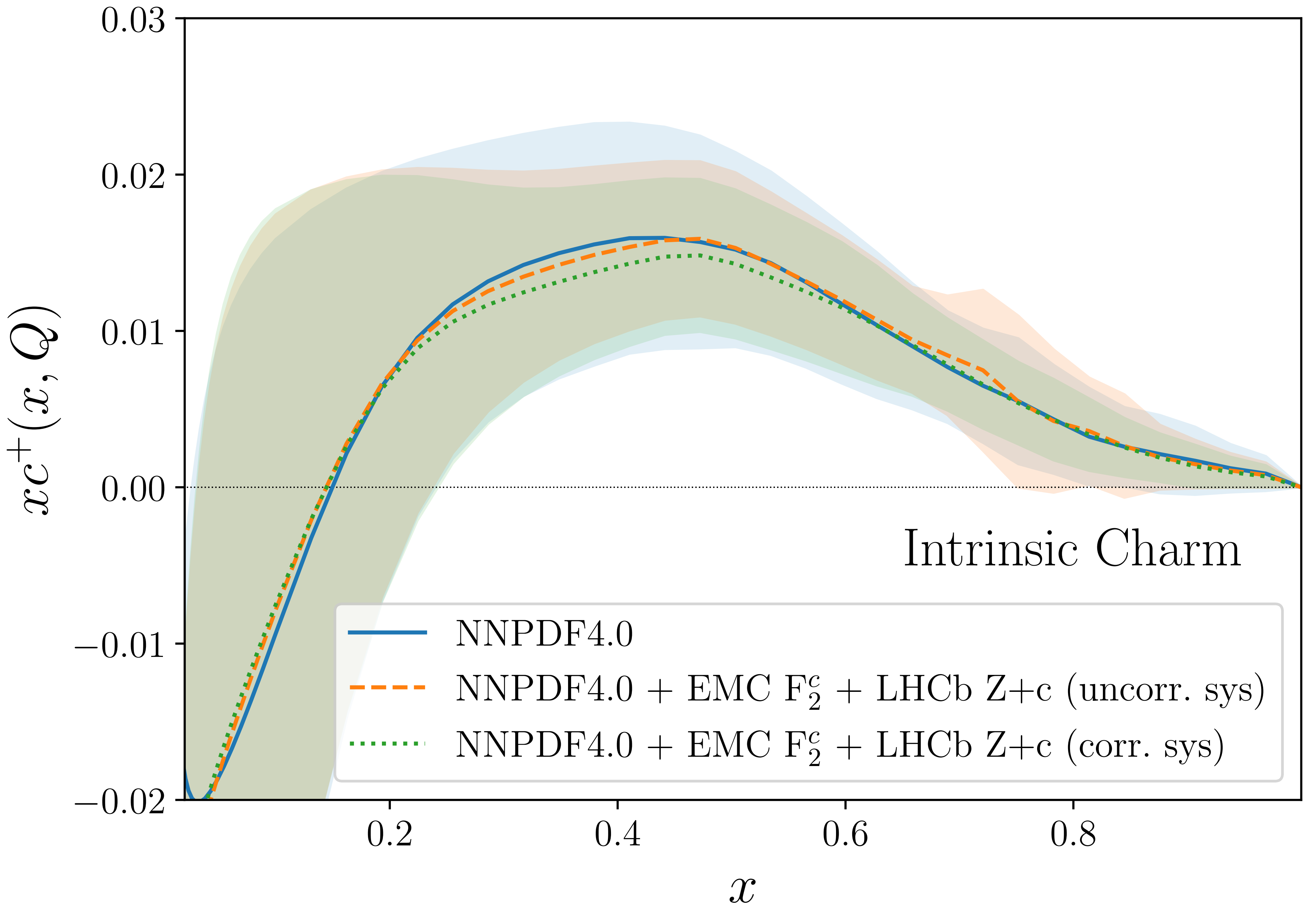
- Evidence of a non zero intrinsic charm c^+ in $n_f = 3$, carrying a momentum fraction total within 1%.
- Our intrinsic charm is in agreement with the most recent LHC results.

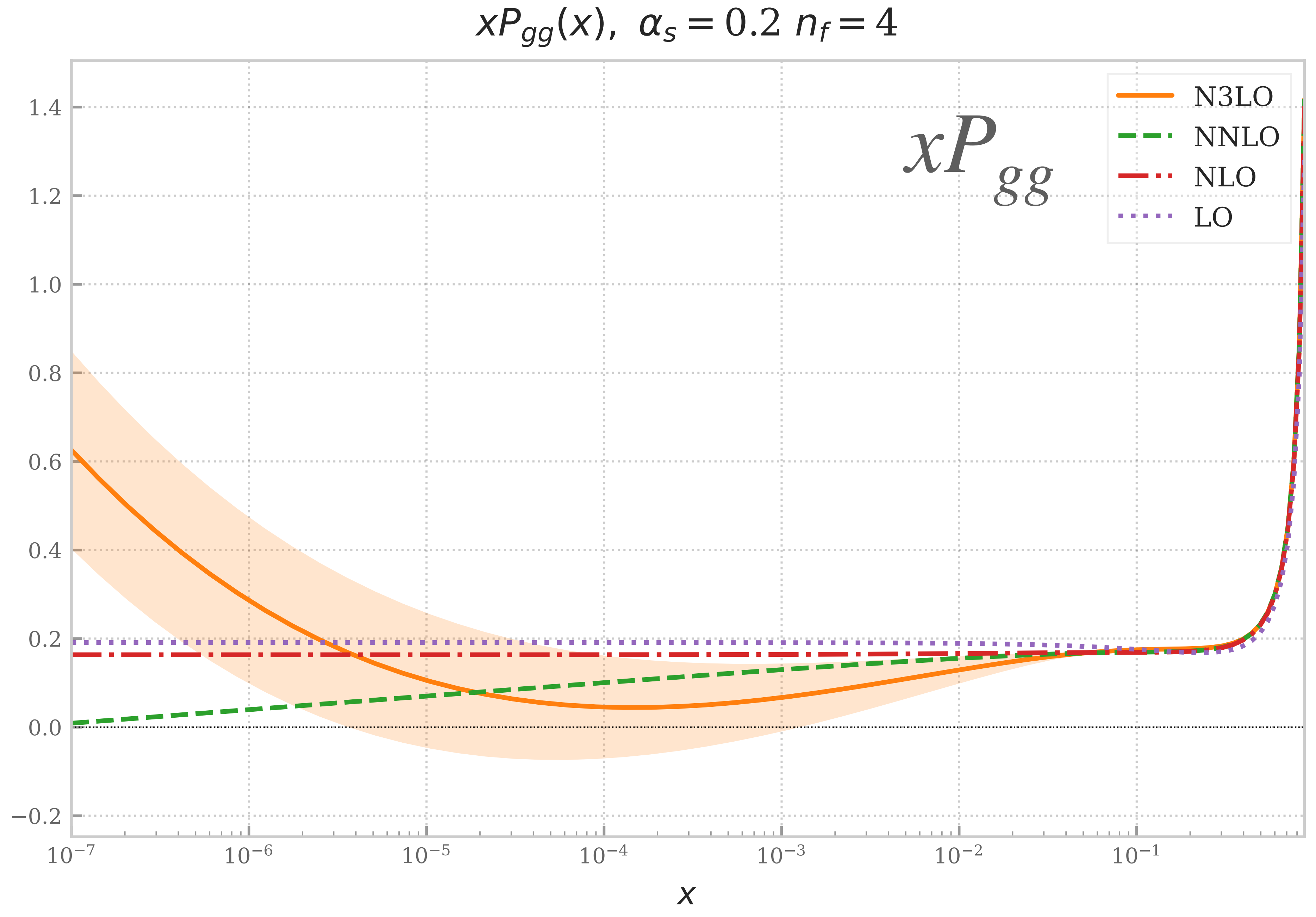
IC study extensions:

- Need to proper quantify MHOU and possibly move towards a full N3LO PDF fit.
- Independent parametrisation of c^-. Any impact of the charm asymmetry?

NNPDF4.0 extensions:

- Faithful estimation of MHOU from scale variations @ NNLO.
- Update the determination of the photon PDF.
- Towards an aN3LO fit.





Backup slides

DGLAP evolution with EKO

EKO
Evolution Kernel Operators

A.Candido, F.Hekhorn, GM:
[arxiv:2202.02338]

$$\mu^2 \frac{d}{d\mu^2} f_i(x,\mu) = \int_x^1 \frac{dz}{z} P_{ij}(\frac{x}{z}, \alpha_s)\, f_j(x,\mu) \quad \longrightarrow \quad \frac{d}{d\alpha_s}\tilde{\mathbf{f}}(\mu_F^2) = -\frac{\gamma(a_s)}{\beta(a_s)} \cdot \tilde{\mathbf{f}}(a_s)$$

- The formal solution of DGLAP can be written as in Mellin space:

$$\tilde{\mathbf{f}}(a_s) = \tilde{\mathbf{E}}(a_s \leftarrow a_s^0) \cdot \tilde{\mathbf{f}}(a_s^0)$$

$$\tilde{\mathbf{E}}(a_s \leftarrow a_s^0) = \mathscr{P} \exp \left[- \int_{a_s^0}^{a_s} \frac{\gamma(a_s')}{\beta(a_s')} da_s' \right]$$

Mellin transformation:

$$\tilde{f}(N) = \int_0^1 x^{N-1} f(x) dx$$

- In x-space:

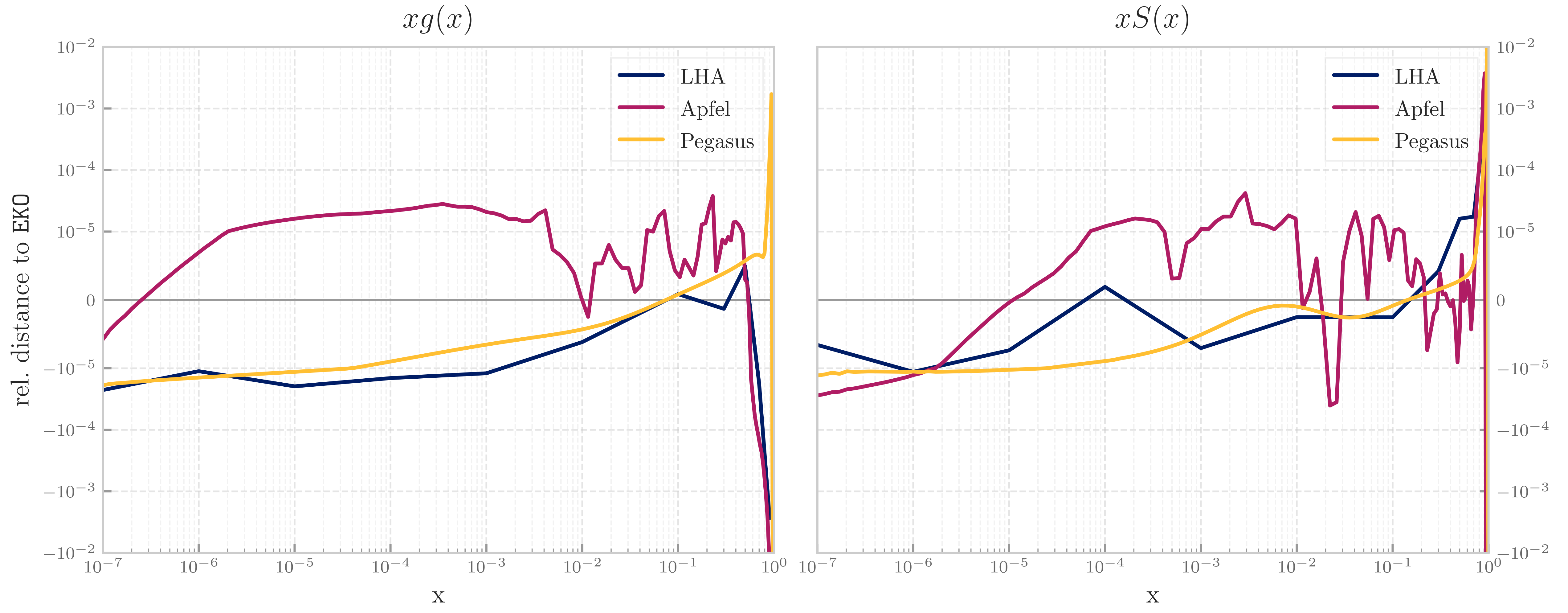
$$\mathbf{f}(x_k, a_s) = \mathbf{E}_{k,j}(a_s \leftarrow a_s^0) \mathbf{f}(x_j, a_s^0)$$

- Evolution is performed in **Intrinsic** Evolution basis:
  $span\{g, \Sigma, V, V_3, T_3, V_8, T_8, c^+, c^-, b^+, b^-, t^+, t^-)\}$

- Solution is available at: LO, NLO, NNLO, aN3LO

- EKO implements various solution methods:
  Exact, Truncated, Expanded (see documentation)

LHA Benchmark comparison

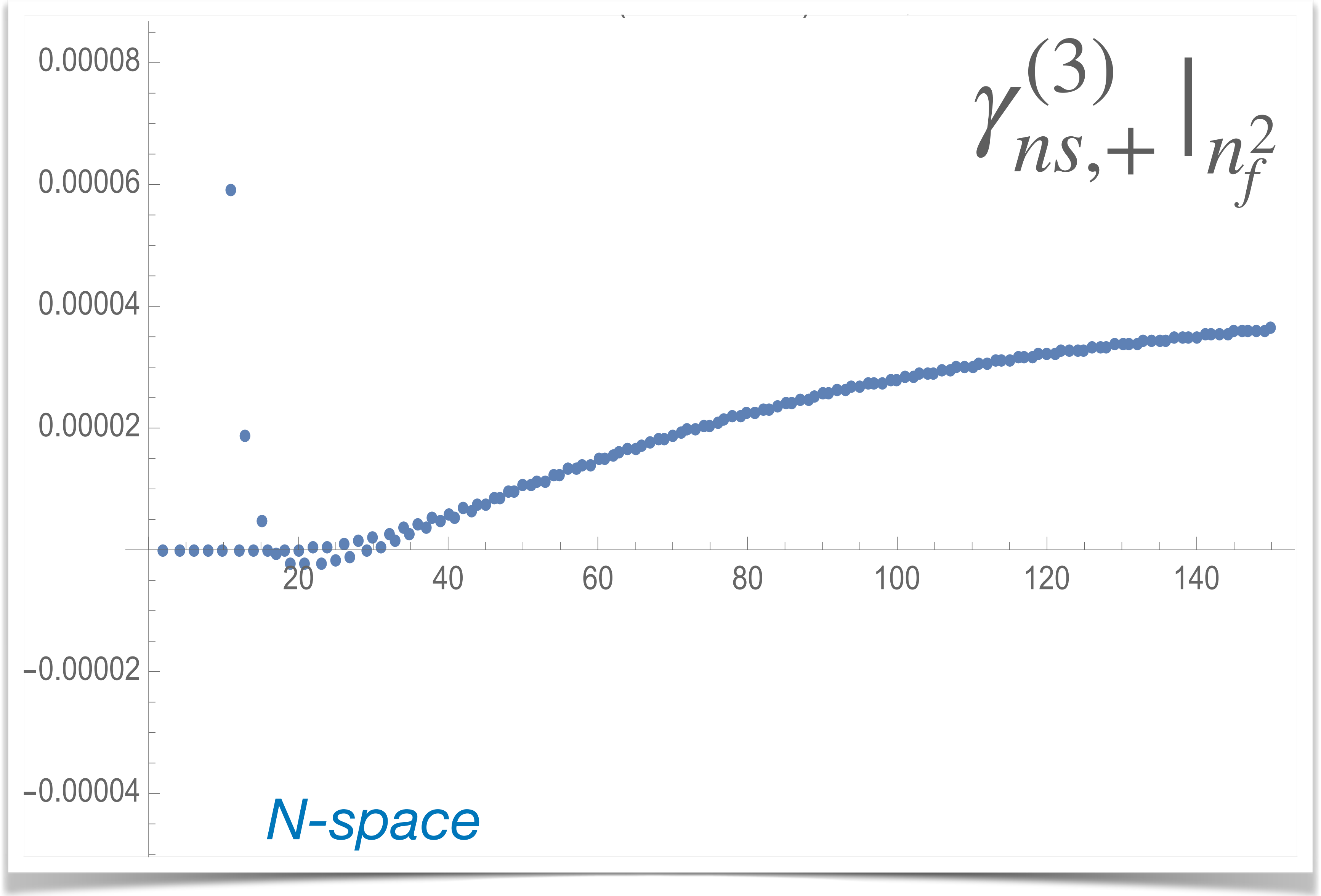


N3LO non singlet sector

non-singlet 4-loop Anomalous Dimensions

	n_f^0	n_f^1	n_f^2	n_f^3
$\gamma_{ns,-}^{(3)}$	✓	✓	✓	✓
$\gamma_{ns,+}^{(3)}$	✓	✓	✓	✓
$\gamma_{ns,s}^{(3)}$		✓	✓	

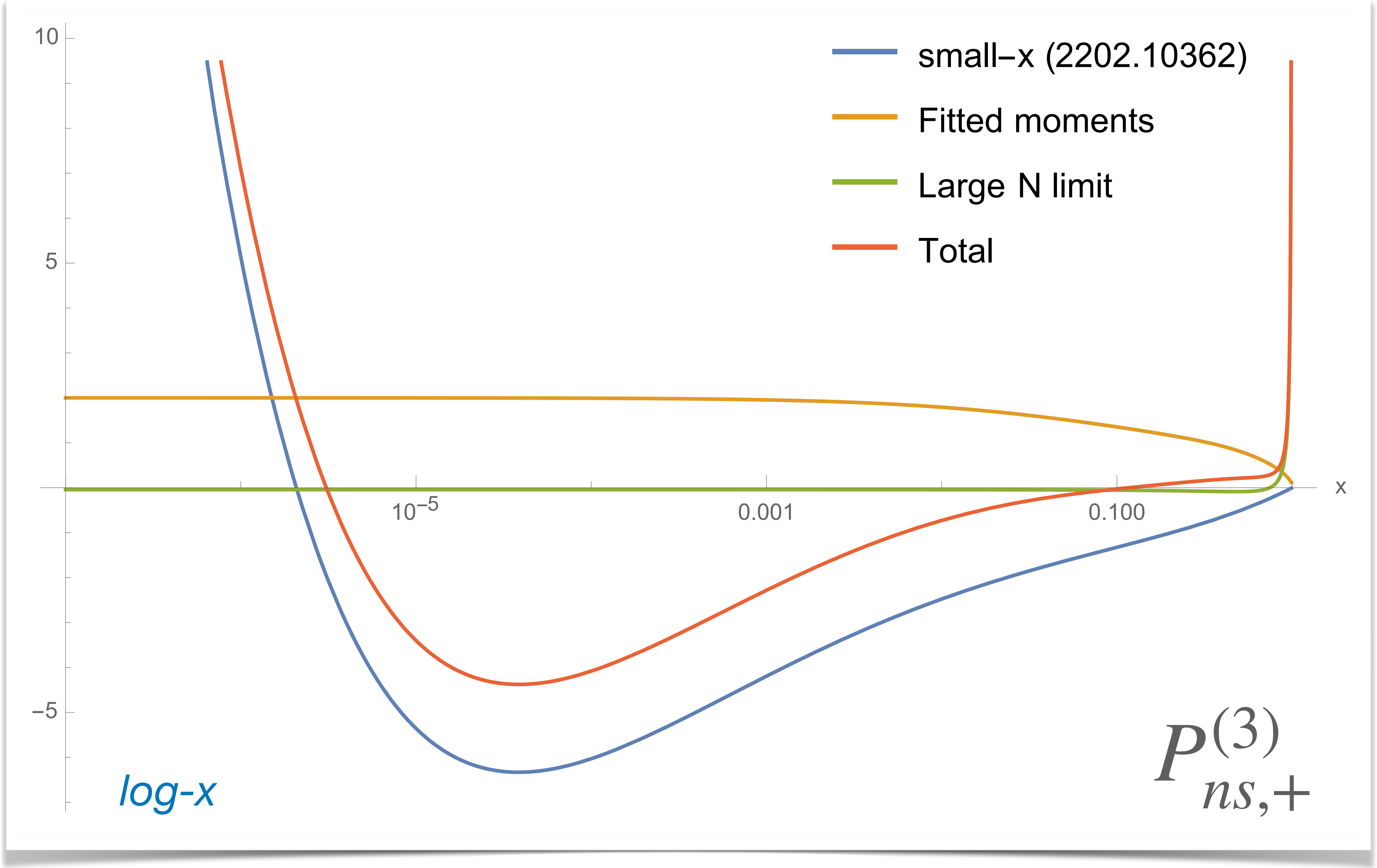
Comparison w.r.t. known analytical part (%)



Main references:

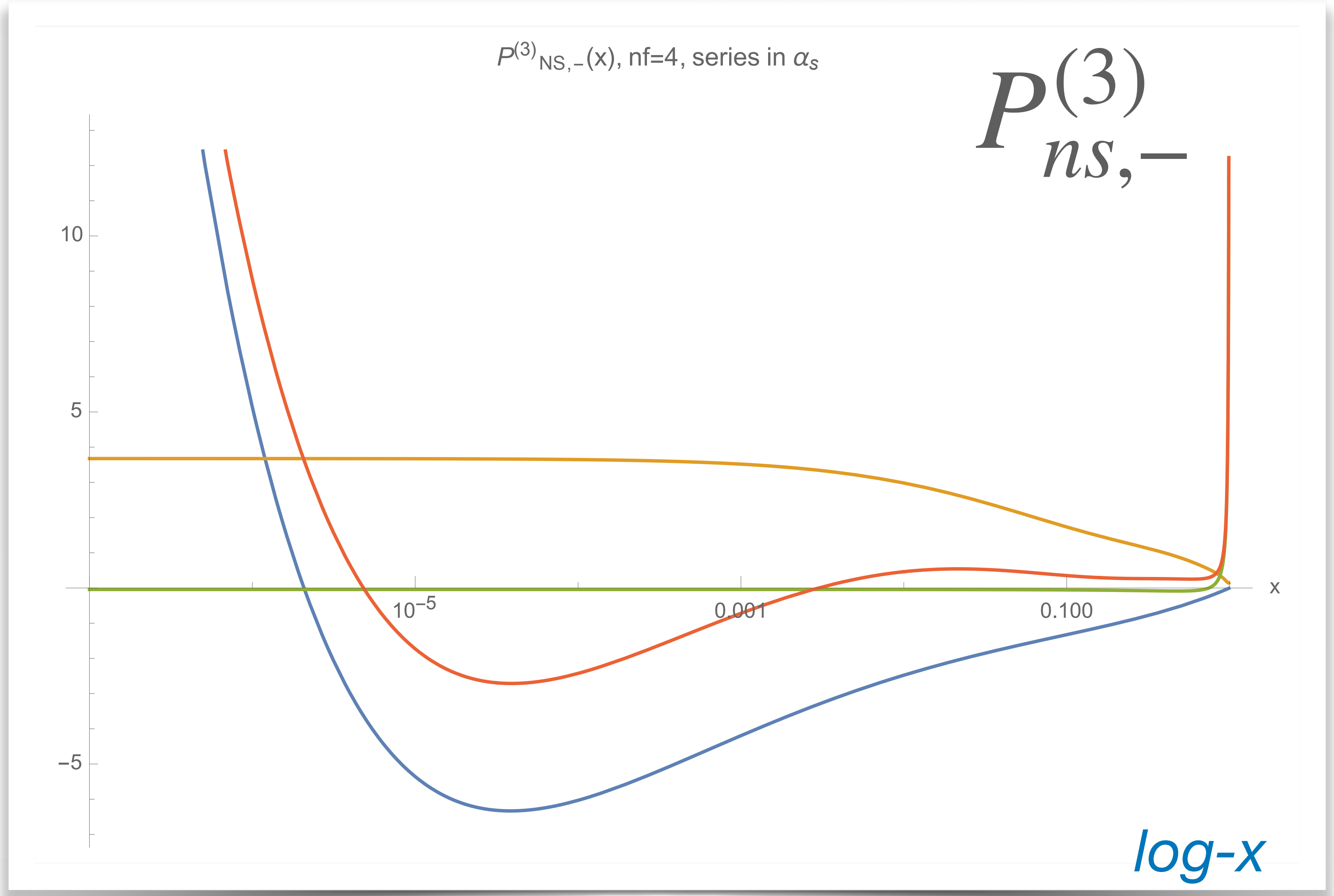
- Moch, Ruijl, Ueda, Vermaseren, Vogt [arXiv:1707.08315].
- Davies, Vogt, Ruijl, Ueda, Vermaseren. [arXiv:1610.07477]
- Davies, Kom, Moch, Vogt . [arXiv:2202.10362].

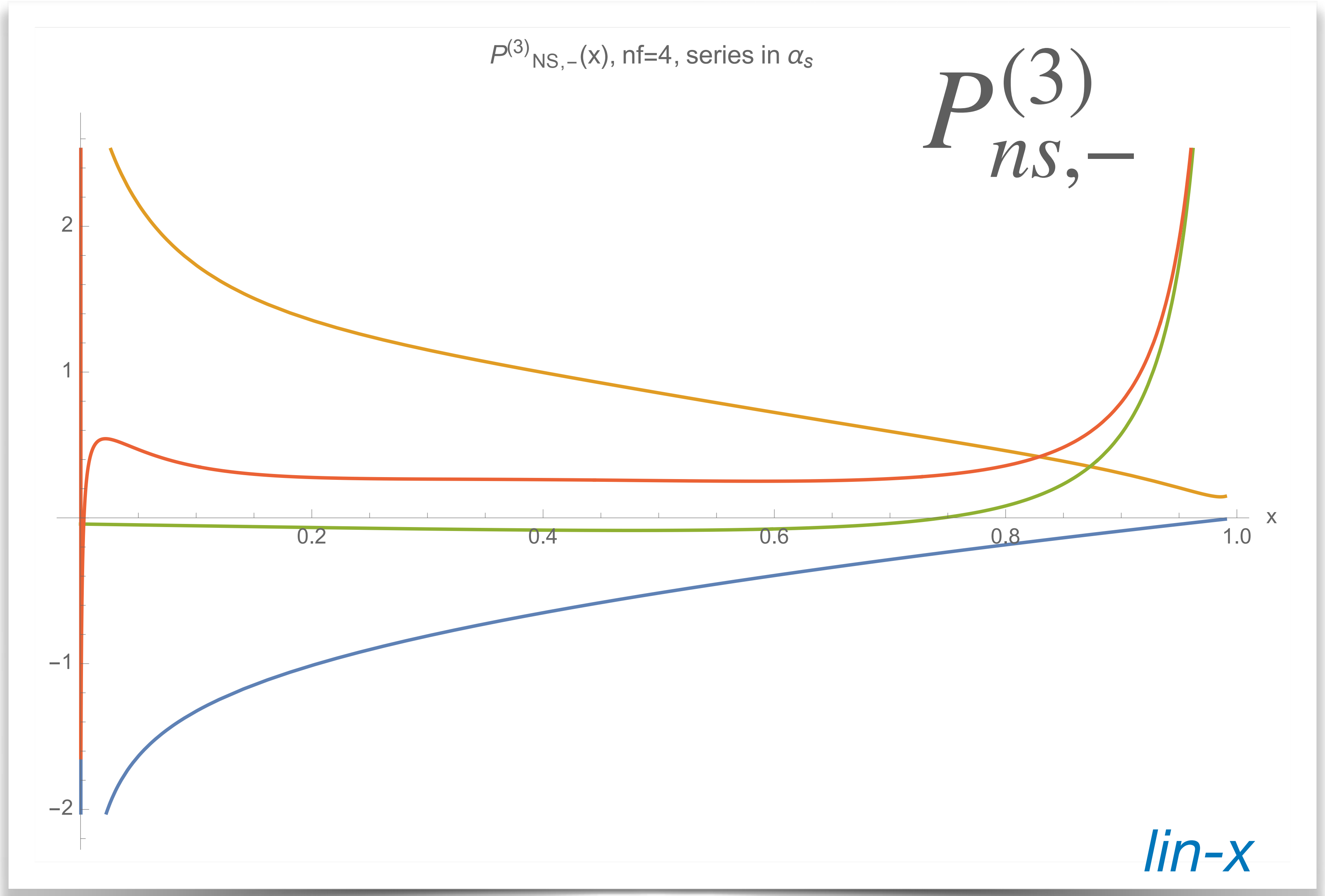
Rule of thumb:
small-N → small-x,
large-N → large-x



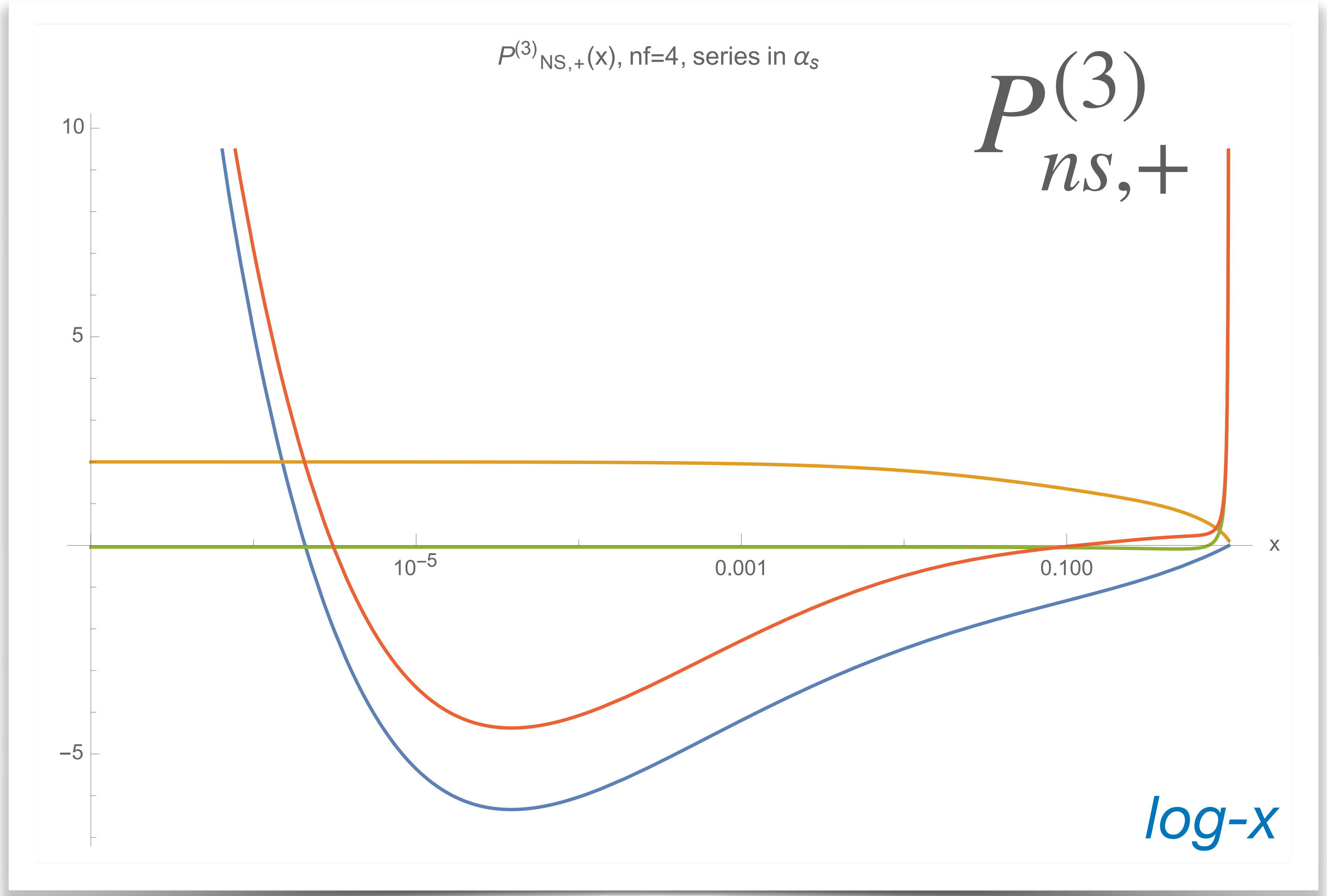
- Estimation of the N3LO anomalous dimensions is based on the best available theoretical constraints:
  - large-N: $$\gamma_{ns}^{(3)}(N \to \infty) \approx \Gamma_f S_1(N) + B + C\frac{S_1(N)}{N} + D\frac{1}{N} + \mathcal{O}(\frac{\ln(N)}{N^2})$$
  - small-N: $$\gamma_{ns}^{(3)}(N \to 0) \approx \sum_{i=1}^{7} C_i \frac{1}{N^i}$$
  - 8 lowest Mellin moments
- For more details on the procedure used see [EKO N3LO ad documentation]
- Non singlet approximated spitting functions are compatible with the known analytical (and much more complex) parts within numerical accuracy.

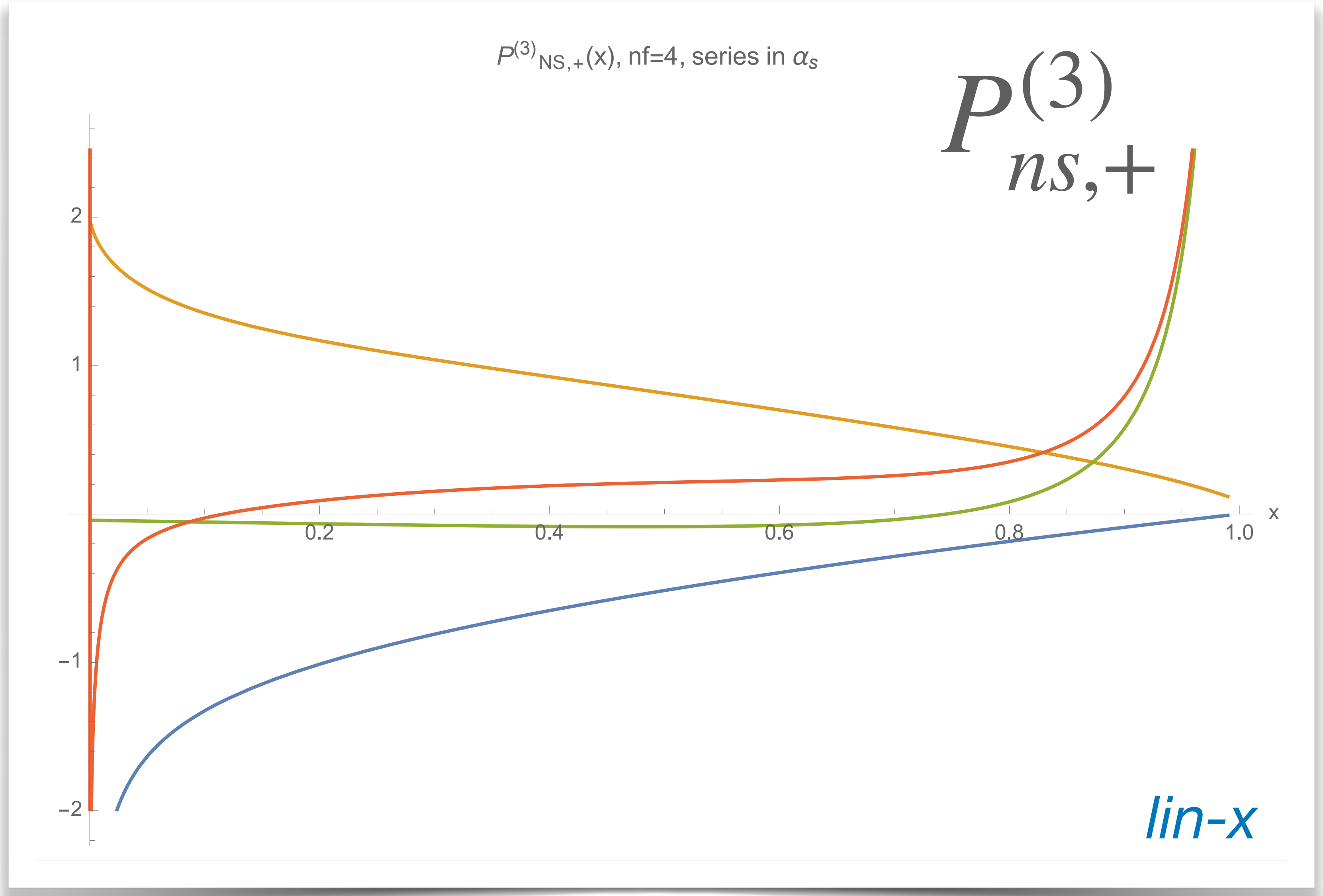
N3LO non singlet





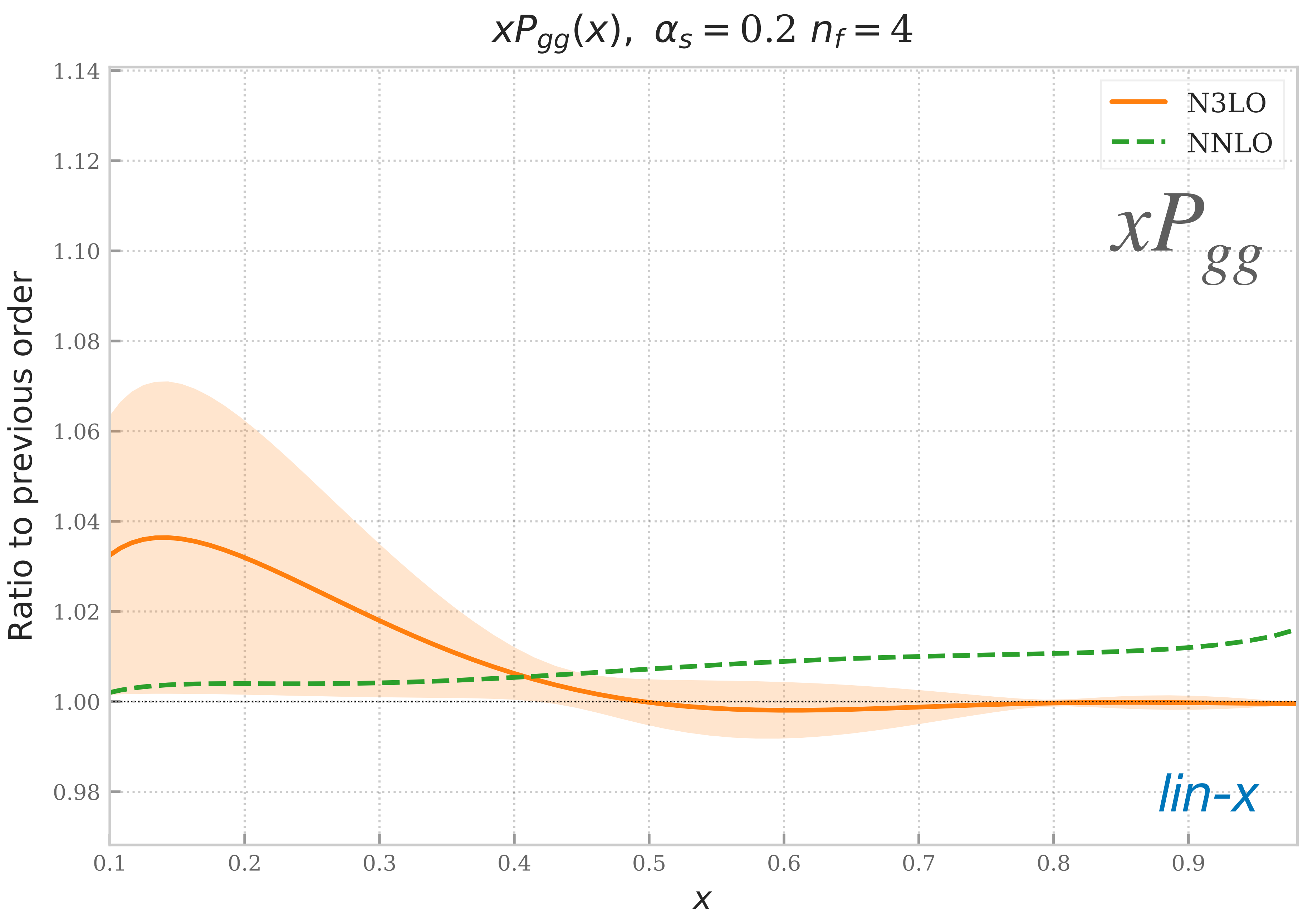
- small–x (2202.10362)
- Fitted moments
- Large N limit
- Total

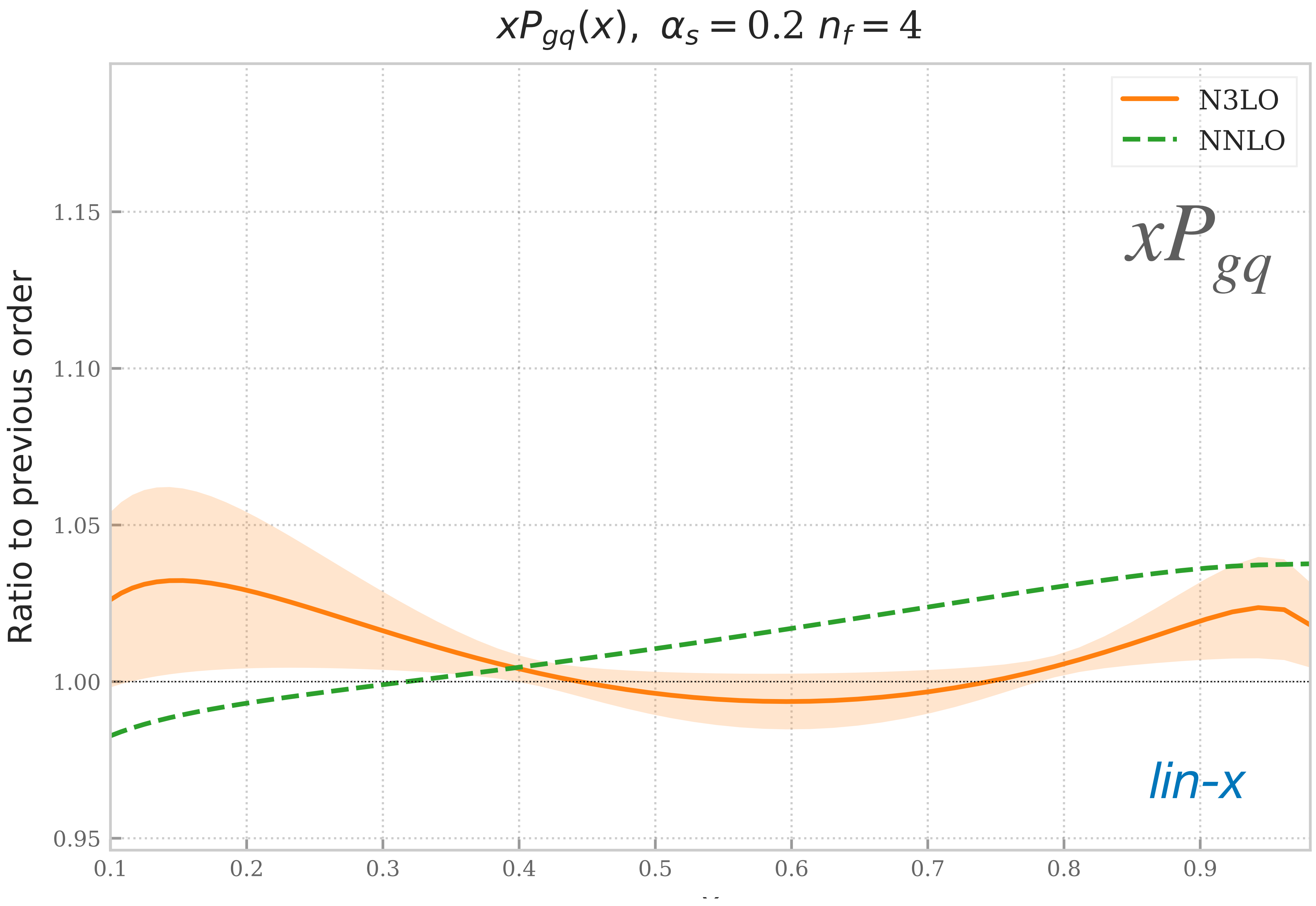


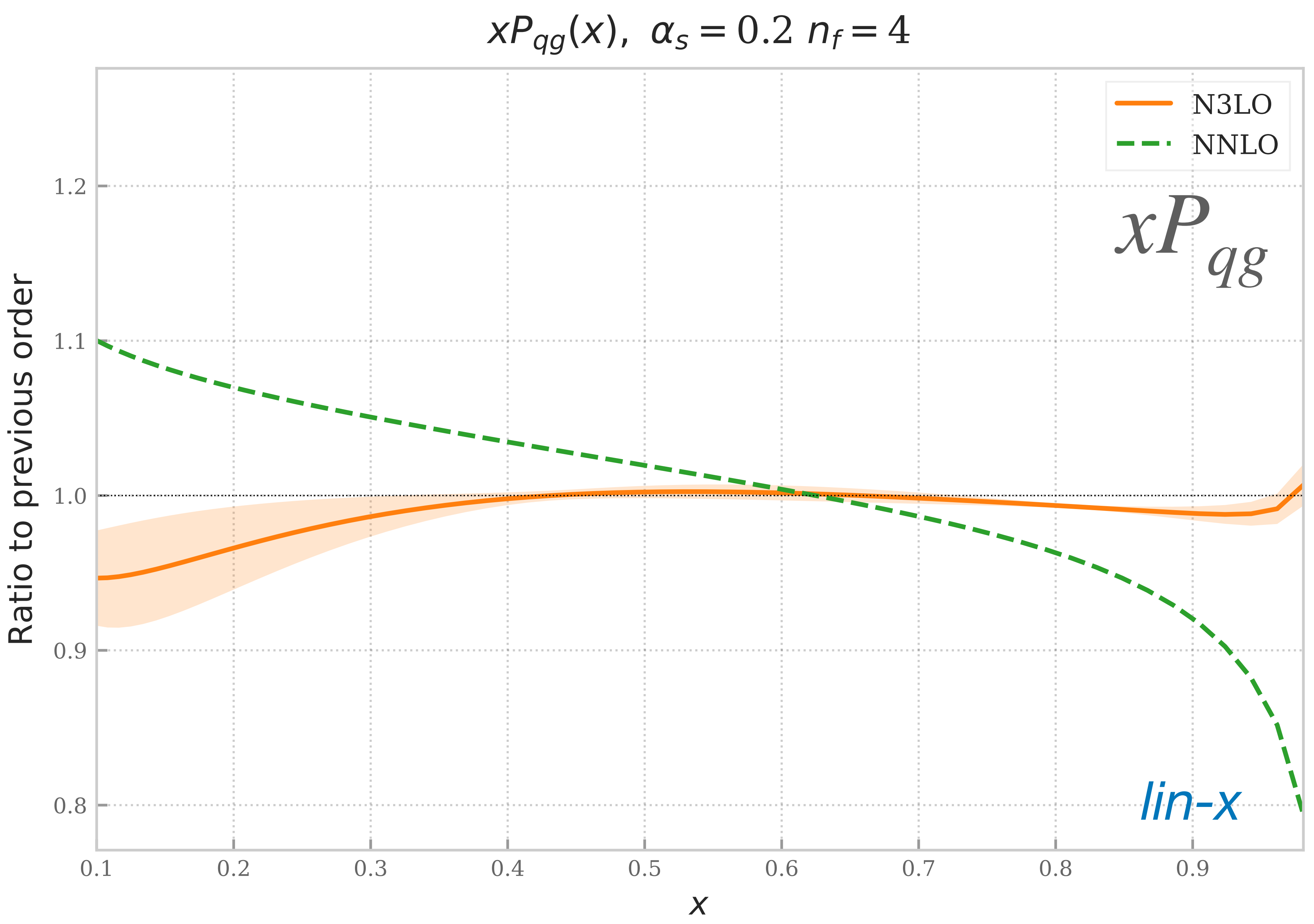


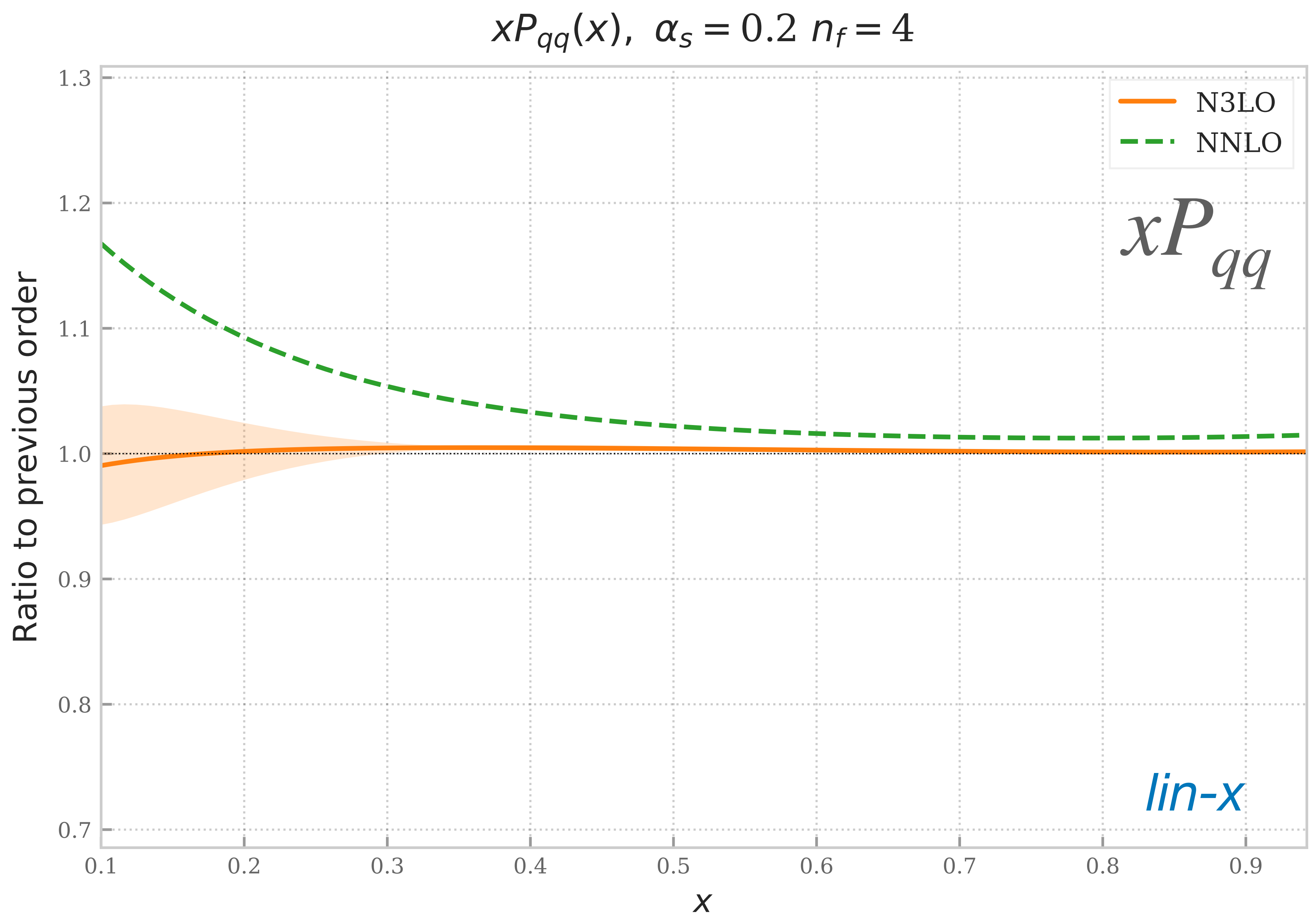
N3LO singlet large-x

PRELIMINARY RESULTS



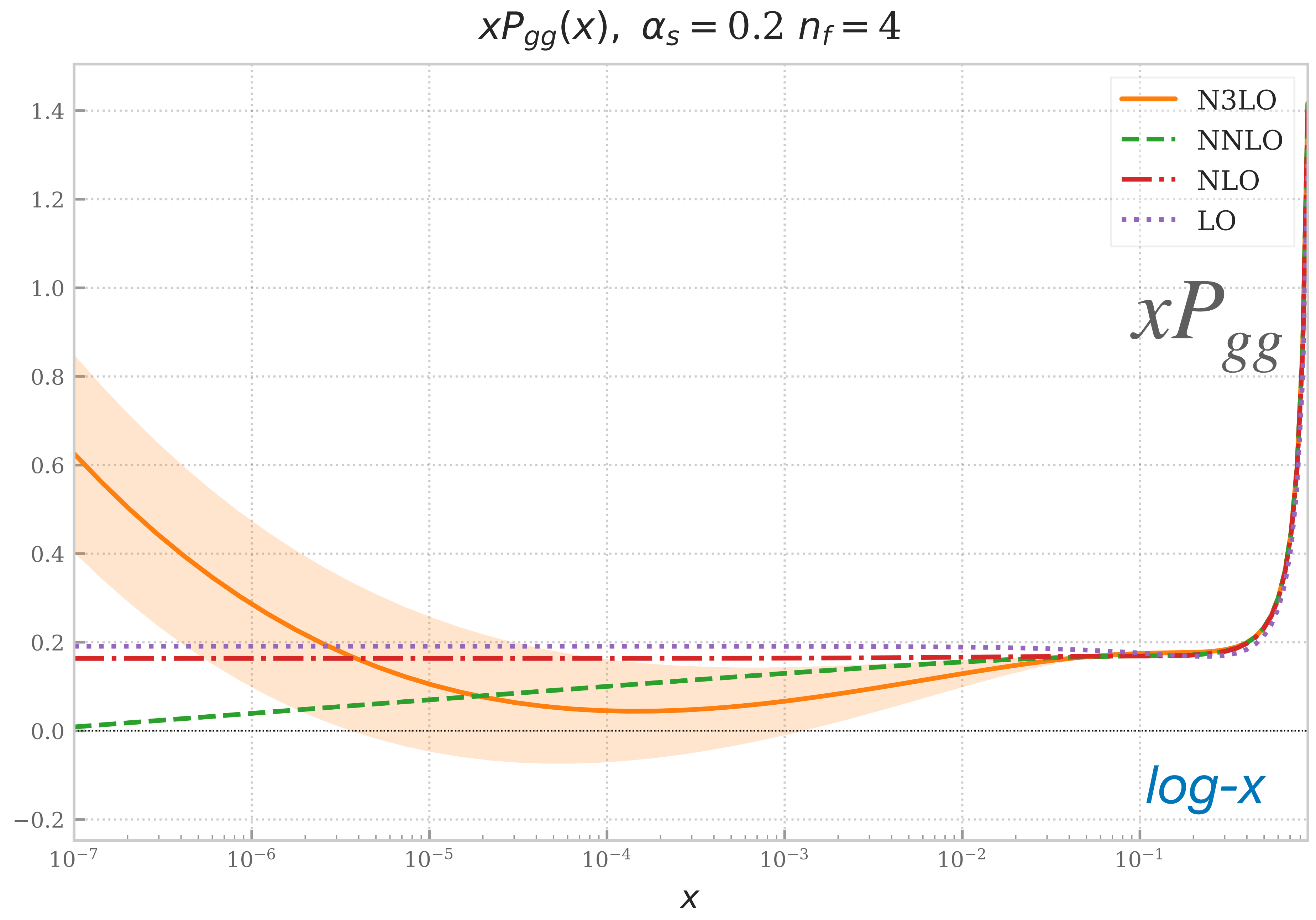


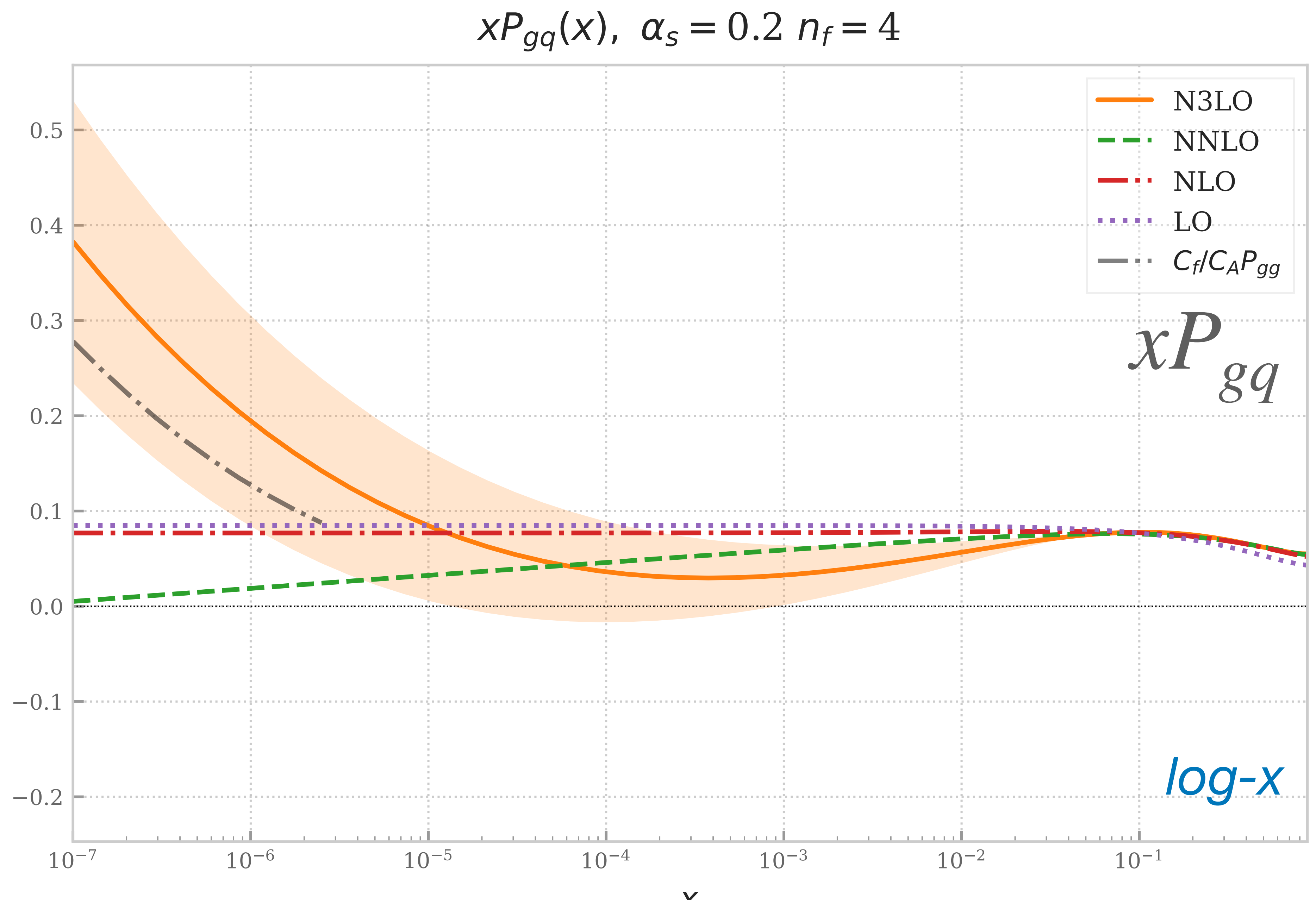


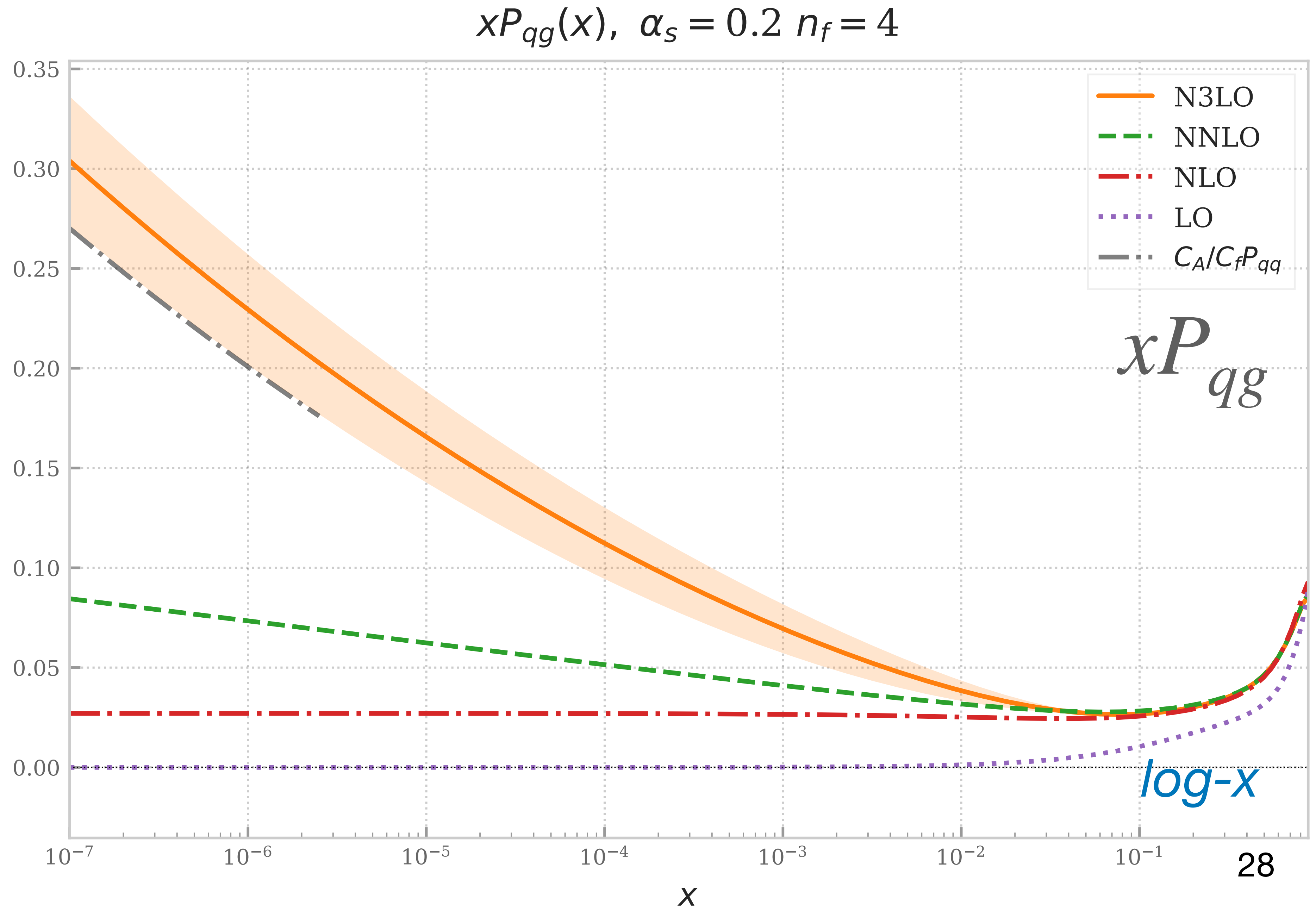


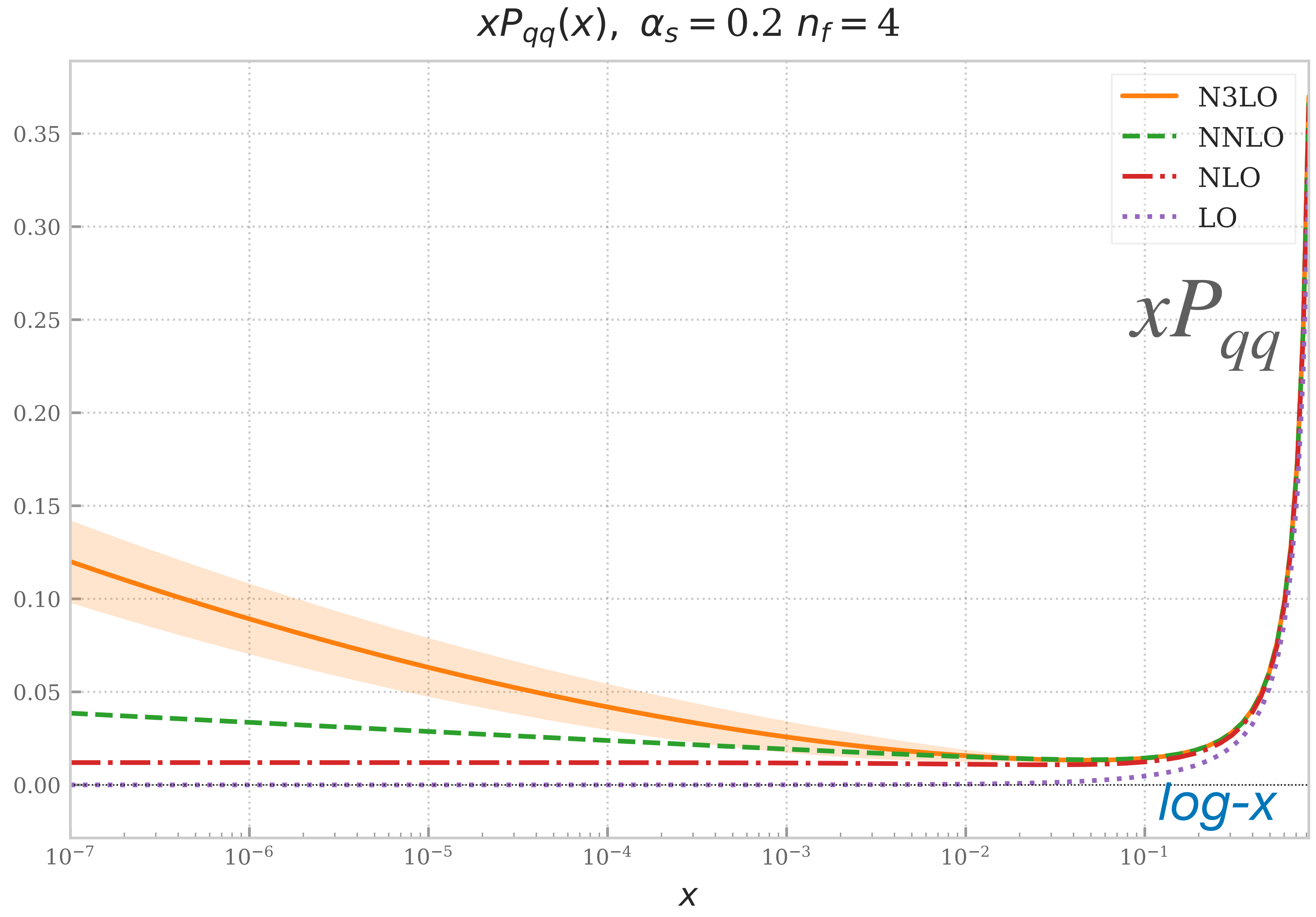
N3LO singlet small-x

PRELIMINARY RESULTS



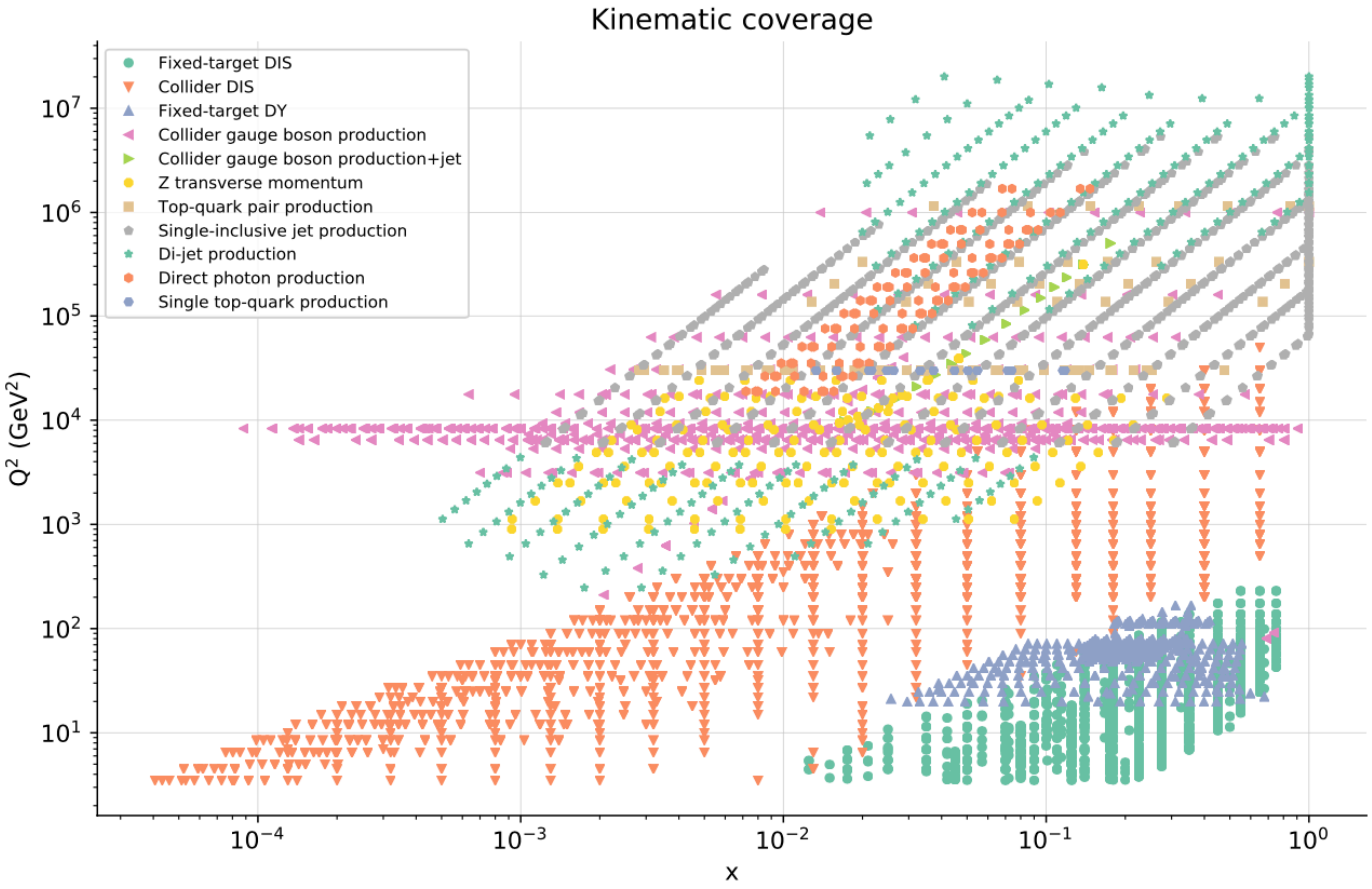






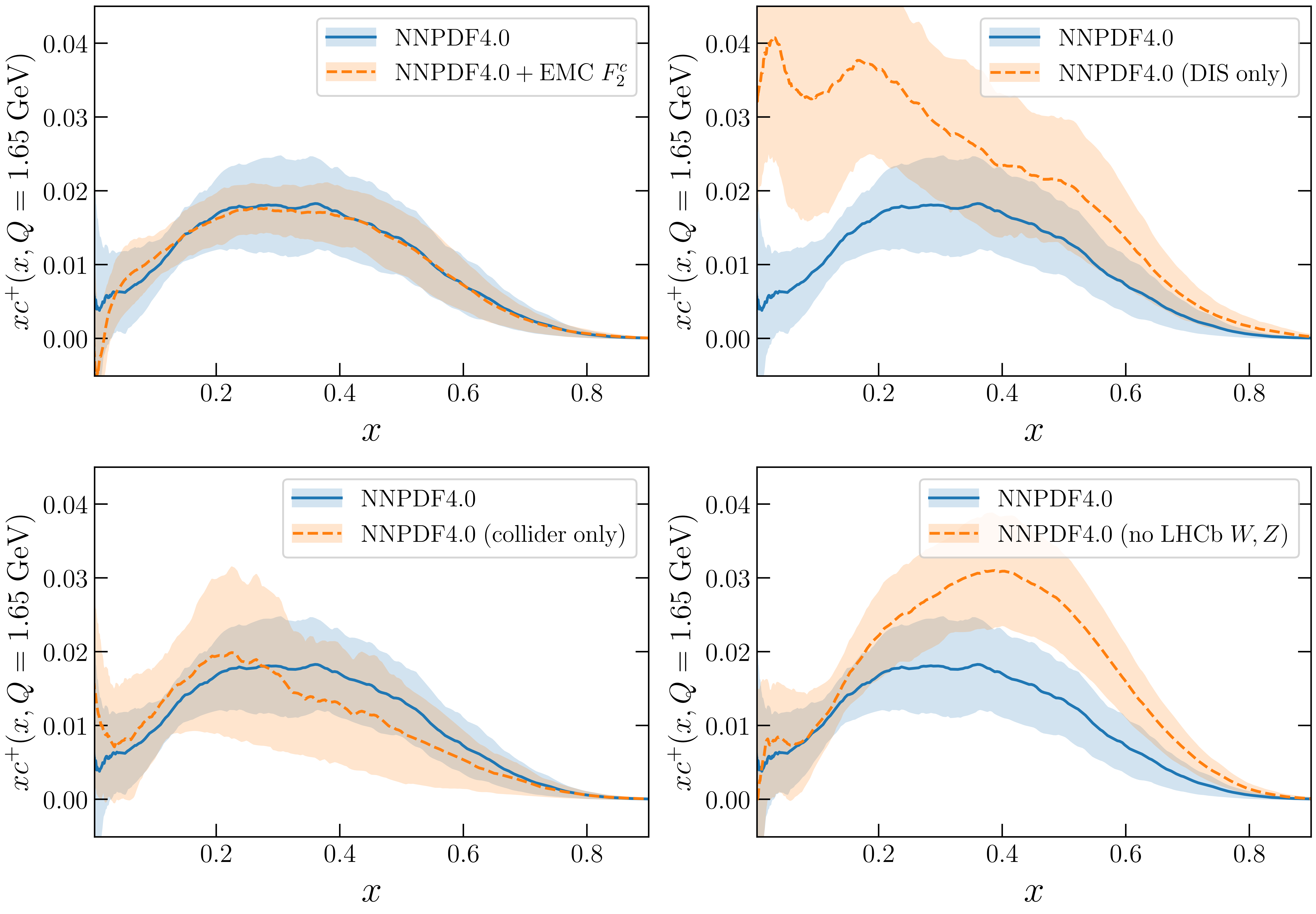
NNPDF4.0 datasets

More than 4000 datapoints in total.



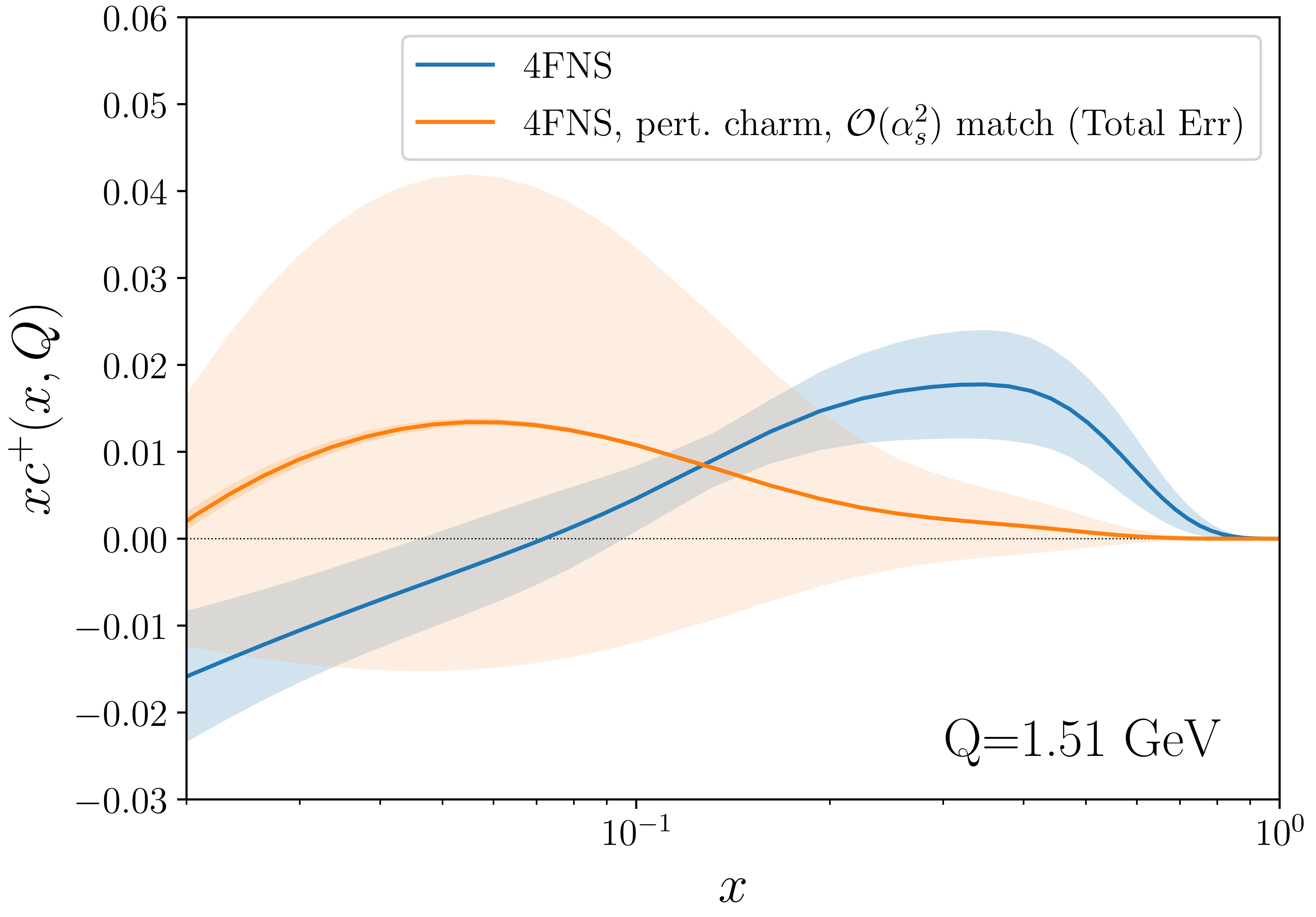
The charm PDF in 4FNS

- $\bar{c} = c$
- c^+ at the fitting scale exhibits a non vanishing peak in the high-x region and vanishes
  at low-x.
- Constrain are coming mainly from collider data.
- NNPDF4.0 is consistent with EMC data.



Fitted vs perturbative only charm

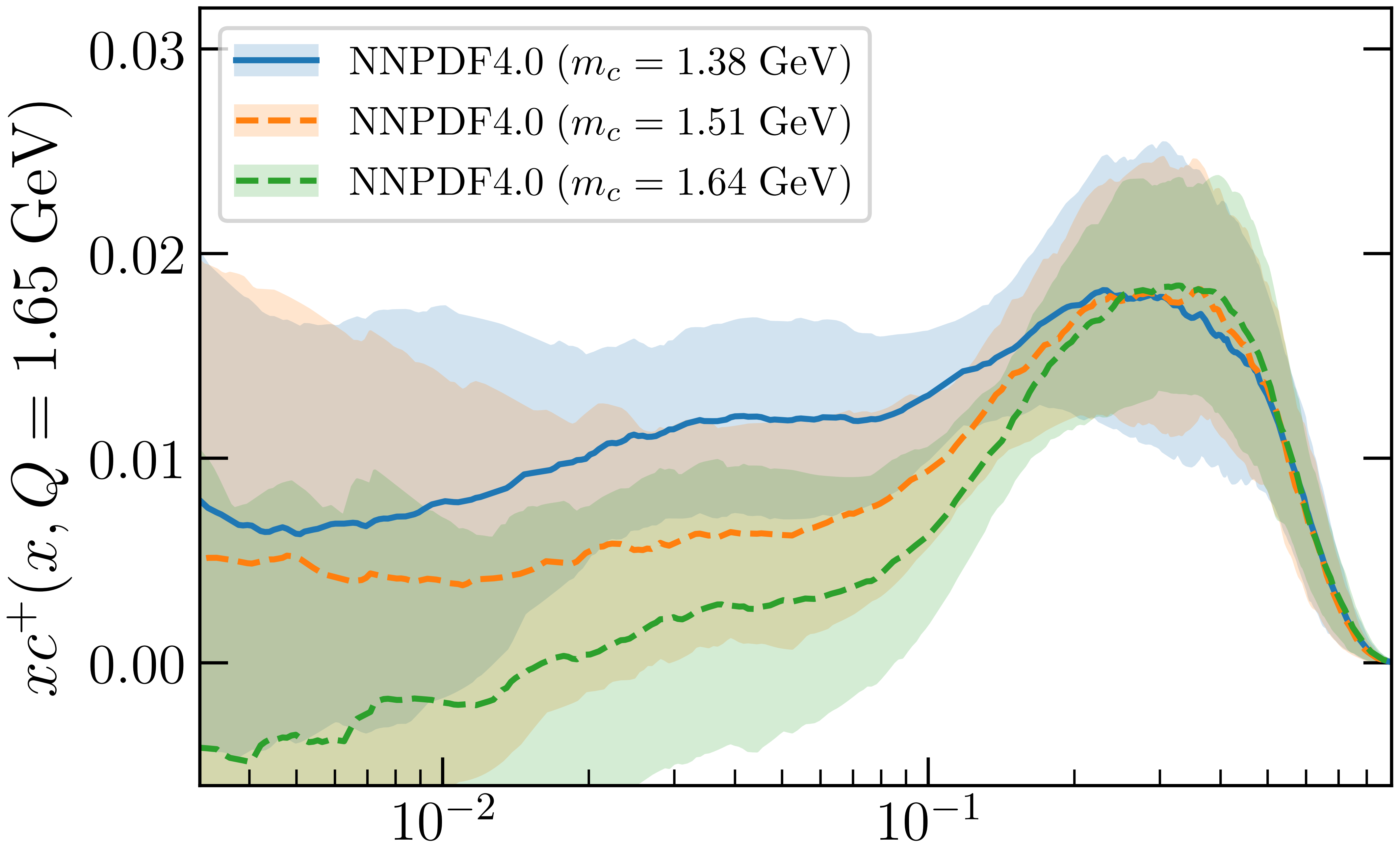
- Perturbative charm is more rigid assumption.
- The matching procedure is perturbative unstable due to the low m_c value. Perturbative charm uncertainty is dominated by MHOU.
- $\chi^2_{fitted\ ch.} < \chi^2_{pert\ ch.}$ mainly due to a worsening of the LHC W, Z and top pair data sets.
- Fully perturbative charm is not compatible with the fitted one.



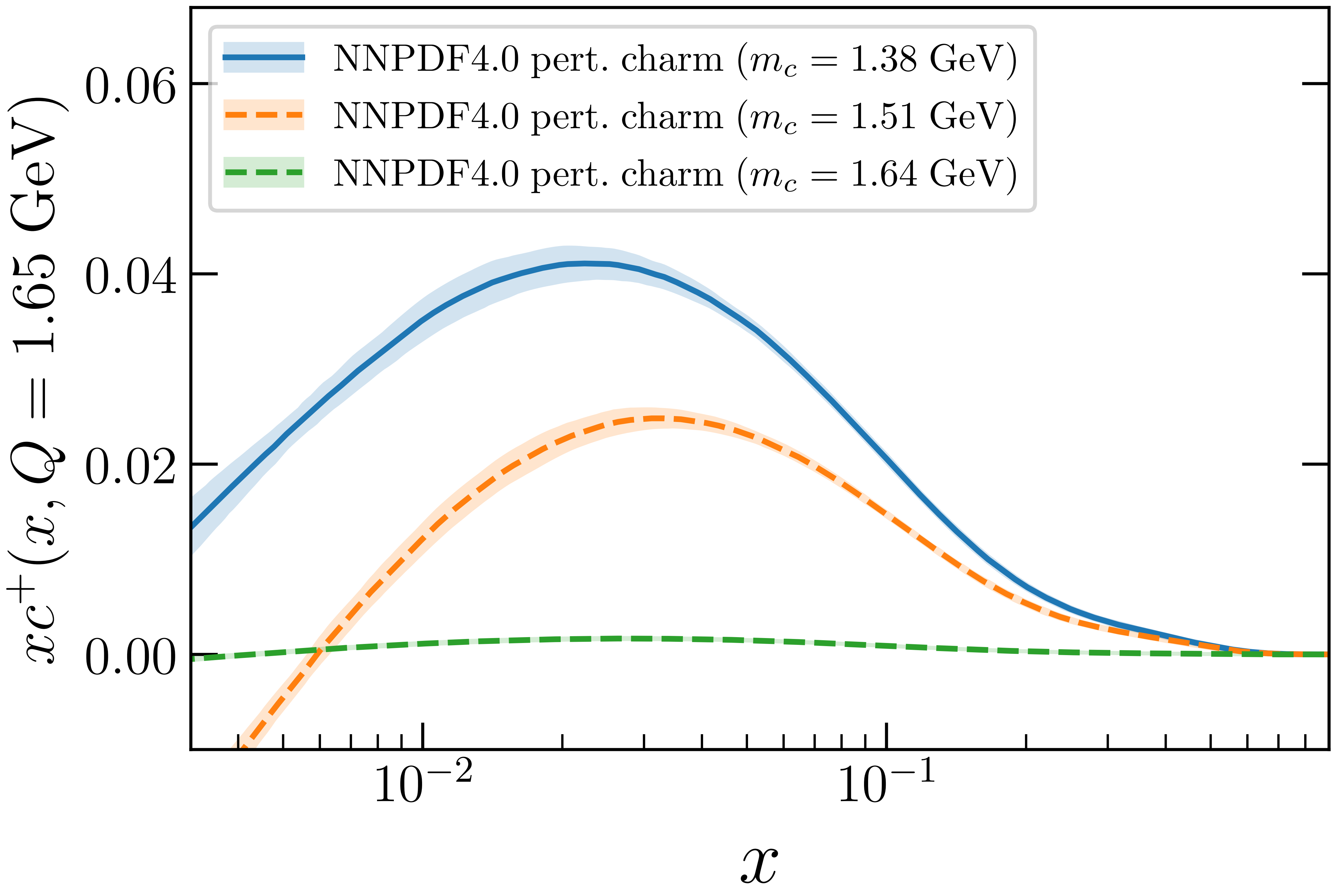
Charm PDF mass dependence

Charm mass is varied in the range: $m_c = 1.51 \pm 0.13\ GeV$

Fitted charm



Perturbative charm only



The matching conditions

$$\mathbf{A}^{(n_f)}(\mu_h^2) = \mathbf{I} + a_s^{(n_f+1)}(\mu_h^2)\mathbf{A}^{(n_f),(1)} + a_s^{(n_f+1),2}(\mu_h^2)\mathbf{A}^{(n_f),(2)} + a_s^{(n_f+1),3}(\mu_h^2)\mathbf{A}^{(n_f),(3)} + \mathcal{O}(\alpha_s^4)$$

NLO

$$\mathbf{A}^{(n_f),(1)}_{S,h^+} = \begin{pmatrix} A^{S,(1)}_{gg,H} & 0 & A^{S,(1)}_{gH} \\ 0 & 0 & 0 \\ A^{S,(1)}_{Hg} & 0 & A^{(1)}_{HH} \end{pmatrix}$$

$$\mathbf{A}^{(n_f),(1)}_{nsv,h^-} = \begin{pmatrix} 0 & 0 \\ 0 & A^{(1)}_{HH} \end{pmatrix}$$

NNLO

$$\mathbf{A}^{(n_f),(2)}_{S,h^+} = \begin{pmatrix} A^{S,(2)}_{gg,H} & A^{S,(2)}_{gq,H} & 0 \\ 0 & A^{ns,(2)}_{qq,H} & 0 \\ A^{S,(2)}_{Hg} & A^{ps,(2)}_{Hq} & 0 \end{pmatrix}$$

$$\mathbf{A}^{(n_f),(2)}_{nsv,h^-} = \begin{pmatrix} A^{ns,(2)}_{qq,H} & 0 \\ 0 & 0 \end{pmatrix}$$

[Eur.Phys.J.C 1 (1998) 301-320, Phys.Lett.B 754 (2016) 49-58]

N3LO

$$\mathbf{A}^{(n_f),(3)}_{S,h^+} = \begin{pmatrix} A^{S,(3)}_{gg,H} & A^{S,(3)}_{gq,H} & 0 \\ A^{S,(3)}_{qg,H} & A^{ns,(3)}_{qq,H} + A^{ps,(3)}_{qq,H} & 0 \\ A^{S,(3)}_{Hg} & A^{ps,(3)}_{Hq} & 0 \end{pmatrix}$$

$$\mathbf{A}^{(n_f),(3)}_{nsv,h^-} = \begin{pmatrix} A^{ns,(3)}_{qq,H} & 0 \\ 0 & 0 \end{pmatrix}$$

[arXiv:0904.3563, doi:10.1016/S0010-4655(00)00156-9, arXiv:1406.4654, doi:10.1016/j.nuclphysb.2014.10.008, arXiv:1405.4259, http://dx.doi.org/10.1016/j.nuclphysb.2014.02.007, arXiv:1403.6356, arXiv:1008.3347, arXiv:1711.07957]

Inversion can be computed exactly or expanding in α_s:

$$\mathbf{A}^{-1}_{exp}(\mu_h^2) = \mathbf{I} - a_s(\mu_h^2)\mathbf{A}^{(1)} + a_s^2(\mu_h^2)\left[\mathbf{A}^{(2)} - \left(\mathbf{A}^{(1)}\right)^2\right] + a_s^3(\mu_h^2)\left[-\mathbf{A}^{(3)} + 2\mathbf{A}^{(1)}\mathbf{A}^{(2)} - \left(\mathbf{A}^{(1)}\right)^3\right] + O(a_s^4)$$

The N3LO matching conditions

- Isabella Bierenbaum, Johannes Blümlein, and Sebastian Klein. Mellin Moments of the $\mathcal{O}(\alpha_s^3)$ Heavy Flavor Contributions to unpolarized Deep-Inelastic Scattering at $Q^2 \gg m^2$ and Anomalous Dimensions. *Nucl. Phys. B*, 820:417–482, 2009. arXiv:0904.3563, doi:10.1016/j.nuclphysb.2009.06.005.
- Johannes Blümlein. Analytic continuation of mellin transforms up to two-loop order. *Computer Physics Communications*, 133(1):76–104, Dec 2000. URL: http://dx.doi.org/10.1016/S0010-4655(00)00156-9, doi:10.1016/s0010-4655(00)00156-9.
- J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, C. Schneider, and F. Wißbrock. The 3-Loop Non-Singlet Heavy Flavor Contributions and Anomalous Dimensions for the Structure Function $F_2(x, Q^2)$ and Transversity. *Nucl. Phys. B*, 886:733–823, 2014. arXiv:1406.4654, doi:10.1016/j.nuclphysb.2014.07.010.
- J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. von Manteuffel, and C. Schneider. The 3-loop pure singlet heavy flavor contributions to the structure function f2(x,q2) and the anomalous dimension. *Nuclear Physics B*, 890:48–151, Jan 2015. URL: http://dx.doi.org/10.1016/j.nuclphysb.2014.10.008, doi:10.1016/j.nuclphysb.2014.10.008.
- J. Ablinger, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, and C. Schneider. The $\mathcal{O}(\alpha_s^3 T_f^2)$ contributions to the Gluonic Operator Matrix Element. *Nucl. Phys. B*, 885:280–317, 2014. arXiv:1405.4259, doi:10.1016/j.nuclphysb.2014.05.028.
- J. Ablinger, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, C. Schneider, and F. Wißbrock. The transition matrix element a_gq(n) of the variable flavor number scheme at o(α_s^3). *Nuclear Physics B*, 882:263–288, May 2014. URL: http://dx.doi.org/10.1016/j.nuclphysb.2014.02.007, doi:10.1016/j.nuclphysb.2014.02.007.
- A. Behring, I. Bierenbaum, J. Blümlein, A. De Freitas, S. Klein, and F. Wißbrock. The logarithmic contributions to the $\mathcal{O}(\alpha_s^3)$ asymptotic massive Wilson coefficients and operator matrix elements in deeply inelastic scattering. *Eur. Phys. J. C*, 74(9):3033, 2014. arXiv:1403.6356, doi:10.1140/epjc/s10052-014-3033-x.
- J. Ablinger, J. Blümlein, S. Klein, C. Schneider, and F. Wissbrock. The $\mathcal{O}(\alpha_s^3)$ Massive Operator Matrix Elements of $\mathcal{O}(n_f)$ for the Structure Function $F_2(x, Q^2)$ and Transversity. *Nucl. Phys. B*, 844:26–54, 2011. arXiv:1008.3347, doi:10.1016/j.nuclphysb.2010.10.021.
- Johannes Blümlein, Jakob Ablinger, Arnd Behring, Abilio De Freitas, Andreas von Manteuffel, Carsten Schneider, and C. Schneider. Heavy Flavor Wilson Coefficients in Deep-Inelastic Scattering: Recent Results. *PoS*, QCDEV2017:031, 2017. arXiv:1711.07957, doi:10.22323/1.308.0031.

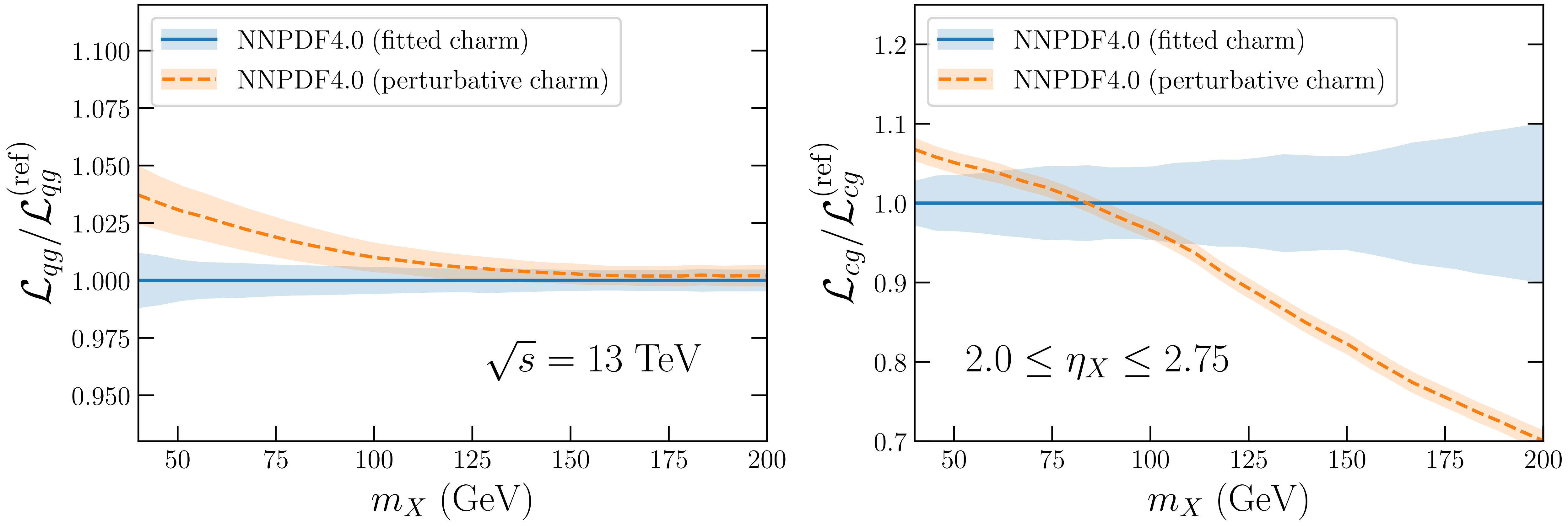
Impact of IC on LHC observables

To see where the differences between fitted and perturbative charm can be evident you can look at partonic lumi of: $pp \to X$

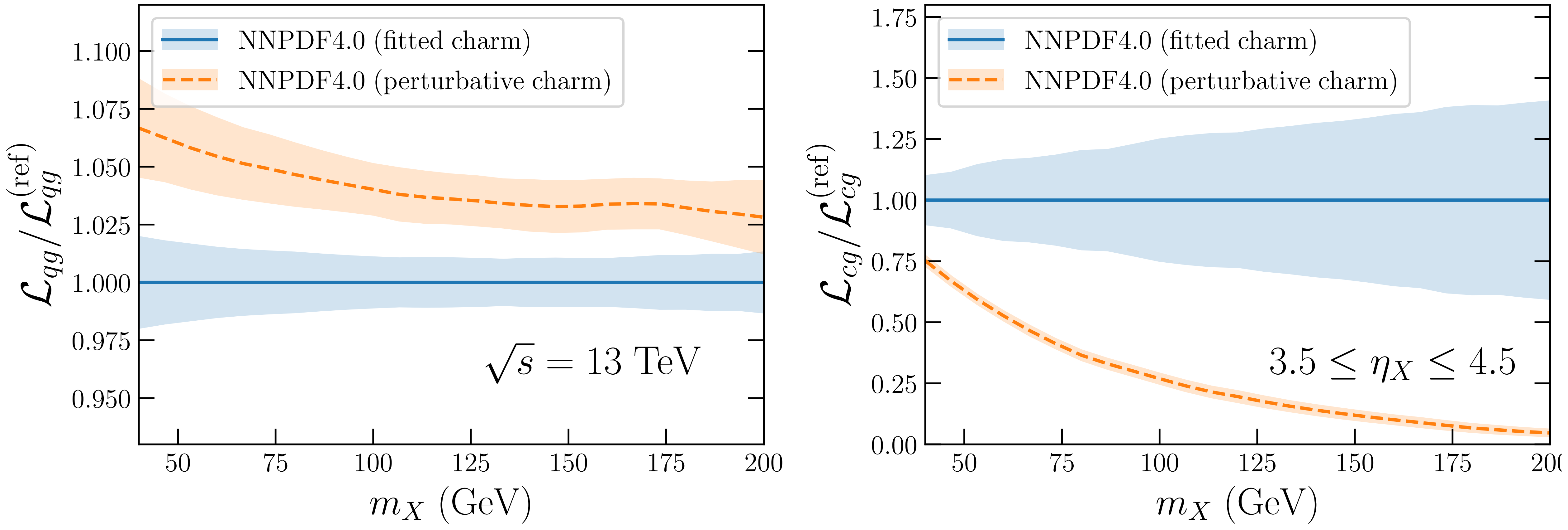
$$\mathscr{L}_{ab} = \frac{1}{s}\int_{\frac{m_X^2}{s}}^{1} \frac{dx}{x} f_a(x, m_X^2) f_b(x, m_X^2)$$

$$\mathscr{L}_{cg} = \mathscr{L}_{cg} + \mathscr{L}_{\bar{c}g} \qquad \mathscr{L}_{qg} = \sum_{i=1}^{n_f} \mathscr{L}_{q_i g} + \mathscr{L}_{\bar{q}_i g}$$

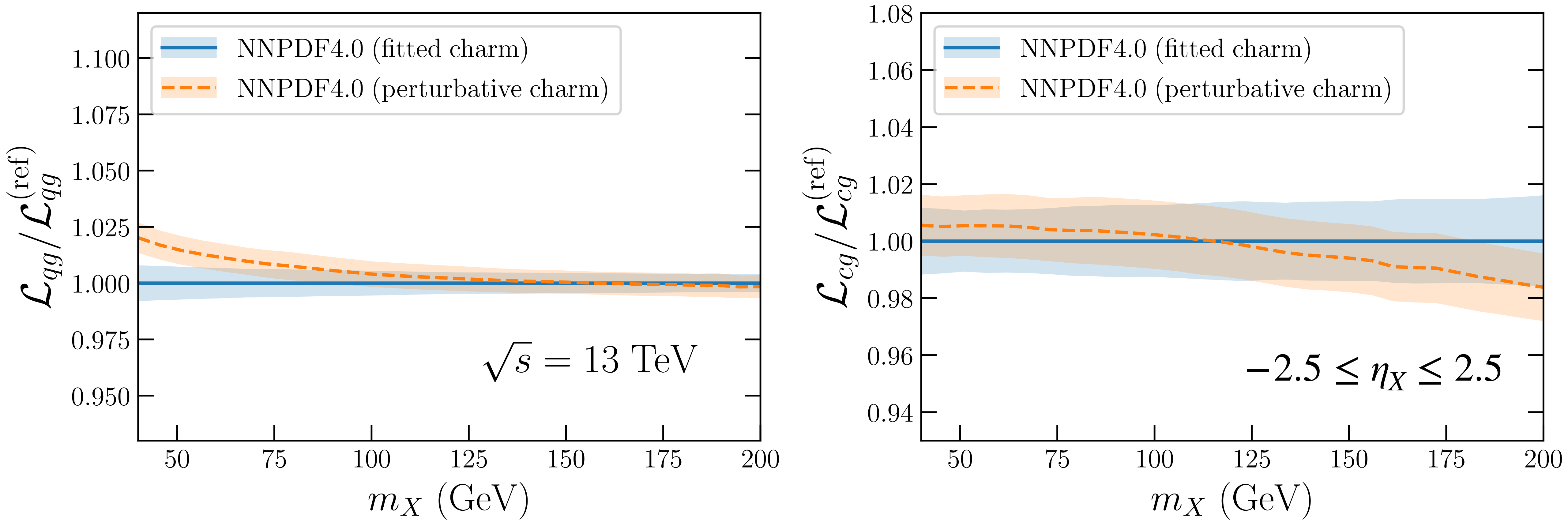
Central region LHCb



Forward region LHCb

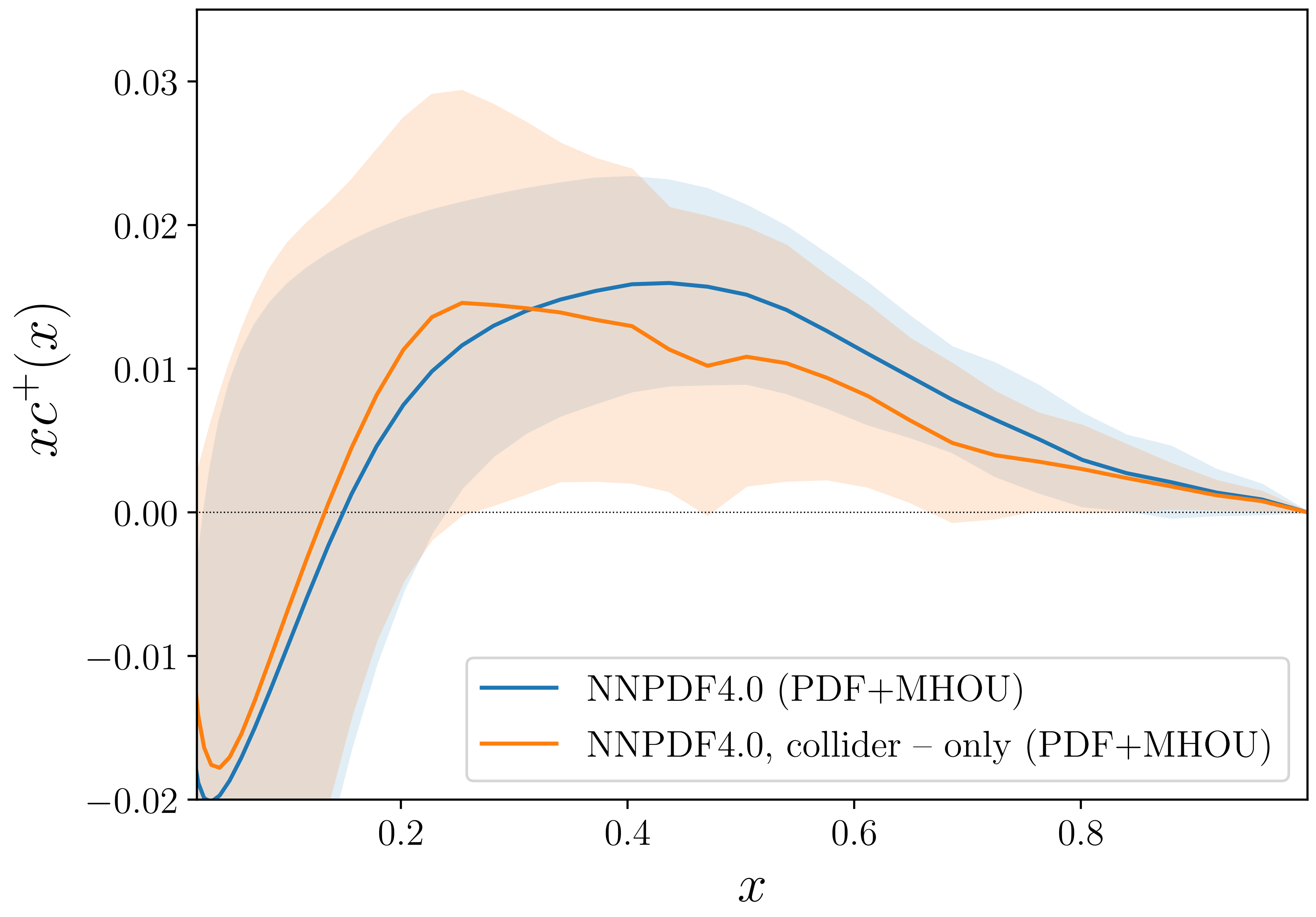


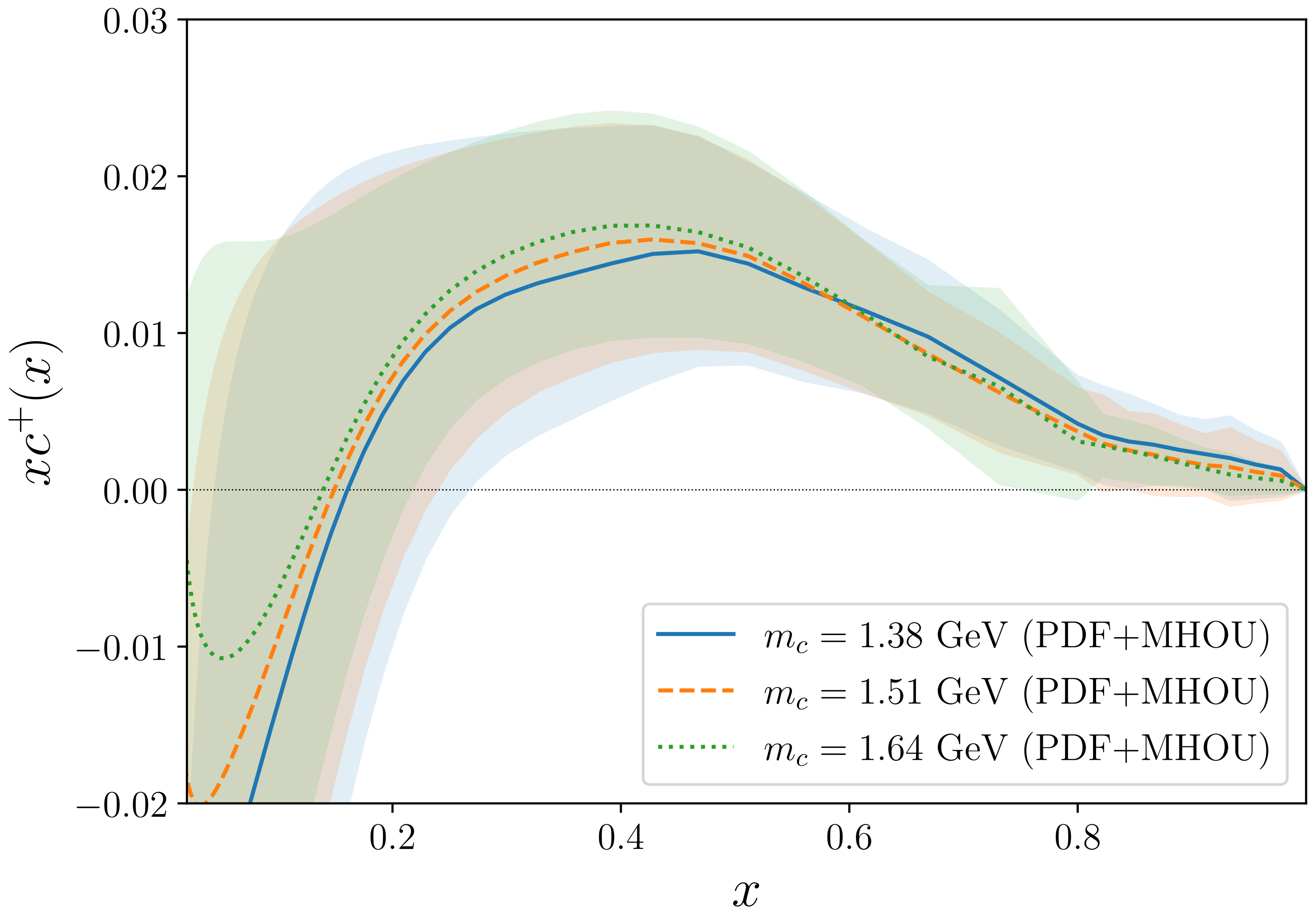
Central region ATLAS-CMS

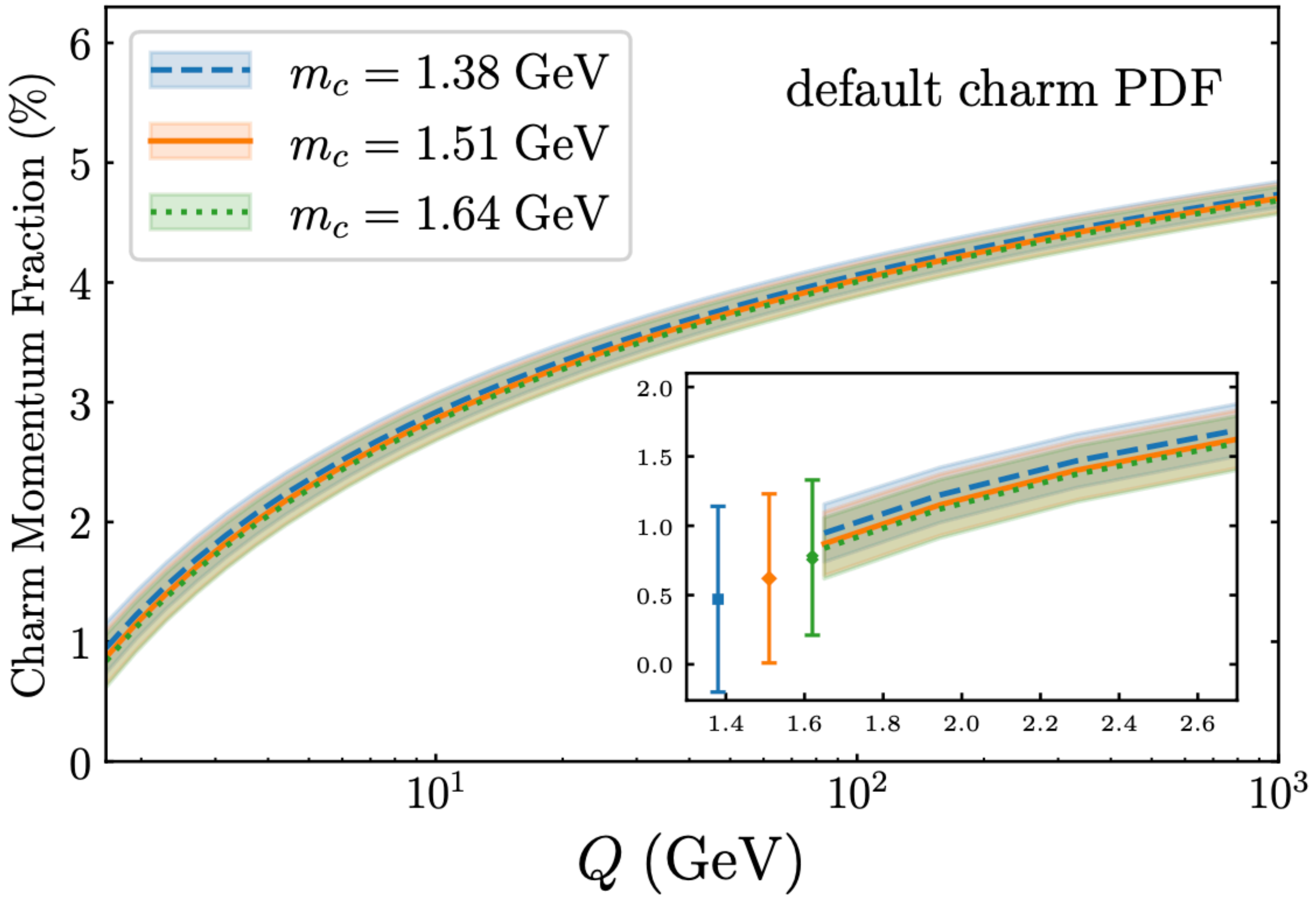


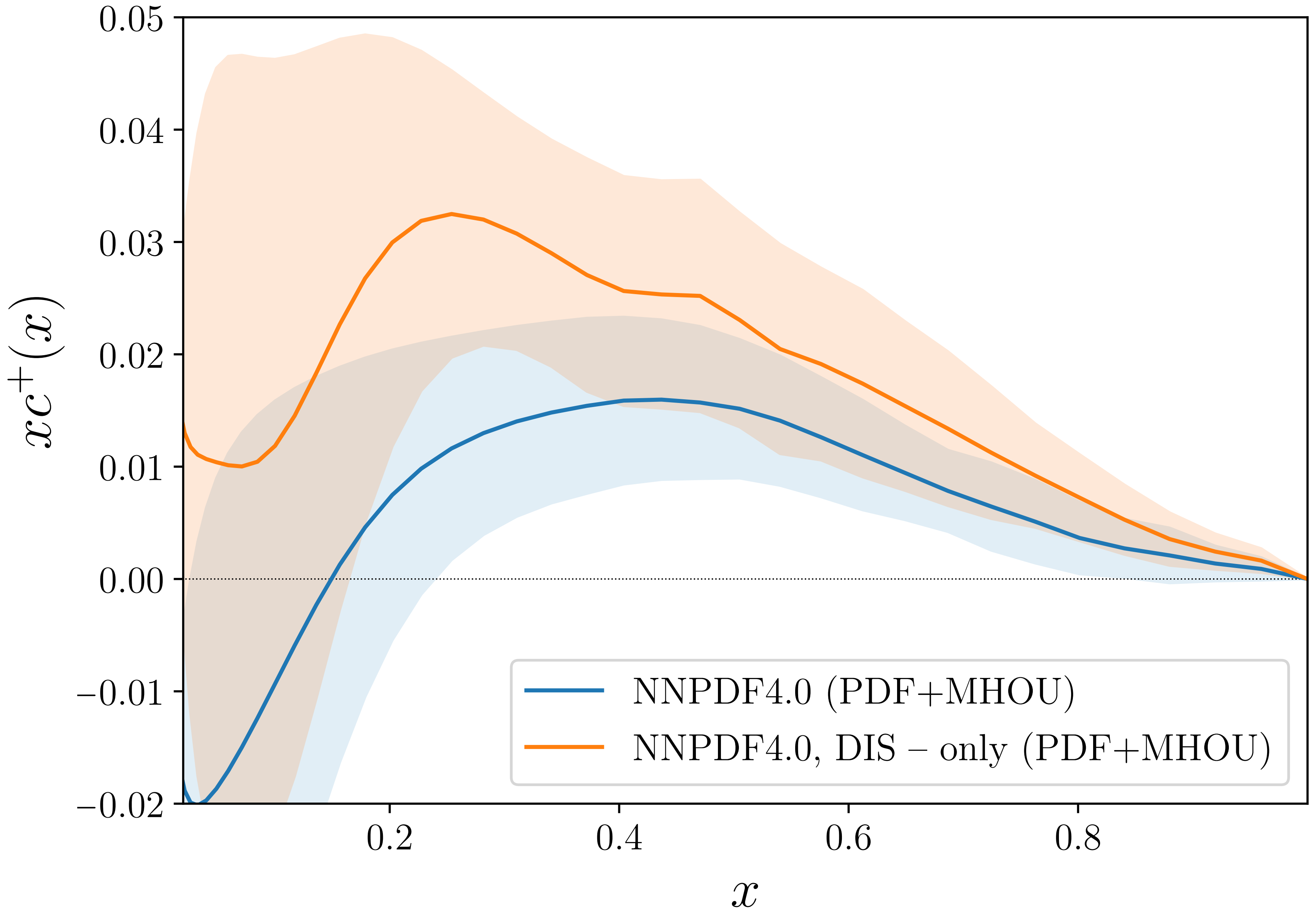
IC mass dependence and dataset variation

- Intrinsic charm is stable upon mass variation
- Scale independecy
- Always vanishing for $x \leq 0.2$









Comparison with CTEQ analysis

CTEQ collaboration recently addressed the IC topic in: [arxiv:2211.01387]

Different BHPS and Meson Baryon clouds models are used as inputs into a PDF fit.

Claims

➡ There is not a clear mapping between the IC models and the charm that you can extract from PDF fits.

➡ The momentum fraction is not resolved enough to justify an evidence for IC.

➡ There might be large FSR effects in the Z+c production measurements, for which NNLO corrections *(+ PS)* should be used.

➡ Need to be sure that PDFs uncertainties are not underestimated.

Possible response

✓ True, but our analysis is model independent. 3FNS heavy quarks PDFs have a definition also as OPE and consistently with operator definition [Collins et al., PhysRevD.18.242, 1978].

✓ There might be better metrics to use in oder to disentangle IC. The two study quote a really similar momentum fraction.

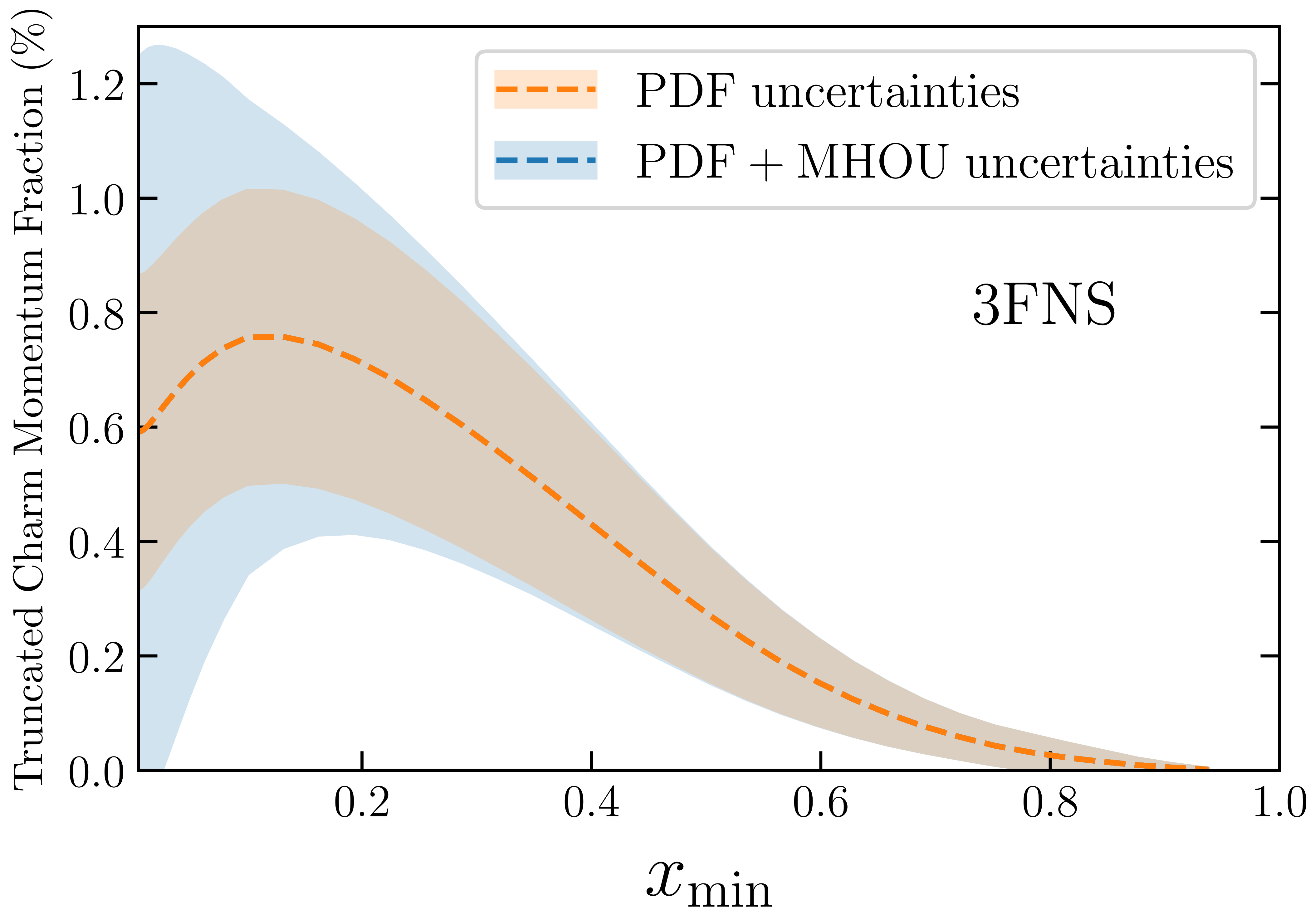
✓ Our calculation reproduces the best knowledge we have on that process so far. Z+c production is not yet included as default in PDFs fits.

✓ PDFU have been validated with many statistical tools *(Closure Tests, Future Tests, Tr/Val ...)*

See E.R.Nocera talk

Comparison with CTEQ analysis

Truncated momentum fraction



Comparison with LHCb data