

NNPDF updates and EW corrections

Stefano Carrazza

University & INFN Milan

PDF4LHC, April 17, 2013



results presented on behalf of the NNPDF collaboration



Outline

1 NNPDF development roadmap

- Code status
- Data status

2 Why electroweak corrections?

3 PDF evolution

- Solution & Benchmark

4 Observables

- How do the observables change by fixing PDFs?

5 Extracting PDFs from real data

- How do the PDFs change by fixing observables?

6 Photon PDF from DIS fit

7 Reweighting the photon PDF with LHC data



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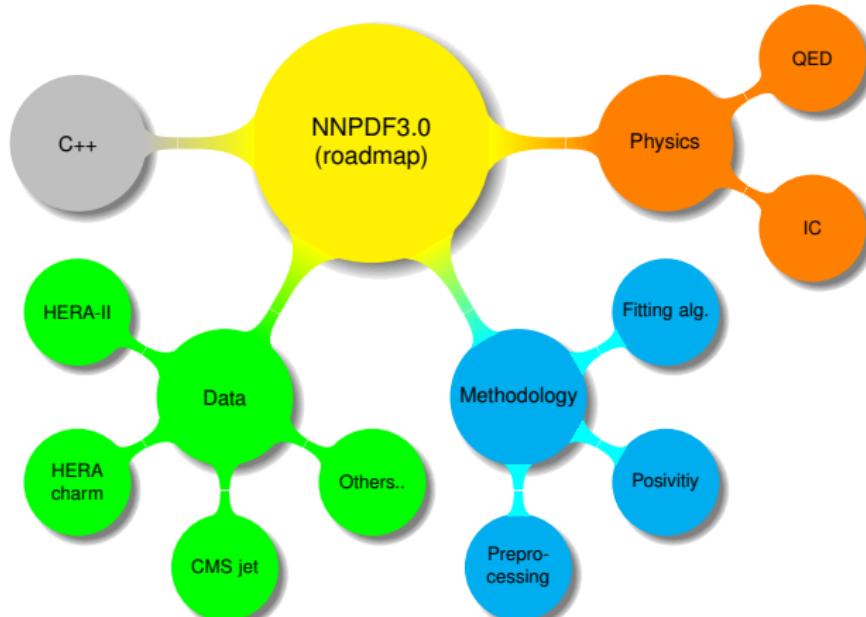


NNPDF development roadmap

- Going towards a NNPDF3.0 release

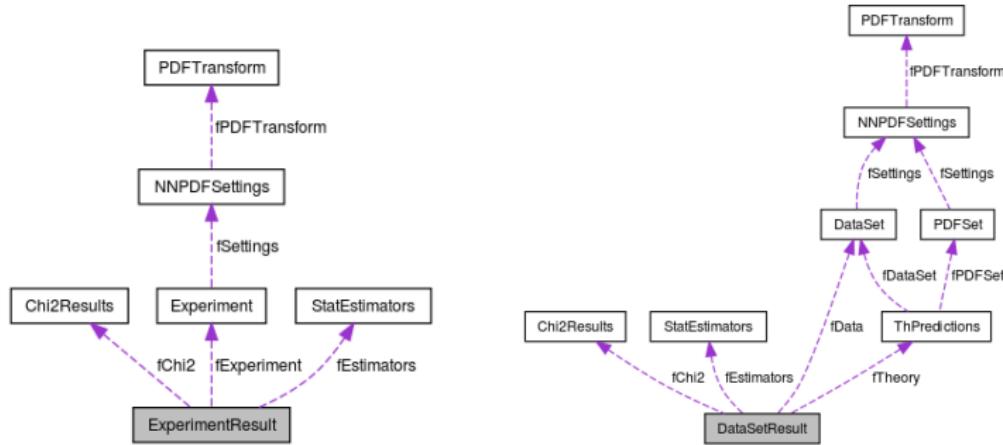
- ▶ Goals:

- ★ new code in C++
- ★ new LHC and HERA data
- ★ methodological improvements



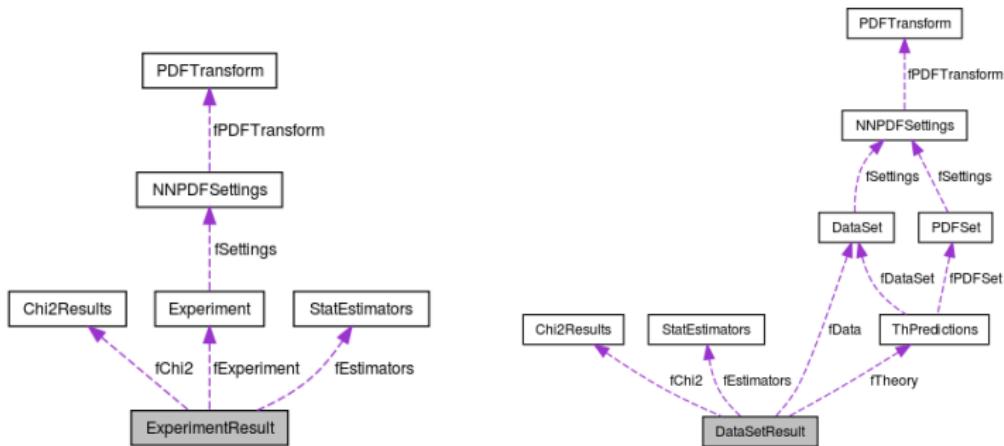
C++ code status

- **NNPDF in C++ is almost complete:**



C++ code status

- **NNPDF in C++** is almost complete:



- **Why bore you with this?**

- 1 New implementation is a **deep crosscheck vs fortran code**
- 2 The new code is extremely optimized ⇒ **fast results**
- 3 Simplicity to perform new **studies, analysis and releases**

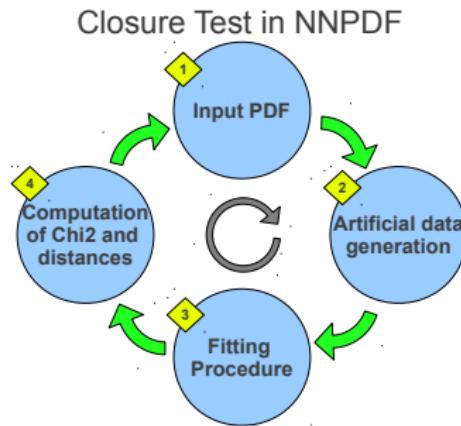
★ e.g. **Photon PDF determination** ⇒ fully fitted by the C++ code



C++ code status

Closure tests

- With the C++ code we can perform **systematic closure tests**.
 - Understand the impact of **methodological choices**:
 - preprocessing, GA, parametrization, etc.



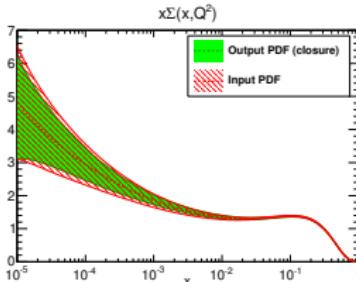
- Idea:** perform this operation when new data is introduced, preprocessing is modified, etc.



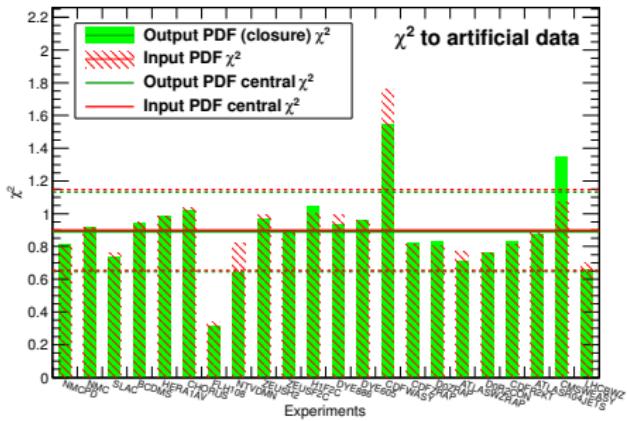
C++ code status

Closure tests

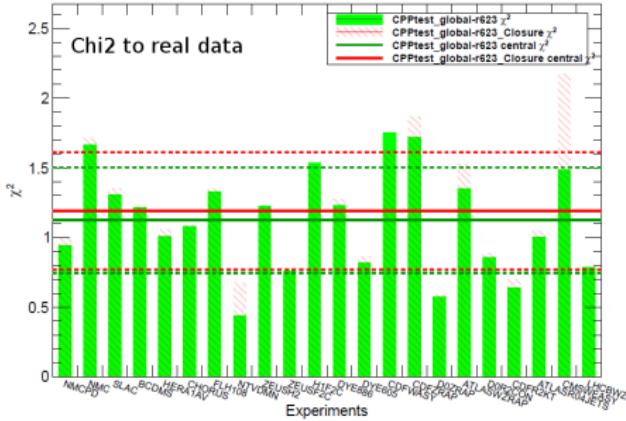
- A first preliminary example of **closure test**:
 - ▶ The procedure is almost working as expected
 - ▶ Distributions tend to **statistical equivalence**
- An example using **NNPDF2.3 NLO as input PDF**:



Distribution of χ^2 for experiments



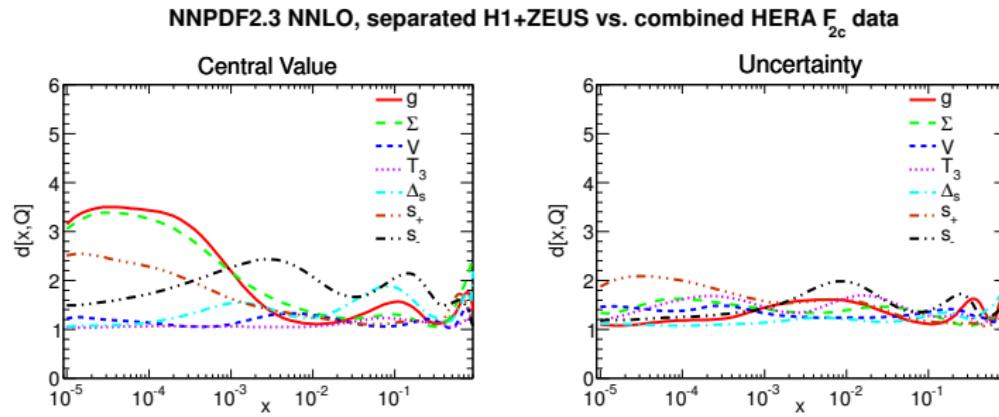
Distribution of χ^2 for experiments



HERA combined charm data

Going towards NNPDF3.0

- Preliminary studies on the inclusion of **HERA combined charm data**¹
 - ▶ H1 and ZEUS F_2^c data ⇒ **replaced by HERA combined data**
 - ★ 47 data points of charm NC reduced cross sections
 - ▶ **Moderate impact** restricted to the small- x region



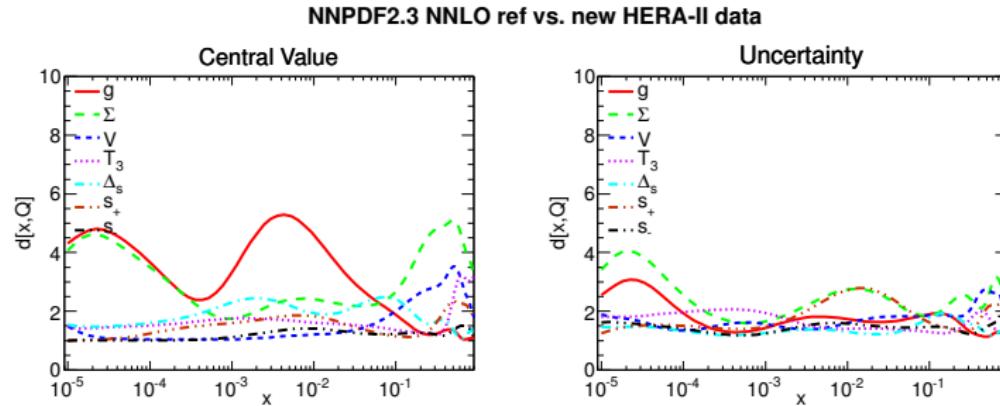
¹arXiv:1211.1182



HERA-II inclusive data

Going towards NNPDF3.0

- Preliminary implementation of the **HERA-II inclusive data**²
 - ▶ +600 new data points
 - ★ H1 inclusive NC and CC structure function
 - ★ H1 inclusive electron and positron NC cross sections
 - ★ ZEUS inclusive positron NC and CC cross sections
 - ▶ Modest but **not negligible** impact vs NNPDF2.3



²arXiv:1206.7007, arXiv:1208.6138

NNPDF outlook

Going towards NNPDF3.0

- Data in progress (in order of completion)
 - ▶ Double differential DY
 - ▶ CMS jet data
 - ▶ W+charm data
 - ▶ Z+jet
 - ▶ others....
- Preprocessing improvements
- Positivity improvements
- Minimization improvements (stopping)



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Motivation

Why electroweak corrections?

- **A naïve argument:**

The QED coupling α can affects processes in which QCD DGLAP is computed at **NLO** and higher orders

$$\frac{\mathcal{O}(\alpha_s^2)}{\mathcal{O}(\alpha)} \rightarrow \frac{\alpha_s^2(M_Z^2)}{\alpha(M_Z^2)} = \frac{0.1184^2}{1/127} \sim 1.78$$

- Leading order QED effects are comparable to NLO QCD corrections.



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- Leading order QED effects are comparable to NLO QCD corrections.

- **Main motivations:**

- ① Provide a first **unbiased determination** of the photon PDF with **faithful uncertainty**.
- ② Assessment of their impact on **theoretical predictions**:
 - ★ EW measurements at the LHC,
 - ★ High-mass Drell-Yan and related searches, m_W determination, etc...
- ③ MRST2004QED is available but **old and based on model assumptions**.



Technical aspects of QED corrections

Step by step: How to obtain the photon PDF.

- In order to achieve our goal we had to implement:

- 1 Modify **PDF evolution** (DGLAP)
 - ★ QCD (NLO/NNLO) + QED (LO)
- 2 Rewrite **observables** including the photon contribution
 - ★ Deep Inelastic Scattering, Drell-Yan and Jet
- 3 Add a new **PDF parametrization for the photon**
 - ★ $\gamma(x, Q^2)$ neural network, imposing PDF positivity
- 4 Perform a fit using the **NNPDF methodology**

Points (3) and (4) were completely written and optimized from scratch.



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PDF evolution (DGLAP)

Solving the coupled evolution

- $\gamma(x, Q^2)$: photon PDF

- ▶ LO QED evolution equations:

$$Q^2 \frac{\partial}{\partial Q^2} \gamma(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} \left[P_{\gamma\gamma}(\xi) \otimes e_\Sigma^2 \gamma\left(\frac{x}{\xi}, Q^2\right) + P_{\gamma q}(\xi) \otimes \sum_j e_j^2 q_j\left(\frac{x}{\xi}, Q^2\right) \right]$$

$$Q^2 \frac{\partial}{\partial Q^2} q_i(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} \left[P_{q\gamma}(\xi) \otimes e_i^2 \gamma\left(\frac{x}{\xi}, Q^2\right) + P_{qq}(\xi) \otimes e_i^2 q_i\left(\frac{x}{\xi}, Q^2\right) \right]$$

with $e_\Sigma^2 = \sum_f^{n_f} N_c^f e_{q_f}^2$ (charges), and the momentum sum rule becomes

$$\int_0^1 dx x \left\{ \sum_i q_i(x, Q^2) + g(x, Q^2) + \gamma(x, Q^2) \right\} = 1$$



PDF evolution (DGLAP)

Multiple methods to solve QCD+QED evolution:

- in a special evolution basis, e.g. in Mellin space:

$$Q^2 \frac{\partial}{\partial Q^2} f(N, Q^2) = P(N) \cdot f(N, Q^2)$$

where $P(N)$ is the splitting function matrix in N space

$$\begin{aligned} P(N) = & \alpha_s(Q^2) P_{\text{LO}}^{\text{QCD}} + \alpha_s^2(Q^2) P_{\text{NLO}}^{\text{QCD}} + \alpha(Q^2) P_{\text{LO}}^{\text{QED}} + \\ & + \mathcal{O}(\alpha\alpha_s) + \mathcal{O}(\alpha_s^3) + \mathcal{O}(\alpha^2) \end{aligned}$$

e.g. Roth, Weinzierl (hep-ph/0403200)



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e.g. Roth, Weinzierl (hep-ph/0403200)

- our method:** combination of QCD and QED evolution solutions

$$f_i(N, Q^2) = \Gamma_{ik}^{\text{QCD}}(Q^2, Q_0^2) \cdot \Gamma_{kj}^{\text{QED}}(Q^2, Q_0^2) \cdot f_j(N, Q_0^2)$$

- Both methods treat the subleading terms in different ways.
- FastKernel implementation of DGLAP evolution.



Current PDF evolution (DGLAP)

- Our DGLAP properties: possibility to switch between **fixed** and **variable** flavor number schemes (**FFNS/VFNS**), running $\alpha(Q^2)$.
- Fast Kernel** implementation in **x-space**, building the interpolation grid.

$$xN_j(x; \mu^2, \nu^2) = \sum_{k=1}^{N_{pdf}} \sum_{\alpha=1}^{N_x} \Gamma_{jk}^{\text{QCD}}(x, x_\alpha | \mu^2, \mu_0^2) \left[x_\alpha N_k(x_\alpha; \mu_0^2, \nu^2) \right],$$

$$x_\alpha N_k(x_\alpha; \mu_0^2, \nu^2) = \sum_{l=1}^{N_{pdf}} \sum_{\beta=1}^{N_x} \Gamma_{kl}^{\text{QED}}(x_\alpha, x_\beta | \nu^2, \nu_0^2) \left[x_\beta N_l(x_\beta; \mu_0^2, \nu_0^2) \right],$$

combining both kernels and setting $\mu = \nu = Q$ we obtain the final expression

$$xN_j(x; Q^2) = \underbrace{\sum_{l=1}^{N_{pdf}} \sum_{\beta=1}^{N_x} \Gamma_{jl}^{\text{QCD.QED}}(x, x_\beta | Q^2, Q_0^2)}_{\text{Fast Kernel}} \underbrace{\left[x_\beta N_l(x_\beta; Q_0^2) \right]}_{\text{Input PDF}}$$

$$\text{where } \Gamma_{jl}^{\text{QCD.QED}}(x, x_\beta | Q^2, Q_0^2) = \sum_{k=1}^{N_{pdf}} \sum_{\alpha=1}^{N_x} \Gamma_{jk}^{\text{QCD}}(x, x_\alpha | Q^2, Q_0^2) \Gamma_{kl}^{\text{QED}}(x_\alpha, x_\beta | Q^2, Q_0^2)$$

Impact of QED corrections to evolution

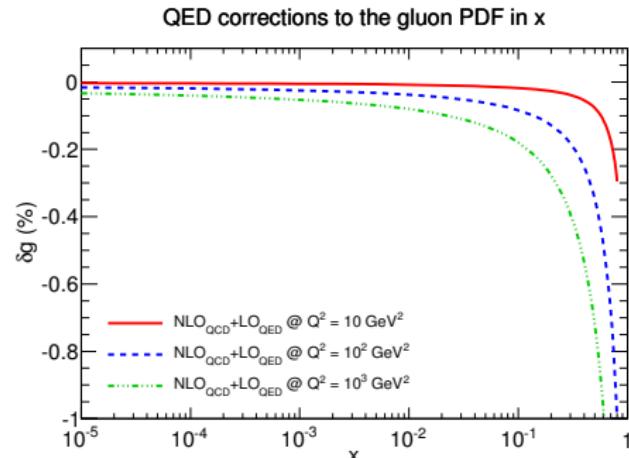
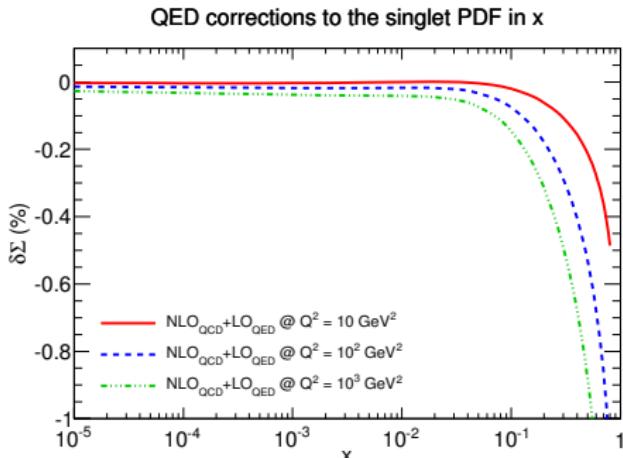
- Input PDF → Les Houches toy PDF +

$$\rightarrow \boxed{x\gamma(x, Q_0^2 = 2 \text{ GeV}^2) = 0}$$

- Relative difference due to QED corrections at $Q^2 = 10, 10^2, 10^3 \text{ GeV}^2$:

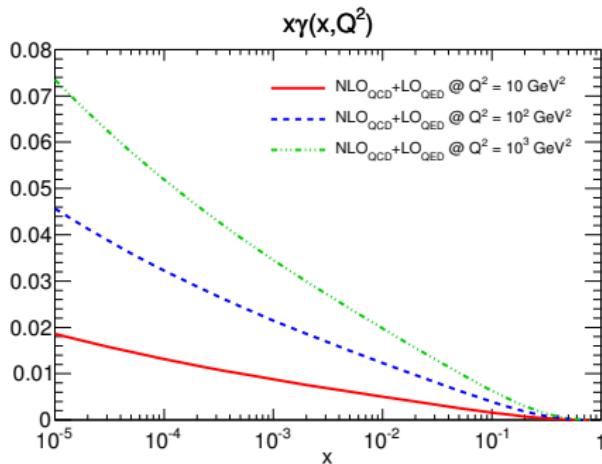
$$\delta f(x, Q^2) = \frac{f_{\text{with QED}}(x, Q^2) - f_{\text{QCD only}}(x, Q^2)}{f_{\text{with QED}}(x, Q^2)}$$

- Singlet and Gluon PDFs



Impact of QED corrections to evolution

- Also for the **photon PDF!**

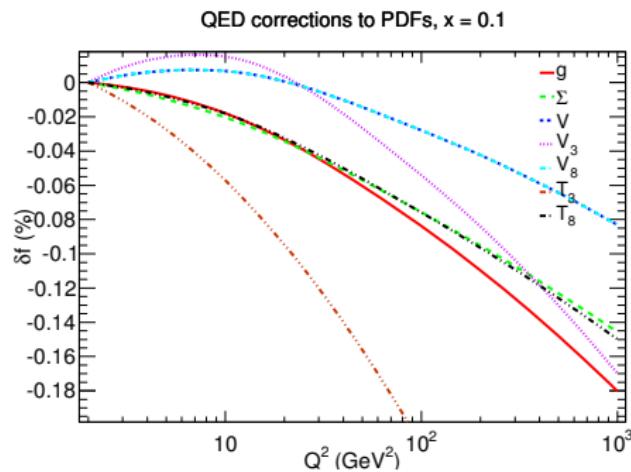
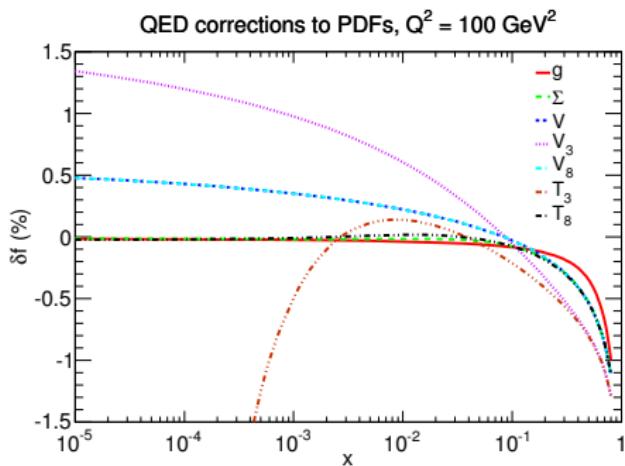


... obtained **dynamically**. $\gamma(x, Q^2)$ is minimally affected by the evolution.



Impact of QED corrections to evolution

- Finally, in the **evolution basis**:



$$\begin{aligned}\Sigma &= u^+ + d^+ + s^+ \\ T_3 &= u^+ - d^+ \\ T_8 &= u^+ + d^+ - 2s^+\end{aligned}$$

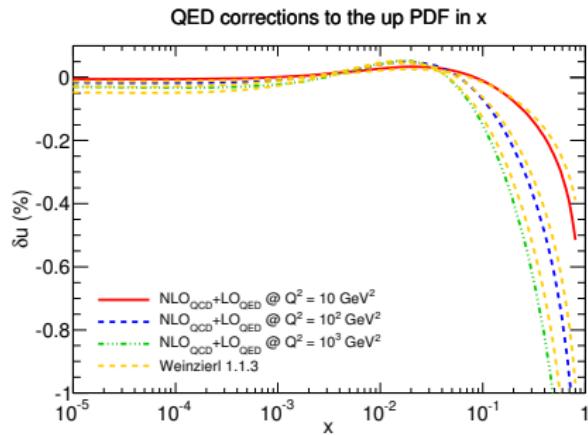
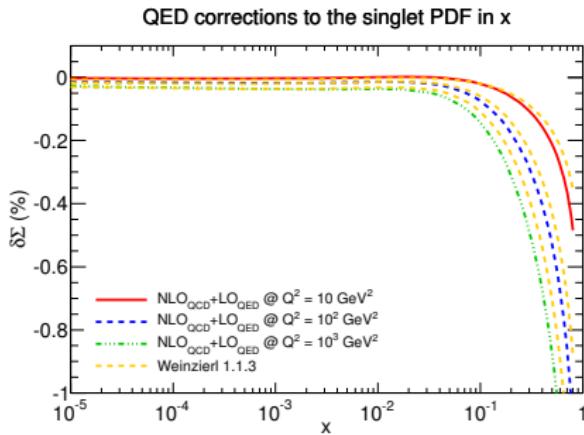
$$\begin{aligned}V &= u^- + d^- + s^- \\ V_3 &= u^- - d^- \\ V_8 &= u^- + d^- - 2s^-\end{aligned}$$



Comparison with other codes

Benchmarking the QCD+QED evolution code

- Good agreement with Weinzierl 1.1.3 code³:
 - ▶ relative differences of the same order of magnitude
 - ▶ differences due to different solution methodology
 - ★ different subleading terms
- Here some examples for the $\Sigma(x, Q^2)$ and $u(x, Q^2)$ PDFs:



³hep-ph/0203112, hep-ph/0403200



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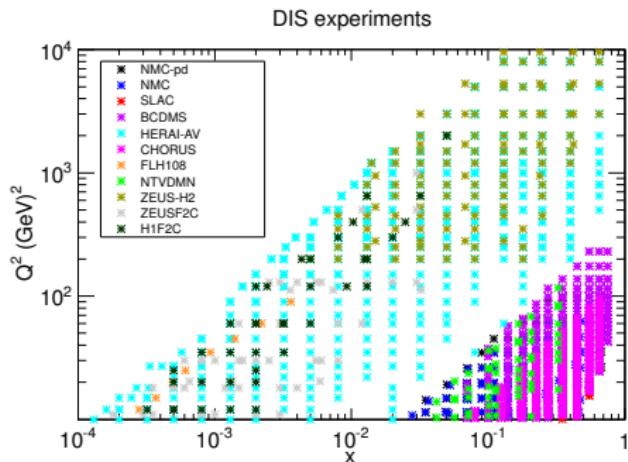
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Observables, current state of the art

- Observables including photon contribution due to evolution:
 - ▶ **2767 DIS data points:** e.g. $F_2^{\gamma,p}$, $F_2^{\gamma,d}$, Dimuon CC cross-section



Isospin symmetry breaking:
When activating QED corrections

$$u^p \neq d^n, d^p \neq u^n$$

so, e.g. $T_3^p(x, Q^2) \neq T_3^n(x, Q^2)$

- Assume no isospin symmetry breaking at initial scale

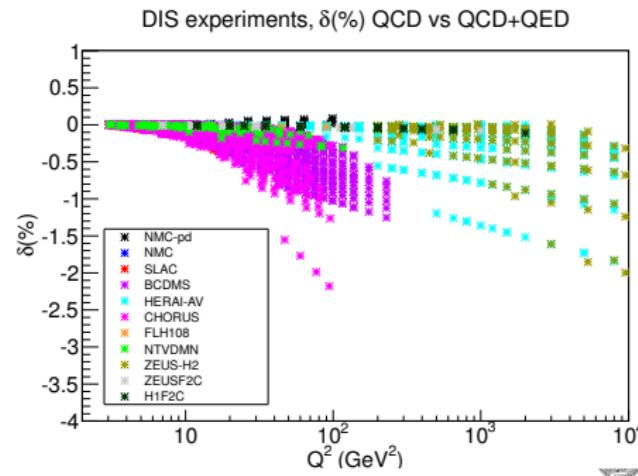
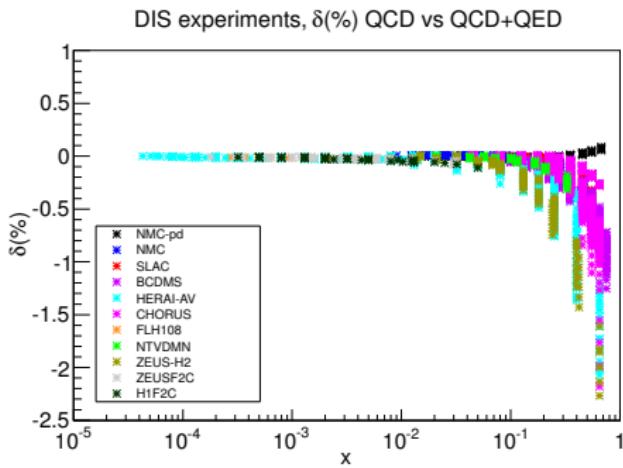
$$T_3^p(x, Q_0^2) = -T_3^n(x, Q_0^2), \quad V_3^p(x, Q_0^2) = -V_3^n(x, Q_0^2)$$

- Isospin is broken dynamically by DGLAP evolution.



Observables, current state of the art

- Observables are codified in Fast Kernel grids
 - ▶ measure the impact on DIS data using NNPDF2.3 NLO
 - ▶ set $\gamma(x, Q_0^2) = 0$
- General behavior very similar to PDFs comparison:
 - ▶ relative differences around -1% for $x \rightarrow 1$



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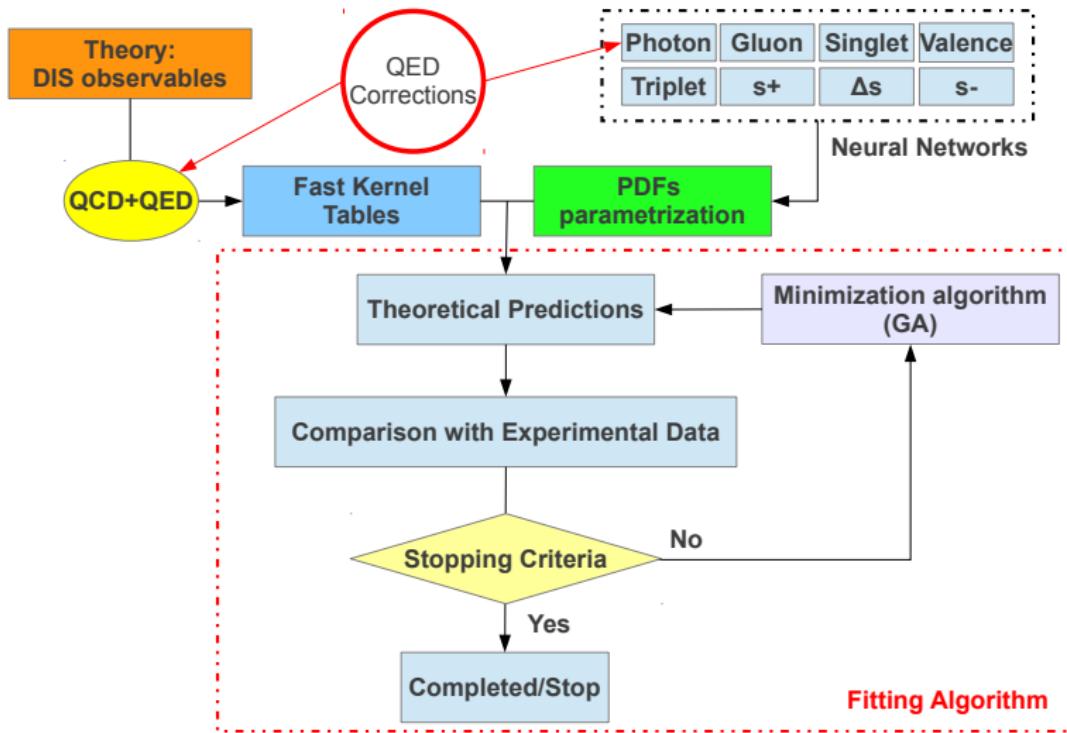
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Fitting algorithm overview



- Fit mechanism also includes momentum sum rule and positivity.



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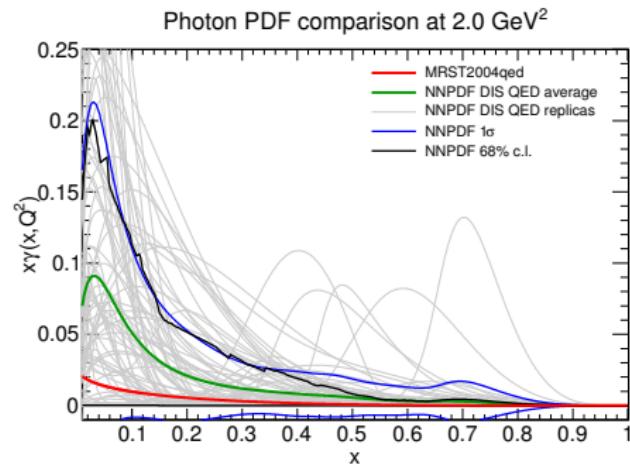
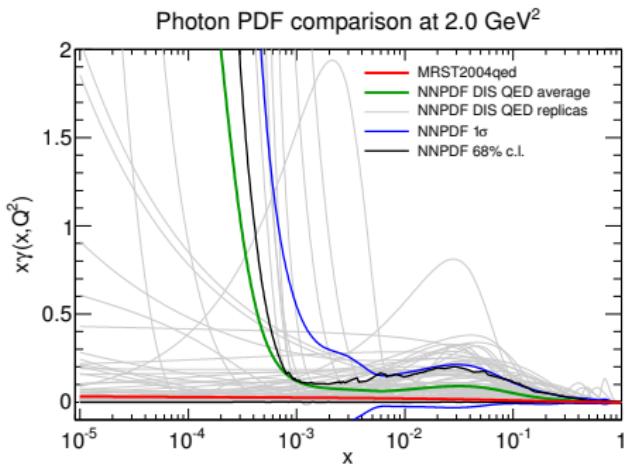
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Photon PDF (preliminary)

- Performing a preliminary **DIS fit** we obtain
 - ▶ the photon PDF **extracted from data** (no toy model)
- Photon from DIS is compatible with zero with large uncertainty.
 - ▶ Z production is a good indicator



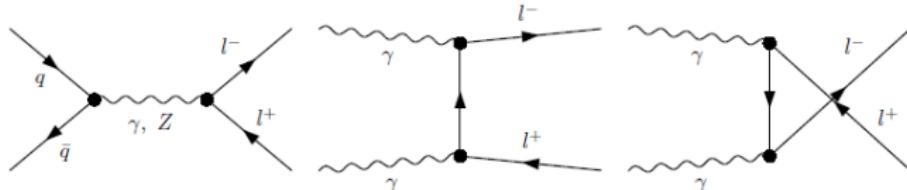
Constrain the Photon with Z production (preliminary)

- Photon PDF: impact on Z production
 - **HORACE**: Drell-Yan EW $\mathcal{O}(\alpha)$, pp @ $\sqrt{s} = 14 \text{ TeV}$
 - cuts: $|\eta'| \leq 2.5$, $p_T' \geq 20 \text{ GeV}$

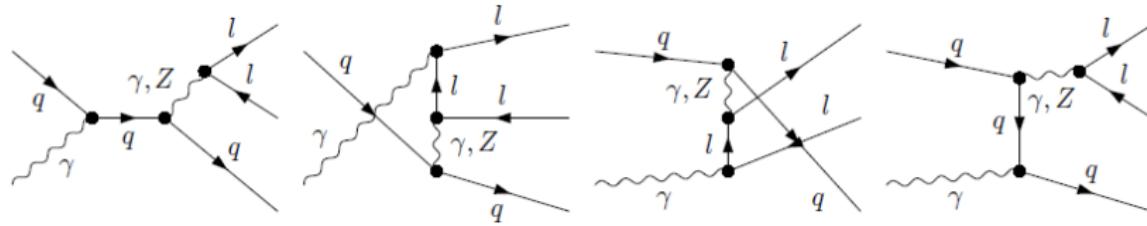


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 - cuts: $|\eta'| \leq 2.5, p_T' \geq 20 \text{ GeV}$
- Born diagrams (from arXiv:0710.1722):

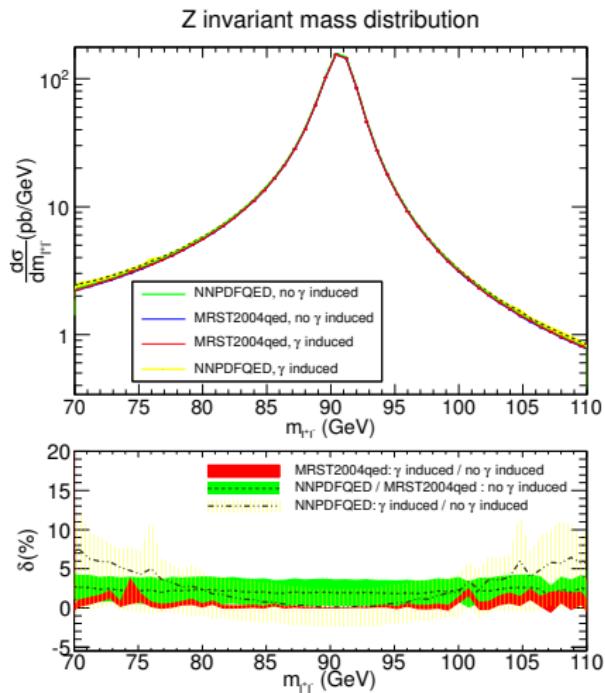


- Photon-induced NLO-EW process diagrams:



Constrain the Photon with Z production (preliminary)

- Example: $Z \rightarrow l^+l^-$ invariant mass

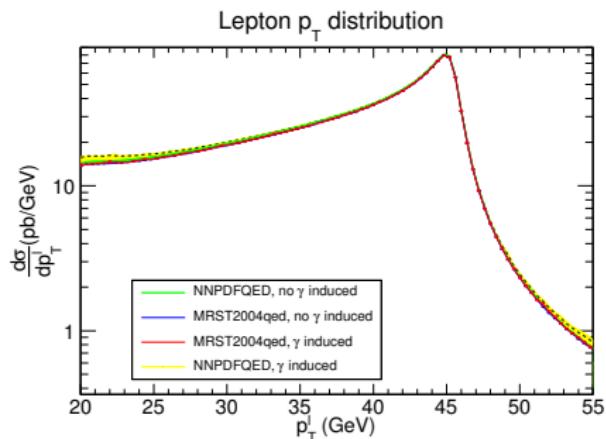


- Effect of photon PDF from DIS data
 - ▶ moderate in the region of the peak
 - ▶ rapidly increases away from the peak
- Potentially huge contribution due to lack of constraints from DIS on small- x
 - ▶ ruins predictions for high m_Z/p_T' !
- **Next step:** use W/Z production data to constraint photon PDF → use for e.g.
 - ▶ predictions for jets & Z' production

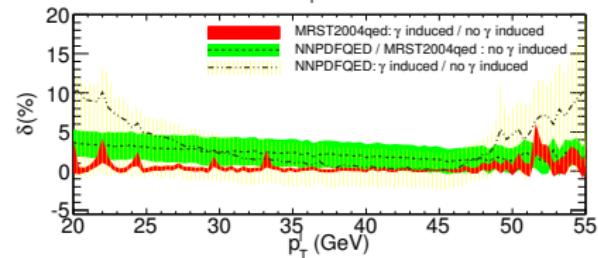


Constrain the Photon with Z production (preliminary)

- Example: $Z \rightarrow l^+l^-$ lepton p_T distribution



- Effect of photon PDF from DIS data
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Reweighting methodology

Adding ATLAS W/Z data

- **Goal:** inclusion of LHC Drell-Yan data to constrain the photon PDF.
 - ▶ **ATLAS W/Z rapidity (2010)** ⇒ **small-x** constraint
 - ▶ **ATLAS DY high mass (2011)** ⇒ **central-x** constraint



Reweighting methodology

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Reweighting, step by step (briefly):

- ➊ Produce N **photon replicas from a DIS fit**
 - ▶ using MSR, dynamical stopping, positivity
- ➋ Build a **NNPDF2.3 NLO + photon DIS set @ $Q^2 = 2 \text{ GeV}^2$**
 - ▶ recomputing evolution with QED corrections
- ➌ Reweight the obtained new set with **ATLAS data**
 - ▶ 43 data points: $d\sigma/d|y_Z|$ (8), $d\sigma/d|\eta_I|$ (22) and $d\sigma/dm_{ee}$ (13)
- ➍ **Unweight the results in a new LHgrid, check the impact.**

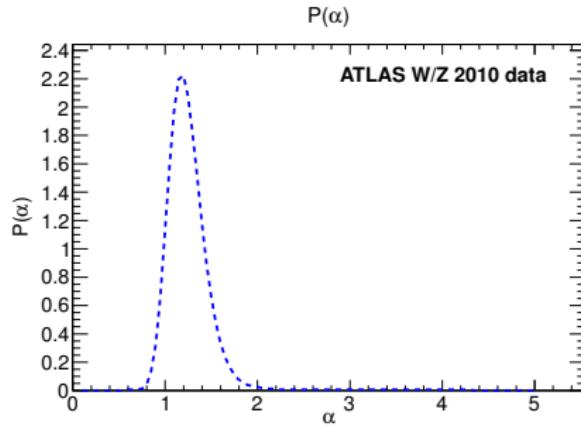
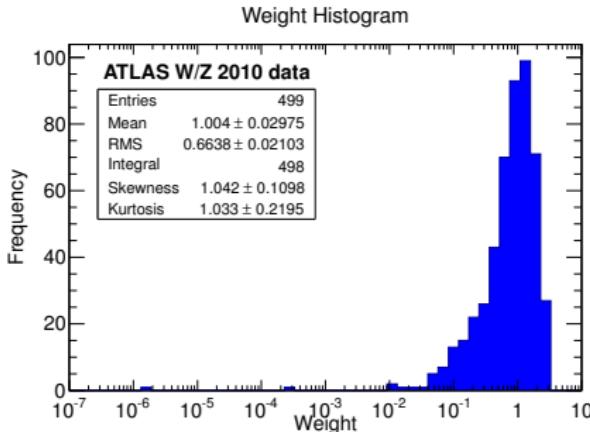


Reweighting with ATLAS W/Z data (preliminary)

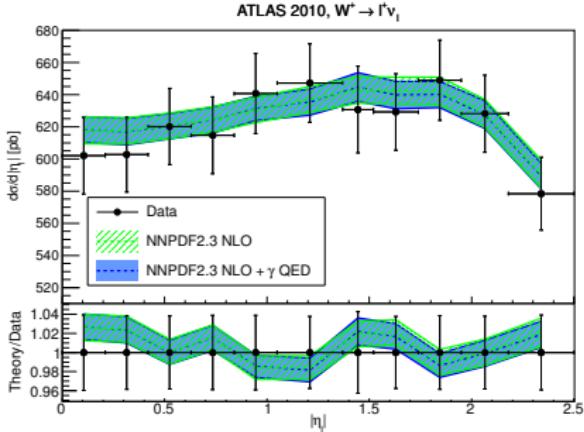
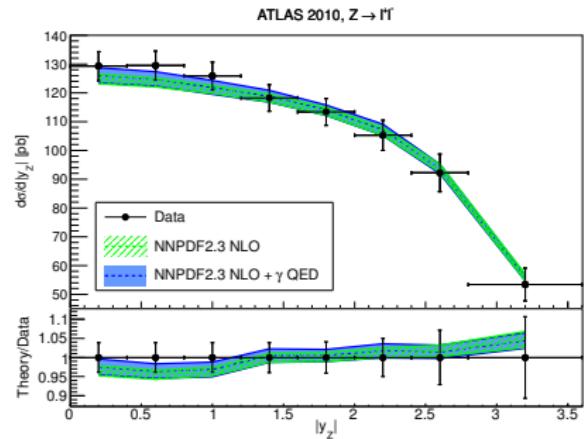
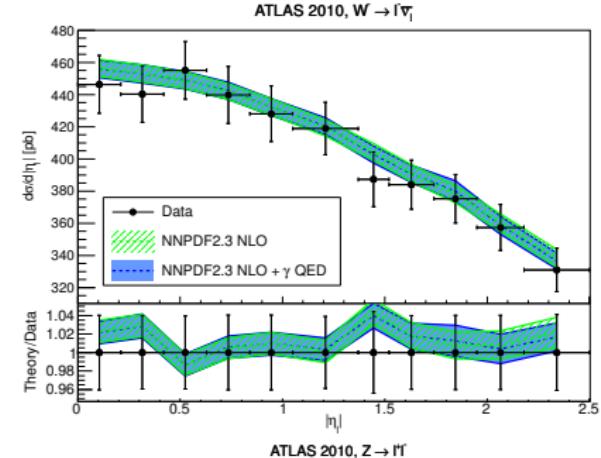
Impact of ATLAS W/Z data

- **Input:** QED DIS fit with $N = 500$ photon replicas.
- For each replica, compute predictions using
 - ▶ **APPLgrid** for the QCD NLO.
 - ▶ **HORACE** for the photon induced (LO and NLO) contribution.
- Compute the weight of replica k (details arXiv:1108.1758)

$$w_k \propto \chi_k^{n-1} e^{-\frac{1}{2}\chi_k^2}$$



Reweighting with ATLAS W/Z data (preliminary)



PDF set	$\chi^2/\text{d.o.f.}$
NLO pure QCD	1.262
NLO + Photon DIS	1.178
NLO + Photon RW	1.097

- $Z \rightarrow l^+ l^-$ channel is more sensitive to the photon PDF:

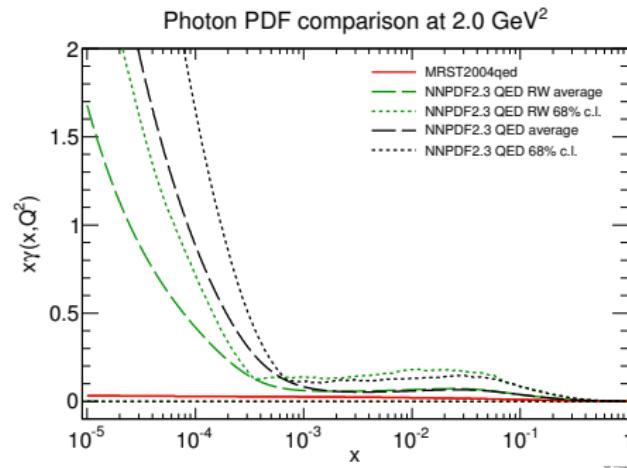
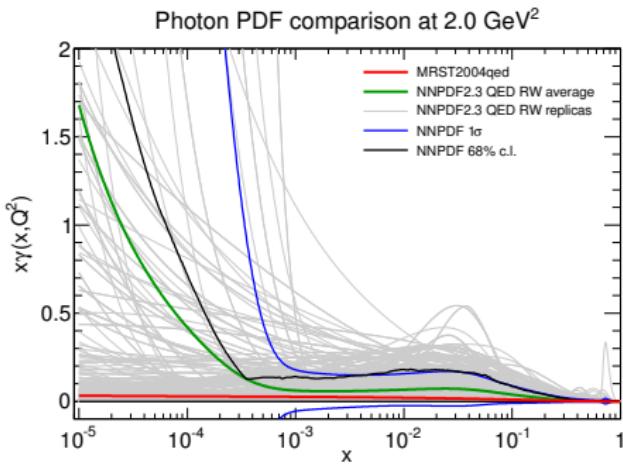
▶ due to $\gamma\gamma \rightarrow e^+ e^-$



Reweighting with ATLAS W/Z data (preliminary)

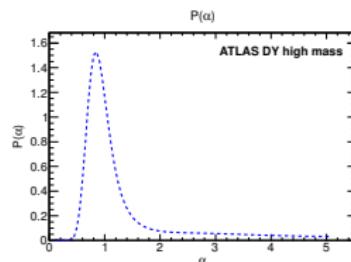
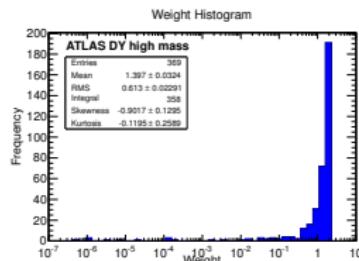
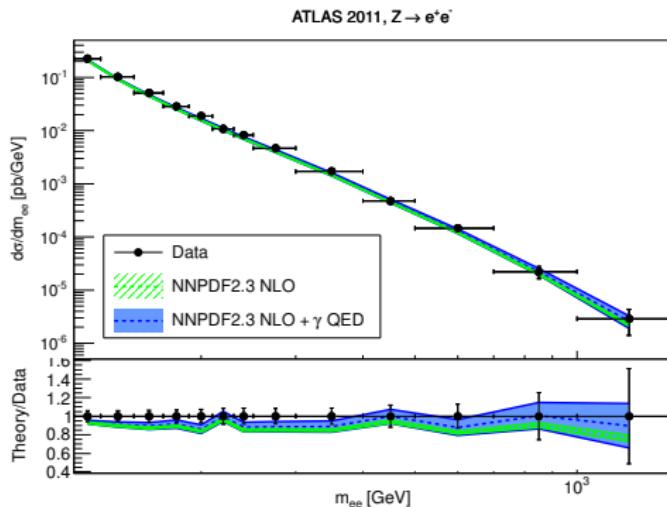
Adding ATLAS W/Z data

- The weighted photon PDF is constrained by the ATLAS W/Z data
 - ▶ Uncertainties are **smaller at small- x**
- From the initial $N = 500$ we obtain $N_{\text{eff}} = 345$.
- **The photon PDF after unweighting** (ATLAS W/Z data):



Reweighting with ATLAS DY high mass data (prel.)

- For each replica, compute with:
 - DYNNLO for the QCD NLO.
 - HORACE for the photon induced contribution.
- $N = 500 \rightarrow N_{\text{eff}} = 300$



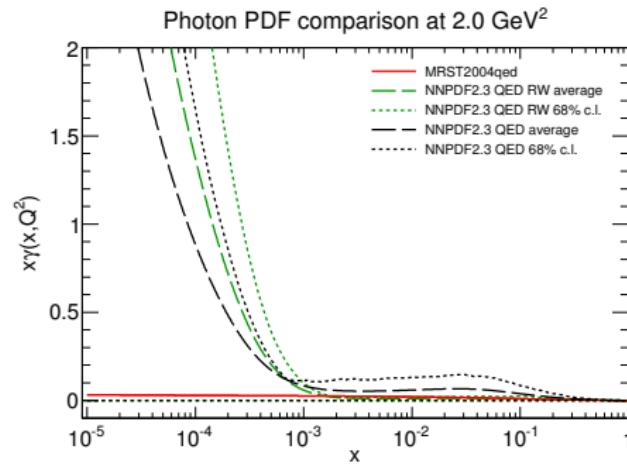
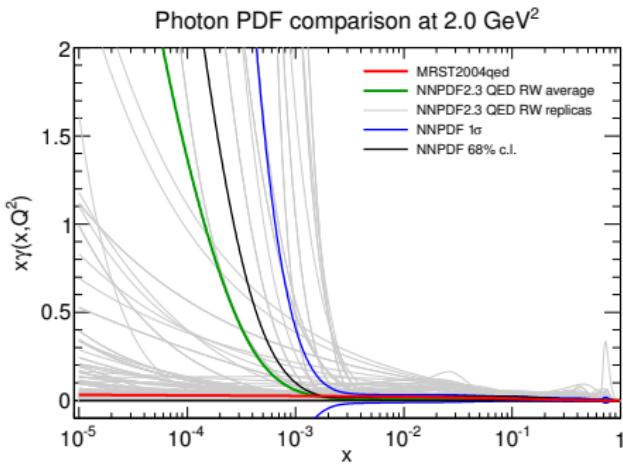
PDF set	$\chi^2/\text{d.o.f.}$
NLO pure QCD	0.830
NLO + Photon DIS	5.715
NLO + Photon RW	0.590



Reweighting with ATLAS DY high mass data (prel.)

Adding ATLAS DY high mass data

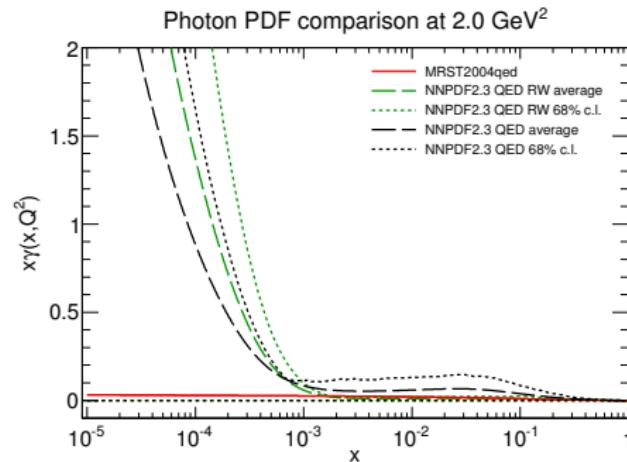
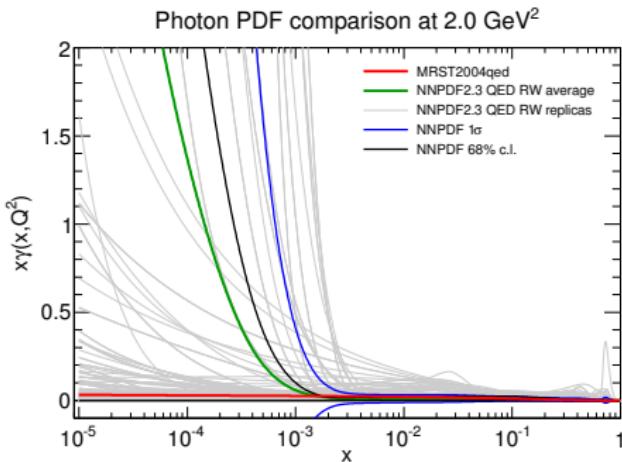
- **Uncertainties are reduced at central/large- x**
- **The photon PDF after unweighting (ATLAS DY high mass data):**



Reweighting with ATLAS DY high mass data (prel.)

Adding ATLAS DY high mass data

- **Uncertainties are reduced at central/large- x**
- **The photon PDF after unweighting (ATLAS DY high mass data):**



Next Step:

Combine both datasets in a single reweighting procedure.

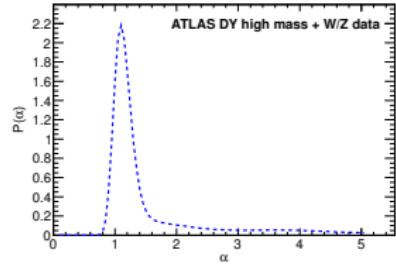
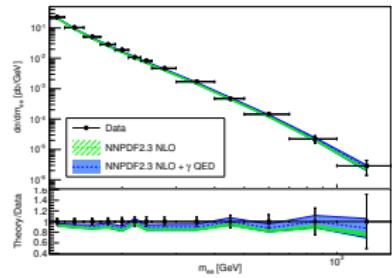
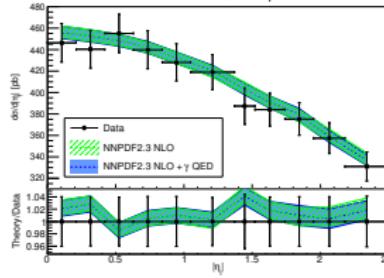
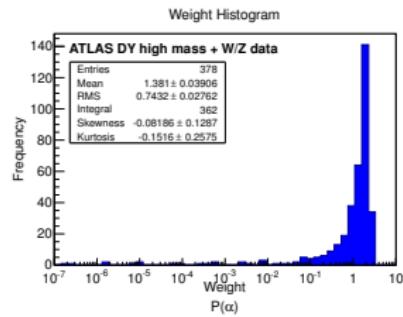
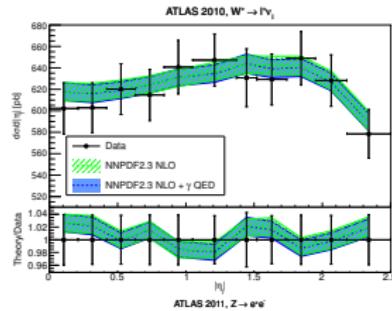
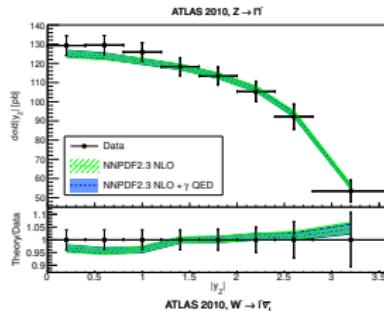


Combined Reweighting (preliminary)

ATLAS W/Z + DY high mass data

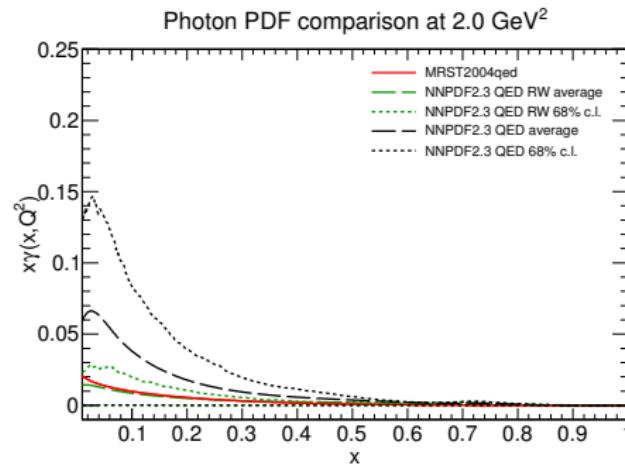
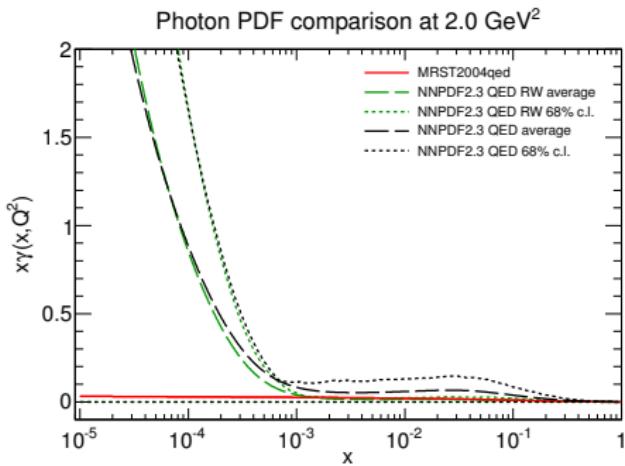
- Full reweighting with **43 data points**:

- ▶ From $N = 500 \rightarrow N_{\text{eff}} = 280$
- ▶ χ^2 from 2.550 to 1.012



Final photon PDF (preliminary)

- Final unweighted photon PDF
 - ▶ constrained at **small** and **central/large-x**.
 - ▶ achieved good precision for **LHC predictions**.

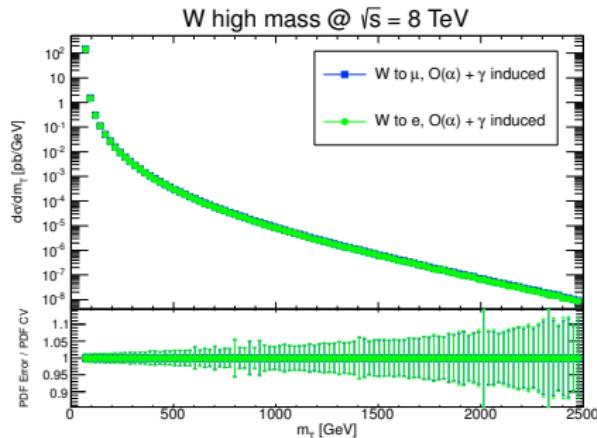
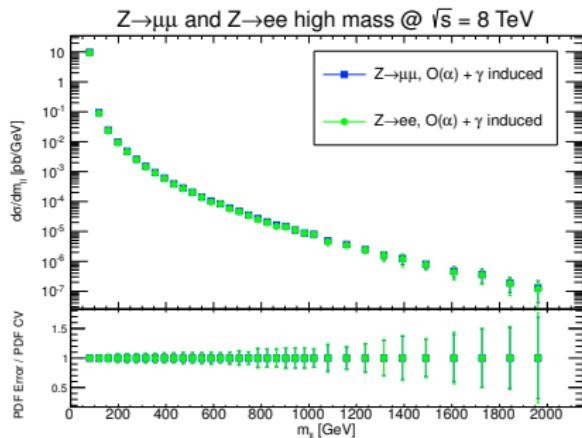


Studying predictions for BSM

Impact on new physics searches

- Simple example test with HORACE:

- 10M events, $p_T^l > 25 \text{ GeV}$, $|\eta| < 2.4$, $\mathcal{O}(\alpha) + \text{photon induced}$



- Photon PDF can improve/change limits for BSM models.



Outlook

● Conclusion

- ▶ Extraction of a preliminary photon PDF from DIS data
- ▶ Study the impact of a DIS photon PDF to Z production
 - ★ **Problem:** too large uncertainties for LHC physics
 - ★ **Solution:** reweighting with ATLAS W/Z and DY high mass data
- ▶ Unweighting the obtained PDF set.

● Outlook

- ▶ Build a NNLO QCD + LO QED fit, using the same methodology.

● Release

- ▶ Release a set with QED corrections in the next LHAPDF release.



Les Houches Toy PDF

- For the benchmark we have used hep-ph/0204316

$$xu_v(x, Q_0^2) = 5.10720 \cdot x^{0.8} (1-x)^3$$

$$xd_v(x, Q_0^2) = 3.06432 \cdot x^{0.8} (1-x)^4$$

$$xg(x, Q_0^2) = 1.70000 \cdot x^{-0.1} (1-x)^5$$

$$x\bar{d}(x, Q_0^2) = .1939875 \cdot x^{-0.1} (1-x)^6$$

$$x\bar{u}(x, Q_0^2) = (1-x)x\bar{d}(x, Q_0^2)$$

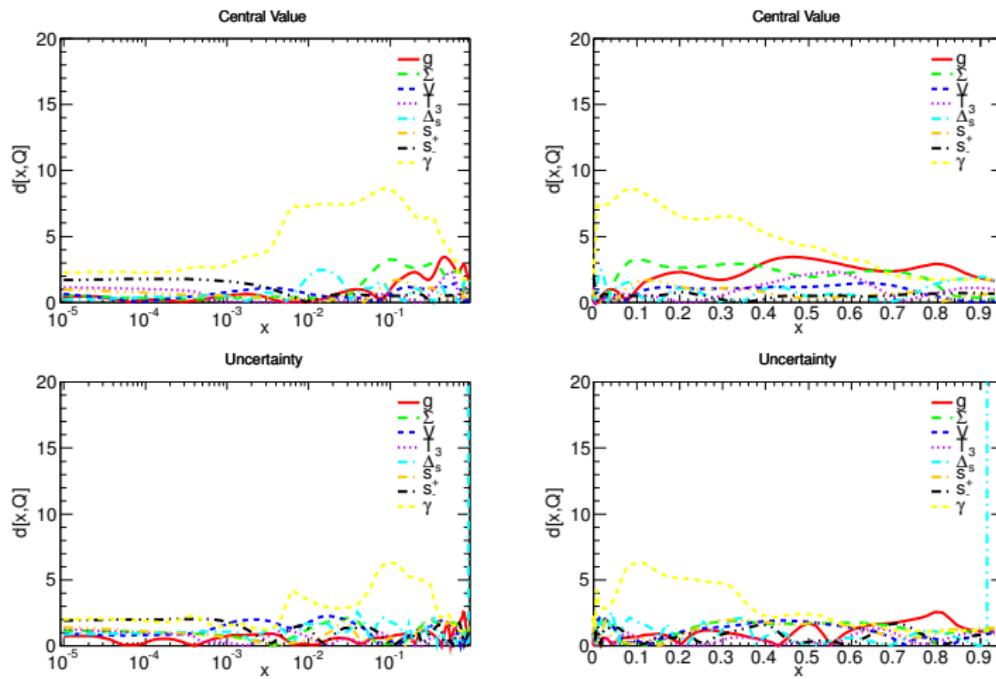
$$xs(x, Q_0^2) = x\bar{s}(x, Q_0^2) = 0.2 \cdot x(\bar{u} + \bar{d})(x, Q_0^2)$$



Photon PDF (preliminary)

- Distances between a pure QCD NLO DIS fit and the respective QED corrected fit.

NNPDF Fit vs Reference Distances



PDF evolution (DGLAP)

Multiple methods to solve QCD+QED evolution:

- (1) in a special evolution basis, e.g. in Mellin space:

$$Q^2 \frac{\partial}{\partial Q^2} f(N, Q^2) = P(N) \cdot f(N, Q^2)$$

where $P(N)$ is the splitting function matrix in N space

$$P(N) = \alpha_s(Q^2) P_{\text{LO}}^{\text{QCD}} + \alpha_s^2(Q^2) P_{\text{NLO}}^{\text{QCD}} + \alpha(Q^2) P_{\text{LO}}^{\text{QED}} + \mathcal{O}(\alpha \alpha_s) + \dots$$

e.g. Roth, Weinzierl (hep-ph/0403200)

- (2) our method: combination of QCD and QED evolution solutions

$$f_i(N, Q^2) = \Gamma_{ik}^{\text{QCD}}(Q^2, Q_0^2) \cdot \Gamma_{kj}^{\text{QED}}(Q^2, Q_0^2) \cdot f_j(N, Q_0^2)$$

Schematically \Rightarrow
$$\begin{cases} \textbf{Method (1): } f(N, Q^2) = \exp[\text{QCD} + \text{QED}] \cdot f(N, Q_0^2) \\ \textbf{Method (2): } f(N, Q^2) = \exp[\text{QCD}] \cdot \exp[\text{QED}] \cdot f(N, Q_0^2) \end{cases}$$

- Methods differ by subleading terms $\mathcal{O}(\alpha \alpha_s)$ (Baker-Campbell-Hausdorff)



Isospin (preliminary)

- We are able to build a **neutron PDF set** for $Q^2 > 2 \text{ GeV}^2$
 - ▶ modify evolution basis $u^p \rightarrow d^n, d^p \rightarrow u^n$
- The ratio Neutron/Proton PDFs
 - ▶ **isospin symmetry breaking on quarks distributions**

