Reweighting NNPDFs

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on behalf of

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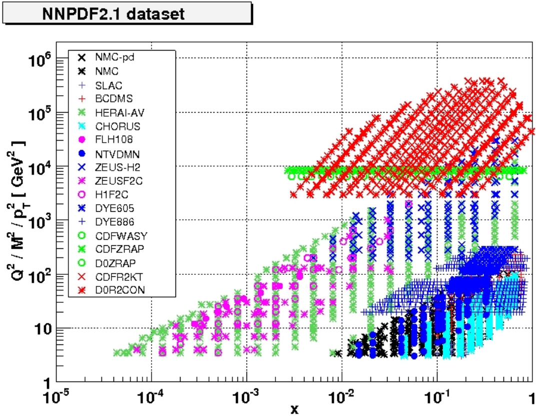
Overview

- NNPDF Approach
- The Reweighting Method
- NNPDF2.2 Parton Set
- Conclusion and Outlook

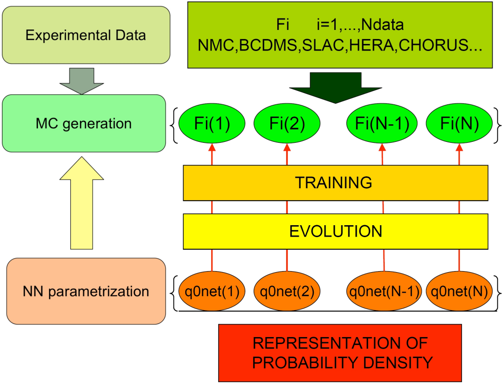
References:

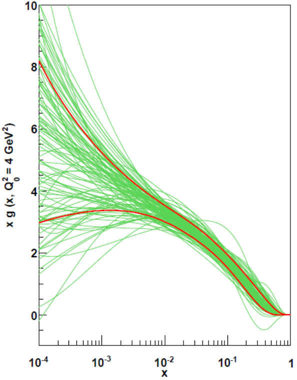
arXiv:1108.1758
arXiv:1107.2652, to be published in Nuclear Physics B
arXiv:1012.0836, Nucl.Phys. B849 (2011) 112-143

- Fixed Target DIS
- Combined HERA-I Data
- HERA F_2^c
- Fixed Target DY
- Tevatron W and Z Production
- Tevatron Jet Production



How does NNPDF work?





NNPDF Approach

NNPDF COLLABORATION

How does NNPDF work?

- Monte Carlo generation:
  textbook methods to evalutate statistical properties:
  $\langle \mathcal{F}[f(x)] \rangle = \frac{1}{N_{rep}} \sum_1^{N_{rep}} \mathcal{F}[f^{(k)}(x)]$ $\quad \sigma_{\mathcal{F}[f(x)]} = \sqrt{\langle \mathcal{F}[f(x)]^2 \rangle - \langle \mathcal{F}[f(x)] \rangle^2}$

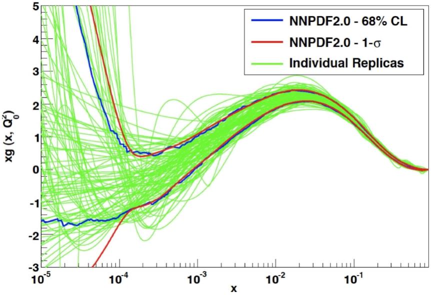
- Neural Network technology:
  universal unbiased interpolant, very redundant parametrization
  $\rightarrow$ O(300) parameters

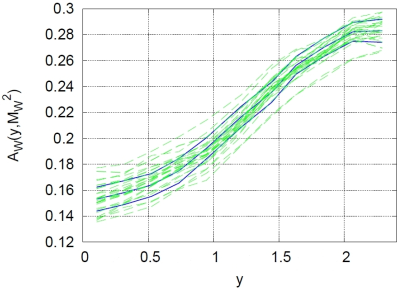
- LO, NLO, NNLO sets:
  $\rightarrow$ Heavy quarks included with FONLL-C scheme
  $\rightarrow$ Fast evolution

What if a new dataset is released?

Usually refitting on old data + new data required:
the N_{rep} replicas of a NNPDF fit give the probability density in PDFs space

$\rightarrow$ with reweighting method new data included without refitting





Reweighting...

If $y = \{y_1, y_2, \cdots, y_n\}$ is the new dataset:

$$\chi^2(y, f_k) = \sum_{i,j=1}^{n} (y_i - y_i[f_k])\sigma_{ij}^{-1}(y_j - y_j[f_k])$$

and the corresponding weights are:

$$\omega_k = \mathcal{N}_\chi (\chi_k^2)^{(n-1)/2} e^{-\frac{1}{2}\chi_k^2} \ \text{ with } \ \mathcal{N}_\chi = \frac{1}{N}\sum_{k=1}^{N} (\chi_k^2)^{(n-1)/2} e^{-\frac{1}{2}\chi_k^2}$$

The observables and their uncertainties can be recomputed like this:

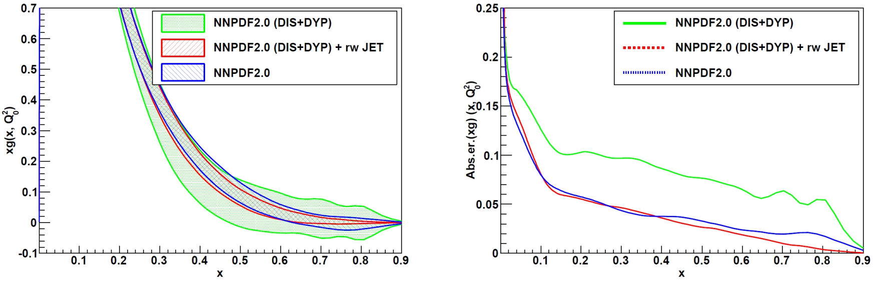
$$\langle \mathcal{F}[f_i(x)] \rangle^{RW} = \int [\mathcal{D}f_i]\mathcal{F}[f_i(x)]\mathcal{P}_{new}[f_i(x)] = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} \omega_k \mathcal{F}[f_i^{(k)}(x)]$$

$$\sigma_{\mathcal{F}_{\text{new}}} = \sqrt{\omega_k \frac{\mathcal{F}^2}{N} - \langle \mathcal{F} \rangle_{\text{new}}^2}$$

Does reweighting work?

- NNPDF2.0 based in DIS+DYP+JET data

→ produce a 2.0 set DIS+DYP only
→ add through reweighting JET data



Differences within statistical fluctuations

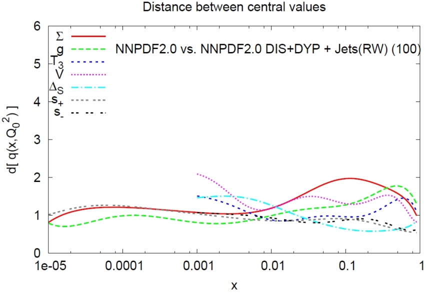
The Reweighting Method

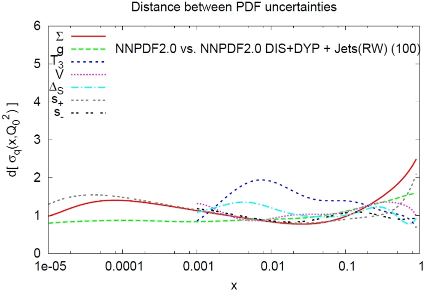
Need to quantify statistical equivalence of two sets

$\rightarrow$ distances between PDFs:

$$d^2\left(\langle q^{(1)}\rangle_{(1)}, \langle q^{(2)}\rangle_{(2)}\right) = \frac{\left(\langle q^{(1)}\rangle_{(1)} - \langle q^{(2)}\rangle_{(2)}\right)^2}{\sigma^2_{(1)}[\langle q^{(1)}\rangle] + \sigma^2_{(2)}[\langle q^{(2)}\rangle]}$$

Average of a hundred random partitions of $N_{rep}/2$ replicas each





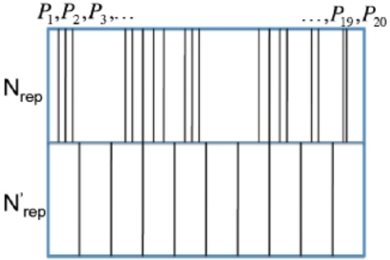
d$\sim$1 $\Leftrightarrow$ equivalence

d$\sim$7 $\Leftrightarrow$ 1-σ level

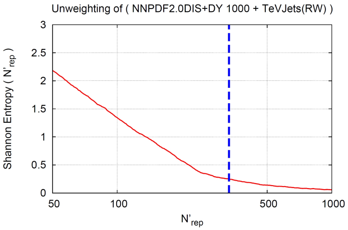
...Unweighting

$$p_k = \frac{\omega_k}{N_{rep}}$$

$$P_k \equiv P_{k-1} + p_k = \sum_{j=0}^{k} p_j$$



By construction new weights are all zero or positive integers:



$$\omega'_k = \sum_{j=1}^{N'_{rep}} \theta\left(\frac{j}{N'_{rep}} - P_{k-1}\right)\theta\left(P_k - \frac{j}{N'_{rep}}\right)$$

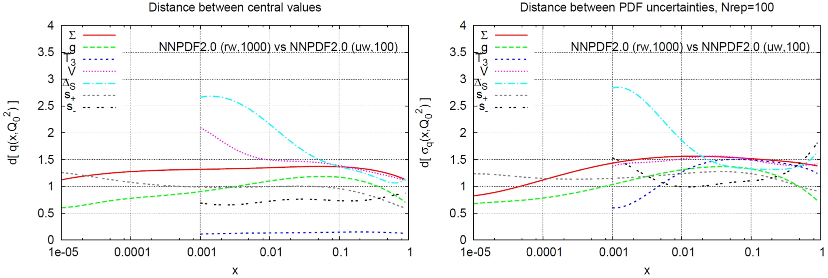
$$N_{eff} \equiv \exp\left\{\frac{1}{N}\sum_{k=1}^{N} \omega_k \ln(N/\omega_k)\right\}$$

Unweighted sets are as easy to use as original “unprocessed” PDF sets

Does unweighting work?

Unweighting of NNPDF2.0(DIS+DY)+rw Tevatron Inclusive Jets

Distances between reweighted and unweighted sets



No significant loss of accuracy

What if new datasets are more than one?

$\rightarrow$ check combination and communtation properties:

$$\hat{W}_{12}=\hat{W}_2\hat{W}_1=\hat{W}_1\hat{W}_2$$

$$\hat{W} \equiv \hat{U}\hat{R} \text{ } (\hat{U}\text{: Unweighting, } \hat{R}\text{: Reweighting})$$

combination $\Longrightarrow$ commutation
(if $\hat{W}_2\hat{W}_1=\hat{W}_{12}$ we have $\hat{W}_2\hat{W}_1=\hat{W}_1\hat{W}_2$)

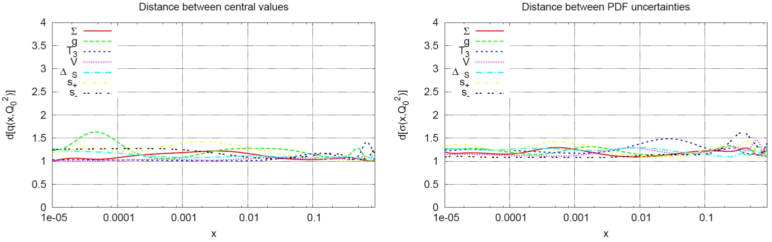
but

commutation $\not\Longrightarrow$ combination
(we might have $\hat{W}_2\hat{W}_1=\hat{W}_1\hat{W}_2\neq\hat{W}_{12}$)

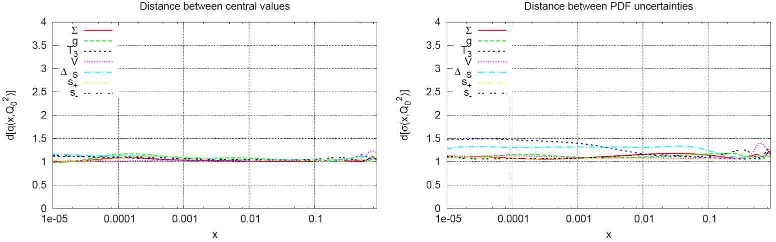
The Reweighting Method

NNPDF COLLABORATION

$\hat{W}$[Jets+E605] vs $\hat{W}$[Jets]$\hat{W}$[E605]



$\hat{W}$[E605]$\hat{W}$[Jets] vs $\hat{W}$[Jets]$\hat{W}$[E605]



	(CDF+D0)	E605	(CDF+D0)+E605
Data points	186	119	305
N_{eff}	627.1	59.5	63.7

Summarizing...

- Reweighting:
  $\rightarrow$ NNPDF2.0(DIS+DY)+rw Jets equivalent to NNPDF2.0
- Unweighting:
  $\rightarrow$ NNPDF2.0(DIS+DY)+rw Jets equivalent to its unweighted set
- Consistency:
  $\rightarrow$ NNPDF2.1 DIS +rw Jets and DY
  $\rightarrow$ NNPDF2.0(DIS+DY)+rw Tevatron inclusive Jets (CDF+D0)

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Summarizing...

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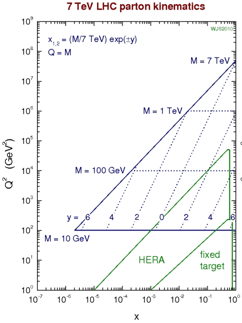
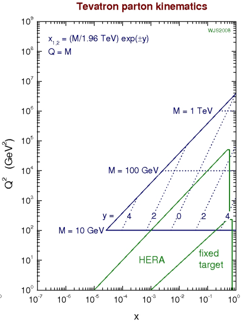
  $\rightarrow$ NNPDF2.0(DIS+DY)+rw Tevatron inclusive Jets (CDF+D0) **V**

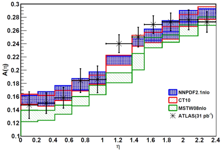
LHC is working very well

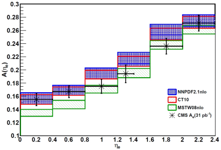
NNPDF2.2 dataset: NNPDF2.1+ATLAS, CMS, D0 W lepton asymmetry data

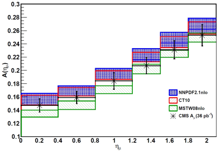
→ NNPDF2.1 only: good description of LHC+D0 data but not optimal ($\chi^2/N_{dat} = 2.22$)











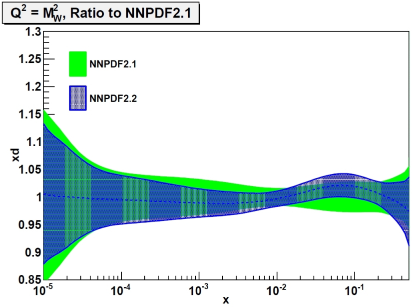
$$A_W^l = \frac{dW_{l^+}/d\eta_l - dW_{l^-}/d\eta_l}{dW_{l^+}/d\eta_l + dW_{l^-}/d\eta_l} \sim \frac{u(x_1,M_W^2)\bar{d}(x_2,M_W^2) - d(x_1,M_W^2)\bar{u}(x_2,M_W^2)}{u(x_1,M_W^2)\bar{d}(x_2,M_W^2) + d(x_1,M_W^2)\bar{u}(x_2,M_W^2)}$$

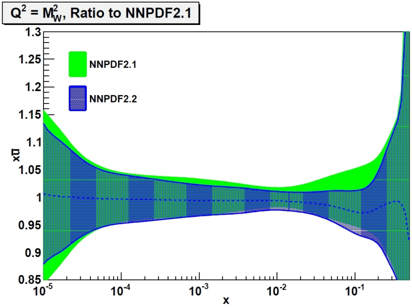
After reweighting $\chi^2/N_{dat} = 0.81$

$N_{eff} = 1000 \rightarrow N_{eff}^{RW} = 181$

Impact mainly in two regions:

- $x \sim 10^{-3} \rightarrow$ up to 20% uncertainties reduction
- $x \sim 10^{-2} - 10^{-1} \rightarrow$ up to 30% uncertainties reduction

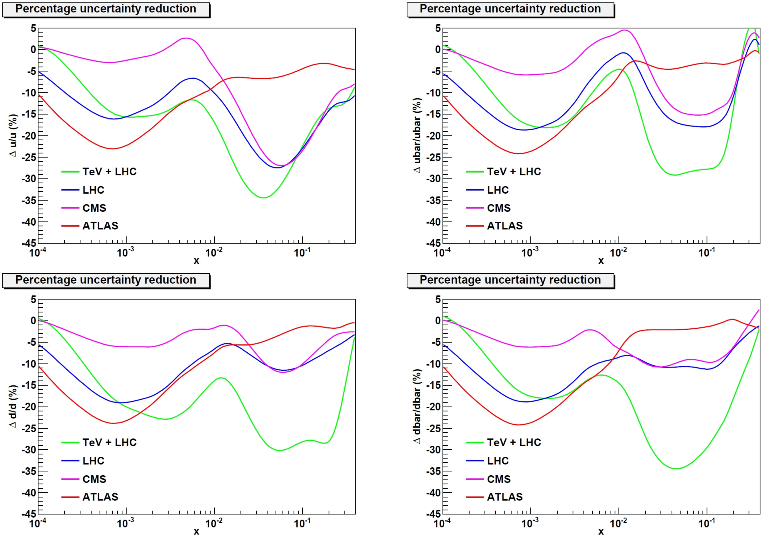




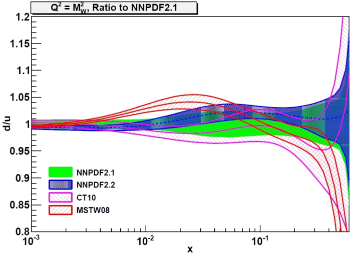
Very significant constraint

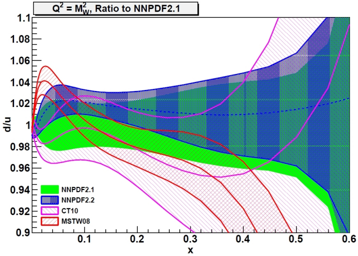
NNPDF2.2 Parton Set

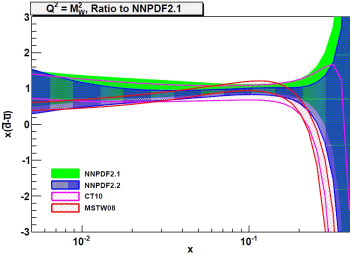
Percentage uncertainty reduction



Comparison between NNPDF2.1, NNPDF2.2, CT10, and MSTW08

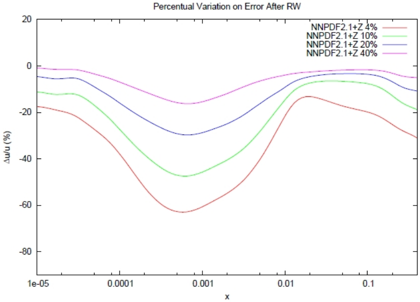






The method is useful to determine pseudo data's impact
$\rightarrow$ possible dialogue with experimentalists

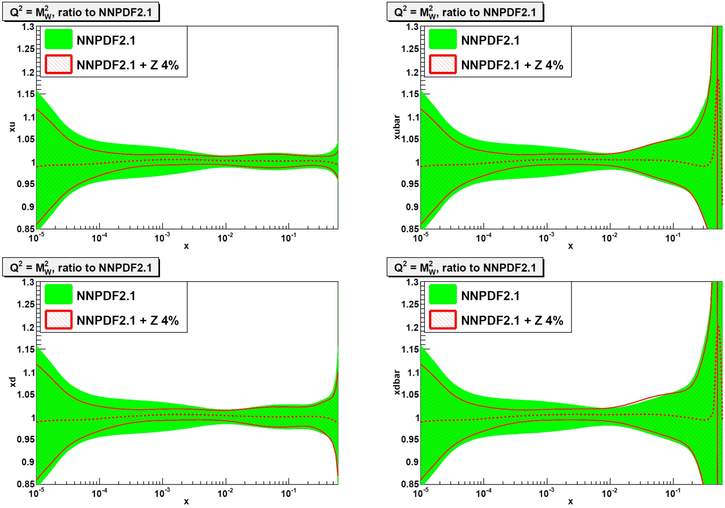
Percentual reduction of PDFs uncertainties:



$$f(p1) + f(p2) \rightarrow Z^0(\rightarrow e^-(p3) + e^+(p4))$$

NNPDF COLLABORATION

Impact on up, down and respective anti-flavor PDFs



$$f(p1) + f(p2) \to Z^0(\to e^-(p3) + e^+(p4))$$

Conclusion and Outlook

NNPDF COLLABORATION

- Reweighting Method
- NNPDF2.2: first Parton Set including LHC data
- LHC is providing us precision information on PDFs
  $\rightarrow$ medium & large x gluon:

  prompt photons available
  (precision) jets in progress

  $\rightarrow$ light flavor separation:

  low-mass DY preliminary
  high-mass DY in progress
  Z rapidity distributions preliminary
  W asymmetries available

  $\rightarrow$ strangeness & heavy flavors:

  strangeness: W+c in progress
  charm: Z+c, γ+c future?
  bottom: Z+b in progress

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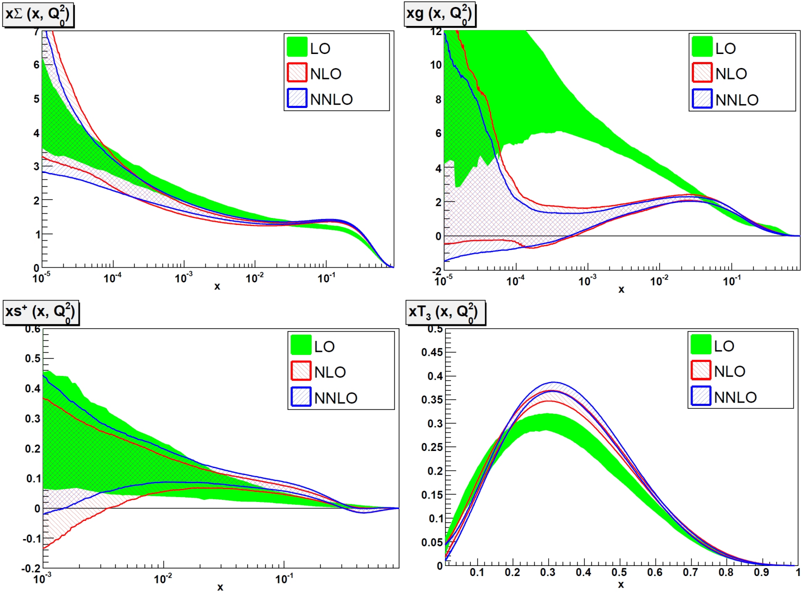
  → strangeness & heavy flavors:

  strangeness: W+c in progress
  charm: Z+c, γ+c future?
  bottom: Z+b in progress

STAY TUNED!

BACKUP SLIDES

Backup Slides

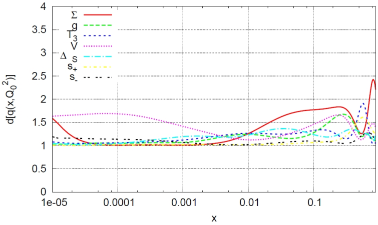


No evidence of instability in PDFs

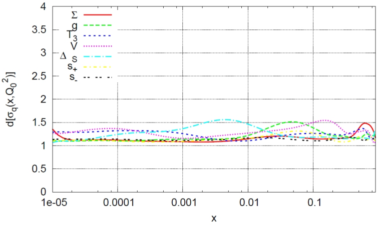
NNPDF COLLABORATION

Ŵ[CDF+D0] vs Ŵ[CDF]Ŵ[D0]

Distance between central values

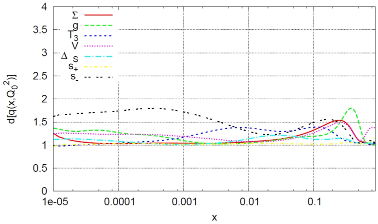


Distance between PDF uncertainties

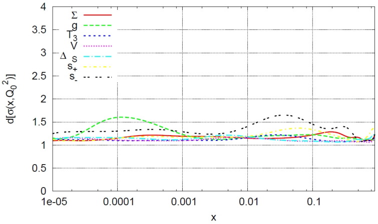


Ŵ[D0]Ŵ[CDF] vs Ŵ[CDF]Ŵ[D0]

Distance between central values



Distance between PDF uncertainties



	CDF	D0	CDF+D0
Data points	76	110	186
N_{eff}	290.8	565.8	334.5

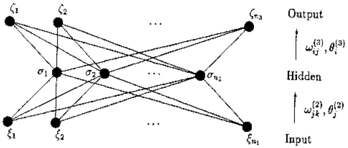
Neural Networks: a non-linear functional form

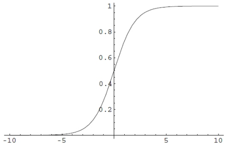
- Each neuron receives input from neurons in preceding layer and feeds output to neurons in subsequent layer
- Activation determined by weights and thresholds

$$\xi_i = g\left(\sum_i \omega_{ij}\xi_j - \theta_i\right)$$

- Sigmoid activation function

$$g(x) = \frac{1}{1 + e^{-\beta x}}$$





An example of 1-2-1 NN:

$$f(x) = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}}}}$$

NNPDF COLLABORATION

→ distances between PDFs:

$$d^2\left(\langle q^{(1)}\rangle_{(1)}, \langle q^{(2)}\rangle_{(2)}\right) = \frac{(\langle q^{(1)}\rangle_{(1)} - \langle q^{(2)}\rangle_{(2)})^2}{\sigma^2_{(1)}[\langle q^{(1)}\rangle] + \sigma^2_{(2)}[\langle q^{(2)}\rangle]}$$

$$\langle q^{(k)}\rangle_{(i)} = \frac{1}{N^{(i)}_{rep}} \sum_{i=1}^{N^(i)_{rep}} q^{(k)}_i,\ \sigma^2_{(i)}[\langle q^{(i)}\rangle] = \frac{1}{N^{(i)}_{rep}} \sigma^2_{(i)}[q^{(i)}]$$

Determination of moments more accurate increasing N_{rep}:
→ if underlying distributions are different, distance grows with $\sqrt{N_{rep}}$

$$\delta(\sigma_{(1)}, \sigma_{(2)}) \equiv \frac{d\left(\sigma_{(1)}, \sigma_{(2)}\right)}{\sqrt{N_{rep}}}$$

NNPDF COLLABORATION

Bayes theorem in terms of probability densities:

$$\mathcal{P}(f|y)\mathcal{D}f\mathcal{P}(y)d^ny = \mathcal{P}(y|f)d^ny\mathcal{P}(f)\mathcal{D}f$$

When fitting we don't demand data to coincide with predictions but we minimize a figure of merit. So:

$$\int \delta(\chi - \chi(y', f))\mathcal{P}(y'|f)d^ny'\mathcal{P}(f)\mathcal{D}f =$$

$$2^{1-n/2}(\Gamma(n/2))^{-1}\Omega_n\chi^{n-1}e^{-\frac{1}{2}\chi^2}\mathcal{P}(f)\mathcal{D}f$$

from which

$$\mathcal{P}(f|\chi)\mathcal{D}f \propto \chi^{n-1}e^{-\frac{1}{2}\chi^2}\mathcal{P}(f)\mathcal{D}f$$

Integrating probability density over a finite volume and sending it to zero:

$$\omega'_k \propto \mathcal{P}(f_k|\chi_k) \propto \chi_k^{n-1}e^{-\frac{1}{2}\chi_k^2}$$