

Unbiased helicity-dependent Parton Distributions with polarized collider data

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and related subjects

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- ① Introduction
- ② Including collider data in a global PDF determination
 - Experimental data sets and new observables
 - Some details on the methodology
- ③ The NNPDFpol1.1 parton set
 - Comparison with NNPDFpol1.0 and DSSV
 - The spin content of the proton
- ④ Conclusions and outlook

1. Introduction

$$\Delta f(x, Q^2) = f^{\rightarrow\rightarrow}(x, Q^2) - f^{\rightarrow\leftarrow}(x, Q^2)$$

1 Understand the spin content of the proton

- what are the patterns of up, down and strange quark and antiquark polarizations?
- how do gluons contribute to the proton spin?

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L_q + L_g$$

$$\Delta\Sigma = \int_0^1 dx \sum_{i=u,d,s} (\Delta q_i + \Delta \bar{q}_i)$$

$$\Delta G = \int_0^1 dx \Delta g$$

2 Describe processes with polarized hadrons in initial states

- explore QCD beyond the helicity-averaged case
- beyond SM studies at a polarized Future Circular Collider [[arXiv:1403.2383](#)]

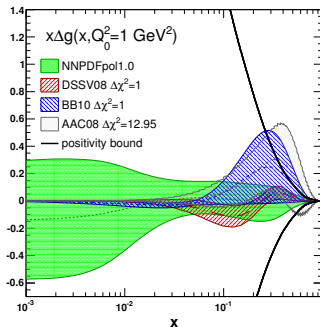
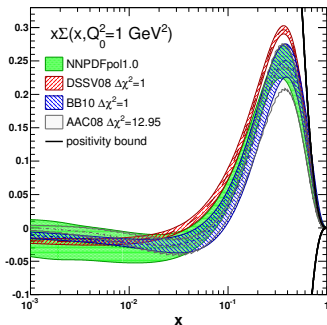
The goal is to provide a statistically sound determination of polarized parton distributions and their uncertainties

Spin-dependent PDF analyses in a nutshell

	Reaction	Partonic subprocess	PDF probed	x	Q^2 [GeV ²]
	$\ell^\pm \{p, d, n\} \rightarrow \ell^\pm X$	$\gamma^* q \rightarrow q$	$\Delta q + \Delta \bar{q}$ Δg	$0.003 \lesssim x \lesssim 0.8$	$1 \lesssim Q^2 \lesssim 70$
	$\ell^\pm \{p, d\} \rightarrow \ell^\pm hX$ $\ell^\pm \{p, d\} \rightarrow \ell^\pm DX$	$\gamma^* q \rightarrow q$ $\gamma^* g \rightarrow c\bar{c}$	$\Delta u \Delta \bar{u}$ $\Delta d \Delta \bar{d}$ Δg Δg	$0.005 \lesssim x \lesssim 0.5$ $0.06 \lesssim x \lesssim 0.2$	$1 \lesssim Q^2 \lesssim 60$ ~ 10
	$\vec{p} \vec{p} \rightarrow jet(s)X$ $\vec{p} p \rightarrow W^\pm X$ $\vec{p} \vec{p} \rightarrow \pi X$	$gg \rightarrow qg$ $qg \rightarrow qg$ $u_L \bar{d}_R \rightarrow W^+$ $d_L \bar{u}_R \rightarrow W^-$ $gg \rightarrow qg$ $qg \rightarrow qg$	Δg $\Delta u \Delta \bar{u}$ $\Delta d \Delta \bar{d}$ Δg	$0.05 \lesssim x \lesssim 0.2$ $0.05 \lesssim x \lesssim 0.4$ $0.05 \lesssim x \lesssim 0.4$	$30 \lesssim p_T^2 \lesssim 800$ $\sim M_W^2$ $1 \lesssim p_T^2 \lesssim 200$
Fit	Data sets	Parton Distributions	Uncertainties	Latest update	
AAC08	DIS, π^0	$\Delta u^+, \Delta d^+, \Delta s^+, \Delta g$	Hessian $\Delta\chi^2 = 12.95$	[arXiv:0808.0413]	
BB10	DIS	$\Delta u^-, \Delta d^-, \Delta \bar{q}, \Delta g$	Hessian $\Delta\chi^2 = 1$	[arXiv:1005.3113]	
LSS10	DIS, SIDIS	$\Delta u^+, \Delta d^+, \Delta \bar{u}, \Delta \bar{d}, \Delta \bar{s}, \Delta g$	Hessian $\Delta\chi^2 = 1$	[arXiv:1010.0574]	
DSSV	DIS, SIDIS, π^0 , Jets	$\Delta u^+, \Delta d^+, \Delta \bar{u}, \Delta \bar{d}, \Delta \bar{s}, \Delta g$	Hessian $\Delta\chi^2 = 1$	[arXiv:0904.3821]	
JAM13	DIS	$\Delta u^+, \Delta d^+, \Delta \bar{u}, \Delta \bar{d}, \Delta \bar{s}, \Delta g$	Lagr. mult. $\Delta\chi^2/\chi^2 = 2\%$	[arXiv:1404.4293]	
			Hessian $\Delta\chi^2 = 1$	[arXiv:1310.3734]	
NNPDFpo11.0	DIS	$\Delta u^+, \Delta d^+, \Delta s^+, \Delta g$	Monte Carlo	[arXiv:1303.7236]	

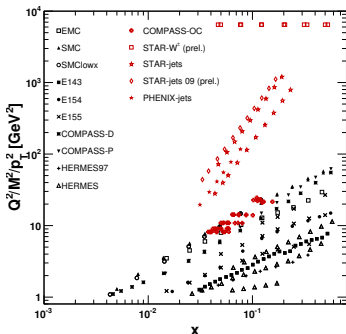
The NNPDFpol1.0 parton set in a nutshell [arXiv:1303.7236]

- 1 **Monte Carlo** error propagation by generation of N_{rep} replicas of experimental data
→ results are delivered as statistical ensembles of equally probable PDF replicas
→ expectation values and uncertainties are computed as means and standard deviations
→ no need to rely on linear propagation of errors (not always adequate)
- 2 **Neural Network** parametrization of each PDF replica
→ only requires smoothness of PDFs, reduce the bias due to the parametrization
→ genetic algorithm for Neural Network training + cross-validation for stopping
- 3 Only **DIS data** included
→ where no data or theoretical constraints are available, uncertainties are large
→ gluon largely uncertain; no handle on Δq and $\Delta \bar{q}$ separation



2. Including collider data in a global PDF determination

Old and new experimental data set



- Include the experimental information from:
→ jet and W production data (collider)
STAR, PHENIX
- Limited kinematic coverage at low- x
- Large Q^2 reached by collider data
- New data sets are included in NNPDFpo11.0 (DIS-only fit) via Bayesian reweighting

REACTION	PARTONIC SUBPROCESS	PDF PROBED	x	Q^2 [GeV 2]
$\ell^\pm \{p, d, n\} \rightarrow \ell^\pm X$	$\gamma^* q \rightarrow q$	$\Delta q + \Delta \bar{q}$	$0.003 \lesssim x \lesssim 0.8$	$1 \lesssim Q^2 \lesssim 70$
$\ell^\pm \{p, d\} \rightarrow \ell^\pm DX$	$\gamma^* g \rightarrow c \bar{c}$	Δg	$0.06 \lesssim x \lesssim 0.2$	$Q^2 \sim 10$
$\vec{p} \vec{p} \rightarrow jet(s)X$	$gg \rightarrow qg$ $qg \rightarrow qg$	Δg	$0.05 \lesssim x \lesssim 0.2$	$30 \lesssim p_T^2 \lesssim 800$
$\vec{p} p \rightarrow W^\pm X$	$u_L \bar{d}_R \rightarrow W^+$ $d_L \bar{u}_R \rightarrow W^-$	$\Delta u \Delta \bar{u}$ $\Delta d \Delta \bar{d}$	$0.05 \lesssim x \lesssim 0.4$	$\sim M_W^2$

- We would like to assess the impact of including a **new data set** $\{y\} = \{y_1, \dots, y_n\}$ (delivered with σ_{ij}) in a **prior ensemble** of PDF replicas $\{f_k\}$, $k = 1, \dots, N_{\text{rep}}$
- We can apply **Bayes theorem** to determine the conditional probability of PDF upon inclusion of the new data and update the probability density in the space of PDFs

$$\mathcal{P}_{\text{new}} = \mathcal{N}_x \mathcal{P}(\chi_k^2 | \{f_k\}) \mathcal{P}_{\text{old}}(\{f_k\}) \quad \mathcal{P}(\chi_k^2 | \{f_k\}) = [\chi_k^2(\{y\}, \{f_k\})]^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi_k^2(\{y\}, \{f_k\})}$$

$$\chi_k^2(\{y\}, \{f_k\}) = \sum_{i,j}^n \{y_i - y_i[f_k]\} \sigma_{ij} \{y_j - y_j[f_k]\}$$

- Replicas are **no longer equally probable**. Expectation values are given by

$$\langle \mathcal{O}[f_i(x, Q^2)] \rangle_{\text{new}} = \sum_{k=1}^{N_{\text{rep}}} w_k \mathcal{O}[f_i^{(k)}(x, Q^2)]$$

$$w_k \propto [\chi_k^2(\{y\}, \{f_k\})]^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi_k^2(\{y\}, \{f_k\})} \quad \text{with} \quad N_{\text{rep}} = \sum_{k=1}^{N_{\text{rep}}} w_k$$

- Loss of efficiency: $N_{\text{eff}} \equiv \exp \left[- \sum_{k=1}^{N_{\text{rep}}} p_k \log p_k \right]$ with $p_k = w_k / N_{\text{rep}}$
 $0 < N_{\text{eff}} < N_{\text{rep}}$; N_{eff} must not be too low $\Rightarrow$ increase the number of replicas in the prior

Reweighting allows us to incorporate new datasets **without** the need of **refitting**

Construction of the prior PDF ensemble: our strategy

New data sets are included in the DIS-only fit, NNPDFpo11.0, via Bayesian reweighting

BUT

The computation of the new observables **requires** light **quark-antiquark separation**

The NNPDFpo11.0 **fit does not provide separate** Δu , $\Delta \bar{u}$ and Δd , $\Delta \bar{d}$ PDFs

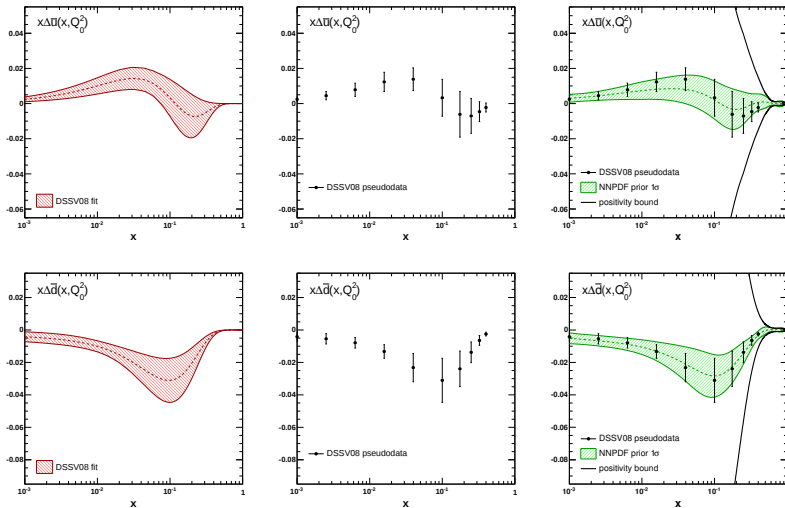
We need to make some assumption on $\Delta \bar{u}$, $\Delta \bar{d}$ and **construct a suitable unbiased prior**

- 1 Take a polarized parton set which provides Δq and $\Delta \bar{q}$ PDFs (from SIDIS)
- 2 Sample their $\Delta \bar{u}$ and $\Delta \bar{d}$ distributions at a given reference scale
- 3 Perform a NN fit to these pseudodata
- 4 Supplement each replica in NNPDFpo11.0 with $\Delta \bar{u}$ and $\Delta \bar{d}$ obtained in this way
- 5 Reweight PDFs and check that observables are properly reproduced

Check that reweighted results are stable upon the choice of different PDF priors:
repeat the procedure from step 2, e.g. by increasing the nominal PDF uncertainty,
until independence from the prior is reached

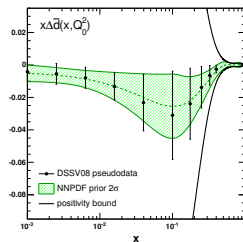
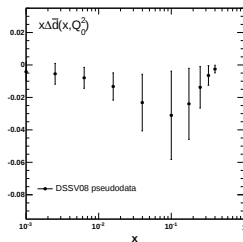
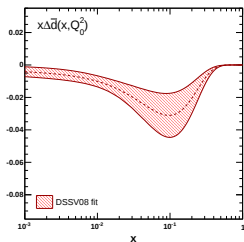
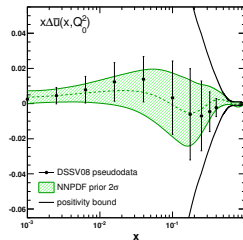
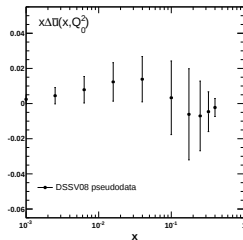
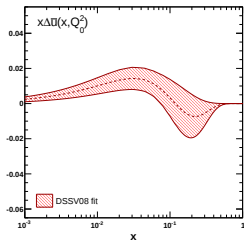
Construction of the prior PDF ensemble: our strategy

We consider four different priors: 1σ , 2σ , 3σ , 4σ corresponding to the different factors by which the DSSV08 nominal $\Delta\chi^2 = 1$ PDF uncertainty has been enlarged



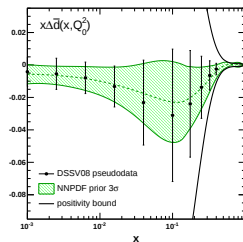
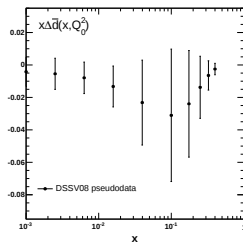
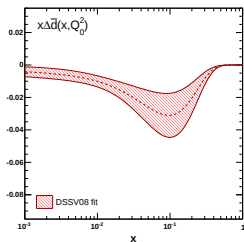
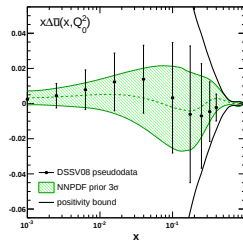
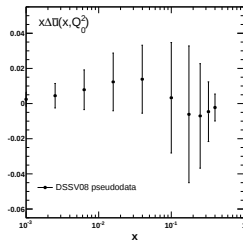
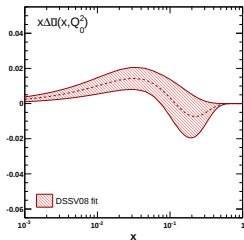
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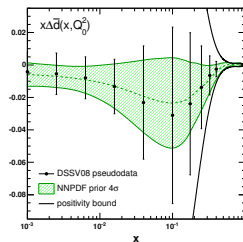
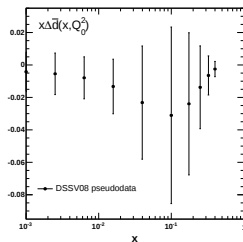
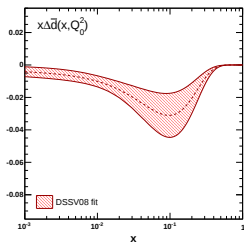
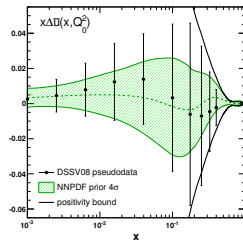
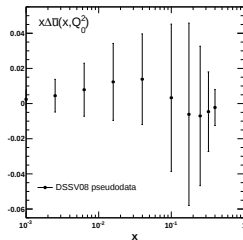
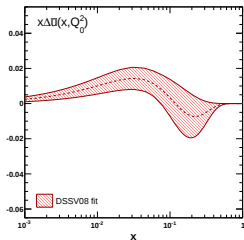
Construction of the prior PDF ensemble: our strategy

We consider four different priors: 1σ , 2σ , 3σ , 4σ corresponding to the different factors by which the DSSV08 nominal $\Delta\chi^2 = 1$ PDF uncertainty has been enlarged



Construction of the prior PDF ensemble: our strategy

We consider four different priors: 1σ , 2σ , 3σ , 4σ corresponding to the different factors by which the DSSV08 nominal $\Delta\chi^2 = 1$ PDF uncertainty has been enlarged



A) Reweighting with inclusive jet production at RHIC

Longitudinal double-spin asymmetry for single-inclusive jet production

[arXiv:hep-ph/9808262] [arXiv:hep-ph/0404057] [arXiv:1209.1785]

$$A_{LL}^{1jet} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$

FEATURES

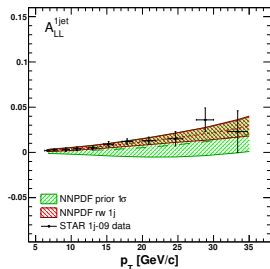
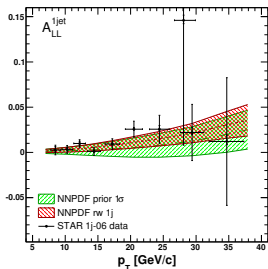
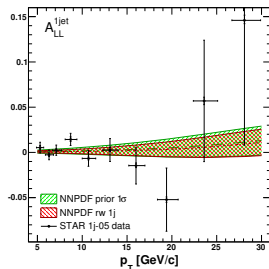
- sensitive to the polarized gluon Δg
(receives leading contribution from $gq \rightarrow qg$ and $qg \rightarrow qg$ partonic subprocesses)

EXPERIMENTAL MEASUREMENT

- STAR 2005, 2006 [arXiv:1205.2735], 2009 (prel.) [arXiv:1303.0543]
- PHENIX [arXiv:1009.4921] at RHIC

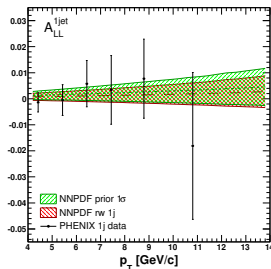
Data set	N_{dat}	jet-algorithm	R	$[\eta_{\min}, \eta_{\max}]$	$\sqrt{s}$ [GeV]	$\mathcal{L}$ [pb ⁻¹]
STAR 1j-05	10	midpoint-cone	0.4	[+0.20, +0.80]	200	2.1
STAR 1j-06	9	midpoint-cone	0.7	[-0.70, +0.90]	200	5.5
STAR 1j-09 (prel.)	11	anti-kt	0.6	[-0.50, +0.50]	200	25
	11	anti-kt	0.6	[-1.00, -0.50] $\cup$ [+0.50, +1.00]	200	25
PHENIX 1j	6	seeded-cone	0.3	[-0.35, +0.35]	200	2.1

A) Reweighting with inclusive jet production at RHIC



Experiment	Set	N_{dat}	χ^2/N_{dat}				$\chi^2_{\text{rw}}/N_{\text{dat}}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR		41	1.19	1.19	1.20	1.20	0.79	0.79	0.79	0.79
	STAR 1j-05	10	1.04	1.04	1.04	1.04	1.01	1.02	1.02	1.01
	STAR 1j-06	9	0.75	0.75	0.75	0.76	0.59	0.59	0.59	0.59
	STAR 1j-09	22	1.69	1.69	1.70	1.71	1.02	1.02	1.03	1.04
PHENIX										
	PHENIX 1j	6	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24

A) Reweighting with inclusive jet production at RHIC



Experiment	Set	N_{dat}	χ^2/N_{dat}				$\chi^2_{\text{rw}}/N_{\text{dat}}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR		41	1.19	1.19	1.20	1.20	0.79	0.79	0.79	0.79
	STAR 1j-05	10	1.04	1.04	1.04	1.04	1.01	1.02	1.02	1.01
	STAR 1j-06	9	0.75	0.75	0.75	0.76	0.59	0.59	0.59	0.59
	STAR 1j-09	22	1.69	1.69	1.70	1.71	1.02	1.02	1.03	1.04
PHENIX	PHENIX 1j	6	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24

A) Reweighting with inclusive jet production at RHIC

Overall **good description** of all **data sets** after they are included in the fit via reweighting
Reweighted results are stable upon the choice of different prior PDF ensembles
since we are looking at an observable which is mostly sensitive to Δg

Different data sets have different power in constraining Δg
Significant reduction of the uncertainty of the gluon PDF in the measured region

Experiment	Set	N_{dat}	χ^2/N_{dat}				$\chi^2_{\text{rw}}/N_{\text{dat}}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR		41	1.19	1.19	1.20	1.20	0.79	0.79	0.79	0.79
	STAR 1j-05	10	1.04	1.04	1.04	1.04	1.01	1.02	1.02	1.01
	STAR 1j-06	9	0.75	0.75	0.75	0.76	0.59	0.59	0.59	0.59
	STAR 1j-09	22	1.69	1.69	1.70	1.71	1.02	1.02	1.03	1.04
PHENIX	PHENIX 1j	6	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24

B) Reweighting with $W^\pm$ production at RHIC

Longitudinal single-spin asymmetry for $W^\pm$ boson production

[arXiv:1003.4533]

$$A_L^{W^\pm} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

$$A_L^{W^+} \sim \frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$A_L^{W^-} \sim \frac{\Delta d(x_1)\bar{u}(x_2) - \Delta\bar{u}(x_1)d(x_2)}{d(x_1)\bar{u}(x_2) + \bar{u}(x_1)d(x_2)}$$

FEATURES

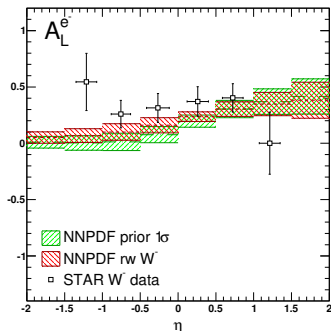
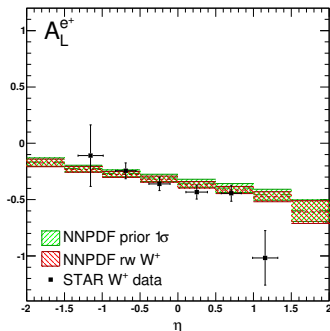
- sensitive to individual quark and antiquark flavours ($\Delta u, \Delta\bar{u}, \Delta d, \Delta\bar{d}$)
(purely weak process coupling q_L with $\bar{q}_R$ at partonic level, $u_L\bar{d}_R \rightarrow W^+$ or $d_L\bar{u}_R \rightarrow W^-$)
- no need for fragmentation functions (instead of SIDIS)

EXPERIMENTAL MEASUREMENT

- STAR and PHENIX at RHIC [arXiv:1009.0326] [arXiv:1009.0505]
(only preliminary measurements from STAR (2012) [arXiv:1302.6639] will be considered here)

Data set	N_{dat}	$[p_{T,\text{min}}, p_{T,\text{max}}]$ [GeV]	$\sqrt{s}$ [GeV]	$\mathcal{L}$ [pb $^{-1}$]
STAR- W^+ (prel.)	6	[25, 50]	510	72
STAR- W^- (prel.)	6	[25, 50]	510	72

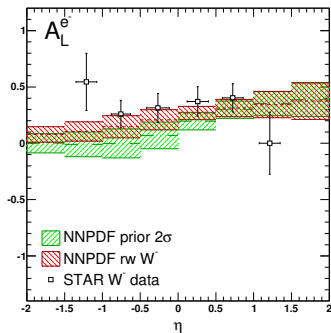
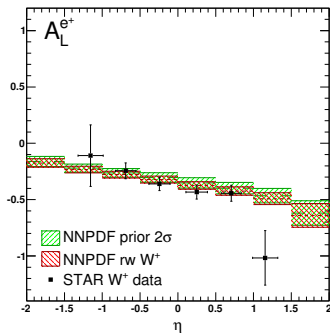
B) Reweighting with $W^\pm$ production at RHIC



Experiment	Set	N_{dat}	χ^2/N_{dat}				$\chi^2_{\text{rw}}/N_{\text{dat}}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR		12	1.93	1.97	1.97	1.91	1.43	1.12	1.03	1.02
	STAR- W^+	6	1.15	1.14	1.22	1.25	1.04	1.01	1.02	1.01
	STAR- W^-	6	2.71	2.80	2.59	2.43	1.78	1.25	1.04	1.02

Overall **good description** of both **data sets** after they are included in the fit via reweighting
Stability of reweighted results reached starting from 3σ prior

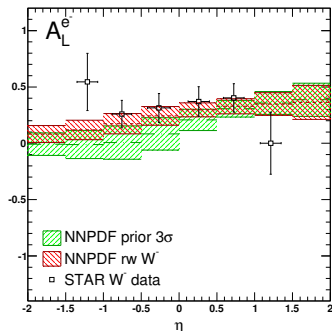
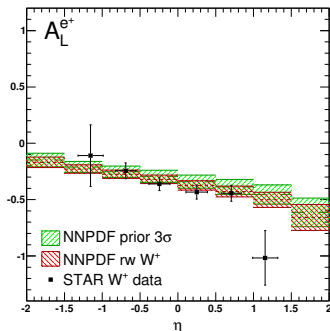
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Experiment	Set	N_{dat}	χ^2/N_{dat}				$\chi^2_{\text{rw}}/N_{\text{dat}}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR		12	1.93	1.97	1.97	1.91	1.43	1.12	1.03	1.02
	STAR- W^+	6	1.15	1.14	1.22	1.25	1.04	1.01	1.02	1.01
	STAR- W^-	6	2.71	2.80	2.59	2.43	1.78	1.25	1.04	1.02

Overall **good description** of both **data sets** after they are included in the fit via reweighting
Stability of reweighted results reached starting from 3σ prior

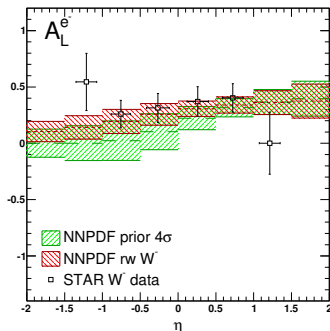
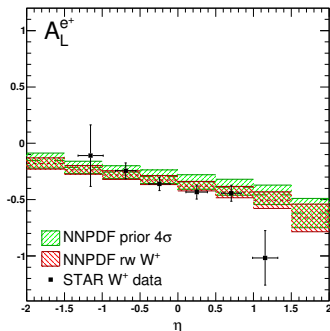
B) Reweighting with $W^\pm$ production at RHIC



Experiment	Set	N_{dat}	χ^2/N_{dat}				$\chi^2_{\text{rw}}/N_{\text{dat}}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR		12	1.93	1.97	1.97	1.91	1.43	1.12	1.03	1.02
	STAR- W^+	6	1.15	1.14	1.22	1.25	1.04	1.01	1.02	1.01
	STAR- W^-	6	2.71	2.80	2.59	2.43	1.78	1.25	1.04	1.02

Overall **good description** of both **data sets** after they are included in the fit via reweighting
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Overall **good description** of both **data sets** after they are included in the fit via reweighting
Stability of reweighted results reached starting from 3σ prior

C) Global reweighting

GLOBAL REWEIGHTING $N_{\text{dat}} = 74$

	1σ	2σ	3σ	4σ
χ^2/N_{dat}	1.22	1.25	1.25	1.24
$\chi^2_{\text{rw}}/N_{\text{dat}}$	1.07	1.04	1.02	1.02
N_{eff}	224	197	177	176

$$\chi^2_{\text{rw}}/N_{\text{dat}} = 1.02$$

$$N_{\text{eff}} \sim 180$$

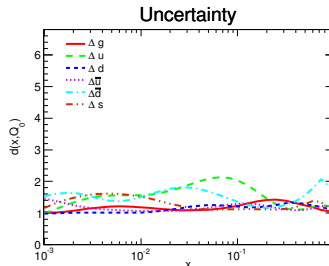
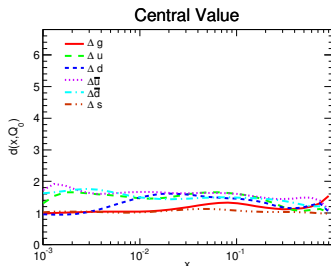
$$d \sim 2$$

Good overall description of new data

The loss of efficiency of the reweighted replicas is under control

Results do not depend on the choice of either the 3σ or 4σ prior

Rewighted NNPDFpol1.1 3σ vs 4σ



3. The NNPDFpol1.1 parton set

Constructing the NNPDFpol1.1 parton set

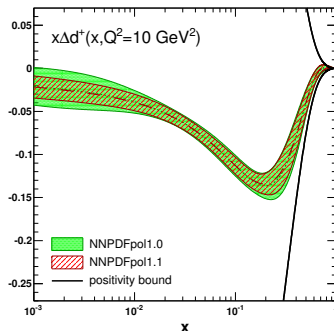
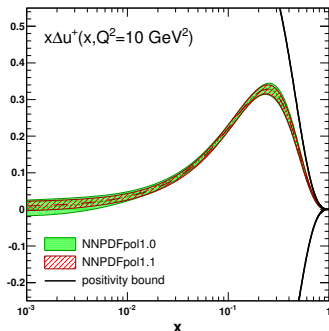
The NNPDFpol1.1 parton set, made of $N_{\text{rep}} = 100$ **equally probable** replicas, is obtained by **unweighting** the result from global reweighting. The idea is to select (possibly more than once) replicas carrying relatively high weight and discard replicas carrying relatively small weight. The unweighted ensemble is **statistically equivalent** to the reweighted ensemble.

Experiment	NNPDFpol1.0 χ^2/N_{dat}	NNPDFpol1.1 χ^2/N_{dat}
EMC	0.44	0.43
SMC	0.93	0.90
SMC _{lowx}	0.97	0.97
E143	0.64	0.67
E154	0.40	0.45
E155	0.89	0.85
COMPASS-D	0.65	0.70
COMPASS-P	1.31	1.38
HERMES97	0.34	0.34
HERMES	0.79	0.82

Similar (good) description of inclusive DIS data in both NNPDF parton sets
How are PDFs affected by the inclusion of polarized collider data sets?

Comparison with NNPDFpol1.0

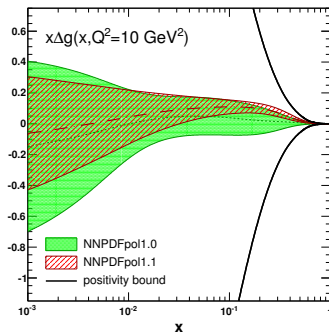
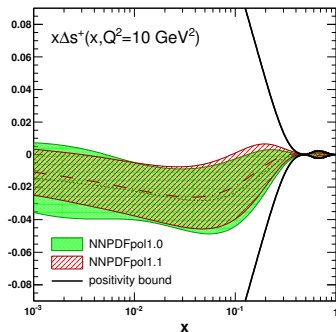
Δu^+ and Δd^+



- $\Delta q^+ \equiv \Delta q + \Delta \bar{q}$, $q = u, d$ are statistically equivalent in the two parton sets
- Δs^+ is almost unaffected by W data (unlike in the unpolarized case)
- the underlying probability distributions for Δg in the two determinations differ up to one sigma in the region covered by jet data, $0.05 \lesssim x \lesssim 0.2$
- Δg from NNPDFpol1.1 is definitely positive in the data region and its uncertainty is reduced up to a factor three w.r.t. NNPDFpol1.0

Comparison with NNPDFpol1.0

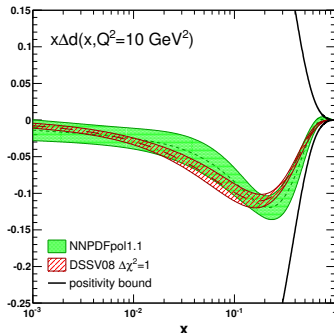
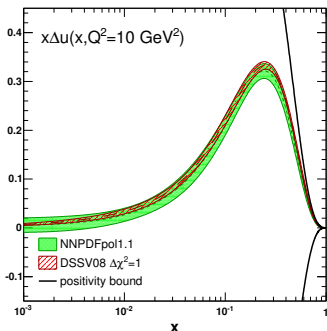
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Comparison with DSSV

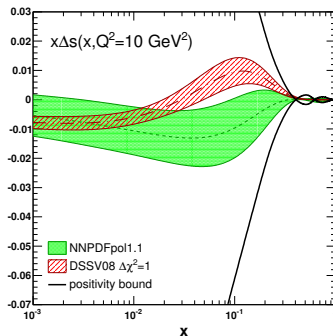
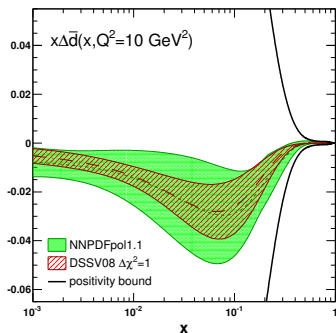
Δu and Δd



- Consistent results are found in the two parton determinations
- NNPDF uncertainties are slightly larger, especially at small- x values (where experimental data are lacking)

Comparison with DSSV

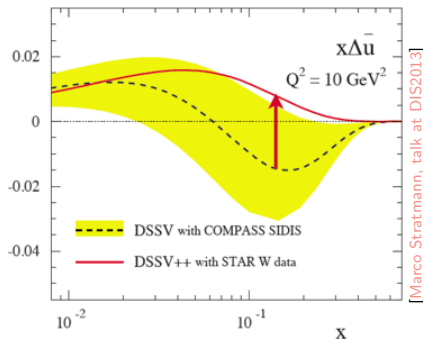
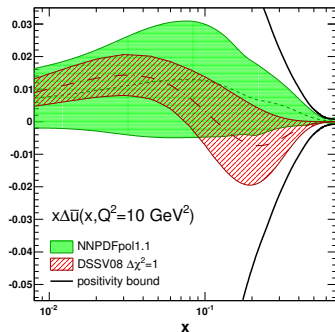
$$\Delta\bar{d} \text{ and } \Delta s = \Delta\bar{s}$$



- Good agreement for the $\Delta\bar{d}$ distribution, but with large uncertainty
- Discrepancy between NNPDF and DSSV determinations of Δs (bias from the kaon fragmentation function used to include SIDIS data?)

Comparison with DSSV

$\Delta\bar{u}$

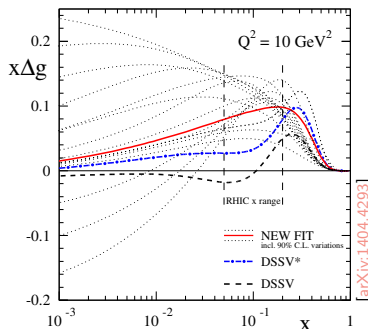
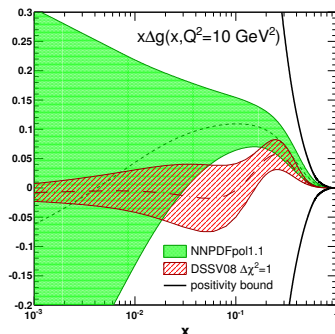


[Marco Stratmann, talk at DIS2013]

- Slight discrepancy for the $\Delta\bar{u}$ distribution at $x \gtrsim 3 \cdot 10^{-2}$
- Recall that $W^\pm$ data were not included in the DSSV08 analysis
- The $\Delta\bar{u}$ distribution was determined in DSSV08 from π production data in SIDIS (bias from the pion fragmentation function?)
- This discrepancy disappears when W data are included in the DSSV analysis

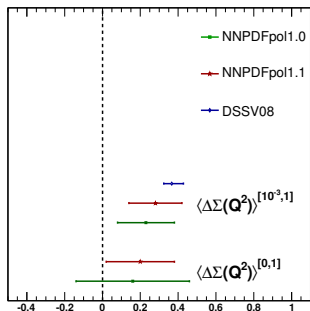
Comparison with DSSV

Δg



- For $x < 0.2$ the gluon has a node in DSSV08, while it is definitely positive in NNPDFpol1.1
- NNPDFpol1.1 and DSSV are in perfect agreement once recent jet data are included
- First evidence of gluon polarization in the proton in the region covered by data, $x \gtrsim 0.2$
- The uncertainty of the gluon blows up in the unmeasured small- x region

The spin content of the proton: quarks and antiquarks

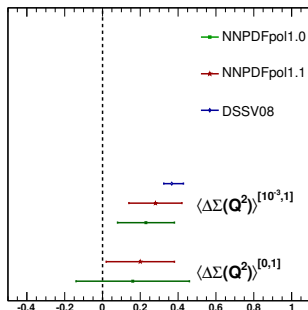


NNPDFpol1.0 vs NNPDFpol1.1

- Moments obtained from NNPDFpol1.0 and NNPDFpol1.1 are consistent with each other
- The uncertainty in the region $x \in [0, 1]$ reduces by a factor two, except for $\Delta \bar{s}$
- The relative contribution to the total momentum uncertainty from the extrapolation region is roughly a half in both parton sets

$Q^2 = 10 \text{ GeV}^2$ Δf	$\langle \Delta f(Q^2) \rangle^{[0,1]}$		$\langle \Delta f(Q^2) \rangle^{[10^{-3},1]}$		
	NNPDFpol1.0	NNPDFpol1.1	NNPDFpol1.0	NNPDFpol1.1	DSSV08 ($\Delta\chi^2/\chi^2 = 2\%$)
$\Delta u + \Delta \bar{u}$	$+0.77 \pm 0.10$	$+0.79 \pm 0.06$	$+0.76 \pm 0.06$	$+0.76 \pm 0.03$	$+0.793^{+0.028}_{-0.034}$
$\Delta d + \Delta \bar{d}$	-0.46 ± 0.10	-0.47 ± 0.06	-0.41 ± 0.06	-0.41 ± 0.04	$-0.416^{+0.035}_{-0.025}$
$\Delta \bar{u}$	—	$+0.06 \pm 0.05$	—	$+0.05 \pm 0.05$	$+0.028^{+0.059}_{-0.059}$
$\Delta \bar{d}$	—	-0.12 ± 0.07	—	-0.10 ± 0.05	$-0.089^{+0.090}_{-0.080}$
$\Delta \bar{s}$	-0.07 ± 0.06	-0.06 ± 0.05	-0.06 ± 0.04	-0.05 ± 0.04	$-0.006^{+0.028}_{-0.031}$
$\Delta \Sigma$	$+0.16 \pm 0.30$	$+0.20 \pm 0.18$	$+0.23 \pm 0.15$	$+0.25 \pm 0.10$	$+0.366^{+0.042}_{-0.062}$

The spin content of the proton: quarks and antiquarks

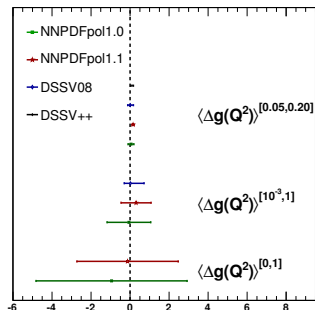


NNPDFpol1.1 vs DSSV08

- Truncated moments are in perfect agreement, both central values and uncertainties
- Slight differences are found for $\Delta \bar{u}$ and $\Delta \bar{s}$ (according to the corresponding PDFs)

$Q^2 = 10 \text{ GeV}^2$ Δf	$\langle \Delta f(Q^2) \rangle^{[0, 1]}$			$\langle \Delta f(Q^2) \rangle^{[10^{-3}, 1]}$	
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The spin content of the proton: the gluon



NNPDFpol1.0 vs NNPDFpol1.1

- Reduction of the uncertainty thanks to jet data
 - by a factor two for $x \in [0, 1]$, $x \in [10^{-3}, 1]$
 - by a factor three for $x \in [0.05, 0.2]$

NNPDFpol1.1 *full* vs *truncated* moments

- The extrapolation uncertainty is still dominant
 - 3 times the uncertainty in $x \in [10^{-3}, 1]$
 - 40 times the uncertainty in $x \in [0.05, 0.2]$

NNPDFpol1.1 vs DSSV++ (same jet data sets)

- Results perfectly consistent for $x \in [0.05, 0.2]$

$Q^2 = 10 \text{ GeV}^2$	$\langle \Delta g(Q^2) \rangle^{[0,1]}$	$\langle \Delta g(Q^2) \rangle^{[10^{-3},1]}$	$\langle \Delta g(Q^2) \rangle^{[0.05,0.2]}$
NNPDFpol1.0	-0.95 ± 3.87	-0.06 ± 1.12	$+0.05 \pm 0.15$
NNPDFpol1.1	-0.13 ± 2.60	$+0.31 \pm 0.77$	$+0.15 \pm 0.06$
DSSV08 ($\Delta\chi^2/\chi^2 = 2\%$)	—	$0.013^{+0.702}_{-0.314}$	$0.005^{+0.129}_{-0.164}$
DSSV++ ($\Delta\chi^2/\chi^2 = 2\%$)	—	—	$0.10^{+0.06}_{-0.07}$

4. Conclusions and outlook

Summary

- 1 The first unbiased global determination of polarized PDFs was presented
 - NNPDFpol1.1: inclusive DIS + open-charm (fixed-target) + jets + W (collider)
 - W data allows for a determination of $\Delta\bar{u}$ and $\Delta\bar{d}$ polarizations
 - jet data from STAR provide the first evidence of gluon polarization, $x \in [0.05, 0.2]$
 - outside this region, the polarized gluon is still largely uncertain
 - large values of the gluon first moment are not completely ruled out
- 2 The NNPDF parton sets (polarized and unpolarized) are available at

<http://nnpdf.hepforge.org/>

NNPDFs together with stand-alone Fortran/C++/*Mathematica* code

- 3 We plan to determine a set of pion and kaon fragmentation functions
 - in view of a full global analysis including SIDIS and hadron production data
- 4 Only an EIC would probe polarized PDFs at small- x values and higher Q^2
 - accurate determination of the longitudinal spin structure [arXiv:1206.6014] [arXiv:1310.0461]
 - quark-antiquark separation from charged-current DIS [arXiv:1309.5327]

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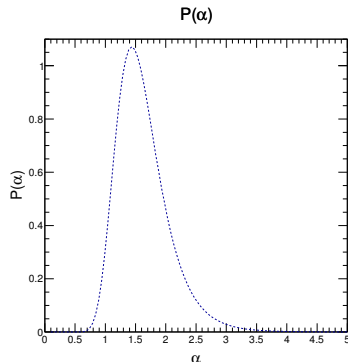
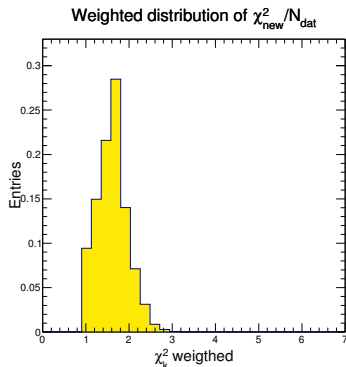
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Thank you for your attention

5. Backup

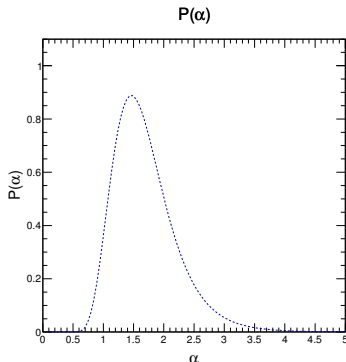
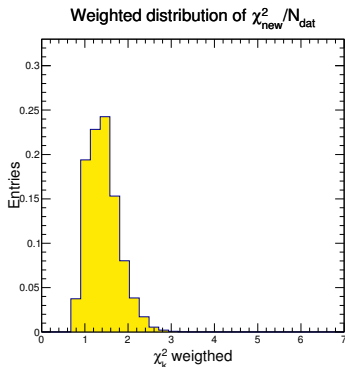
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	STAR- W^+	12	1.43	1.12	1.03	1.02	1.45	1.43	1.40	1.39
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Overall **good description** of both **data sets** after they are included in the fit via reweighting
Stability of reweighted results reached starting from 3σ prior

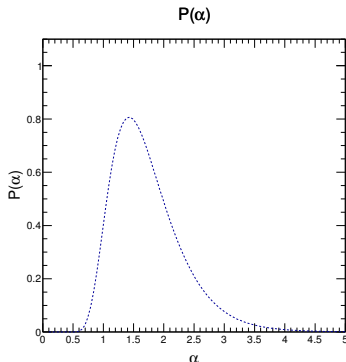
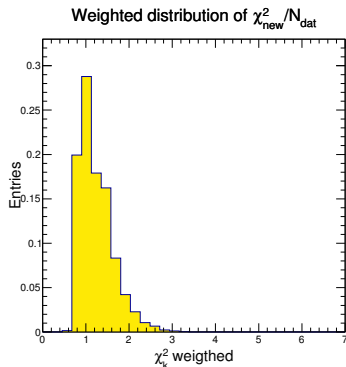
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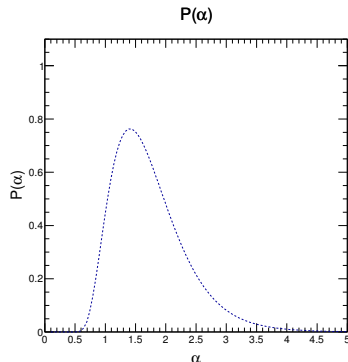
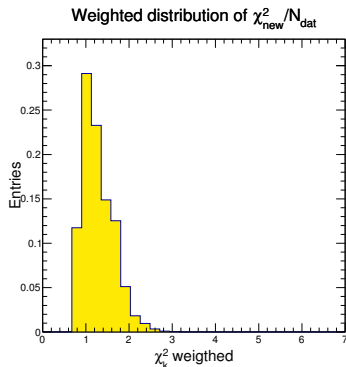
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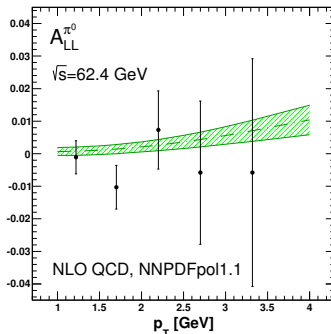
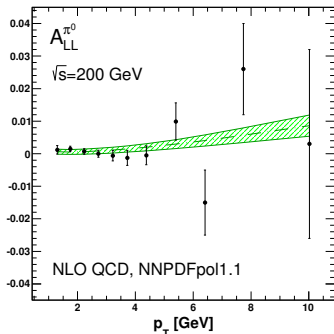
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Predictions for single-hadron production asymmetry

$$A_{LL}^H = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}} = \frac{\sum_{a,b,c=q,\bar{q},g} f_a \otimes f_b \otimes D_c^H \otimes \Delta \hat{\sigma}_{ab}^c}{\sum_{a,b,c=q,\bar{q},g} f_a \otimes f_b \otimes D_c^H \otimes \hat{\sigma}_{ab}^c}$$

PHENIX [arXiv:0810.0701] [arXiv:0810.0694] [arXiv:1402.6296] STAR [arXiv:1309.1800]

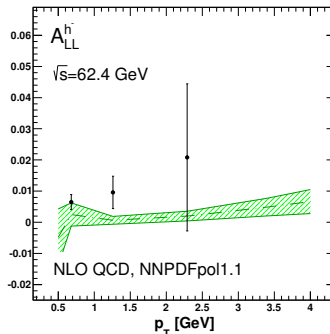
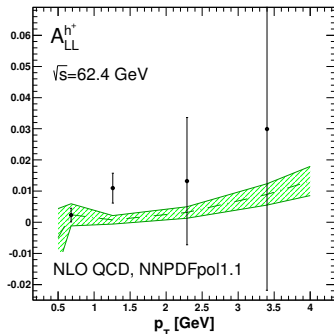


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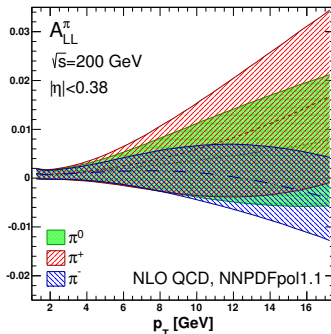
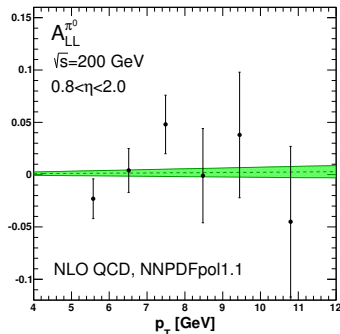


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PHENIX [arXiv:0810.0701] [arXiv:0810.0694] [arXiv:1402.6296] STAR [arXiv:1309.1800]

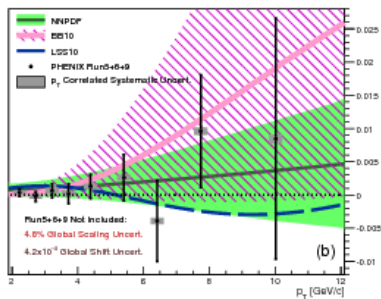
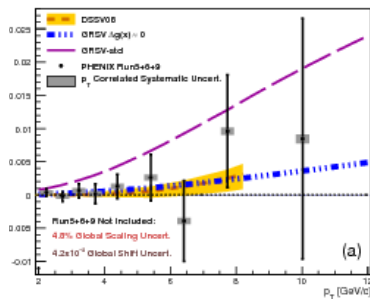


- Good agreement between experimental data and theoretical predictions
- Experimental uncertainties are larger than those of the corresponding predictions
- We expect a slight impact on the gluon PDF from these data

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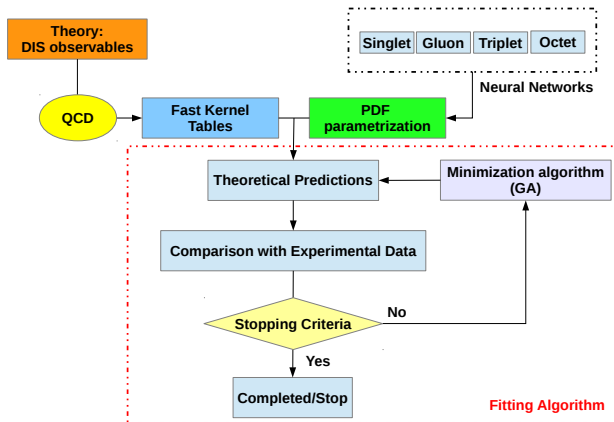
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A general overview on the methodology



Ingredients:
Monte Carlo sampling and Neural Networks

Ingredient 1: Monte Carlo sampling of experimental data

MONTE CARLO SAMPLING

- Sample the probability density $\mathcal{P}[\Delta q]$ in the space of functions assuming **multi-Gaussian** data probability distribution

$$g_{1,p}^{(\text{art}), (k)}(x, Q^2) = \left[1 + \sum_c r_{c,p}^{(k)} \sigma_{c,p} + r_{s,p}^{(k)} \sigma_{s,p} \right] g_{1,p}^{(\text{exp})}(x, Q^2)$$

$\sigma_{c,p}$: correlated systematics $\sigma_{s,p}$: statistical errors (also uncorrelated systematics)
 $r_{c,p}^{(k)}, r_{s,p}^{(k)}$: Gaussian random numbers

- Generate MC ensemble of N_{rep} replicas with the data probability distribution

MAIN FEATURES

- Expectation values** for observables are **Monte Carlo integrals**

$$\langle \mathcal{O}[\Delta q] \rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \mathcal{O}[\Delta q_k]$$

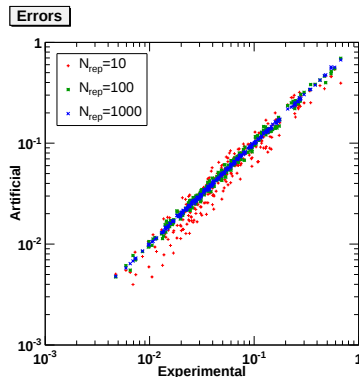
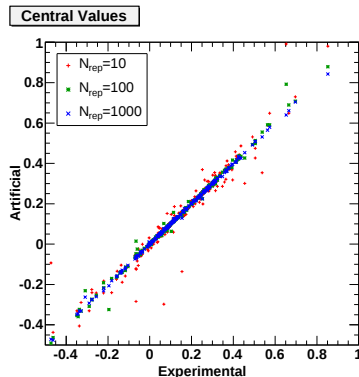
... and the same is true for errors, correlations etc.

- No need to rely on **linear propagation** of errors
- Possibility to test for **non-Gaussian** behaviour in fitted PDFs

Ingredient 1: Monte Carlo sampling of experimental data

DETERMINING THE SAMPLE SIZE

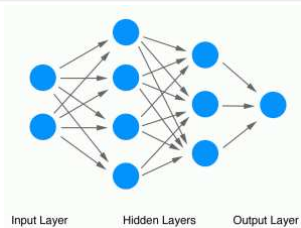
- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy



Accuracy of few % requires ~ 100 replicas

Ingredient 2: Neural Networks

A convenient **functional form**
providing **redundant** and **flexible** parametrization
used as a generator of random functions in the PDF space



$$\xi_i^{(l)} = g \left(\sum_j^{n_l-1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)} \right)$$

$$g(x) = \frac{1}{1 + e^{-x}}$$

- made of neurons grouped into layers (define the architecture)
- each neuron receives input from neurons in preceding layer (feed-forward NN)
- activation determined by parameters (**weights** and **thresholds**)
- activation determined according to a **non-linear function** (except the last layer)

Distances

Compare two sets of $N_{\text{rep}}^{(1)}$ and $N_{\text{rep}}^{(2)}$ replicas coming from different fits
Do they have belong to the same underlying probability distribution?

MEAN VALUE

$$d^2 \left(\langle q^{(k)} \rangle_{(1)}, \langle q^{(k)} \rangle_{(2)} \right) = \frac{\left(\langle q^{(k)} \rangle_{(1)} - \langle q^{(k)} \rangle_{(2)} \right)^2}{\sigma^2 \left[\langle q^{(k)} \rangle_{(1)} \right] + \sigma^2 \left[\langle q^{(k)} \rangle_{(2)} \right]}$$

$$\langle q^{(k)} \rangle_{(i)} = \frac{1}{N_{\text{rep}(i)}} \sum_{l=1}^{N_{\text{rep}(i)}} q_l^{(k)}$$

$$\sigma^2 \left[\langle q^{(k)} \rangle_{(i)} \right] = \frac{1}{N_{\text{rep}(i)}} \sigma^2 \left[q_{(i)}^{(k)} \right] = \frac{1}{N_{\text{rep}(i)} - 1} \sum_{l=1}^{N_{\text{rep}(i)}} \left(q_{l,(i)} - \langle q \rangle_{(i)} \right)^2$$

UNCERTAINTY

$$d^2 \left(\sigma_{(1)}^2, \sigma_{(2)}^2 \right) = \frac{\left(\bar{\sigma}_{(1)}^2 - \bar{\sigma}_{(2)}^2 \right)}{\sigma^2 \left[\sigma_{(1)}^2 \right] + \sigma^2 \left[\sigma_{(2)}^2 \right]}$$

$$\bar{\sigma}_{(i)}^2 \equiv \sigma^2 \left[q_{(i)}^{(k)} \right]$$

$$\sigma^2 \left[\sigma_{(i)}^2 \right] = \frac{1}{N_{\text{rep}(i)}} \left[\frac{1}{N_{\text{rep}(i)}} \sum_{l=1}^{N_{\text{rep}(i)}} \left(q_{l,(i)} - \langle q \rangle_{(i)} \right)^4 - \frac{N_{\text{rep}(i)} - 3}{N_{\text{rep}(i)} - 1} \left(\bar{\sigma}_{(i)}^2 \right)^2 \right]$$

By definition, the distances have a χ^2 probability distribution with one degree of freedom
mean $\langle d \rangle = 1$ and $d \lesssim 2.3$ at 90% confidence level

Unweighting allows for constructing an ensemble of equally probable PDFs **statistically equivalent** to a given reweighted set
Hence, the new set can be given without weights

IDEA

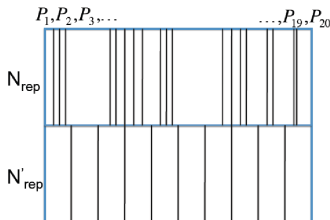
Given a weighted set of N_{rep} replicas, select (possibly more than once) replicas carrying relatively high weight and discard replicas carrying relatively small weight

CONSTRUCTION OF THE UNWEIGHTED SET

- 1 Set the number of replicas N'_{rep} in the unweighted set
(pointless to choose $N'_{\text{rep}} > N_{\text{rep}}$: no gain of information)
- 2 Compute, for the k -th replica of the reweighted set, the integer non negative number

$$w'_k = \sum_{j=1}^{N'_{\text{rep}}} \theta \left(\frac{j}{N'_{\text{rep}}} - P_{k-1} \right) \theta \left(P_k - \frac{j}{N'_{\text{rep}}} \right), \quad P_k = \sum_{j=0}^k \frac{w_j}{N_{\text{rep}}}, \quad \sum_{k=1}^{N_{\text{rep}}} w'_k = N'_{\text{rep}}$$

- 3 Construct the unweighted set taking w'_k copies of the k -th replica, for $k = 1, \dots, N_{\text{rep}}$



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C) Reweighting with open-charm production at COMPASS

Virtual photon-nucleon asymmetry for open-charm production

[arXiv:1212.1319]

$$A^{\gamma N \rightarrow D^0 X} = \frac{\Delta g \otimes \Delta \hat{\sigma}_{\gamma g} \otimes D_c^H}{g \otimes \hat{\sigma}_{\gamma g} \otimes D_c^H}$$

FEATURES

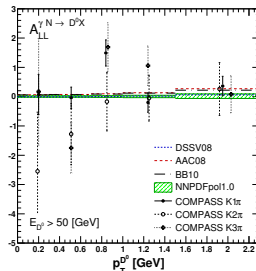
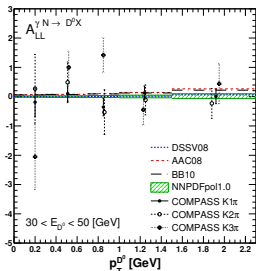
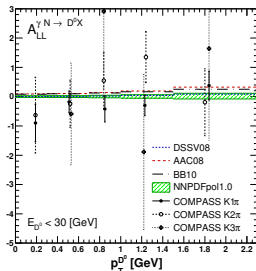
- Δg is probed *directly* through the photon-gluon fusion process
(in DIS Δg is mostly probed through scaling violations instead)
- the fragmentation functions for heavy quarks are computable in perturbation theory
(and only introduce a very moderate uncertainty in the fit)

EXPERIMENTAL MEASUREMENT

- COMPASS (2002-2007) [arXiv:1211.6849]

Experiment	Set	N_{dat}	NNPDFpo11.0	χ^2/N_{dat} DSSV08	AAC08	BB10
COMPASS		45	1.23	1.23	1.27	1.25
	COMPASS $K1\pi$	15	1.27	1.27	1.43	1.38
	COMPASS $K2\pi$	15	0.51	0.51	0.56	0.55
	COMPASS $K3\pi$	15	1.90	1.90	1.81	1.82

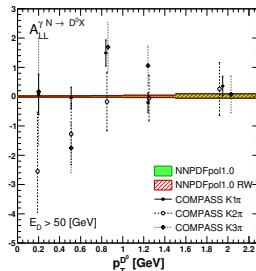
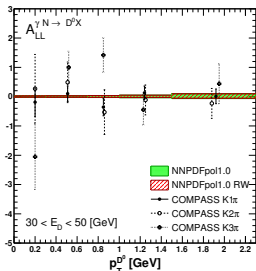
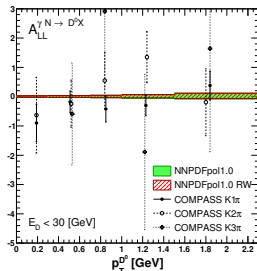
C) Reweighting with open-charm production at COMPASS



Data are affected by large uncertainties w.r.t. the uncertainty due to PDFs
They do not show a clear trend

Experiment	Set	N_{dat}	NNPDFpol1.0	χ^2/N_{dat}		
				DSSV08	AAC08	BB10
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C) Reweighting with open-charm production at COMPASS



The impact of open-charm data from COMPASS is mostly negligible, as we notice from the value of the χ^2/N_{dat} and the reweighted observable

Experiment	Set	N_{dat}	χ^2/N_{dat}	$\chi^2_{\text{rw}}/N_{\text{dat}}$
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