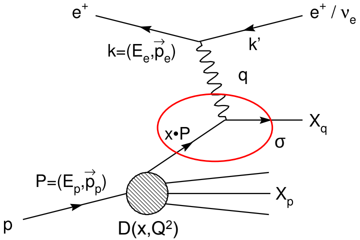
Monte Carlo techniques, Neural Networks and Parton Densities: the NNPDF approach to PDF fitting

Alberto Guffanti
Albert-Ludwigs-Universität Freiburg

LPTHE Jussieu, Paris,
March 18, 2011

What are Parton Distribution Functions?

- Consider a process with one hadron in the initial state



- According to the **Factorization Theorem** we can write the cross section as

$$d\sigma = \sum_a \int_0^1 \frac{d\xi}{\xi} D_a(\xi, \mu^2) d\hat{\sigma}_a \left(\frac{x}{\xi}, \frac{\hat{s}}{\mu^2}, \alpha_s(\mu^2) \right) + \mathcal{O}\left(\frac{1}{Q^p} \right)$$

What are Parton Distribution Functions?

- The **absolute value** of PDFs at a given x and Q^2 **cannot be computed** in QCD Perturbation Theory
  (Lattice? In principle yes, but ...)

- ... but the **scale dependence** is governed by **DGLAP** evolution equations

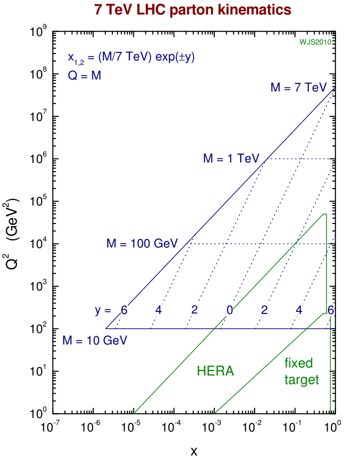
$$\frac{\partial}{\ln Q^2} q^{NS}(\xi, Q^2) = P^{NS}(\xi, \alpha_s) \otimes q^{NS}(\xi, Q^2)$$

$$\frac{\partial}{\ln Q^2} \begin{pmatrix} \Sigma \\ g \end{pmatrix} (\xi, Q^2) = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} (\xi, \alpha_s) \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix} (\xi, Q^2)$$

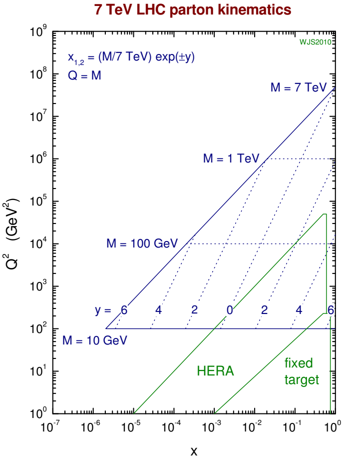
- ... and the **splitting functions** P can be computed in PT and are known up to **NNLO**

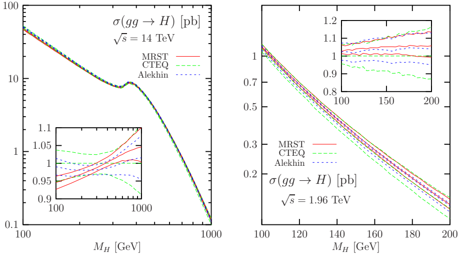
(**LO** - Dokshitzer; Gribov, Lipatov; Altarelli, Parisi; 1977)
(**NLO** - Floratos, Ross, Sachrajda; Gonzalez-Arroyo, Lopez, Yndurain; Curci, Furmanski, Petronzio, 1981)
(**NNLO** - Moch, Vermaseren, Vogt; 2004)

Why care about PDFs (and their uncertainties)?



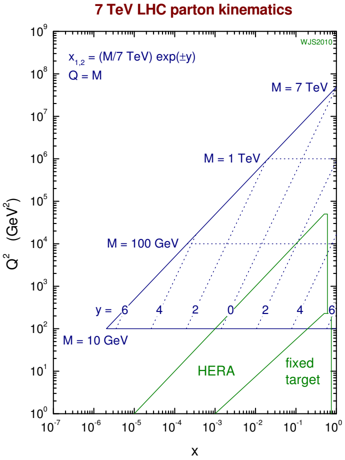
Why care about PDFs (and their uncertainties)?

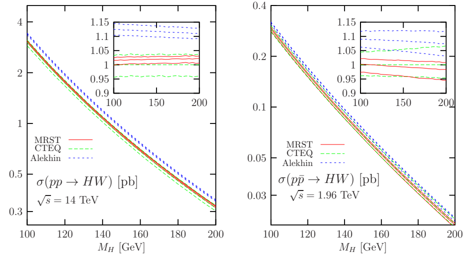




[A. Djouadi and S. Ferrag, hep-ph/0310209]

Why care about PDFs (and their uncertainties)?





[A. Djouadi and S. Ferrag, hep-ph/0310209]

Why care about PDFs (and their uncertainties)?

- Errors on PDFs are in some cases the **dominating theoretical error** on precision observables

Ex. $\sigma(Z^0)$ at the LHC: $\delta_{PDF} \sim 3\%$, $\delta_{NNLO} \sim 2\%$

[J. Campbell, J. Huston and J. Stirling, (2007)]

Why care about PDFs (and their uncertainties)?

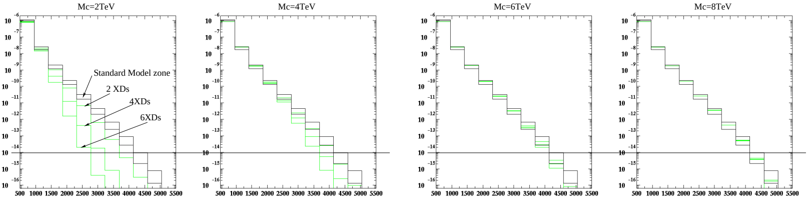
- Errors on PDFs are in some cases the **dominating theoretical error** on precision observables

Ex. $\sigma(Z^0)$ at the LHC: $\delta_{PDF} \sim 3\%$, $\delta_{NNLO} \sim 2\%$

[J. Campbell, J. Huston and J. Stirling, (2007)]

- Errors on PDFs might **reduce sensitivity to New Physics**

Ex. Extra Dimensions discovery in dijet cross section at the LHC:



[S. Ferrag (ATLAS), hep-ph/0407303]

Problem

Faithful estimation of errors on PDFs

- Single quantity: **1-σ error**
- Multiple quantities: **1-σ contours**
- Function: need an **"error band" in the space of functions**
  (*i.e.* the probability density $\mathcal{P}[f]$ in the space of functions $f(x)$)

Expectation values are **Functional integrals**

$$\langle \mathcal{F}[f(x)] \rangle = \int \mathcal{D}f \mathcal{F}[f(x)] \mathcal{P}[f(x)]$$

Problem

Faithful estimation of errors on PDFs

- Single quantity: **1-σ error**
- Multiple quantities: **1-σ contours**
- Function: need an **"error band" in the space of functions**
  (*i.e.* the probability density $\mathcal{P}[f]$ in the space of functions $f(x)$)

Expectation values are **Functional integrals**

$$\langle \mathcal{F}[f(x)] \rangle = \int \mathcal{D}f \mathcal{F}[f(x)] \mathcal{P}[f(x)]$$

Determine a function from a finite set of data points

Solution

Standard Approach

- Introduce a simple functional form with enough free parameters

$$q(x, Q^2) = x^\alpha (1 - x)^\beta P(x; \lambda_1, ..., \lambda_n).$$

- Fit parameters minimizing χ^2.

Solution

Standard Approach

- Introduce a simple functional form with enough free parameters

$$q(x, Q^2) = x^{\alpha}(1-x)^{\beta} P(x; \lambda_1, ..., \lambda_n).$$

- Fit parameters minimizing χ^2.

Open problems:

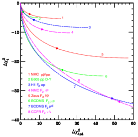
- **Error propagation** from data to parameters and from parameters to observables is **not trivial**.
- **Theoretical bias** due to the chosen **parametrization** is difficult to assess.

Shortcomings of the Standard approach

What is the meaning of a one-σ uncertainty?

- Standard $\Delta\chi^2 = 1$ criterion is **too restrictive** to account for large discrepancies among experiments in a **global fit**.

  [Collins & Pumplin, 2001]

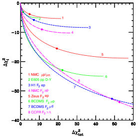


Shortcomings of the Standard approach

What is the meaning of a one-σ uncertainty?

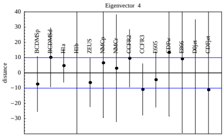
- Standard $\Delta\chi^2 = 1$ criterion is **too restrictive** to account for large discrepancies among experiments in a **global fit**.

  [Collins & Pumplin, 2001]



- Introduce a **TOLERANCE** criterion, i.e. take the envelope of uncertainties of experiments to determine the $\Delta\chi^2$ to use for the global fit (CTEQ).

  [Tung et al., 2006]

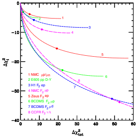


Shortcomings of the Standard approach

What is the meaning of a one-σ uncertainty?

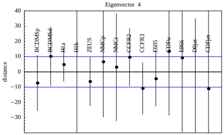
- Standard $\Delta\chi^2 = 1$ criterion is **too restrictive** to account for large discrepancies among experiments in a **global fit**.

  [Collins & Pumplin, 2001]



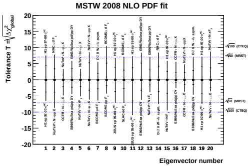
- Introduce a **TOLERANCE** criterion, i.e. take the envelope of uncertainties of experiments to determine the $\Delta\chi^2$ to use for the global fit (CTEQ).

  [Tung et al., 2006]



- Make it **DYNAMICAL**, i.e. determine $\Delta\chi^2$ separately for each hessian eigenvector (MSTW).

  [Thorne et. al, 2008]

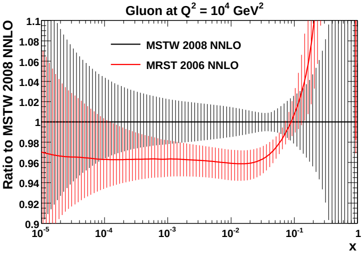


Shortcomings of the standard approach

What determines PDF uncertainties?

- **Uncertainties** in standard fits often **increase when adding data** (i.e. when adding information) even if they are compatible with the old data.
- **Reason**: need change the parametriztion in order to accomodate the new data.

Smaller high-x gluon (and slightly smaller α_S) results in larger small-x gluon – now shown at NNLO.



Larger small-x uncertainty due to extrat free parameter.

[R. Thorne, PDF4LHC]

THE NNPDF METHODOLOGY

[R. D. Ball, V. Bertone, F. Cerutti, L. Del Debbio, S. Forte, J. I. Latorre, A. Piccione, J. Rojo, M. Ubiali and AG]

NNPDF Methodology

Main Ingredients

- **Monte Carlo** determination of errors
  - No need to rely on linear propagation of errors
  - Possibility to test for the impact of non gaussianly distributed errors
  - Possibility to test for non-gaussian behaviour in fitted PDFs ($1 - \sigma$ vs. 68% CL)
- **Neural Networks**
  - Provide an **unbiased** parametrization
- **Stopping based on Cross-Validation**
  - Ensures proper fitting avoiding overlearning

NNPDF Methodology

... in a Nutshell

- Generate N_{rep} **Monte-Carlo replicas** of the experimental data (sampling of the probability density in the space of data)
- Fit a set of Parton Distribution Functions on each replica (sampling of the probability density in the space of PDFs)
- **Expectation values** for observables are **Monte Carlo integrals**

$$\langle \mathcal{F}[f_i(x,Q^2)] \rangle = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} \mathcal{F}\left(f_i^{(net)(k)}(x,Q^2) \right)$$

... the same is true for errors, correlations, etc.

NNPDF Methodology

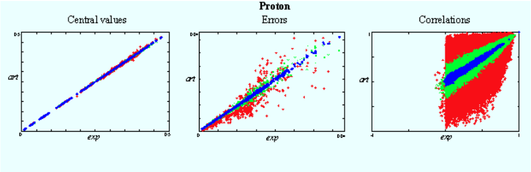
Monte Carlo replicas generation

- **Generate** artificial data according to distribution

$$O_i^{(art)\,(k)} = (1 + r_N^{(k)}\,\sigma_N)\left[O_i^{(exp)} + \sum_{p=1}^{N_{sys}} r_p^{(k)}\,\sigma_{i,p} + r_{i,s}^{(k)}\,\sigma_s^i\right]$$

  where r_i are univariate (gaussianly distributed) random numbers

- **Validate** Monte Carlo replicas against experimental data (statistical estimators, faithful representation of errors, convergence rate increasing N_{rep})

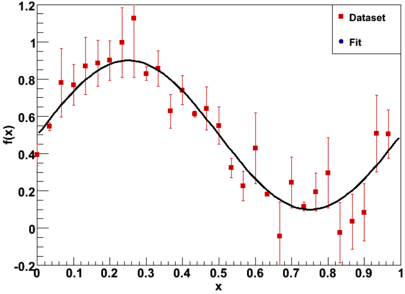


- $\mathcal{O}(1000)$ replicas needed to reproduce correlations to percent accuracy

Proper Fitting avoiding Overlearning

Addressing parametrization bias

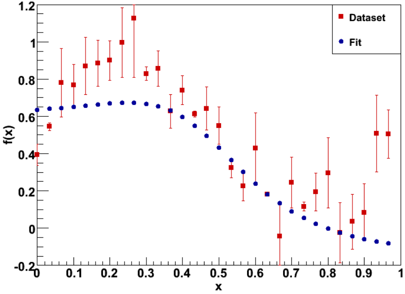
- Let's see how proper fitting works in a toy model



Proper Fitting avoiding Overlearning

Addressing parametrization bias

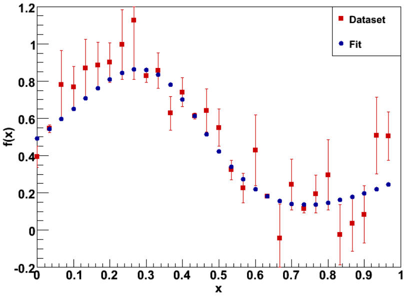
- Let's see how proper fitting works in a toy model



Proper Fitting avoiding Overlearning

Addressing parametrization bias

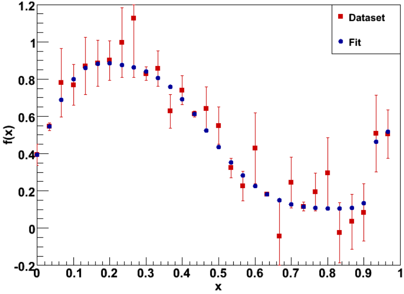
- Let's see how proper fitting works in a toy model



Proper Fitting avoiding Overlearning

Addressing parametrization bias

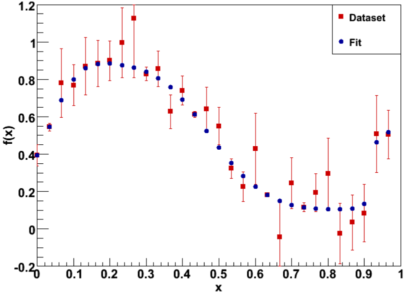
- Let's see how proper fitting works in a toy model



Proper Fitting avoiding Overlearning

Addressing parametrization bias

- Let's see how proper fitting works in a toy model



- Need a **redundant parametrization** to avoid parametrization bias.
- Need a way of **stopping the fit before overlearning** sets in to avoid fitting statistical noise.

Neural Networks

... a suitable basis of functions

- We use **Neural Networks** as **functions** to represent **PDFs at the starting scale**
- We employ **Multilayer Feed-Forward** Neural Networks trained using a **Genetic Algorithm**
- Activation determined by weights and thresholds

$$\xi_i = g\left(\sum_j \omega_{ij}\xi_j - \theta_i\right) , \qquad g(x) = \frac{1}{1+e^{-\beta x}}$$

Neural Networks

... a suitable basis of functions

- We use **Neural Networks** as **functions** to represent **PDFs at the starting scale**
- We employ **Multilayer Feed-Forward** Neural Networks trained using a **Genetic Algorithm**
- Activation determined by weights and thresholds

$$\xi_i = g\left(\sum_j \omega_{ij}\xi_j - \theta_i\right) , \qquad g(x) = \frac{1}{1+e^{-\beta x}}$$

Ex.: 1-2-1 NN:

$$\xi_1^{(3)}(\xi_1^{(1)}) = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - \xi_1^{(1)}\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - \xi_1^{(1)}\omega_{21}^{(1)}}}}}$$

Neural Networks

... a suitable basis of functions

- We use **Neural Networks** as **functions** to represent **PDFs at the starting scale**
- We employ **Multilayer Feed-Forward** Neural Networks trained using a **Genetic Algorithm**
- Activation determined by weights and thresholds

$$\xi_i = g\left(\sum_j \omega_{ij}\xi_j - \theta_i\right) , \qquad g(x) = \frac{1}{1+e^{-\beta x}}$$

Ex.: 1-2-1 NN:

$$\xi_1^{(3)}(\xi_1^{(1)}) = \frac{1}{1+e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - \xi_1^{(1)}\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - \xi_1^{(1)}\omega_{21}^{(1)}}}}}$$

- They provide a parametrization which is **redundant** and **robust** against variations

Genetic Algorithm

1. Set network parameters randomly.
2. Make *clones* of the set of parameters.
3. Mutate each clone.
4. Evaluate χ^2 for all the clones.
5. Select the clone that has the lowest χ^2.
6. Back to 2, until stability in χ^2 is reached.

Genetic Algorithm

1. Set network parameters randomly.
2. Make *clones* of the set of parameters.
3. Mutate each clone.
4. Evaluate χ^2 for all the clones.
5. Select the clone that has the lowest χ^2.
6. Back to 2, until stability in χ^2 is reached.

Pros:

- Allows to minimize the fully correlated χ^2
- Explores the full parameter space, reducing the risk of being trapped in a local minimum

Cons:

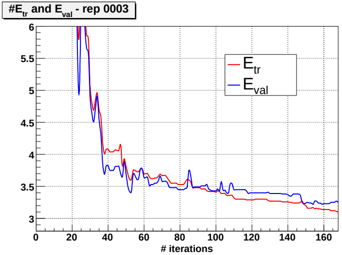
- Slow convergence
- χ^2 decreases monotonically - need to find a suitable stopping criterion

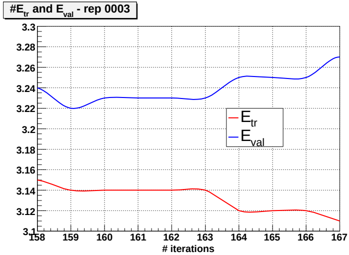
Stopping criterion based on Training-Validation separation

- Divide the data in two sets: Training and Validation
- Minimize the χ^2 of the data in the Training set
- Compute the χ^2 for the data in the Validation set
- When validation χ^2 stops decreasing, **STOP** the fit

Stopping criterion based on Training-Validation separation

- Divide the data in two sets: Training and Validation
- Minimize the χ^2 of the data in the Training set
- Compute the χ^2 for the data in the Validation set
- When validation χ^2 stops decreasing, **STOP** the fit



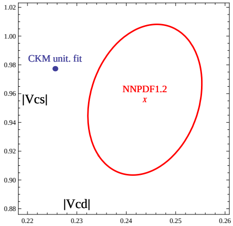


RESULTS

The Past
NNPDF1.0/1.2

- **NNPDF 1.0** [R. D. Ball et al., arXiv:0808.1231]
  - Global DIS fit
  - First application of the full NNPDF Methodology (multiple exps., multiple PDFs)

- **NNPDF 1.2** [R. D. Ball et al., arXiv:0906.1958]
  - Constraining strangeness (dimuon data)
  - Extraction of physical parameters (CKM matrix elements)



- Result for the combined fit

$$
\begin{aligned}
|V_{cs}| &= 0.96 \pm 0.07 \\
|V_{cd}| &= 0.244 \pm 0.019 \\
\rho[V_{cs}, V_{cd}] &= 0.21
\end{aligned}
$$

The (recent) Past

NNPDF2.0

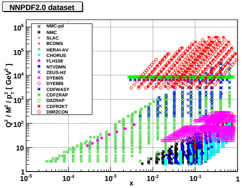
- **NNPDF 2.0**

  [R. D. Ball et al., arXiv:1002.4407]

  - First NNPDF **global fit** (DIS, DY, EWVB and jet data)
  - **NLO QCD** corrections included **without** resorting to **K-factors**.
  - Improved treatment of normalization errors (**t_0-method**).

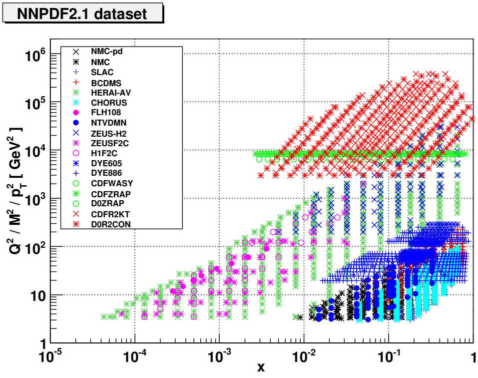
    [R. D. Ball et al., arXiv:0912.2276]

  - Zero Mass Variable Falvour Number Scheme (**ZM-VFNS**)



NNPDF 2.1

Dataset



3415 data points

(for comparison MSTW08 includes 2699 data points)

[R. D. Ball et. al, arXiv:1101.1300]

OBS	*Data set*
Deep Inelastic Scattering	
F_2^d / F_2^p	NMC-pd
F_2^p	NMC, SLAC, BCDMS
F_2^d	SLAC, BCDMS
$\sigma_{NC}^{\pm}$	HERA-I, ZEUS (HERA-II)
$\sigma_{CC}^{\pm}$	HERA-I, ZEUS (HERA-II)
F_L	H1
$\sigma_\nu, \sigma_{\bar{\nu}}$	CHORUS
dimuon prod.	NuTeV
F_2^c	**ZEUS (99,03,08,09)**
F_2^c	**H1 (01,09,10)**
Drell-Yan & Vector Boson prod.	
$d\sigma^{\mathrm{DY}}/dM^2dy$	E605
$d\sigma^{\mathrm{DY}}/dM^2dx_F$	E866
W asymm.	CDF
Z rap. distr.	D0/CDF
Inclusive jet prod.	
Incl. $\sigma^{(\mathrm{jet})}$	CDF (k_T) - Run II
Incl. $\sigma^{(\mathrm{jet})}$	D0 (cone) - Run II

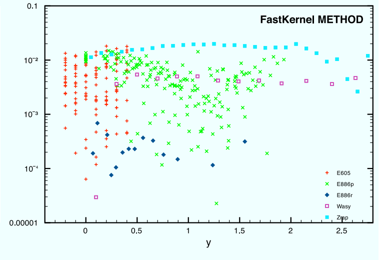
NNPDF 2.x

Inclusion of Higher Order corrections - *FastKernel*

- NLO computation of hadronic observables too slow for parton global fits.
- MSTW08 and CTEQ include Drell-Yan NLO as (local) K factors rescaling the LO cross section
- K-factor depends on PDFs and it is not always a good approximation.

* NNPDF2.0 includes full NLO calculation of hadronic observables.
* Use available fastNLO interface for jet inclusive cross-sections.[hep-ph/0609285]
* Built up our own **FastKernel** computation of DY observables.

$$\int_{x_{0,1}}^{1} dx_1 \int_{x_{0,2}}^{1} dx_2\, f_a(x_1) f_b(x_2) C^{ab}(x_1, x_2) \to \sum_{\alpha,\beta=1}^{N_x} f_a(x_{1,\alpha}) f_b(x_{2,\beta}) \int_{x_{0,1}}^{1} dx_1 \int_{x_{0,2}}^{1} dx_2\, \mathcal{I}^{(\alpha,\beta)}(x_1, x_2) C^{ab}(x_1, x_2)$$



- Both PDFs evolution and double convolution sped up by
  - Use high-orders polynomial interpolation
  - Precompute all Green Functions

A truly NLO analysis

NNPDF 2.1

Heavy Flavour treatment - FONLL

- We adopt the **FONLL** General Mass-Variable Flavour Number Scheme

  [M. Cacciari, M. Greco and P. Nason, (1998)]

  [S. Forte, P. Nason E. Laenen and J. Rojo, (2010)]

- FONLL gives a prescription to **combine FFN** (Massive) and **ZM-VFN** (Massless) computations, at any given order, **avoiding double counting**.
- With results available three implementations of FONLL are possibile:
  - **FONLL-A**: $\mathcal{O}(\alpha_s)$ Massless + $\mathcal{O}(\alpha_s)$ Massive
  - **FONLL-B**: $\mathcal{O}(\alpha_s)$ Massless + $\mathcal{O}(\alpha_s^2)$ Massive
  - **FONLL-C**: $\mathcal{O}(\alpha_s^2)$ Massless + $\mathcal{O}(\alpha_s^2)$ Massive
- **Fixed Flavour Number Scheme** (3-, 4-, 5-) fits **available**.

NNPDF 2.1

Parametrization

Parton Distributions Combination		NN architechture
Singlet ($\Sigma(x)$)	$\Longrightarrow$	2-5-3-1 (37 pars)
Gluon ($g(x)$)	$\Longrightarrow$	2-5-3-1 (37 pars)
Total valence ($V(x) \equiv u_V(x) + d_V(x)$)	$\Longrightarrow$	2-5-3-1 (37 pars)
Non-singlet triplet ($T_3(x)$)	$\Longrightarrow$	2-5-3-1 (37 pars)
Sea asymmetry ($\Delta_S(x) \equiv \bar{d}(x) - \bar{u}(x)$)	$\Longrightarrow$	2-5-3-1 (37 pars)
Total Strangeness ($s^+(x) \equiv (s(x) + \bar{s}(x))/2$)	$\Longrightarrow$	2-5-3-1 (37 pars)
Strange valence ($s^-(x) \equiv (s(x) - \bar{s}(x))/2$)	$\Longrightarrow$	2-5-3-1 (37 pars)

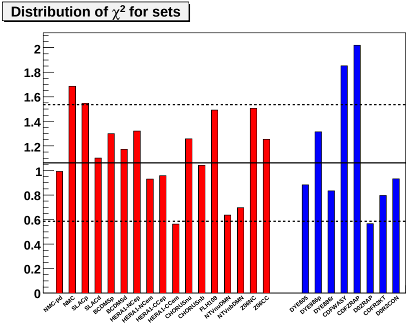
259 parameters
Standard fits have $\sim$ 25 parameters in total

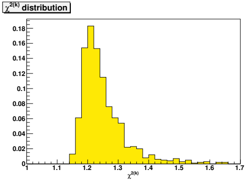
No change in the parametrization since NNPDF1.2 ... despite substantial **enlargement of the dataset**.

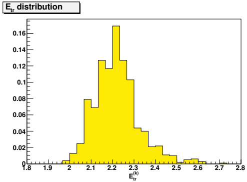
NNPDF 2.1

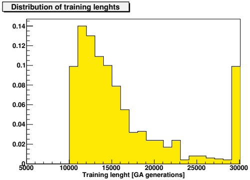
Results - General features of the fit

χ^2_{tot}	1.16
$\langle E \rangle \pm \sigma_E$	2.24 ± 0.09
$\langle E_{\mathrm{tr}} \rangle \pm \sigma_{E_{\mathrm{tr}}}$	2.22 ± 0.11
$\langle E_{\mathrm{val}} \rangle \pm \sigma_{E_{\mathrm{val}}}$	2.28 ± 0.12
$\langle \mathrm{TL} \rangle \pm \sigma_{\mathrm{TL}}$	$(1.6 \pm 0.6)\cdot 10^4$
$\langle \chi^{2(k)} \rangle \pm \sigma_{\chi^2}$	1.25 ± 0.09



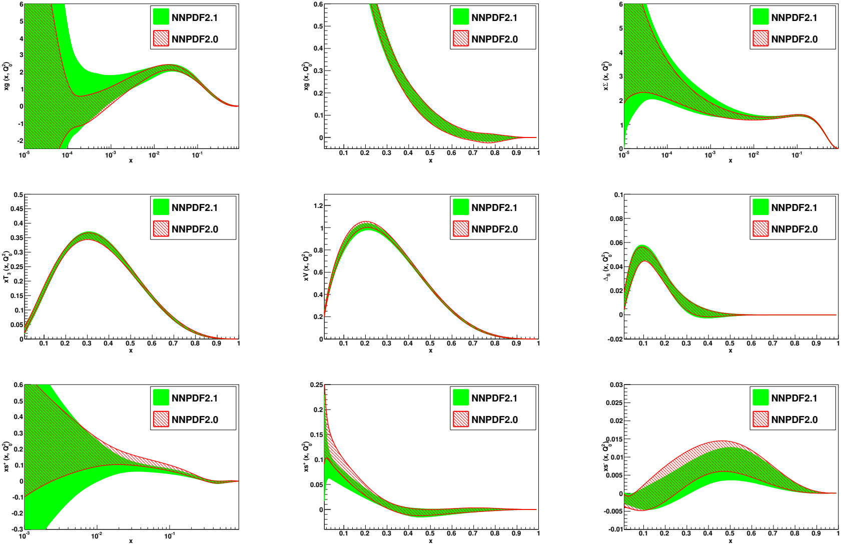






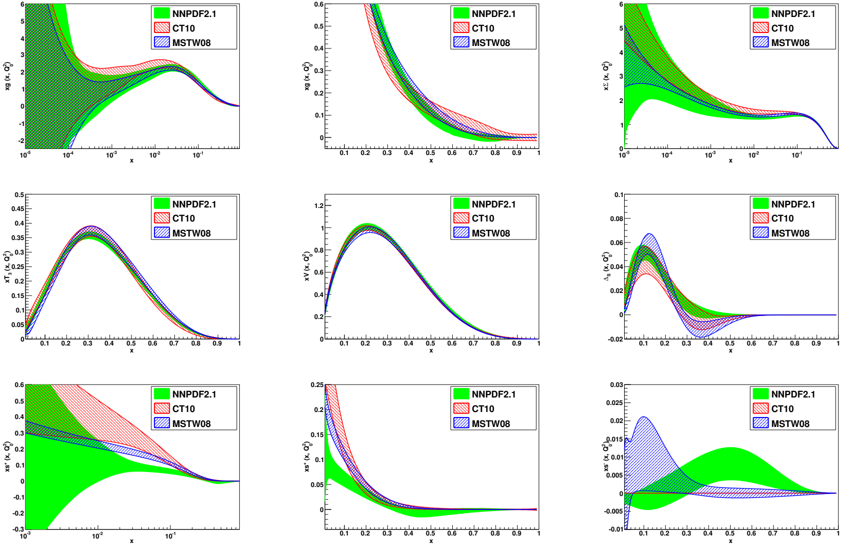
NNPDF 2.1

Partons - Comparison to NNPDF2.0



NNPDF 2.1

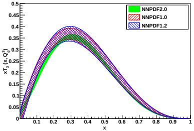
Partons - Comparison to CT10 and MSTW08



NNPDF 2.1

Partons - A couple of upshots

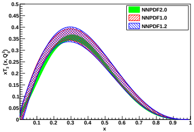
- **Reduction of uncertainties** with respect to older NNPDF sets due to **inclusion of new data**.



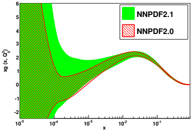
NNPDF 2.1

Partons - A couple of upshots

- **Reduction of uncertainties** with respect to older NNPDF sets due to **inclusion of new data**.



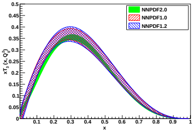
- When **uncertainties increase** we know it is **not a parametrzation effect**.



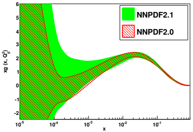
NNPDF 2.1

Partons - A couple of upshots

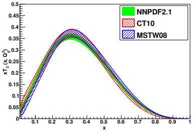
- **Reduction of uncertainties** with respect to older NNPDF sets due to **inclusion of new data**.



- When **uncertainties increase** we know it is **not a parametrzation effect**.



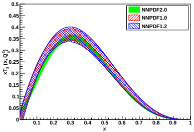
- **Uncertainties** on PDFs have **size comparable** to those obtained by other groups in kinematic regions where there are significant **contraints from data** ...



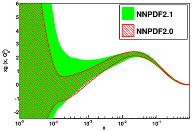
NNPDF 2.1

Partons - A couple of upshots

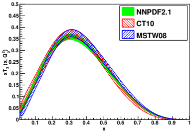
- **Reduction of uncertainties** with respect to older NNPDF sets due to **inclusion of new data**.



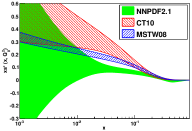
- When **uncertainties increase** we know it is **not a parametrzation effect**.



- **Uncertainties** on PDFs have **size comparable** to those obtained by other groups in kinematic regions where there are significant **contraints from data** ...



- ... but still retain **unbiasedness** in kinematic regions where there are little or **no experimental constraints**.



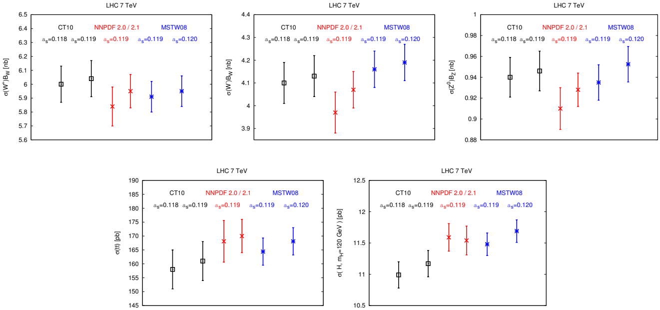
NNPDF 2.1

Phenomenology - LHC Standard Candles

- Updated predictions for **LHC Standard Candles**

	$\sigma(W^+)B_{l\nu}$ [nb]	$\sigma(W^-)B_{l\nu}$ [nb]	$\sigma(Z^0)B_{ll}$ [nb]	$\sigma(t\bar{t})$ [pb]	$\sigma(H)$ [pb]
NNPDF2.1	5.99 ± 0.14	4.09 ± 0.09	0.93 ± 0.02	170 ± 5	11.64 ± 0.17
NNPDF2.0	5.84 ± 0.14	3.97 ± 0.09	0.91 ± 0.02	168 ± 7	11.59 ± 0.22
CT10	6.00 ± 0.13	4.10 ± 0.09	0.94 ± 0.02	158 ± 7	10.99 ± 0.21
MSTW08	5.95 ± 0.11	4.19 ± 0.08	0.95 ± 0.02	168 ± 5	11.69 ± 0.18

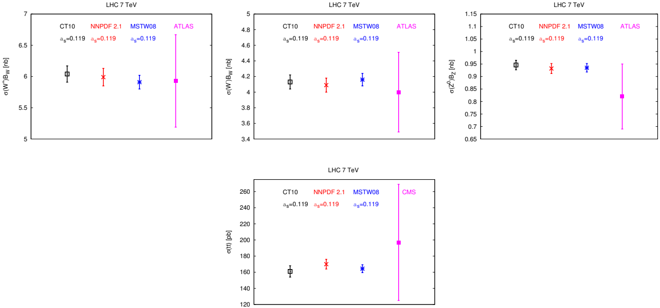
- ... which can be compared to **results from other groups**



- Updated predictions for **LHC Standard Candles**

	$\sigma(W^+)B_{l\nu}$ [nb]	$\sigma(W^-)B_{l\nu}$ [nb]	$\sigma(Z^0)B_{ll}$ [nb]	$\sigma(t\bar{t})$ [pb]	$\sigma(H)$ [pb]
NNPDF2.1	5.99 ± 0.14	4.09 ± 0.09	0.93 ± 0.02	170 ± 5	11.64 ± 0.17
NNPDF2.0	5.84 ± 0.14	3.97 ± 0.09	0.91 ± 0.02	168 ± 7	11.59 ± 0.22
CT10	6.00 ± 0.13	4.10 ± 0.09	0.94 ± 0.02	158 ± 7	10.99 ± 0.21
MSTW08	5.95 ± 0.11	4.19 ± 0.08	0.95 ± 0.02	168 ± 5	11.69 ± 0.18

- ... or to **DATA**



The W lepton asymmetry at the LHC

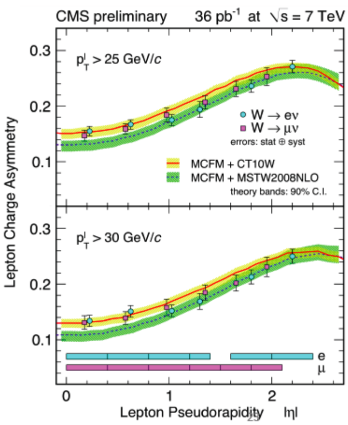
First constraints on PDFs from the LHC

- Run-II Tevatron data on the $p\bar{p} \to W^{\pm} \to l^{\pm}\nu$ asymmetry available

$$A_\ell(y_\ell) = \frac{d\sigma(W^+)/dy_\ell \; - \; d\sigma(W^-)/dy_\ell}{d\sigma(W^+)/dy_\ell \; + \; d\sigma(W^-)/dy_\ell}$$

- NNPDF (arXiv:1012.0836): D0 $\mu + e$ inclusive data (not binned in p_t^l) perfectly consistent with global fit, reduction in valence PDF uncertainty, no tension exclusive to deuteron data found
- Related studies by CT (arXiv:1007.2241) and MSTW (arXiv:1006.2753)
- Now precise LHC measurements also available from CMS (CMS PAS EWK 10 -006) and ATLAS

CMS PAS EWK 10 -006

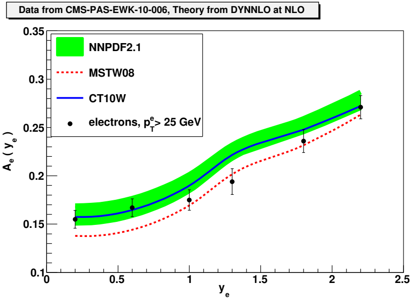


The W lepton asymmetry at the LHC

- First data on the $p\bar{p} \to W^{\pm} \to l^{\pm}\nu$ asymmetry at the LHC available from ATLAS and CMS

$$A_\ell(y_\ell) = \frac{d\sigma(W^+)/dy_\ell \ - \ d\sigma(W^-)/dy_\ell}{d\sigma(W^+)/dy_\ell \ + \ d\sigma(W^-)/dy_\ell}$$

- CMS measurement (CMS PAS EWK 10 -006): e and μ asymmetries with two cuts $p_T^l \geq 25$ GeV and $p_T^l \geq 30$ GeV
- Compare CMS data with predictions from DYNNLO at NLO Discriminating power on PDF sets



Preliminary Results

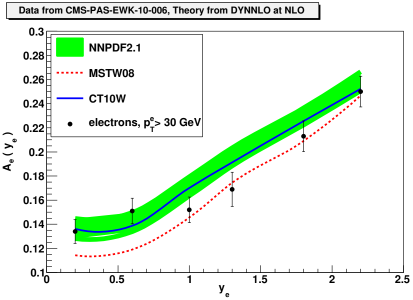
$\chi^2/N_{\rm dat}$ (el)	$p_T^l \geq 25$ GeV	$p_T^l \geq 30$ GeV
NNPDF2.1	**1.8**	1.6
MSTW08	**1.8**	2.3
CT10W	**1.2**	1.3

The W lepton asymmetry at the LHC

- First data on the $p\bar{p} \to W^{\pm} \to l^{\pm}\nu$ asymmetry at the LHC available from ATLAS and CMS

$$A_\ell(y_\ell) = \frac{d\sigma(W^+)/dy_\ell \; - \; d\sigma(W^-)/dy_\ell}{d\sigma(W^+)/dy_\ell \; + \; d\sigma(W^-)/dy_\ell}$$

- CMS measurement (CMS PAS EWK 10 -006): e and μ asymmetries with two cuts $p_T^l \geq 25$ GeV and $p_T^l \geq 30$ GeV
- Compare CMS data with predictions from DYNNLO at NLO Discriminating power on PDF sets



Preliminary Results

$\chi^2/N_{\rm dat}$ (el)	$p_T^l \geq 25$ GeV	$p_T^l \geq 30$ GeV
NNPDF2.1	1.8	**1.6**
MSTW08	1.8	**2.3**
CT10W	1.2	**1.3**

Reweighting PDFs

Assessing the impact of new data on PDF fits

[R. D. Ball et al., arXiv:1012.0836]

- Inspired by Giele and Keller [hep-ph/9803393]
- The N_{rep} **replicas** of a NNPDF fit give the **probability density** in the space of PDFs
- **Expectation values** for observables are **Monte Carlo integrals**

$$\langle \mathcal{F}[f_i(x,Q^2)] \rangle = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} \mathcal{F}\left(f_i^{(net)(k)}(x,Q^2) \right)$$

(... the same is true for errors, correlations, etc.)

- We can **assess the impact** of including **new data** in the fit updating the probability density distribution.

Reweighting PDFs

Assessing the impact of new data on PDF fits

- According to **Bayes Theorem** we have

$$\mathcal{P}_{\rm new}(\{f\}) = \mathcal{N}_\chi \mathcal{P}(\chi^2|\{f\}) P_{\rm init}(\{f\}) \,, \quad \mathcal{P}(\chi^2|\{f\}) = [\chi^2(y,\{f\})]^{\frac{n_{dat}}{2}-1} e^{-\frac{\chi^2(y,\{f\})}{2}}$$

- **Monte Carlo integrals** are now **weighted sums**

$$\langle \mathcal{F}[f_i(x,Q^2)] \rangle = \sum_{k=1}^{N_{rep}} w_k \mathcal{F}\left(f_i^{(net)(k)}(x,Q^2) \right)$$

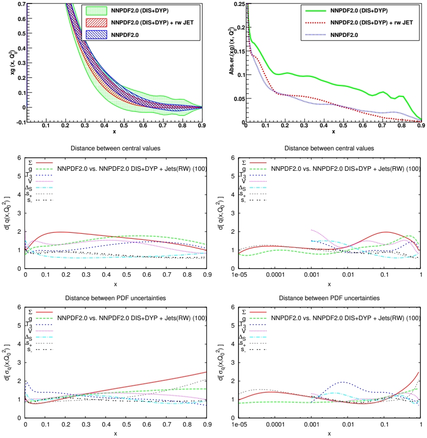
where the **weights** are

$$w_k = \frac{[\chi^2(y,f_k)]^{\frac{n_{dat}}{2}-1} e^{-\frac{\chi^2(y,f_k)}{2}}}{\sum_{i=1}^{N_{\rm rep}} [\chi^2(y,f_i)]^{\frac{n_{dat}}{2}-1} e^{-\frac{\chi^2(y,f_i)}{2}}}$$

Reweighting PDFs

Proof-of-concept: Inclusive Jet data, reweighting vs. refitting

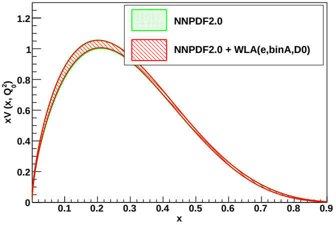
- Use **DIS+DY-fit** as **prior** probability distribution
- Add Tevatron Inclusive Jet data through refitting and through reweighting
- **Reweighting** and **refitting** yield **statistically equivalent** results

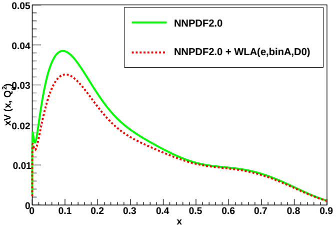


Reweighting PDFs

Reweighting real data: W lepton asymmetry

- In the NNPDF2.0 fit we only included CDF W asymmetry data
- We evaluated W electron asymmetry with NNPDF20 1000 replicas set using **DYNNLO** [Catani et al., arXiv:0903.2120].
- .. and included D0 W electron asymmetry data points through reweighting.
- Main impact on reduction of middle-x Valence uncertainty.
- **No** need of **refitting**

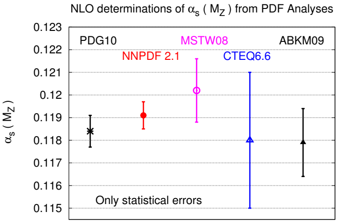


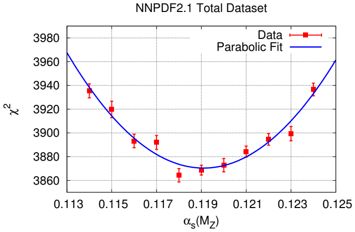


$\alpha_s(M_Z)$ *from PDF analysis*

[S. Lionetti et al., arXix:1103.2369]

- **The Good**: Large dataset
  $\Longrightarrow$ **Small statistical errors**
- **The Bad**: Best fit α_S and PDFs are correlated
  $\Longrightarrow$ **Parametrization bias**? **Dataset Dependence**?
- **The Ugly**: Need to tame statistical fluctuations in χ^2
  $\Longrightarrow$ **Large replica samples!**

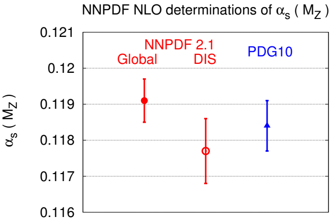




$\alpha_s(M_Z)$ from PDF analysis

Dependence of $\alpha_s\,(M_Z)$ on the dataset

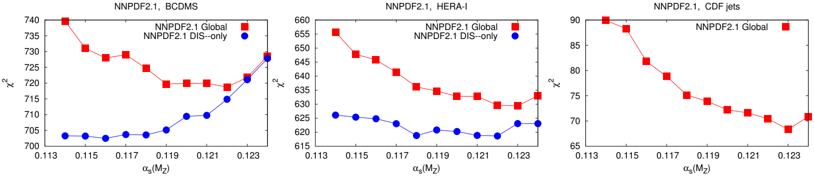
	$\alpha_s\,(M_Z)$
NNPDF2.1	**$0.1191 \pm 0.0006^{\rm stat}$**
NNPDF2.1 DIS–only	**$0.1177 \pm 0.0009^{\rm stat}$**
NNPDF2.0	$0.1168 \pm 0.0007^{\rm stat}$
NNPDF2.0 DIS–only	$0.1145 \pm 0.0010^{\rm stat}$



- Do DIS data prefer a smaller value of α_s?

  Maybe (but not much smaller), and anyway **compatible with the value from global fit** and with **larger uncertainties**.

- Theoretical uncertainties likely dominant (*Ex.* $\Delta\alpha_s^{\rm HQ} \sim 0.002$)



- BCDMS in a DIS–only fit sometimes has runaway direction at small $\alpha_s\,(M_Z)$, absent in the global fit
- HERA rather flat in α_s in DIS–only fit
- Tevatron jet experiments exclude small $\alpha_s\,(M_Z)$ values

Significant interplay between DIS and hadronic data

Conclusions

- A **reliable** estimation of **PDF uncertainties** is crucial in order to exploit the full physics potential of the LHC experiments.
- The **NNPDF2.1** set fulfills most of the requirement of an ideal PDF set for precision (NLO) phenomenology at the LHC
  - it is based on a comprehensive global dataset,
  - it is (almost) free of parametrization bias,
  - it is provided with a reliable, statistically meaningful estimation of uncertainties,
  - it includes NLO corrections without resorting to K-factor approximations,
  - it includes a consistent treatment of heavy quark effects,
  - it is available for a variety of values of α_s and quark masses.
- NNPDF sets are **available** within the **LHAPDF** interface.
- Next steps:
  - Inclusion of higher order contributions (NNLO QCD/EW effects)
  - Study of theoretical uncertainties (quark masses, fact./ren. scales...)
  - Inclusion of resummation (small-/large-x) effects

BACKUP SLIDES

Distances between fits

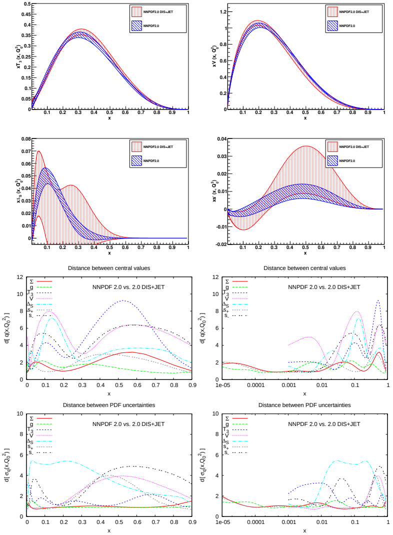
Quantitative comparison of different fits

- We define the **distance** between central values of different PDF fits

$$d(q_j) = \sqrt{\left\langle \frac{\left(\langle q_j\rangle_{(1)} - \langle q_j\rangle_{(2)}\right)^2}{\sigma_1^2[q_j] + \sigma_2^2[q_j]} \right\rangle_{N_{\text{part}}}}$$

  and the similarly for Standard Deviations.

- $d \sim 1$ indicates **statistically equivalent** fits.
- Much **stronger check** of (dis)agreement between fits than “overlap of error bands”.



- All **valence-type PDF** combinations are **affected** by these data.
- **Singlet and gluon** PDFs essentially **unaffected**.
- Sizable reduction of the **uncertainty on the strange valence**. (possible impact on NuTeV anomaly)

PDF Uncertainties and Correlations

A practitioner's guide to NNPDF predictions

Central Value

$$\langle \mathcal{F} \rangle = \frac{1}{N_{\rm set}} \sum_{k=1}^{N_{\rm set}} \mathcal{F}[q^{(k)}]$$

Standard Deviation

$$\sigma_{\mathcal{F}} = \left(\frac{1}{N_{N_{\rm set}}} \sum_{k=1}^{N_{\rm set}} \left(\mathcal{F}[\{q^{(k)}\}] - \langle \mathcal{F}[\{q\}] \rangle \right)^2 \right)^{1/2}$$

Correlation

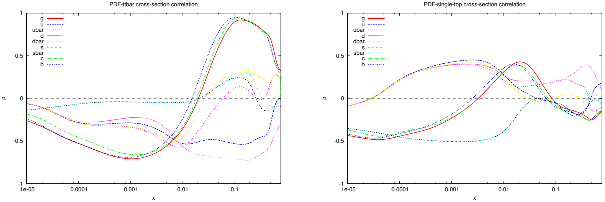
$$\rho \equiv \cos\varphi(\mathcal{F},\mathcal{G}) = \frac{< \mathcal{F}\,\mathcal{G} >_{\rm rep} - < \mathcal{F} >_{\rm rep} < \mathcal{G} >_{\rm rep}}{\sqrt{< \mathcal{F}^2 >_{\rm rep} - < \mathcal{F} >_{\rm rep}^2}\sqrt{< \mathcal{G}^2 >_{\rm rep} - < \mathcal{G} >_{\rm rep}^2}}$$

PDF induced correlations

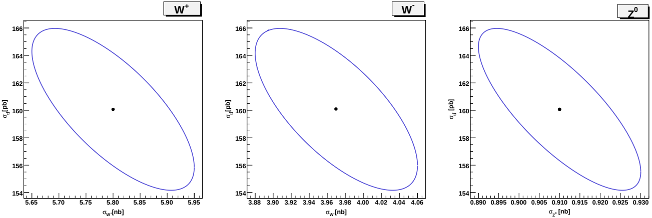
Ex.: Top-quark studies within the NNPDF framework

[J. Rojo and AG, arXiv:1008.4671]

- It is easy to compute **correlations** between **PDFs and observables**



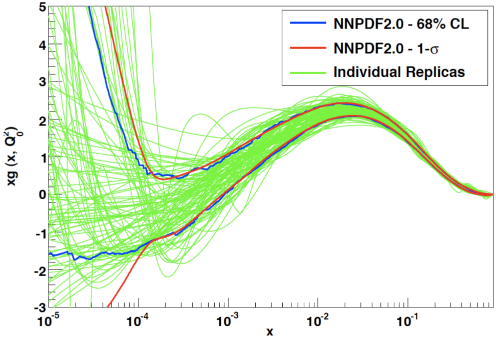
- ... or **pairs of observbles**



Confidence Level Intervals

Testing for non gaussian distribution of fitted PDFs

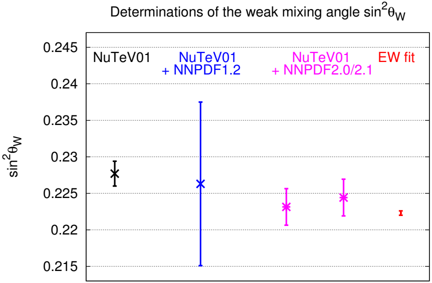
- **Confidence Level intervals** can be computed directly from the replicas distribution
- Comparison of 68% C.L. and symmetric 1σ especially in extrapolation regions where theory constraints dominate on experimental information



Phenomenology

The "NuTeV Anomaly"

- Can we explain the "NuTev Anomaly" simply allowing for non-vanishing strange valence distribution?
- Impact on **NuTeV** determination of $\sin^2\theta_W$



NNPDF 2.1

FONLL - The gory details

- A **generic DIS observable** in the FONLL scheme is written as:

$$F^{FONLL}(x,Q^2) = \mathcal{D}(Q^2) F^{(d)}(x,Q^2) + F^{(n_l)}(x,Q^2)$$

where the **threshold damping factor** is given by

$$\mathcal{D}(Q^2) = \theta(Q^2 - m^2) \left(1 - \frac{m^2}{Q^2}\right)$$

and the **subtraction term** is

$$F^{(d)} = \left[F^{(n_l+1)}(x,Q^2) - F^{(n_l,0)}(x,Q^2) \right]$$

with the massless limit of the massive contributions being

$$F^{n_l,0}(x,Q^2) = x \int_x^1 \frac{dy}{y} \sum_{i=q,\bar{q},g} B_i^{(0)} \left(\frac{x}{y}, \frac{Q^2}{m^2}, \alpha_S^{(n_l+1)}(Q^2) \right) f_i^{(n_l+1)}(y,Q^2)$$

with

$$\lim_{m \to 0} \left[B_i\left(x, \frac{Q^2}{m^2}\right) - B_i^{(0)}\left(x, \frac{Q^2}{m^2}\right) \right] = 0$$