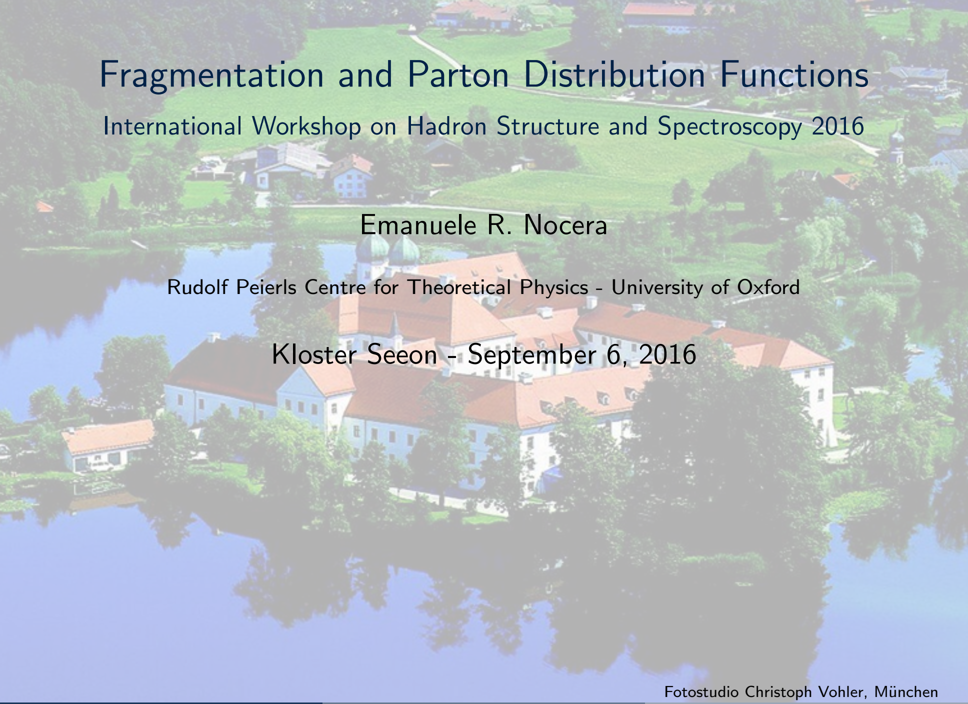

Fragmentation and Parton Distribution Functions

International Workshop on Hadron Structure and Spectroscopy 2016

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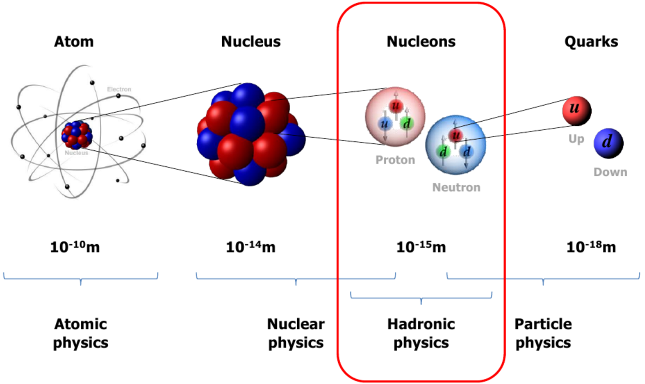
Fotostudio Christoph Vohler, München

Hadron physics, or the quest for the nucleon structure

Nucleons, protons and neutrons, are those bound states which make up all nuclei and hence most of the visible matter in the Universe

Understanding their structure and dynamics in terms of their partonic constituents, and their emergence from quarks and gluons, is a challenge in hadron physics

This talk is about Fragmentation Functions and Parton Distribution Functions, some of the tools which bring such an understanding from high-energy particle physics



Outline

DISCLAIMER

The subject is extremely vast, impossible to give a comprehensive review in a single talk
Inevitably, this talk contains a partial subjective selection of topics/results
The focus is on FFs/PDFs in connection with the hadron structure
(rather than on precision physics at the LHC)
Apologies in advance if your favorite subject has been omitted

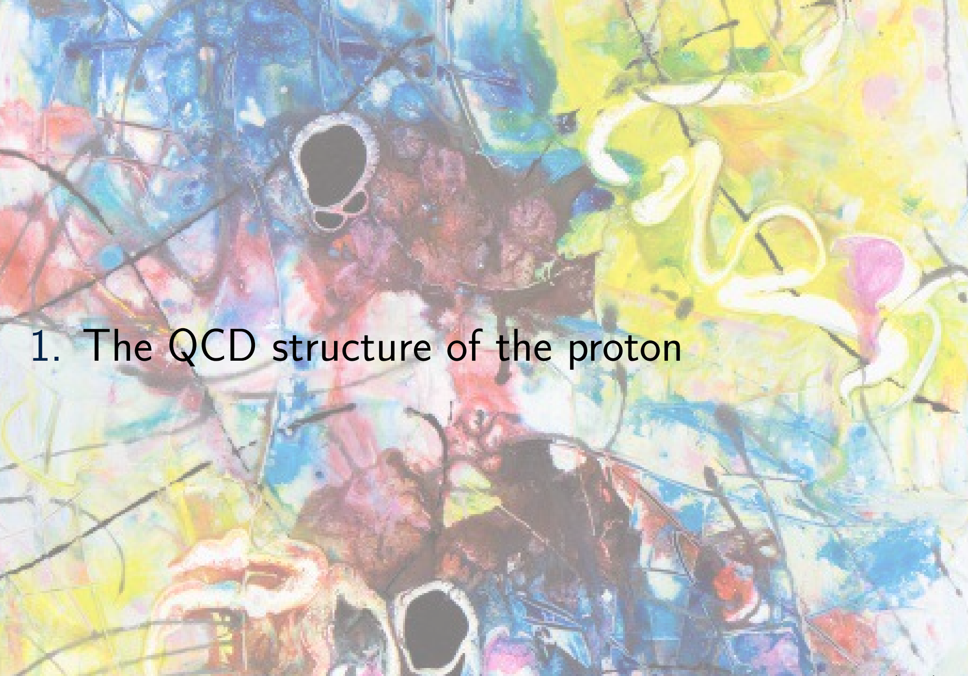
1. The QCD structure of the proton
   - Theory: factorization, evolution, distributions
2. A global analysis of FFs/PDFs
   - Practice: combining theory and data with a proper methodology
3. Some selected, recent results
   - A Fragmentation functions
   - B Parton distribution functions (mostly helicity)
4. Drawing conclusions

Some excellent reviews are available in the literature

FFs: [Rev.Mod.Phys. 82 (2010) 2489; arXiv:1607.02521]

PDFs: [Ann.Rev.Nucl.Part.Sci. 63 (2013) 291;J.Phys. G40 (2013) 093102] + PDF4LHC working group reports

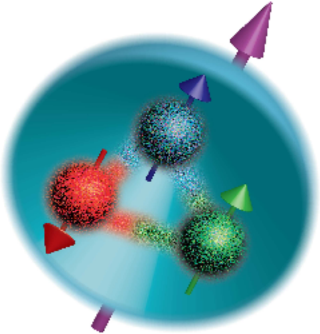
1. The QCD structure of the proton



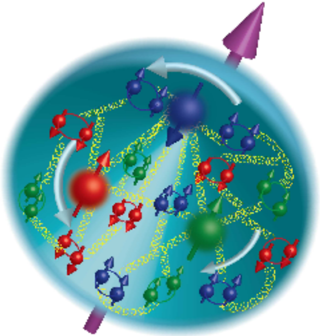
Blake Brasher, *Deep Inelastic Scattering*, Acrylic, ink, and holographic glitter on canvas (2014)

The QCD picture of the nucleon

naive picture



realistic picture



three non-relativistic quarks $\xleftrightarrow[\text{factorization,evolution}]{\text{QCD}}$ indefinite number of relativistic quarks and gluons

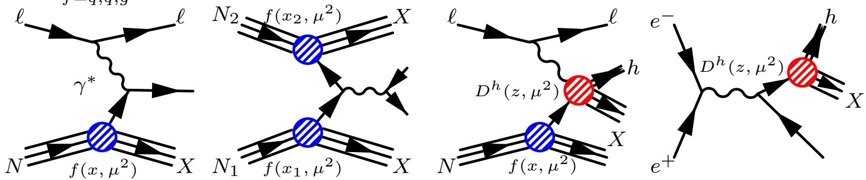
Factorization of physical observables [Adv.Ser.Direct.HEP 5 (1988) 1]

1. A variety of sufficiently inclusive processes allow for a factorized description

short-distance part	$\xleftrightarrow[\text{scheme \& scale } \mu]{\text{factorization}}$	long-distance part
hard interaction of partons		nucleon structure
process-dependent kernels		universal parton distributions

2. Physical observables are written as a convolution of coefficient functions and PDFs

$$\mathcal{O}_I = \sum_{f=q,\bar{q},g} C_{If}(y, \alpha_s(\mu^2)) \otimes f(y,\mu^2) + \text{p.s. corrections} \qquad f \otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$



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3. Coefficient functions allow for a perturbative expansion

$$C_{If}(y,\alpha_s) = \sum_{k=0} a_s^k C_{If}^{(k)}(y), \qquad a_s = \alpha_s/(4\pi)$$

4. After factorization, all quantities (including FFs/PDFs) depend on μ

Evolution of FFs/PDFs: DGLAP equations [NP B126 (1977) 298]

1. A set of $(2n_f + 1)$ integro-differential equations, n_f is the number of active flavors

$$\frac{\partial}{\partial \ln \mu^2} f_i(x,\mu^2) = \sum_j^{n_f} \int_x^1 \frac{dz}{z} P_{ji}\left(z, \alpha_s(\mu^2)\right) f_j\left(\frac{x}{z}, \mu^2\right)$$

2. Often written in a convenient basis of PDFs

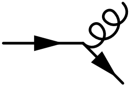
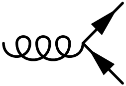
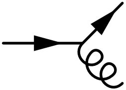
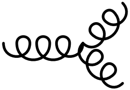
$$q_{\mathrm{NS};\pm} = (q_i \pm \bar{q}_i) - (q_j \pm \bar{q}_j) \qquad q_{\mathrm{NS;v}} = \sum_i^{n_f} (q_i - \bar{q}_j) \qquad \Sigma = \sum_i^{n_f} (q_i + \bar{q}_j)$$

$$\frac{\partial}{\partial \ln \mu^2} q_{\mathrm{NS};\pm,v}(x,\mu^2) = P^{\pm,\mathrm{v}}(x,\mu_F^2) \otimes q_{\mathrm{NS};\pm,v}(x,\mu^2)$$

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} \Sigma(x,\mu^2) \\ g(x,\mu^2) \end{pmatrix} = \begin{pmatrix} P^{qq} & P^{gq} \\ P^{qg} & P^{gg} \end{pmatrix} \otimes \begin{pmatrix} \Sigma(x,\mu^2) \\ g(x,\mu^2) \end{pmatrix}$$

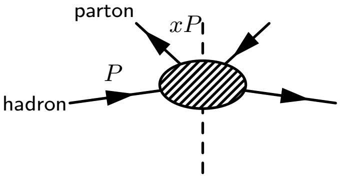
3. With perturbative computable splitting functions

$$P_{ji}(z,\alpha_s) = \sum_{k=0} a_s^{k+1} P_{ji}^{(k)}(z), \qquad a_s = \alpha_s/(4\pi)$$

$P_{qq}^{(0)}$  $P_{gq}^{(0)}$  $P_{qg}^{(0)}$  $P_{gg}^{(0)}$ 

Field-theoretic definition on the light-cone [Rev.Mod.Phys. 67 (1995) 157]

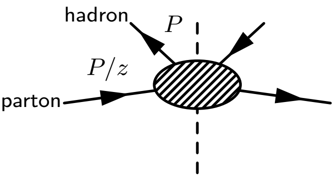
Parton Distribution functions (PDFs)



collinear transition of a massles hadron h into a massless parton i with fractional momentum x

local OPE $\Longrightarrow$ lattice formulation

Fragmentation Functions (FFs)



collinear transition of a massless parton i into a massless hadron h with fractional momentum z

no local OPE $\Longrightarrow$ no lattice formulation

Expectation values (matrix elements) of certain (bilocal) operators in hadronic states

$$f_i^h(x) = \frac{1}{4\pi}\int dy^- e^{-ixP^+y^-}\langle h(P)|\bar{\psi}_i(0,y^-,\mathbf{0}_\perp)\gamma^+\mathcal{P}\psi_i(0)|h(P)\rangle$$

$$D_i^h(z) = \frac{1}{12\pi}\sum_X\int dy^- e^{i\frac{P^+}{z}y^-}\mathrm{Tr}\left[\gamma^+\langle 0|\psi(0,y,\mathbf{y}_\perp)\mathcal{P}|h(P)\,X\rangle\langle h(P)\,X|\mathcal{P}'\bar{\psi}(0)|0\rangle\right]$$

$$y=(y^+,y^-,\mathbf{y}_\perp)\,,\qquad y^+=(y^0+y^z)/\sqrt{2}\,,\qquad y^-=(y^0-y^z)/\sqrt{2}\,,\qquad \mathbf{y}_\perp=(y^x,y^y)$$

All these definitions have ultraviolet divergences which must be renormalized to define finite PDFs and FFs to be used in the factorization formulas

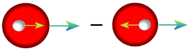
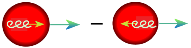
(PDF/FFs are scheme dependent)

All these definitions can be generalized to include longitudinal/transverse polarizations

Helicity-dependent PDFs and the proton spin

The momentum densities of partons with spin ($^\uparrow$) or ($^\downarrow$) *w.r.t* the nucleon

$$\Delta f(x) \equiv f^{\uparrow}(x) - f^{\downarrow}(x)\,, \qquad f = u, \bar{u}, d, \bar{d}, s, \bar{s}, g$$

$\Delta q(x) =$  $\Delta g(x) =$ 

$$\Delta q(x) = \frac{1}{4\pi}\int dy^- e^{-ixP^+y^-}\langle h(P,S)|\bar{\psi}(0,y^-,\mathbf{0}_\perp)\gamma^+\gamma^5\psi(0)|h(P,S)\rangle$$

$$\Delta g(x) = \frac{1}{4\pi x P^+}\int dy^- e^{-ixP^+y^-}\langle h(P,S)|G^{+\alpha}(0,y^-,\mathbf{0}_\perp)\tilde{G}^{+}_{\alpha}(0)|h(P,S)\rangle$$

$$G^{\alpha}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$$

A realization of the total proton angular momentum decomposition

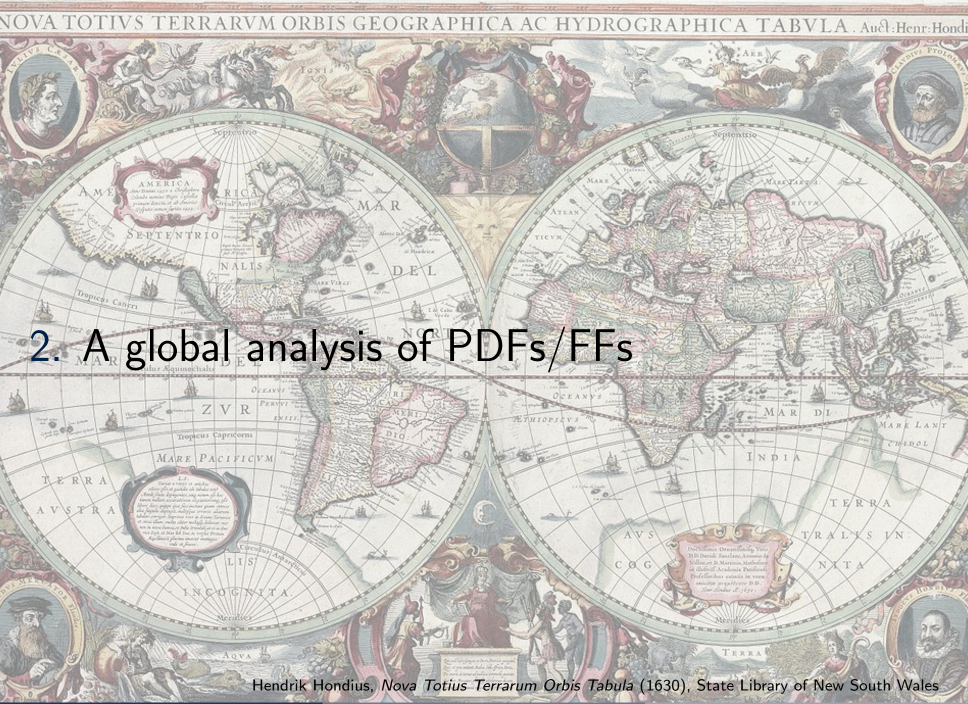
$$\mathcal{S}(\mu^2) = \sum_f \left\langle P;S|\hat{J}^z_f(\mu^2)|P;S\right\rangle = \frac{1}{2} = \frac{1}{2}\Delta\Sigma(\mu^2) + \Delta G(\mu^2) + \mathcal{L}_q(\mu^2) + \mathcal{L}_g(\mu^2)$$

$$\Delta\Sigma(\mu^2) = \sum_{q=u,d,s}\int_0^1[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)] \qquad \Delta G(\mu^2) = \int_0^1 dx\Delta g(x,\mu^2)$$

$$a_0 = \left\langle P;S|\hat{J}^z_\Sigma(\mu^2)|P;S\right\rangle \xrightarrow{\text{naive p.m.}} 2\langle S_z^{q+\bar{q}}\rangle \simeq 1 \qquad \text{EMC 1988 a}_0 = 0.098 \pm 0.076 \pm 0.113$$

$$a_0 = \left\langle P;S|\hat{J}^z_\Sigma(\mu^2)|P;S\right\rangle \stackrel{\overline{\text{MS}}}{=} \Delta\Sigma(\mu^2) - n_f\frac{\alpha_s(\mu^2)}{2\pi}\Delta G(\mu^2) \qquad \Delta G(\mu^2) \propto \left[\alpha_s(\mu^2)\right]^{-1}$$

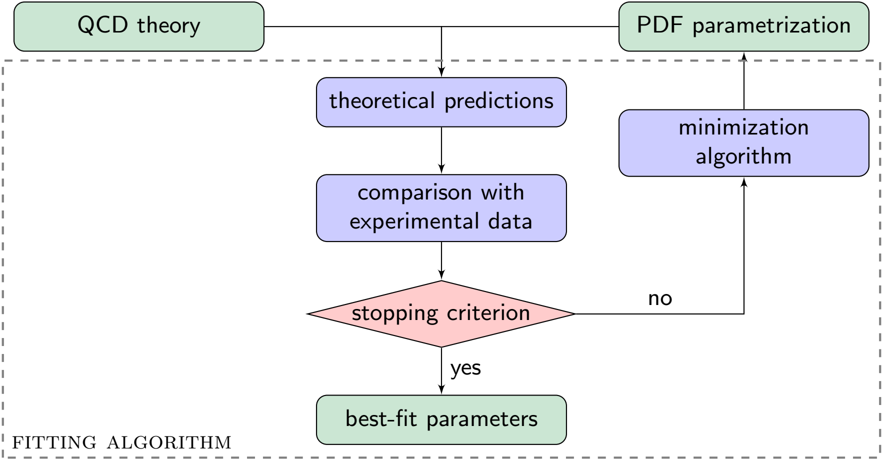
2. A global analysis of PDFs/FFs



Hendrik Hondius, *Nova Totius Terrarum Orbis Tabula* (1630), State Library of New South Wales

A global FF/PDF determination: the underlying strategy

A statistically ill-posed problem: determine a set of functions from a finite set of data



Assume a reasonable FF/PDF parametrization

Obtain theoretical predictions for various processes and compare predictions to data

Determine the best-fit parameters via minimization of a proper figure of merit (*e.g.* χ^2)

A global FF/PDF determination: the ingredients we need

theory	methodology	data
DGLAP evolution	parametrization	fixed-target data
partonic cross sections	uncertainty estimation	collider data
heavy quark treatment	error propagation	(DIS, SIDIS, DY, ...)
QED/EW corrections	minimization strategy	(e^+e^-, ep, pp, $p\bar{p}$)

Need for a choice of

1. **theory**, or the theoretical details of the QCD analysis
   (perturbative order, treatment of heavy quarks, treatment of α_s, theoretical constraints)
2. **methodology**, or a prescription to determine PDFs and their uncertainties
   (uncertainty estimates are crucial to make reliable predictions based on PDFs)
3. **data**, or the set of observables to be included in the analysis
   (constrain all possible PDFs in the widest range of Bjorken-x)

Each of these ingredients is a source of uncertainty on the PDF determination

Theory: perturbative accuracy (QCD, fixed order)

PDFs	Polarized PDFs	FFs
	Usual perturbative accuracy in global fits	
NNLO (do we need N^{3}LO?)	NLO (comfortably precise)	NLO (roughly OK)
	Splitting functions known up to NNLO	
$P_{ji}^{(k)} \propto \frac{a_s^{k+1}}{x} \log^{k+1-m} \frac{1}{x}$	$\Delta P_{ji}^{(k)} \propto \frac{a_s^{k+1}}{x} \log^{k+1-m} \frac{1}{x}$	$P_{ji}^{(k)} \propto \frac{a_s^{k+1}}{z} \log^{2(k+1)-m-1} z$

with $m = 1, \ldots, 2k+1$: soft gluon logarithms diverge more rapidly in the time-like case than in space-like case

as z decreases, the SGLs will spoil the convergence of the fixed-order series for P_{ji} once $\log \frac{1}{z} \geq \mathcal{O}\left(a_s^{-1/2}\right)$

numerical implementation of space- and time-like evolution in `APFEL-MELA` [JHEP 1503 (2015) 046]

`https://apfel.hepforge.org/mela.html`

Almost all relevant processes known at NLO, increasing progress in NNLO (LHC) [PDG]

PDFs	Polarized PDFs	FFs
DIS: N^{3}LO	DIS: NNLO	
DY: NNLO	SIDIS: NLO	SIA: NNLO
jets: NNLO (partial)	(di-)jets in pp: NLO	SIDIS: NLO
W in pp: NNLO	W in pp: NLO	π in pp: NLO
$t\bar{t}$: NNLO	π in pp: NLO	

Partonic hard cross sections are precomputed in such a way that the standard numerical convolution with any set of PDFs can be approximated by means of interpolation techniques (`APPLgrid`, `APFELgrid`) [EPJ C66 (2010) 503; arXiv:1605.02070]

Theory: theoretical constraints

NOT NEGOTIABLE: must be fulfilled by a reliable determination of PDFs/FFs

1. Momentum sum rule (follows from energy-momentum conservation)

$$\mathcal{M}(\mu^2) = \int_0^1 dx\, x \sum_q \left\{ \left[q(x,\mu^2) + \bar{q}(x,\mu^2) \right] + g(x,\mu^2) \right\} = 1 \qquad \text{for PDFs}$$

$$\mathcal{M}_i^h(\mu^2) = \sum_h \int_0^1 dz\, z D_i^h(z,\mu^2) = 1 \qquad \text{for FFs, for each parton i}$$

PDFs: almost fulfilled by experimental data ($\mathcal{M}^{\rm NNLO} = 1.002 \pm 0.014$, `NNPDF2.1`)
FFs: limited practical use in fits (evolution and mass corrections)

2. Integrability (the nucleon matrix element of the axial current must be finite $\forall$ flavors)
PDFs/FFs cannot grow arbitrarily at small values of x/z

3. Positivity (PDFs and FFs must not lead to negative cross sections)
often built-in in the parametrization; PDFs/FFS should not be strictly positive beyond LO

DEBATABLE: usually included in a determination of PDFs/FFs

1. Symmetries
isospin for sea quark FFs: *e.g.* $D_{\bar{u}}^{\pi^+} = D_{d}^{\pi^-}$
SU(2) and SU(3) for polarized PDFs
(relate the first moments of the $\mathcal{C}$-even combinations to baryon octect decay constants)

2. *Reasonable* assumptions
e.g. symmetric sea/unfavored quarks: $\Delta s = \Delta \bar{s}$, $D_{\bar{u}}^{K^+} = D_{s}^{K^+} = D_{\bar{d}}^{K^+} = D_{d}^{K^+}$

Methodology: parametrization

A (general, smooth, flexible) parametrisation at an initial scale Q_0^2 is chosen

$$xf_i(x,Q_0^2) = A_{f_i}\, x^{a_{f_i}}\, (1-x)^{b_{f_i}}\, \mathscr{F}(x,\{c_{f_i}\})$$

small x

$$xf_i(x,Q^2) \xrightarrow{x\to 0} x^{a_{f_i}}$$

(Regge theory)

$$\mathscr{F}(x,\{c_{f_i}\}) \xrightarrow[x\to 1]{x\to 0} \text{finite}$$

$$\xrightarrow[\text{smooth interpolation in between}]{}$$

large x

$$xf_i(x,Q^2) \xrightarrow{x\to 1} (1-x)^{b_{f_i}}$$

(quark counting rules)

1. Simple parametrization: small number of parameters ($\mathcal{O}(30)$ per set)

$$\mathscr{F}(x,\{c_{f_i}\}) = \eta_{f_i}\left(1+\rho_{f_i}x^{\frac{1}{2}}+\gamma_{f_i}x\right) \qquad \{\mathbf{a}\} = \{\eta_{f_i},\rho_{f_i},\gamma_{f_i}\}\cup\{a_{f_i},b_{f_i}\}$$

↑ smooth behavior (a desirable feature for a PDF)

↓ potential source of bias if the parametrization is too rigid

2. Redundant parametrization: huge set of parameters ($\mathcal{O}(200)$ per PDF set)

$$\mathscr{F}(x,\{c_{f_i}\}) = \begin{cases} \text{Chebyschev polynomials} \\ \text{Bernstein polynomials} \\ \text{Neural Networks} \end{cases} \qquad \{\mathbf{a}\} = \{\omega_{f_i}^{(L-1,n)},\theta_{f_i}^{(L,n)}\}(\cup\{a_{f_i},b_{f_i}\})$$

↓ potential non-smoothness, extra flexibility requires a careful minimization

↑ bias due to the parametrization reduced as much as possible

Methodology: error propagation (Hessian method)

1. Expand the χ^2 about its global minimum at first (nontrivial) order

$$\chi^2\{\mathbf{a}\} \approx \chi^2\{\mathbf{a_0}\} + \delta a^i H_{ij} \delta a^j , \qquad H_{ij} = \left. \frac{\partial^2 \chi^2(\{\mathbf{a}\})}{\partial a_i \partial a_j} \right|_{\{\mathbf{a}\}=\{\mathbf{a_0}\}}$$

2. A C.L. about the best fit is obtained as the volume (in parameter space) about $\chi^2\{\mathbf{a_0}\}$ that corresponds to a fixed increase of the χ^2; for Gaussian uncertainties:

$$68\% \text{ C.L.} \Longleftrightarrow \Delta\chi^2 = \chi^2\{\mathbf{a}\} - \chi^2\{\mathbf{a_0}\} = 1$$

↑ the C.L. is entirely determined by $H_{ij} = \mathrm{cov}^{-1}_{ij}$
↑ compact representation and computation of FFs/PDFs and their uncertainties
↓ the Hessian matrix becomes unmanageable if the number of parameters is huge
↓ is linear approximation adequate? do we need a tolerance $\Delta\chi^2 > 1$?
↓ what if uncertainties deviate from Gaussian behavior?

$$\langle \mathcal{O}[f(x,Q^2)] \rangle = \mathcal{O}[f_0(x,Q^2)]$$

$$\sigma_{\mathcal{O}}[f(x,Q^2)] = \frac{1}{2} \left[\sum_{i=1}^{N_{\mathrm{par}}} \left(\mathcal{O}[f_i(x,Q^2)] - \mathcal{O}[f_0(x,Q^2)] \right)^2 \right]^{1/2}$$

Methodology: error propagation (LM and MC methods)

1. Minimize the function (for fixed values of the Lagrange multipliers $\{\boldsymbol{\lambda}\}$)

$$\Psi(\{\mathbf{a}\},\{\boldsymbol{\lambda}\}) = \chi^2(\{\mathbf{a}\}) + \sum_j \lambda_j \mathcal{O}(\{\mathbf{a}\})$$

2. Map out how the fit to data deteriorates as the observable $\mathcal{O}_j$ is forced to change
   - ↑ no need to rely on any assumption on the χ^2 dependence on $\{\mathbf{a}\}$
   - ↓ no compact representation and computation of FFs/PDFs and their uncertainties

---

1. Generate (art) replicas of each (exp) data point (Monte Carlo sampling of data)

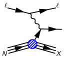
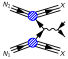
$$\mathcal{O}_i^{(art)(k)} = \mathcal{O}_i^{(exp)} + r_i^{(k)}\sigma_{\mathcal{O}_i} \qquad \text{for each } i = 1,\dots,N_{\rm dat},\ k = 1,\dots,N_{\rm rep}$$

2. Perform a fit to each replica $k = 1,\dots,N_{\rm rep}$
   - ↑ no need to rely on linear approximation, test for non-Gaussian uncertainties
   - ↑ compact representation and computation of FFs/PDFs and their uncertainties
   - ↓ computational expensive: need to perform k fits instead of one

$$\langle \mathcal{O}[f(x,Q^2)]\rangle = \frac{1}{N_{\rm rep}}\sum_{k=1}^{N_{\rm rep}} \mathcal{O}[f^{(k)}(x,Q^2)]$$

$$\sigma_{\mathcal{O}}[f(x,Q^2)] = \left[\frac{1}{N_{\rm rep}-1}\sum_{k=1}^{N_{\rm rep}}\left(\mathcal{O}[f^{(k)}(x,Q^2)] - \langle \mathcal{O}[f(x,Q^2)]\rangle\right)^2\right]^{1/2}$$

Data: PDFs (unpolarized)

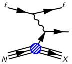
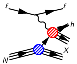
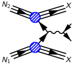
Process	Reaction	Subprocess	PDFs probed	x
	$\ell^{\pm}\{p,n\} \to \ell^{\pm} + X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
	$\ell^{\pm} n/p \to \ell^{\pm} + X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
	$\nu(\bar{\nu})N \to \mu^-(\mu^+) + X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
	$\nu N \to \mu^-\mu^+ + X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \to \mu^+\mu^- + X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
	$e^{\pm} p \to e^{\pm} + X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
	$e^{+} p \to \bar{\nu} + X$	$W^+\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
	$e^{\pm} p \to e^{\pm} c\bar{c} + X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.1$
	$e^{\pm} p \to jet(s) + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
	$pp \to \mu^+\mu^- + X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
	$pn/pp \to \mu^+\mu^- + X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
	$p\bar{p}(pp) \to jet(s) + X$	$gg, qg, qq \to 2jets$	g, q	$0.005 \lesssim x \lesssim 0.5$
	$p\bar{p} \to (W^{\pm} \to \ell^{\pm}\nu) + X$	$ud \to W^+, \bar{u}\bar{d} \to W^-$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
	$pp \to (W^{\pm} \to \ell^{\pm}\nu) + X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d}, (g)$	$x \gtrsim 0.001$
	$p\bar{p}(pp) \to (Z \to \ell^+\ell^-) + X$	$uu, dd(u\bar{u}, d\bar{d}) \to Z$	$u, d(g)$	$x \gtrsim 0.001$
	$pp \to (W + c) + X$	$gs \to W^- c, g\bar{s} \to W^+\bar{c}$	$s, \bar{s}$	$x \sim 0.01$
	$pp \to t\bar{t} + X$	$gg \to t\bar{t}$	g	$x \sim 0.01$

CERN	SLAC	DESY	FERMILAB
NMC, BCDMS, ATLAS, CMS, LHCb	E142, E143, E154, E155	HERA, ZEUS, H1	NuTev, E605, E866, CDF, D0

kinematic cuts: $Q^2 \geq Q^2_{\rm cut}$ (pQCD) and $W^2 = m_p^2 + \frac{1-x}{x}Q^2 \geq W^2_{\rm cut}$ (no HT)

after kinematic cuts $N_{\rm dat} \sim \mathcal{O}(4000)$

Data: PDFs (polarized)

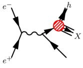
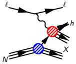
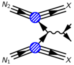
Process	Reaction	Subprocess	PDFs probed	x
 DIS	$\ell^\pm\{p,d,n\} \to \ell^\pm + X$	$\gamma^* q \to q$	$\Delta q + \Delta\bar{q}$ Δg	$0.003 \lesssim x \lesssim 0.8$
 SIDIS	$\ell^\pm\{p,d\} \to \ell^\pm h + X$	$\gamma^* q \to q$	$\Delta u\ \Delta\bar{u}$ $\Delta d\ \Delta\bar{d}$ Δg	$0.005 \lesssim x \lesssim 0.5$
	$\ell^\pm\{p,d\} \to \ell^\pm D + X$	$\gamma^* g \to c\bar{c}$	Δg	$0.06 \lesssim x \lesssim 0.2$
 pp	$\vec{p}\vec{p} \to jet(s) + X$	$gg \to qg$ $qg \to qg$	Δg	$0.05 \lesssim x \lesssim 0.2$
	$\vec{p}p \to W^\pm + X$	$u_L\bar{d}_R \to W^+$ $d_L\bar{u}_R \to W^-$	$\Delta u\ \Delta\bar{u}$ $\Delta d\ \Delta\bar{d}$	$0.05 \lesssim x \lesssim 0.4$
	$\vec{p}\vec{p} \to \pi + X$	$gg \to qg$ $qg \to qg$	Δg	$0.05 \lesssim x \lesssim 0.4$

CERN	SLAC	DESY	JLAB	BNL
EMC, SMC, COMPASS	E142, E143, E154, E155	HERMES	HALL-A, HALL-B, CLAS	PHENIX, STAR

kinematic cuts: $Q^2 \geq Q^2_{\rm cut}$ (pQCD) and $W^2 = m_p^2 + \frac{1-x}{x}Q^2 \geq W^2_{\rm cut}$ (no HT)

after kinematic cuts $N_{\rm dat} \sim \mathcal{O}(400)$

Data: FFs

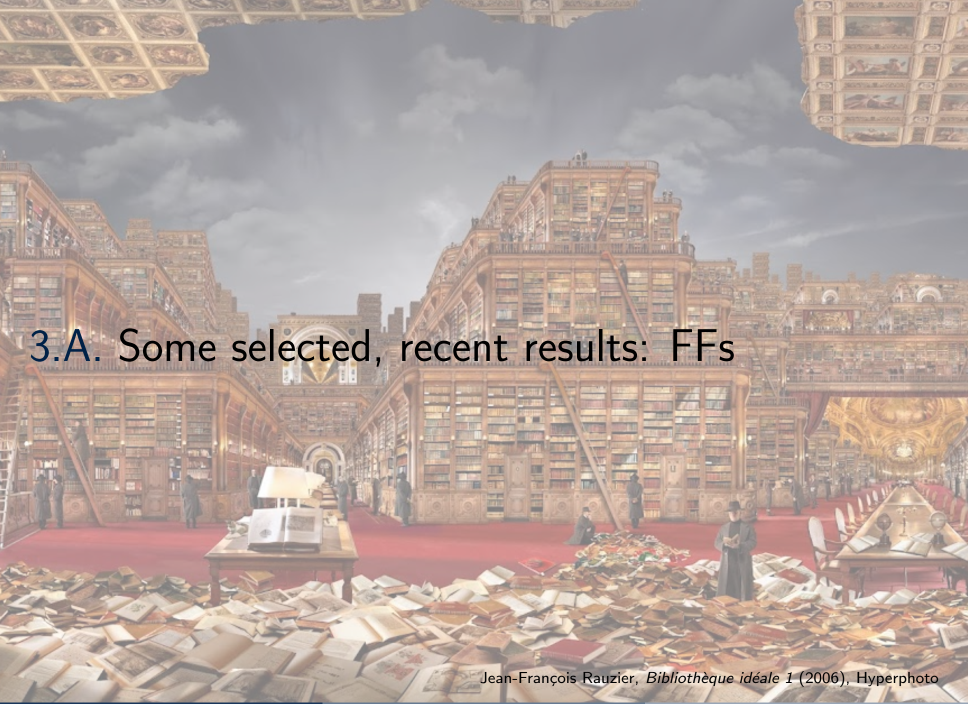
Process	Reaction	Subprocess	PDFs probed	x
 SIA	$e^+e^- \to h + X$	$\gamma/Z^0 \to q\bar{q}$	$D_q + D_{\bar{q}}$ D_g	$0.001 \lesssim z \lesssim 0.95$
 SIDIS	$\ell^\pm\{p,d\} \to \ell^\pm h + X$	$\gamma^* q \to q$	$D_u \; D_{\bar{u}}$ $D_d \; D_{\bar{d}}$	$0.005 \lesssim z \lesssim 0.8$
 pp	$pp \to \pi + X$	$gg \to qg$ $qg \to qg$	D_g	$0.05 \lesssim z \lesssim 0.4$

CERN	SLAC	DESY	KEK	BNL
ALEPH, OPAL, DELPHI, COMPASS, ALICE	TPC, HRS, SLD	TASSO, HERMES	TOPAZ, BELLE	PHENIX, STAR

kinematic cuts: $z \geq z_{\rm cut}$ (HM effects + singularities); $p_T \geq p_{T_{\rm cut}}$ (RHIC vs LHC)

after kinematic cuts $N_{\rm dat} \sim \mathcal{O}(200)$ (kaons) to $N_{\rm dat} \sim \mathcal{O}(900)$ (pions)

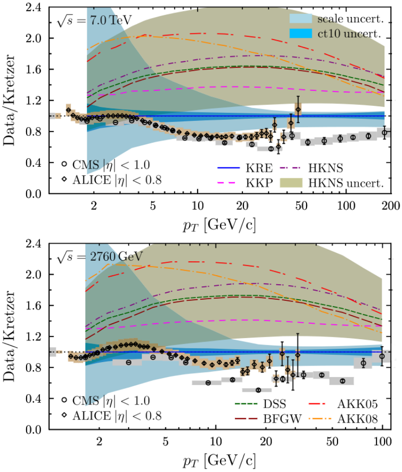
3.A. Some selected, recent results: FFs



Jean-François Rauzier, *Bibliothèque idéale 1* (2006), Hyperphoto

Fragmentation functions: why should we bother?

Example 1: Ratio of the inclusive charged-hadron spectra measured by CMS and ALICE



Figures taken from [NPB 883 (2014) 615]

Example 2: The strange polarized parton distribution at $Q^2 = 2.5$ GeV^2 ($\Delta s = \Delta \bar{s}$)

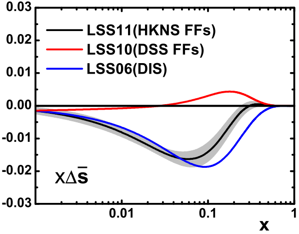


Figure taken from [PRD D84 (2011) 014002]

1 Predictions from all available FF sets are not compatible with CMS and ALICE data, not even within scale and PDF/FF uncertainties $\longrightarrow$ input for nuclear medium modifications

2 If SIDIS data are used to determine Δs, $K^{\pm}$ FFs for different sets lead to different results. Such results may differ significantly among them and w.r.t. the results obtained from DIS $\longrightarrow$ input for polarized PDFs and TMDs

Fragmentation Functions: available sets

Process	DSS	HKNS	KRE	AKK08
SIA	☑	☑	☑	☑
SIDIS	☑	☒	☒	☒
PP	☑	☒	☒	☑
statistical treatment	Lagr. mult. $\Delta\chi^2/\chi^2 = 2\%$	Hessian $\Delta\chi^2 = 15.94$	no uncertainty determination	no uncertainty determination
hadron species	$\pi^\pm$, $K^\pm$, $p/\bar{p}$, $h^\pm$	$\pi^\pm$, $K^\pm$, $p/\bar{p}$	$\pi^\pm$, $K^\pm$, $h^\pm$ $h^\pm$	$\pi^\pm$, $K^\pm$, $p/\bar{p}$, K^0_S, $\Lambda/\bar{\Lambda}$
latest update	PRD 91 (2015) 014035	arXiv:1608.04067	PR D62 (2000) 054001	NP B803 (2008) 42

$+$ some others (including anlyses for specific hadrons)

BKK95 [ZPB 65 (1995) 471]	$\pi^\pm, K^\pm$	AESS11 [PRD 83 (2011) 034002]	η
BKK96 [PRD 53 (1996) 3553]	K^0	SKMNA13 [PRD 88 (2013) 054019]	$\pi^\pm, K^\pm$
DSV97 [PRD 57 (1998) 5811]	Λ^0	LSS15 [PRD 96 (2016) 074026]	SIDIS only
BFGW00 [EPJ C19 (2001) 89]	$h^\pm$	NNPDF [in progress]	e^+e^- only
SSZ10 [PRD 81 (2010) 054001]	nFFs	JAM [arXiv:1609:00899]	e^+e^- only

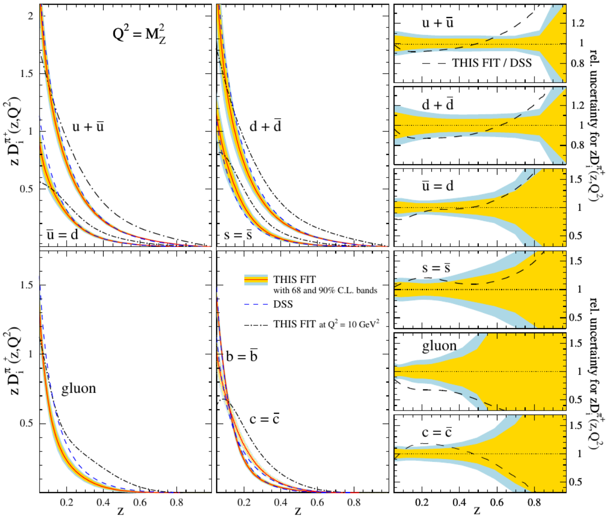
some of these determinations are publicly available at

`http://lapth.cnrs.fr/ffgenerator/`

Focus on π and K which constitute the largest fraction in measured yields (small room left for other hadrons)

Fragmentation Functions from a global determination

A global analysis of FFs from hadro-production data in SIA, SIDIS and pp collisions



π

DSS14 [PRD 91 (2015) 014035]

$\chi^2_{\text{tot}} = 1189.5$
$(N_{\text{dat}} = 973)$

$D_{u+\bar{u}} \sim D_{d+\bar{d}}$
(little c.s. breaking)

D_g largely uncertain

sizable D_c and D_b

K

DSS07 [PRD 75 (2007) 114010]

$\chi^2_{\text{tot}} = 394.1$
$(N_{\text{dat}} = 232)$

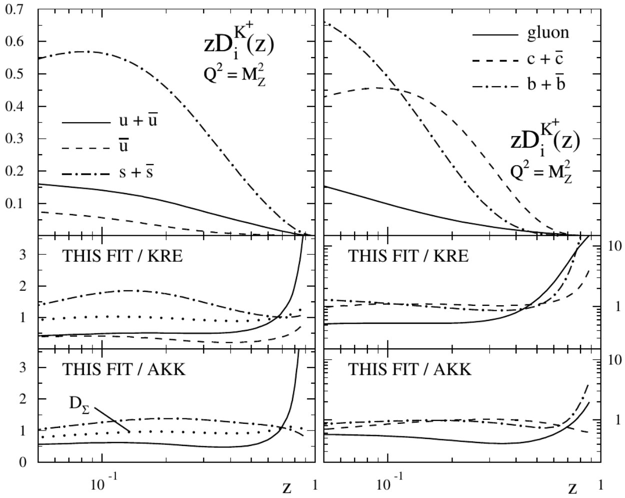
worse data description w.r.t the π fit

no uncertainty estimates

sizable D_c and D_b

Fragmentation Functions from a global determination

A global analysis of FFs from hadro-production data in SIA, SIDIS and pp collisions



π

DSS14 [PRD 91 (2015) 014035]

$\chi^2_{\rm tot} = 1189.5$
$(N_{\rm dat} = 973)$

$D_{u+\bar{u}} \sim D_{d+\bar{d}}$
(little c.s. breaking)

D_g largely uncertain

sizable D_c and D_b

K

DSS07 [PRD 75 (2007) 114010]

$\chi^2_{\rm tot} = 394.1$
$(N_{\rm dat} = 232)$

worse data description w.r.t the π fit

no uncertainty estimates

sizable D_c and D_b

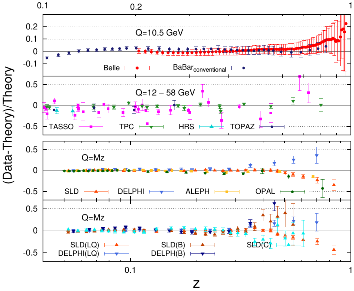
Impact of data: SIA (NLO, $\pi^{\pm}$ and $K^{\pm}$)

↑ clean process (only FFs involved)
↑ very precise data (mainly LEP and B-factories); tagged data for D_c and D_b
↓ only information on $D_q + D_{\bar{q}}$ (with partial separation of u and d due to electroweak couplings)

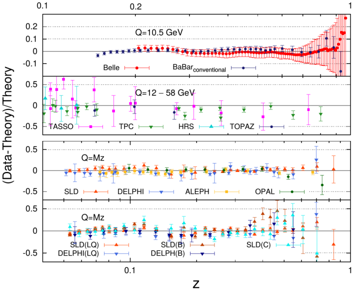
$$\frac{d\sigma^h}{dz} = \frac{4\pi\alpha_{\rm em}^2}{Q^2}\langle e^2\rangle \left\{ D_\Sigma^h \otimes \mathcal{C}_q^{\rm S} + n_f D_g^h \otimes \mathcal{C}_g^{\rm S} + D_{\rm NS}^h \otimes \mathcal{C}_q^{\rm NS} \right\}$$

$$\langle e^2\rangle = \frac{1}{n_f}\sum_{p=1}^{n_f}\hat{e}_p^2 \qquad D_\Sigma^h = \sum_{p=1}^{n_f}\left(D_p^h + D_{\bar{p}}^h\right) \qquad D_{\rm NS}^h = \sum_{p=1}^{n_f}\left(\frac{\hat{e}_p^2}{\langle e^2\rangle} - 1\right)\left(D_p^h + D_{\bar{p}}^h\right)$$

↓ D_g determined only via scaling violations



pions

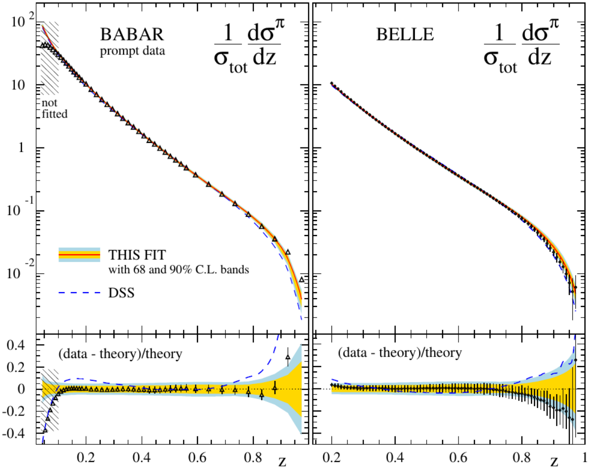


kaons

overall good descriptions of all data sets (for illustrative purposes, plots from [arXiv:1608.04067])

Impact of data: SIA (NLO, $\pi^{\pm}$ BELLE and BABAR)

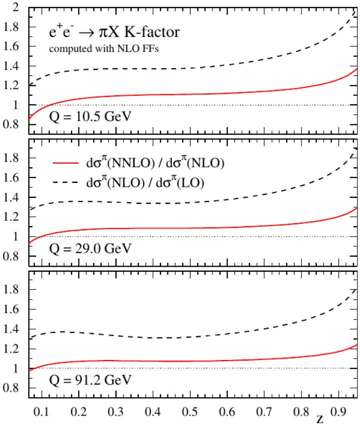
↑ substantial improvement in the description of the data after their inclusion in the fit
↑ sensible constraint on D_g through evolution up to LEP scale
↑ significant reduction of the uncertainties
↓ be careful when comparing BABAR and BELLE data (*e.g.* different normalizations)
BABAR [PR D88 (2013) 032011] BELLE [PRL 111 (2013) 062002] + re-analysis of BELLE [PRD 92 (2015) 092007]



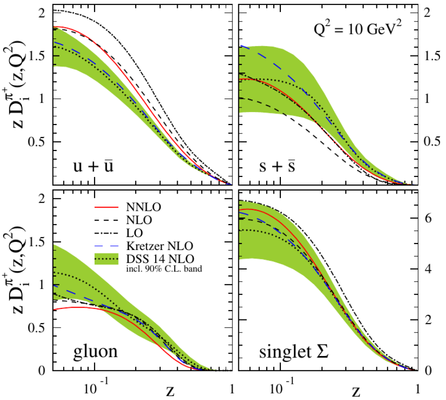
More on SIA: NNLO, $\pi^{\pm}$ [PRD 92 (2015) 114017]

$$K \equiv \frac{d\sigma^{\pi}(N^m LO)}{d\sigma^{\pi}(N^{m-1}LO)}$$

$$\chi^2 = \sum_{i=1}^{N_{\rm set}} \left[\left(\frac{1-\mathcal{N}_i}{\delta \mathcal{N}_i} \right)^2 + \sum_{j=1}^{N_{\rm dat}} \frac{(\mathcal{N}_i T_j - E_j)^2}{\delta E_j^2} \right]$$



40% correction from LO to NLO
10% correction from NLO to NNLO



Improvement of the χ^2 (BABAR)

$\chi^2_{\rm LO} = 241$ $\quad \chi^2_{\rm NLO} = 190$ $\quad \chi^2_{\rm NNLO} = 175$

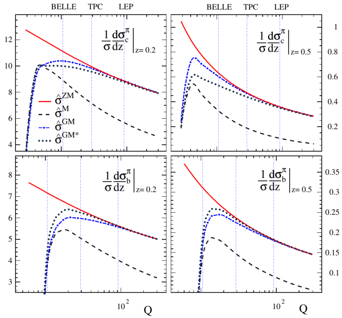
$(N_{\rm dat} = 288)$

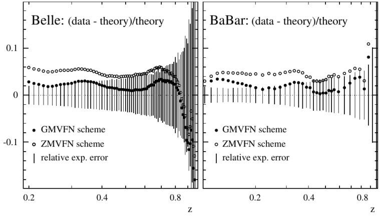
More on SIA: NLO, $\pi^\pm$, heavy quark masses [PRD 94 (2016) 034037]

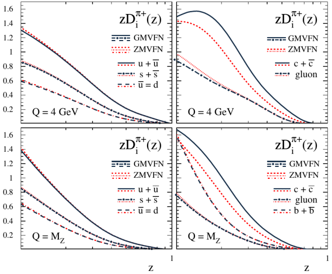
$$\frac{d\sigma^{\rm ZM}}{dz} = \sum_{i=q,g,h} \hat{\sigma}_i^{\rm ZM}(Q) \otimes D_i^{\rm ZM}(Q)$$

$$\frac{d\sigma^{\rm M}}{dz} = \sum_{i=q,g} \hat{\sigma}^{\rm M}(Q,m_h) \otimes D_i^{\rm M}(Q) + \hat{\sigma}^{\rm M}(Q,m_h) \otimes D_h^{\rm M}$$

$$\frac{d\sigma^{\rm GM}}{dz} = \sum_{i=q,g,h} \hat{\sigma}^{\rm GM}(Q,m_h) \otimes D_i^{\rm GM}(Q)$$







$\chi^2_{\rm ZMVFN} = 966 \quad \chi^2_{\rm GMVFN} = 876 \quad (N_{\rm dat} = 924)$

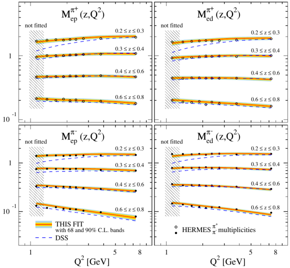
Impact of data: SIDIS (NLO, $\pi^\pm$, $K^\pm$)

↑ separate information on D_q and $D_{\bar{q}}$
↑ new accurate data on $\pi^\pm$ and $K^\pm$ multiplicities (HERMES and COMPASS)
↓ additional convolution with PDFs

$$\frac{d\sigma^h}{dxdydz} = \frac{2\pi\alpha_{em}^2}{Q^2}\left[\frac{1+(1-y)^2}{y}2F_1^h + \frac{2(1-y)}{y}F_L^h\right]$$

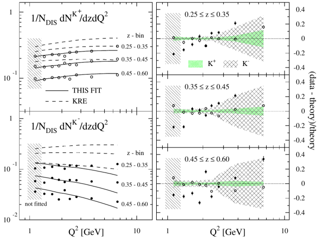
$$2F_1^h = \sum_{q,\bar{q}} e_q^2 \left\{ q \otimes D_q^h + \frac{\alpha_s}{2\pi}\left[q\otimes C_{qq}^1 \otimes D_q^h + q \otimes C_{gq}^1 \otimes D_g^h + g \otimes C_{qg}^1 \otimes D_q^h\right]\right\}$$

$$F_L^h = \frac{\alpha_s}{2\pi}\sum_{q,\bar{q}} e_q^2\left[q\otimes C_{qq}^L \otimes D_q^h + q\otimes C_{gq}^L \otimes D_g^h + g \otimes C_{qg}^L \otimes D_q^h\right]$$



pions

HERMES [PRD 87 (2013) 074029]



kaons

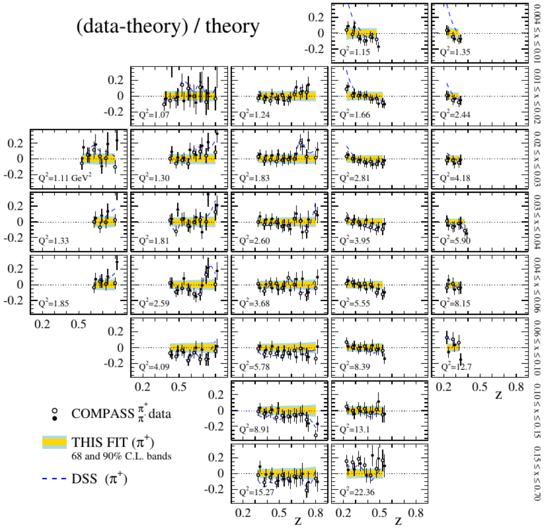
$$\chi^2_{\pi^+p} = 25.8;\ \chi^2_{\pi^-p} = 52.4;\ \chi^2_{\pi^+d} = 44.7;\ \chi^2_{\pi^-d} = 58.2;\ (N_{\rm dat} = 32)$$
$$\chi^2_{K^+} = 23.9;\ \chi^2_{K^-} = 131.2;\ (N_{\rm dat} = 24)$$

More on SIDIS: new COMPASS multiplicities [Details in N. Makke's talk]

Need of a combined analysis of HERMES and COMPASS data (kinematic complementarity)

↑ new COMPASS multiplicities for π^+, π^- [arXiv:1604:02695] and K^+, K^- [arXiv:1608.06760]

↑ so far, good agreement between HERMES and COMPASS (preliminary) π multiplicities



$\chi^2_{\pi^+d} = 175.8$

$\chi^2_{\pi^-d} = 221.1$

$(N_{\rm dat} = 199)$

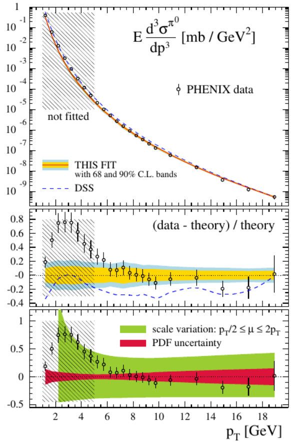
Impact of data: π production in pp collisions

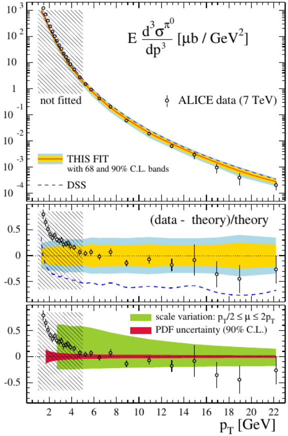
$\uparrow$ direct sensitivity to D_g

$\uparrow$ scale (p_T scan + universality check)

$\downarrow$ uncertainties from PDFs (up to 10%) and from HO QCD corrections ($> 50\%$for $p_T < 5$ GeV)

$$E_h \frac{d^3\sigma}{dp_h^3} = \sum_{a,b,c} f_a \otimes f_b \otimes \hat{\sigma}_{ab}^c \otimes D_c^h$$

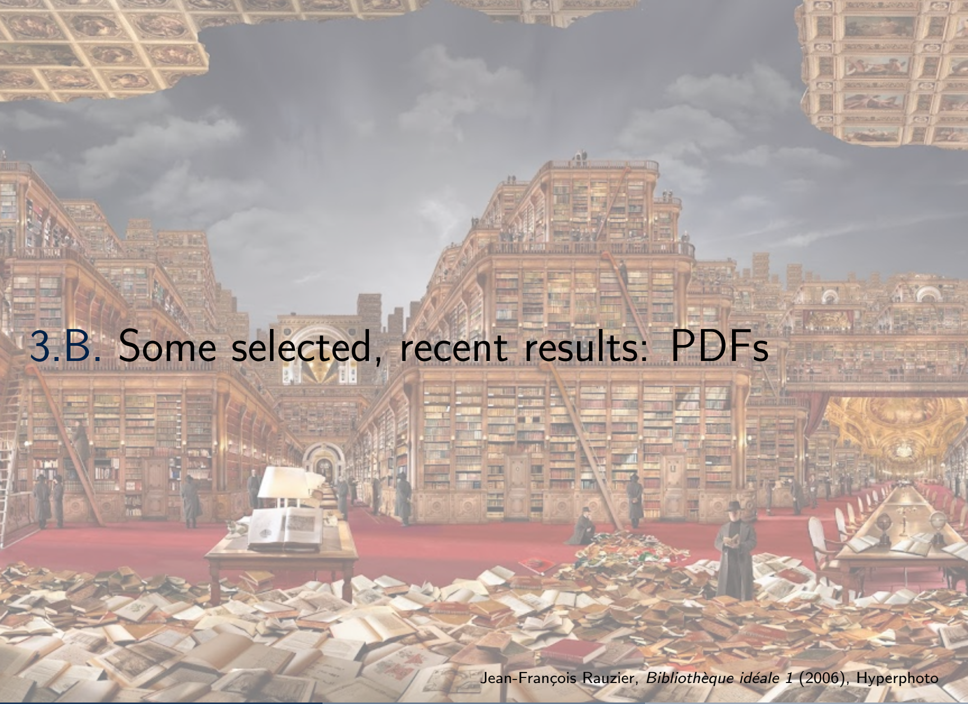




PHENIX ($\sqrt{s} = 200$ GeV)
[PRD 76 (2007) 051106]
$\chi^2 = 13.9$ ($N_{\rm dat} = 15$)
$\longrightarrow$ harder D_g

ALICE ($\sqrt{s} = 7$ TeV)
[PLB 717 (2012) 162]
$\chi^2 = 32.1$ ($N_{\rm dat} = 11$)
$\longrightarrow$ softer D_g

cut $p_T \geq 5$ GeV in order to reconcile RHIC and LHC data
(different treatment of pion decays in the two experiments?)



3.B. Some selected, recent results: PDFs

Jean-François Rauzier, *Bibliothèque idéale 1* (2006), Hyperphoto

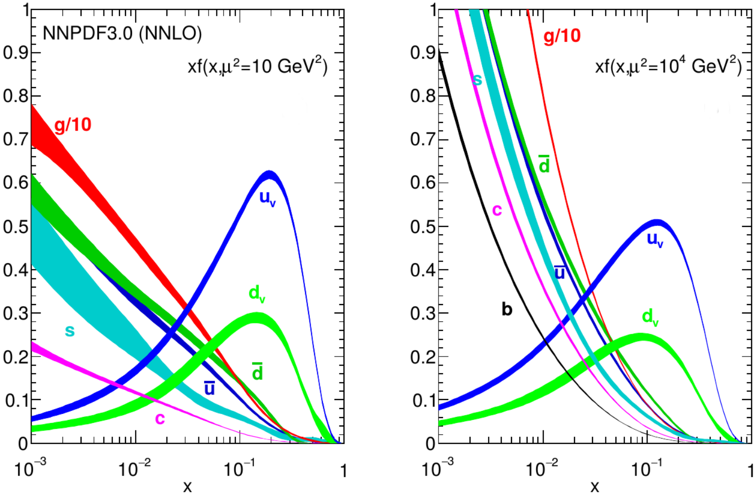
Parton Distribution Functions: available sets

	CT14	MMHT14	NNPDF3.0	ABM12	HERAPDF1.5
fixed-target DIS	✓	✓	✓	✓	✗
HERA	✓	✓	✓	✓	✓
fixed-target DY	✓	✓	✓	✓	✗
Tevatron (W, Z)	✓	✓	✓	✗	✗
Tevatron (jets)	✓	✓	✓	✗	✗
LHC	✓	✓	✓	✓ (W, Z)	✗
statistical treatment	Hessian $\Delta\chi^2 = 100$	Hessian $\Delta\chi^2$ dynamical	Monte Carlo	Hessian $\Delta\chi^2 = 1$	Hessian $\Delta\chi^2 = 100$
parametrization	Bernstein pol. (26 pars)	Chebyschev pol. (20 pars)	neural network (259 pars)	polynomial (14 pars)	polynomial (14 pars)
HQ scheme	ACOT-χ	TR$'$	FONLL	FFN	TR$'$
α_s	varied	fitted+varied	varied	fitted	varied
latest update	PRD 89 (2014) 033009	EPJC C75 (2015) 204	JHEP 1504 (2015) 040	PRD 89 (2014) 054028	PoS EPS-HEP2011 (2011) 320

Focus on precision physics and searches at the Large Hadron Collider

1. Higgs boson characterization (PDFs are becoming the dominant source of uncertainty)
2. Consistency stress tests of SM (PDFs dominant source of systematics, *e.g.* W mass)
3. Search for BSM physics (accurate PDFs to discriminate between various models)

PDFs from a global determination: NNPDF3.0 [JHEP 1504 (2015) 040]



A lot of extra sophistication in dedicated analyses recently:
QED correction and a determination of the photon PDF [NPB 877 (2013) 290];
PDFs with threshold resummation [JHEP 1509 (2015) 191];
intrinsic charm PDF of the proton [arXiv:1605.06515]; . . .

Helicity-dependent PDFs: available sets

	DSSV	NNPDF	JAM	LSS	BB
DIS	☑	☑	☑	☑	☑
SIDIS	☑	☒	☒	☑	☒
pp	☑ (jets, π^0)	☑ (jets, $W^\pm$)	☒	☒	☒
statistical treatment	Lagr. mult. $\Delta\chi^2/\chi^2 = 2\%$	Monte Carlo	Monte Carlo	Hessian $\Delta\chi^2 = 1$	Hessian $\Delta\chi^2 = 1$
parametrization	polynomial (23 pars)	neural network (259 pars)	polynomial (10 pars)	polynomial (20 pars)	polynomial (15 pars)
features	global fit	minimally biased fit	large-x effects	higher-twist effects	simultaneous fit of α_s
latest update	PRL 113 (2014) 012001	NP B887 (2014) 276	PR D93 (2016) 074005	PR D82 (2010) 114018	NP B841 (2010) 205

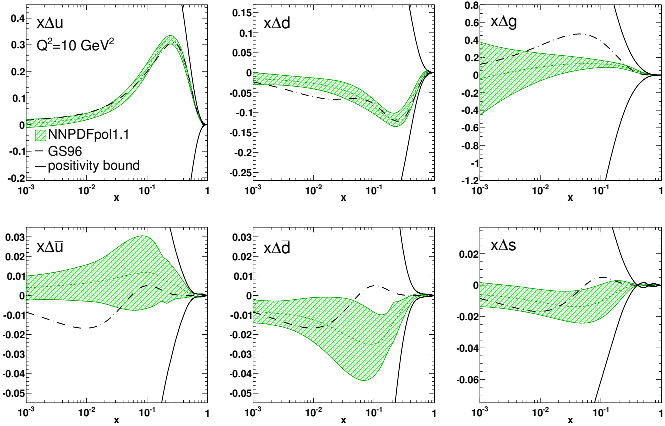
$$\text{DIS}: \quad g_1 = \frac{\sum_q^{n_f} e_q^2}{2n_f} \left(\mathcal{C}_{\rm NS} \otimes \Delta q_{\rm NS} + \mathcal{C}_{\rm S} \otimes \Delta\Sigma + 2n_f \mathcal{C}_g \otimes \Delta g \right)$$

$$\text{SIDIS}: \quad g_1^h = \sum_{q,\bar{q}} e_q^2 \left[\Delta q \otimes C_{qq}^{1,h} \otimes D_q^h + \Delta q \otimes C_{gq}^{1,h} \otimes D_g^h + \Delta g \otimes C_{qg}^{1,h} \otimes D_q^h \right]$$

$$pp: \quad \Delta\sigma = \sigma^{(+)+} - \sigma^{(+)-} = \sum_{a,b,(c)} \Delta f_a \otimes (\Delta) f_b (\otimes D_c^h) \otimes \Delta\hat{\sigma}_{ab}^{(c)}$$

Helicity PDFs from a global determination

A global analysis of helicity PDFs from DIS, SIDIS and pp collision data from (GS96) 1996 [PRD 53 (1996) 6100] to 2014 (NNPDFpol1.1) [NPB 887 (2014) 276]



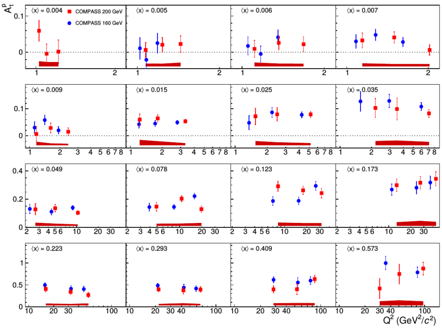
↑ first hint of a symmetry breaking in the light sea quark sector

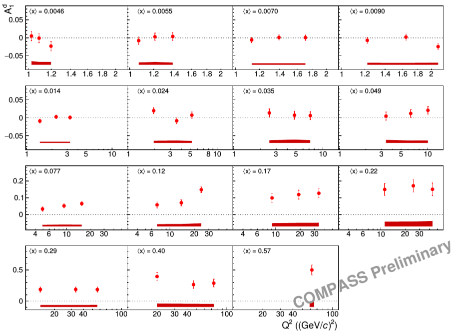
↑ first hint of a sizable, positive polarization of the gluon in the proton

↓ large uncertainties in the small-x extrapolation region (lack of data)

New inclusive DIS data: COMPASS and JLAB

Final COMPASS measurements of single spin asymmetries
A_1^p [PL B753 (2016) 18] and A_1^d [M. Wilfert's talk at DIS2016]
including 2011 (p, $E_\mu = 200$ GeV) and 2006 (d, $E_\mu = 160$ GeV) data





↑ increased statistics w.r.t previous COMPASS 2007 (p) and 2002-2004 (d)

↑ reduced uncertainties (up to a factor 2) ↑ small values of x reached (down to $x \sim 0.004$)

New JLAB measurements of single spin asymmetries A_1^p and A_1^d

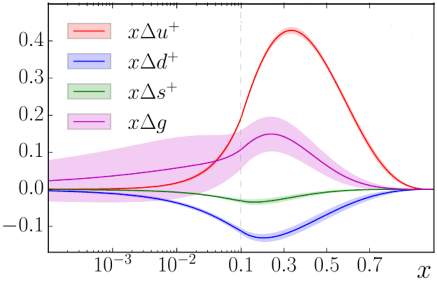
[PRC 90 (2014) 025212; PLB 744 (2015) 309; PRC 92 (2015) 055201]

↑ very accurate measurements ↓ limited kinematic coverage (small Q, large x)

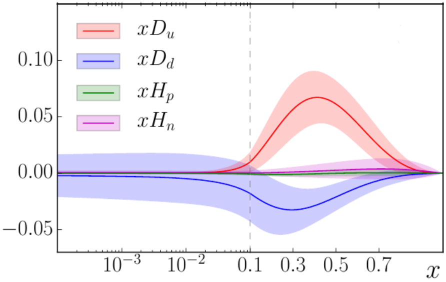
More on DIS: nuclear corrections and higher twists

$$g_1(x,Q^2) = \underbrace{\frac{\sum_q^{n_f} e_q^2}{2n_f}\left(\mathcal{C}_{\rm NS}\otimes\Delta q_{\rm NS} + \mathcal{C}_{\rm S}\otimes\Delta\Sigma + 2n_f\mathcal{C}_g\otimes\Delta g\right)}_{\text{leading-twist factorization}} + \underbrace{\frac{h^{\rm TMC}}{Q^2} + \frac{h^{\rm HT}}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)}_{\text{power-suppressed TMCs and HT}}$$

Helicity PDFs (DIS only) including TMCs and HT (up to $\tau = 4$) [PRD 93 (2016) 074005]



LT PDFs (contributing to $g_1^{\rm LT}$)



$g_1^{\tau=3} \propto D$ and $g_1^{\tau=4} = H/Q^2$

↑ remarkably accuracy of all distributions (except the gluon)

↑ nonzero twist-3 quark distributions

↑ twist-4 quark distributions compatible with zero

Impact of data: W production in pp collisions

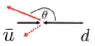
Observable

$$A_L = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

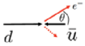
Features

- at RHIC,
  $\langle x_{1,2} \rangle \simeq \frac{M_W}{\sqrt{s}} e^{-\eta_l/2} \approx [0.04, 0.4]$
- A_L sensitive to Δq, $\Delta\bar{q}$ at $Q \sim M_W$
  (no need of fragmentation functions)

$$A_L^{W^-} \sim \frac{\Delta\bar{u}_{x_1} d_{x_2} (1-\cos\theta)^2 - \Delta d_{x_1} \bar{u}_{x_2} (1+\cos\theta)^2}{\bar{u}_{x_1} d_{x_2} (1-\cos\theta)^2 - d_{x_1} \bar{u}_{x_2} (1+\cos\theta)^2}$$



backward lepton rapidity



forward lepton rapidity

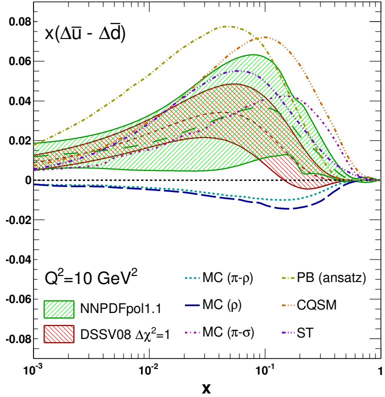
- for W^+, $d \longleftrightarrow u$ and $\Delta d \longleftrightarrow \Delta u$
- no access to strangeness ($W^\pm$ + c)

Measurements

- STAR [PRL 113 (2014) 072301]
- PHENIX [PRD 93 (2016) 051103]

Effects

First evidence of broken flavor symmetry for polarized light sea quarks



- $\Delta\bar{u} > 0 > \Delta\bar{d}$, $|\Delta\bar{d}| > |\Delta\bar{u}|$
- $|\Delta\bar{u} - \Delta\bar{d}| \sim |\bar{u} - \bar{d}|$
- some models are disfavored

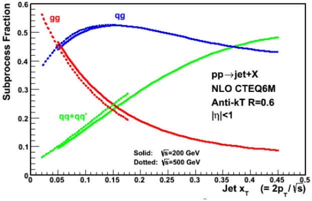
Impact of data: jet and π production

Observable

$$A_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$

Features

- at RHIC, $\langle x_{1,2} \rangle \simeq \frac{2p_T}{\sqrt{s}} e^{-\eta/2} \approx [0.05, 0.2]$
- qg,gg initiated subprocesses dominate (for most of the RHIC kinematics)
- A_{LL} sensitive to Δg

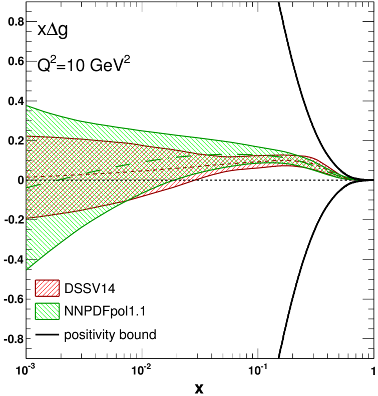


Measurements

- STAR (jets) [PRL 115 (2015) 092002]
- PHENIX (π) [PRD 90 (2014) 012007]

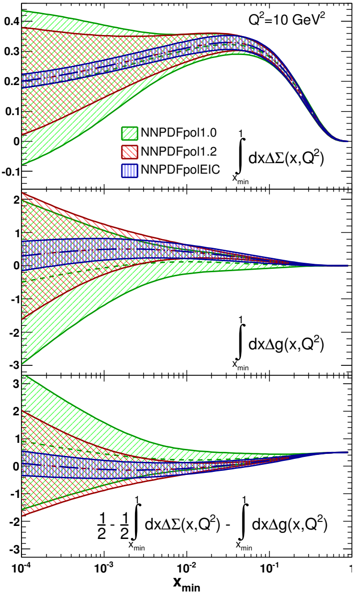
Effects

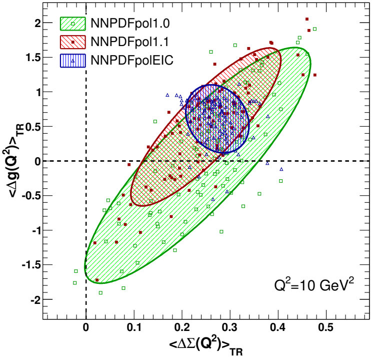
First evidence of a sizable, positive gluon polarization in the proton



$Q^2 = 10$ GeV^2	$\int_{0.05}^{0.2} dx \Delta g(x, Q^2)$
NNPDFpol1.1	$+0.15 \pm 0.06$
DSSV14	$0.10^{+0.06}_{-0.07}$

The spin content of the proton

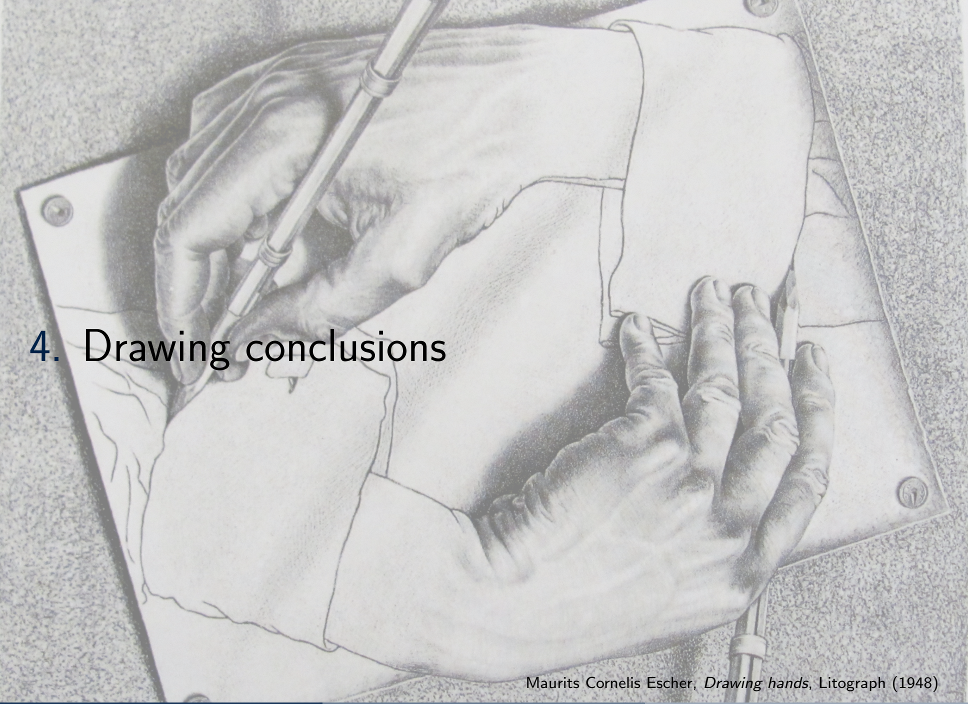




$Q^2 = 10$ GeV^2	$\int_{10^{-3}}^{1} dx\, \Delta\Sigma$	$\int_{10^{-3}}^{1} dx\, \Delta g$
NNPDFpol1.0	$+0.23 \pm 0.15$	-0.06 ± 1.12
NNPDFpol1.1	$+0.25 \pm 0.10$	$+0.49 \pm 0.75$
NNPDFpolEIC	$+0.24 \pm 0.04$	$+0.49 \pm 0.25$

quarks and antiquarks $\sim 20\% - 30\%$
gluons $\sim 50\%$ [PLB 728 (2014) 524] - $\sim 70\%$ [PRD 92 (2015) 094030]

4. Drawing conclusions



Maurits Cornelis Escher, *Drawing hands*, Litograph (1948)

Summary and final remarks

Fragmentation and Parton Distribution Functions play a leading role
in our understanding of how hadrons emerge from quark and gluon dynamics
and how quark and gluon dynamics makes up nucleons

Global determinations of FFs and PDFs support the validity
of the underlying theoretical framework, based on perturbative QCD,
and, in particular, the notions of factorization and universality of collinear PDFs and FFs

There has been significant progress in the sophistication of the determination
of FFs and PDFs, which are being known with increasing accuracy

Theoretical efforts try to keep up interesting physics questions
specifically in the gluon and sea quark regime

Forthcoming experimental results will provide further input
for the improvement of our knowledge of FFs and PDFs

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Thank you