Unpolarized and Helicity Parton Distribution Functions at an Electron-Ion Collider

7th International Conference on Physics Opportunities at an Electron-Ion Collider

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Foreword

Dual role of Parton Distribution Functions

constitutive: they enhance our understanding of QCD as a fundamental cornerstone of the SM
operational: they are fundamental tools in the study of high energy scattering processes

Unpolarized PDFs
Precision physics at the LHC
Higgs boson characterization,
SM parameters, BSM signatures

Polarized PDFs
Longitudinal spin structure of the nucleon
how the proton spin is built from
the spin and dynamics of quarks and gluons

Outline

1. Parton distributions in the QCD framework: theoretical and phenomenological status
2. Unpolarized Parton Distributions: open issues addressed at an EIC
3. Polarized Parton Distributions open issues addressed at an EIC
4. Conclusions: summary of PDF-related measurements at an EIC

This talk is mostly based on
the EIC White Paper [EPJA 52 (2016) 268] and on the joint BNL/INT/JLab report [arXiv:11081713]
(+ [PRD 86 (2012) 054020; PRD 88 (2013) 114025; PLB 728 (2014) 524; PRD 92 (2015) 094030])

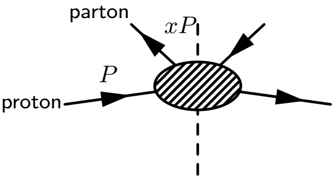
1. Parton Distributions in the QCD framework

Parton distributions on the light cone [Rev.Mod.Phys. 67 (1995) 157]

1. The densites of partons $f = q, \bar{q}, g$ with momentum fraction x

$$f(x) \equiv f^{\uparrow}(x) + f^{\downarrow}(x) \qquad\qquad \Delta f(x) \equiv f^{\uparrow}(x) - f^{\downarrow}(x)$$

2. Allow for a proper field-theoretic definition as matrix elements of bilocal operators



collinear transition of a massles proton h
into a massless parton i
with fractional momentum x
local OPE $\Longrightarrow$ lattice formulation
[see also the talks by K.-F. Liu, G. Koutsou and F. Steffens]

$$q(x) = \frac{1}{4\pi}\int dy^- e^{-iy^- xP^+} \langle P|\bar{\psi}(0, y^-, \mathbf{0}_\perp)\gamma^+\psi(0)|P\rangle$$

$$\Delta q(x) = \frac{1}{4\pi}\int dy^- e^{-iy^- xP^+} \langle P, S|\bar{\psi}(0, y^-, \mathbf{0}_\perp)\gamma^+\gamma^5\psi(0)|P, S\rangle$$

with light-cone coordinates

$$y = (y^+, y^-, \mathbf{y}_\perp)\,, \quad y^+ = (y^0 + y^z)/\sqrt{2}\,, \quad y^- = (y^0 - y^z)/\sqrt{2}\,, \quad \mathbf{y}_\perp = (v^x, v^y)$$

3. All these definitions have ultraviolet divergences which must be renormalized

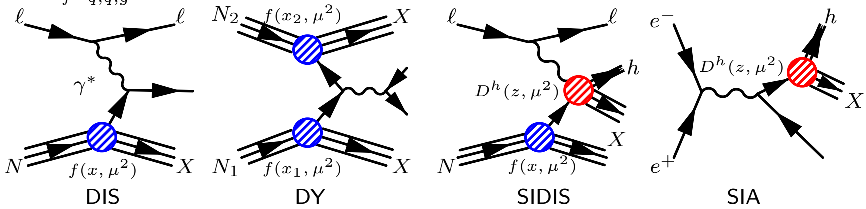
Factorization of physical observables [Adv.Ser.Direct.HEP 5 (1988) 1]

1. A variety of sufficiently inclusive processes allow for a factorized description

short-distance part		long-distance part
hard interaction of partons	$\xleftrightarrow[\text{scheme \& scale } \mu]{\text{factorization}}$	nucleon structure
process-dependent kernels		universal parton distributions

2. Physical observables are written as a convolution of coefficient functions and PDFs

$$\mathcal{O}_I = \sum_{f=q,\bar{q},g} C_{If}(y,\alpha_s(\mu^2)) \otimes f(y,\mu^2) + \text{p.s. corrections} \qquad f \otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$



3. Coefficient functions allow for a perturbative expansion

$$C_{If}(y,\alpha_s) = \sum_{k=0} a_s^k C_{If}^{(k)}(y), \qquad a_s = \alpha_s/(4\pi)$$

4. After factorization, all quantities (including PDFs) depend on μ

Evolution of PDFs: DGLAP equations [NPB 126 (1977) 298]

1. A set of $(2n_f+1)$ integro-differential equations, n_f is the number of active flavors

$$\frac{\partial}{\partial \ln \mu^2} f_i(x,\mu^2) = \sum_j^{n_f} \int_x^1 \frac{dz}{z} P_{ji}\left(z,\alpha_s(\mu^2)\right) f_j\left(\frac{x}{z},\mu^2\right)$$

2. Often written in a convenient basis of PDFs

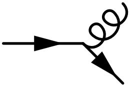
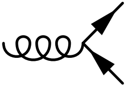
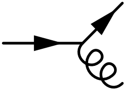
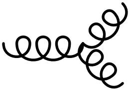
$$q_{\mathrm{NS};\pm} = (q_i \pm \bar{q}_i) - (q_j \pm \bar{q}_j) \qquad q_{\mathrm{NS;v}} = \sum_i^{n_f}(q_i - \bar{q}_j) \qquad \Sigma = \sum_i^{n_f}(q_i + \bar{q}_j)$$

$$\frac{\partial}{\partial \ln \mu^2} q_{\mathrm{NS};\pm,v}(x,\mu^2) = P^{\pm,\mathrm{v}}(x,\mu_F^2) \otimes q_{\mathrm{NS};\pm,v}(x,\mu^2)$$

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} \Sigma(x,\mu^2) \\ g(x,\mu^2) \end{pmatrix} = \begin{pmatrix} P^{qq} & P^{gq} \\ P^{qg} & P^{gg} \end{pmatrix} \otimes \begin{pmatrix} \Sigma(x,\mu^2) \\ g(x,\mu^2) \end{pmatrix}$$

3. With perturbative computable splitting functions

$$P_{ji}(z,\alpha_s) = \sum_{k=0} a_s^{k+1} P_{ji}^{(k)}(z), \qquad a_s = \alpha_s/(4\pi)$$

$P_{qq}^{(0)}$  $P_{gq}^{(0)}$  $P_{qg}^{(0)}$  $P_{gg}^{(0)}$ 

A global determination of parton distribution functions

A mathematically ill-posed problem: determine a set of functions from a finite set of data

METHODOLOGY

1. Parametrization: general, smooth, flexible at an initial scale Q_0^2

$$xf_i(x,Q_0^2) = A_{f_i}\, x^{a_{f_i}}\,(1-x)^{b_{f_i}}\,\mathscr{F}(x,\{c_{f_i}\})$$

small x		large x
$xf_i(x,Q^2) \xrightarrow{x\to 0} x^{a_{f_i}}$	$\mathscr{F}(x,\{c_{f_i}\}) \underset{x\to 1}{\xrightarrow{x\to 0}}$ finite $\xrightarrow[\text{smooth interpolation in between}]{}$	$xf_i(x,Q^2) \xrightarrow{x\to 1} (1-x)^{b_{f_i}}$
(Regge theory)	(polynomials, neural networks)	(quark counting rules)

2. A prescription to determine/compute expectation values and uncertainties

$$E[\mathcal{O}] = \int \mathcal{D}\Delta f \mathcal{P}(\Delta f|data)\mathcal{O}(\Delta f) \qquad V[\mathcal{O}] = \int \mathcal{D}\Delta f \mathcal{P}(\Delta f|data)[\mathcal{O}(\Delta f) - E[\mathcal{O}]]^2$$

Monte Carlo: $\mathcal{P}(\Delta f|data) \longrightarrow \{\Delta f_k\}$

$$E[\mathcal{O}] \approx \frac{1}{N}\sum_k \mathcal{O}(\Delta f_k)$$

$$V[\mathcal{O}] \approx \frac{1}{N}\sum_k [\mathcal{O}(\Delta f_k) - E[\mathcal{O}]]^2$$

Maximum likelihood: $\mathcal{P}(\Delta f|data) \longrightarrow \Delta f_0$

$$E[\mathcal{O}] \approx \mathcal{O}(\Delta f_0)$$

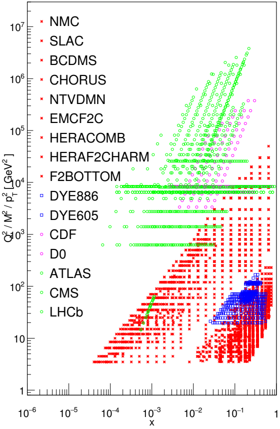
$$V[\mathcal{O}] \approx \text{Hessian}, \Delta\chi^2 \text{envelope}, \ldots$$

COMBINED WITH THEORY AND DATA TO FIND BEST-FIT PDFs

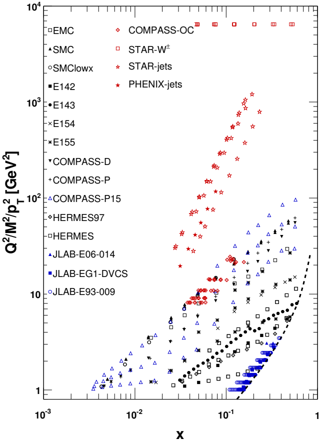
[More on theory in G. Salam's (LHC) and J. Qiu's (EIC) talks]

Data: kinematic coverage

Unpolarized PDFs



Polarized PDFs

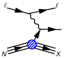
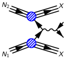
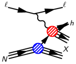


$\mathcal{O}(4000)$ data points after cuts
$Q^2_{\rm cut} = 1$ GeV^2 $W^2_{\rm cut} = 3 - 12.5$ GeV^2

$\mathcal{O}(400)$ data points after cuts
$Q^2_{\rm cut} = 1$ GeV^2 $W^2_{\rm cut} = 4 - 6.25$ GeV^2

kinematic cuts: $Q^2 \geq Q^2_{\rm cut}$ (pQCD) and $W^2 = m_p^2 + \frac{1-x}{x}Q^2 \geq W^2_{\rm cut}$ (no HT)

Data: relevant processes

Process	Reaction	Subprocess	PDFs probed	x
 DIS	$\ell^{\pm}\{p,n\} \to \ell^{\pm} + X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
	$\ell^{\pm} n/p \to \ell^{\pm} + X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
	$\nu(\bar{\nu})N \to \mu^-(\mu^+) + X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
	$\nu N \to \mu^- \mu^+ + X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \to \mu^+ \mu^- + X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
	$e^{\pm} p \to e^{\pm} + X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
	$e^+ p \to \bar{\nu} + X$	$W^+\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
	$e^{\pm} p \to e^{\pm} c\bar{c} + X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.1$
	$e^{\pm} p \to jet(s) + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
	$\overrightarrow{\ell}^{\pm}\{\overrightarrow{p}, \overrightarrow{d}, \overrightarrow{n}\} \to \ell^{\pm} + X$	$\gamma^* q \to q$	$\Delta q + \Delta\bar{q}$, Δg	$0.003 \lesssim x \lesssim 0.8$
 pp	$pp \to \mu^+\mu^- + X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
	$pn/pp \to \mu^+\mu^- + X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
	$p\bar{p}(pp) \to jet(s) + X$	$gg, qg, qq \to 2jets$	g, q	$0.005 \lesssim x \lesssim 0.5$
	$p\bar{p} \to (W^{\pm} \to \ell^{\pm}\nu) + X$	$ud \to W^+$, $\bar{u}\bar{d} \to W^-$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
	$pp \to (W^{\pm} \to \ell^{\pm}\nu) + X$	$u\bar{d} \to W^+$, $d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d}, (g)$	$x \gtrsim 0.001$
	$p\bar{p}(pp) \to (Z \to \ell^+\ell^-) + X$	$uu, dd(u\bar{u}, d\bar{d}) \to Z$	$u, d(g)$	$x \gtrsim 0.001$
	$pp \to (W + c) + X$	$gs \to W^- c$, $g\bar{s} \to W^+ \bar{c}$	$s, \bar{s}$	$x \sim 0.01$
	$pp \to t\bar{t} + X$	$gg \to t\bar{t}$	g	$x \sim 0.01$
	$\overrightarrow{p} p \to W^{\pm} + X$	$u_L \bar{d}_R \to W^+$, $d_L \bar{u}_R \to W^-$	$\Delta u\ \Delta\bar{u}\ \Delta d\ \Delta\bar{d}$	$0.05 \lesssim x \lesssim 0.4$
	$\overrightarrow{p}\,\overrightarrow{p} \to \pi + X$	$gg \to qg$, $qg \to qg$	Δg	$0.05 \lesssim x \lesssim 0.4$
 SIDIS	$\overrightarrow{\ell}^{\pm}\{\overrightarrow{p}, \overrightarrow{d}\} \to \ell^{\pm} h + X$	$\gamma^* q \to q$	$\Delta u\ \Delta\bar{u}\ \Delta d\ \Delta\bar{d}$ Δg	$0.005 \lesssim x \lesssim 0.5$
	$\overrightarrow{\ell}^{\pm}\{\overrightarrow{p}, \overrightarrow{d}\} \to \ell^{\pm} D + X$	$\gamma^* g \to c\bar{c}$	Δg	$0.06 \lesssim x \lesssim 0.2$

Overview of recent PDF determinations

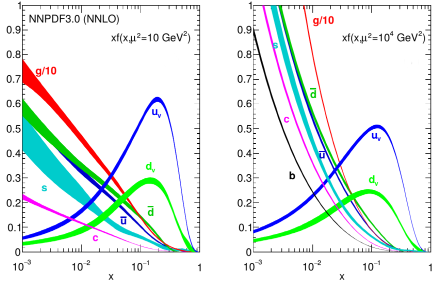
Unpolarized PDFs	CT14	MMHT14	NNPDF3.0	ABM16	HERAPDF2.0
fixed-target DIS	☑	☑	☑	☑	☒
HERA	☑	☑	☑	☑	☑
fixed-target DY	☑	☑	☑	☑	☒
Tevatron (W, Z)	☑	☑	☑	☑	☒
Tevatron (jets)	☑	☑	☑	☒	☒
LHC	☑	☑	☑	☑ (W, Z)	☒
statistical treatment	Hessian $\Delta\chi^2 = 100$	Hessian $\Delta\chi^2$ dynamical	Monte Carlo	Hessian $\Delta\chi^2 = 1$	Hessian $\Delta\chi^2 = 100$
parametrization	Bernstein pol.	Chebyschev pol.	neural network	polynomial	polynomial
latest update	PRD 89 (2014) 033009	EPJC C75 (2015) 204	JHEP 1504 (2015) 040	arXiv:1609.03327	EPJC 75 (2015) 580

Polarized PDFs	DSSV	NNPDF	JAM	LSS	BB
DIS	☑	☑	☑	☑	☑
SIDIS	☑	☒	☒	☑	☒
pp	☑ (jets,π^0)	☑ (jets,$W^\pm$)	☒	☒	☒
statistical treatment	Lagr. mult. $\Delta\chi^2/\chi^2 = 2\%$	Monte Carlo	Monte Carlo	Hessian $\Delta\chi^2 = 1$	Hessian $\Delta\chi^2 = 1$
parametrization	polynomial	neural network	polynomial	polynomial	polynomial
latest update	PRL 113 (2014) 012001	NP B887 (2014) 276	PR D93 (2016) 074005	PR D82 (2010) 114018	NP B841 (2010) 205

Unpolarized vs Polarized PDFs

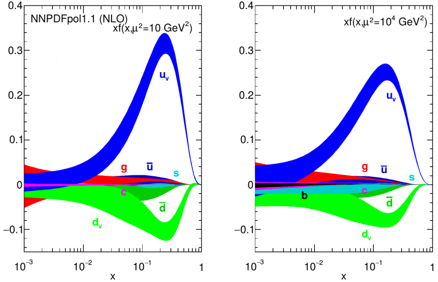
Unpolarized PDFs [PDG, 2016]

Largely benefit from HERA and LHC programs



Polarized PDFs [PDG, 2016]

Are starting to benefit from RHIC program



Aspects of PDFs which could be addressed at an EIC

- the gluon PDF at small and large x
- the light sea asymmetry at small x
- the strange-antistrange asymmetry
- heavy flavor contributions in DIS

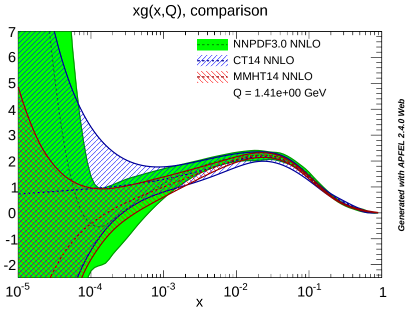
- the gluon PDF at small x
- the individual quark-antiquark PDFs
- the strange PDF and SU(3) breaking
- quarks and gluons in the spin sum rule

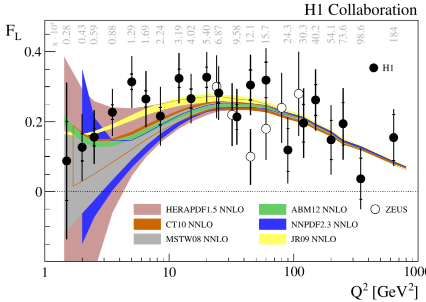
EIC key assets

large kinematic reach	precision of EM probes	spin	versatility
high-energy collider	electron beams	polarized hadron beams	ion beams

2. Unpolarized Parton Distributions

The gluon PDF at small x: the structure function F_L





[Figure taken from EPJC 74 (2014) 2814]

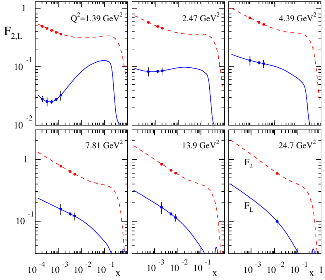
The gluon PDF is affected by large uncertainties below $x \sim 10^{-4}$
difficult to test for DGLAP vs BFKL and saturation dynamics and/or color dipole models

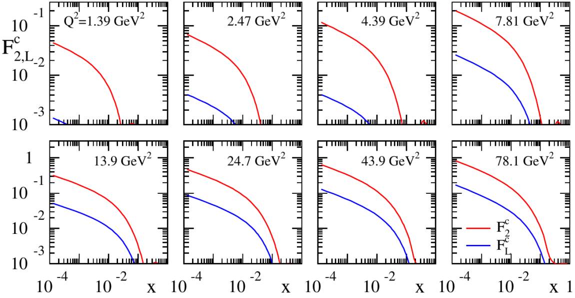
Golden measurement so far: HERA (combined) data
DIS structure functions and reduced cross sections

The leading contribution to $F_L(x, Q^2)$ is $\mathcal{O}(\alpha_s)$ and is dominated by $\gamma^* g$ fusion
F_L is particularly sensitive to the gluon PDF, corrections are known up to $\mathcal{O}(\alpha_s^3)$ [NPB 724 (2005) 3]

Mild tension between experimental data and theory predictions (small x and small Q^2)
achieved statistical precision of the combined H1 and ZEUS measurements rather limited

The gluon PDF at small x: F_L and F_L^c in DIS at an EIC





[Figures taken from arXiv:1108:1713]

An EIC could make precise measurements of F_L in a HERA-like kinematic region

advantage w.r.t. HERA: possibility to vary $\sqrt{s}$ in a wide range for high luminosity collisions

competitive measurements require carefully designed detector and data analyses

pseudodata: LEPYO+JETSET, ABKM09 PDFs at NNLO, $0.01 < y < 0.90$, $\sqrt{s} = 70.7$ GeV

Study the transition to the high parton density regime [A. Statso's talk] (with eA)

Good complementarity with LHC and LHeC measurements [more in A. Cooper-Sarkar's talk]

LHC: direct photon production (NLO only) + charm production in the forward region (c frag.)

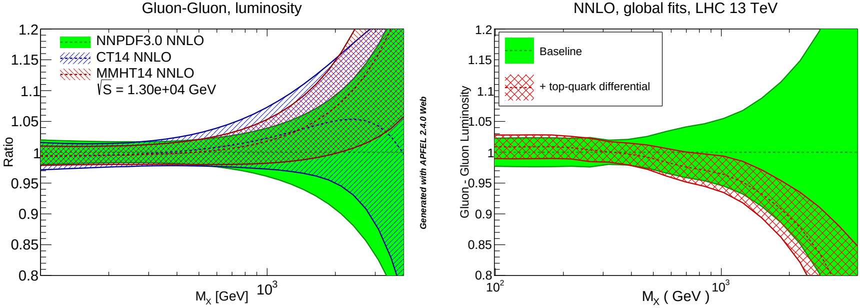
LHeC: precise measurement of F_L down to $x \sim 10^{-5}$, relative PDF uncertainty down to 5%

An EIC could make a first measurement of F_L^c (further handle in addition to F_2^c)

The gluon PDF at large x: LHC and J/ψ, Υ at an EIC

Large-x gluon relevant in several BSM scenarios

affected by large uncertainties at $x \gtrsim 0.4$, see also $\mathcal{L}_{gg}$ at high M_X



[Preliminary figure taken from Czakon, Hartland, Mitov, ERN and Rojo, in preparation]

Important LHC constraints from high-p_T jet production and top-pair production

substantial impact of $t\bar{t}$ differential distributions at $\sqrt{s} = 8$ TeV

An EIC could probe the large-x gluon via c and b pair production of J/ψ and Υ mesons

the EIC detectors will have excellent charm tagging efficiency

DIS is a relatively clean environment as compared to pp at the LHC

Complementary program at the LHeC [more in A. Cooper-Sarkar's]

extend the gluon probe up to $x \sim 0.7$, relative PDF uncertainty from $\sim 30\%$ to $\sim 10\%$

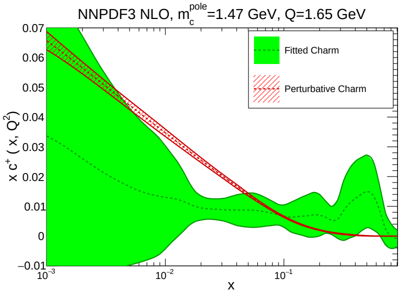
A determination of the charm content of the proton

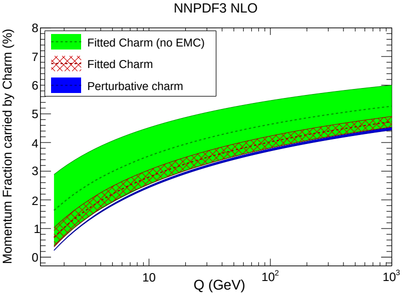
Parametrize the $c^+(x, Q_0^2)$, quark and gluon PDFs on the same footing

stabilize the dependence of LHC processes upon variations of m_c

quantify the nonperturbative charm component in the proton (BHPS? sea-like?)

take into account massive charm-initiated contribution to the DIS structure functions





[Figure taken from R.D. Ball et al., accepted for publication in EPJC]

Fitted charm found to differ from perturbative charm at scales $Q \sim m_c$

preference for a BHPS-like shape

significant improvement of the $\chi^2/N_{\rm dat}$ for the EMC data: 7.3 (PC) w.r.t. 1.09 (FC)

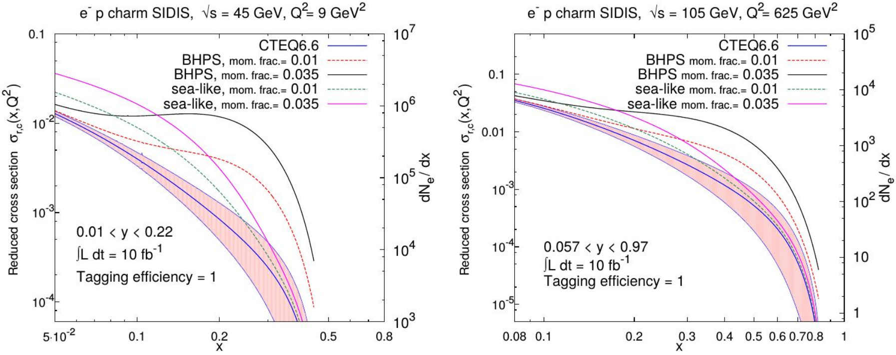
At $Q = 1.65$ GeV charm carry 0.7 ± 0.3 % of the proton momentum

but it mainly relies on EMC data

but it is affected by large uncertainties, especially if no EMC data are included

Probing intrinsic charm at an EIC

Intrinsic charm may be probed at an EIC by measuring
the charm contribution to the (reduced) DIS structure functions $\sigma_{r,c}$
the longitudinal structure function F_L or angular distributions

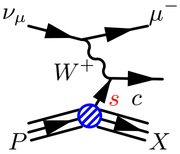


[Figure taken from arXiv:1108:1713]

Estimate of the reduced cross section $\sigma_{r.c}$ and of the number of events dN_e/dx
(assumed $\mathcal{L} = 10$ fb^{-1}, $dN_e/dx = \mathcal{L}\langle\sigma_c/dx\rangle$ in a Q bin of size 0.15 GeV, NLO)
$\sigma_{r,c}$ exceeds perturbative charm (CTEQ6.6) by a sizable amount for both BHPS and sea-like
momentum fractions of 3.5% are easily distinguished, fractions down to 1% can be also resolved

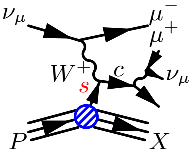
The strange PDF: current knowledge and limitations

Several processes are (in principle) sensitive to strange/antistrange quarks



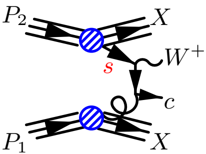
CHORUS

[PLB B632 (2006) 65]



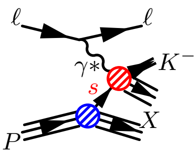
NuTeV

[PRD 64 (2001) 112006]



CMS

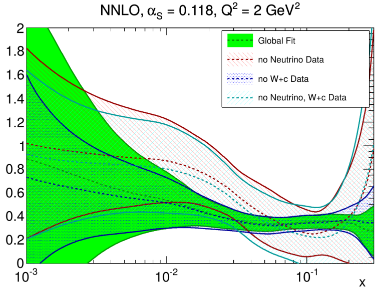
[JHEP 1402 (2014) 013]

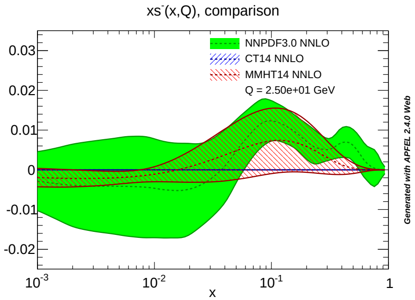


HERMES

[PLB 666 (2008) 446]

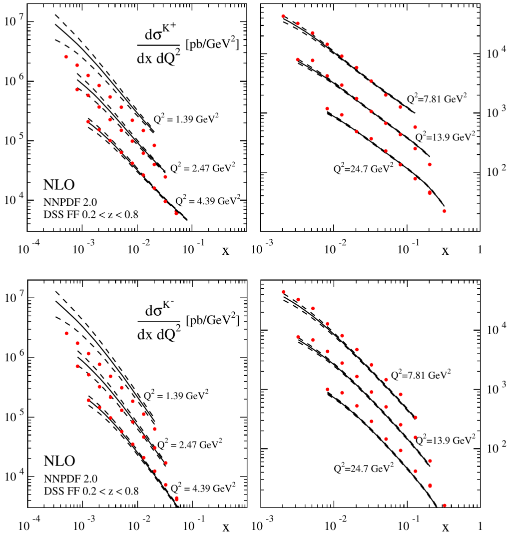
LHC data not competitive w.r.t neutrino-induced DIS data, large uncertainty on s^-





[Figures taken from JHEP 1504 (2015) 040]

The strange PDF: $K^{\pm}$ production in SIDIS at an EIC



[figure taken from arXiv:1108:1713]

red points: pseudodata at an EIC (based on PYTHIA + JETSET)

black curves: theory predictions (NNPDF2.0 + DSS07, NLO)

$0.01 \leq y \leq 0.95$, $\sqrt{s} = 70.7$ GeV
z integrated in the range $[0.2, 0.8]$

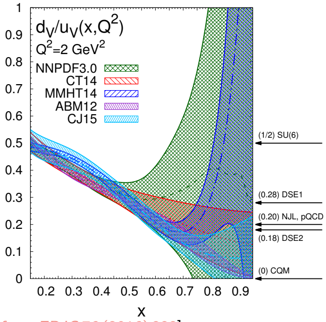
small x: $d\sigma^{K^+} \approx d\sigma^{K^-}$
large x: $d\sigma^{K^+} \gg d\sigma^{K^-}$
may constrain s^+ and s^-

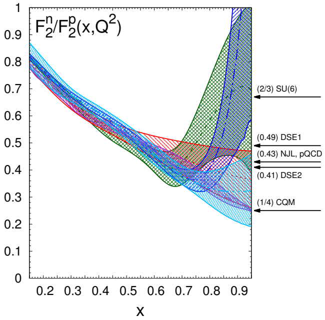
drawback: $K^{\pm}$ fragmentation
a) study FFs separately
b) analyze PDFs and FFs simultaneously [more in N. Sato's talk]

LHeC: direct sensitivity to s
charm tagging in CC DIS $(W + s \to c)$

$\pi^{\pm}$ production in SIDIS at an EIC allow for a determination of $\bar{u} - \bar{d}$

The PDF ratio d_V/u_V at large x: F_2^n/F_2^p at an EIC





[Figure taken from EPJC 76 (2016) 383]

$$\frac{d_V}{u_V} \xrightarrow{x\to 1} (1-x)^{b_{d_V}-b_{u_V}} \xrightarrow[\text{c. r.}]{b_{d_V}=b_{u_V}} k \qquad \frac{F_2^n}{F_2^p} \xrightarrow{x\to 1} \frac{4(1-x)^{b_{u_V}}+(1-x)^{b_{d_V}}}{(1-x)^{b_{u_V}}+4(1-x)^{b_{d_V}}} \xrightarrow[\text{c. r.}]{b_{d_V}=b_{u_V}} 1$$

case $b_{u_V} \gg b_{d_V}$: $\frac{d_V}{u_V} \xrightarrow{x\to 1} \infty$; $\frac{F_2^n}{F_2^p} \xrightarrow{x\to 1} 4$ case $b_{u_V} \ll b_{d_V}$: $\frac{d_V}{u_V} \xrightarrow{x\to 1} 0$; $\frac{F_2^n}{F_2^p} \xrightarrow{x\to 1} \frac{1}{4}$

No predictive power from current PDF determinations, no discrimination among models unless $\frac{d_V}{u_V} \xrightarrow{x\to 1} k$ is built in the parametrization (CT14, CJ16, ABM12)

The EIC may measure the ratio F_2^n/F_2^p with high accuracy, provided neutron beams expected to be less prone to nuclear and/or higher twist corrections than fixed-target DIS

Complementary measurements from the LHC (DY) and (particularly) the LHeC (DIS)

Additional opportunities at an EIC

1. Nuclear PDFs [N. Armesto's talk]

$$f_i^A(x,\mu^2) = \frac{Z}{A} f_i^{p,A}(x,\mu^2) + \frac{N}{A} f_i^{n,A}(x,\mu^2) \qquad f_i^{p,A}(x,\mu^2) = R_i^A(x,\mu^2) f_i^p(x,\mu^2)$$

   An EIC can distinguish between properties of the proton and of the nuclear medium
   A variety of nuclear beams needed to map the A-dependent nuclear corrections
   Important piece of information to quantify nuclear correction in DIS
   (important for dimuon production at NuTeV)

2. Behavior at the boundary of perturbative and nonperturbative QCD
   The EIC would span across both the perturbative and the nonperturbative regions
   Precise EIC data might enable us to connect the different pictures in those two regions

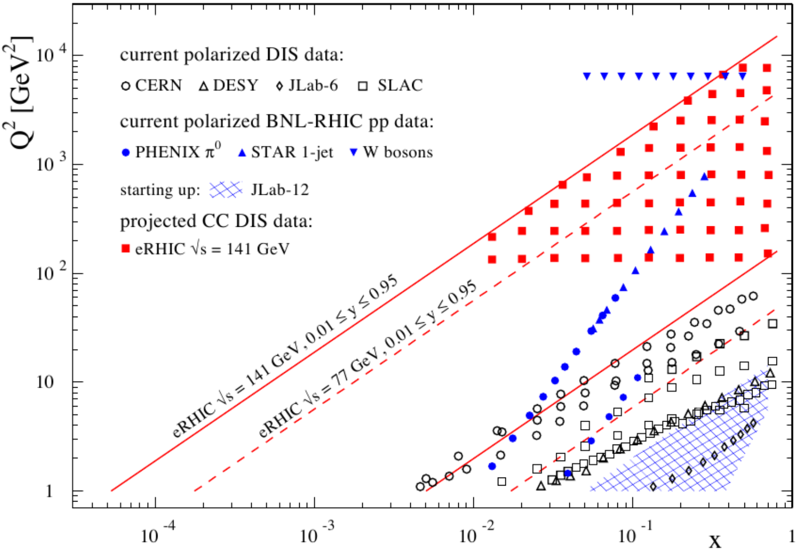
3. Electroweak contributions to the PDFs [Y. Zhao's talk]
   An EIC may allow for a thorough assessment of EW corrections
   (NLO EW corrections $\sim$ NNLO QCD corrections, routinely included)
   (accurate determination of the photon PDF, with *proton brightness* [arXiv:1607.04266])
   Test of potential isospin symmetry violations *e.g.* via a measurement of

$$\Delta F_2 \equiv \frac{5}{18} F_2^{CC}(x,Q^2) - F_2^{NC}(x,Q^2)$$

   together with other combinations of EW SFs measured in other experiments
   (*e.g.* $\Delta F_3 = F_3^{W^+} - F_3^{W^-}$ in neutrino-nucleon DIS)

3. Polarized Parton Distributions

The key asset of a polarized EIC: the kinematic coverage



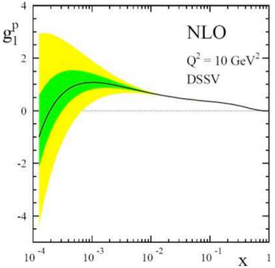
[Figure taken from EPJA 52 (2016) 268]

The structure function g_1^p at an EIC

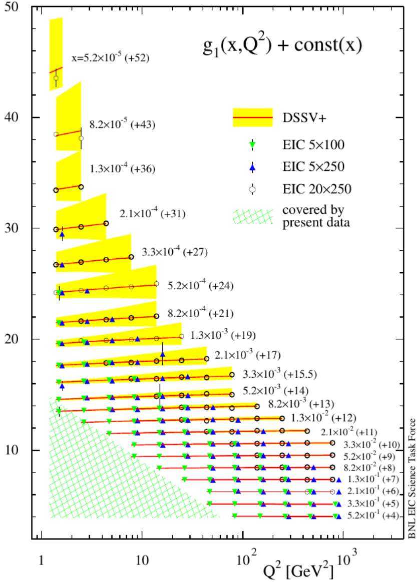
Golden measurement in DIS: g_1^p

$$g_1^p = \frac{\langle e^2 \rangle}{2}[C_{NS}\otimes\Delta q_{NS} + C_S\otimes\Delta\Sigma + 2n_f C_g\otimes\Delta g]$$

large spread at $x \lesssim 10^{-3}$



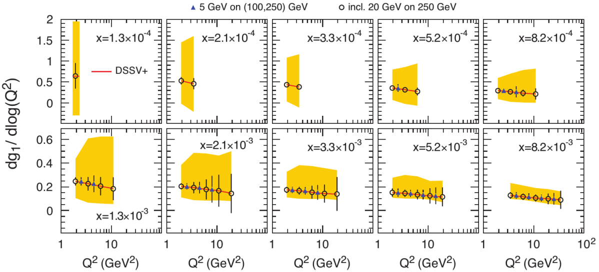
$E_e \times E_p$ [GeV]	$\sqrt{s}$ [GeV]	$x_{\min}$
5×100	44.7	5.2×10^{-4}
5×250	70.7	2.1×10^{-4}
20×250	141	5.2×10^{-5}



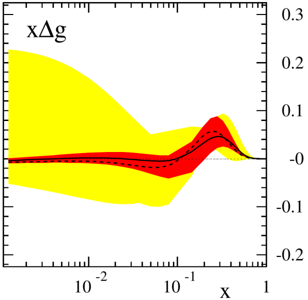
pseudodata ⇔ impact of g_1^p at an EIC

[Figure taken from PRD 86 (2012) 054020; see also EPJA 52 (2016) 268 and PLB 728 (2014) 524]

The gluon at small x: scaling violations in DIS



$$\frac{dg_1}{d\ln Q^2} \propto -\Delta g(x,Q^2)$$



Constrain Δg through scaling violations of g_1

full NNLO [NPB 417 (1994) 61; NPB 889 (2014) 351]

map Δg with an accuracy of 10% (or better) at $x \gtrsim 10^{-4}$

may be advantageous to measure $\Delta\sigma$ instead of A_1^p or g_1^p

Study possible deviations from DGLAP evolution

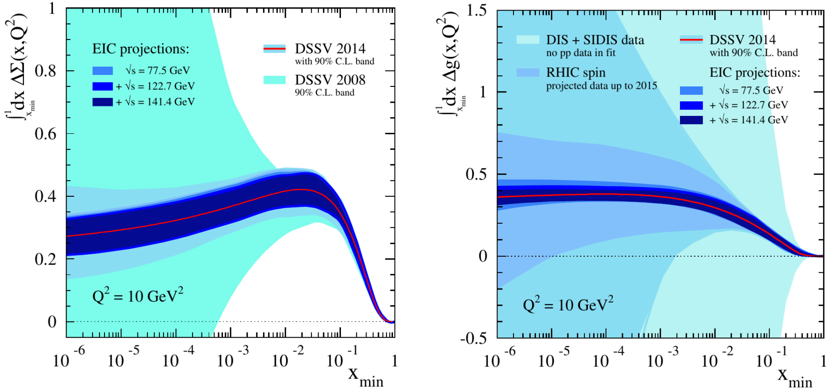
not clear if EIC kinematic range is large enough

the shape of Δg at small x may change significantly

[Figures taken from PRD 86 (2012) 054020]

The spin sum rule

$$\mathcal{J}(\mu^2) = \sum_f \left\langle P; S | \hat{J}_f^z(\mu^2) | P; S \right\rangle = \frac{1}{2} = \frac{1}{2}\Delta\Sigma(\mu^2) + \Delta G(\mu^2) + \mathcal{L}_q(\mu^2) + \mathcal{L}_g(\mu^2)$$

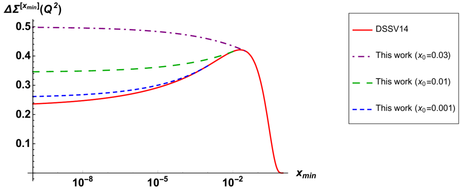


[Figures taken from PRD 92 (2015) 094030]

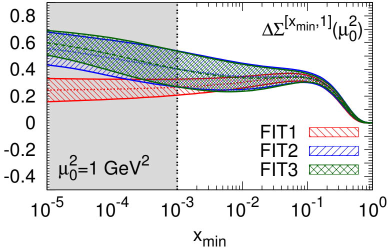
An EIC is expected to control $\Delta\Sigma$ within 15% and Δg within 10% relative accuracy

An EIC may provide an indirect constraint on the orbital angular momentum

Small-x asymptotics of the quark helicity [See Y. Kovchegov and M. Sievert]



[Figure taken from arXiv:1610.06188]



[Preliminary figure from ERN and E. Santopinto, in preparation]

Small-x evolution equations for g_1

based on the dipole model

resum powers of $\alpha_s \ln^2(1/x)$

become closed for N_C, n_f large

a solution for the flavor-singlet is

$$g_1 \sim \Delta\Sigma \sim \left(\frac{1}{x}\right)^{\alpha_h}, \qquad \alpha_h \sim 2.31\sqrt{\frac{\alpha_s N_C}{2\pi}}$$

Potential solid amount of spin at small x

attach $\Delta\hat{\Sigma}(x,Q^2) = N x^{-\alpha_h}$ at x_0 to DSSV

detailed phenomenology needed

Should be tested at an EIC

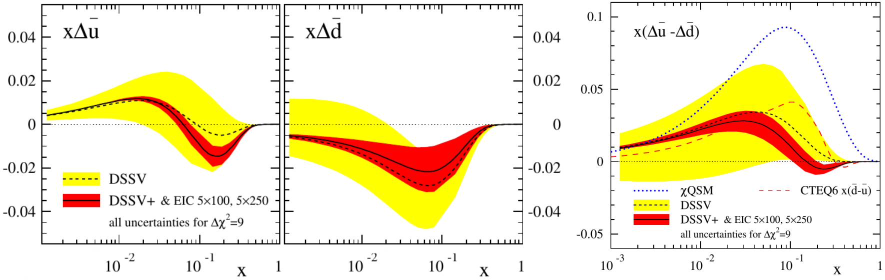
FIT1: $\Delta T_3 = 1.2701 \pm 0.0025$
$\Delta T_8 = 0.585 \pm 0.176$

FIT2: $\Delta U^+ = +1.098 \pm 0.220$
$\Delta D^+ = -0.417 \pm 0.084$
$\Delta S^+ = -0.005 \pm 0.001$

FIT 3 $\Delta U^+ = +1.132 \pm 0.226$
$\Delta D^+ = -0.368 \pm 0.074$
$\Delta S^+ = 0$

	FIT1	FIT2	FIT3
$\chi^2_{\rm dat}$	0.74	0.76	0.79
$\Delta\Sigma$	$+0.23 \pm 0.09$	$+0.64 \pm 0.14$	$+0.73 \pm 0.16$

Sea quark PDFs: SIDIS at an EIC



[Figures taken from PRD 86 (2012) 054020]

Full flavor separation and accurate determination of individual Δu, $\Delta\bar{u}$, Δd, $\Delta\bar{d}$

the EIC kinematic coverage will turn SIDIS in a precision tool to study light sea quarks

projections shown are based only on *stage 1* pseudodata, down to $x \sim 10^{-4}$ after *stage 2*

need a careful control of systematics (lumi, polarimetry, ...) and particle ID (large phase space)

need a sensible determination of pion fragmentation functions

progress expected by the time of an EIC, see *e.g.* new COMPASS multiplicities [arXiv:1604.02695]

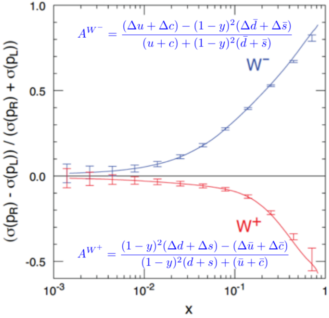
a simultaneous analysis of FFs and polarized PDFs may be beneficial given their cross-talk

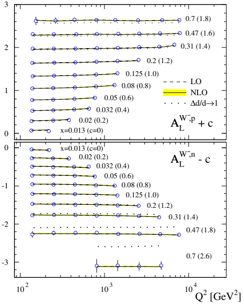
Emergence of a polarized light sea asymmetry

driven by $W^{\pm}$ production at RHIC, but limited x range and large uncertainites [NPB 887 (2014) 276]

several nonperturbative models predict a large sea asymmetry (χQM, MC, PB)

New electroweak probes at an EIC





[Figures taken from arXiv:1108.1713 and PRD 88 (2013) 114025]

At sufficiently high Q^2, one can exploit CC DIS (mediated by $W^{\pm}$) at an EIC

$$\frac{d\Delta\sigma^{e^{\pm},i}}{dxdy} = \frac{4\pi\alpha^2}{xyQ^2}\left[\pm y(2-y)x\hat{g}_1^i - (1-y)g_4^i - y^2 x\hat{g}_5^i\right] \qquad i = \mathrm{CC}, \mathrm{NC}$$

measure CC asymmetries $A_L^{W^{\pm}}$, which require a positron beam not neccessarily polarized

$$A_L^{W^+,p} \xrightarrow[y\to 0]{\text{LO}} \frac{\Delta u - \Delta\bar{d}}{u+\bar{d}} \qquad A_L^{W^+,p} \xrightarrow[y=1/2]{\text{LO}} \frac{4\Delta u - \Delta\bar{d}}{4u+\bar{d}} \qquad A_L^{W^+,p} \xrightarrow[y\to 1]{\text{LO}} \frac{\Delta u}{u} \qquad \longleftrightarrow \text{ for } A_L^{W^-,n}$$

NNLO corrections known [see e.g. PRD 53 (1996) 138], can be easily put into global QCD analyses

novel Bjorken-like sum rules, *e.g.* $g_5^{W^-,p} - g_5^{W^+,n} = [1 - 2\alpha_s/(3\pi)]g_A$ (*neutron* beams needed)

SU(3) breaking and strangeness

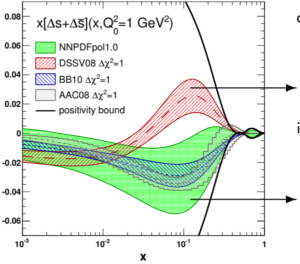
NNPDFpol1.0 [NPB 874 (2013) 36]
$\int_0^1 dx[\Delta s + \Delta\bar{s}] = -0.13 \pm 0.09$

Lattice [PRL 108 (2012) 222001]
$\int_0^1 dx[\Delta s + \Delta\bar{s}] = -0.020(10)(1)$

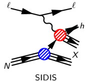
First moment constrained by

$a_3 = \int_0^1 dx[\Delta u^+ - \Delta d^+]$
$= 1.2701 \pm 0.0025$

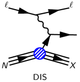
$a_8 = \int_0^1 dx[\Delta u^+ + \Delta d^+ - 2\Delta s^+]$
$= 0.585 \pm 0.025$



directly from SIDIS Kaon data



indirectly from DIS + SU(3)



All PDF determinations based only on DIS data (+ SU(3)) find a negative Δs^+
PDF determinations based on DIS+SIDS data (+SU(3)) find a negative or a positive Δs^+ depending on the K FF set [PRD 91 (2015) 054017]

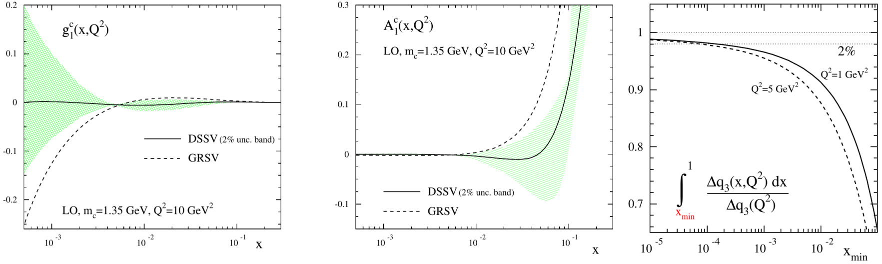
Tension between DIS and SIDIS data can be ficticious

$SU(3)$ may be broken [PRD 58 (1998) 094028, Ann.Rev.Nucl.Part.Sci. 53 (2003) 39], but how much?
$\longrightarrow$ in NNPDFpol, the nominal uncertainty on a_8 in inflated by 30% of its value to allow for a $SU(3)$ symmetry violation ($a_8 = 0.585 \pm 0.025 \longrightarrow a_8 = 0.585 \pm 0.176$)
$\longrightarrow$ but *e.g.* lattice finds a larger $SU(3)$ symmetry violation [PRL 108 (2012) 222001]

Opportunities at an EIC

one could study kaon multiplicities in SIDIS $\longrightarrow$ further constraint on kaon FFs
one could study CC charm production $W^+ s \to c$ in DIS $\longrightarrow$ direct handle on s, $\bar{s}$

Additional opportunities at an EIC



[Figures taken from arXiv:1108.1713]

1. Heavy flavor contribution to g_1, specifically from charm
   irrelevant ($\ll 1\%$) so far in fixed-target DIS, relevant at an EIC depending on Δg
   small Δg (DSSV07) $\Rightarrow g_1^c$ negligible; $A_1^c \sim \mathcal{O}(10^{-5})$ too small to be measured
   large Δg (GRSV) $\Rightarrow g_1^c \sim 10 - 15\%$ of g_1 at $x = 10^{-3}$, $Q^2 \simeq 10$ GeV^2; $A_1^c \sim \mathcal{O}(10^{-3})$
   charm not massless at the EIC kinematics: relevant NLO corrections are needed
2. The Bjorken sum rule $\int_0^1 dx[g_1^p(x,Q^2) - g_1^n(x,Q^2)] = \frac{1}{6}\Delta C_{\rm NS}\alpha_s(Q^2)g_A$
   currently verified within 10% accuracy, fix the target down to 2% (isospin violations)
   expect to need data on g_1^p and g_1^n down to $x \sim 10^{-4}$ to constrain $g_A = \int_0^1 dx\Delta T_3$
   corrections to $\Delta C_{\rm NS}$ known up to $\mathcal{O}(\alpha_s^4)$ [PRL 104 (2010) 132004]
   need longitudinally polarized *neutron* beams (challenging R&D task)

4. Conclusions and final remarks

Summary of PDF-related measurements at an EIC

	Measurement	Process	What we learn
Unpolarized	unpolarized structure functions F_L and F_L^c	scaling violations in inclusive DIS	unpolarized gluon distribution at small x
	heavy mesons J/ψ and Υ charm contribution to the cross section	heavy-quark production in (semi-inclusive) DIS	unpolarized gluon at large x intrinsic charm contribution in the proton
	kaon multiplicities	charged kaon production in semi-inclusive DIS	unpolarized strange and antistrange distributions
Polarized	polarized structure function g_1	scaling violations in inclusive DIS	gluon contribution to proton spin
	polarized structure function g_1^h	semi-inclusive DIS for pions and kaons	quark contribution to proton spin sea asymmetry $\Delta\bar{u} - \Delta\bar{d}$; Δs
	novel electroweak spin structure functions	inclusive DIS at high Q^2	flavor separation at medium x and large Q^2

UNP Excellent complementarity with the LHC (discovery) and LHeC (ultra-precision)
training ground for future colliders as HERA has been for the EIC

POL Unique machine to address the spin structure of the proton
the EIC might save unexpected surprises, like the SPS-EMC did in the 80s

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A favorite theory of mine - to wit, that no occurrence is sole and solitary, but is merely a repetition of a thing which has happened before, and perhaps often.
(M. Twain, *The Celebrated Jumping Frog of Calaveras County*, 1865)