Longitudinally-polarized parton distributions with faithful uncertainty estimates

XVI workshop on high energy spin physics

Emanuele R. Nocera

Università di Genova & INFN, Genova

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Foreword

1. How the nucleon spin is built up from the quark and gluon spin and OAM?

$$\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma(\mu^2) + \Delta G(\mu^2)}_{\text{quark and gluon spin fractions}} + \underbrace{\mathcal{L}_q(\mu^2) + \mathcal{L}_q(\mu^2)}_{\text{quark and gluon OAM}} \quad \text{[NP B337 (1990) 509]}$$

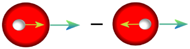
2. Does each of these terms allow for a unique field-theoretic definition in QCD? (possibly gauge-invariant, physically meaningful and related to a measurable quantity)
[Phys.Rept. 541 (2014) 163, see also the talk by Bo-Qiang Ma]

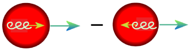
3. This talk is about an accurate determination of $\Delta\Sigma(\mu^2)$ and $\Delta G(\mu^2)$ in QCD

$$\Delta\Sigma(\mu^2) = \sum_q \int_0^1 dx\, \left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] \qquad \Delta g(\mu^2) = \int_0^1 dx\, \Delta g(x,\mu^2)$$

4. Namely, a determination of the longitudinally-polarized PDFs of the proton (*i.e.* the momentum densities of partons with spin ($\uparrow$) or ($\downarrow$) *w.r.t* the nucleon)

$$\Delta f(x) \equiv f^{\uparrow}(x) - f^{\downarrow}(x)\,, \qquad f = u, \bar{u}, d, \bar{d}, s, \bar{s}, g$$

$\Delta q(x) =$ 

$\Delta g(x) =$ 

Outline

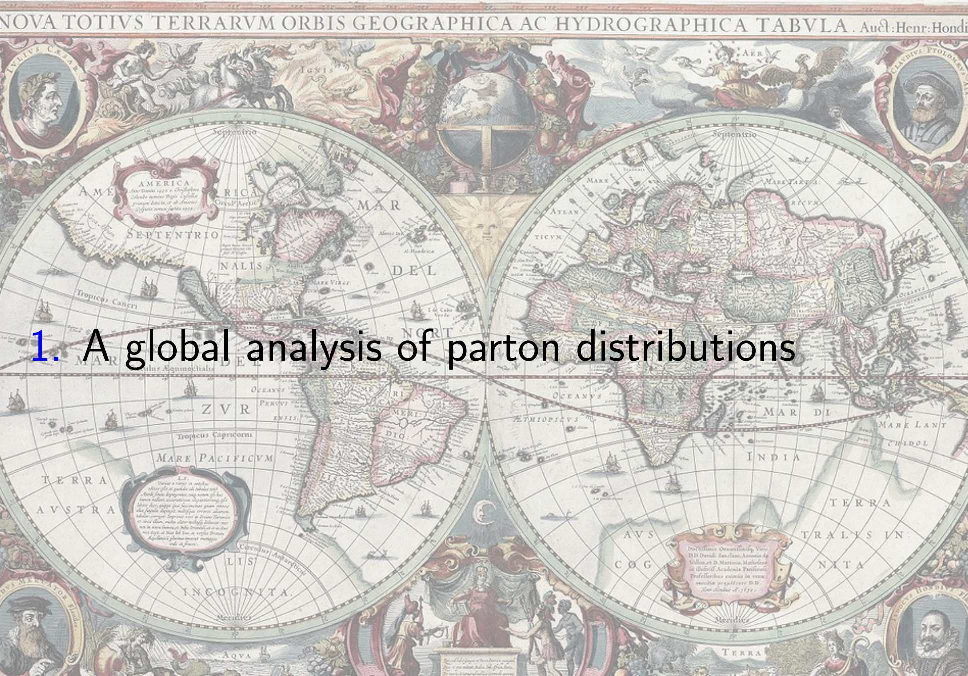
1. A global analysis of parton distributions
   - Theory: perturbative accuracy, theoretical constraints
   - Methodology: *standard* vs NNPDF routes
   - Data: spin observables and accessible PDFs
2. Longitudinally polarized PDFs from the NNPDF family
   - Evolution of `NNPDFpol` fits: kinematic coverage and fit quality
   - Impact of new data: RHIC data, new DIS data
   - The emerging picture of the polarized nucleon
3. Drawing conclusions

Results shown in this presentation are based on the following papers

[NP B874 (2013) 36] [PL B728 (2014) 524] [NP B887 (2014) 276] [PL B742 (2015) 117]

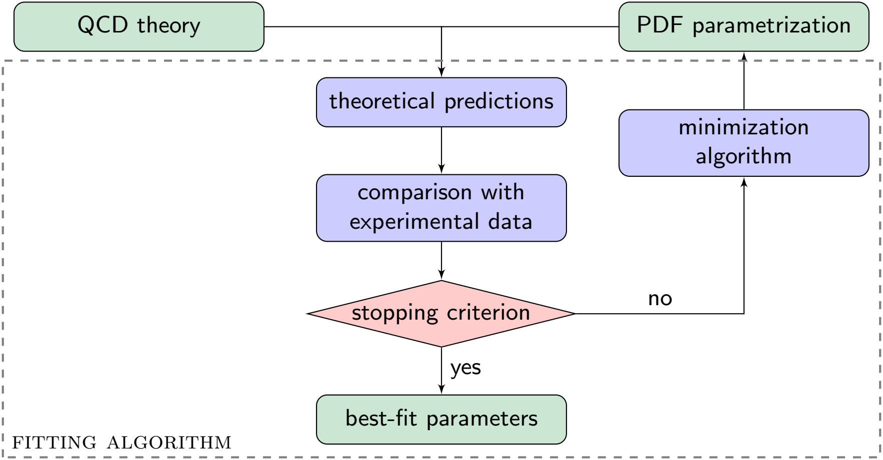
in collaboration with R.D. Ball, S. Forte, G. Ridolfi and J. Rojo

1. A global analysis of parton distributions



Hendrik Hondius, *Nova Totius Terrarum Orbis Tabula* (1630), State Library of New South Wales

A global PDF determination: the underlying strategy



Assume a reasonable PDF parametrization

Obtain theoretical predictions for various processes and compare predictions to data

Determine the best-fit parameters via minimization of a proper figure of merit (*e.g.* χ^2)

A global PDF determination: the ingredients we need

theory	methodology	data
DGLAP evolution	parametrization	fixed-target data
partonic cross sections	uncertainty estimation	(DIS, SIDIS)
heavy quark treatment	error propagation	collider data
QED/EW corrections	minimization strategy	(*ep pp*)

Need for a choice of

1. **theory**, or the theoretical details of the QCD analysis
   (perturbative order, treatment of heavy quarks, treatment of α_s, theoretical constraints)
2. **methodology**, or a prescription to determine PDFs and their uncertainties
   (uncertainty estimates are crucial to make reliable predictions based on PDFs)
3. **data**, or the set of observables to be included in the analysis
   (constrain all possible PDFs in the widest range of Bjorken-x)

Each of these ingredients is a source of uncertainty on the PDF determination

Theory: perturbative QCD

1. Factorization of physical observables $\mathcal{O}_I$ [Adv.Ser.Direct.High Energy Phys. 5 (1988) 1]
   - a convolution between coefficient functions $\mathcal{C}_{If}(x, \alpha_s(\mu^2))$ and PDFs $f(x, \mu^2)$

$$\mathcal{O}_I = \sum_{f=q,\bar{q},g} \mathcal{C}_{If}(y, \alpha_s(\mu^2)) \otimes f(y, \mu^2) + \text{p.s. corrections} \qquad f \otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$

   - coefficient functions allow for a perturbative expansion in terms of $a_s = \alpha_s/(4\pi)$

$$\mathcal{C}_{If}(y, \alpha_s) = \sum_{k=0} a_s^k C_{If}^{(k)}(y) \begin{cases} \text{DIS (up to NNLO)} & \text{[NP B417 (1994) 61]} \\ \text{SIDIS (up to NLO)} & \text{[PR D57 (1998) 5811, NP B539 (1999) 455]} \\ pp \text{ (up to NLO)} & \begin{cases} \text{[NP B539 (1999) 455, PR D70 (2004) 034010]} \\ \text{[PR D67 (2003) 054004, ibidem 054005]} \\ \text{[PR D81 (2010) 094020]} \end{cases} \end{cases}$$

2. Evolution of parton distributions [NP B126 (1977) 298]
   - a set of $(2n_f + 1)$ integro-differential equations, n_f is the number of active flavors

$$\frac{\partial}{\partial \ln \mu^2} f_i(x, \mu^2) = \sum_j^{n_f} \int_x^1 \frac{dz}{z} \mathcal{P}_{ji}\left(z, \alpha_s(\mu^2)\right) f_j\left(\frac{x}{z}, \mu^2\right)$$

   - with perturbative computable splitting functions

$$\mathcal{P}_{ji}(z, \alpha_s) = \sum_{k=0} a_s^{k+1} P_{ji}^{(k)}(z) \qquad \begin{cases} \text{LO} & \text{[NP B126 (1977) 298]} \\ \text{NLO} & \text{[ZP C70 (1996) 637, PR D54 (1996) 2023]} \\ \text{NNLO} & \text{[NP B889 (2014) 351]} \end{cases}$$

Theory: theoretical constraints

1. Polarized PDFs must lead to positive cross sections
   - at LO, polarized PDFs are bounded by their unpolarized counterparts

$$|\Delta f(x,\mu^2)| \leq f(x,\mu^2)$$

   - beyond LO, other relations hold, but are of limited effect [NP B534 (1998) 277]
2. Polarized PDFs must be integrable
   - *i.e.* require that the axial matrix elements of the nucleon are finite

$$\langle P,S|\bar{\psi}_q\gamma^\mu\gamma_5\psi_q|P,S\rangle \longrightarrow \text{finite for each flavor } q$$

3. Assume SU(2) and SU(3) symmetry
   - relate the octet of axial-vector currents to β-decay of spin-1/2 hyperons

$$a_3 = \int_0^1 dx\,\Delta T_3 = 1.2701\pm 0.0025 \qquad a_8 = \int_0^1 dx\,\Delta T_8 = 0.585\pm 0.025 \qquad \text{[PDG 2014]}$$

$$\Delta T_3 = (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d}) \qquad \Delta T_8 = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) - 2(\Delta s + \Delta\bar{s})$$

   - note: violations of SU(3) symmetry are advocated in the literature [ARNPS 53 (2003) 39]

Methodology: the standard route

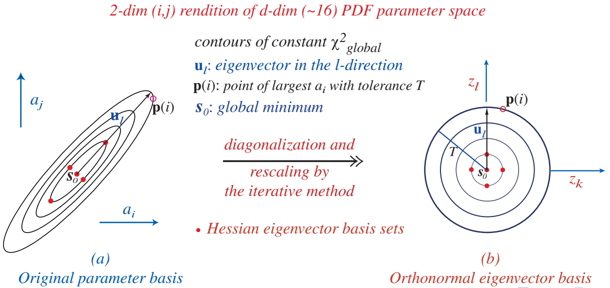
1. Simple analytical parametrization of PDFs, *e.g.*

$$xf(x,\mu_0^2) = \eta_f x^{a_f}(1-x)^{b_f}\left(1+\rho_f x^{\frac{1}{2}}+\gamma_f x\right) \qquad \{\mathbf{a}\} = \{a,b,\eta,\rho,\gamma\}$$

$\Rightarrow$ **potential bias if the parametrization is too rigid**

2. Hessian propagation of errors
   - expand the χ^2 about its global minimum at first order, $\chi^2\{\mathbf{a}\} \approx \chi^2\{\mathbf{a_0}\} + \delta a^i H_{ij} \delta a^j$
   - diagonalize the Hessian matrix and take the hypersphere of radius $\sqrt{\chi^2} = 1$

$\Rightarrow$ **is linear approximation adequate? do we need a tolerance $\mathbf{T} = \sqrt{\chi^2} > 1$?**



Methodology: the NNPDF route

1. Neural network parametrization of PDFs
   - redundant and flexible parametrization, $\mathcal{O}(200)$ parameters
   - requires a proper minimization algorithm and stopping criterion

   $\Rightarrow$ **reduce the theoretical bias due to the parametrization**

2. Monte Carlo propagation of errors
   - generate experimental data replicas assuming multi-Gaussian probability distribution
   - validate against experimental data to determine the sample size ($N_{\mathrm{rep}} \sim 100$)

   $\Rightarrow$ **no need to rely on linear error propagation, no tolerance needed**

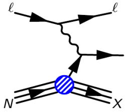
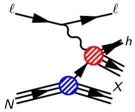
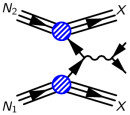
PDF replicas are equally probable members of a statistical ensemble
which samples the probability density $\mathcal{P}[f_i]$ in the space of PDFs

$$\langle \mathcal{O} \rangle = \int \mathcal{D}f_i \mathcal{P}[f_i] \mathcal{O}[f_i]$$

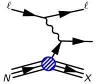
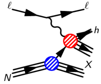
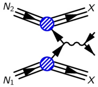
Expectation values for observables are Monte Carlo integrals

$$\langle \mathcal{O}[f_i(x,Q^2)] \rangle = \frac{1}{N_{\mathrm{rep}}} \sum_{k=1}^{N_{\mathrm{rep}}} \mathcal{O}[f_i^{(k)}(x,Q^2)]$$

Data: spin asymmetries

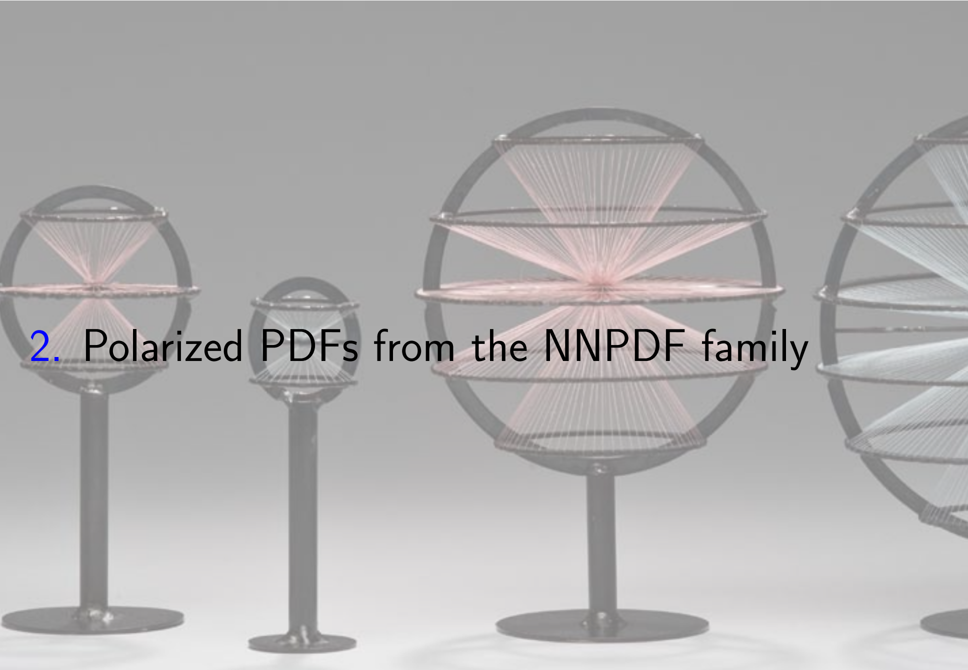
PROCESS	OBSERVED ASYMMETRIES	SUBPROCESSES	PROBED PDFS
 DIS $\ell^{\pm} + N \to \ell^{\pm} + X$	$A_1 \approx \frac{\sum_q \Delta q(x)+\Delta\bar{q}(x)}{\sum_{q'} q'(x)+\bar{q}'(x)}$	$\gamma^* q \to q$	$\Delta q + \Delta\bar{q}$ Δg (NLO)
 SIDIS $\ell^{\pm} + N \to \ell^{\pm} h + X$	$A_1^h \approx \frac{\sum_q \Delta q(x)\otimes D_q^h(z)}{\sum_{q'} q'(x)\otimes D_{q'}^h(z)}$	$\gamma^* q \to q$	Δu $\Delta\bar{u}$ Δd $\Delta\bar{d}$ Δg (NLO)
	$A_{LL}^{\gamma N \to D_0 X} \approx \frac{\Delta g \otimes D_c^{D^0}(z)}{g(x)\otimes D_c^{D^0}(z)}$	$\gamma^* g \to c\bar{c}$	Δg
 pp	$A_{LL}^{jet} \approx \frac{\sum_{a,b=q,\bar{q},g} \Delta f_a(x_1)\otimes \Delta f_b(x_2)}{\sum_{a,b,c=q,\bar{q},g} f_a(x_1)\otimes f_b(x_2)}$	$gg \to qg$ $qg \to qg$	Δg
	$A_L^{W^+} \approx \frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2)+\bar{d}(x_1)u(x_2)}$	$u_L\bar{d}_R \to W^+$ $d_L\bar{u}_R \to W^+$	Δu $\Delta\bar{u}$ Δd $\Delta\bar{d}$
$N_1 + N_2 \to \{jet(s), W^{\pm}, \pi\} + X$	$A_{LL}^h \approx \frac{\sum_{a,b,c=q,\bar{q},g} \Delta f_a(x_1)\otimes \Delta f_b(x_2)\otimes D_c^h(z)}{\sum_{a,b,c=q,\bar{q},g} f_a(x_1)\otimes f_b(x_2)\otimes D_c^h(z)}$	$gg \to qg$ $qg \to qg$	Δg

Overview of available polarized PDF sets

	DSSV	NNPDF	JAM	LSS
 DIS	☑	☑	☑	☑
 SIDIS	☑	☒	☒	☑
 pp	☑ (jets, π^0)	☑ (jets, $W^\pm$)	☒	☒
statistical treatment	Lagr. mult. $\Delta\chi^2/\chi^2 = 2\%$	Monte Carlo	Hessian $\Delta\chi^2 = 1$	Hessian $\Delta\chi^2 = 1$
parametrization	polynomial (23 pars)	neural network (259 pars)	polynomial (10 pars)	polynomial (20 pars)
features	global fit	minimally biased fit	large-x effects	higher-twist effects
latest update	PRL 113 (2014) 012001	NP B887 (2014) 276	PR D89 (2014) 034025	PR D91 (2015) 054017

+ some others: AAC [NP B813 (2009) 106] BB [NP B841 (2010) 205], AKS [PR D89 (2014) 034006], ...

2. Polarized PDFs from the NNPDF family



Julian Voss-Andreae, *Spin Family (Bosons and Fermions)*, Steel and silk (2009)

Evolution of NNPDFpol fits

NNPDFpol1.0 [NP B87 (2013) 36]

- inclusive DIS data from CERN, SLAC and DESY on $g_1^{p,d,n}$

$$g_1(x,Q^2) = \underbrace{\frac{\sum_q^{n_f} e_q^2}{2n_f}\left(\mathcal{C}_{\rm NS}\otimes\Delta q_{\rm NS} + \mathcal{C}_{\rm S}\otimes\Delta\Sigma + 2n_f\mathcal{C}_g\Delta g\right)}_{\text{leading-twist factorization}} + \underbrace{\frac{h^{\rm TMC}}{Q^2} + \frac{h^{\rm HT}}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)}_{\text{power-suppressed TMCs and HT}}$$

- TMCs included exactly [NP B513 (1998) 301]
- kinematic cut $W^2 \geq 6.25$ GeV^2 to remove sensitivity to dynamical HTs [arXiv:0807.1501]
- inflated uncertainty on a_8 (up to 30% of its exp value) to allow for SU(3) violation
- NLO perturbative accuracy, $\overline{\rm MS}$ renormalization scheme, ZM-VFN scheme

NNPDFpol1.1 [NP B877 (2014) 276]

- + new collider data from RHIC, included via reweighting:

→ jet production: STAR [PRD 86 (2012) 032006, PRL 115 (2015) 092002], PHENIX [PRD 84 (2011) 012006]

→ W-boson production from STAR [PRL 11 (2014) 072301]

- + open-charm production: COMPASS [PRD 87 (2013) 052018], included via reweighting

NNPDFpol1.2 [in preparation]

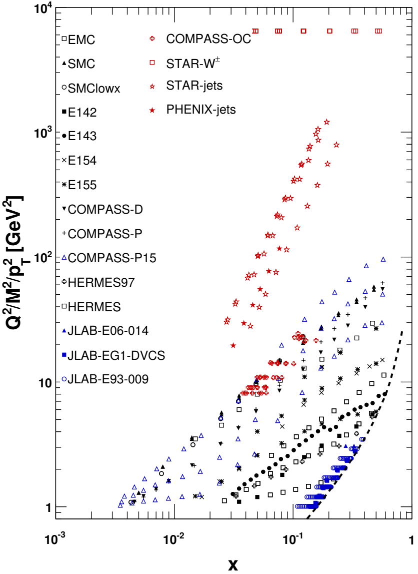
- + new inclusive DIS data, included via a complete refit:

→ COMPASS [arXiv:1503.08935] (p)

→ JLAB [PLB 641 (2006) 11, PRC 90 (2014) 025212, PLB 744 (2015) 309, arXiv:1505.07877] (p, d)

- the new unpolarized fit NNPDF3.0 [JHEP 1504 (2015) 040] is used as baseline

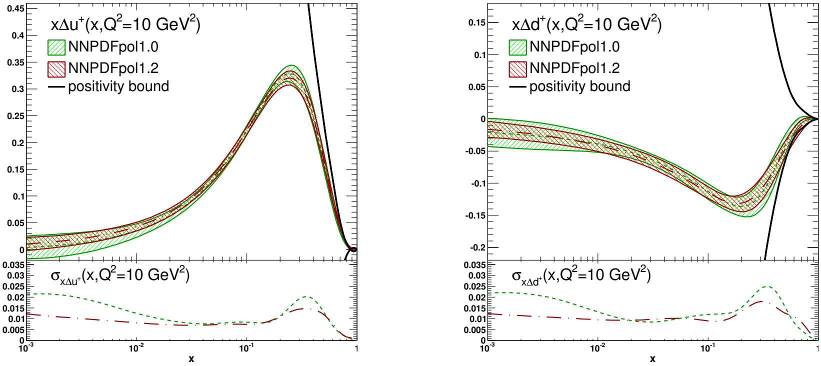
Kinematic coverage and fit quality



EXPERIMENT	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$ 1.0	1.1	1.2
EMC	10	0.44	0.43	0.43
SMC	24	0.93	0.90	0.92
SMClowx	16	0.97	0.97	0.94
E142	8	0.67	0.66	0.55
E143	50	0.64	0.67	0.63
E154	11	0.40	0.45	0.34
E155	40	0.89	0.85	0.98
COMPASS-D	15	0.65	0.70	0.57
COMPASS-P	15	1.31	1.38	0.93
HERMES97	8	0.34	0.34	0.23
HERMES	56	0.79	0.82	0.69
COMPASS-P-15	51	0.98*	0.99*	0.65
JLAB-E93-009	148	1.26*	1.23*	0.94
JLAB-EG1-DVCS	18	0.45*	0.59*	0.29
JLAB-E06-014	2	2.81*	3.20*	1.33
TOTAL DIS		0.77	0.78	0.74
COMPASS (OC)	45	1.22*	1.22	1.22
STAR (jets)	41	—	1.05	1.06
PHENIX (jets)	6	—	0.24	0.24
STAR-A_L	24	—	1.05	1.05
STAR-A_{LL}	12	—	0.95	0.94
TOTAL		0.77	1.05	1.01

* data set not included in the corresponding fit

Impact of new DIS data: total up and down

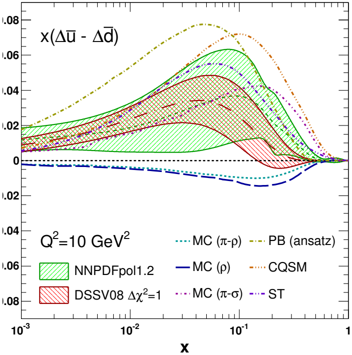


- Improved accuracy at small x: new COMPASS data (+ improved unpolarized F_L and F_2 from NNPDF3.0)
- Improved accuracy at large x: new JLAB data (also note that the positivity bound is slightly different)
- A lower cut on W^2 will allow for exploiting the full potential of JLAB data (if we replace $W^2 \geq 6.25$ GeV^2 with $W^2 \geq 4.00$ GeV^2 the χ^2 deteriorates significantly) (need to include and fit dynamic higher twists, in progress)

Impact of RHIC data: sea asymmetry and gluon

$W^{\pm}$ boson production

first evidence of broken flavor symmetry for polarized light sea quarks



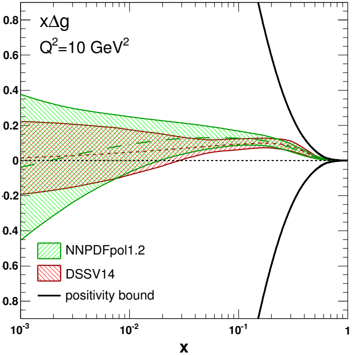
$$\langle x_{1,2} \rangle \simeq \frac{M_W}{\sqrt{s}} e^{-\eta_l/2} \approx [0.04, 0.4]$$

$$\Delta\bar{u} > 0 > \Delta\bar{d},\ |\Delta\bar{d}| > |\Delta\bar{u}|$$

$$\int_{0.04}^{0.4} dx\, \Delta_{sea}(x, Q^2 = 10\,\mathrm{GeV}^2) = +0.06 \pm 0.03$$

High-p_T jet production

first evidence of a sizable, positive gluon polarization in the proton

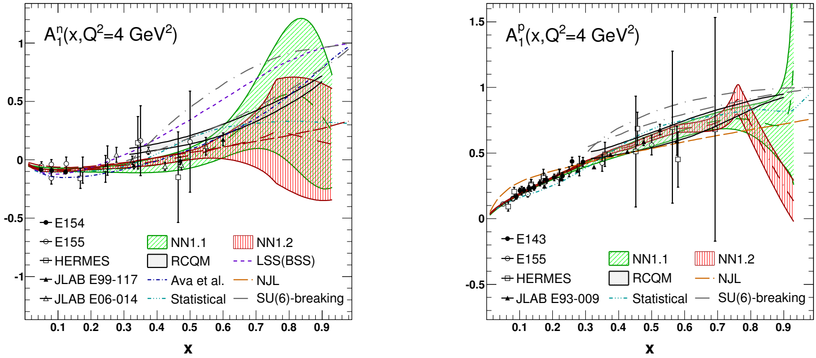


$$\langle x_{1,2} \rangle \simeq \frac{2p_T}{\sqrt{s}} e^{-\eta/2} \approx [0.05, 0.2]$$

NNPDF and DSSV results well compatible

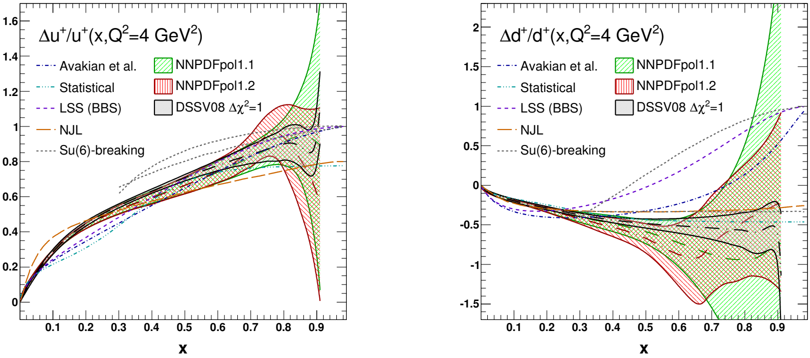
$$\int_{0.05}^{0.2} dx\, \Delta g(x, Q^2 = 10\,\mathrm{GeV}^2) = +0.15 \pm 0.07$$

Behavior at large-x values: $A_1^{n,p}$ [PL B742 (2015) 117]



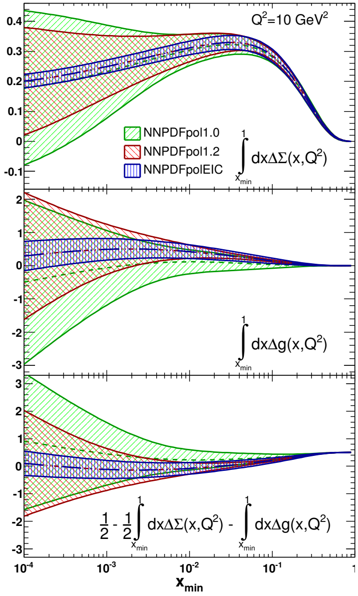
Model	A_1^n	A_1^p	Model	A_1^n	A_1^p
SU(6)	0	5/9	NJL	0.35	0.77
RCQM	1	1	DSE (*realistic*)	0.17	0.59
QHD ($\sigma_{1/2}$)	1	1	DSE (*contact*)	0.34	0.88
QHD (ψ_ρ)	1	1	pQCD	1	1
NNPDFpol1.1 ($x=0.7$)	0.41 ± 0.31	0.75 ± 0.07	NNPDFpol1.1 ($x=0.9$)	0.36 ± 0.61	0.74 ± 0.34
NNPDFpol1.2 ($x=0.7$)	0.18 ± 0.26	0.74 ± 0.06	NNPDFpol1.2 ($x=0.9$)	0.15 ± 0.59	0.24 ± 0.15

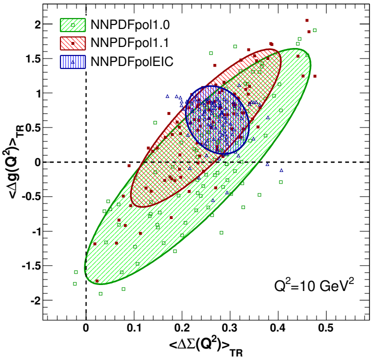
Behavior at large-x values: PDF ratios [PL B742 (2015) 117]



Model	$\Delta u^+/u^+$	$\Delta d^+/d^+$	Model	$\Delta u^+/u^+$	$\Delta d^+/d^+$
SU(6)	$2/3$	$-1/3$	NJL	0.80	-0.25
RCQM	1	$-1/3$	DSE (*realistic*)	0.65	-0.26
QHD ($\sigma_{1/2}$)	1	1	DSE (*contact*)	0.88	-0.33
QHD (ψ_ρ)	1	$-1/3$	pQCD	1	1
NNPDFpol1.1 ($x=0.7$)	0.82 ± 0.08	-0.88 ± 0.68	NNPDFpol1.1 ($x=0.9$)	0.91 ± 0.65	-0.74 ± 3.57
NNPDFpol1.2 ($x=0.7$)	0.86 ± 0.08	-0.75 ± 0.62	NNPDFpol1.2 ($x=0.9$)	0.62 ± 0.48	-0.23 ± 1.06

The spin content of the proton

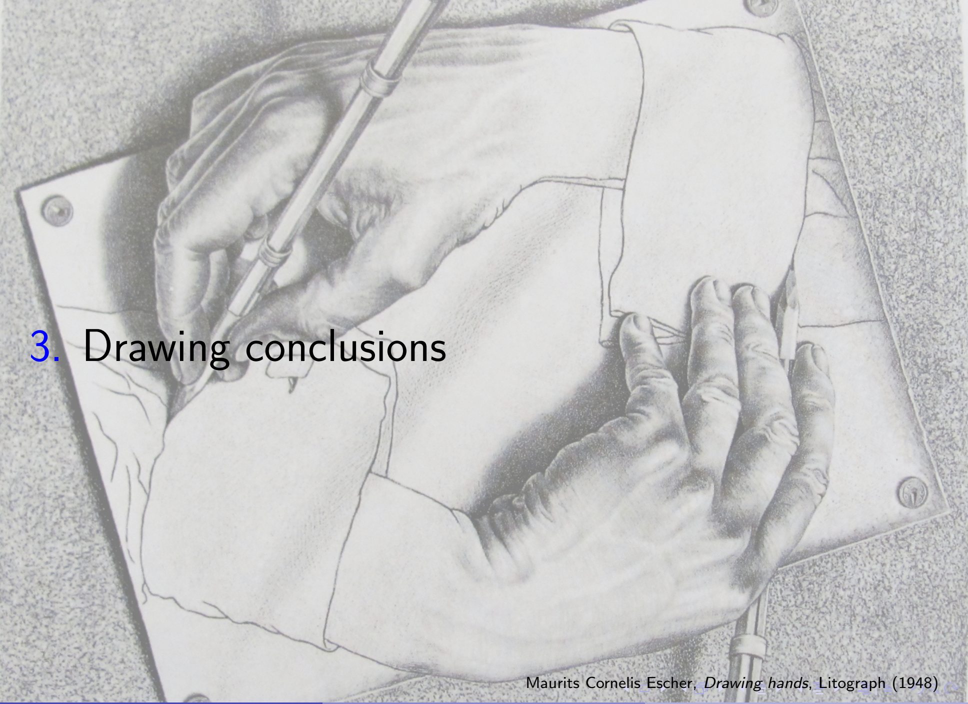




$Q^2 = 10$ GeV^2	$\int_{10^{-3}}^{1} dx\, \Delta\Sigma$	$\int_{10^{-3}}^{1} dx\, \Delta g$
NNPDFpol1.0	$+0.23 \pm 0.15$	-0.06 ± 1.12
NNPDFpol1.2	$+0.25 \pm 0.10$	$+0.49 \pm 0.75$
NNPDFpolEIC	$+0.24 \pm 0.04$	$+0.49 \pm 0.25$

quarks and antiquarks $\sim 20\% - 30\%$
gluons $\sim 70\%$
OAM $\sim 0\%$

3. Drawing conclusions



Maurits Cornelis Escher, *Drawing hands*, Litograph (1948)

Final remarks

After three decades of experimental and theoretical activity,
we cannot really say we know $\Delta\Sigma$ and Δg
Main culprit: small-x behavior of polarized PDFs

Spin experiments continue to produce high impact results (RHIC, JLAB, ...)
First evidence of a sizable, positive gluon polarization in the proton
First evidence of broken flavor symmetry for polarized light sea quarks

Theory efforts and global QCD analyses try to keep up interesting physics questions
(*e.g.* sea, large-x behavior, higher-twist, perturbative accuracy ...)

The `NNPDF` collaboration regularly delivers sets of unpolarized/polarized PDF sets
They are determined within a mutually consistent methodology
which allows for faithful uncertainty estimates

A brand-new machine (an EIC?) is however required to
push forward our knowledge of the nucleon spin content significantly

Final remarks

After three decades of experimental and theoretical activity,
we cannot really say we know $\Delta\Sigma$ and Δg
Main culprit: small-x behavior of polarized PDFs

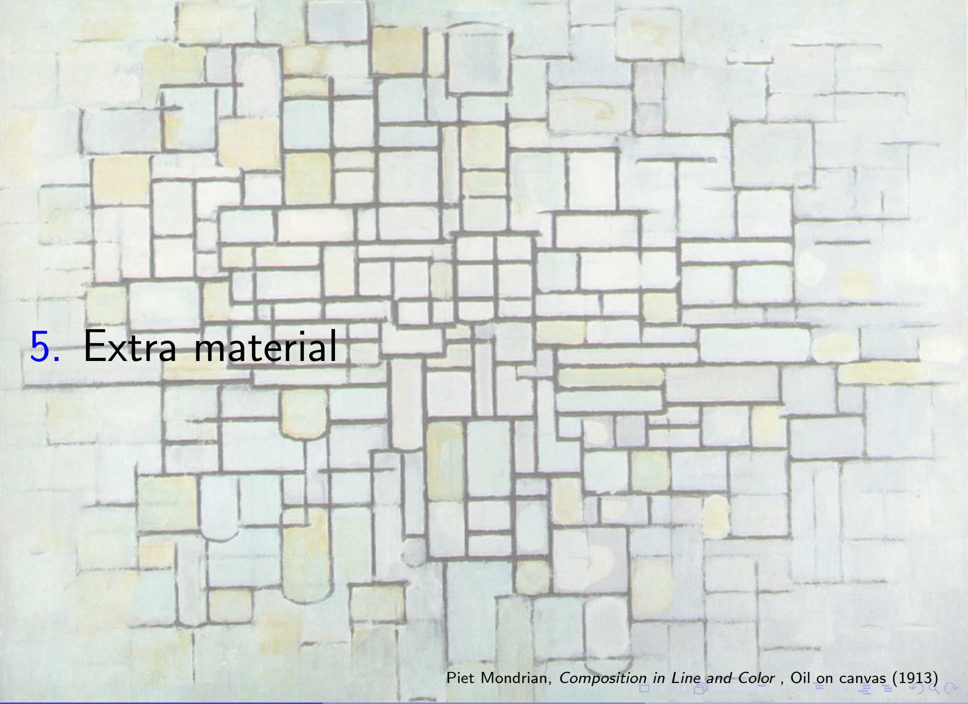
Spin experiments continue to produce high impact results (RHIC, JLAB, ...)
First evidence of a sizable, positive gluon polarization in the proton
First evidence of broken flavor symmetry for polarized light sea quarks

Theory efforts and global QCD analyses try to keep up interesting physics questions
(*e.g.* sea, large-x behavior, higher-twist, perturbative accuracy ...)

The `NNPDF` collaboration regularly delivers sets of unpolarized/polarized PDF sets
They are determined within a mutually consistent methodology
which allows for faithful uncertainty estimates

A brand-new machine (an EIC?) is however required to
push forward our knowledge of the nucleon spin content significantly

Thank you for your attention



5. Extra material

Piet Mondrian, *Composition in Line and Color* , Oil on canvas (1913)

Methodology: Monte Carlo sampling

MONTE CARLO SAMPLING

- Sample the probability density $\mathcal{P}[\Delta q]$ in the space of functions assuming multi-Gaussian data probability distribution

$$g_{1,p}^{(\text{art}),(k)}(x, Q^2) = \left[1 + \sum_c r_{c,p}^{(k)}\sigma_{c,p} + r_{s,p}^{(k)}\sigma_{s,p}\right] g_{1,p}^{(\text{exp})}(x, Q^2)$$

$\sigma_{c,p}$: correlated systematics $\sigma_{s,p}$: statistical errors (also uncorrelated systematics)

$r_{c,p}^{(k)}$, $r_{s,p}^{(k)}$: Gaussian random numbers

- Generate MC ensemble of N_{rep} replicas with the data probability distribution

MAIN FEATURES

- Expectation values for observables are Monte Carlo integrals

$$\langle \mathcal{O}[\Delta q] \rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \mathcal{O}[\Delta q_k]$$

...and the same is true for errors, correlations etc.

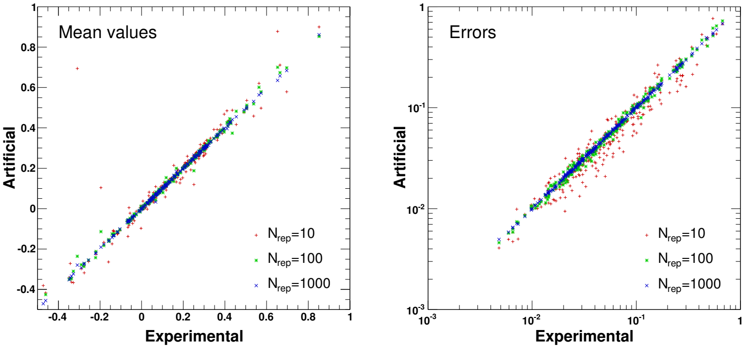
- No need to rely on linear propagation of errors
- Possibility to test for non-Gaussian behaviour in fitted PDFs

Methodology: Monte Carlo sampling

DETERMINING THE SAMPLE SIZE

- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy

Qualitative approach: look at the scatter plots



Accuracy of few % requires $\sim$ 100 replicas

Methodology: Monte Carlo sampling

DETERMINING THE SAMPLE SIZE

- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy

Quantitative approach: devise proper statistical estimators

	$\left\langle \mathrm{PE}\left[\langle g_1^{(\mathrm{art})}\rangle\right]\right\rangle$ [%]			$r\left[g_1^{(\mathrm{art})}\right]$		
N_{rep}	10	100	1000	10	100	1000
EMC	23.7	3.5	2.9	.76037	.99547	.99712
SMC	19.4	5.6	1.2	.94789	.99908	.99993
...	...	...	...	...	...	...

$$\left\langle PE\left[\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right]\right\rangle_{\mathrm{dat}} = \frac{1}{N_{\mathrm{dat}}}\sum_{i=1}^{N_{\mathrm{dat}}}\left|\frac{\langle F_i^{(\mathrm{art})}\rangle_{\mathrm{rep}} - F_i^{(\mathrm{exp})}}{F_i^{(\mathrm{exp})}}\right|$$

Percentage Error

$$r\left[F^{(\mathrm{art})}\right] = \frac{\left\langle F^{(\mathrm{exp})}\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right\rangle_{\mathrm{dat}} - \langle F^{(\mathrm{exp})}\rangle_{\mathrm{dat}}\left\langle\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right\rangle_{\mathrm{dat}}}{\sigma_s^{(\mathrm{exp})}\sigma_s^{(\mathrm{art})}}$$

Scatter Correlation

Accuracy of few % requires $\sim$ 100 replicas

Methodology: Monte Carlo sampling

DETERMINING THE SAMPLE SIZE

- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy

Quantitative approach: devise proper statistical estimators

	$\left\langle \mathrm{PE}\left[\langle\delta g_1^{(\mathrm{art})}\rangle\right]\right\rangle$ [%]			$r\left[\delta g_1^{(\mathrm{art})}\right]$		
N_{rep}	10	100	1000	10	100	1000
EMC	12.8	4.9	2.0	.97397	.99521	.99876
SMC	22.4	5.4	1.7	.96585	.99489	.99980
...	...	...	...	...	...	...

$$\left\langle PE\left[\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right]\right\rangle_{\mathrm{dat}} = \frac{1}{N_{\mathrm{dat}}}\sum_{i=1}^{N_{\mathrm{dat}}}\left|\frac{\langle F_i^{(\mathrm{art})}\rangle_{\mathrm{rep}} - F_i^{(\mathrm{exp})}}{F_i^{(\mathrm{exp})}}\right|$$

Percentage Error

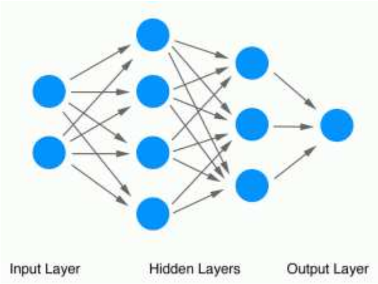
$$r\left[F^{(\mathrm{art})}\right] = \frac{\left\langle F^{(\mathrm{exp})}\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right\rangle_{\mathrm{dat}} - \langle F^{(\mathrm{exp})}\rangle_{\mathrm{dat}}\left\langle\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right\rangle_{\mathrm{dat}}}{\sigma_s^{(\mathrm{exp})}\sigma_s^{(\mathrm{art})}}$$

Scatter Correlation

Accuracy of few % requires $\sim$ 100 replicas

Methodology: neural networks

A convenient functional form
providing redundant and flexible parametrization
used as a generator of random functions in the PDF space



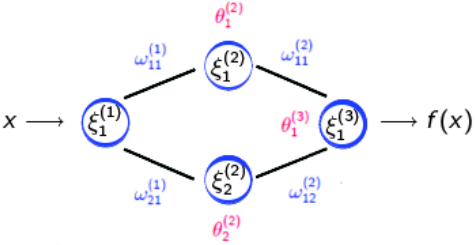
$$\xi_i^{(l)} = g\left(\sum_j^{n_l-1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)}\right)$$

$$g(x) = \frac{1}{1+e^{-x}}$$

- made of neurons grouped into layers (define the architecture)
- each neuron receives input from neurons in preceding layer (feed-forward NN)
- activation determined by parameters (weights and thresholds)
- activation determined according to a non-linear function (except the last layer)

Methodology: neural networks

EXAMPLE: THE SIMPLEST 1-2-1 NN

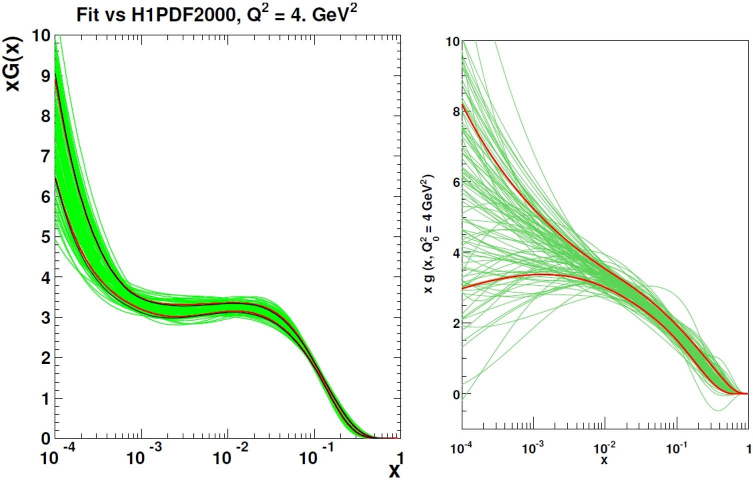


$$f(x) \equiv \xi_1^{(3)} = \left\{ 1 + \exp\left[\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}} \right] \right\}^{-1}$$

Recall: $$\xi_i^{(l)} = g\left(\sum_j^{n_l - 1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)} \right) \quad ; \qquad g(x) = \frac{1}{1 + e^{-x}}$$

Methodology: standard *vs* neural network parametrization

HERA-LHC 2009 PDF benchmark



simple functional forms

neural networks

Methodology: minimization and stopping

GENETIC ALGORITHM

Standard minimization unefficient owing to the large parameter space
and non-local x-dependence of the observables
Genetic algorithm provides better exploration of the whole parameter space

- Set Neural Network parameters randomly
- Make clones of the parameter vector and mutate them
- Define a figure of merit or error function for the k-th replica

$$E^{(k)} = \frac{1}{N_{\text{rep}}} \sum_{i,j=1}^{N_{\text{rep}}} \left(g_{1,i}^{(\text{art})(k)} - g_{1,i}^{(\text{net})(k)} \right) \left((\text{cov})^{-1} \right)_{ij} \left(g_{1,j}^{(\text{art})(k)} - g_{1,j}^{(\text{net})(k)} \right)$$

$g_{1,i}^{(\text{art})(k)}$: generated from Monte Carlo sampling
$g_{1,i}^{(\text{net})(k)}$: computed from Neural Network PDFs

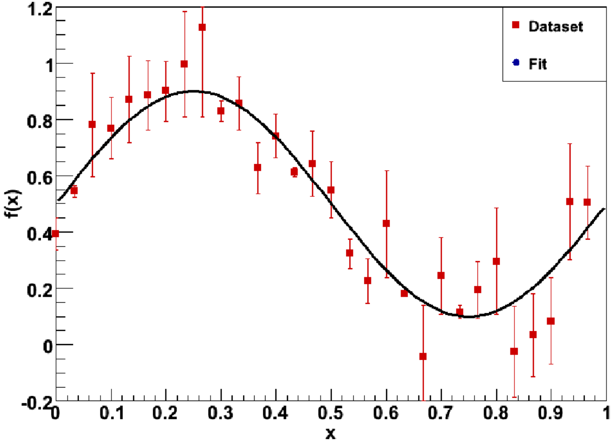
- Select the best set of parameters and perform other manipulations (crossing, mutating, . . .) until stability is reached.

Methodology: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

UNDERLYING PHYSICAL LAW

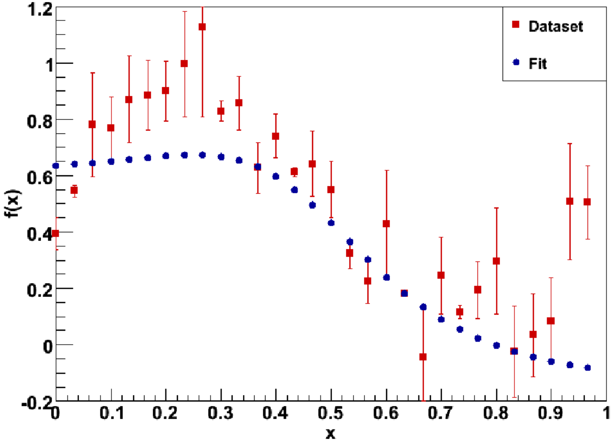


Methodology: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

UNDERLEARNING

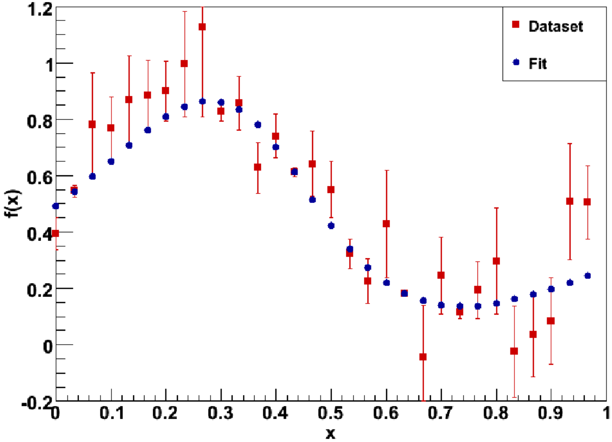


Methodology: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

PROPER LEARNING

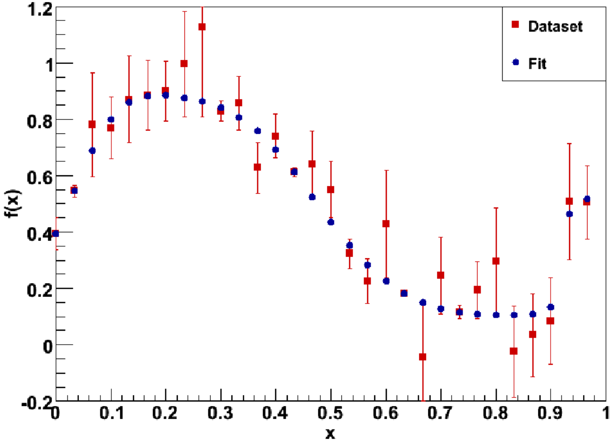


Methodology: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

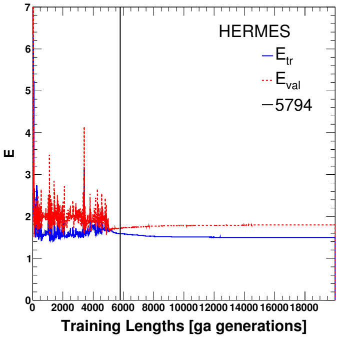
OVERLEARNING



Methodology: minimization and stopping

CROSS-VALIDATION METHOD

- divide data into two subsets (training & validation)
- train the NN on training subset and compute χ^2 for each subset
- stop when χ^2 of validation subset no longer decreases (NN are learning noise!)



The best fit does not coincide with the χ^2 absolute minimum

Methodology: reweighting [PR D58 (1998) 094023]

Assess the impact of including a new data set $\{y\} = \{y_1, \ldots, y_n\}$ in an old PDF set

Bayesian reweighting [NP B849 (2011) 112] [NP B855 (2012) 608]

1. Evaluate the agreement between new data and each replica f_k in a prior ensemble

$$\chi^2_k(\{y\},\{f_k\}) = \sum_{i,j}^{n} \{y_i - y_i[f_k]\}\,\sigma_{ij}\,\{y_j - y_j[f_k]\}$$

2. Apply Bayes theorem to determine the conditional probability of PDF upon the inclusion of the new data and update the probability density in the space of PDFs

$$\mathcal{P}_{\rm new} = \mathcal{N}_\chi \mathcal{P}(\chi^2_k|\{f_k\})\mathcal{P}_{\rm old}(\{f_k\}) \qquad \mathcal{P}(\chi^2_k|\{f_k\}) = [\chi^2_k(\{y\},\{f_k\}]^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi^2_k(\{y\},\{f_k\})}$$

3. Replicas are no longer equally probable. Expectation values are given by

$$\langle \mathcal{O}[f_i(x,Q^2]\rangle_{\rm new} = \sum_{k=1}^{N_{\rm rep}} w_k \mathcal{O}[f_i^{(k)}(x,Q^2)]$$

$$w_k \propto [\chi^2_k(\{y\},\{f_k\})]^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi^2_k(\{y\},\{f_k\})} \quad \text{with} \quad N_{\rm rep} = \sum_{k=1}^{N_{\rm rep}} w_k$$

Methodology: reweighting [PR D58 (1998) 094023]

Assess the impact of including a new data set $\{y\} = \{y_1, \dots, y_n\}$ in an old PDF set

Bayesian reweighting with Hessian PDF sets or Hessian reweighting [JHEP 1412 (2014) 100]

1. Define the function $\chi^2_{\rm new}$

$$\chi^2_{\rm new} \equiv \chi^2\{\mathbf{a}\} + \sum_{i,j}^{n} \{y_i - y_i[f]\}\,\sigma_{ij}\,\{y_j - y_j[f]\}$$

$$y_i[f] \approx y_i[S_0] + \sum_{k=1}^{n_{\rm eig}} D_{ik} w_k \qquad D_{ik} \equiv (y_i[S_k^+] - y_i[S_k^-])/2 \qquad w_k \equiv \sqrt{\epsilon_k}\sum_j^n v_j^{(k)}\delta a_j/\sqrt{\Delta\chi^2}$$

2. The components of $\mathbf{w}_{\rm min}$ specify the set of PDFs corresponding to the new global minimum of $\chi^2_{\rm new}$, which is a continuous, quadratic function of the parameters w_k

$$f^{\rm new} \approx f_{S_0} + \sum_{k=1}^{n_{\rm eig}} \left(\frac{f_{S_k^+} - f_{S_k^-}}{2}\right) w_k^{\rm min}$$

$$\mathbf{w}^{\rm min} = -B^{-1}\mathbf{a} \qquad B_{kn} = \sum_{i,j}^{n} D_{ik}\sigma_{ij}D_{jn} + \Delta\chi^2\delta_{kn} \qquad a_k = \sum_{i,j}^{n} D_{ik}\sigma_{ij}\,(y_i[S_0] - y_i)$$

Reweighting allows for incorporating new datasets without the need of refitting

Methodology: unweighting [NP B855 (2012) 608]

Unweighting allows for constructing an ensemble of equally probable PDFs
statistically equivalent to a given reweighted set
Hence, the new set can be given without weights

IDEA

Given a weighted set of $N_{\rm rep}$ replicas, select (possibly more than once) replicas carrying relatively hight weight and discard replicas carrying relatively small weight

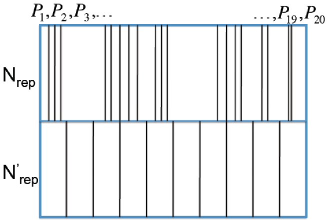
CONSTRUCTION OF THE UNWEIGHTED SET

1. Set the number of replicas $N'_{\rm rep}$ in the unweighted set
   (pointless to choose $N'_{\rm rep} > N_{\rm rep}$: no gain of information)
2. Compute, for the k-th replica of the reweighted set, the integer number

$$w'_k = \sum_{j=1}^{N'_{\rm rep}} \theta\left(\frac{j}{N'_{\rm rep}} - P_{k-1}\right)\theta\left(P_k - \frac{j}{N'_{\rm rep}}\right)\,, \qquad P_k = \sum_{j=0}^{k} \frac{w_j}{N_{\rm rep}}\,, \qquad \sum_{k=1}^{N_{\rm rep}} w'_k = N'_{\rm rep}$$

3. Construct the unweighted set taking w'_k copies of the k-th replica, $k = 1, \ldots, N_{\rm rep}$

Methodology: unweighting [NP B855 (2012) 608]



CONSTRUCTION OF THE UNWEIGHTED SET

1. Set the number of replicas $N'_{\rm rep}$ in the unweighted set
   (pointless to choose $N'_{\rm rep} > N_{\rm rep}$: no gain of information)
2. Compute, for the k-th replica of the reweighted set, the integer number

$$w'_k = \sum_{j=1}^{N'_{\rm rep}} \theta\left(\frac{j}{N'_{\rm rep}} - P_{k-1}\right)\theta\left(P_k - \frac{j}{N'_{\rm rep}}\right) , \quad P_k = \sum_{j=0}^{k} \frac{w_j}{N_{\rm rep}} , \quad \sum_{k=1}^{N_{\rm rep}} w'_k = N'_{\rm rep}$$

3. Construct the unweighted set taking w'_k copies of the k-th replica, $k = 1, \ldots, N_{\rm rep}$

NNPDFpol1.1: open-charm production at COMPASS

$$A^{\gamma N \to D^0 X} = \frac{\Delta g \otimes \Delta\hat{\sigma}_{\gamma g} \otimes D_c^H}{g \otimes \hat{\sigma}_{\gamma g} \otimes D_c^H}$$

Virtual photon-nucleon asymmetry for open-charm production
[arXiv:1212.1319]

FEATURES

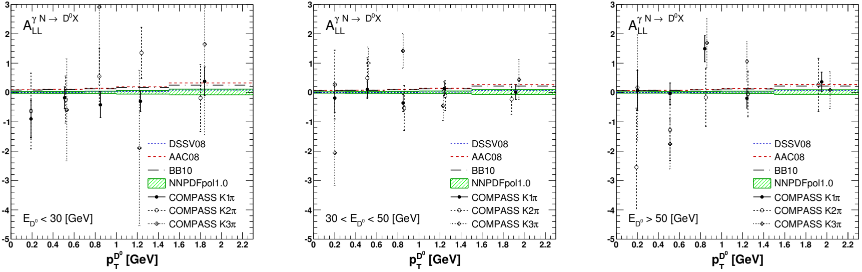
- Δg is probed *directly* through the photon-gluon fusion process
  (in DIS Δg is mostly probed through scaling violations instead)
- the fragmentation functions for heavy quarks are computable in perturbation theory
  (and only introduce a very moderate uncertainty in the fit)

EXPERIMENTAL MEASUREMENT

- COMPASS (2002-2007) [arXiv:1211.6849]

Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$ NNPDFpol1.0	DSSV08	AAC08	BB10
COMPASS		45	1.23	1.23	1.27	1.25
	COMPASS $K1\pi$	15	1.27	1.27	1.43	1.38
	COMPASS $K2\pi$	15	0.51	0.51	0.56	0.55
	COMPASS $K3\pi$	15	1.90	1.90	1.81	1.82

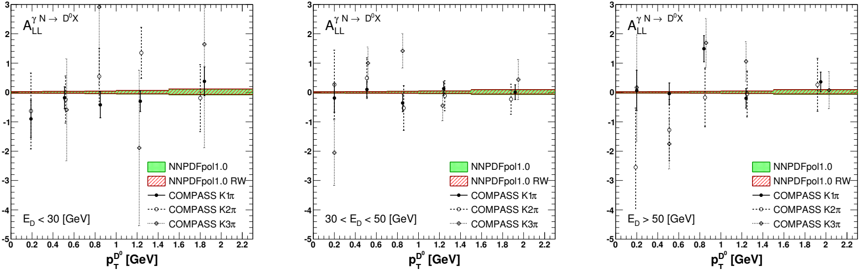
NNPDFpol1.1: open-charm production at COMPASS



Data are affected by large uncertainties w.r.t. the uncertainty due to PDFs
They do not show a clear trend

Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$ NNPDFpol1.0	DSSV08	AAC08	BB10
COMPASS		45	1.23	1.23	1.27	1.25
	COMPASS $K1\pi$	15	1.27	1.27	1.43	1.38
	COMPASS $K2\pi$	15	0.51	0.51	0.56	0.55
	COMPASS $K3\pi$	15	1.90	1.90	1.81	1.82

NNPDFpol1.1: open-charm production at COMPASS



The impact of open-charm data from COMPASS is mostly negligible, as we notice from the value of the $\chi^2/N_{\rm ndat}$ and the reweighted observable

Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$	$\chi^2_{\rm rw}/N_{\rm dat}$
COMPASS		45	1.23	1.23
	COMPASS $K1\pi$	15	1.27	1.27
	COMPASS $K2\pi$	15	0.51	0.51
	COMPASS $K3\pi$	15	1.90	1.89

NNPDFpol1.1: inclusive jet production at RHIC

$$A_{LL}^{1jet} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$

Longitudinal double-spin asymmetry for single-inclusive jet production
[arXiv:hep-ph/9808262] [arXiv:hep-ph/0404057] [arXiv:1209.1785]

FEATURES

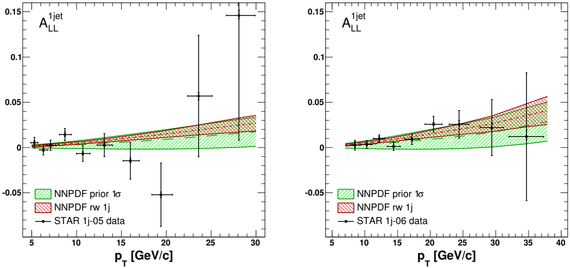
- sensitive to the polarized gluon Δg
  (receives leading contribution from $gq \to qg$ and $qg \to qg$ partonic subrocesses)

EXPERIMENTAL MEASUREMENT

- STAR 2005, 2006 [arXiv:1205.2735], 2009 [arXiv:1405.5134]
- PHENIX [arXiv:1009.4921] at RHIC

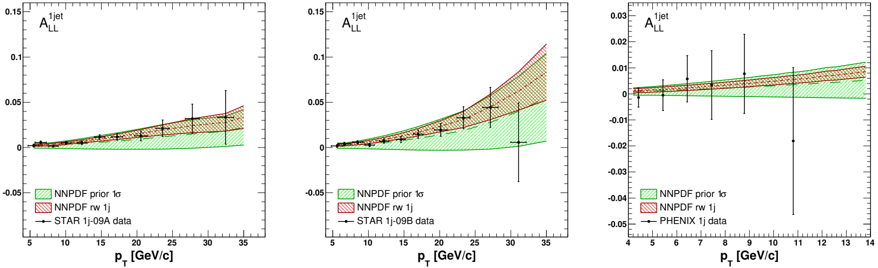
Data set	$N_{\rm dat}$	jet-algorithm	R	$[\eta_{\rm min}, \eta_{\rm max}]$	$\sqrt{s}$ [GeV]	$\mathcal{L}$ [pb^{-1}]
STAR 1j-05	10	midpoint-cone	0.4	$[+0.20, +0.80]$	200	2.1
STAR 1j-06	9	midpoint-cone	0.7	$[-0.70, +0.90]$	200	5.5
STAR 1j-09A	11	anti-k_t	0.6	$[-0.50, +0.50]$	200	25
STAR 1j-09B	11	anti-k_t	0.6	$[-1.00, -0.50]$ $[+0.50, +1.00]$	200	25
PHENIX 1j	6	seeded-cone	0.3	$[-0.35, +0.35]$	200	2.1

NNPDFpol1.1: inclusive jet production at RHIC



Experiment	Set	N_{dat}	χ^2/N_{dat}				$\chi^2_{\mathrm{rw}}/N_{\mathrm{dat}}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR		41	1.50	1.49	1.50	1.50	1.05	1.04	1.04	1.04
	STAR 1j-05	10	1.04	1.05	1.04	1.04	1.01	1.02	1.02	1.02
	STAR 1j-06	9	0.75	0.76	0.76	0.76	0.59	0.58	0.59	0.59
	STAR 1j-09A	11	1.40	1.39	1.39	1.40	0.98	0.99	0.98	0.98
	STAR 1j-09B	11	3.04	3.05	3.03	3.05	1.18	1.17	1.17	1.18
PHENIX										
	PHENIX 1j	6	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
		47	1.35	1.35	1.35	1.36	1.00	1.01	1.01	1.00

NNPDFpol1.1: inclusive jet production at RHIC



Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$				$\chi^2_{\rm rw}/N_{\rm dat}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR		41	1.50	1.49	1.50	1.50	1.05	1.04	1.04	1.04
	STAR 1j-05	10	1.04	1.05	1.04	1.04	1.01	1.02	1.02	1.02
	STAR 1j-06	9	0.75	0.76	0.76	0.76	0.59	0.58	0.59	0.59
	STAR 1j-09A	11	1.40	1.39	1.39	1.40	0.98	0.99	0.98	0.98
	STAR 1j-09B	11	3.04	3.05	3.03	3.05	1.18	1.17	1.17	1.18
PHENIX										
	PHENIX 1j	6	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
		47	1.35	1.35	1.35	1.36	1.00	1.01	1.01	1.00

NNPDFpol1.1: $W^{\pm}$ production at RHIC

$$A_L^{W^\pm} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \qquad A_{LL}^{W^\pm} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$

$$A_L^{W^+} \sim \frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)} \qquad A_L^{W^-} \sim \frac{\Delta d(x_1)\bar{u}(x_2) - \Delta\bar{u}(x_1)d(x_2)}{d(x_1)\bar{u}(x_2) + \bar{u}(x_1)d(x_2)}$$

Longitudinal single-spin asymmetry for $W^{\pm}$ boson production

[arXiv:1003.4533]

FEATURES

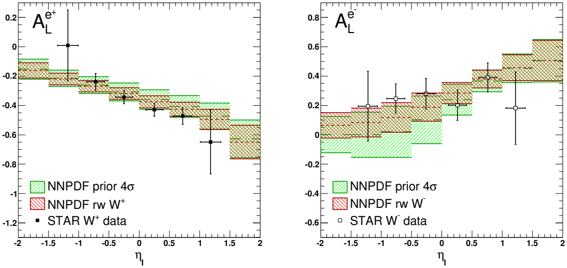
- sensitive to individual quark and antiquark flavours $(\Delta u, \Delta\bar{u}, \Delta d, \Delta\bar{d})$
  (purely weak process coupling q_L with $\bar{q}_R$ at partonic level, $u_L\bar{d}_R \to W^+$ or $d_L\bar{u}_R \to W^-$)
- no need for fragmentation functions (instead of SIDIS)

EXPERIMENTAL MEASUREMENT

- STAR and PHENIX at RHIC [arXiv:1009.0326] [arXiv:1009.0505] [arXiv:1404.6880]

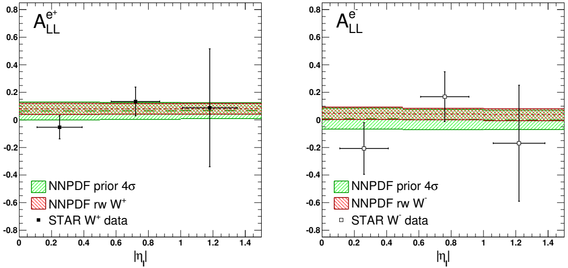
Data set	$N_{\rm dat}$	$[p_{T,\rm min}, p_{T,\rm max}]$ [GeV]	$\sqrt{s}$ [GeV]	$\mathcal{L}$ [pb^{-1}]
STAR-W^+ (prel.)	6	[25, 50]	510	72
STAR-W^- (prel.)	6	[25, 50]	510	72

NNPDFpol1.1: $W^{\pm}$ production at RHIC



Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$				$\chi^2_{\rm rw}/N_{\rm dat}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR-A_L		12	1.38	1.44	1.39	1.33	1.08	0.88	0.74	0.74
	STAR-$A_L^{W^+}$	6	0.75	0.75	0.86	0.90	0.75	0.75	0.68	0.70
	STAR-$A_L^{W^-}$	6	1.92	2.03	1.82	1.67	1.32	1.08	0.83	0.82
STAR-A_{LL}		6	0.82	0.81	0.78	0.78	0.82	0.80	0.76	0.76
	STAR-$A_{LL}^{W^+}$	3	0.92	0.88	0.81	0.80	0.90	0.85	0.77	0.76
	STAR-$A_{LL}^{W^-}$	3	0.73	0.74	0.75	0.76	0.73	0.74	0.75	0.76
		18	1.19	1.20	1.15	1.15	1.00	0.87	0.78	0.77

NNPDFpol1.1: $W^{\pm}$ production at RHIC



Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$				$\chi^2_{\rm rw}/N_{\rm dat}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR-A_L		12	1.38	1.44	1.39	1.33	1.08	0.88	0.74	0.74
	STAR-$A_L^{W^+}$	6	0.75	0.75	0.86	0.90	0.75	0.75	0.68	0.70
	STAR-$A_L^{W^-}$	6	1.92	2.03	1.82	1.67	1.32	1.08	0.83	0.82
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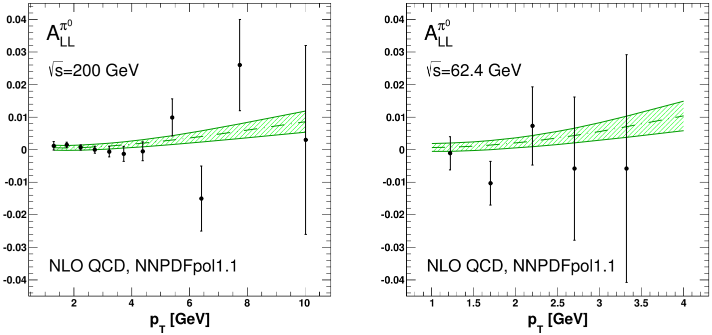
NNPDFpol1.1: single-hadron production at RHIC

$$A_{LL}^{H} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma{+-}} = \frac{\sum_{a,b,c=q,\bar{q},g} f_a \otimes f_b \otimes D_c^H \otimes \Delta\hat{\sigma}_{ab}^c}{\sum_{a,b,c=q,\bar{q},g} f_a \otimes f_b \otimes D_c^H \otimes \hat{\sigma}_{ab}^c}$$

PHENIX [arXiv:0810.0701] [arXiv:0810.0694] [arXiv:1402.6296] STAR [arXiv:1309.1800]



- Good agreement between experimental data and theoretical predictions
- Experimental uncertainties are larger than than those of the corresponding predictions
- We expect a slight impact on the gluon PDF from these data

NNPDFpol1.1: single-hadron production at RHIC

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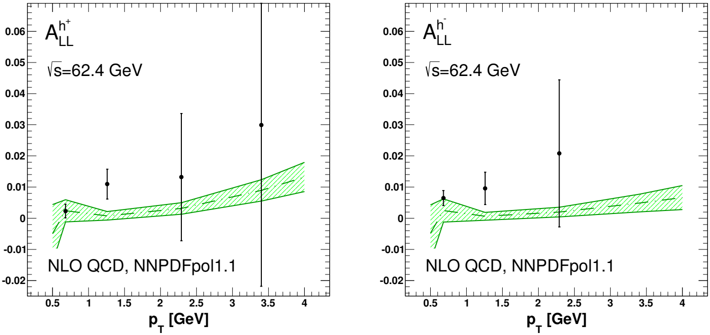

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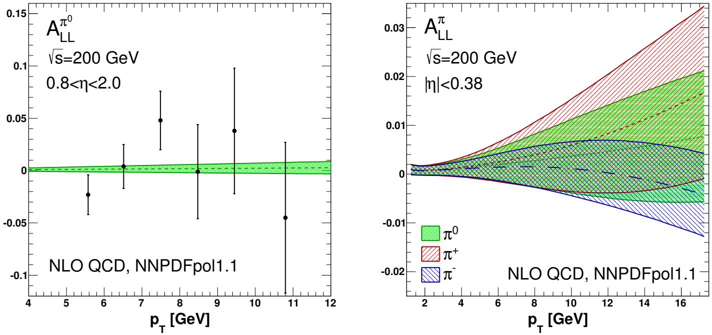

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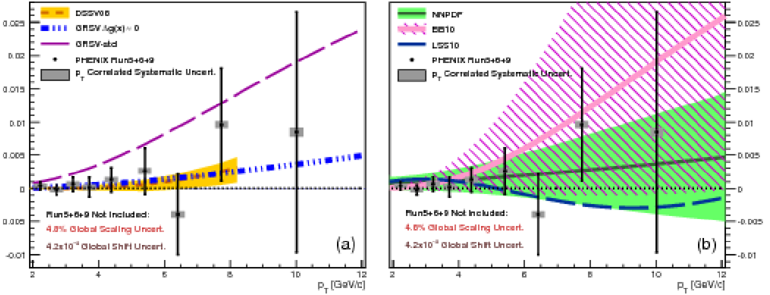

[arXiv:1402.6296]

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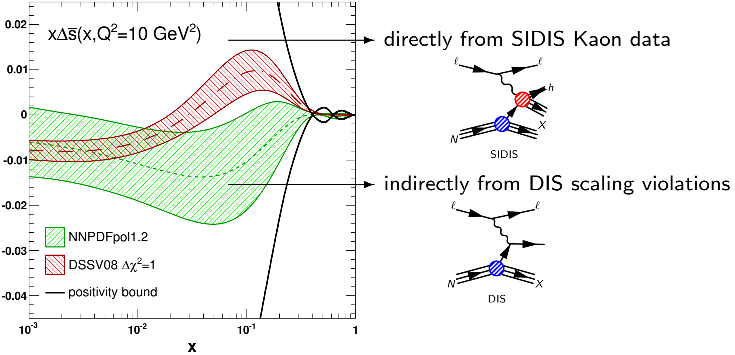
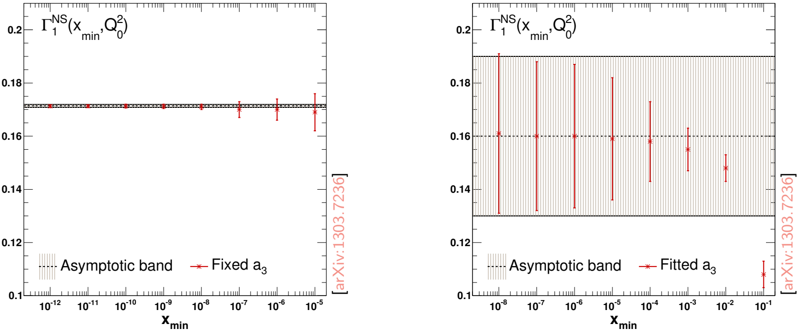
Open issues: strangeness



NNPDFpol1.2: DIS ☑, SIDIS ($K^{\pm}$) ☒; DSSV08: DIS ☑, SIDIS ($K^{\pm}$) ☑;

- assume $\Delta s = \Delta\bar{s}$, which may not be true [PR D71 (2005) 094014]
- DIS data ⇒ negative $x\Delta\bar{s}$; SIDIS data ⇒ changing-sing $x\Delta\bar{s}$
- New, very precise, JLAB data (DIS) point to negative $x\Delta s$ [PR D91 (2015) 054017]
- Is there mounting tension between DIS and SIDIS data?
- How well do we know K fragmentation functions? [PR D84 (2011) 014002]

Open issues: the Bjorken sum rule

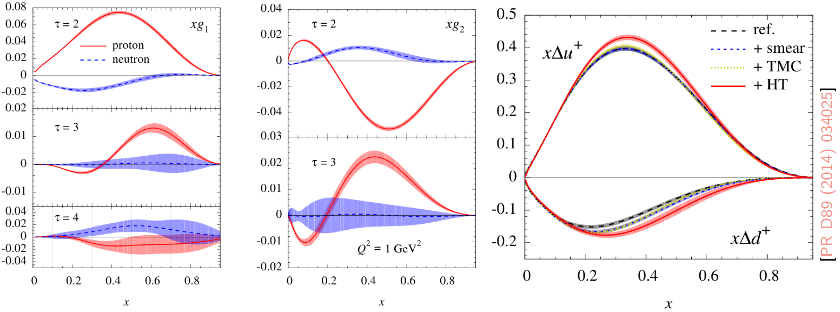


fixed $a_3 = 1.2701 \pm 0.0025$ fitted $a_3 = 1.19 \pm 0.22$

$$\Gamma_1^{\rm NS}(x_{\rm min}, Q^2) \equiv \int_{x_{\rm min}}^1 dx[g_1^p(x,Q^2) - g_1^n(x,Q^2)] \xrightarrow{x_{\rm min}=0} \frac{1}{6} a_3(Q^2)\Delta C_{\rm NS}[\alpha_s(Q^2)]$$

$$a_3(Q^2) = \int_0^1 dx \left[\Delta u(x,Q^2) + \Delta\bar{u}(x,Q^2) - \Delta d(x,Q^2) - \Delta\bar{d}(x,Q^2)\right]$$

Theory: higher-twist corrections and JAM13 [PR D89 (2014) 034025]



- leading-twist factorization of g_1 and g_2 receives contributions from higher-twist terms

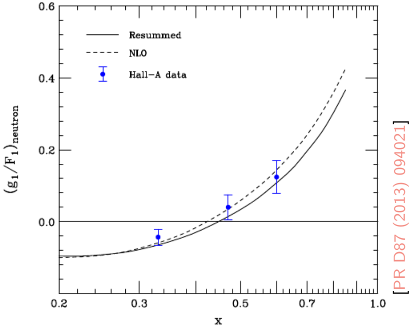
$$g_1 = g_1^{\tau=2} + g_1^{\tau=3} + g_1^{\tau=4} \qquad g_2 = g_2^{\tau=2} + g_2^{\tau=3}$$

→ $g_2^{\tau=2}$ can be related to $g_1^{\tau=2}$ via Wandzura-Wilckzek relation [PL B72 (1977) 195]
→ $g_2^{\tau=3}$ can be related to $g_1^{\tau=3}$ via Blümlein-Tkablaze identity [NP B553 (1999) 427]
→ $g_2^{\tau=3}$ can be parametrized (using *e.g.* the form by Braun *et al.*) [PR D83 (2011) 094023]
→ $g_1^{\tau=4}$ can be parametrized as $g_1^{\tau=4}(x, Q^2) = h(x)/Q^2$ (D. Hui)

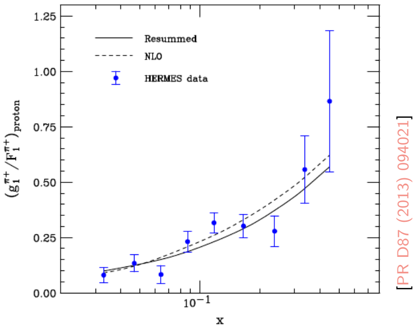
- higher twists to both g_1 and g_2 are included in JAM13
- higher twist contributions are sizable and are needed for describing JLAB data properly

Theory: all-order resummation [PR D87 (2013) 094021]

DIS: Hall A data [PR C70 (2004) 065207]

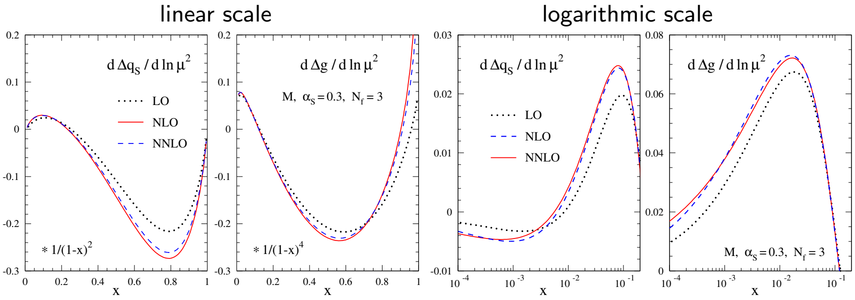


SIDIS: HERMES data [PR D71 (2005) 012003]



- resummation of large logarithm corrections to spin asymmetries in DIS and SIDIS
- asymmetries are rather insensitive to the inclusion of resummed higher-order terms
- modest decrease of spin asymmetries at fairly high x values, more pronounced for SIDIS
- most relevant for JLAB kinematics, important for future high statistic JLAB12

Theory: higher-order computations ($\overline{\mathrm{MS}}$) [NP B889 (2014) 351]

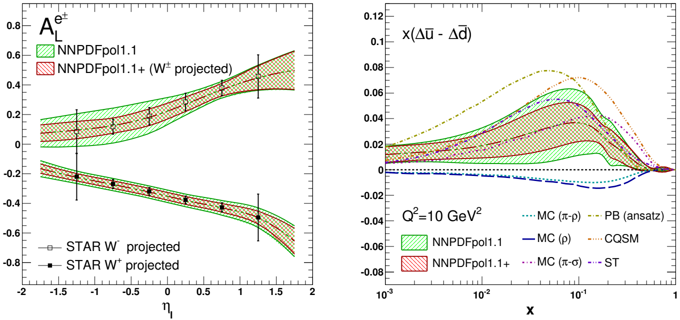


[NP B889 (2014) 351]

- NNLO (three-loop) corrections to spin-dependent splitting functions have been computed
- NNLO corrections to the splitting functions are small outside the region of small x
- corrections to the evolution of the PDFs can be unproblematic down to $x \approx 10^{-4}$
- QCD analyses of polarized PDFs are now feasible up to NNLO accuracy
  → only in a FFN scheme (VFN would require non-trivial unknown matching conditions)
  → only including DIS data (coefficient functions are know at NNLO only for DIS)

Opportunities at RHIC

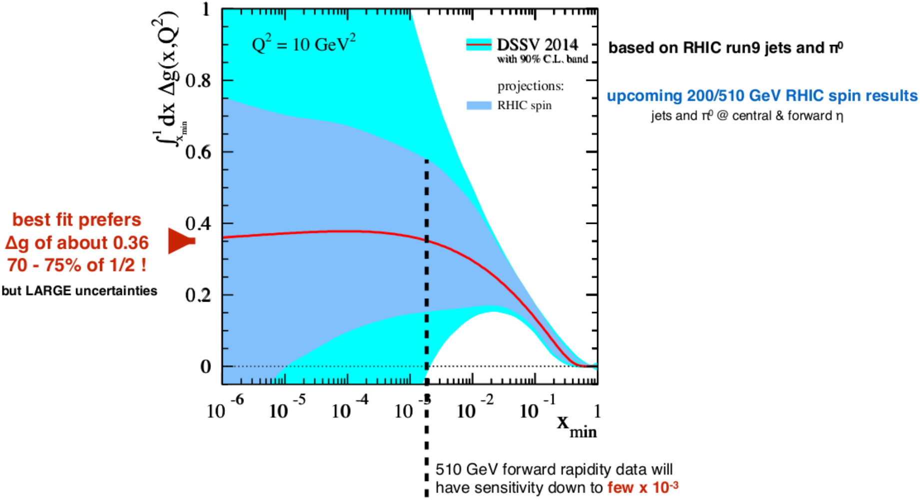
PINNING DOWN THE LIGHT POLARIZED SEA ASYMMETRY



	$\int_{10^{-3}}^{1} dx \Delta f(x, Q^2)$			$\int_{0.05}^{0.4} dx \Delta f(x, Q^2)$		
	cv	*unc* (pol1.1)	*unc* (pol1.1+)	*cv*	*unc* (pol1.1)	*unc* (pol1.1+)
Δu^+	$+0.764$	± 0.035	± 0.034	$+0.523$	± 0.014	± 0.013
Δd^+	-0.407	± 0.037	± 0.036	-0.231	± 0.018	± 0.018
$\Delta \bar{u}$	$+0.044$	± 0.046	± 0.030	$+0.019$	± 0.023	± 0.012
$\Delta \bar{d}$	-0.088	± 0.067	± 0.032	-0.037	± 0.021	± 0.013
$\Delta_{\rm sea}$	$+0.123$	± 0.076	± 0.038	$+0.056$	± 0.030	± 0.016

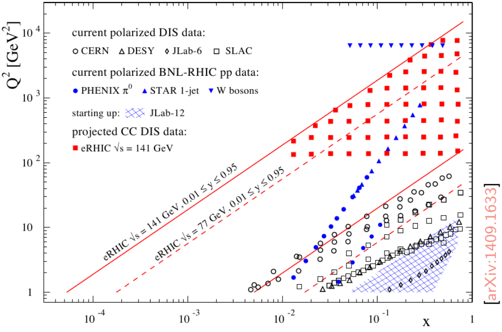
Opportunities at RHIC

PINNING DOWN THE GLUON POLARIZATION



[M. Stratmann, *Talk at HiX2014*]

Opportunities at a future Electron-Ion Collider



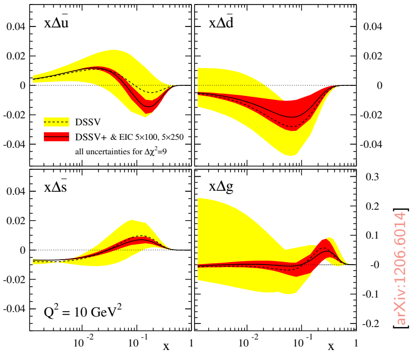
REQUIREMENTS & FEATURES

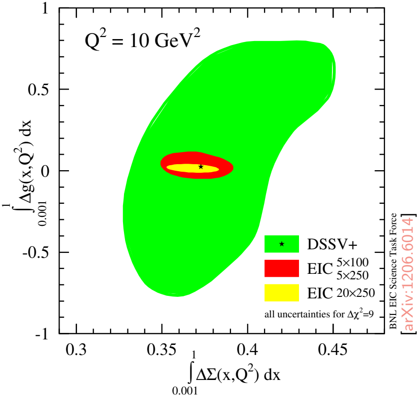
- large kinematic reach
  → high-energy collider
- precision of electromagnetic probes
  → electron beams
- spin
  → polarized hadron beams
- versatility
  → heavy ion beams

DELIVERABLES	OBSERVABLES	WHAT WE LEARN
Δg	scaling violations in DIS	gluon contribution to proton spin
Δq, $\Delta\bar{q}$	SIDIS for pions and kaons	quark contribution to proton spin; flavor asymmetry $\Delta\bar{u} - \Delta\bar{d}$; strangeness Δs
$g_1^{W^-}$, $g_5^{W^-}$	inclusive CC DIS at high Q^2	flavor separation at medium x and high Q^2

Opportunities at a future Electron-Ion Collider

- Dramatic reduction of uncertainties of both PDFs and their moments [arXiv:1206.6014]
- Accurate determination of Δg via scaling violations in DIS [arXiv:1206.6014] [arXiv:1310.0461]
- Accurate determination of $\Delta\bar{u}$, $\Delta\bar{d}$ via SIDIS and CC DIS [arXiv:1309.5327]
- Access to unknown electroweak structure functions [arXiv:1309.5327]





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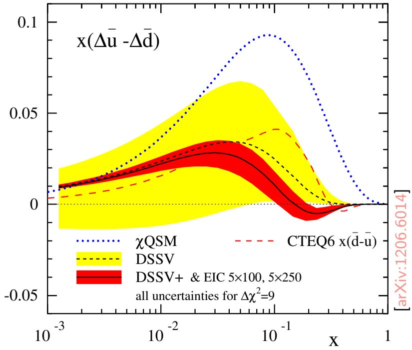

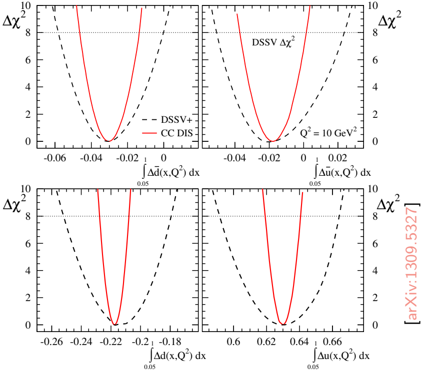

The emerging picture of the polarized nucleon