Towards Small-x Phenomenology

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Related to work with the NNPDF collaboration

However....

I've been infected!

presented by

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Oxford Physics

UNIVERSITY OF OXFORD

PDF4BSM

Parton Distributions in the Higgs Boson Era

erc European Research Council

Logarithmic enhancement: resummation

Single (double) logarithmic enhancements

$$\alpha_s^k \log^j \qquad 0 \le j \le (2)k$$

If/when

$$\alpha_s \log^{(2)} \sim 1$$

all such terms in the perturbative series are equally important:

all-order RESUMMATION

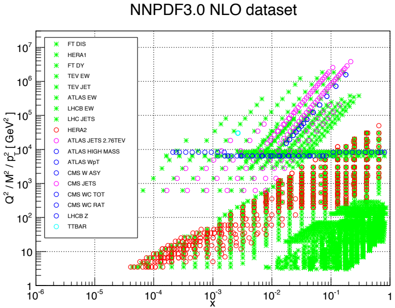
resummed phenomenology $\rightarrow$ requires resummed PDFs

Goals of resummations in PDF fits:

- provide PDFs consistent with resummed computations
- improve the quality of PDF fits
- investigate the impact of higher orders (and thus estimate the uncertainty from missing higher orders)
- getting closer to "all-order PDFs"

Large-x: threshold resummation

- $x \to 1$
- due to soft gluon emissions
- resums double logs $\left(\frac{\log^k(1-x)}{1-x}\right)_+$
- in Mellin space, $\log N$ at $N \to \infty$
- [MB,Marzani,Rojo,Rottoli,Ubiali,Ball,Bertone, Carrazza,Hartland 1507.01006]



Small-x: high-energy (BFKL) resummation

- $x \to 0$
- due to high-energy gluon emissions
- resums single logs $\frac{1}{x}\log^k x$
- in Mellin space, poles $1/(N-1)$ in the limit $N \to 1$
- [MB,Marzani,Peraro 1607.02153]
  [MB,Marzani,Muselli,Peraro 17xx.xxxxx] ← **this talk**
  [NNPDF (in progress)]

Resum what?

Observable: $$\sigma = \sigma_0\, C(\alpha_s(\mu)) \otimes f(\mu) \left[\otimes f(\mu)\right]$$

Evolution: $$\mu^2 \frac{d}{d\mu^2} f(\mu) = P(\alpha_s(\mu)) \otimes f(\mu)$$

Any object with a perturbative expansion and a log enhancement:

- coefficient functions $C(\alpha_s(\mu))$ (observable)
- splitting functions $P(\alpha_s(\mu))$ (evolution)

	observable coefficient functions $C(\alpha_s(\mu))$	evolution splitting functions $P(\alpha_s(\mu))$
large-x	(N)NNLL	—
small-x	LLx (means largest non-vanishing log order)	NLLx

Small-x resummation: overview

Small-x resummation based on k_t-factorization and BFKL

Developed in the 90s-00s [Catani,Ciafaloni,Colferai,Hautmann,Salam,Stasto] [Altarelli,Ball,Forte] [Thorne,White]

Affects both evolution (known to LLx and NLLx) and coefficient functions (known only at lowest logarithmic order, which is often NLLx) in the **singlet** sector

We follow the ABF [Altarelli,Ball,Forte 1995,...,2008] procedure to resum splitting functions and develop a new formalism for coefficient functions [MB,Marzani,Peraro 1607.02153]
We are now proposing further improvements to the ABF formalism [MB,Marzani,Muselli,Peraro 17xx.xxxxx]

We published (and keep developing) a public code

`HELL: High-Energy Large Logarithms` `www.ge.infn.it/∼bonvini/hell`

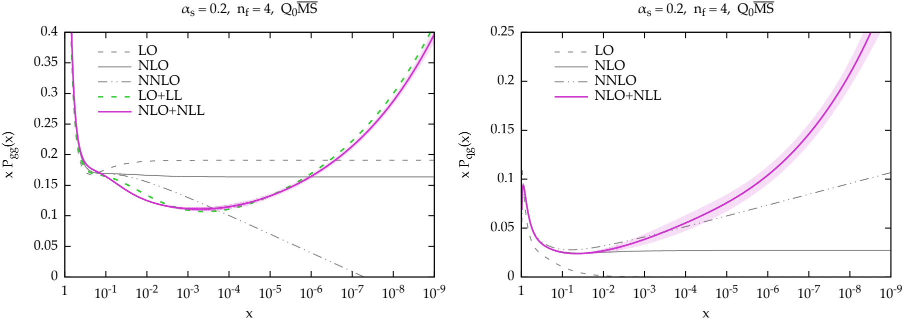
which delivers resummed splitting functions and coefficient functions

`HELL` has been interfaced to `APFEL` `apfel.hepforge.org`
opening the door to its usage for PDF fitting

Small-x resummation of DGLAP evolution

Ingredients (ABF):

- duality with BFKL evolution
- symmetry of the BFKL kernel
- momentum conservation
- resummation of (subleading, but fundamental) running coupling effects

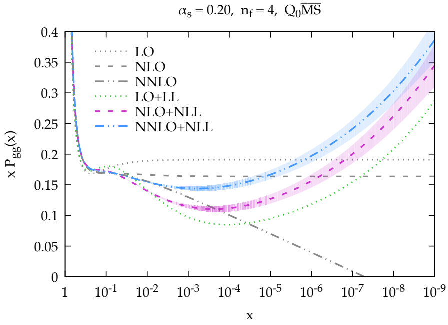


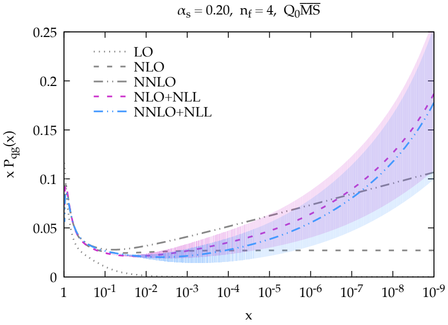
[MB,Marzani,Peraro 1607.02153]

Until yesterday, resummation matched only to NLO
NNLO+NLLx is practically complicated

We have matched NLLx resummation to NNLO [MB,Marzani,Muselli,Peraro 17xx.xxxxx]

En passant, several theoretical improvements have been implemented, and a couple of issues solved (details are very technical)





PRELIMINARY

Small-x resummation of coefficient functions

High-energy (k_T) factorization:

$$\sigma \sim \int \frac{dz}{z} \int d^2\boldsymbol{k}\, \hat\sigma_g\left(\frac{x}{z}, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s(Q^2)\right) \mathcal{F}_g(z, \boldsymbol{k})$$

$$\begin{cases} \mathcal{F}_g(x,\boldsymbol{k}) : \text{unintegrated PDF} \\ \hat\sigma_g\left(z, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) : \text{off-shell xs} \end{cases}$$

Collinear factorization

$$\sigma \sim \int \frac{dz}{z}\, C_g\left(\frac{x}{z}, \alpha_s(Q^2)\right) f_g(z, Q^2)$$

$$\begin{cases} f_g(x,Q^2) : \text{standard PDF} \\ C_g(z,\alpha_s) : \text{on-shell coefficient function} \end{cases}$$

Defining

$$\mathcal{F}_g(N,\boldsymbol{k}) = U\left(N, \frac{\boldsymbol{k}^2}{Q^2}\right) f_g(N, Q^2)$$

we get

[MB,Marzani,Peraro 1607.02153]

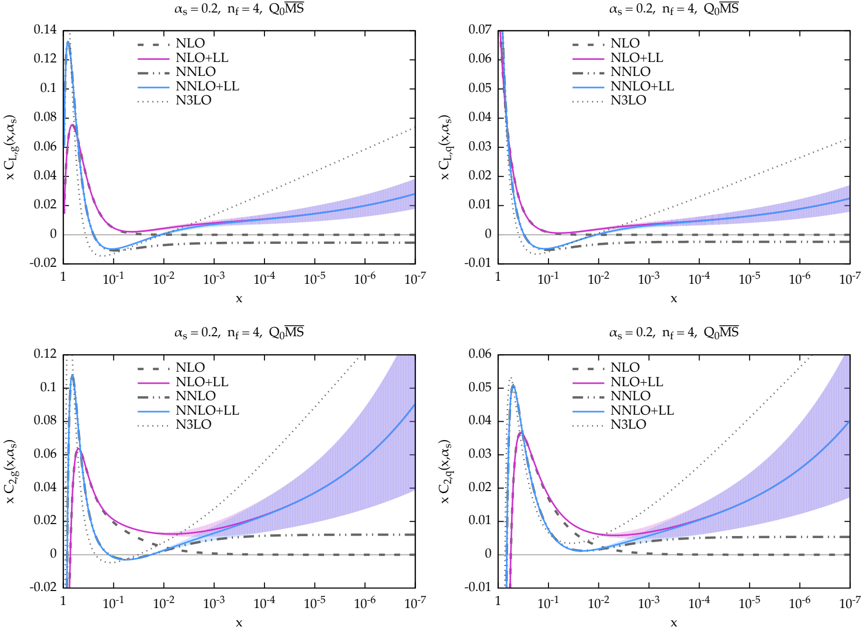
$$C_g(N,\alpha_s) = \int d^2\boldsymbol{k}\, \hat\sigma_g\left(N, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) U\left(N, \frac{\boldsymbol{k}^2}{Q^2}\right)$$

At LLx accuracy, U has a simple form, in terms of small-x resummed anom dim γ

$$U\left(N, \frac{\boldsymbol{k}^2}{Q^2}\right) \approx \boldsymbol{k}^2 \frac{d}{d\boldsymbol{k}^2} \exp \int_{Q^2}^{\boldsymbol{k}^2} \frac{d\nu^2}{\nu^2} \gamma(N, \alpha_s(\nu^2))$$

- Only known at LLx
- Just uses the off-shell cross sections $\hat\sigma(N, Q^2/\boldsymbol{k}^2, \alpha_s)$ (one for each process)
- Formally equivalent to ABF (practically easier and numerically stabler)

Small-x resummation in massless DIS (new results)



We now have all the ingredients for a PDF fit to DIS data:

- massive DIS coefficient functions
- implementation of VFNS (à la FONLL = S-ACOT)
- resummed matching conditions

The off-shell coefficients for heavy-quark production in DIS were already available [Catani,Hautmann 1994], now implemented in HELL

VFNS rather simple at LLx (see also [Thorne,White 0603030, 0611204] in DIS scheme)

$$C_{L,g}^{[n_f+1]}(m) = C_{L,g}^{[n_f]}(m)$$
$$C_{2,g}^{[n_f+1]}(m) = C_{2,g}^{[n_f]}(m) - K_{hg}(m)$$

Matching conditions K_{ij}

$$f_i^{[n_f+1]} = \sum_{j=g,q_1,\bar{q}_1,\ldots,q_{n_f},\bar{q}_{n_f}} K_{ij}(m) f_j^{[n_f]}, \qquad i = g, q_1, \bar{q}_1, \ldots, q_{n_f}, \bar{q}_{n_f}, q_{n_f+1}, \bar{q}_{n_f+1},$$

now resummed in $\overline{\text{MS}}$ and implemented in HELL

Towards a global small-x resummed fit

For a global fit we need additional process:

- Drell-Yan
- jets
- ...

Drell-Yan rapidity distributions have never been resummed in the ABF formalism due to technical difficulties $\rightarrow$ should be simpler and doable with the new formalism (no need to compute M-Mellin transform)

Do we need small-x resummation for jets? And for other processes?

Future work will be oriented in this direction

In the meantime, we have performed some **PRELIMINARY** fits to DIS-only data at NLO+NLLx and NNLO+NLLx

(thanks to Luca Rottoli, Valerio Bertone and Nathan Hartland!)

Fit results: χ^2

PRELIMINARY

Experiment	Dataset	DOF	Current ^{2}	Reference ^{2}	Current ^{2}	Reference ^{2}
NMC		325	1.28627	1.28758	1.27370	1.28742
	NMCPD	121	0.92561	0.91453	0.89964	0.91669
	NMC	204	1.50019	1.50885	1.49558	1.50731
SLAC		67	1.00929	1.01376	0.87304	0.83805
	SLACP	33	1.02654	1.02160	0.86848	0.84938
	SLACD	34	0.94462	0.96066	0.83496	0.78457
BCDMS		581	1.18540	1.19748	1.20455	1.21112
	BCDMSP	333	1.23390	1.25020	1.26626	1.27350
	BCDMSD	248	1.11995	1.12657	1.12252	1.12718
CHORUS		832	0.97194	0.97820	0.98387	0.97908
	CHORUSNU	416	0.93686	0.93564	0.94292	0.94093
	CHORUSNB	416	0.97409	0.98321	0.99881	0.99085
NTVDMN		76	0.64439	0.67227	0.69993	0.69213
	NTVNUDMN	39	0.62988	0.55987	0.63087	0.70683
	NTVNBDMN	37	0.64793	0.78956	0.76609	0.67187
HERACOMB		1145	1.12111	1.13084	1.12411	1.17376
	HERACOMBNCEM	159	1.45607	1.44595	1.44561	1.44855
	HERACOMBNCEP460	204	1.07735	1.09569	1.07618	1.09723
	HERACOMBNCEP575	254	0.87031	0.87236	0.86894	0.91757
	HERACOMBNCEP820	70	1.00489	1.04616	1.04623	1.18655
	HERACOMBNCEP920	377	1.17811	1.18217	1.18983	1.27363
	HERACOMBCCEM	42	0.94844	0.96002	0.96945	1.00185
	HERACOMBCCEP	39	1.30369	1.29350	1.23654	1.21963
HERAF2CHARM		47	2.15652	1.75245	1.75765	1.62864
F2BOTTOM		29	1.00797	1.01885	1.05043	1.10405
	H1HERAF2B	12	0.77889	0.76393	0.75769	0.81308
	ZEUSHERAF2B	17	1.16968	1.19879	1.25708	1.30944
Total (exps)		**3102**	**1.11098**	**1.11341**	**1.10824**	**1.12602**

NLO+NLLx NLO NNLO+NLLx NNLO

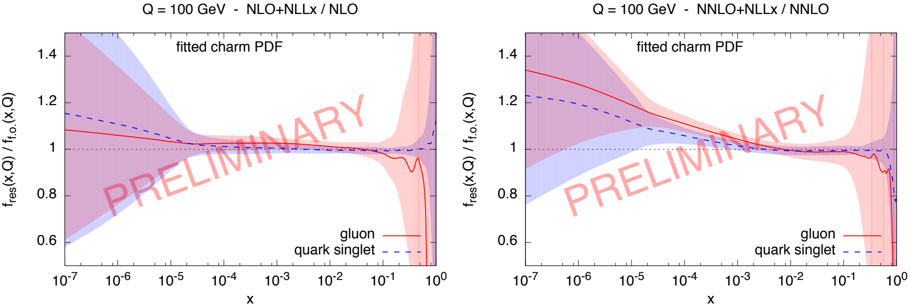
Hierarchy as expected:

$\chi^2_{\text{NNLO+NLLx}}$ smallest

$\chi^2_{\text{(N)NLO+NLLx}} < \chi^2_{\text{(N)NLO}}$

$\chi^2_{\text{NLO}} < \chi^2_{\text{NNLO}}$

PRELIMINARY



Higgs production very sensitive to gluon distribution.

$gg \to H$ inclusive N^{3}LO cross section at $\sqrt{s} = 13$ TeV:

	$\sigma_{\mathrm{N^3LO}}(ggH)$
Preliminary NNLO PDFs	47.9 pb
Preliminary NNLO+NLLx PDFs	48.8 pb

Using resummed PDFs at small-x may lead to a $+1$ pb effect at LHC!!
Much more dramatic effect at FCC

For consistency, small-x resummation should be included in Higgs production
Work in progress [MB,Marzani 17xx.xxxxx]

Conclusions

Theoretical side:

- resummation formalism developed long ago
- recent developments of the ABF formalism
- now both NLO+NLLx and NNLO+NLLx available
- resummed matching conditions and VFNS coefficient functions
- we have all the ingredients for a consistent DIS-only PDF fit

PDF fitting side:

- `HELL` code interfaced to `APFEL`, so resummation in NNPDF machinery available
- very preliminary fits performed with NNPDF3.1 methodology/setting
- NNLO+NLLx improves wrt NNLO
- sizeable impact — relevant for future collider phenomenology

Outlook:

- finalize PDF fits with small-x resummation
- produce global fits (need to resum more processes)
- PDF fit with joint (threshold + high-energy) resummation?

Backup slides

Fit results: χ^2 with perturbative charm PDF

PRELIMINARY

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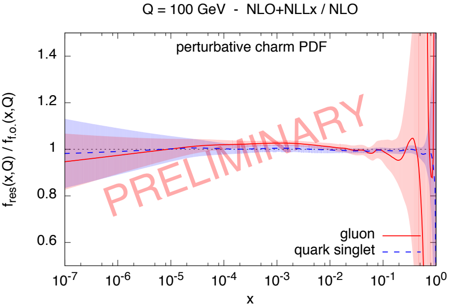
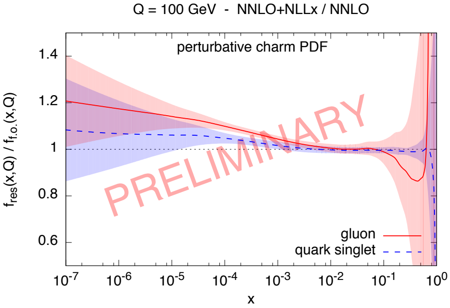
NLO+NLLx NLO NNLO+NLLx NNLO


Q = 100 GeV - NLO+NLLx / NLO
perturbative charm PDF
PRELIMINARY
$f_{res}(x,Q) / f_{f.o.}(x,Q)$
1.4
1.2
1
0.8
0.6
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x
gluon
quark singlet


Q = 100 GeV - NNLO+NNLx / NNLO
perturbative charm PDF
PRELIMINARY
$f_{res}(x,Q) / f_{f.o.}(x,Q)$
1.4
1.2
1
0.8
0.6
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x
gluon
quark singlet