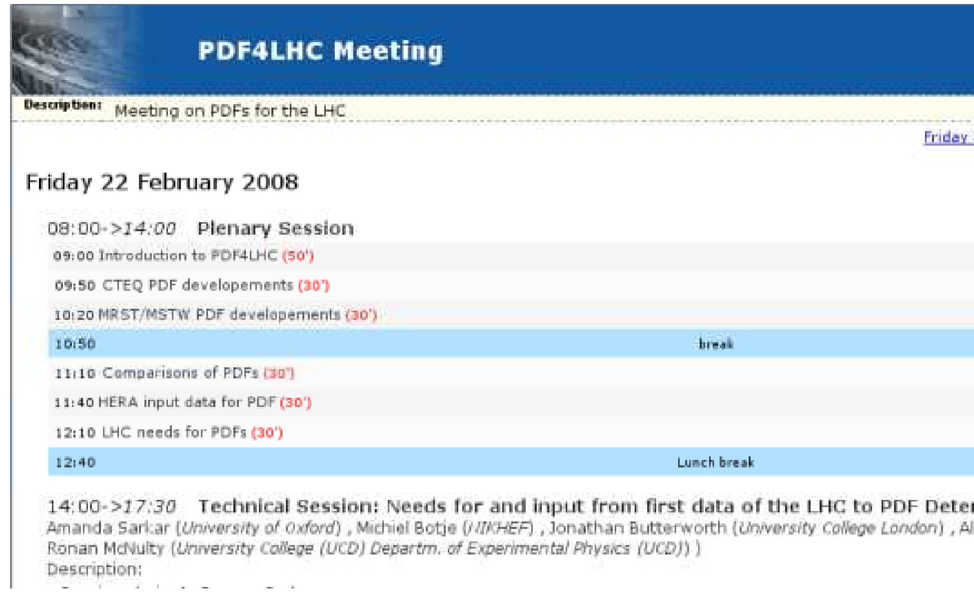
PARTON DISTRIBUTIONS FOR THE LHC

STEFANO FORTE
UNIVERSITÀ DI MILANO

CERN, FEBRUARY 8, 2008

AN ONGOING EFFORT



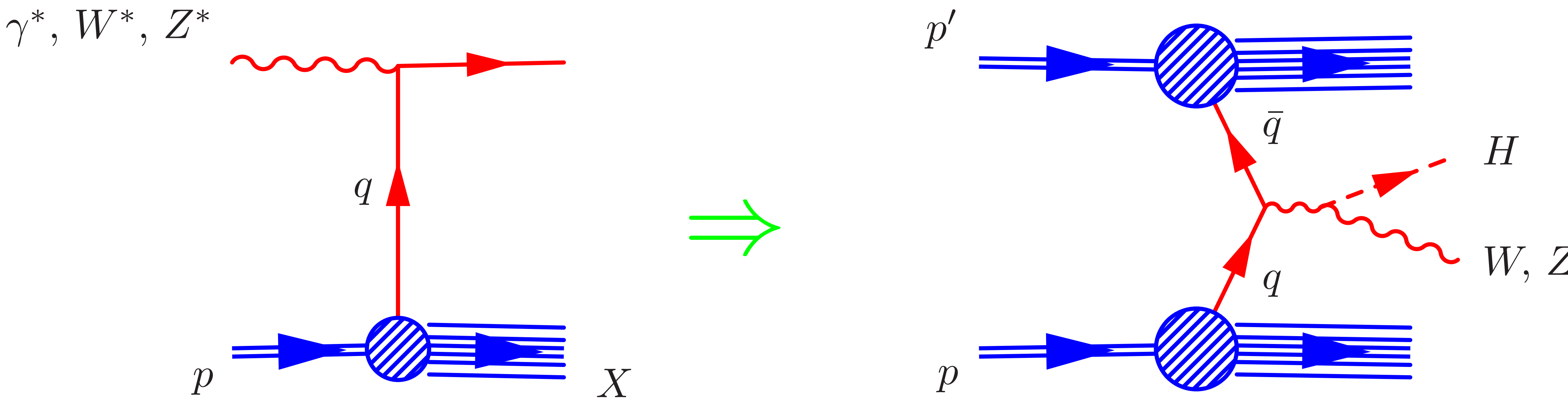
HERALHC: CERN, MAY 26-30, 2008 (LAST MEETING)

PDF4LHC: CERN, FEBRUARY 22-23, 2008 (FIRST MEETING)

PARTONS FOR LHC:

THE ACCURATE COMPUTATION OF PHYSICAL PROCESS AT A HADRON COLLIDER REQUIRES GOOD KNOWLEDGE OF PARTON DISTRIBUTIONS OF THE NUCLEON

FACTORIZATION



IN ORDER TO EXTRACT THE RELEVANT PHYSICS SIGNAL,

WE NEED TO KNOW THE PARTON DISTRIBUTIONS AND THEIR UNCERTAINTY

- IS THIS ASPECT OF LHC PHYSICS UNDER CONTROL?
- WILL LHC TEACH US SOMETHING ABOUT QCD TOO?

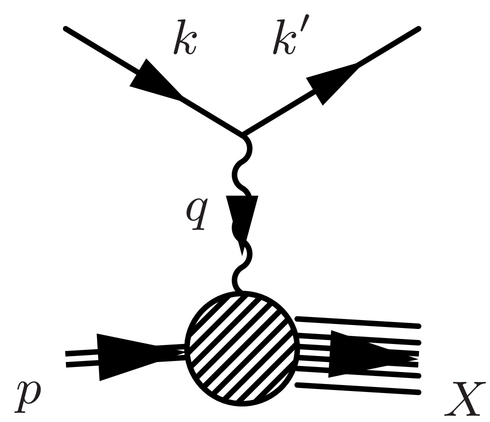
SUMMARY

- **DETERMINING PDFs**
  - factorization
  - disentangling PDFs
  - the state of the art
- **CURRENT ISSUES & LHC NEEDS**
  - experimental information
  - theoretical issues
  - towards LHC
- **PDFS WITH ERRORS**
  - the problem of PDF uncertainties
  - incompatible data
  - the standard approach and its limitations
- **THE NEURAL NETWORK APPROACH**
  - the neural monte carlo strategy
  - unbiased neural fitting
  - nonsinglet and singlet fits

DETERMINING PDFs

FACTORIZATION I: DEEP-INELASTIC SCATTERING

Structure Functions. . .



Lepton fractional energy loss: $y = \frac{p \cdot q}{p \cdot k}$;

gauge boson virtuality: $q^2 = -Q^2$

Bjorken x: $x = \frac{Q^2}{2p \cdot q}$

lepton-nucleon CM energy: $s = \frac{Q^2}{xy}$;

virtual boson-nucleon CM energy $W^2 = Q^2 \frac{1-x}{x}$;

$$\frac{d^2\sigma^{\lambda_p \lambda_\ell}(x,y,Q^2)}{dxdy} = \frac{G_F^2}{2\pi(1+Q^2/m_W^2)^2} \frac{Q^2}{xy} \left\{ \left[-\lambda_\ell\, y \left(1 - \frac{y}{2}\right) x F_3(x,Q^2) + (1-y) F_2(x,Q^2) \right.\right.$$

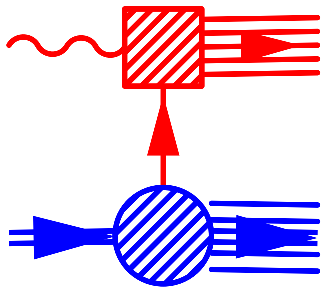
$$\left.\left. + y^2 x F_1(x,Q^2) \right] - 2\lambda_p \left[-\lambda_\ell\, y(2-y) x g_1(x,Q^2) - (1-y) g_4(x,Q^2) - y^2 x g_5(x,Q^2) \right] \right\}$$

$\lambda_l \;\to$ lepton helicity

$\lambda_p \;\to$ proton helicity

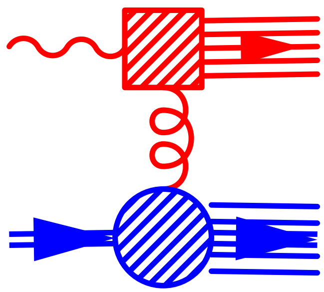
	PARITY CONS.	PARITY VIOL.
UNPOL.	$F_1,\ F_2$	F_3
POL.	g_1	$g_4,\ g_5$

...and Parton Distributions



Structure Function=Hard coeff. (partonic structure function)

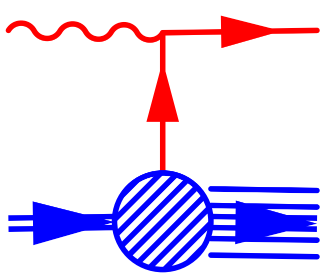
⊗parton distn.



$$F_2^{\rm NC}(x,Q^2) = x \sum_{\text{flav. } i} e_i^2 (q_i + \bar{q}_i) + \alpha_s \left[C_i[\alpha_s] \otimes (q_i + \bar{q}_i) + C_g[\alpha_s] \otimes g \right]$$

q_i quark, $\bar{q}_i$ antiquark, g gluon

Leading Parton Content (up to $O[\alpha_s]$ corrections)



$q_i \equiv q_i^{\uparrow\uparrow} + q_i^{\uparrow\downarrow}$ $\qquad$ $\Delta q_i \equiv q_i^{\uparrow\uparrow} - q_i^{\uparrow\downarrow}$

NC $\quad F_1{}^{\gamma,\,Z} = \sum_i e_i^2 \, (q_i + \bar{q}_i)$ $\qquad$ $g_1{}^{\gamma,\,Z} = \sum_i e_i^2 \, (\Delta q_i + \Delta\bar{q}_i)$

CC $\quad F_1^{W^+} = \bar{u} + d + s + \bar{c}$ $\qquad$ $g_1^{W^+} = \Delta\bar{u} + \Delta d + \Delta s + \Delta\bar{c}$

CC $\quad -F_3^{W^+}/2 = \bar{u} - d - s + \bar{c}$ $\qquad$ $g_5^{W^+} = \Delta\bar{u} - \Delta d - \Delta s + \Delta\bar{c}$

$F_2 = 2xF_1$ $\qquad$ $g_4 = 2xg_5$

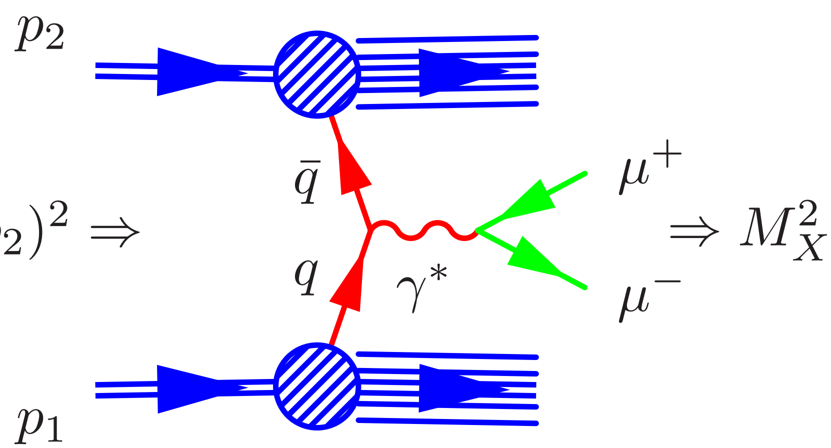
$W^+ \to W^- \Rightarrow u \leftrightarrow d,\ c \leftrightarrow s$; more combinations using Isospin: $p \to n \Rightarrow u \leftrightarrow d$

FACTORIZATION II: HADRONIC PROCESSES

$$\sigma_X(s, M_X^2) = \sum_{a,b} \int_{x_{\min}}^1 dx_1\, dx_2\, f_{a/h_1}(x_1) f_{b/h_2}(x_2) \hat{\sigma}_{q_a q_b \to X}\left(x_1 x_2 s, M_X^2\right)$$

LEAD. ORD. $= \sigma_0 \sum_{a,b} \int_\tau^1 \frac{dx}{x} f_{a/h_1}(x) f_{b/h_2}(\tau/x) \equiv \sigma_0 \mathcal{L}(\tau)$ $\Rightarrow \mathcal{L}$ PARTON LUMI

$s = (p_1 + p_2)^2 \Rightarrow$



LEADING ORDER

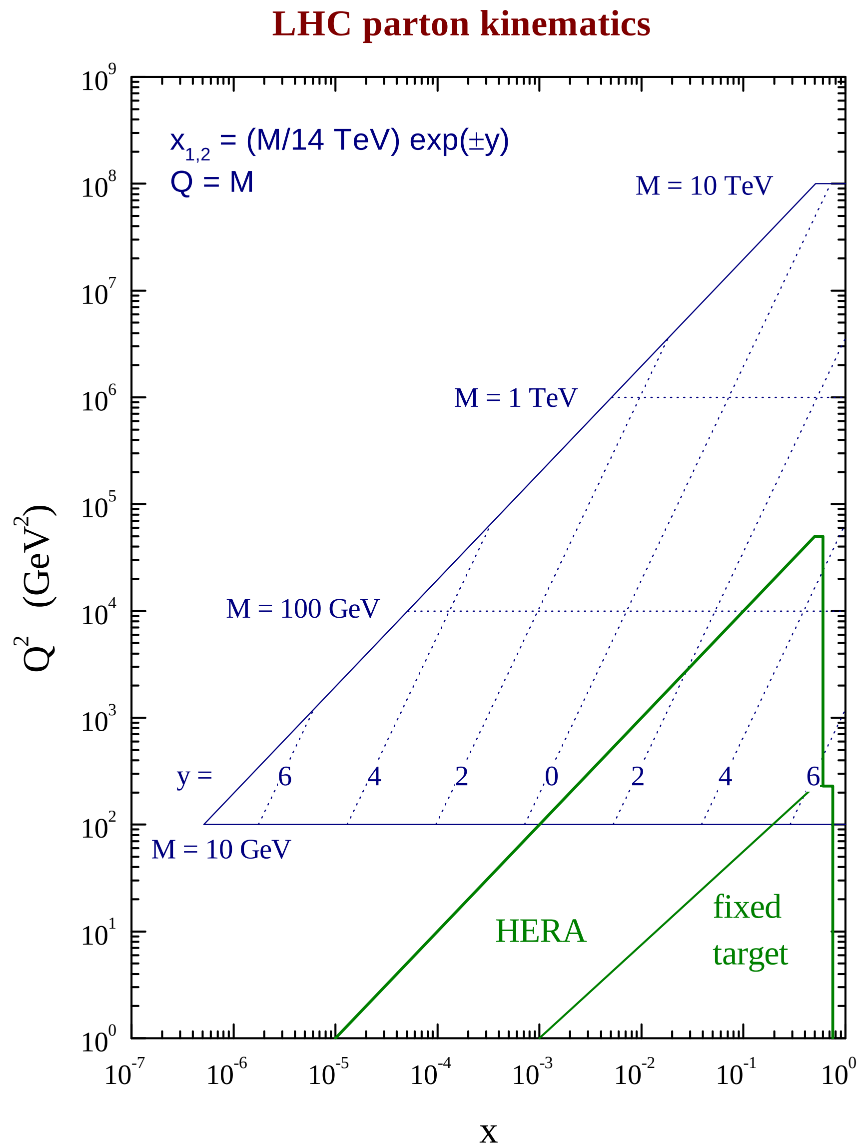
DRELL-YAN

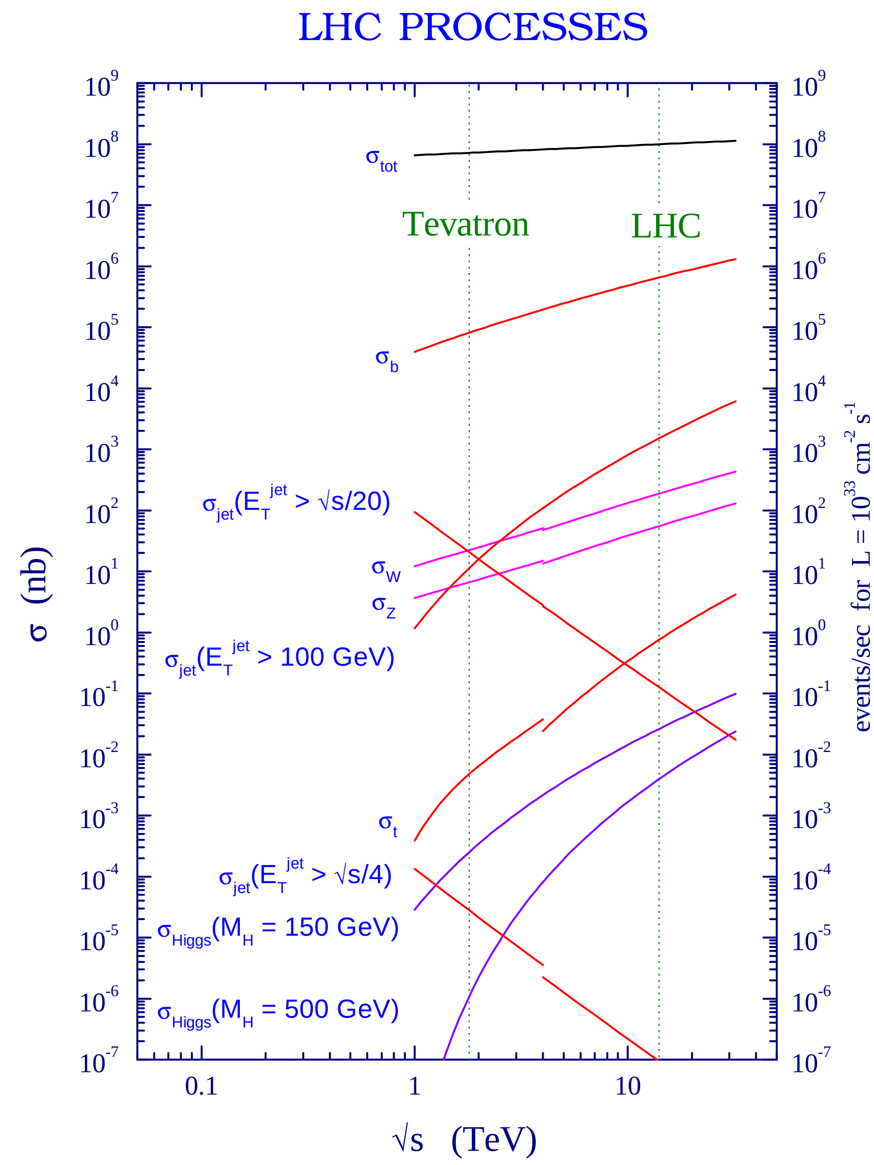
- Hadronic c.m. energy: $s = (p_1 + p_2)^2$
- Momentum fractions $x_{1,\,2} = \sqrt{\frac{\hat{s}}{s}}\, \exp \pm$
  Lead. Ord. $\hat{s} = M^2$
- Partonic c.m. energy: $\hat{s} = x_1 x_2 s$
- Invariant mass of final state X (dilepton, Higgs,...):
  $M_W^2 \Rightarrow$ scale of process
- Scaling variable $\tau = \frac{M_X^2}{s}$

- $\hat{\sigma}_{q_a q_b \to X} = \sigma_0 C\left(x, \alpha_s(M_H^2)\right)$; $\quad C\left(x, \alpha_s(M_H^2)\right) = \delta(1 - x) + O(\alpha_s)$
- $\sigma_X(s, M^2) = \sigma_0 \sum_{a,b} \int_{x_{\min}}^1 dx_1\, dx_2\, f_{a/h_1}(x_1) f_{b/h_2}(x_2) \delta(x_1 x_2 x - \tau) C\left(x, \alpha_s(M_H^2)\right)$
  $= \sigma_0 \sum_{a,b} \int_{x_2}^1 \frac{dx_1}{x_1} \int_\tau^1 \frac{dx_2}{x_2} f_{a/h_1}(x_1) f_{b/h_2}(x_2) C\left(\frac{\tau}{x_1 x_2}, \alpha_s(M_H^2)\right)$

EXAMPLE: DRELL-YAN $\sigma_X \to M^2 \frac{d\sigma}{dM^2}$; $\sigma_0 = \frac{4}{9}\pi\alpha\frac{1}{s}$

LHC: KINEMATICS AND PHYSICAL PROCESSES





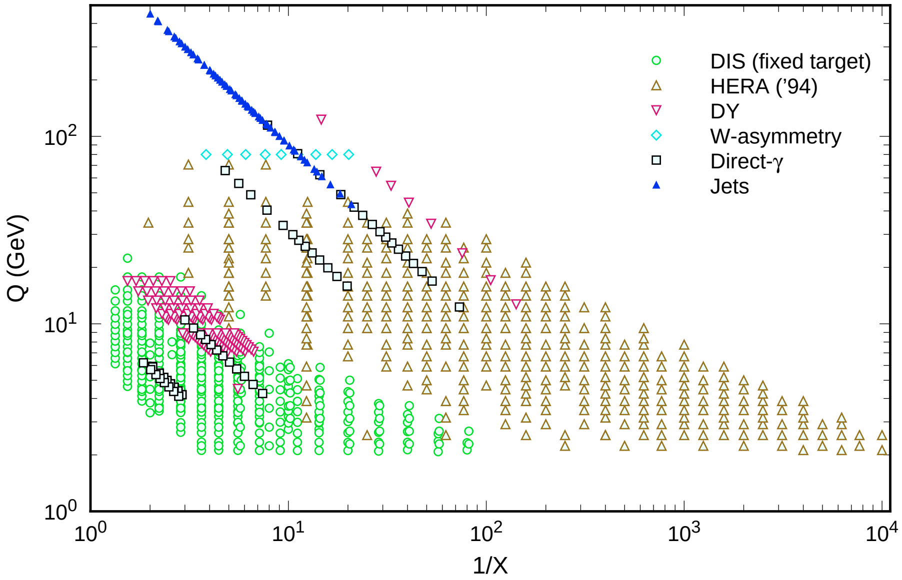
PARTON FITS

DATA $\longrightarrow$ PARTON DISTRIBUTIONS

STRATEGY:

- CHOOSE SET OF OBSERVABLES (DIS, DRELL-YAN, W PRODUCTION...) & COMPUTE THEM IN PERT. THEORY
- CHOOSE A SET OF BASIS PARTON DISTRIBUTIONS (SINGLET, VALENCE, SEA...)
- FIT THE OBSERVABLES WITH THE PDFS AS FREE PARAMETERS

DATA INCLUDED IN CTEQ5 PARTON FIT



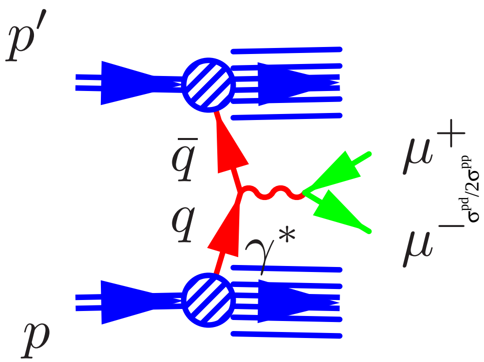
TASKS:

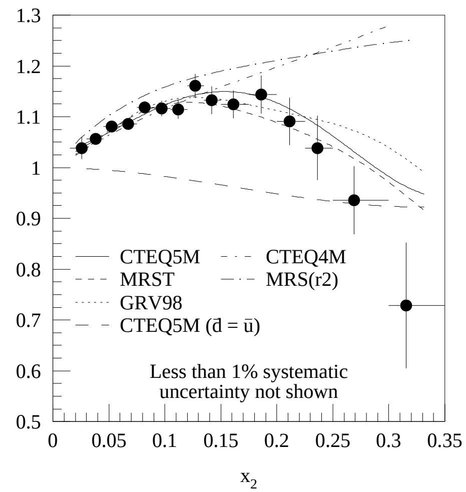
- STRUCTURE FUNCTION (OR XSECT) IS A CONVOLUTION OVER x OF PARTON DISTNS. AND PERTURBATIVE CROSS SECTION
  $\longrightarrow$ MUST DECONVOLUTE
- EACH STRUCTURE FUNCTION (OR XSECT) IS A LINEAR COMBINATION OF MANY PARTON DISTNS ($2N_f$ QUARKS + 1 GLUON)
  $\longrightarrow$ MUST COMBINE DIFFERENT PROCESSES
- DATA GIVEN AT VARIOUS SCALES, WANT PARTON DISTNS. AS FCTN OF x AT COMMON SCALE Q^2
  $\longrightarrow$ MUST EVOLVE
- TH UNCERTAINTIES: HIGHER ORDERS, RESUMMATIONS, HEAVY QUARK THRESHOLDS, NUCLEAR CORRECTIONS, HIGHER TWIST, ...

DISENTANGLING QUARKS FROM ANTIQUARKS

γ^* DIS ONLY MEASURES $q + \bar{q}$ COMBINATION!

DRELL-YAN p/d ASYMMETRY



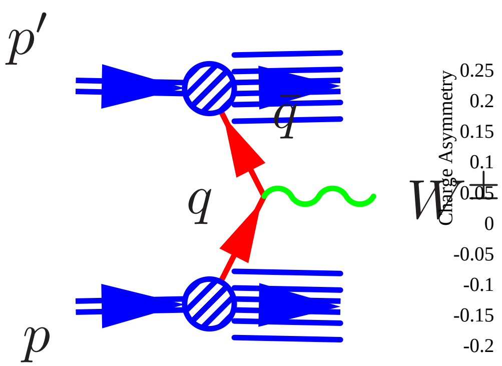


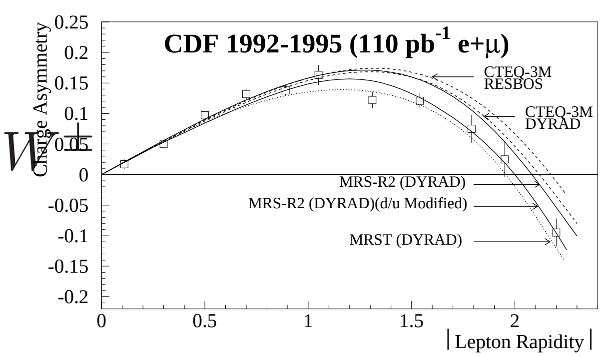
E866 (2001)

LIGHT ANTIQUARK ASYMMETRY

$$\frac{\sigma^{pn}}{\sigma^{pp}} \sim \left. \frac{\frac{4}{9}\, u^p\, \bar{d}^p + \frac{1}{9}\, d^p\, \bar{u}^p}{\frac{4}{9}\, u^p\, \bar{u}^p + \frac{1}{9}\, d^p\, \bar{d}^p} \right|_{\text{large } x} \approx \frac{\bar{d}}{\bar{u}}$$

$W^\pm$ ASYMMETRY





CDF (1998)

LIGHT QUARK ASYMMETRY

$$\frac{\sigma^{p\bar{p}}_{W+}}{\sigma^{p\bar{p}}_{W-}} \sim \frac{u^p\, d^p}{d^p\, u^p} \qquad (q^p = \bar{q}^{\bar{p}})$$

DETERMINING THE GLUON

EVOLUTION:

SINGLET SCALING VIOLATIONS

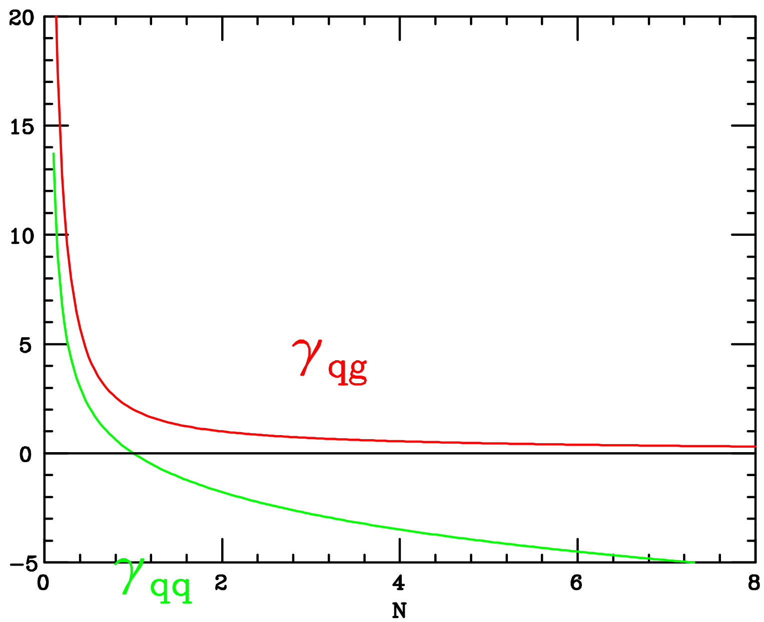
$$\frac{d}{dt}F_2^s(N,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\left[\gamma_{qq}(N)F_2^s + 2\,n_f\gamma_{qg}(N)g(N,Q^2)\right] + O(\alpha_s^2)$$

$$F_2(N,Q^2) \equiv \int_0^1 dx\, x^{N-1}F_2(x,Q^2); \qquad \gamma_{ij}(N) \equiv \int_0^1 dx\, x^{N-1}P_{ij}(x,Q^2)$$

LARGE/SMALL X $\Leftrightarrow$ LARGE/SMALL N

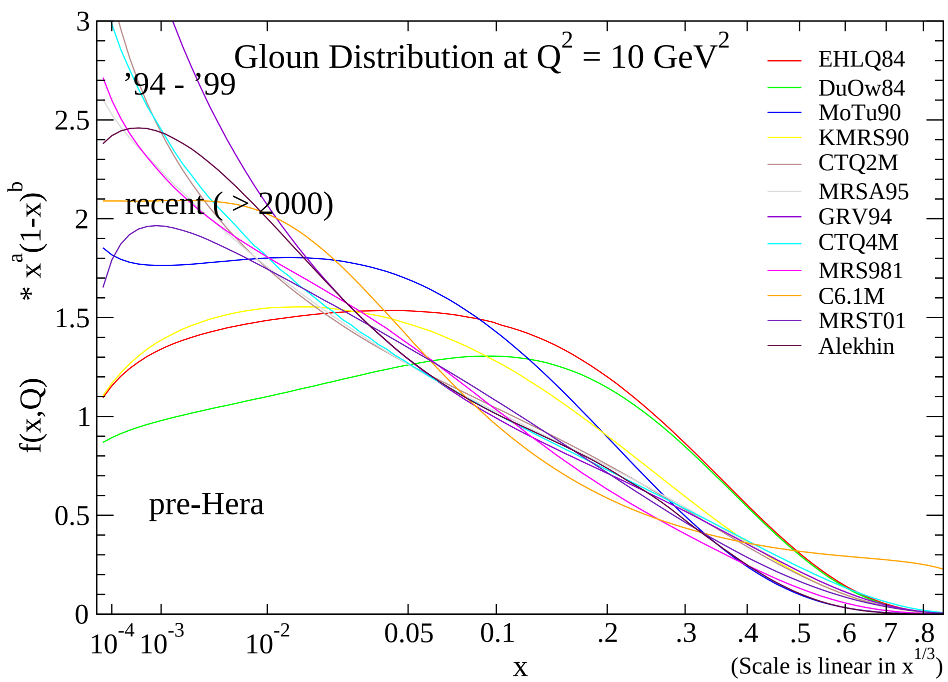
AT LARGE N

$\gamma_{qg} << \gamma_{qq}$



AT LARGE x

$\Rightarrow$ GLUON HARD TO DETERMINE



W.K. Tung, 2004

THE STANDARD APPROACH:

FUNCTIONAL PARTON FITTING

- CHOOSE A FIXED FUNCTIONAL FORM:
  - MRST: 24 PARMS., SOME FIXED → 15 PARMS.

$$xq(x,Q_0^2) = A(1-x)^\eta(1+\epsilon x^{0.5}+\gamma x)x^\delta, \quad x[\bar{u}-\bar{d}](x,Q_0^2) = A(1-x)^\eta(1+\gamma x+\delta x^2)x^\delta.$$

$$xg(x,Q_0^2) = A_g(1-x)^{\eta_g}(1+\epsilon_g x^{0.5}+\gamma_g x)x^{\delta_g} - A_-(1-x)^{\eta_-}x^{-\delta_-},$$

  - CTEQ: 20 PARMS.

$$x\,f(x,Q_0) = A_0\,x^{A_1}\,(1-x)^{A_2}\,e^{A_3 x}\,(1+e^{A_4}\,x)^{A_5}$$

with independent params for combinations $u_v \equiv u-\bar{u}$, $d_v \equiv d-\bar{d}$, g, and $\bar{u}+\bar{d}$, $s=\bar{s}=0.2\,(\bar{u}+\bar{d})$ at Q_0; NORM. FIXED BY SUM RULES

  - ALEKHIN: 17 PARMS.

$$xu_{\rm V}(x,Q_0) = \frac{2}{N_{\rm u}^{\rm V}}x^{a_{\rm u}}(1-x)^{b_{\rm u}}(1+\gamma_2^{\rm u}x); \quad xu_{\rm S}(x,Q_0) = \frac{A_{\rm S}}{N_{\rm S}}\eta_{\rm u}x^{a_{\rm s}}(1-x)^{b_{\rm su}}$$

$$xd_{\rm V}(x,Q_0) = \frac{1}{N_{\rm d}^{\rm V}}x^{a_{\rm d}}(1-x)^{b_{\rm d}}; \quad xd_{\rm S}(x,Q_0) = \frac{A_{\rm S}}{N^{\rm S}}x^{a_{\rm s}}(1-x)^{b_{\rm sd}},$$

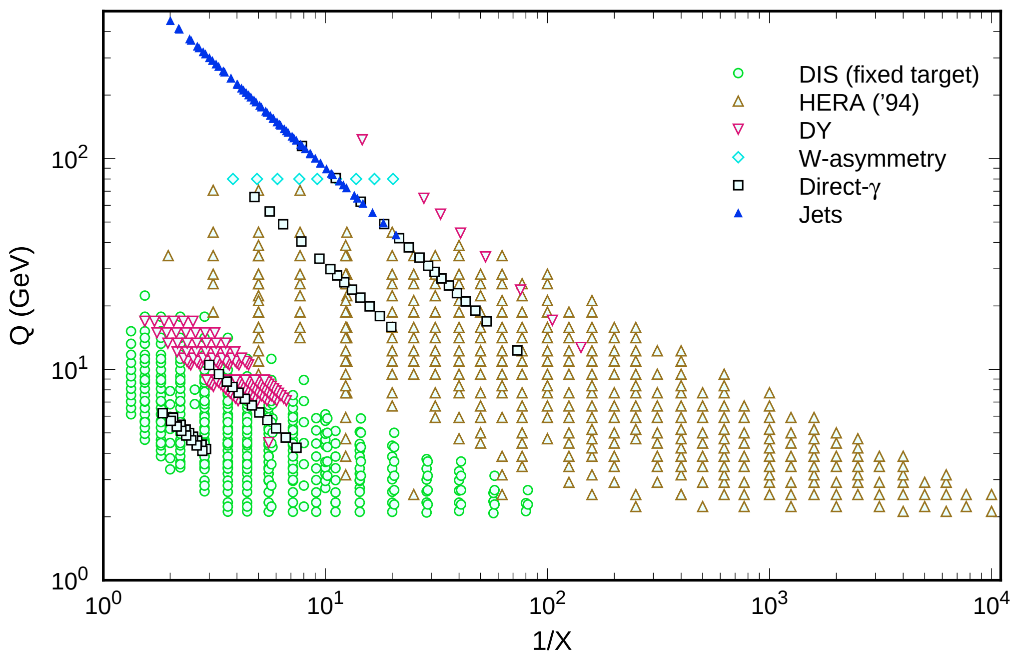
$$xs_{\rm S}(x,Q_0) = \frac{A_{\rm S}}{N^{\rm S}}\eta_{\rm s}x^{a_{\rm s}}(1-x)^{(b_{\rm su}+b_{\rm sd})/2}; \quad xG(x,Q_0) = A_{\rm G}x^{a_{\rm G}}(1-x)^{b_{\rm G}}(1+\gamma_1^{\rm G}\sqrt{x}+\gamma_2^{\rm G}x),$$

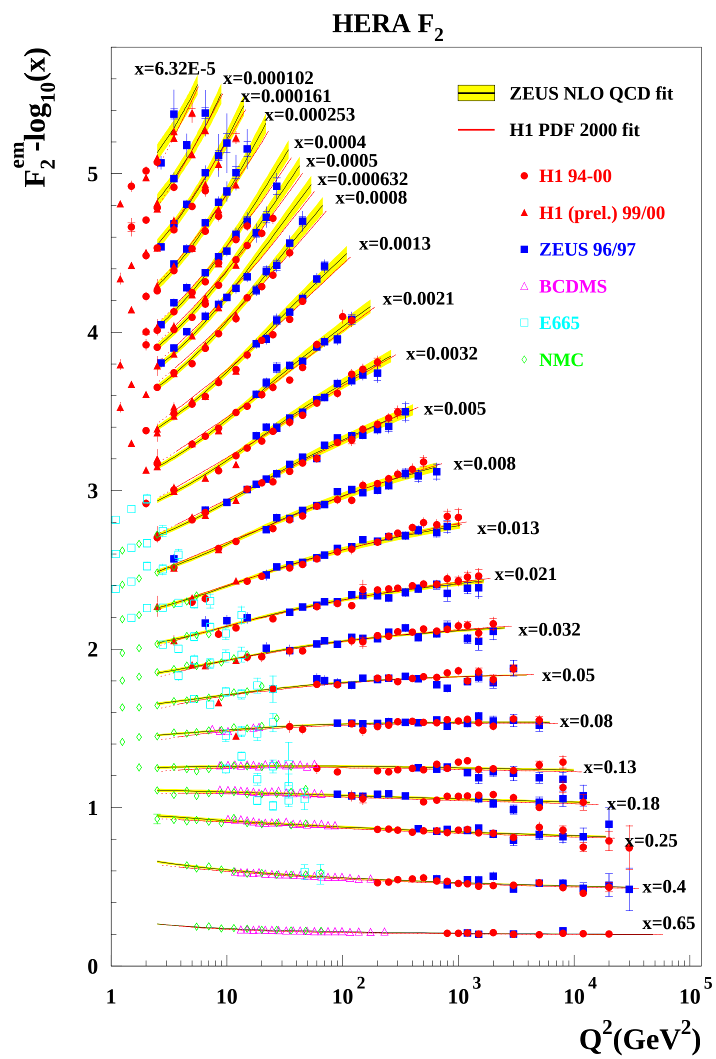
- EVOLVE TO DESIRED SCALE & COMPUTE PHYSICAL OBSERVABLES
- DETERMINE BEST-FIT VALUES OF PARAMETERS
- DETERMINE ERROR BY PROPAGATION OF ERROR ON PARMS ('HESSIAN METHOD') OR BY PARM. SCANS ('LAGRANGE MULTIPLIER METHOD')

HOW WELL DOES IT WORK?

NOMINALLY, RATHER WELL INDEED...

DATA INCLUDED IN CTEQ5 PARTON FIT

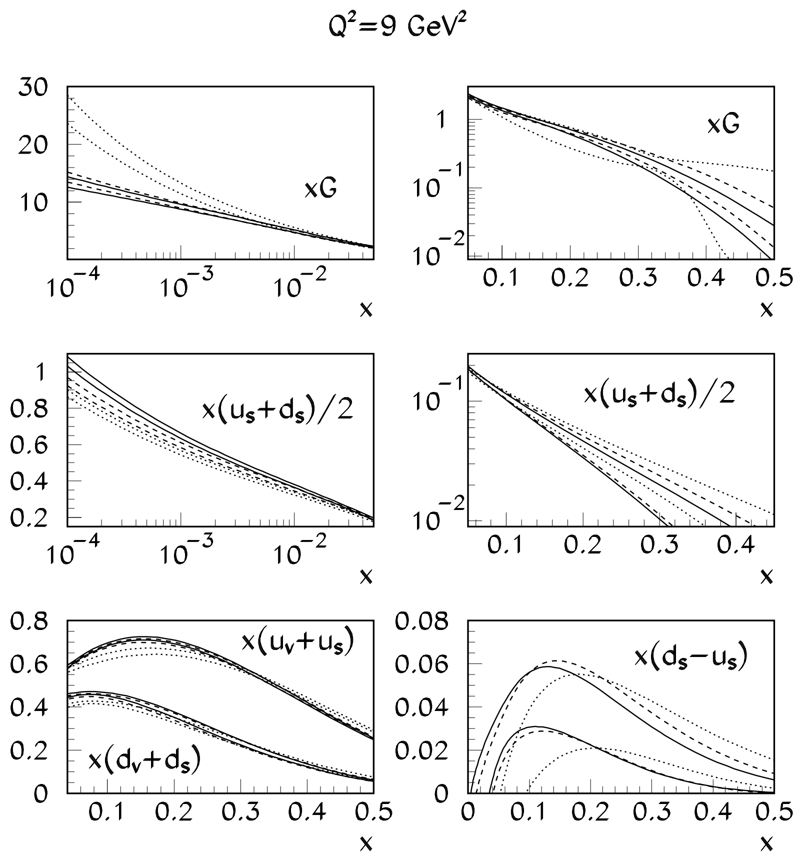




HOW WELL DOES IT WORK?

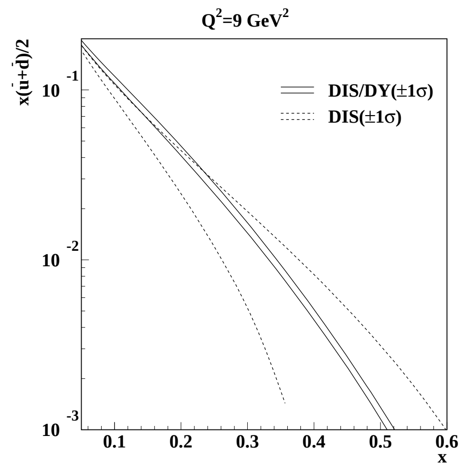
DIS+DY only (Alekhin 2003-2006)

DIS total error bands for
LO (dots), NLO (dashes), NNLO (solid)
valence $u^v \equiv u - \bar{u}$, $d^v \equiv d - \bar{d}$,
sea $u^s = \bar{u}^s = d^s = \bar{d}^s$

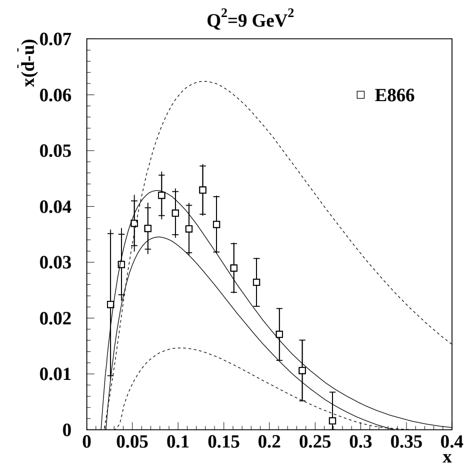


TOTAL ERROR BANDS:
DIS (dots) vs. DIS+DY (solid)

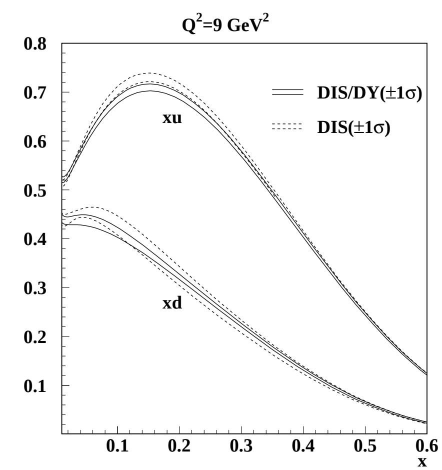
$\bar{u} + \bar{d}$



$\bar{u} - \bar{d}$



$u,\ d$



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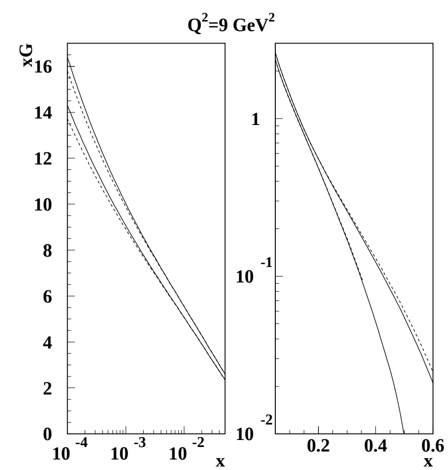
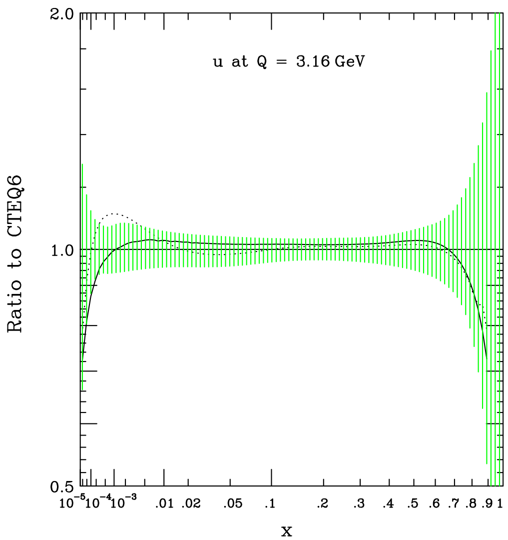


HOW WELL DOES IT WORK?

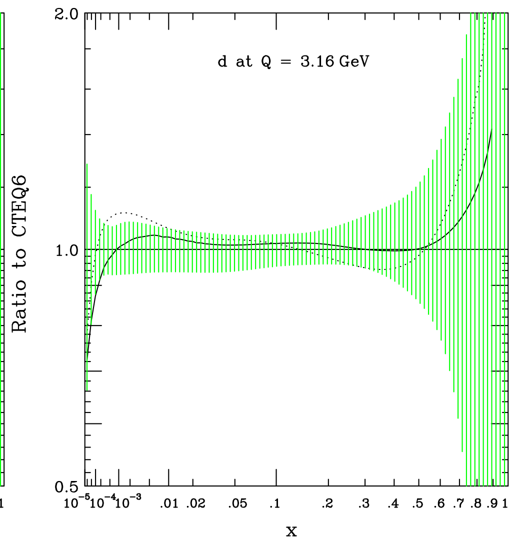
GLOBAL FITS (MRST-CTEQ 2002-2006)

CTEQ ERROR BAND & MRST/CTEQ CURVE

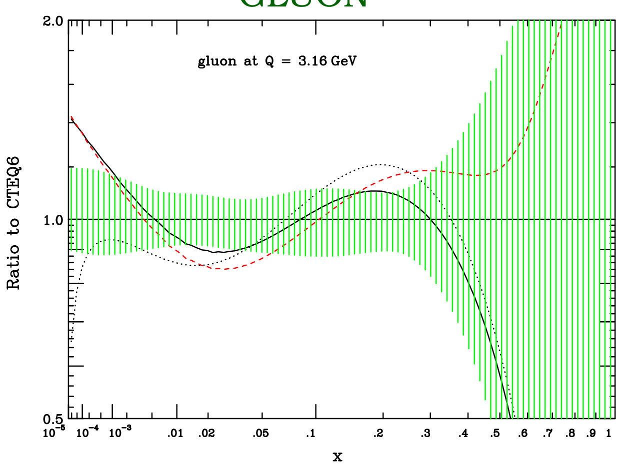
$u - \bar{u}$



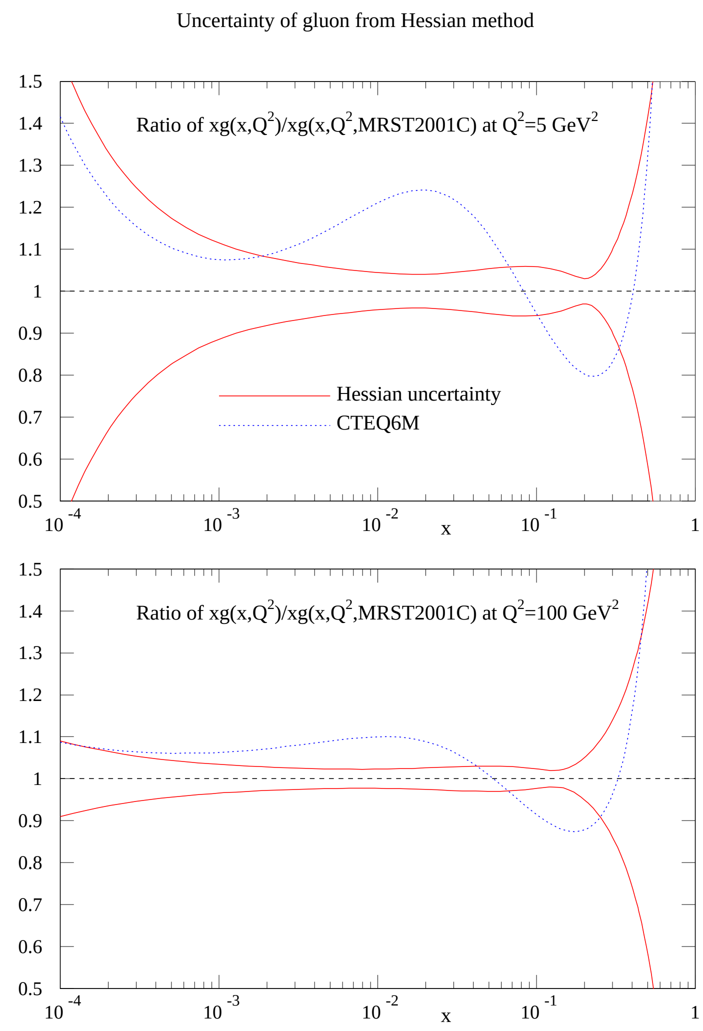
$d - \bar{d}$



GLUON



MRST GLUON ERROR BAND &CTEQ/MRST CURVE



–FEW PERCENT ERROR ON VALENCE & GLUE

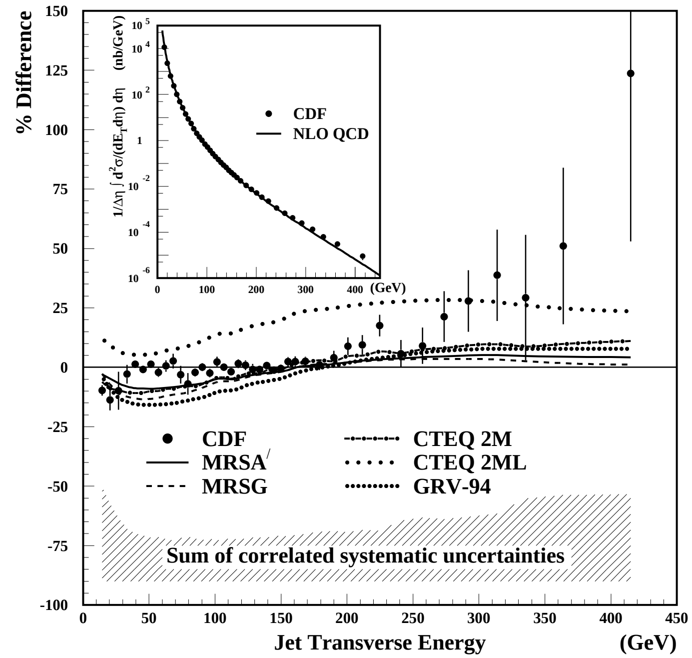
–OTHER PDFS:

ERROR NOT WELL CONTROLLED

KNOWN ISSUES
& LHC NEEDS

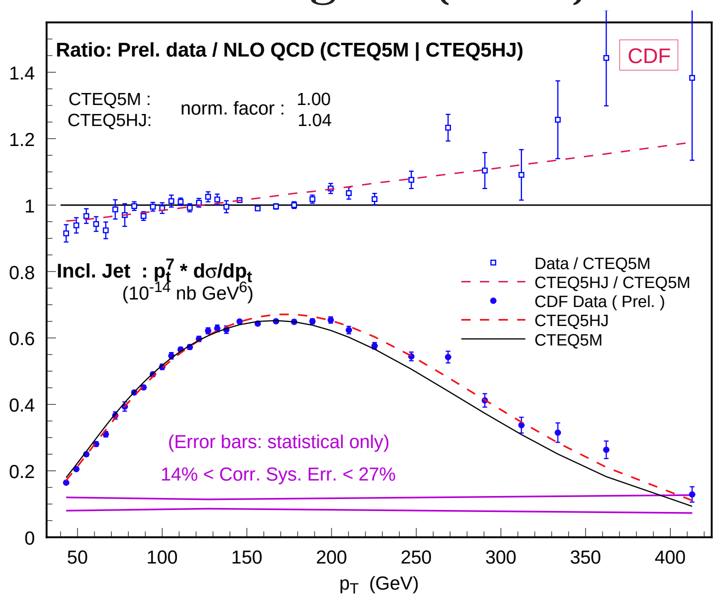
CASE STUDY I: THE CDF LARGE E_T JETS
CDF 1995

- DISCREPANCY BETWEEN QCD CALCULATION AND CDF JET DATA (1995)
- EVIDENCE FOR QUARK COMPOSITENESS?
- BUT NO INFO ON PARTON UNCERTAINTY $\Rightarrow$ RESULT STRONGLY DEPENDS ON GLUON AT $x \gtrsim 0.1$

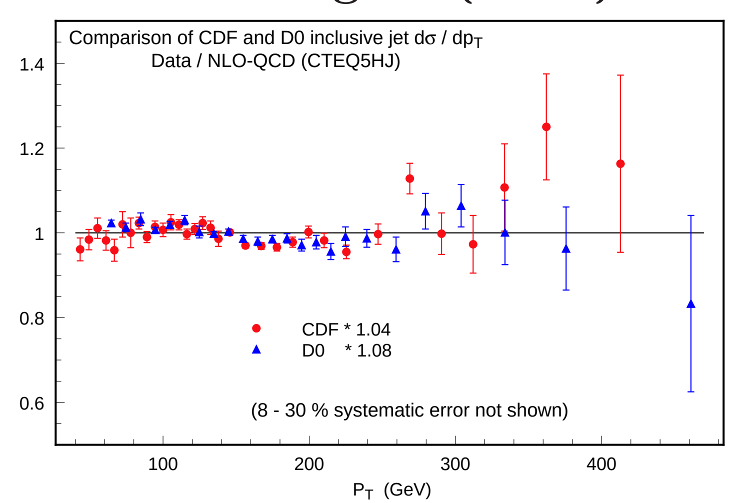


DISCREPANCY REMOVED IF JET DATA INCLUDED IN THE FIT

NEW CTEQ FIT (1996)



FINAL CTEQ FIT (1998)



CASE STUDY II: THE NuTeV ANOMALY
THE PASCHOS-WOLFENSTEIN RATIO: DATA...

NuTeV 2001 $\sin^2\theta_W(\mathrm{OS}) = 0.2272 \pm 0.0013(\mathrm{stat}) \pm 0.0009(\mathrm{syst}) \pm 0.0002(M_t, M_H)$

Global Fit 2003 $\sin^2\theta_W(\mathrm{OS}) = 0.2229 \pm 0.0004$

...VS. THEORY

$$R^- = \frac{\sigma_{NC}(\nu) - \sigma_{NC}(\bar\nu)}{\sigma_{CC}(\nu) - \sigma_{CC}(\bar\nu)}$$

$$= \left(\frac{1}{2} - \sin^2\theta_W\right) + 2\left[\frac{(u-\bar u) - (d-\bar d)}{u-\bar u + d - \bar d} - \frac{s - \bar s}{u - \bar u + d - \bar d}\right] \times \left[\left(\frac{1}{2} - \frac{7}{6}\sin^2\theta_W\right) + \frac{4}{9}\frac{\alpha_s}{2\pi}\left(\frac{1}{2} - \sin^2\theta_W\right) + O(\alpha_s^2)\right] + O(\delta(u-d)^2,\ \delta s^2$$

u,d...denote momentum fractions carried by corresp. quark flavors

NuTeV RESULT OBTAINED NEGLECTING:

- ISOSPIN VIOLATION $\longrightarrow$ ISOSPIN KNOWN TO BE GOOD TO $\sim .1\%$
- STRANGE ASYM. $\longrightarrow$ EXPECT SEA TO BE FLAVOUR/ANTIFLAVOUR SYMMETRIC
- QCD CORRECTIONS $\longrightarrow$ TINY (ONLY ENTER THROUGH SYM. VIOLATING TERMS)

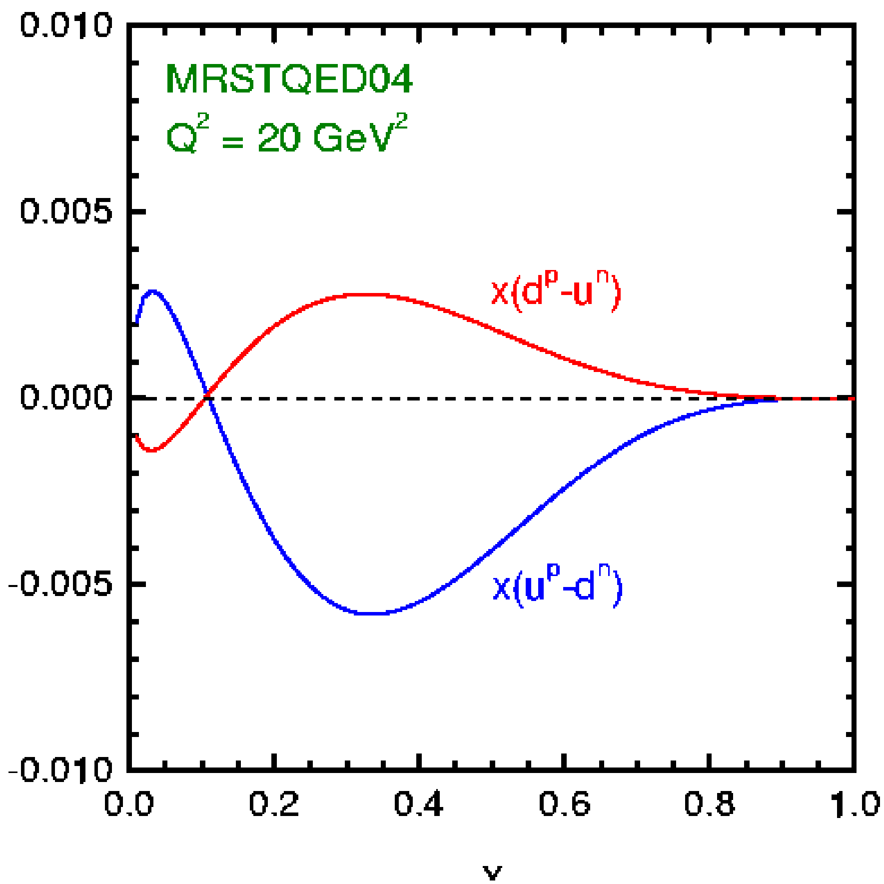
ISOSPIN VIOLATION

QED EFFECTS LEAD TO ISOSPIN VIOLATION:

$u - \bar{u}$ radiate more photons than $d - \bar{d}$: $\frac{d}{dt} q_i \propto e_i^2 q_i$

$\Rightarrow$ MORE PHOTON MOMENTUM IN PROTON THAN NEUTRON

$\Rightarrow |u(x) - \bar{u}(x)| < |d(x) - \bar{d}(x)|$ AT LARGE x



- SIGN OF EFFECT AS REQUIRED TO EXPLAIN NUTEV
- SIZE OF EFFECTS WITH REASONABLE ASSUMPTIONS ABOUT 1/2 OF NUTEV ANOMALY
- THEORETICAL RESULTS AGREES WITH FIT IF ISOSPIN VIOLATION ALLOWED

MRST 2005: "QED" PARTON SET

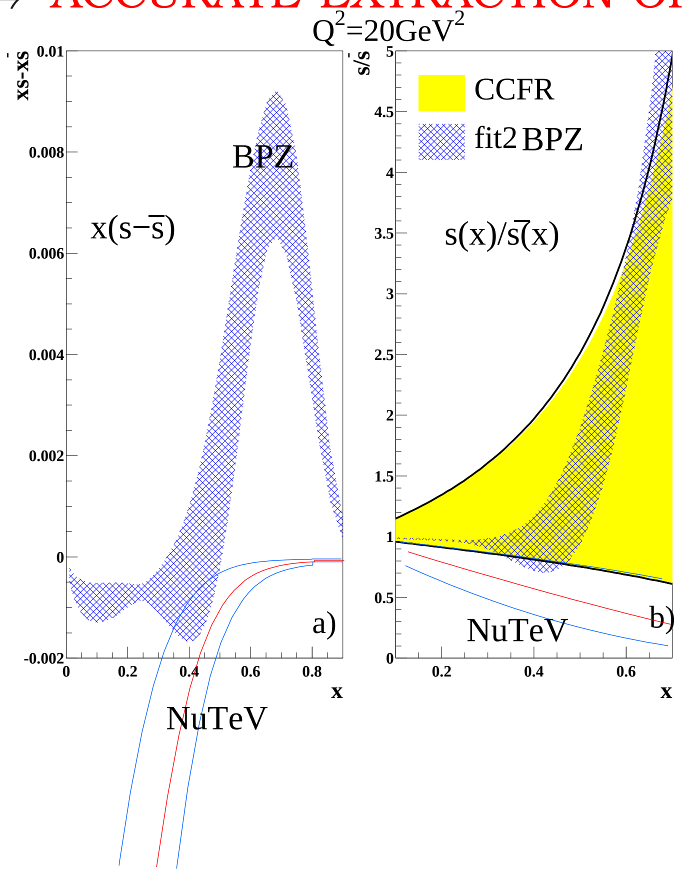
STRANGENESS ASYMMETRY

Q: ARE WE SURE THAT MOMENTUM FRACTION $s - \bar{s} = 0$?

A: MEASURE IT!: CHARM IS COPIOUSLY PRODUCED IN $W^+ + s \to c$

easily tagged through dimuon signal, 2nd muon from subsequent c decay

$\Rightarrow$ ACCURATE EXTRACTION OF THE STRANGE DISTRIBUTION



CCFR/NuTeV $s - \bar{s}$ DETERMINATION

5000 ν & 1500 $\bar{\nu}$ DIMUON EVENT SAMPLE:

ASSUMED PARM.: $s(x) = \kappa \frac{\bar{u}(x)+\bar{d}(x)}{2}(1-x)^{\alpha}$

NEGATIVE $s - \bar{s}$ AT SMALL x

$\Rightarrow$ MOM. FRACT. $s - \bar{s} = -0.003 \pm 0.001$

NuTeV ANOMALY WORSE!

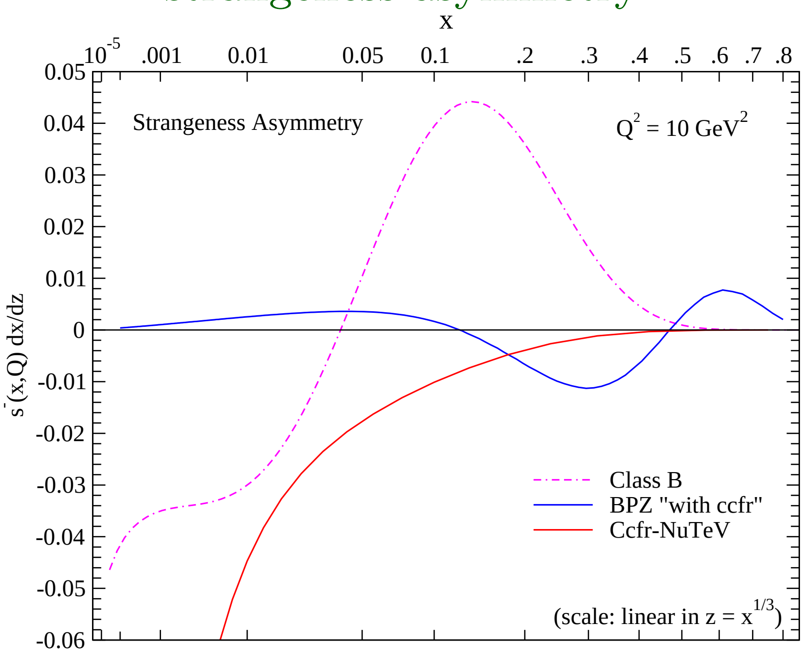
HOWEVER, BPZ GLOBAL FIT TO NEUTRINO INCLUSIVE DIS (Barone et al 2003) $\Rightarrow$ POSITIVE (TINY) ASYMMETRY

COMBINING INCLUSIVE AND EXCLUSIVE INFORMATION

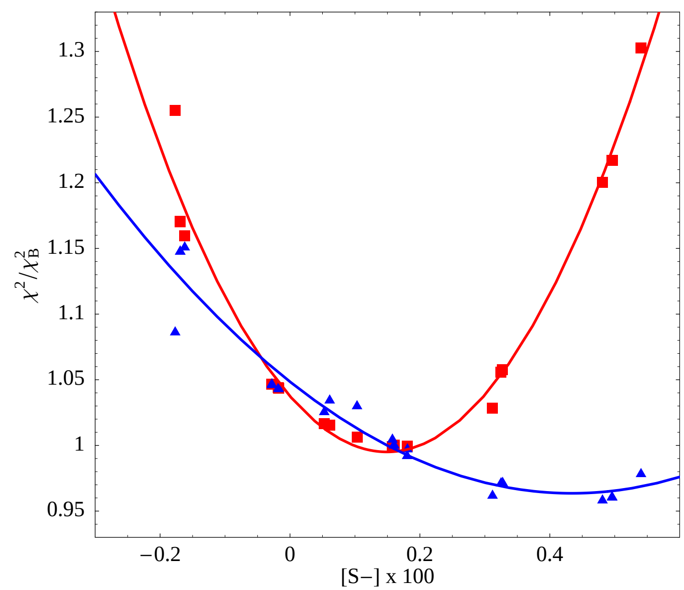
CTEQ DEDICATED DIMUON ANALYSIS

- $\int_0^1 (s(x) - \bar{s}(x))dx = 0$ IN PROTON
  $\Rightarrow$ EITHER $s(x) - \bar{s}(x)$ HAS A NODE OR IT VANISHES EVERYWHERE
- $[s(x) - \bar{s}(x)] < 0$ FOR SMALL $x \lesssim 0.05$ CONSTRAINED BY DIMUON
- LARGE x REGION WEIGHS MORE IN MOMENTUM FRACTION
- POSITIVE MOM. FRACTION $s - \bar{s} \approx 0.02$:

strangeness asymmetry



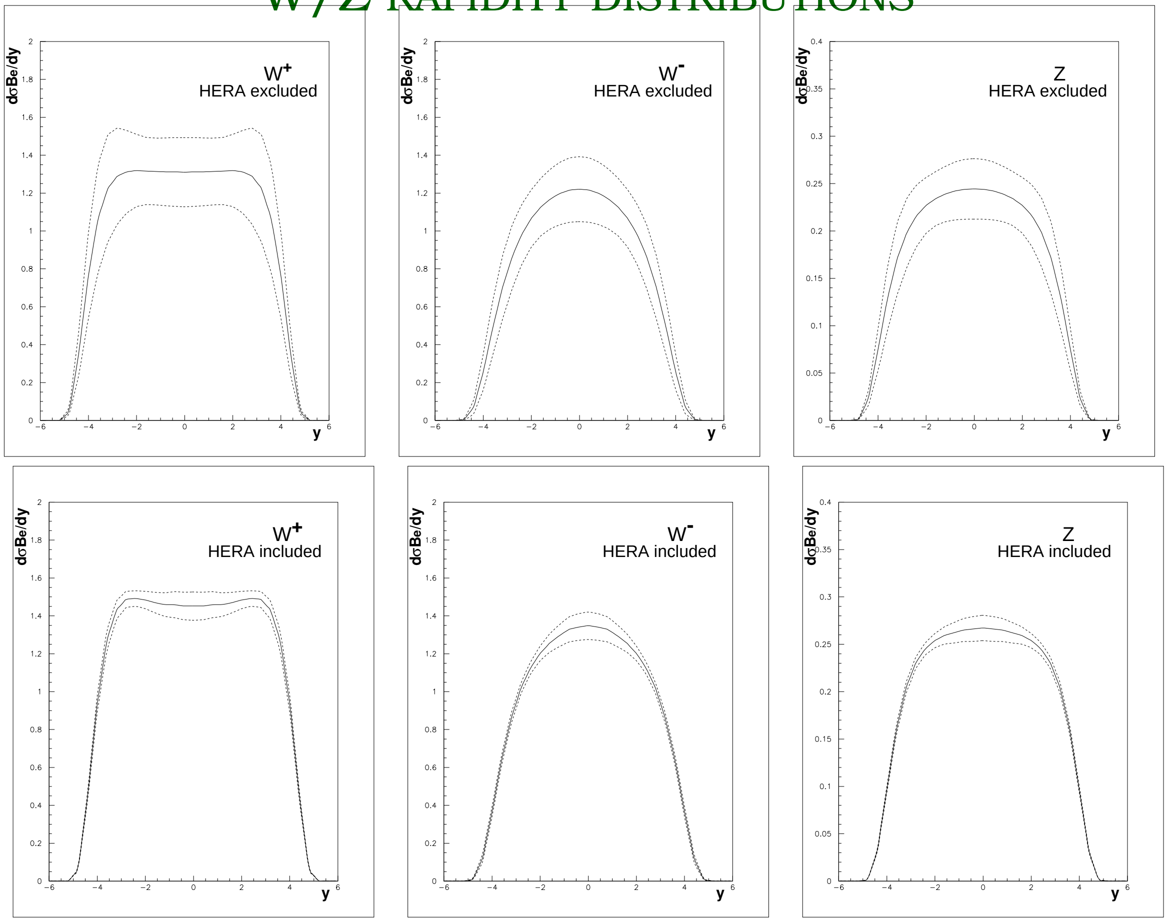
χ^2 dimuon vs. inclusive



STRANGE QUARK PDF FITTED IN FORTHCOMING MRST & CTEQ SETS

CASE STUDY III W PRODUCTION @ LHC: THE GOOD NEWS

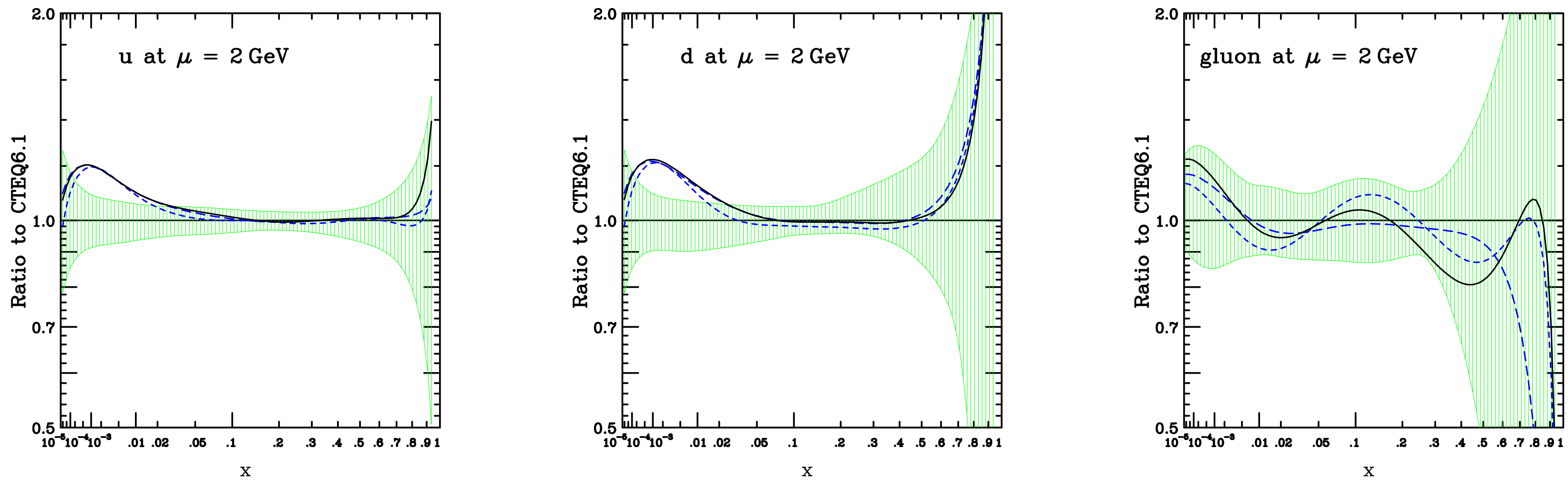
W/Z RAPIDITY DISTRIBUTIONS

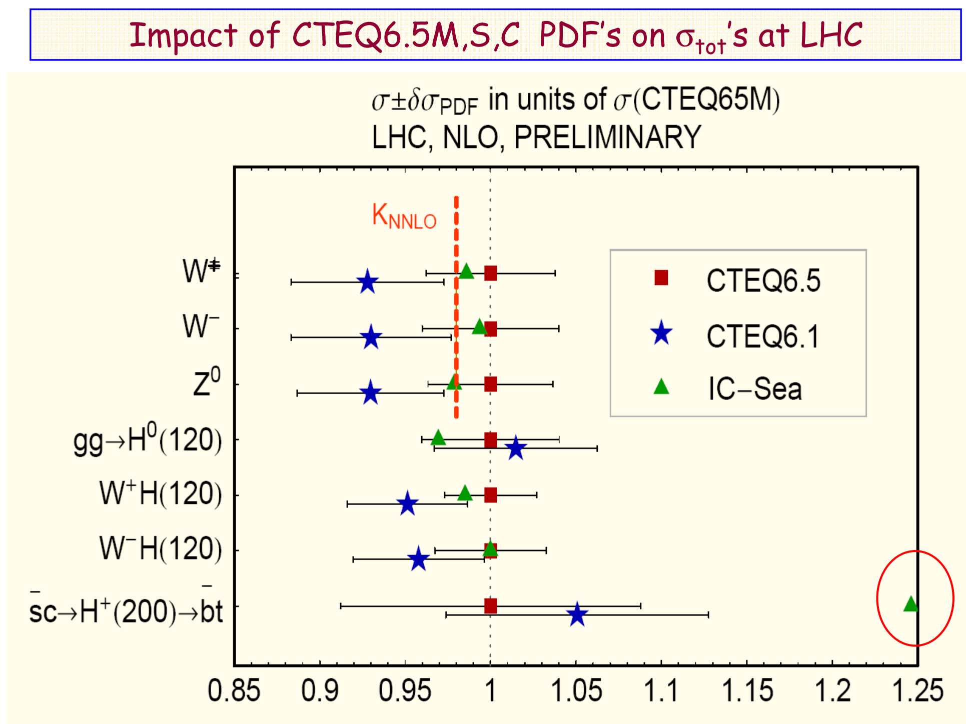


- W/Z RAPDITY SPECTRA & TOTAL CROSS SECTIONS: $\sim 15\%$ PRE-HERA ACCURACY $\sim 3-5\%$ POST-HERA ACCURACY
- GOOD AGREEMENT BETWEN DIFFERENT PDF SETS

PDF SET	$\sigma(W^+).B(W^+ \to l^+\nu_l)$	$\sigma(W^-).B(W^- \to l^-\bar{\nu}_l)$	$\sigma(Z).B(Z \to l^+l$
ZEUS-S NO HERA	10.63 ± 1.73 NB	7.80 ± 1.18 NB	1.69 ± 0.23 NB
ZEUS-S	12.07 ± 0.41 NB	8.76 ± 0.30 NB	1.89 ± 0.06 NB
CTEQ6.1	11.66 ± 0.56 NB	8.58 ± 0.43 NB	1.92 ± 0.08 NB
MRST01	11.72 ± 0.23 NB	8.72 ± 0.16 NB	1.96 ± 0.03 NB

W PRODUCTION @ LHC: THE NOT SO GOOD NEWS





- NEW (CTEQ6.5) PARTON SET INCLUDES HQ MATCHING
- EFFECT OF IMPROVED HW MASS FELT MOSTLY IN SMALL x QUARK SUPPRESSION OF CHARM $\Rightarrow$ ENHANCEMENT OF LIGHT SEA
- IN COMPARISON TO PREVIOUS (CTEQ76.1), SIGNIFICANT CHANGE OF u, d QUARK DISTNS AT $x \sim 0.01$
- W, Z TOTAL XSECT NO LONGER AGREES WITH MRST THOUGH MRST INCLUDES HQ MATCHING!
- EFFECT OF INTRINSIC CHARM (IC) MINOR

Where do we stand now?

WHAT WE HAVE LEARNT

- LIGHT QUARK STRUCTURE IN "VALENCE" REGION $0.1 \lesssim x \lesssim 0.5$ (old fixed target dis data)
- SINGLET AND GLUON AT SMALL $x < 10^{-2}$ (HERA)
- SEA ASYMMETRY AT MEDIUM $x \sim 0.1 \div 0.2$ (Drell-Yan)
- HINTS ON STRANGENESS (neutrinos)

WHAT WE ARE STILL MISSING

- GLUONS AT LARGE x (cfr large E_T jet problem)
- NONSINGLET & VALENCE AT SMALL x
- DETAILED INFO ON STRANGENESS (cfr NuTeV problem)
- INFO ON HEAVY QUARKS (cfr small x W xsect problem)

IS IT A PROBLEM?

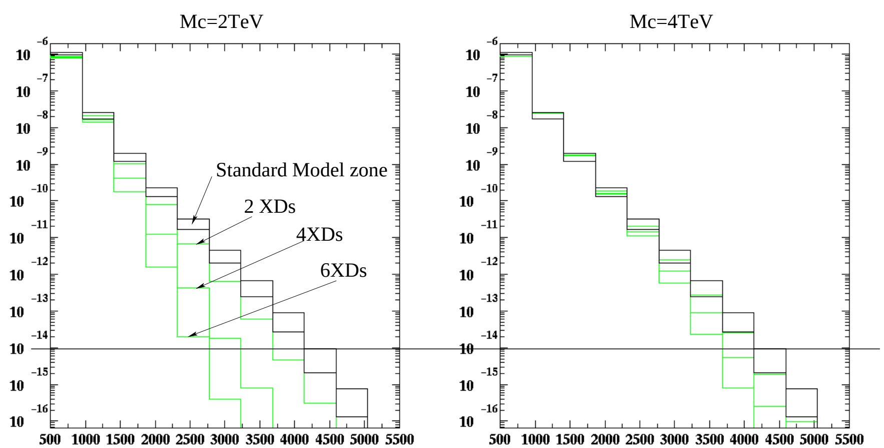
EXAMPLE: LACK OF KNOWLEDGE OF LARGE x GLUON LIMITS DISCOVERY POTENTIAL FOR EXTRA DIMENSIONS

UPPER LIMIT ON COMPACTIFICATION SCALE FROM DIJET CROSS SECTIONS FROM 100 FB^{-1} AT LHC Ferrag (ATLAS, 2006)

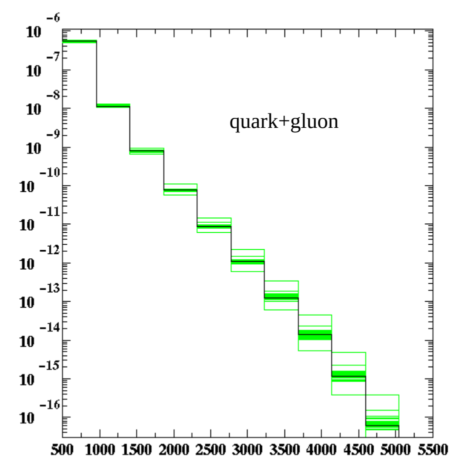
	2 extra dimensions	4 extra dimensions	6 extra dimensions
THEORETICALLY	5 TEV	5 TEV	5 TEV
INCLUDING PDF UNCERTAINTIES	$<$ 2 TEV	$<$ 3 TEV	$<$ 4 TEV

CROSS–SECTION IN FIXED p_t BINS

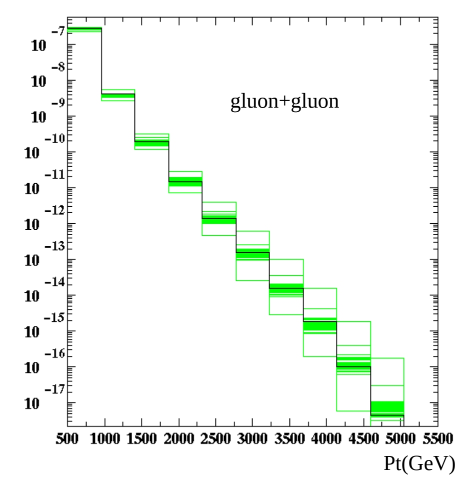
EXTRA DIMENSIONS VS STANDARD MODEL



PDF UNC.: QG CHANNEL

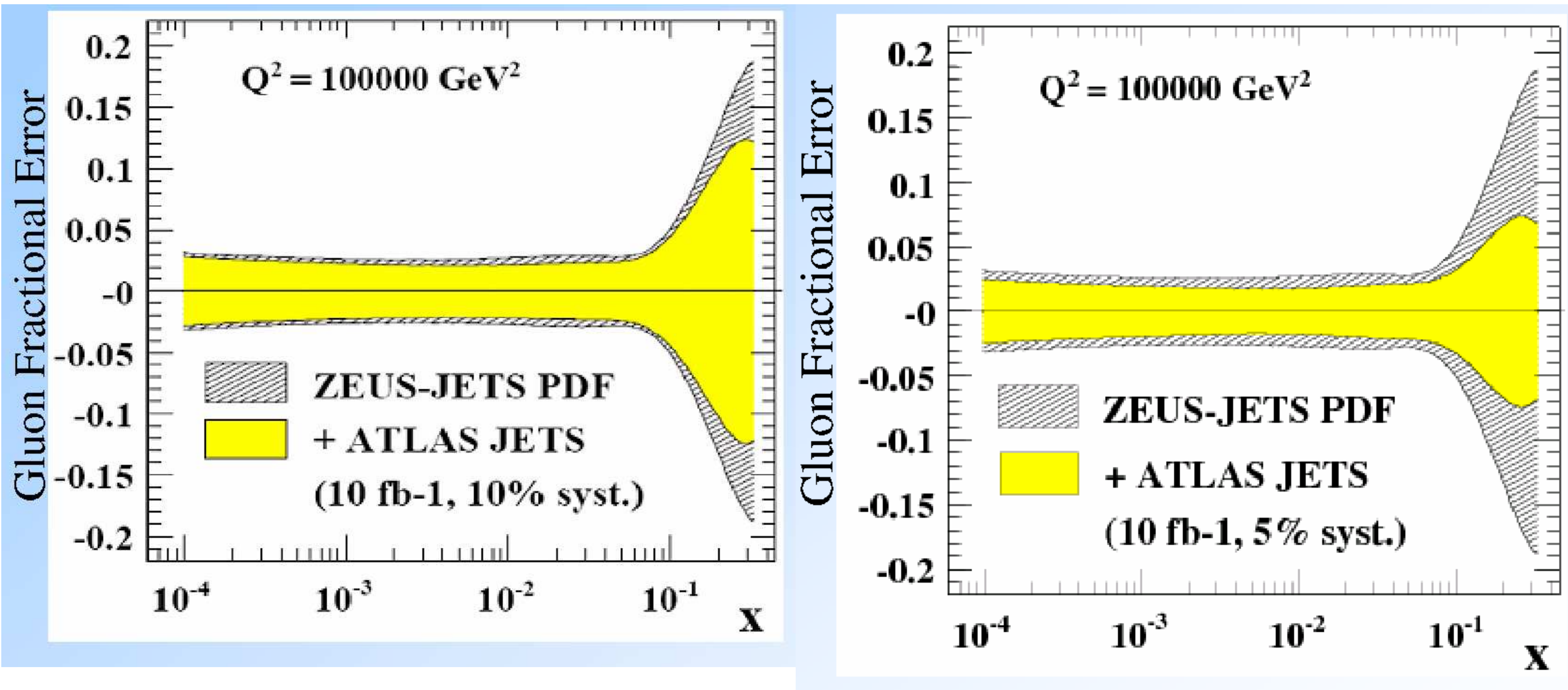


GG CHANNEL



SOLUTIONS: LARGE E_T JETS @ LHC
DETERMINING THE GLUON AT LARGE x

UNCERTAINTY IN THE GLUON GREATLY REDUCED
PROVIDED SYSTEMATICS CAN BE KEPT AT FEW PERCENT LEVEL

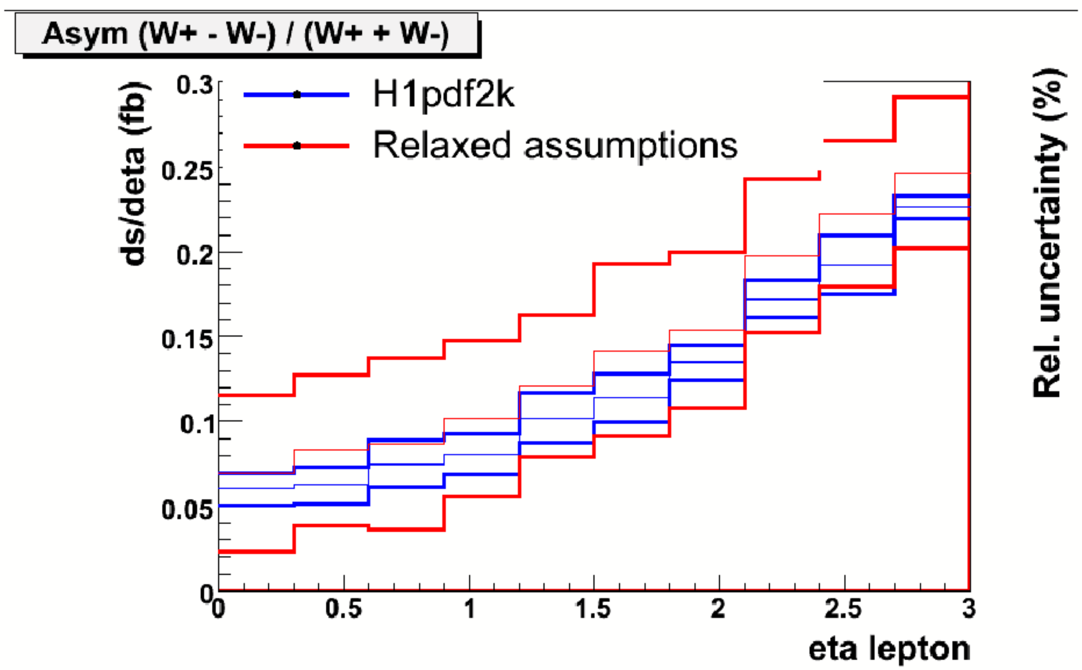


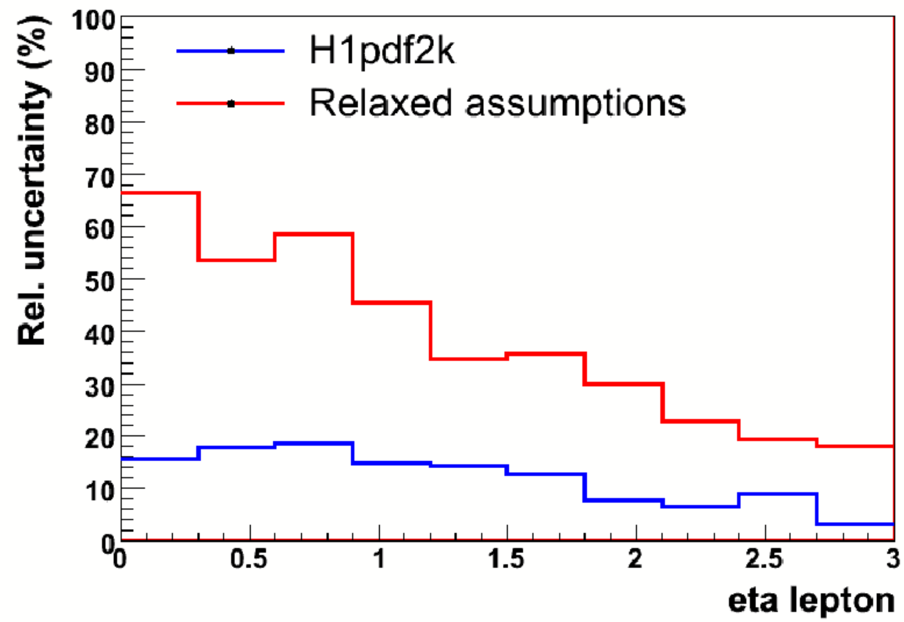
D. Clements (Atlas 2006)

SOLUTIONS: W ASYMMMETRY @ LHC

DETERMINING QUARKS AT SMALL x

- W PRODUCTION AT LHC PROBES $x \sim 10^{-2}$
- $W^{\pm}$ ASYMMETRIES SENSITIVE TO $\bar{u}/\bar{d}$
- $\Rightarrow$ IF SMALL x BEHAVIOUR IS NOT AS CURRENTLY ASSUMED ("REGGE"), $W^{\pm}$ ASYMMETRY CHANGES BY UP TO FACTOR 5!



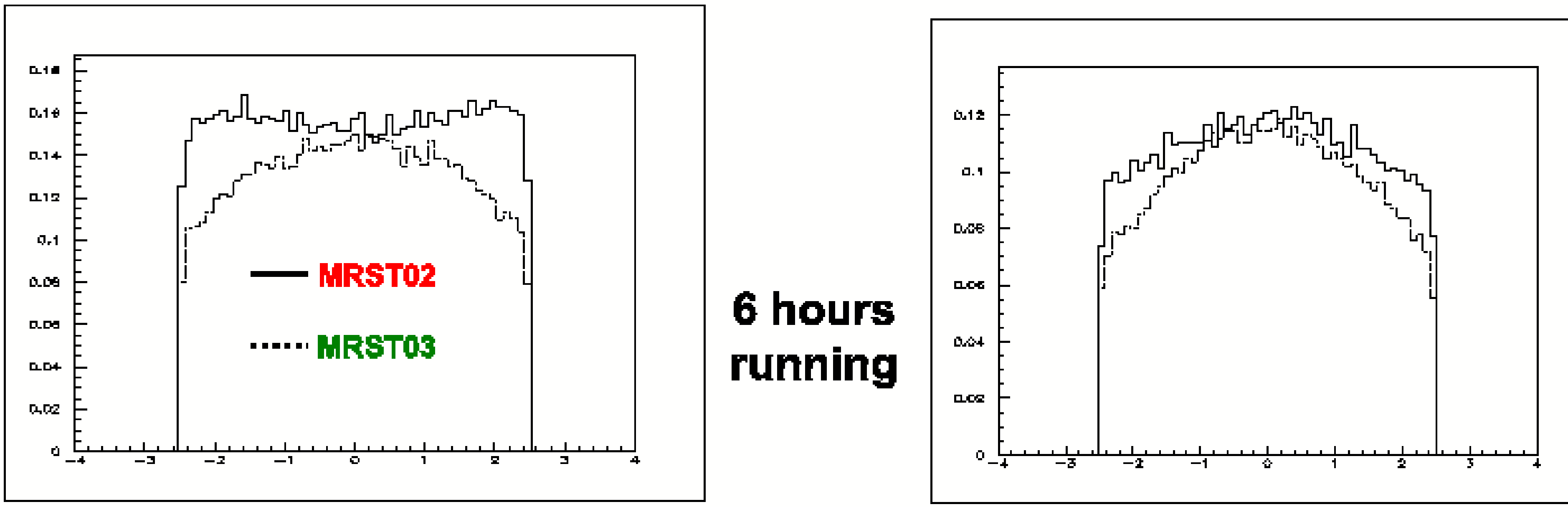


E. Perez (CMS 2006)

SOLUTIONS: W DISTRIBUTION @LHC

PRECISION PHYSICS @ SMALL x

- MRST03 $\Rightarrow$ BEST FIT VS.
  MRST02 $\Rightarrow$ VERY SMALL x HERA DATA NOT INCLUDED
- DIFFERENCE IN ASYMMETRY SEEN AFTER FEW HOURS OF RUNNING!



A. Cooper-Sarkar (Atlas 2006)

PDF UNCERTAINTIES

WHAT'S THE PROBLEM?

- FOR A SINGLE QUANTITY, WE QUOTE 1 SIGMA ERRORS: VALUE$\pm$ ERROR
- FOR A PAIR OF NUMBERS, WE QUOTE A 1 SIGMA ELLIPSE
- FOR A FUNCTION, WE NEED AN "ERROR BAR" IN A SPACE OF FUNCTIONS

MUST DETERMINE THE PROBABILITY DENSITY (MEASURE) $\mathcal{P}[f_i(x)]$
IN THE SPACE OF PARTON DISTRIBUTION FUNCTIONS $f_i(x)$ (i=quark, antiquark, gluon)

EXPECTATION VALUE OF $\sigma[f_i(x)] \Rightarrow$ FUNCTIONAL INTEGRAL

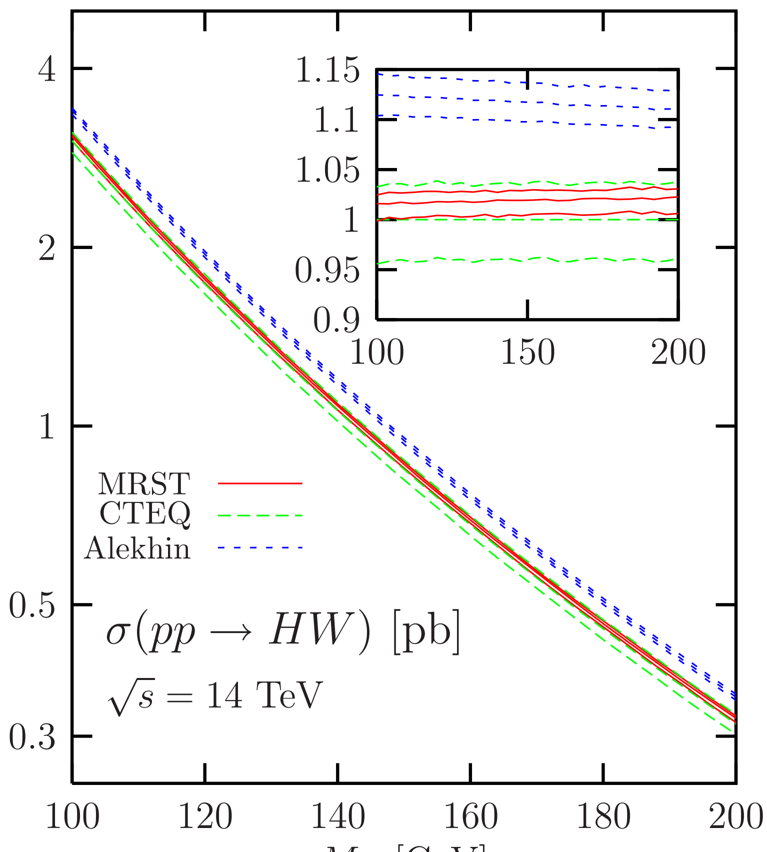
$$\left\langle \sigma[f_i(x)] \right\rangle = \int \mathcal{D}f_i \, \sigma[f_i(x)] \, \mathcal{P}[f_i],$$

MUST DETERMINE AN INFINITE–DIMENSIONAL OBJECT FROM A FINITE SET OF DATA POINTS

CAN WE TRUST PDF UNCERTAINTIES?

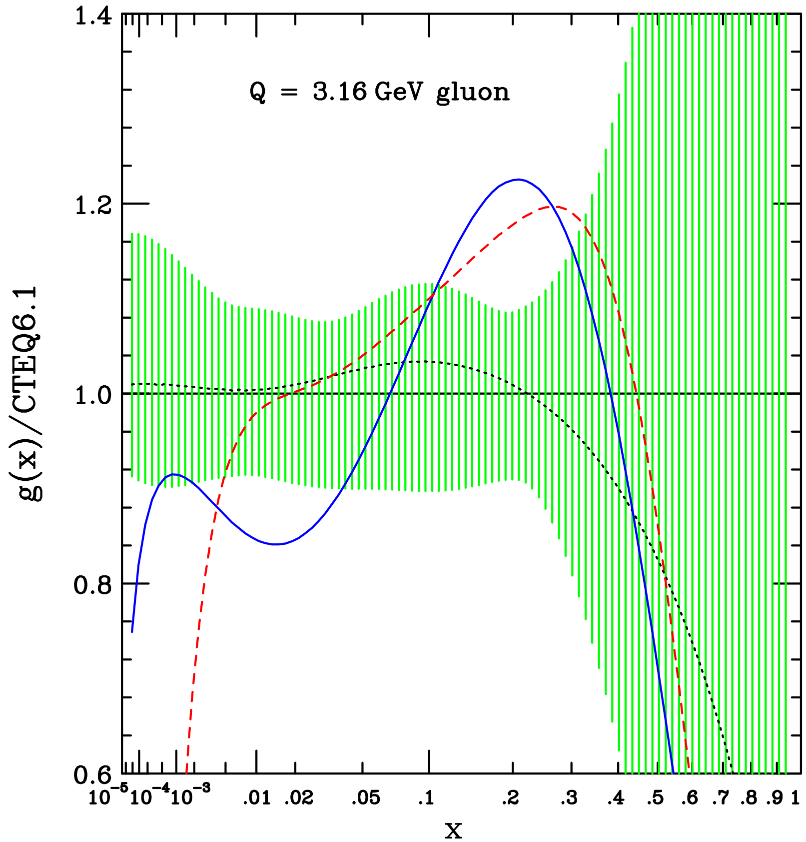
PARTON SETS DO NOT AGREE WITHIN RESPECTIVE ERRORS...

PHYSICAL OBSERVABLE:
HIGGS PRODUCTION AT LHC



DJOUADI AND FERRAG, 2004

PARTON DISTRIBUTIONS:
MRST/CTEQ GLUON



- ALEKHIN VS. MRST/CTEQ → PREDICTIONS FOR ASSOCIATE HIGGS W PRODUCTION @ LHC DO NOT AGREE WITHIN RESPECTIVE ERRORS
- MRST VS. CTEQ GLUONS DO NOT AGREE WITHIN RESPECTIVE ERRORS

ARE MORE DATA ENOUGH TO RESOLVE THE DISCREPANCIES?

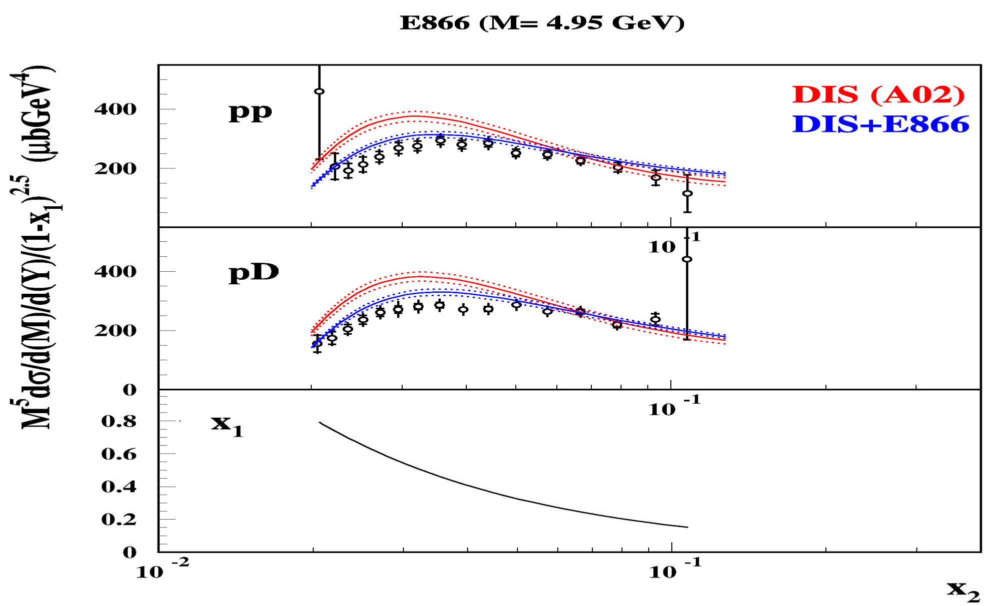
INCOMPATIBLE DATA?

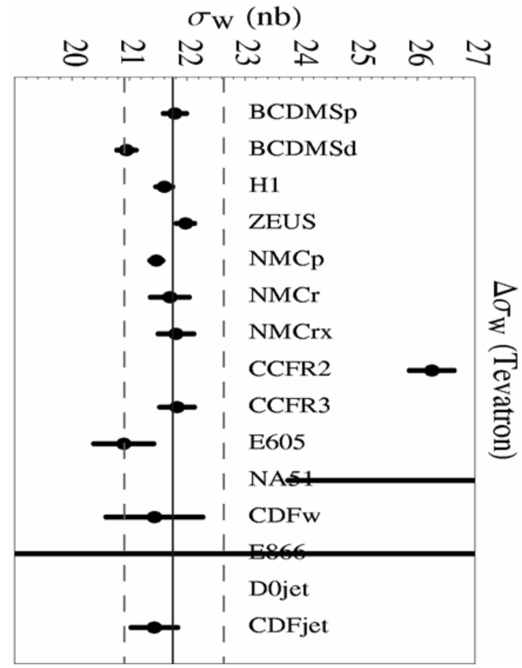
E866 DY DATA DISAGREE WITH DIS DATA:

$\sigma_{DY} \sim q(x_1)q(x_2)$ DISAGREES
WITH DIS QUARK AT SAME x AND Q^2

σ_W PREDICTION UNSTABLE

ONE σ ERROR BAND FOR PHYSICAL
PREDICTIONS BASED ON DIFFERENT
UNDERLYING DATASETS DISAGREE





ALEKHIN 2005

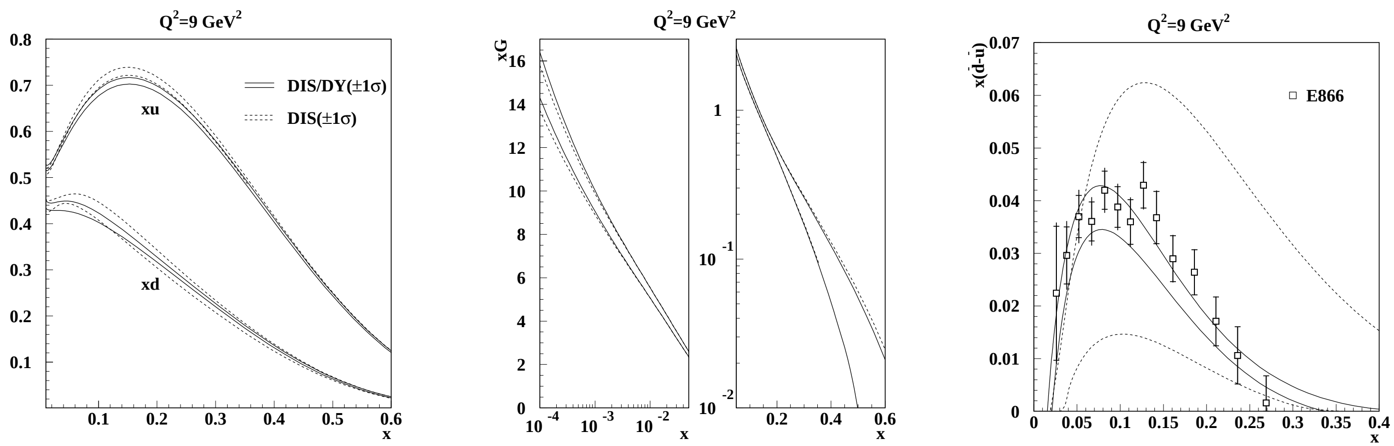
CTEQ 2004

CONSERVATIVE SOLUTION:

SELECT COHERENT SET OF DATA: ALEKHIN PARTONS

- ONLY DIS + SUBSET OF DY DATA INCLUDED
- $\Delta\chi^2 = 1$ PROVIDES GOOD 1-σ CURVES

UP AND DOWN GLUE QUARK SEA ASYM.



- ALEKHIN 2003-2006 PARTON UNCERTAINTIES COMPARABLE TO CTEQ6
- ERROR ON σ_W COMPARABLE TO CTEQ, MRST

CONSERVATIVE SOLUTION:

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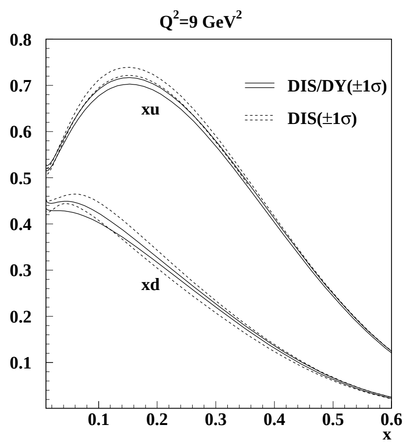
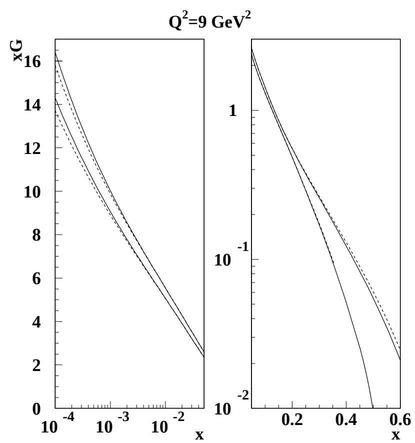
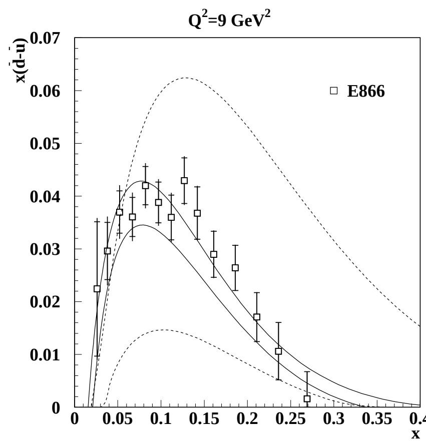
UP AND DOWN



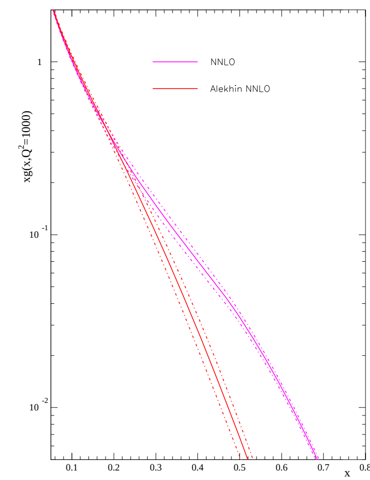
GLUE



QUARK SEA ASYM.



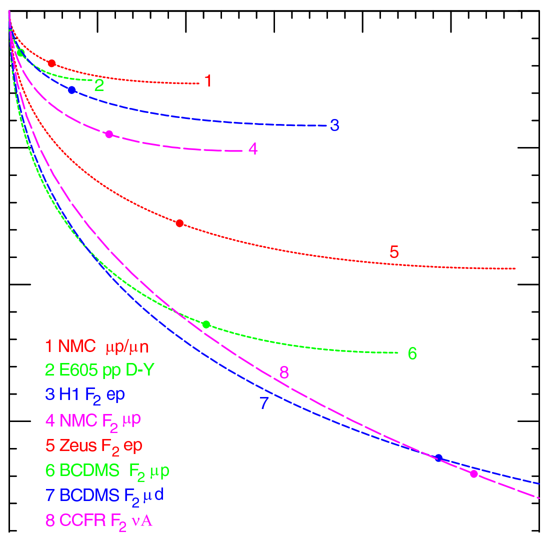
GLUE
A VS MRST



- ALEKHIN 2003-2006 PARTON UNCERTAINTIES COMPARABLE TO CTEQ6
- ERROR ON σ_W COMPARABLE TO CTEQ, MRST
- LOTS OF MISSING INFORMATION (E.G. LARGE x GLUON)

STANDARD SOLUTION: CTEQ TOLERANCE CRITERION

MINIMUM χ_i^2 VS GLOBAL χ^2

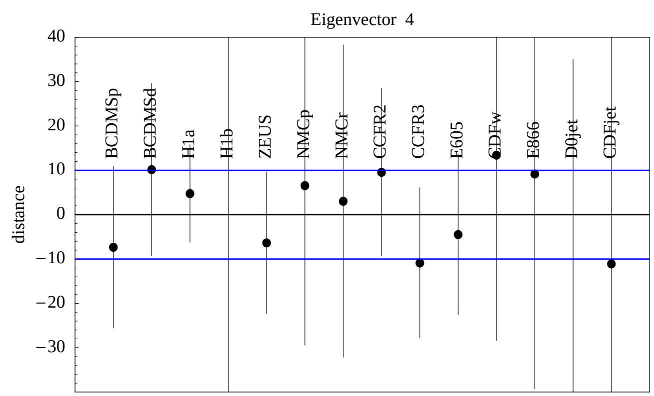


Collins, Pumplin 2001

CCFR, BCDMS INCOMPATIBLE

- DETERMINE EIGENVECTORS OF χ^2 PARABOLOID
- DETERMINE 90% C.L. FOR EACH EXPT. ALONG EACH EIGENVECTOR
- DETERMINE MOST RESTRICTIVE INTERVAL ABOUT GLOBAL MINIMUM (TOLERANCE)

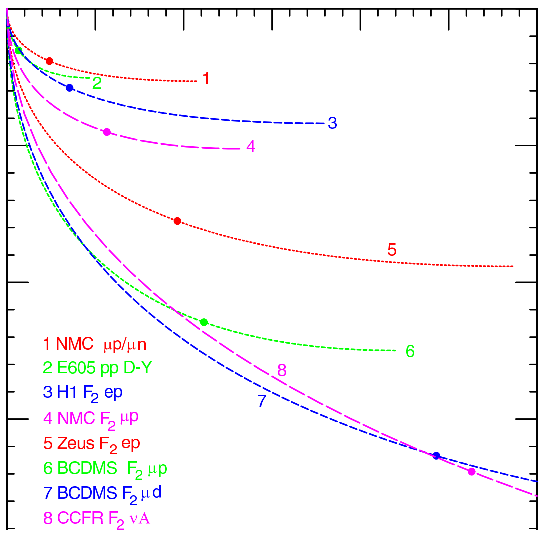
TOLERANCE PLOT
FOR 4TH EIGENVEC.



STANDARD SOLUTION: CTEQ TOLERANCE CRITERION

- DETERMINE EIGENVECTORS OF χ^2 PARABOLOID
- DETERMINE 90% C.L. FOR EACH EXPT. ALONG EACH EIGENVECTOR
- DETERMINE MOST RESTRICTIVE INTERVAL ABOUT GLOBAL MINIMUM (TOLERANCE)
- $\Delta\chi^2 = 100$

MINIMUM χ_i^2 VS GLOBAL χ^2



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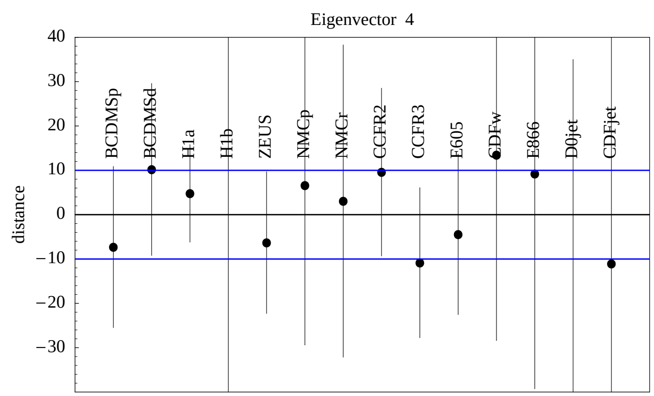
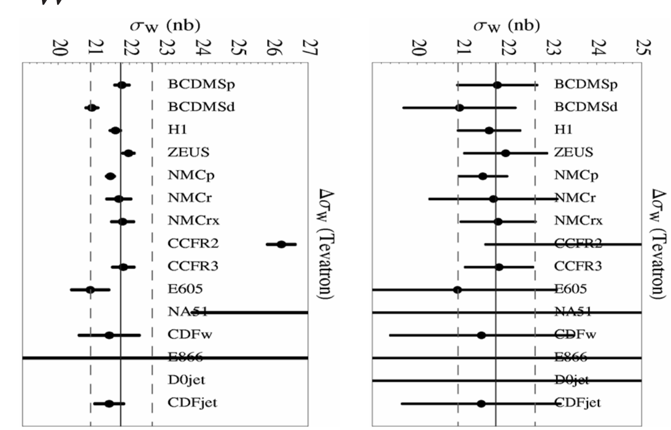


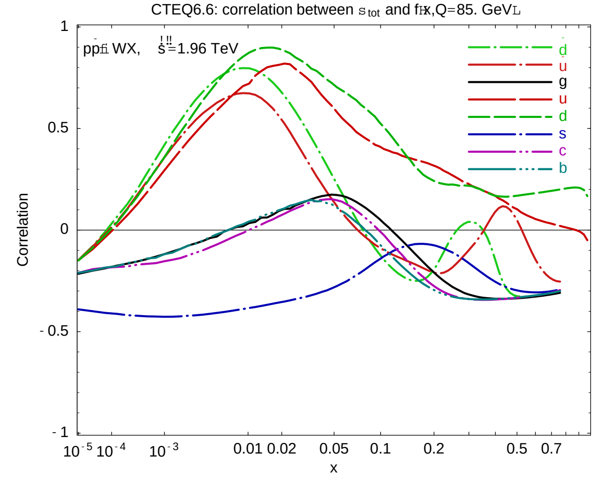
σ_W: ONE σ VS. TOLERANCE



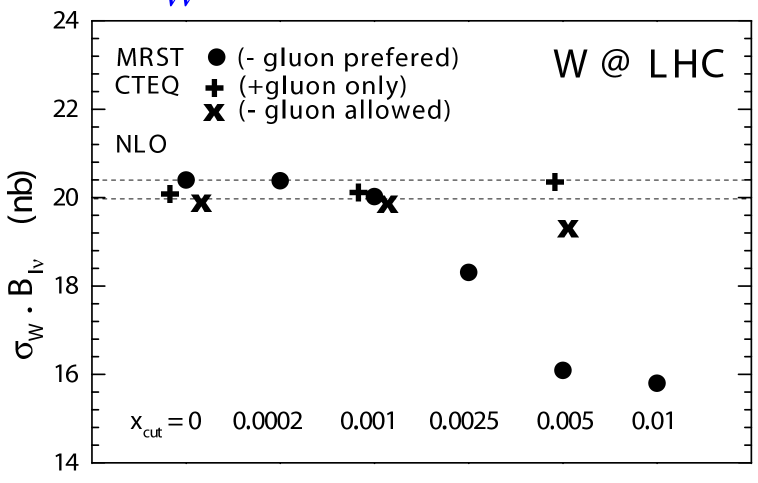
(CTEQ6, 2002-2007)

STABILITY(?) CORRELN. σ_W - PDFS (CTEQ 2008)

- STUDY CORRELATION BETWEEN DATA & PDFS
- REMOVE TROUBLESOME DATA BY CUTTING LOW x, LOW Q^2 ("CONSERVATIVE PARTONS", MRST 2003)
- RESULTS UNSTABLE $\longrightarrow$ MISSING INFO (MRST)
- OR STABLE WITH "PROPER" ASSUMPTIONS (CTEQ)



σ_W AS CUTS ARE RAISED



STABILITY(?)

- STUDY CORRELATION BETWEEN DATA & PDFS
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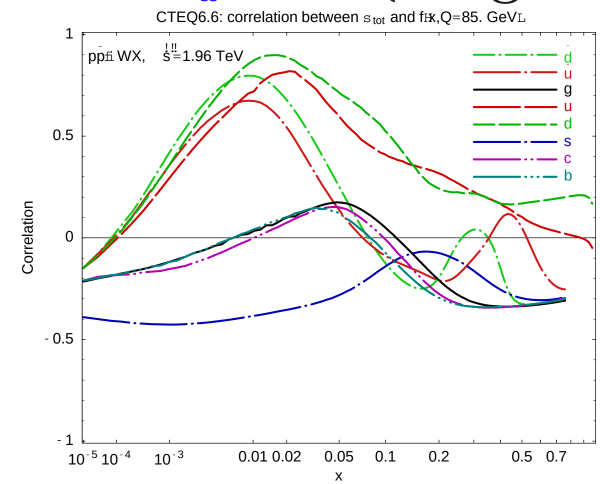
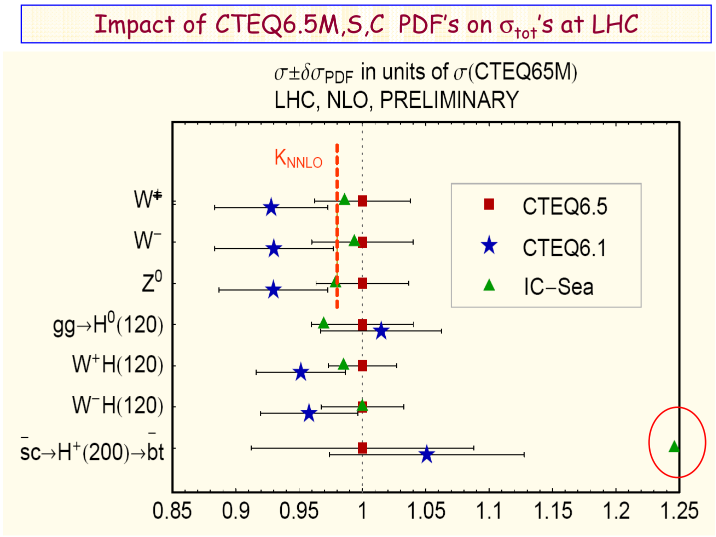
- RESULTS UNSTABLE $\longrightarrow$ MISSING INFO (MRST)
- OR STABLE WITH "PROPER" ASSUMPTIONS (CTEQ)
- IS A STABLE RESULT RELIABLE?

CORRELN. σ_W - PDFS (CTEQ 2008)



EFFECT OF HQ MATCHING



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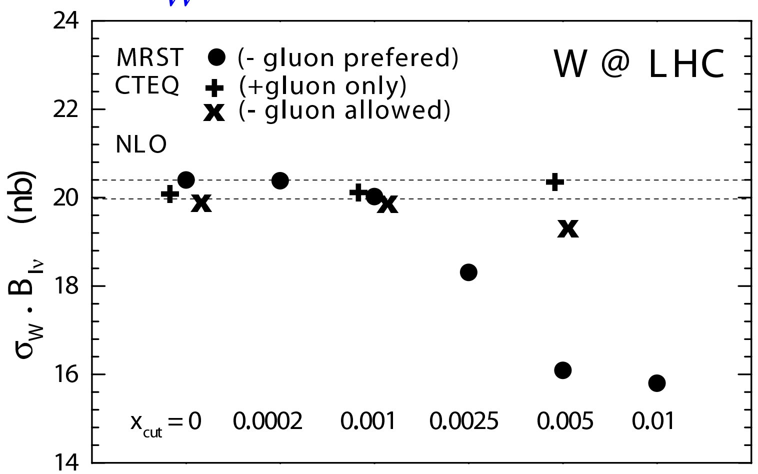
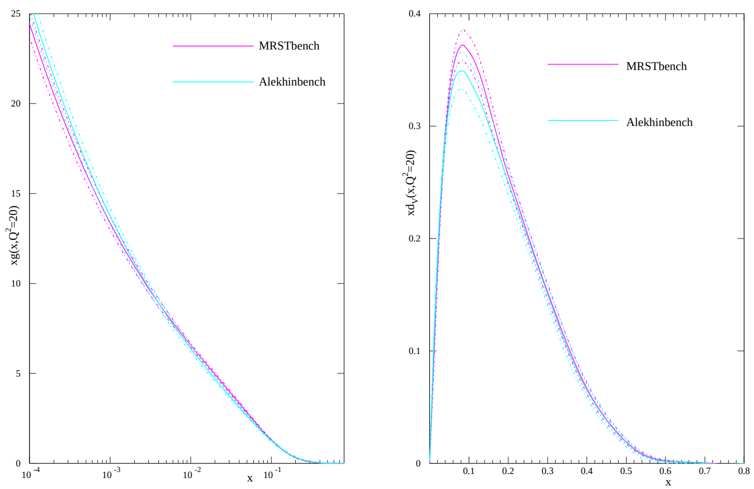


THE HERA-LHC BENCHMARK: AN IMPASSE

- HERA-LHC BENCHMARK PARTONS OBTAINED FROM NC DIS DATA ONLY, $Q^2 > 9\ \mathrm{GeV}^2$
- FITTED WITH RESPECTIVE METHODS BY A & MRST
- ALL ERRORS DETERMINED AT ONE σ

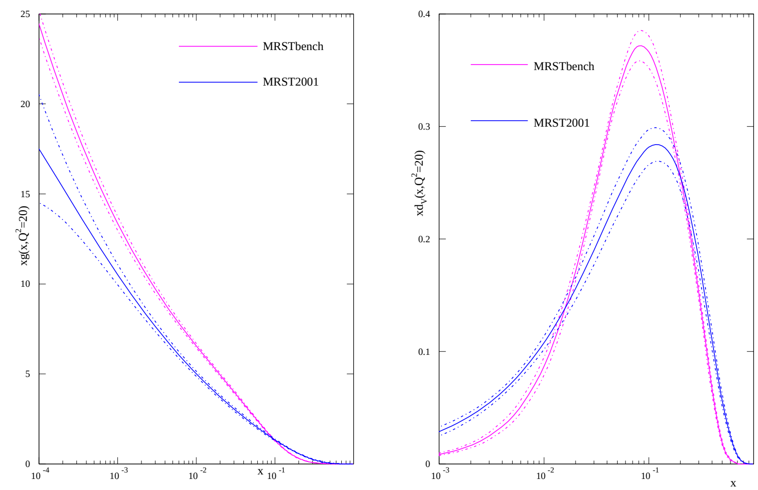
GLUON AND d_V

A BENCH VS. MRST BENCH



AGREEMENT!

MRST VS. BENCH



THE HERA-LHC BENCHMARK: AN IMPASSE

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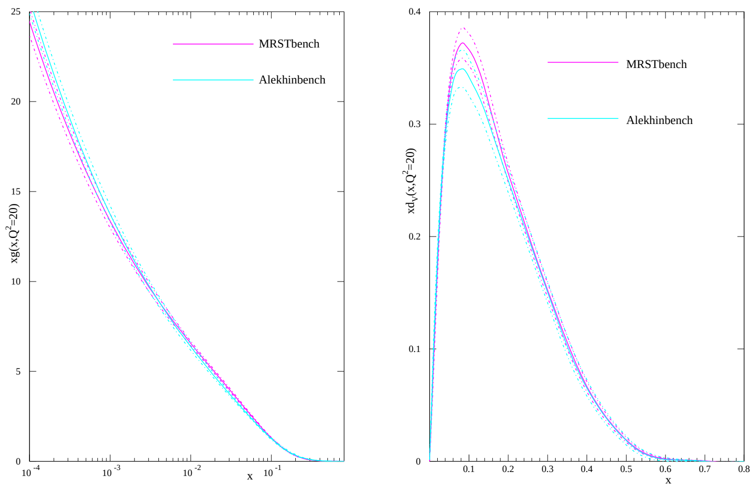
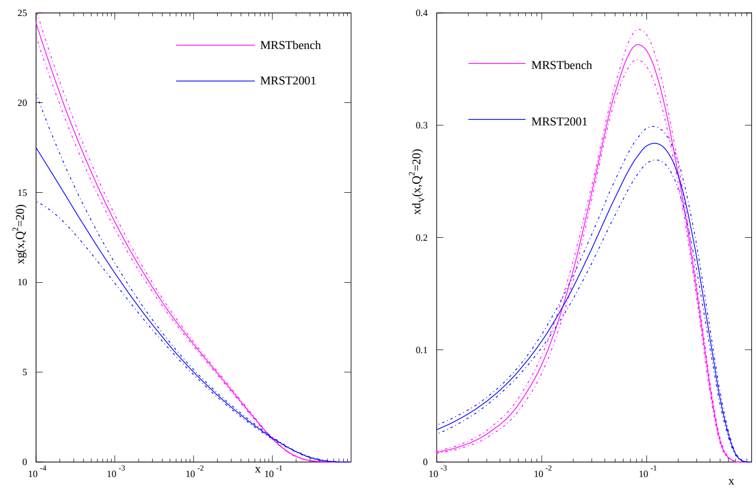


AGREEMENT!

MRST VS. BENCH



DISAGREEMENT!

- IT IS UNSURPRIZING THAT CENTRAL VALUES DEPEND STRONGLY ON THE DATASET
- BUT IT IS VERY WORRISOME THAT THE RESULT WITH THE FULL DATA SET IS NOT WITHIN THE ERROR BAND OF THE RESULT FROM A DATA SUBSET

THE NNPDF APPROACH

THE NEURAL MONTE CARLO

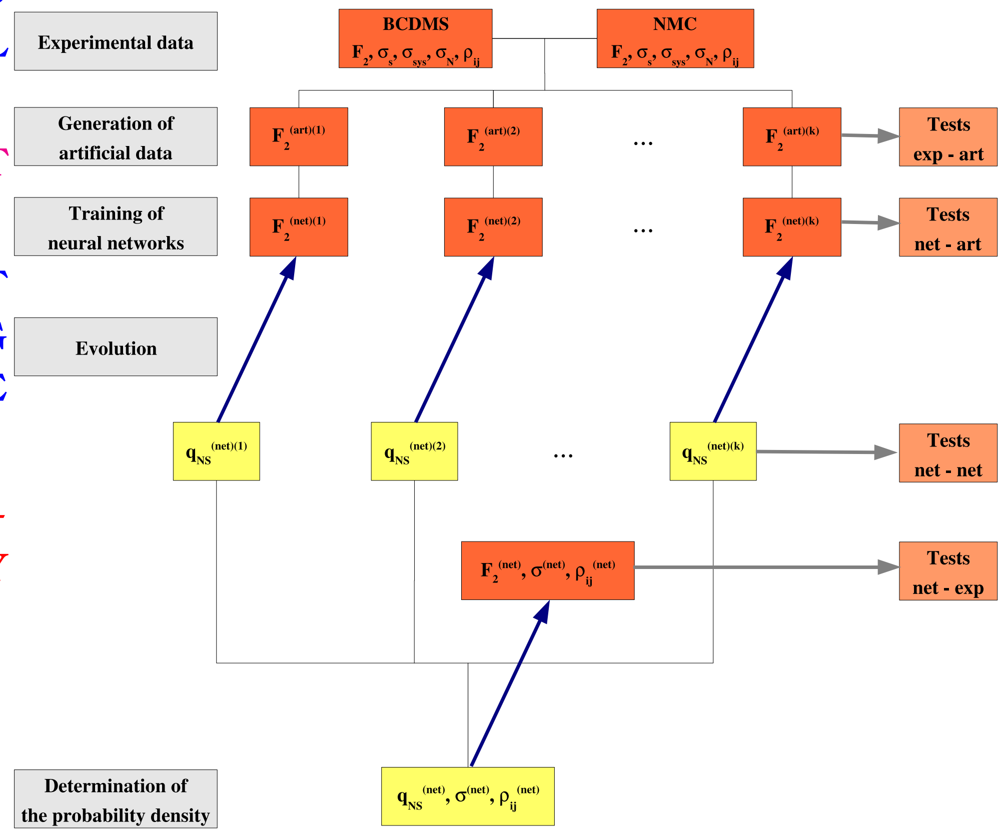
THE NNPDF COLLABORATION

(2004: Del Debbio, SF, Latorre, Piccione, Rojo; 2007: +Ball, Guffanti, Ubiali)

BASIC IDEA: USE NEURAL NETWORKS AS UNIVERSAL UNBIASED INTERPOLANTS

- GENERATE A SET OF MONTE CARLO REPLICAS $\sigma^{(k)}(p_i)$ OF THE ORIGINAL DATASET $\sigma^{(\mathrm{data})}(p_i)$
  $\Rightarrow$ REPRESENTATION OF $\mathcal{P}[\sigma(p_i)]$ AT DISCRETE SET OF POINTS p_i
- TRAIN A NEURAL NET FOR EACH PDF ON EACH REPLICA, THUS OBTAINING A NEURAL REPRESENTATION OF THE PDFS $f_i^{(net),(k)}$
- THE SET OF NEURAL NETS IS A REPRESENTATION OF THE PROBABILITY DENSITY:

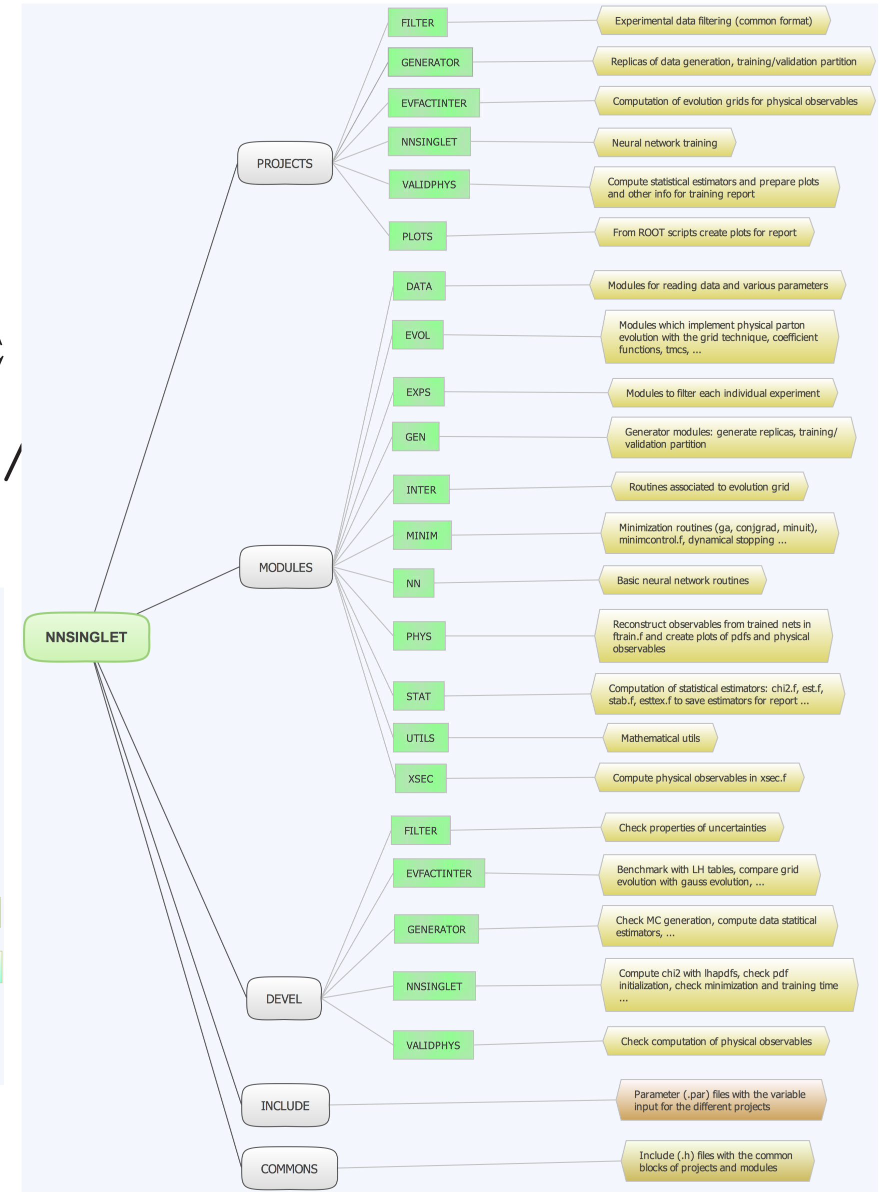
$$\left\langle \sigma\left[f_i\right] \right\rangle = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} \sigma\left[f_i^{(net)(k)}\right]$$



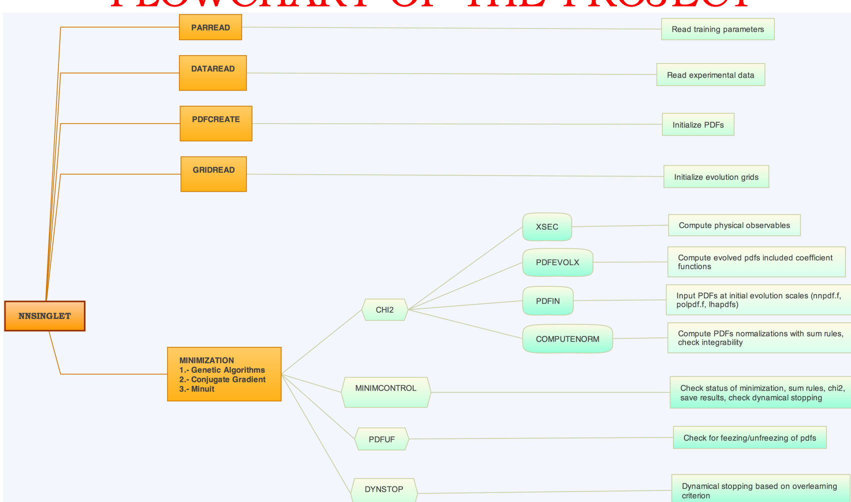
THE PROJECT AND ITS STRUCTURE

- ABOUT 20.000 LINES OF CODE, ABOUT 200 MODULES/ROUTINES
- OBJECT-ORIENTED STRUCTURE, SVN
- FULL DOCUMENTATION AVAILABLE AT `http://sophia.ecm.ub.es/nnpdf/`

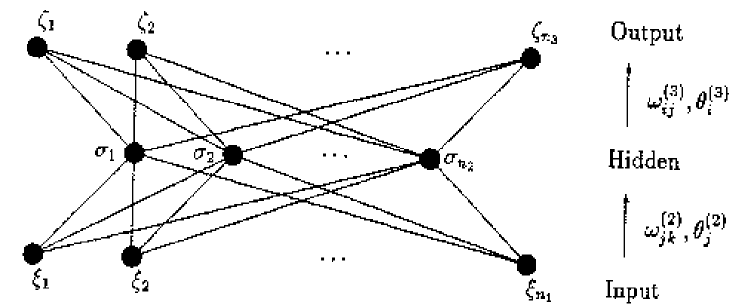
STRUCTURE OF THE CODE

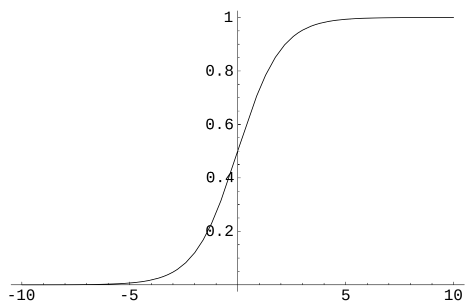


FLOWCHART OF THE PROJECT



WHAT ARE NEURAL NETWORKS?





MULTILAYER FEED-FORWARD NETWORKS

- Each neuron receives input from neurons in preceding layer and feeds output to neurons in subsequent layer
- Activation determined by weights and thresholds
  $\xi_i = g\left(\sum_j \omega_{ij}\xi_j - \theta_i\right)$
- Sigmoid activation function
  $g(x) = \frac{1}{1+e^{-\beta x}}$

JUST ANOTHER SET OF BASIS FUNCTIONS!

A 1-2-1 NN: $\xi_1^{(3)}(\xi_1^{(1)}) = \frac{1}{1+e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - \xi_1^{(1)}\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - \xi_1^{(1)}\omega_{21}^{(1)}}}}}$

ANY FUNCTION CAN BE REPRESENTED BY A SUFFICIENTLY BIG NEURAL NETWORK

LESS PARAMETERS → SMOOTHER FUNCTIONS

WHY NEURAL NETWORKS?

IN A STANDARD FIT, ONE LOOKS FOR MINIMUM χ^2 WITH GIVEN FINITE PARM.

- IF THE BASIS IS TOO LARGE, THE FIT NEVER CONVERGES
- IF THE BASIS IS TOO SMALL, THE FIT IS BIASED

Q: HOW CAN ONE BE SURE THAT THE COMPROMISE IS UNBIASED?

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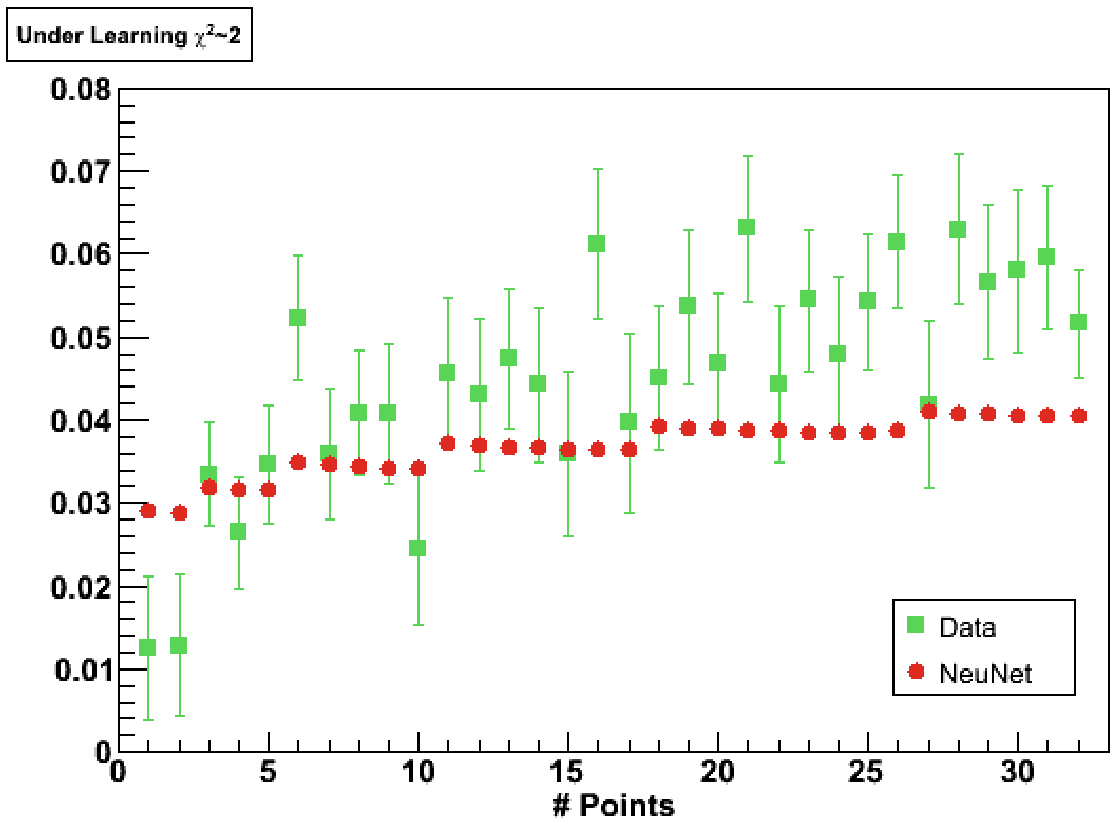
UNDERLEARNING



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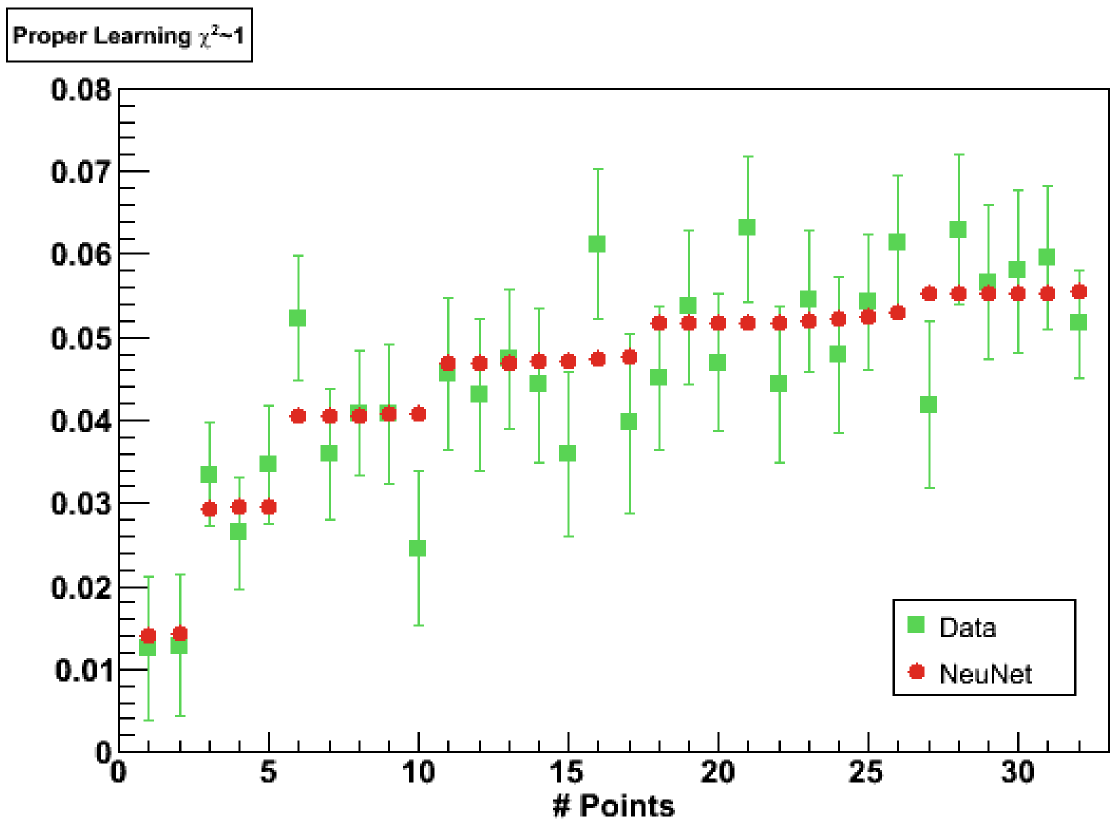
PROPER LEARNING



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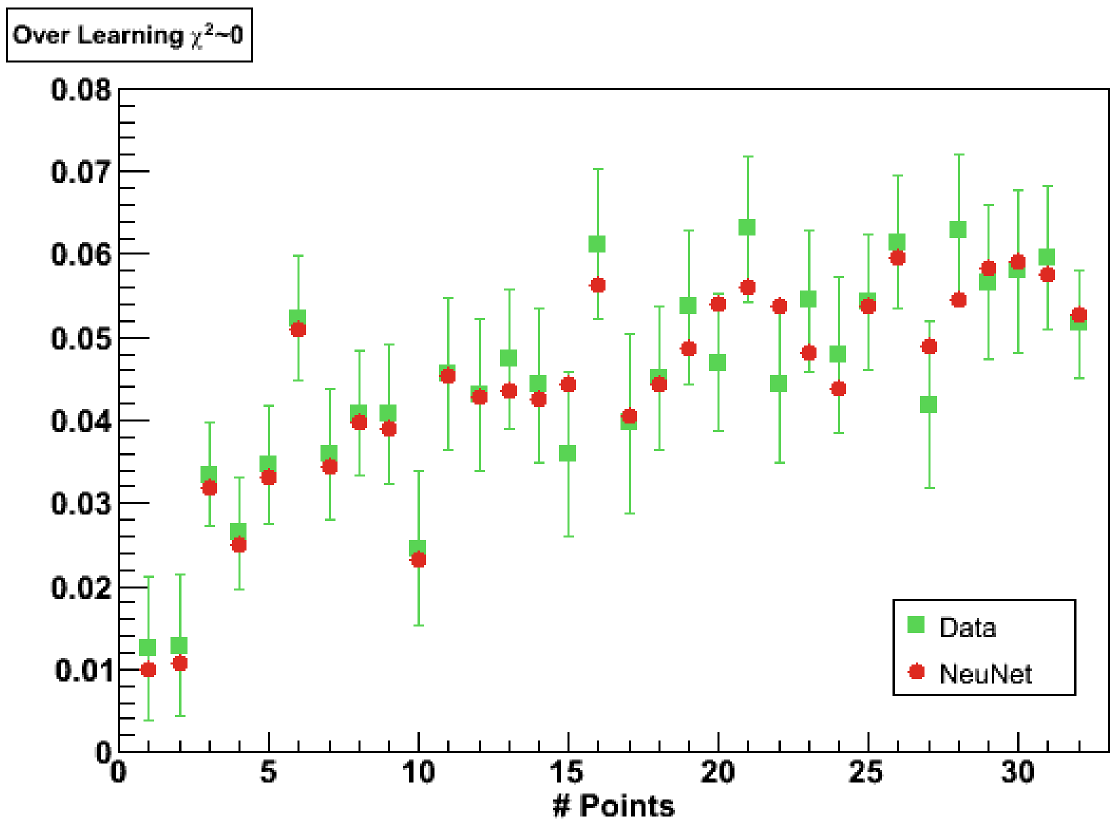

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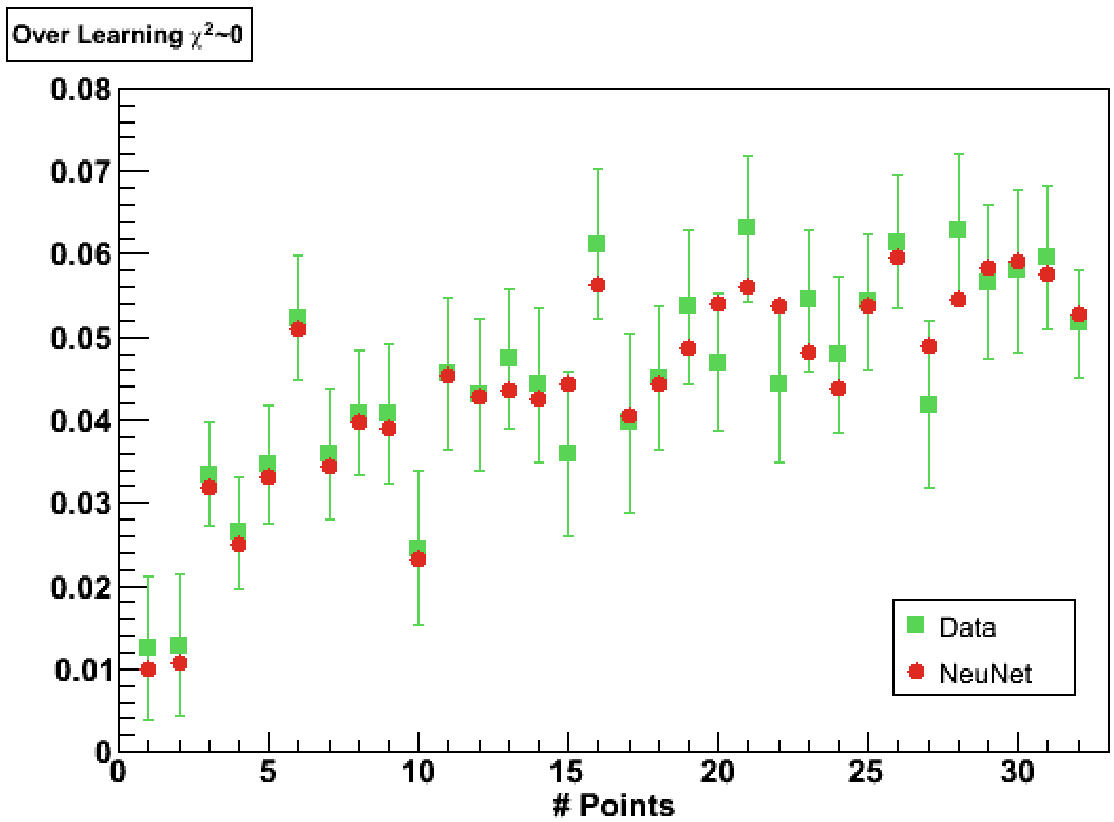

A: STOP THE FIT BEFORE OVERLEARNING SETS IN!

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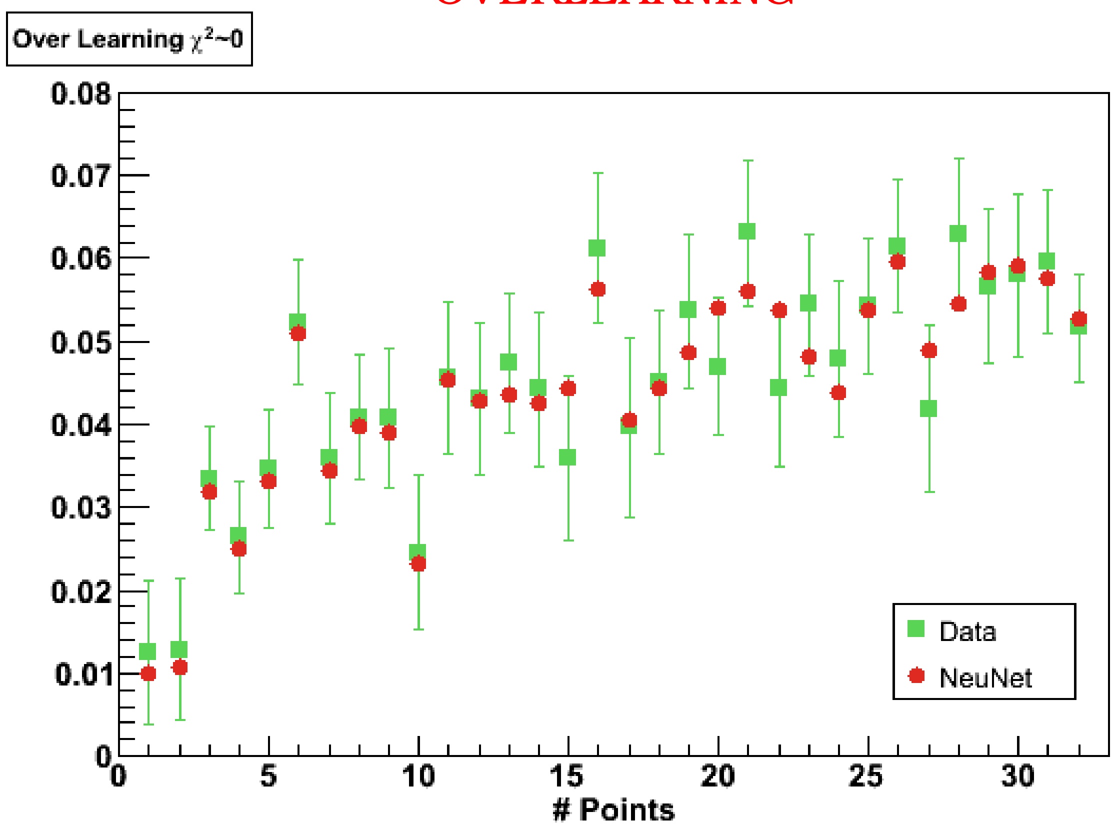
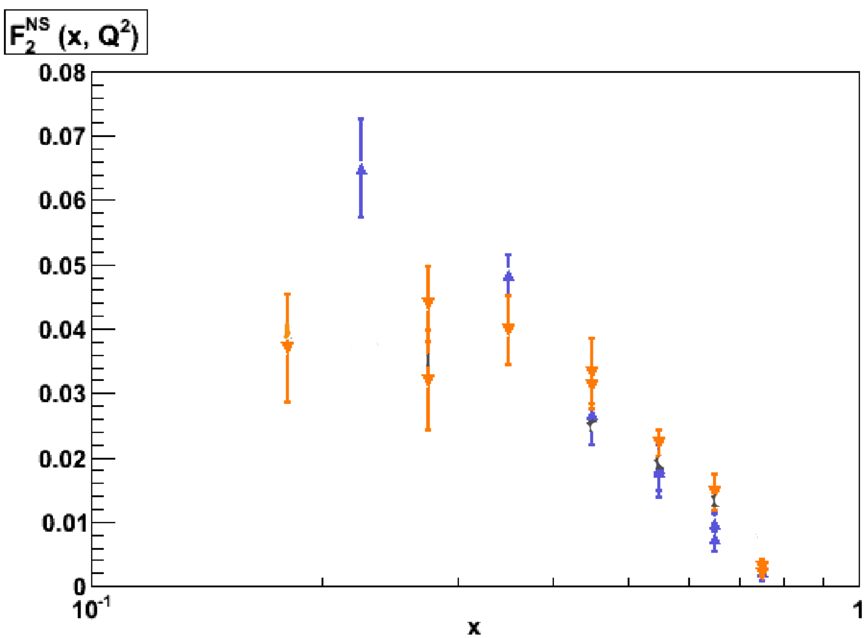


A: STOP THE FIT BEFORE OVERLEARNING SETS IN!
COULD BE DONE WITH STANDARD PARAMETRIZATIONS, BUT VERY INEFFICIENTLY

THE STOPPING CRITERION

MINIMIZE BY GENETIC ALGORITHM:
AT EACH GENERATION, THE χ^2 EITHER UNCHANGED OR DECREASING

- DIVIDE THE DATA IN TWO SETS: TRAINING AND VALIDATION
- MINIMIZE THE χ^2 OF THE DATA IN THE TRAINING SET
- AT EACH ITERATION, COMPUTE THE χ^2 FOR THE DATA IN THE VALIDATION SET (NOT USED FOR FITTING)
- WHEN THE VALIDATION χ^2 STOPS DECREASING, STOP THE FIT



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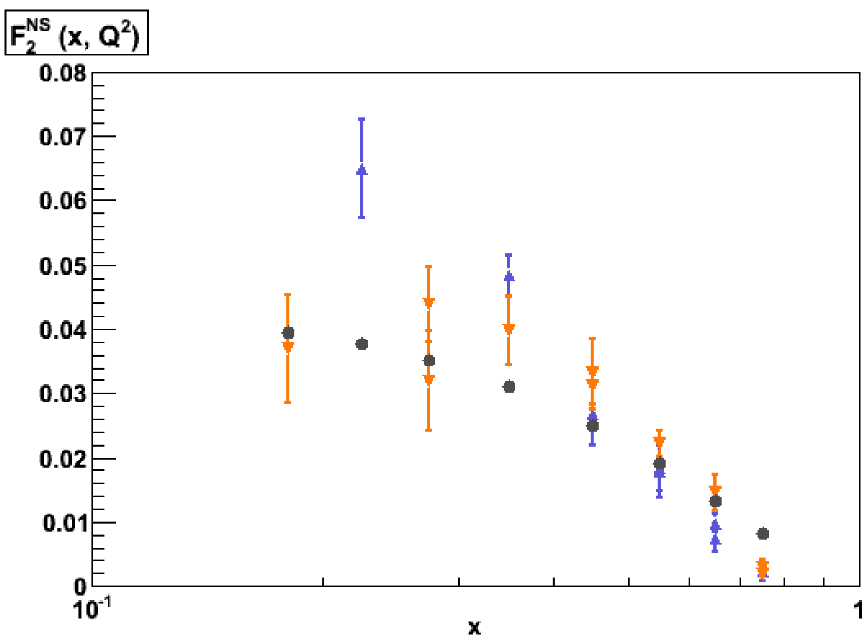
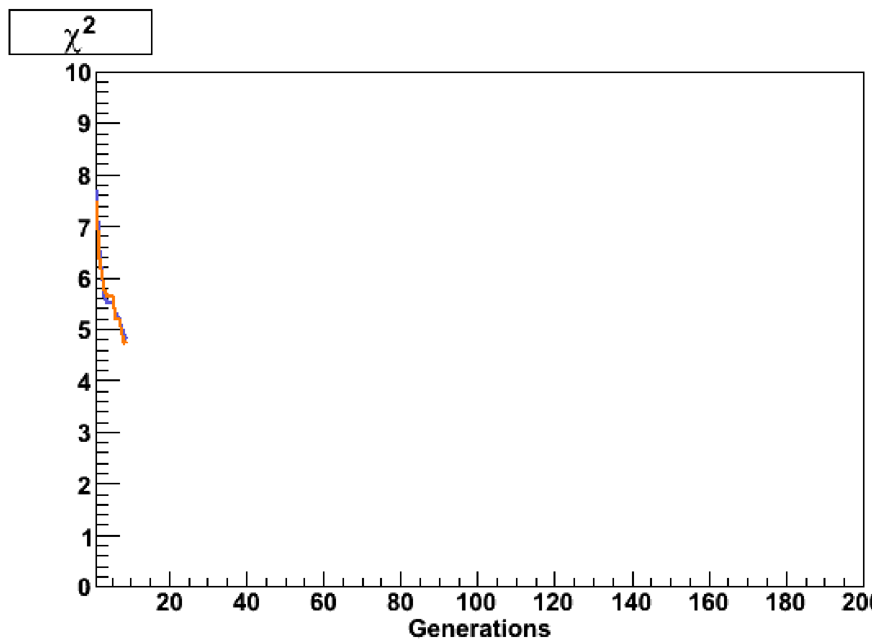
GO!





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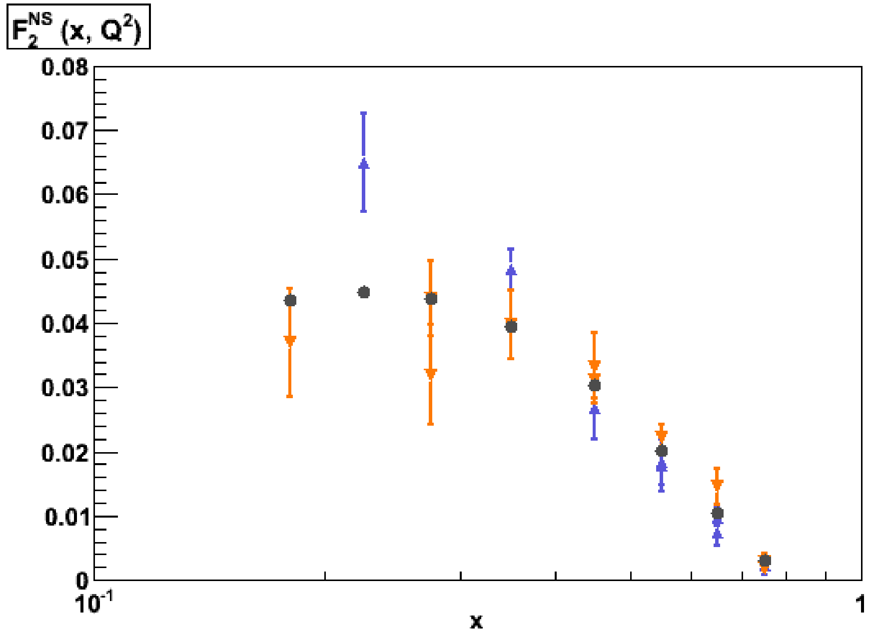
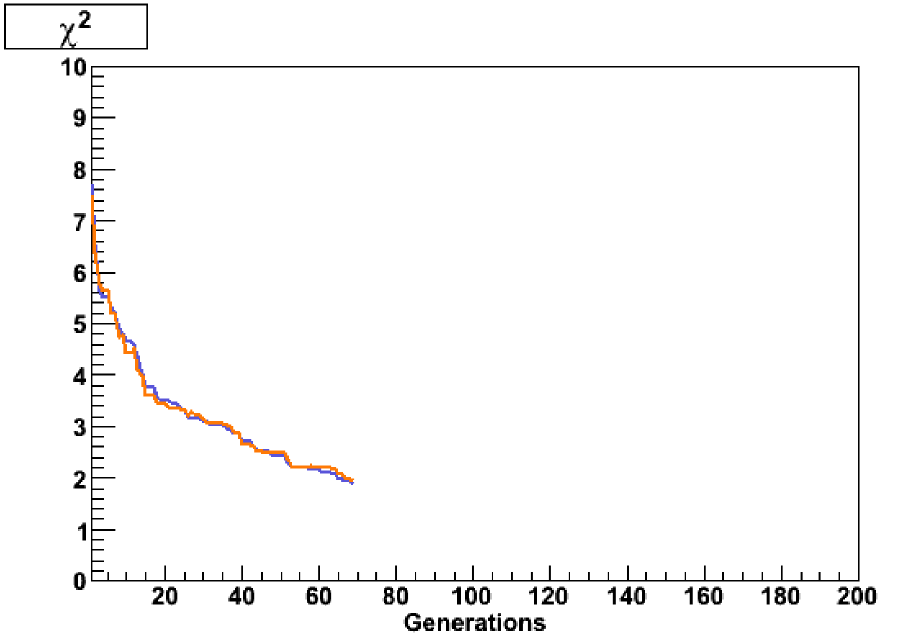
STOP!





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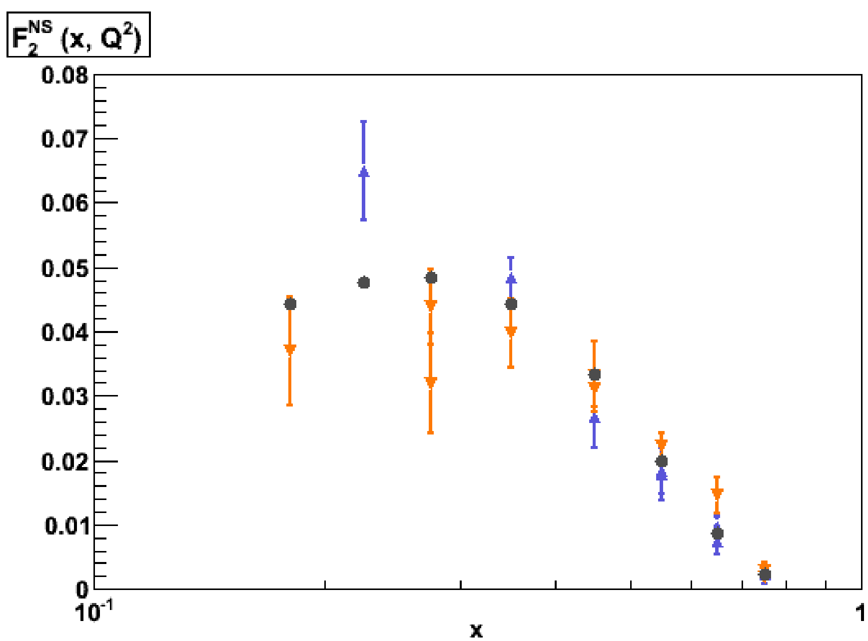
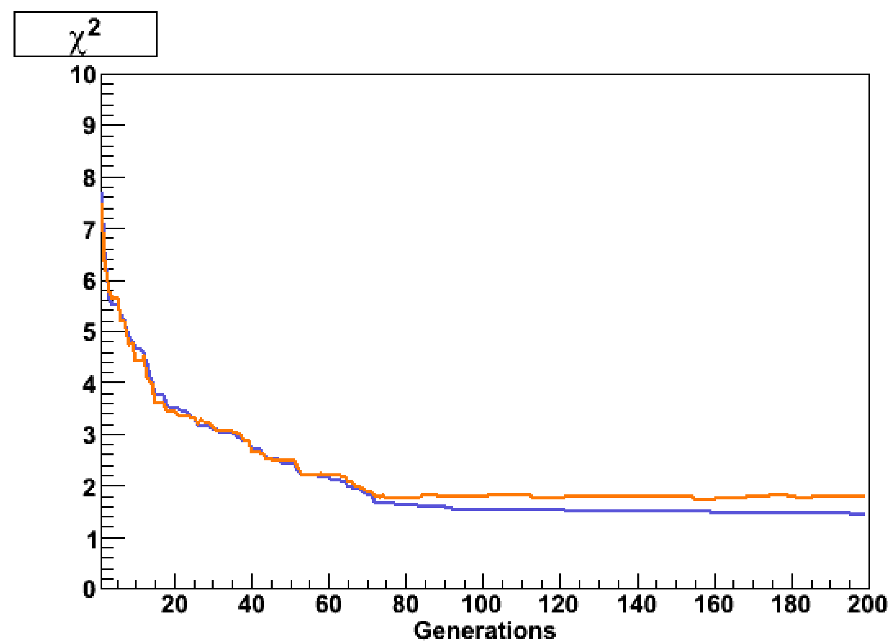
TOO LATE!





STATUS OF THE PROJECT

NONSINGLET FIT

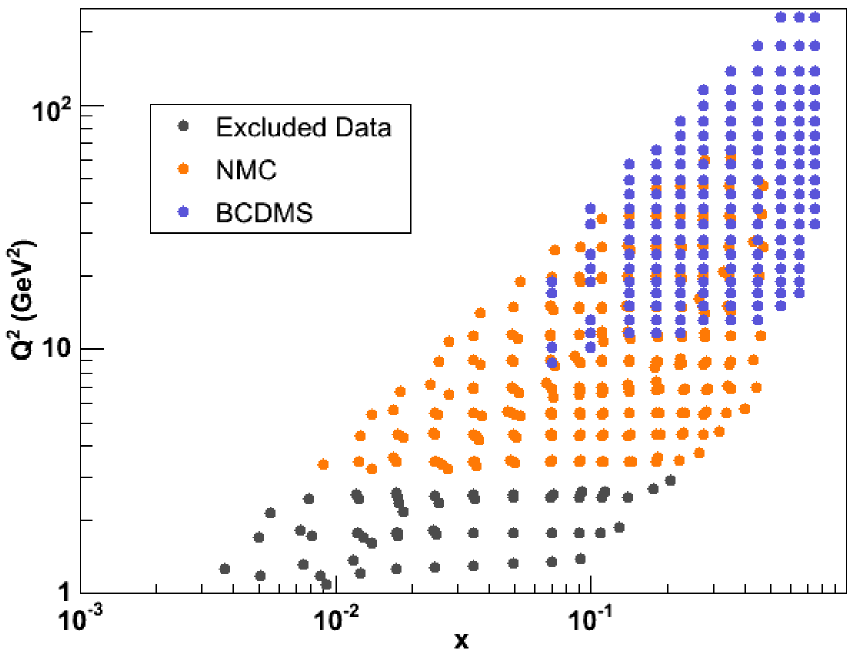
- FIT TO $F_2^p - F_2^d$ DIS DATA FROM BCDMS & NMC (ABOUT 500 DATAPOINTS)
- DETERMINATION OF $u + \bar{u} - (d + \bar{d})$ (ISOTRIPLET) PARTON DISTRIBUTION
- PUBLISHED IN JHEP 0703:039,2007

FULL FIT

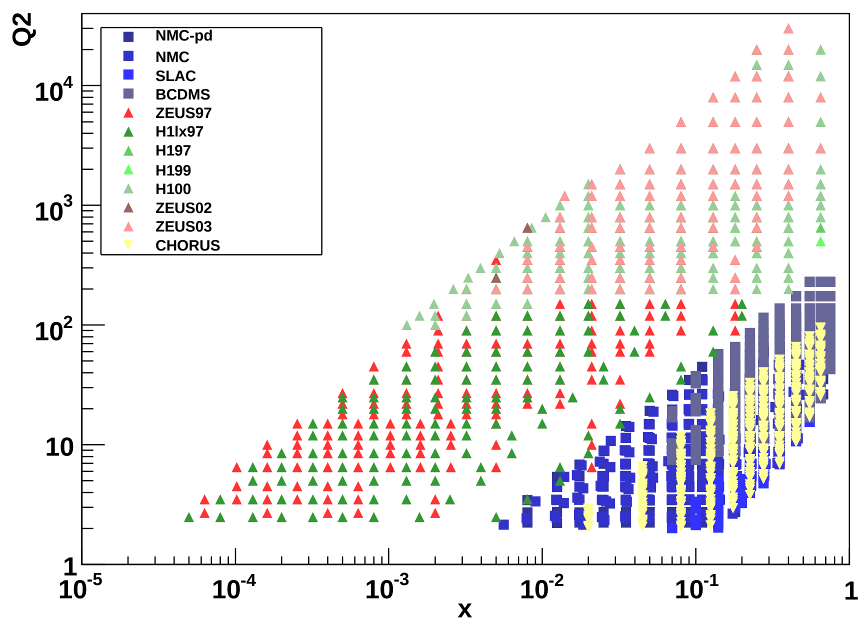
- FIT TO AVAILABLE UNPOLARIZED DIS DATA: ELECTRON AND NEUTRINO BEAMS, NC & CC SCATTERING, PROTON AND DEUTERIUM TARGETS FROM 12 EXPERIMENTS (ABOUT 3000 DATAPOINTS)
- DETERMINATION OF A SET OF FIVE INDEP. PDFS (SAME AS MRST, CTEQ)
- TO APPEAR BEFORE THE SPRING 2008

THE DATA

NONSINGLET



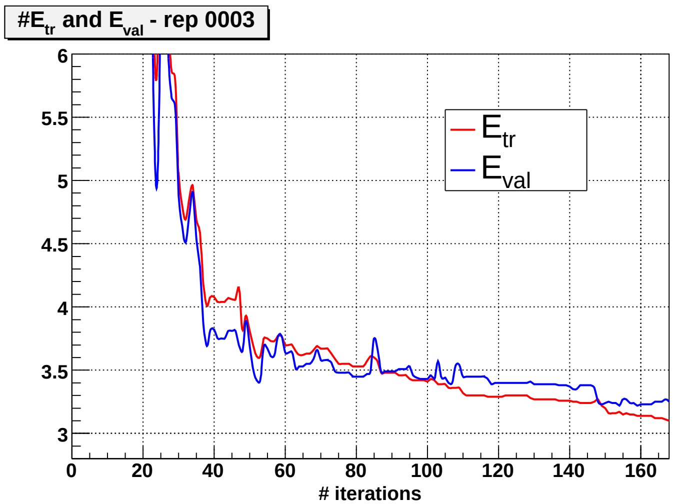
FULL

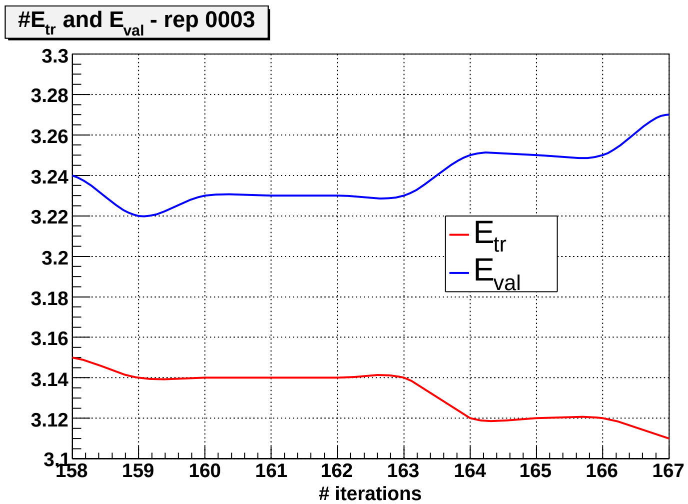


STOPPING I

- EACH NEURAL NET IS FITTED TO A PSEUDODATA REPLICA BY MINIMIZING THE χ^2 TO SUBSET OF DATA (TRAINING SET)
- FIT STOPS WHEN THE χ^2 OF THE REMAINING DATA STARTS TO GROW (VALIDATION SET)

STOPPING FOR THE χ^2 OF ONE REPLICA (FULL FIT)

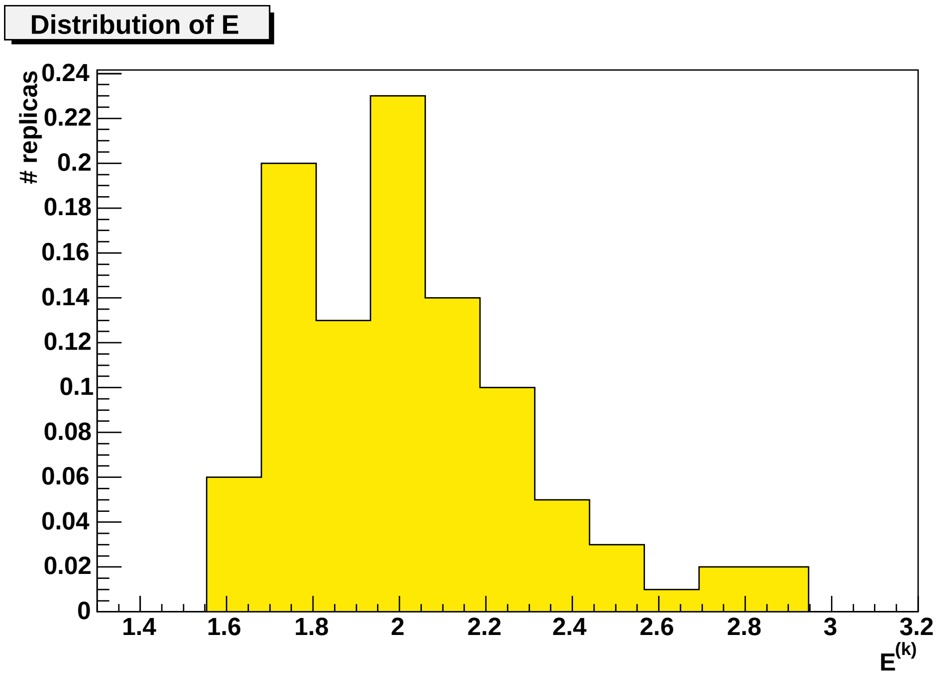




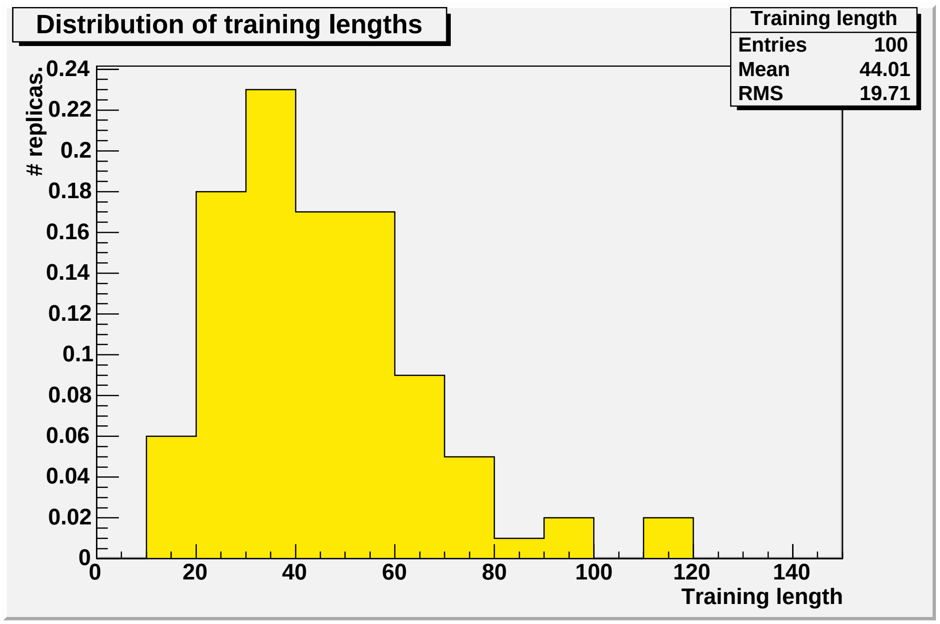
STOPPING II

AFTER STOPPING CRITERION IMPLEMENTED (NONSINGLET FIT)

DISTRIBUTION OF χ^2 AT STOPPING



DISTRIBUTION OF TRAINING LENGTHS



- POISSONIAN DISTRIBUTION OF TRAINING LENGTHS
- BEST FIT $\chi^2 = 0.75$ (BCDMS: 0.75, NMC: 0.72): EXPT. ERRORS SOMEWHAT OVERESTIMATED?

STABILITY

(NONSINGLET FIT)

CAN CHECK STABILITY BY COMPARING RESULTS IF THE WHOLE PROCEDURE IS REPEATED WITH A DIFFERENT SET OF REPLICAS

DEFINE R.M.S. DISTANCE $\langle d[q]\rangle = \sqrt{\left\langle \frac{\left(\langle q_i\rangle_{(1)} - \langle q_i\rangle_{(2)}\right)^2}{\sigma^2[q_i^{(1)}] + \sigma^2[q_i^{(2)}]} \right\rangle_{\rm dat}}$

NOTE σ $\Rightarrow$ ERROR ON AVERAGE = (ERROR ON q_i)/$\sqrt{N}$

$\Rightarrow$ TESTS BOTH ACCURACY OF CENTRAL VALUE & ERRORS

SELF-STABILITY:
DIFFERENT SETS OF 100 REPLICAS

$\langle d\,[q]\rangle_{\rm dat}$	0.96
$\langle d\,[q]\rangle_{\rm extra}$	0.99
$\langle d\,[\sigma_q]\rangle_{\rm dat}$	0.88
$\langle d\,[\sigma_q]\rangle_{\rm extra}$	0.97

CHANGE OF ARCHITECTURE:
2-4-3-1 VS. 2-5-3-1

$\langle d\,[q]\rangle_{\rm dat}$	0.9
$\langle d\,[q]\rangle_{\rm extra}$	0.9
$\langle d\,[\sigma_q]\rangle_{\rm dat}$	0.9
$\langle d\,[\sigma_q]\rangle_{\rm extra}$	1.4

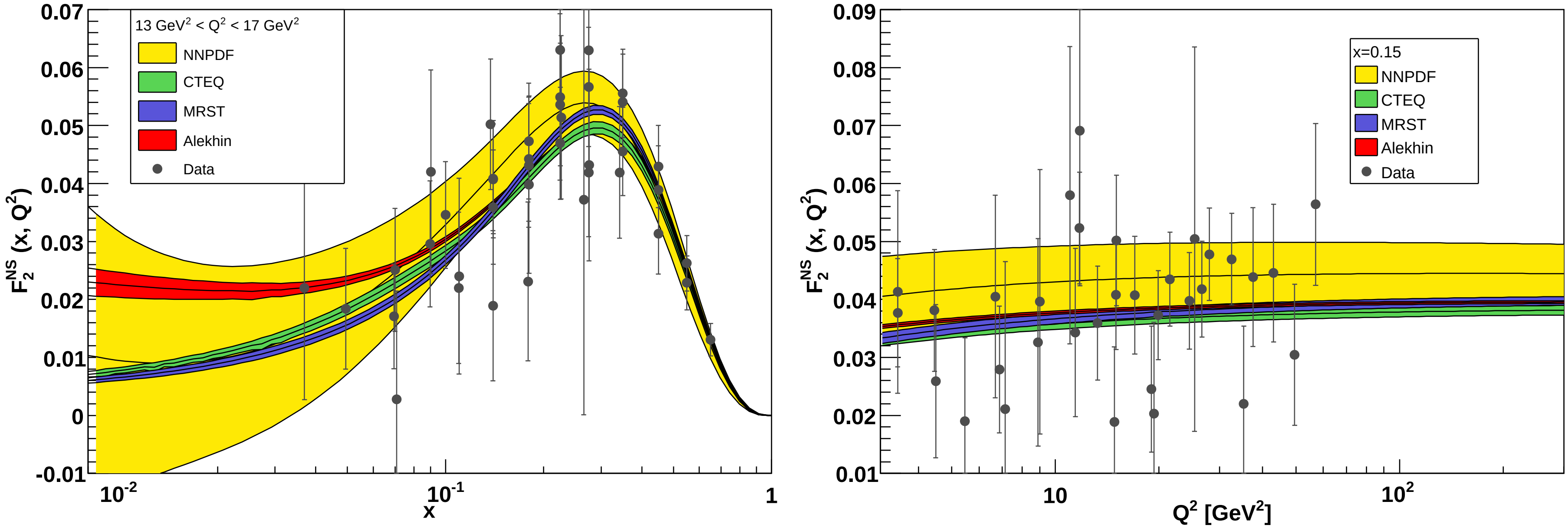
DISTANCE COMPUTED FOR 14 POINTS LINEARLY SPACED IN THE DATA REGION ($0.05 \le x \le 0.75$)
& 14 POINTS LOG SPACED IN THE EXTRAPOLATION REGION ($10^{-3} \le x \le 10^{-2}$)

RESULTS & COMPARISON TO OTHER APPROACHES

(NONSINGLET FIT)

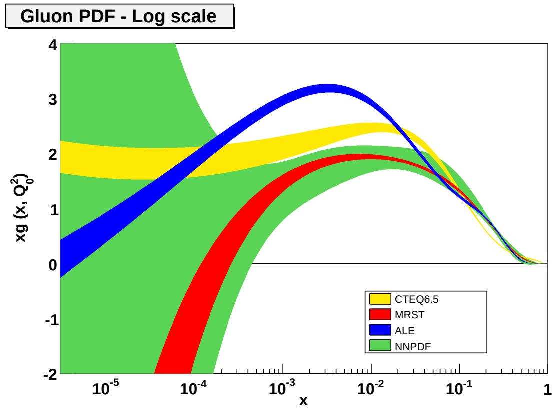
NLO RESULTS: THE STRUCTURE FUNCTION $F_2^{\rm NS}(x,Q^2)$

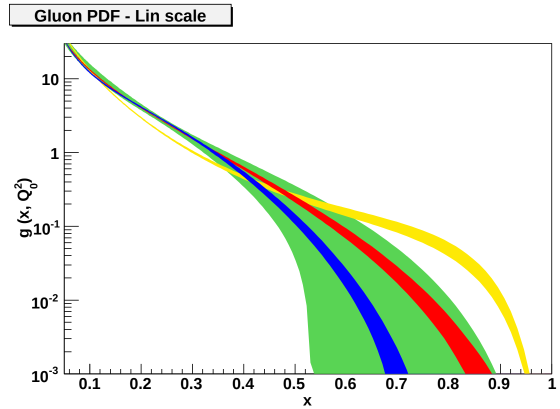
VS x AT $Q^2 = 15$ GEV^2 VS Q^2 AT $x = 0.15$

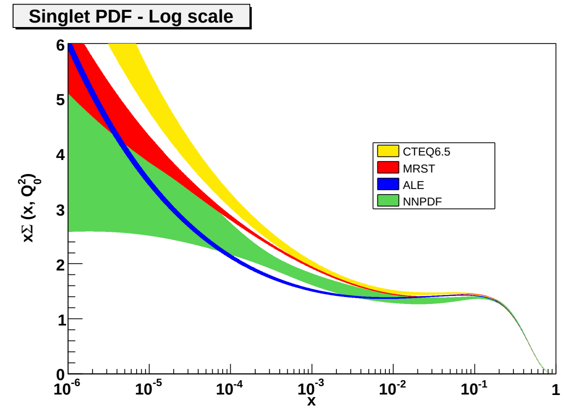


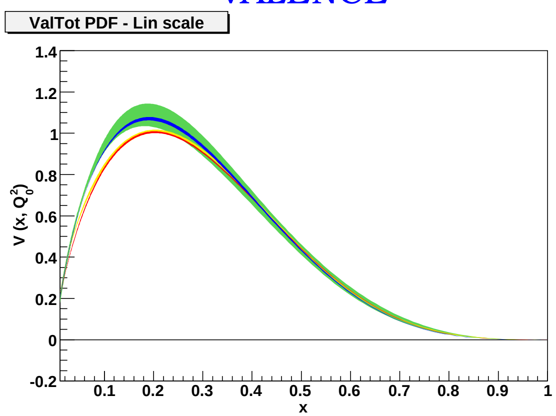
- COMPATIBLE WITH EXISTING FITS WITHIN ERROR
(even when they disagee with each other)
- UNCERTAINTY MUCH LARGER IN EXTRAPOLATION BUT ALSO IN DATA REGION
(note no other global fit data constrain $q_{\rm NS}$)
- CENTRAL FIT DISAGREES WITH EXISTING FITS IN VALENCE REGION
$0.1 \leq x \leq 0.3$

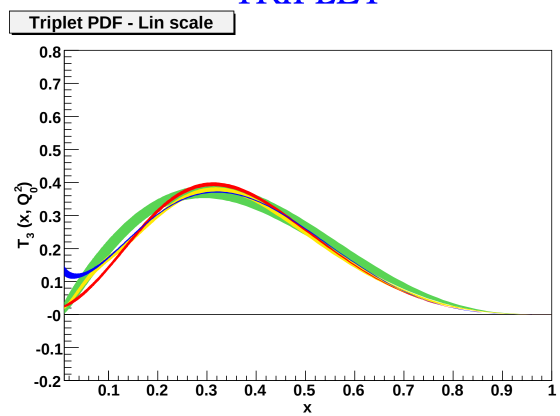
RESULTS (SINGLET FIT)
PRELIMINARY: 25 REPLICAS

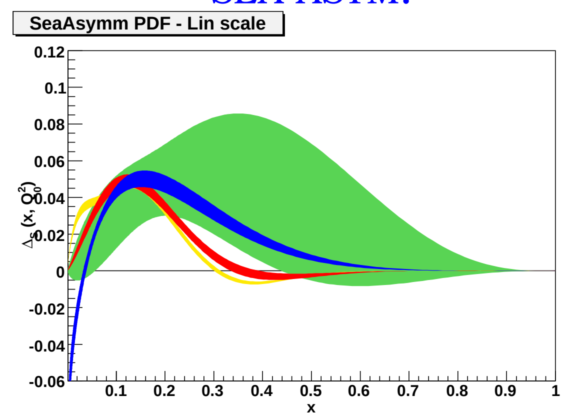
GLUON


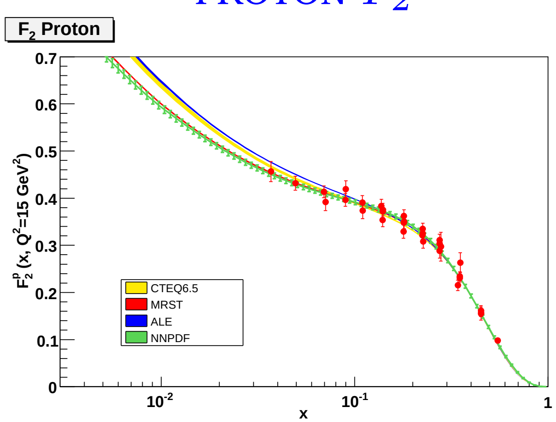
GLUON


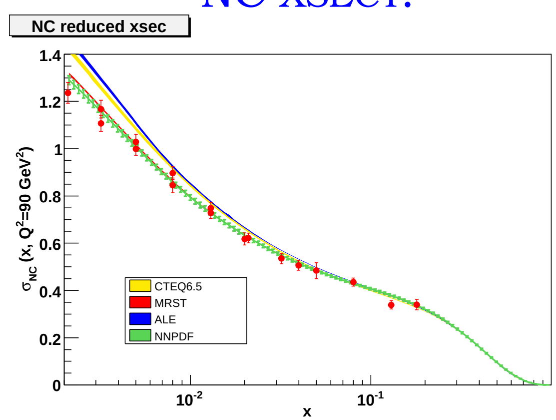
SINGLET


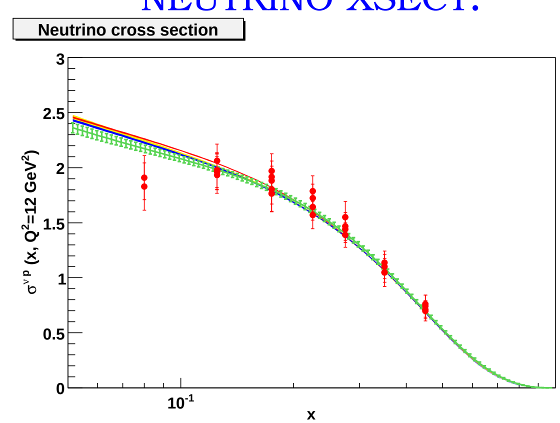
VALENCE


TRIPLET


SEA ASYM.


PROTON F_2


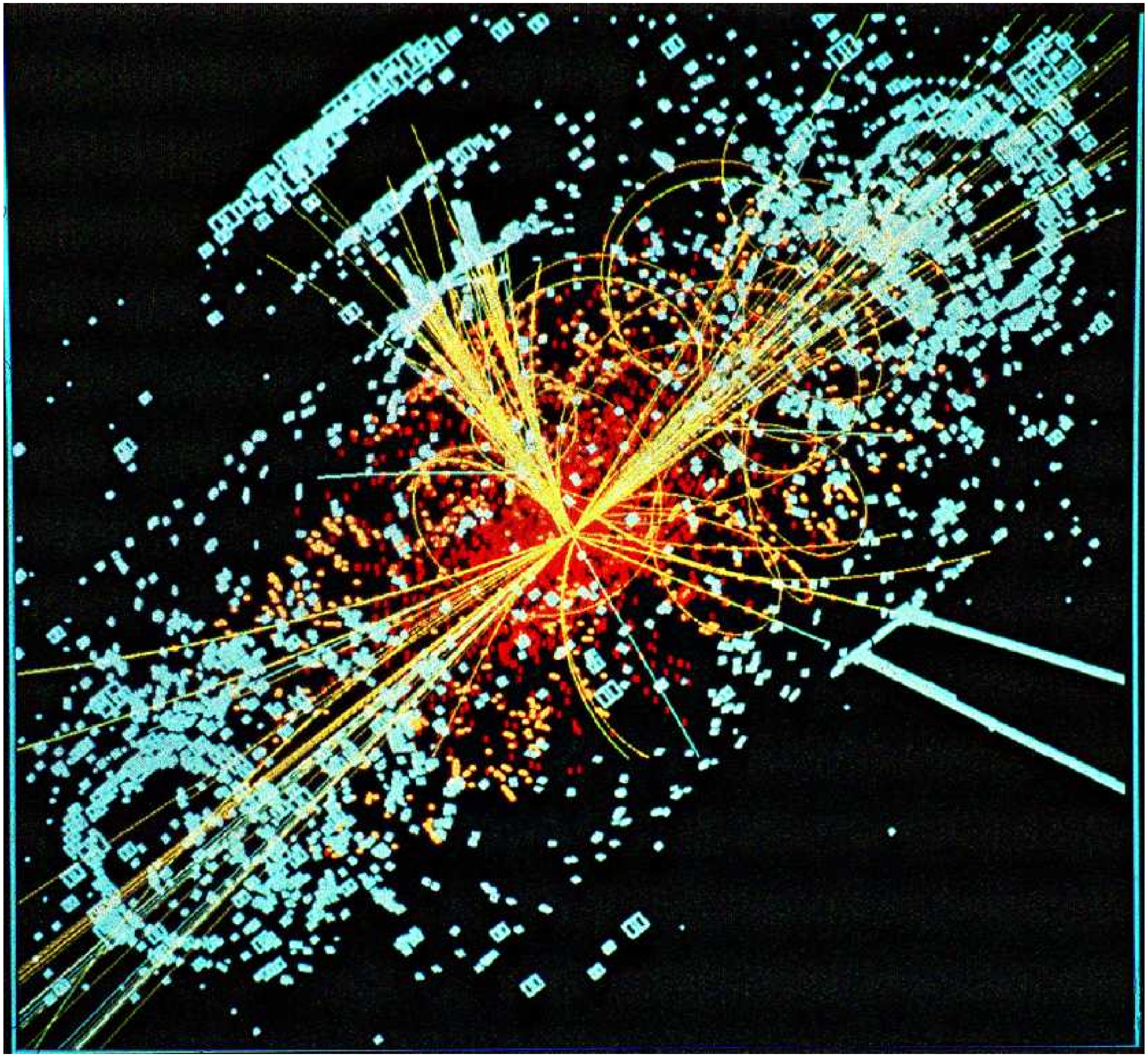
NC XSECT.


NEUTRINO XSECT.


CONCLUSION

AT LHC, WE NEED PRECISION PHYSICS

FOR DISCOVERY PHYSICS



Higgs decay in $e^+e^- + 2$ jets at CMS

EXTRAS

NNLO CORRECTIONS

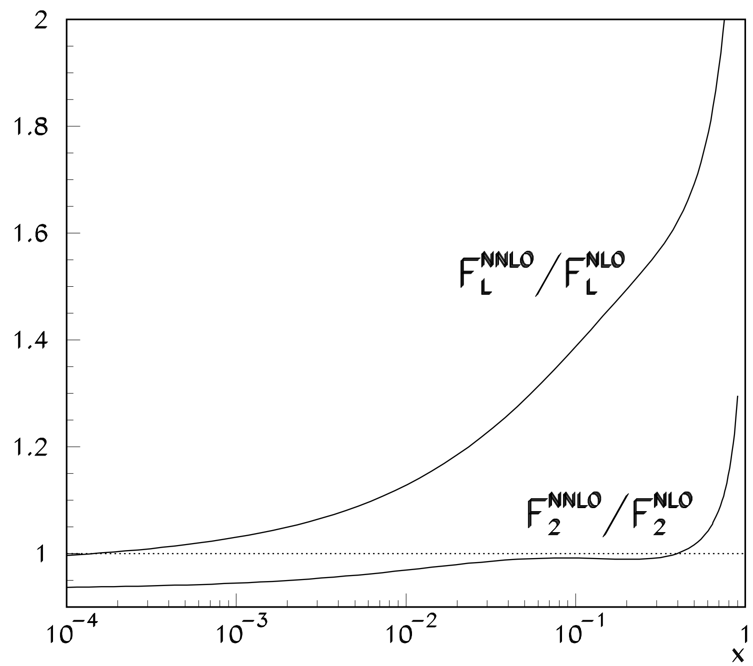
HOW BIG IS THE IMPACT OF HIGHER ORDER PERTURBATIVE CORRECTIONS?

NNLO SPLITTING FUNCTIONS COMPUTED RECENTLY (Moch, Vermaseren and Vogt, 2004)

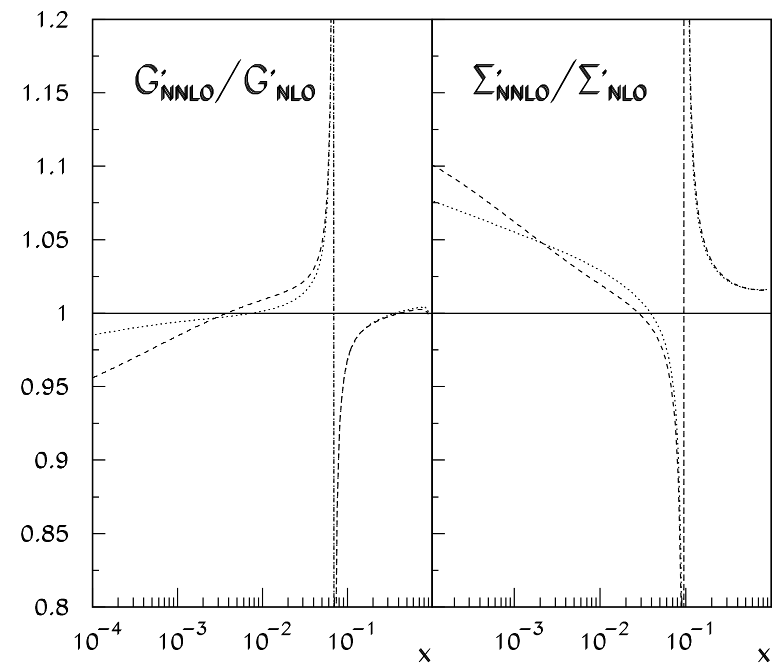
NNLO HARD XSECTS AVAILABLE FOR DIS, DY, W AND HIGGS PRODUCTION (INCL.)

$\Rightarrow$ NNLO GLOBAL FITS AROUND THE CORNER

PERTURBATIVE COEFFICIENTS



EVOLUTION



Alekhin

The Results

Anomalous dimensions in Mellin space

– One-loop : Gross, Wilczek '73

$$\gamma_{\rm ns}^{(0)}(N) \quad = \quad C_F\big(2(\mathbf{N}_- + \mathbf{N}_+)S_1 - 3\big)$$

– Two-loop : Floratos, Ross, Sachrajda '79 ; Gonzalez-Arroyo, Lopez, Ynduráin '79

$$\gamma_{\rm ns}^{(1)+}(N) = 4C_AC_F\Bigg(2\mathbf{N}_+S_3 - \frac{17}{24} - 2S_{-3} - \frac{28}{3}S_1 + (\mathbf{N}_- + \mathbf{N}_+)\bigg[\frac{151}{18}S_1 + 2S_{1,-2} - \ldots$$

$$+4C_Fn_f\Bigg(\frac{1}{12} + \frac{4}{3}S_1 - (\mathbf{N}_- + \mathbf{N}_+)\bigg[\frac{11}{9}S_1 - \frac{1}{3}S_2\bigg]\Bigg) + 4C_F{}^2\Bigg(4S_{-3} + 2S_1 + 2S_2 - \frac{3}{8} \ldots$$

$$+\mathbf{N}_-\Big[S_2 + 2S_3\Big] - (\mathbf{N}_- + \mathbf{N}_+)\Big[S_1 + 4S_{1,-2} + 2S_{1,2} + 2S_{2,1} + S_3\Big]\Bigg)$$

$$\gamma_{\rm ns}^{(1)-}(N) = \gamma_{\rm ns}^{(1)+}(N) + 16C_F\left(C_F - \frac{C_A}{2}\right)\Big((\mathbf{N}_- - \mathbf{N}_+)\Big[S_2 - S_3\Big] - 2(\mathbf{N}_- + \mathbf{N}_+ - 2) \ldots$$

– Compact notation : $\mathbf{N}_{\pm} f(N) = f(N \pm 1)$, $\quad \mathbf{N}_{\pm\mathbf{i}} f(N) = f(N \pm i)$

– Three-loop :

S.M., Vermaseren, Vogt '04

$$\gamma_{\rm ns}^{(2)+}(N) = 16 C_A C_F n_f \Big(\frac{3}{2}\zeta_3 - \frac{5}{4} + \frac{10}{9} S_{-3} - \frac{10}{9} S_3 + \frac{4}{3} S_{1,-2} - \frac{2}{3} S_{-4} + 2 S_{1,1} - \frac{25}{9} S_2 + \frac{257}{27} S_1 - \frac{2}{3} S_{-3,1} - \mathbf{N}_+ \Big[S_{2,1} - \frac{2}{3} S_{3,1} -$$

$$- (\mathbf{N}_+ - 1) \Big[\frac{23}{18} S_3 - S_2 \Big] - (\mathbf{N}_- + \mathbf{N}_+) \Big[S_{1,1} + \frac{1237}{216} S_1 + \frac{11}{18} S_3 - \frac{317}{108} S_2 + \frac{16}{9} S_{1,-2} - \frac{2}{3} S_{1,-2,1} - \frac{1}{3} S_{1,-3} - \frac{1}{2} S_{1,3} - \frac{1}{2} S_{2,1}$$

$$- \frac{1}{3} S_{2,-2} + S_1 \zeta_3 + \frac{1}{2} S_{3,1} \Big] \Big) + 16 C_F {C_A}^2 \Big(\frac{1657}{576} - \frac{15}{4}\zeta_3 + 2 S_{-5} + \frac{31}{6} S_{-4} - 4 S_{-4,1} - \frac{67}{9} S_{-3} + 2 S_{-3,-2} + \frac{11}{3} S_{-3,1} + \frac{3}{2} S_{-2}$$

$$- 6 S_{-2}\zeta_3 - 2 S_{-2,-3} + 3 S_{-2,-2} - 4 S_{-2,-2,1} + 8 S_{-2,1,-2} - \frac{1883}{54} S_1 - 10 S_{1,-3} - \frac{16}{3} S_{1,-2} + 12 S_{1,-2,1} + 4 S_{1,3} - 4 S_{2,-2} - \frac{5}{2} S_4 +$$

$$+ \frac{176}{9} S_2 + \frac{13}{3} S_3 + (\mathbf{N}_- + \mathbf{N}_+ - 2) \Big[3 S_1 \zeta_3 + 11 S_{1,1} - 4 S_{1,1,-2} \Big] + (\mathbf{N}_- + \mathbf{N}_+) \Big[\frac{9737}{432} S_1 - 3 S_{1,-4} + \frac{19}{6} S_{1,-3} + 8 S_{1,-3,1} + \frac{91}{9} S_{1,}$$

$$- 6 S_{1,-2,-2} - \frac{29}{3} S_{1,-2,1} + 8 S_{1,1,-3} - 16 S_{1,1,-2,1} - 4 S_{1,1,3} - \frac{19}{4} S_{1,3} + 4 S_{1,3,1} + 3 S_{1,4} + 8 S_{2,-2,1} + 2 S_{2,3} - S_{3,-2} + \frac{11}{12} S_{3,1} - S_{4,}$$

$$+ \frac{1}{6} S_{2,-2} - \frac{1967}{216} S_2 + \frac{121}{72} S_3 \Big] - (\mathbf{N}_- - \mathbf{N}_+) \Big[3 S_2 \zeta_3 + 7 S_{2,1} - 3 S_{2,1,-2} + 2 S_{2,-2,1} - \frac{1}{4} S_{2,3} - \frac{3}{2} S_{3,-2} - \frac{29}{6} S_{3,1} + \frac{11}{4} S_{4,1} + \frac{1}{2} S_{2,}$$

$$+ \mathbf{N}_+ \Big[\frac{28}{9} S_3 - \frac{2376}{216} S_2 - \frac{8}{3} S_4 - \frac{5}{2} S_5 \Big] \Big) + 16 C_F n_f^2 \Big(\frac{17}{144} - \frac{13}{27} S_1 + \frac{2}{9} S_2 + (\mathbf{N}_- + \mathbf{N}_+) \Big[\frac{2}{9} S_1 - \frac{11}{54} S_2 + \frac{1}{18} S_3 \Big] \Big) + 16 {C_F}^2 C_A \Big(\frac{45}{4}$$

$$- \frac{151}{64} - 10 S_{-5} - \frac{89}{6} S_{-4} + 20 S_{-4,1} + \frac{134}{9} S_{-3} - 2 S_{-3,-2} - \frac{31}{3} S_{-3,1} + 2 S_{-3,2} - \frac{9}{2} S_{-2} + 18 S_{-2}\zeta_3 + 10 S_{-2,-3} - 6 S_{-2,-2}$$

$$+ 8 S_{-2,-2,1} - 28 S_{-2,1,-2} + 46 S_{1,-3} + \frac{26}{3} S_{1,-2} - 48 S_{1,-2,1} + \frac{28}{3} S_{1,2} - \frac{185}{6} S_3 - 8 S_{1,3} + 2 S_{3,-2} - 4 S_5 - (\mathbf{N}_- + \mathbf{N}_+ - 2) \Big[9 S_1 \zeta_3$$

$$+ \frac{209}{6} S_{1,1} - 14 S_{1,1,-2} - \frac{242}{18} S_2 + 9 S_{2,-2} + \frac{33}{4} S_4 - 3 S_{3,1} + \frac{14}{3} S_{2,1} \Big] + (\mathbf{N}_- + \mathbf{N}_+) \Big[17 S_{1,-4} - \frac{107}{6} S_{1,-3} - 32 S_{1,-3,1} - \frac{173}{9} S_{1,-}$$

$$+16S_{1,-2,-2}+\frac{103}{3}S_{1,-2,1}-2S_{1,-2,2}-36S_{1,1,-3}+56S_{1,1,-2,1}+8S_{1,1,3}-\frac{109}{9}S_{1,2}-4S_{1,2,-2}+\frac{43}{3}S_{1,3}-8S_{1,3,1}-11S_{1}$$

$$+21S_{2,-3}-30S_{2,-2,1}-4S_{2,1,-2}-5S_{2,3}-S_{4,1}+\frac{31}{6}S_{2,-2}-\frac{67}{9}S_{2,1}\Big]+(\mathbf{N}_- -\mathbf{N}_+)\Big[9S_2\zeta_3+2S_{2,-3}+4S_{2,-2,1}-12S_{2,1,}$$

$$+13S_{4,1}+\frac{1}{2}S_{2,-2}+\frac{11}{2}S_4-\frac{33}{2}S_{3,1}+\frac{59}{9}S_3+\frac{127}{6}S_{2,1}-\frac{1153}{72}S_2\Big]+\mathbf{N}_+\Big[8S_{3,-2}+\frac{4}{3}S_{3,1}-2S_{3,2}+14S_5+\frac{23}{6}S_4+\frac{73}{3}S_3$$

$$+16{C_F}^2n_f\Big(\frac{23}{16}-\frac{3}{2}\zeta_3+\frac{4}{3}S_{-3,1}-\frac{59}{36}S_2+\frac{4}{3}S_{-4}-\frac{20}{9}S_{-3}+\frac{20}{9}S_1-\frac{8}{3}S_{1,-2}-\frac{8}{3}S_{1,1}-\frac{4}{3}S_{1,2}+\mathbf{N}_+\Big[\frac{25}{9}S_3-\frac{4}{3}S_{3,1}-\frac{1}{3}S$$

$$-(\mathbf{N}_+-1)\Big[\frac{67}{36}S_2-\frac{4}{3}S_{2,1}+\frac{4}{3}S_3\Big]+(\mathbf{N}_-+\mathbf{N}_+)\Big[S_1\zeta_3-\frac{325}{144}S_1-\frac{2}{3}S_{1,-3}+\frac{32}{9}S_{1,-2}-\frac{4}{3}S_{1,-2,1}+\frac{4}{3}S_{1,1}+\frac{16}{9}S_{1,2}-\frac{4}{3}S$$

$$+\frac{11}{18}S_2-\frac{2}{3}S_{2,-2}+\frac{10}{9}S_{2,1}+\frac{1}{2}S_4-\frac{2}{3}S_{2,2}-\frac{8}{9}S_3\Big]\Big)+16{C_F}^3\Big(12S_{-5}-\frac{29}{32}-\frac{15}{2}\zeta_3+9S_{-4}-24S_{-4,1}-4S_{-3,-2}+6S_{-3}$$

$$-4S_{-3,2}+3S_{-2}+25S_3-12S_{-2}\zeta_3-12S_{-2,-3}+24S_{-2,1,-2}-52S_{1,-3}+4S_{1,-2}+48S_{1,-2,1}-4S_{3,-2}+\frac{67}{2}S_2-17S_4$$

$$+(\mathbf{N}_-+\mathbf{N}_+-2)\Big[6S_1\zeta_3-\frac{31}{8}S_1+35S_{1,1}-12S_{1,1,-2}+S_{1,2}+10S_{2,-2}+S_{2,1}+2S_{2,2}-2S_{3,1}-3S_5\Big]+(\mathbf{N}_-+\mathbf{N}_+)\Big[23S$$

$$-22S_{1,-4}+32S_{1,-3,1}-2S_{1,-2}-8S_{1,-2,-2}-30S_{1,-2,1}-6S_{1,3}+4S_{1,-2,2}+40S_{1,1,-3}-48S_{1,1,-2,1}+8S_{1,2,-2}+4S_{1,2,2}$$

$$+4S_{1,4}+28S_{2,-2,1}+4S_{2,1,2}+4S_{2,2,1}+4S_{3,1,1}-4S_{3,2}+8S_{2,1,-2}-26S_{2,-3}-2S_{2,3}-4S_{3,-2}-3S_{2,-2}-3S_{2,2}+\frac{3}{2}S_4\Big]$$

$$+(\mathbf{N}_- -\mathbf{N}_+)\Big[12S_{2,1,-2}-6S_2\zeta_3-2S_{2,-3}+3S_{2,3}+2S_{3,-2}-\frac{81}{4}S_{2,1}+14S_{3,1}-5S_{2,-2}-\frac{1}{2}S_{2,2}+\frac{15}{8}S_2+\frac{1}{2}S_3-13S_{4,1}$$

$$+\mathbf{N}_+\Big[14S_4-\frac{265}{8}S_2-\frac{87}{4}S_3-4S_{4,1}-4S_5\Big]\Big)$$

High Energy (small x) resummation

At $O(\alpha_s^n)$, $O\left[ln(\frac{1}{x})^n\right]$ contributions:

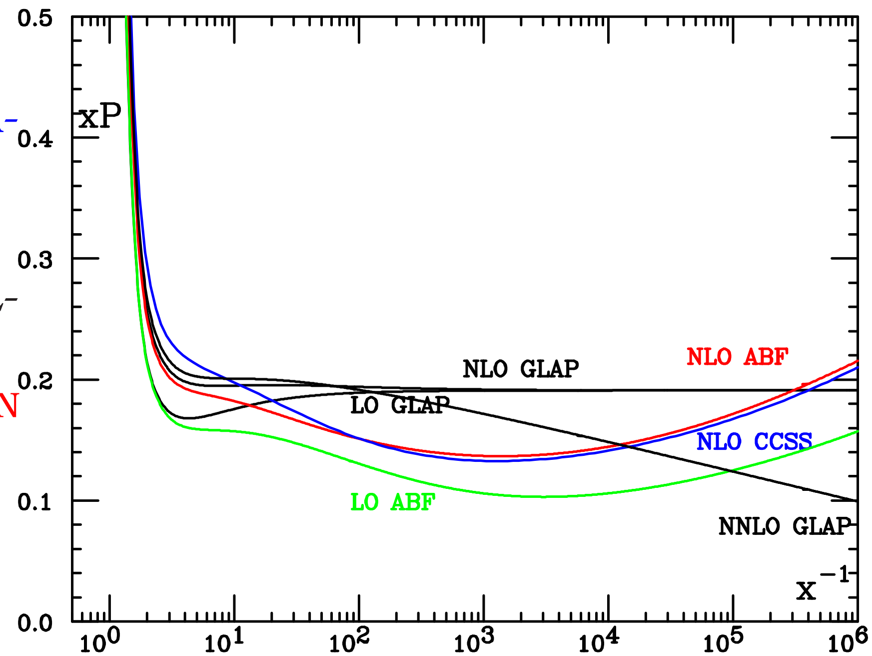
$\Rightarrow$ perturbation theory unstable at small x (c.m. energy >> final state mass)

x_{cut} :	0	0.0002	0.001	0.0025	0.005	0.01
# data points	2097	2050	1961	1898	1826	1762
$\chi^2(x > 0)$	2267					
$\chi^2(x > 0.0002)$	2212	2203				
$\chi^2(x > 0.001)$	2134	2128	2119			
$\chi^2(x > 0.0025)$	2069	2064	2055	2040		
$\chi^2(x > 0.005)$	2024	2019	2012	1993	1973	
$\chi^2(x > 0.01)$	1965	1961	1953	1934	1917	1916
Δ_i^{i+1}	0.19	0.10	0.24	0.28	0.02	

Data-theory agreement for evolution of F_2 improves if small x data removed (MRST 2003)

χ^2 improves with fixed # of pts (same row)

- Considerable progress in full resummation of small x terms (Ciafaloni, Colferai, Salam, Staśto; Altarelli, Ball, S.F.)
  $\Rightarrow$ stable resummed splitting fctns available
- Resummed spl. fctn. closer to LO than to NNLO
- Not yet included in parton fits

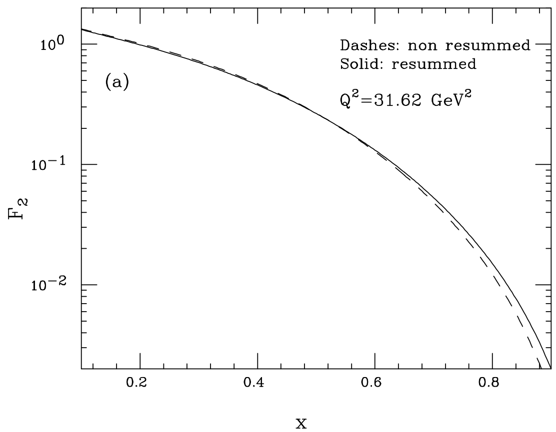


THRESHOLD (LARGE x) RESUMMATION

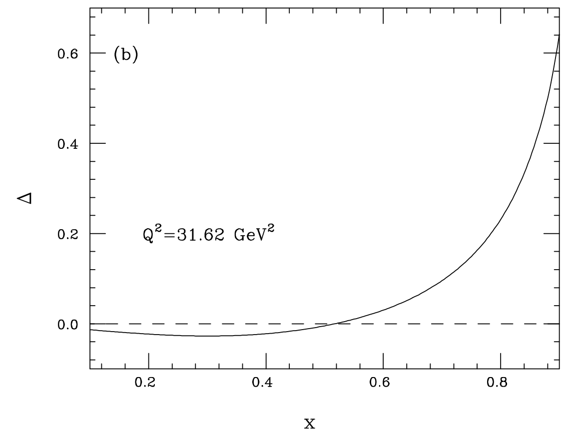
AT $O(\alpha_s^n)$, $O\left[ln^{2m}(1-x)\right]$ CONTRIBUTIONS:
$\Rightarrow$ PERTURBATION THEORY UNSTABLE AT LARGE x
(C.M. ENERGY $\sim$ FINAL STATE MASS)

- DIS: SIZABLE ONLY @ VERY LARGE x, WHERE XSECT & PDF TINY (Corcella, Magnea 2005)
- DY: CAN HAVE SIZABLE EFFECTS, ESPECIALLY ON RAP. DISTN. (Bolzoni 2006)
- NOT INCLUDED IN CURRENT FITS

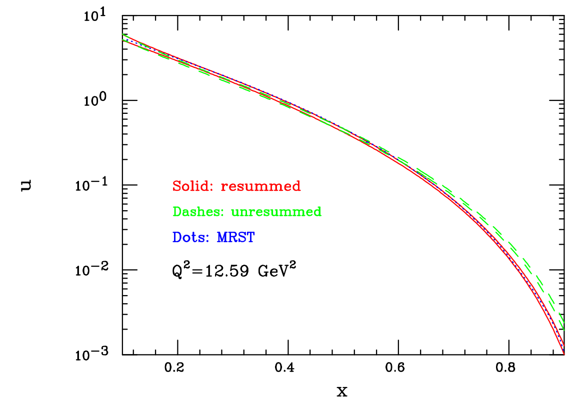
F_2 RES. VS. UNRES.



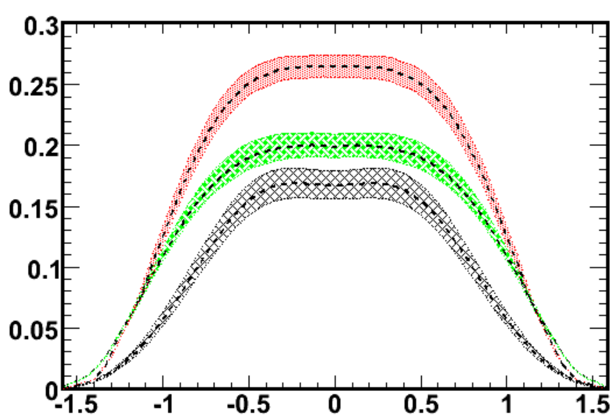
F_2 PERCENTAGE RES. CORRN.



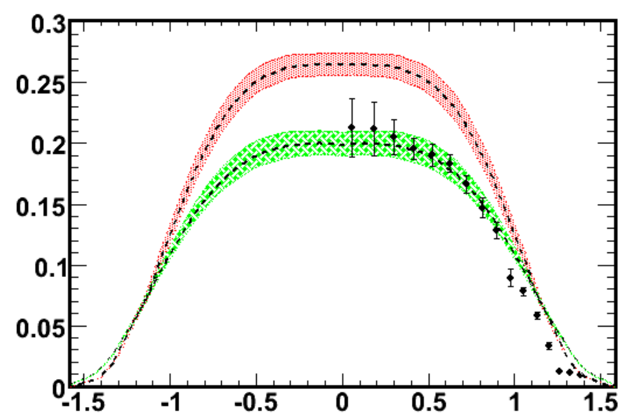
UP QUARK RES. VS. UNRES.



DY $d\sigma/dQ^2dy$ VS. y LO NLO RES.



INCL. E866 DATA



HEAVY QUARKS

HOW CAN ONE ACCOUNT FOR HEAVY FLAVOURS (CHARM, BEAUTY...)?

SIMPLE OPTION: (CTEQ6, Alekhin) CHARM PDF VANISHES BELOW THRESHOLD, INCLUDED ALONG OTHER PDFS ABOVE THRESHOLD ⇒ EFFECTIVELY, $m_c \approx 0$ FOR $Q^2 > Q^2_{th}$.

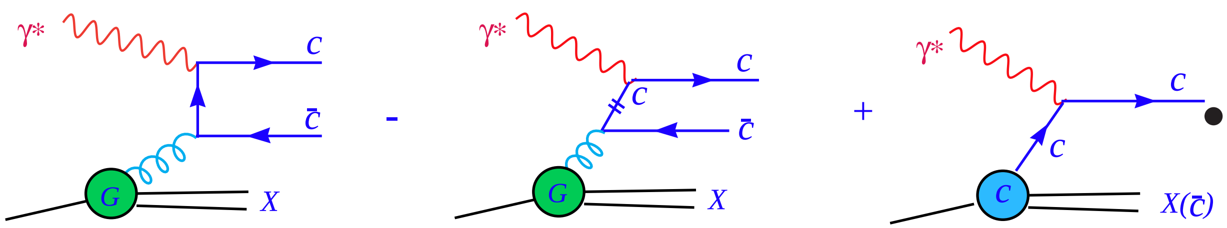
- HQ PDF GENERATED DYNAMICALLY BY PERTURBATIVE EVOLUTION (HQ PAIR-PRODUCED BY RADIATION FROM GLUONS)
- TREATMENT NOT ACCURATE IN $Q^2 \approx Q^2_{th}$ REGION

MORE REFINED TREATMENT OF THRESHOLD: [Collins, Tung et al (ACOT) 1986-2006]

$m_c \neq 0$, LO — CHARM RADIATION

$m_c = 0$, LO — CHARM RADIATION

$m_c = 0$ — CHARM PDF



- $m_c \neq 0$ IN HARD XSECT ⇒ RELEVANT AROUND THRESHOLD
- $m_c = 0$ IN CHARM PDF ⇒ RELEVANT AT LARGE SCALE
- SUBTRACT DOUBLE COUNTING

THE BAYESIAN MONTE CARLO (GIELE, KOSOWER, KELLER 2001)

- generate a Monte-Carlo sample of fcts. with “reasonable” prior distn. (e.g. an available parton set) → representation of probability functional $\mathcal{P}[f_i]$
- calculate observables with functional integral
- update probability using Bayesian inference on MC sample: better agreement with data → more functions in sample
- iterate until convergence achieved

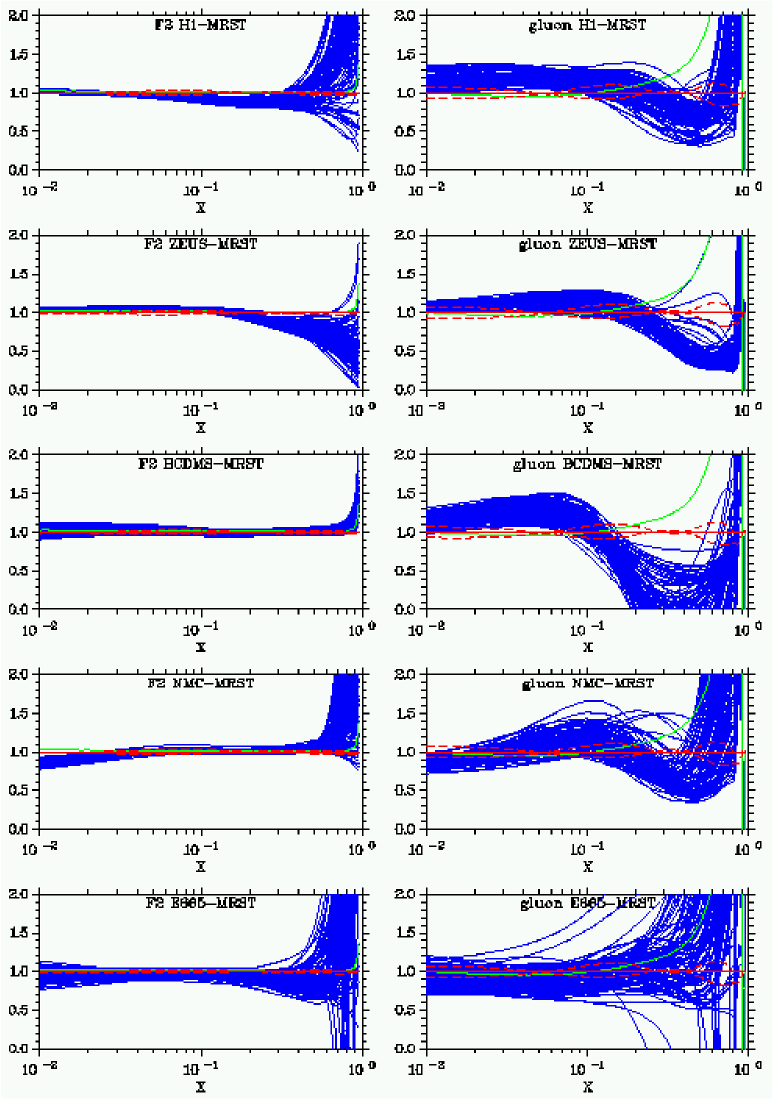
PROBLEM IS MADE FINITE-DIMENSIONAL BY THE CHOICE OF PRIOR, BUT RESULT DO NOT DEPEND ON THE CHOICE IF SUFFICIENTLY GENERAL

HARD TO HANDLE “FLAT DIRECTIONS” (Monte Carlo replicas which lead to same agreement with data);

COMPUTATIONALLY VERY INTENSIVE;

DIFFICULT TO ACHIEVE INDEP. FROM PRIOR

Result: Fermi partons



F_2^{singlet} and gluon ratios Fermi/MRST

only subset of data fitted (H1, E665, BCDMS DIS data)

good agreement with Tevatron W xsect

trouble with value of α_s

Monte Carlo data generation

- BCDMS+ NMC proton & deuteron F_2 data (full correlated systematics available), taken at 4 beam energies

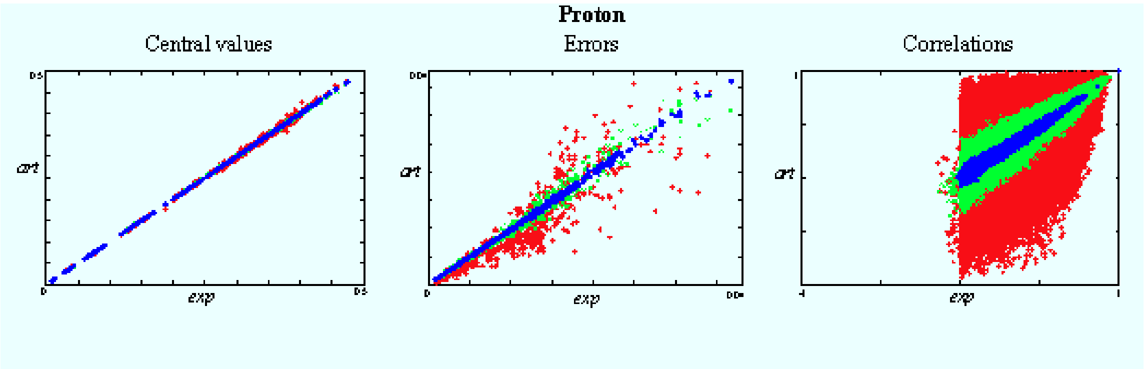
- on top of stat. errors, 4 systematics + 1 normalization (NMC) or 6 systematics + 1 absolute & 2 relative normalizations (BCDMS), with various forms of correlation (full, or for each target, or for each beam energy)

Generate data according to a multigaussian distribution

$$F_i^{(art)\,(k)} =$$

$$(1 + r_5^{(k)}\,\sigma_N)\sqrt{1 + r_{i,6}^{(k)}\,\sigma_{N_t}}\sqrt{1 + r_{i,7}^{(k)}\,\sigma_{N_b}}\left[F_i^{(exp)} + \frac{r_{i,1}^{(k)}\,f_b + r_{i,2}^{(k)}\,f_{i,s} + r_{i,3}^{(k)}\,f_{i,r}}{100}F_i^{(exp)} + r_{i,s}^{(k)}\,\sigma_s^i\right]$$

r univariate gaussian random nos., one $r_{i,s}$ for each data, but single $r_{i,j}$ for all correlated data



scatter plot art. vs. exp. for 10 (red) 100 (green) and 1000 (blue) replicas

need 1000 replicas to reproduce correlations to percent accuracy

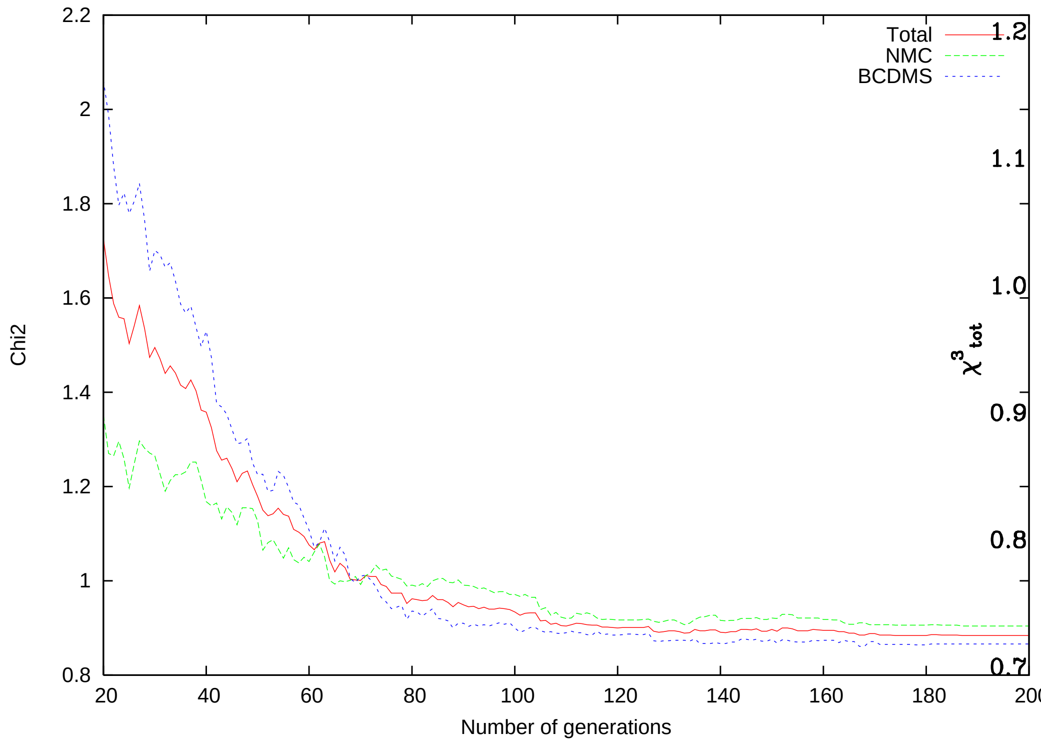
PERTURBATIVE EVOLUTION

- PARAMETRIZE INITIAL PDFS AS A FUNCTION OF x
- DETERMINE GREEN'S FUNCTION FOR ALTARELLI-PARISI EVOLUTION
  $\Gamma(x, \alpha_s\left(Q^2\right), \alpha_s\left(Q_0^2\right))$ (note it is a distribution)
- DETERMINE EVOLVED PDF AS
  $q(x, Q^2) = Gq(x, Q_0^2) + \int_x^1 \frac{dy}{y} \Gamma^{(+)}(y, \alpha_s\left(Q^2\right), \alpha_s\left(Q_0^2\right)) q\left(\frac{x}{y}, Q_0^2\right)$
- GREEN FUNCTION CAN BE INTERPOLATED OR COMPUTED ON A GRID AND STORED
- EVOLUTION AND INTERPOLATION FULLY BENCHMARKED

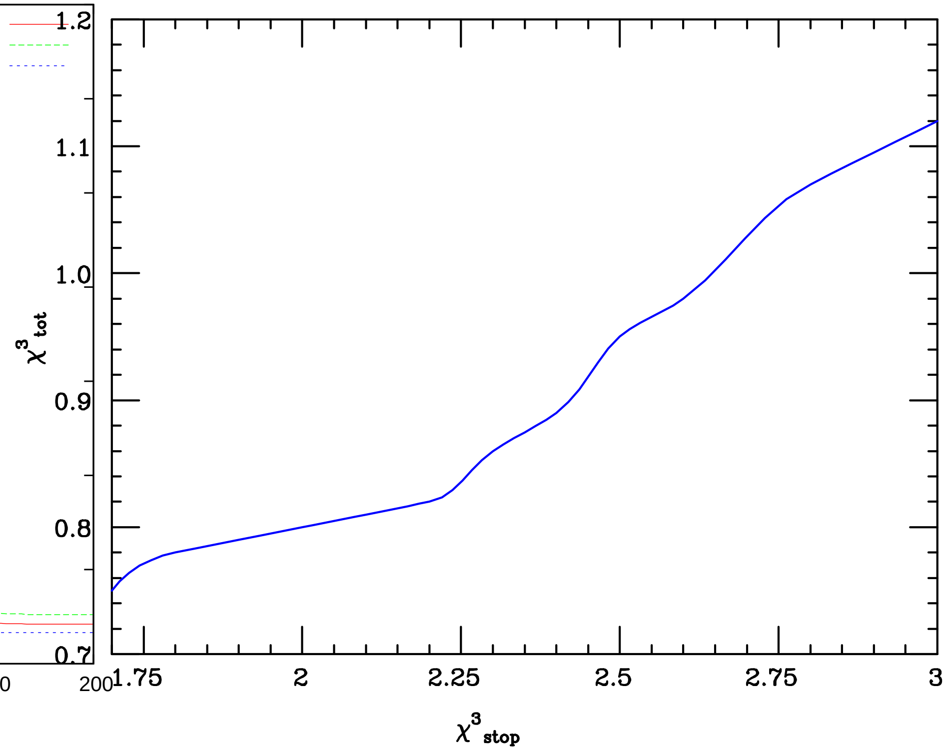
TRAINING...

- EACH NEURAL NET IS FITTED TO A PSEUDODATA REPLICA BY MINIMIZING ITS χ^2
- MINIMIZATION THROUGH GENETIC ALGORITHM + REWEIGHTING OF EXPERIMENTS
- QUALITY OF FIT MEASURED BY χ^2 OF AVERAGE OF NN COMPARED TO DATA

χ^2 OF BEST FIT



χ^2 OF BEST FIT VS. AVERAGE χ^2



TRAINING...

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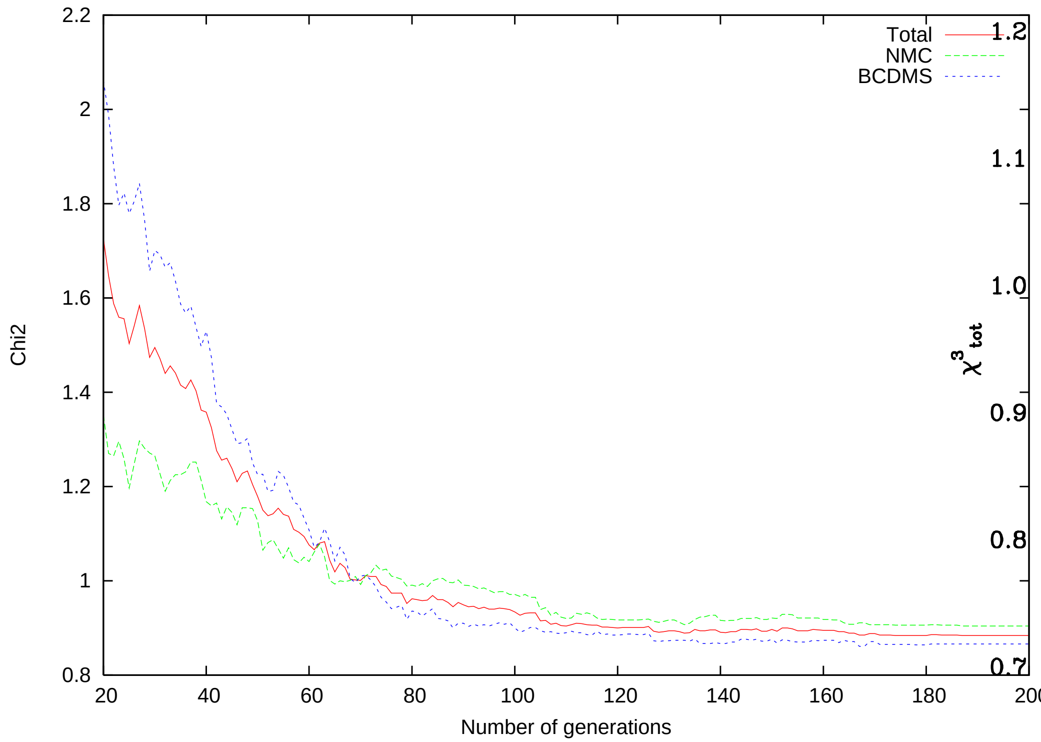
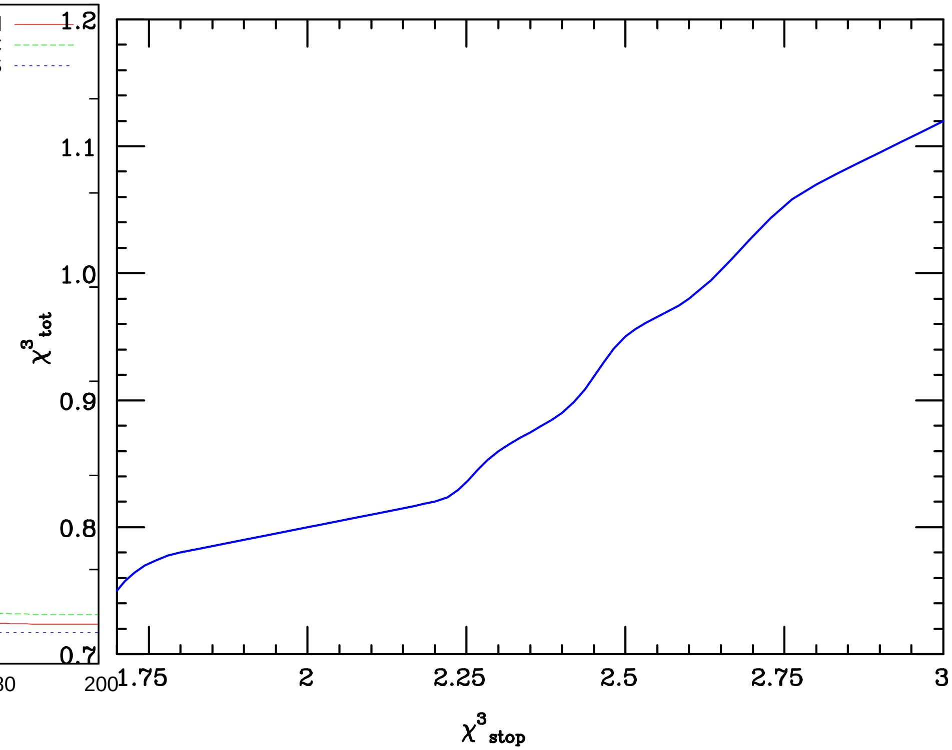


χ^2 OF BEST FIT VS. AVERAGE χ^2



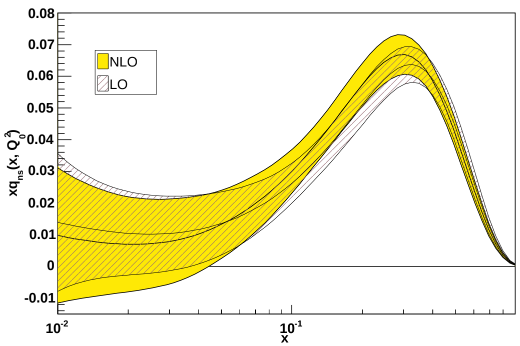
- IF NO STOPPING IMPLEMENTED, χ^2 OF THE AVERAGE DECREASES AS A FUNCTION OF AVERAGE χ^2 OF REPLICAS
- AT BEST FIT, AVERAGE χ^2 OF REPLICAS ~ 2; χ^2 OF AVERAGE TO DATA ~ 1

RESULTS:

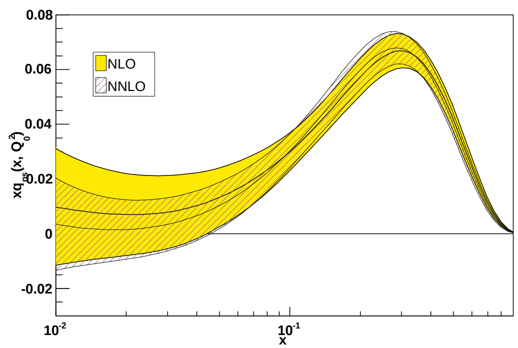
THE NONSINGLET QUARK PDF $q^{\mathrm{NS}}(x, Q^2)$

LO, NLO & NNLO

LO VS. NLO



NLO VS. NNLO



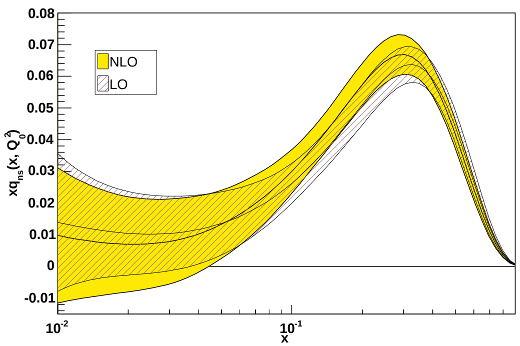
- quality of fit (χ^2)
  same at LO, NLO, NNLO
- NLO & NNLO agree within one σ
  NNLO terms negligible within errors
- LO & NLO agree within three σ
  NLO terms absorbed in b.c.

RESULTS:

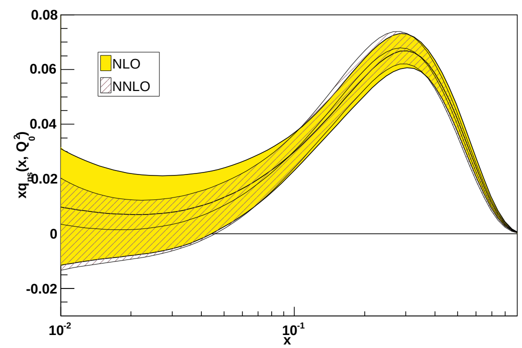
THE NONSINGLET QUARK PDF $q^{\mathrm{NS}}(x, Q^2)$

LO, NLO & NNLO

LO VS. NLO



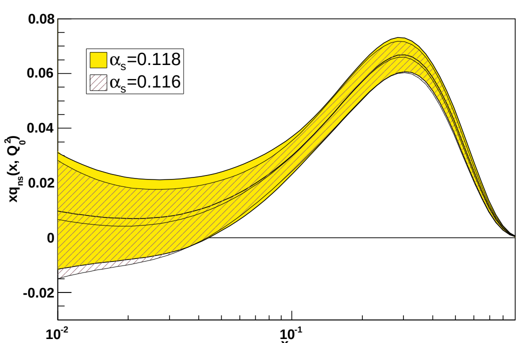
NLO VS. NNLO



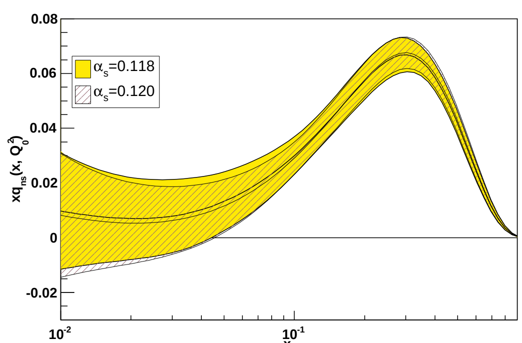
- quality of fit (χ^2) same at LO, NLO, NNLO
- NLO & NNLO agree within one σ
  NNLO terms negligible within errors
- LO & NLO agree within three σ
  NLO terms absorbed in b.c.

VARIATION OF α_s

LOW $\alpha_s = 0.116$



HIGH $\alpha_s = 0.120$



- quality of fit (χ^2) unchanged with $\alpha_s = 0.118 \pm 0.002$
- all fits agree within one σ
  $\Rightarrow$ α_s cannot be determined with good accuracy