Statistical issues in global fits: Lessons from PDF determinations

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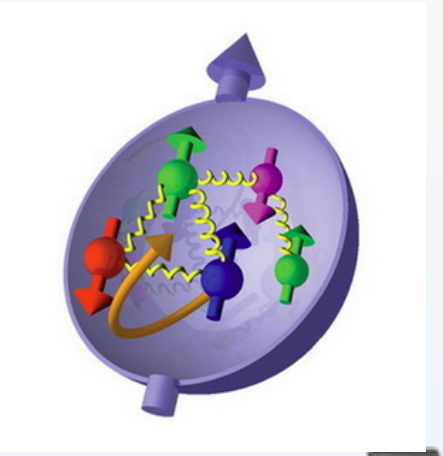
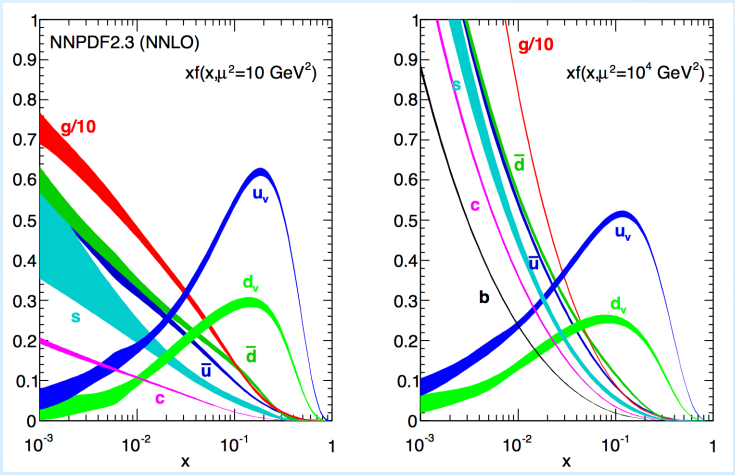
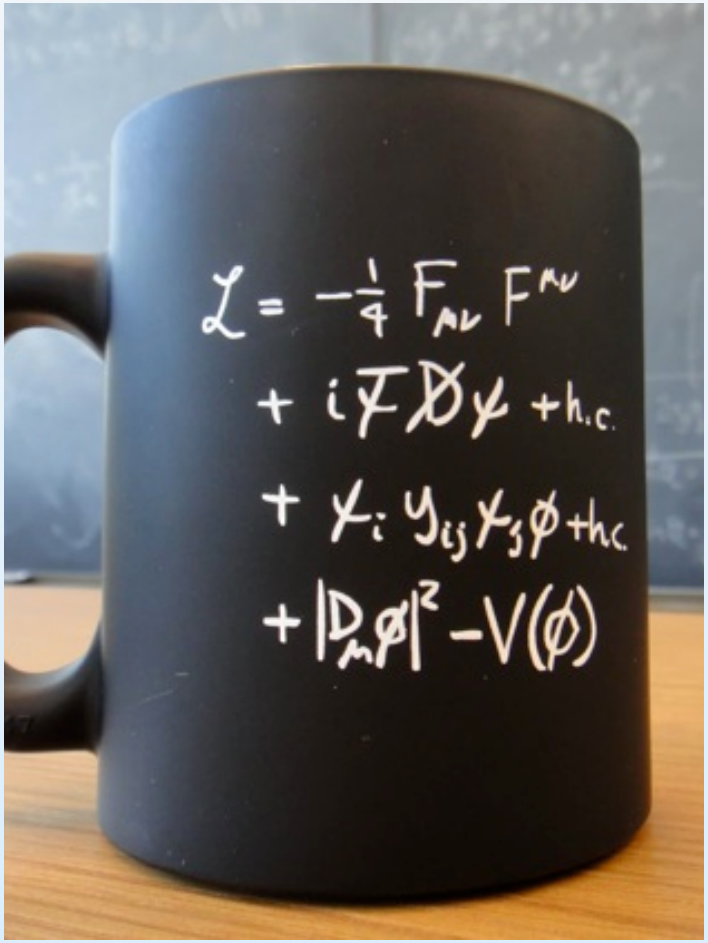


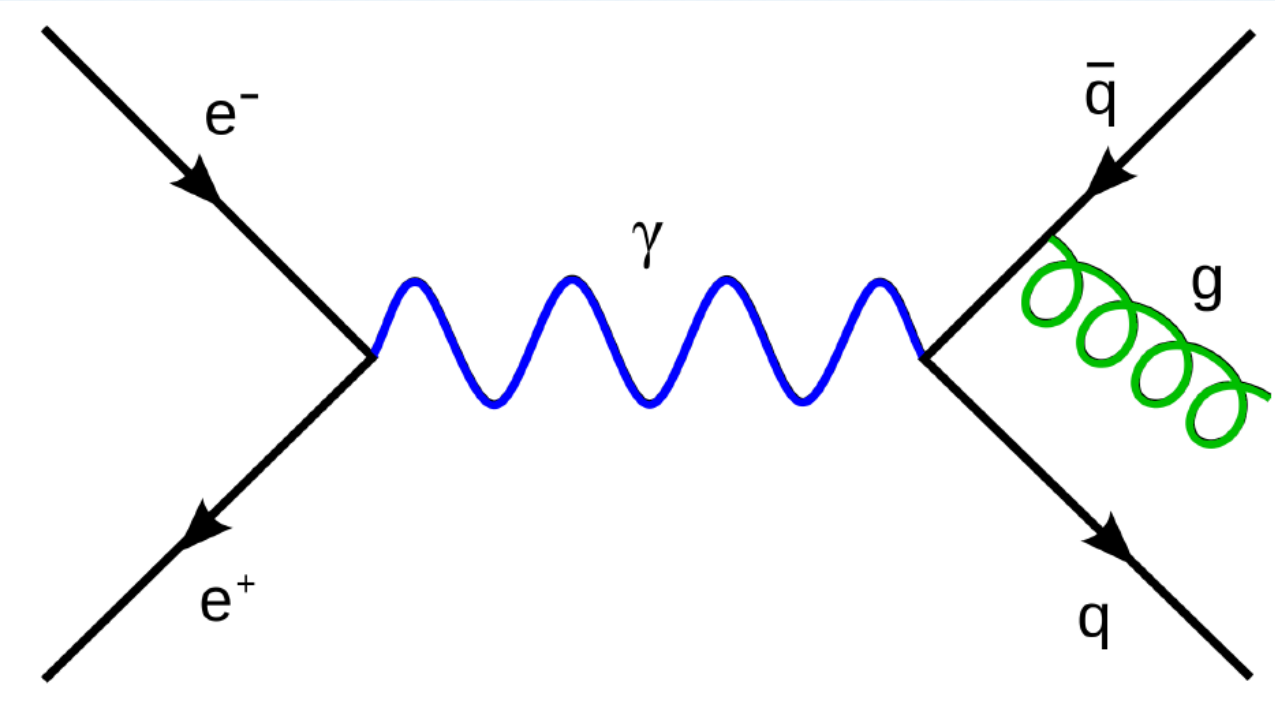
The inner life of protons : Parton Distribution Functions



Lepton vs Hadron Colliders

In high-energy **lepton colliders**, such as the **Large Electron-Positron Collider** (LEP) at CERN, the collisions involve **elementary particles** without substructure

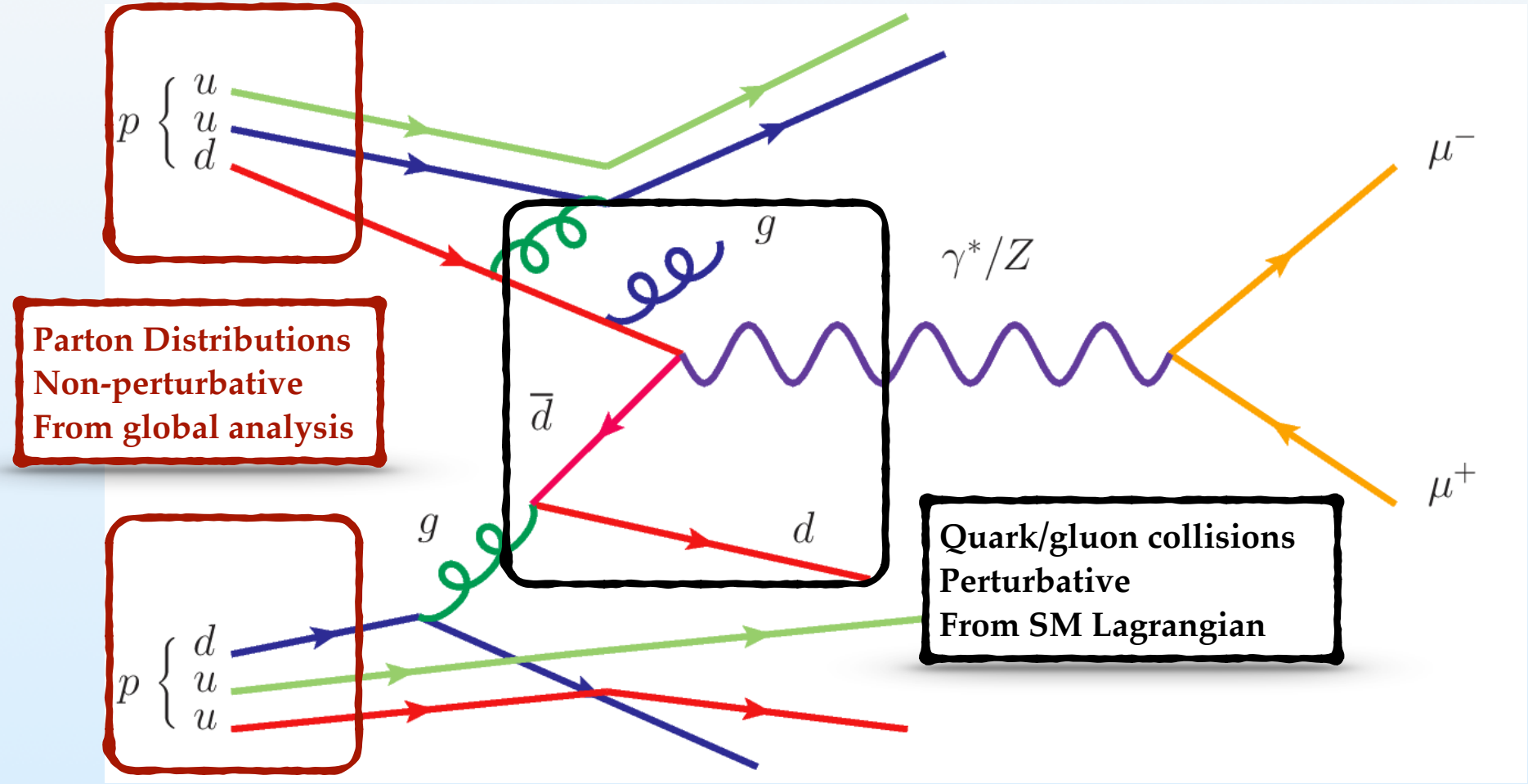




Cross-sections in lepton colliders can be computed in perturbation theory using the **Feynman rules of the Standard Model Lagrangian**

Lepton vs Hadron Colliders

In high-energy **hadron colliders**, such as the LHC, the collisions involve **composite particles** (protons) with internal structure (quarks and gluons)



Calculations of c**ross-sections** in hadron collisions require the combination of **perturbative, quark/gluon-initiated processes**, and **non-perturbative, parton distributions**, information

Parton Distributions

The distribution of energy that **quarks and gluons carry inside the proton** is quantified by the **Parton Distribution Functions (PDFs)**

$$g(x,Q)$$

Q: **Energy of the quark/gluon collision**
Inverse of the resolution length

g(x,Q): **Probability** of **finding a gluon inside a proton**, carrying a fraction *x* of the proton momentum, when probed with energy *Q*

x: **Fraction of the proton's momentum**

PDFs are determined by **non-perturbative QCD dynamics:** cannot be computed from first principles, and need to be **extracted from experimental data** with a **global analysis**

- **Energy conservation**

$$\int_0^1 dx \left(g(x,Q) + \sum_q q(x,Q) \right) = 1$$

- **Dependence with quark/gluon collision energy Q** determined in perturbation theory

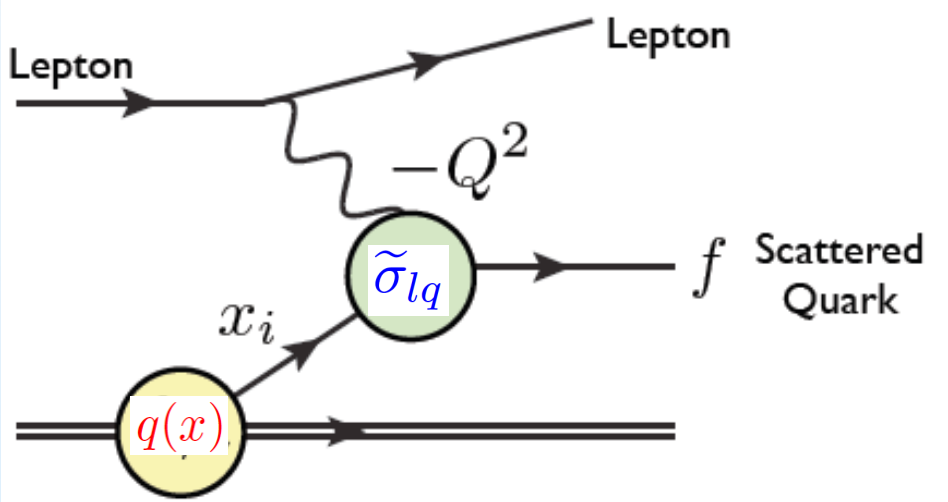
$$\frac{\partial g(x,Q)}{\partial \ln Q} = P_g(\alpha_s) \otimes g(x,Q) + P_q(\alpha_s) \otimes q(x,Q)$$

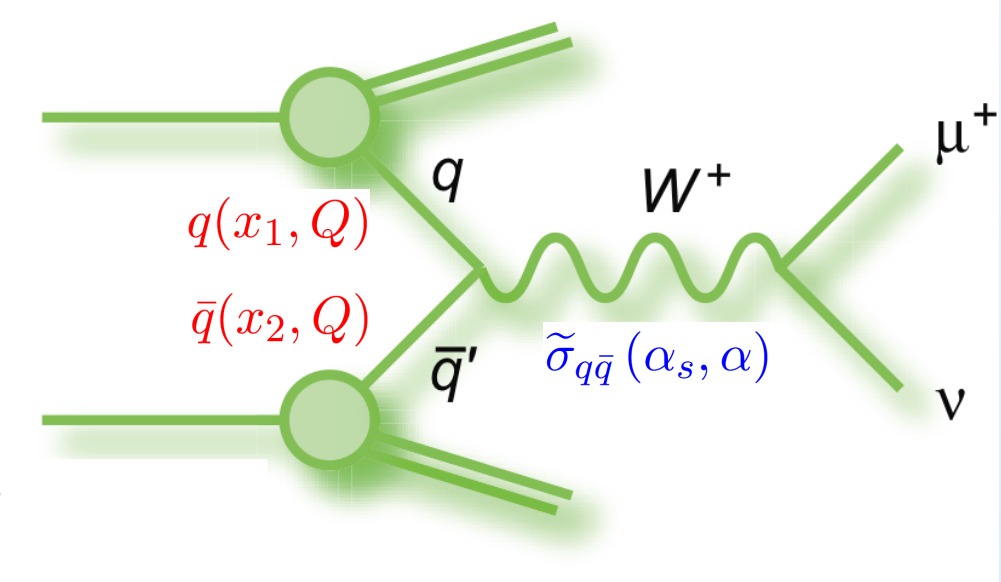
The Factorization Theorem

The **QCD Factorization Theorem** guarantees **PDF universality: extract them from a subset of** process and use them to provide pure predictions for new processes

$$\sigma_{lp} \simeq \widetilde{\sigma}_{lq}\left(\alpha_s, \alpha\right) \otimes q(x, Q)$$

$$\sigma_{pp} \simeq \widetilde{\sigma}_{q\bar{q}}\left(\alpha_s, \alpha\right) \otimes q(x_1, Q) \otimes \bar{q}(x_2, Q)$$



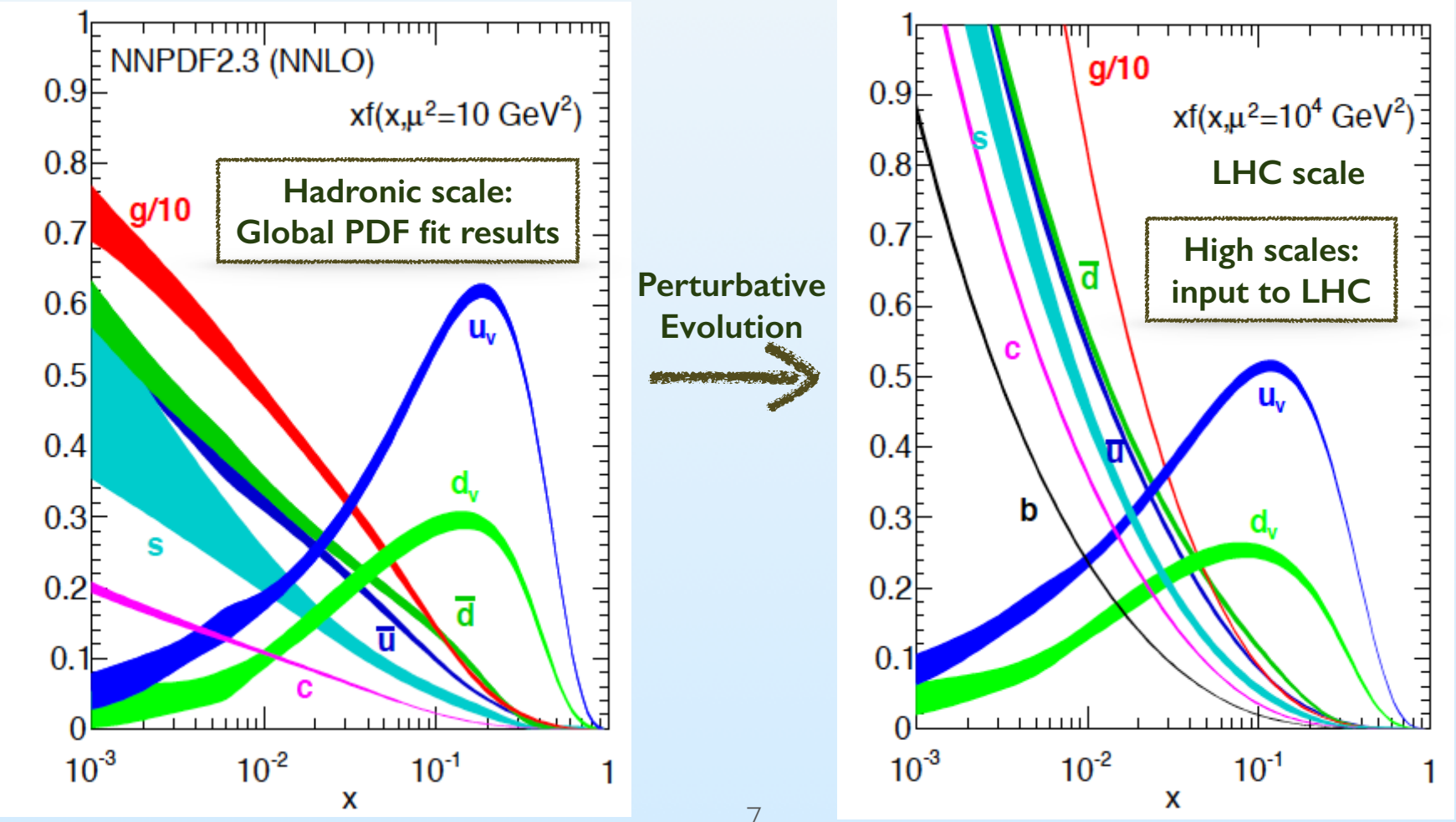


Determine PDFs in lepton-proton collisions ….

And use them to compute cross-sections in proton-proton collisions at the LHC

The global PDF analysis

- Combine **state-of-the-art theory calculations,** the **constraints from PDF-sensitive measurements** from different processes and colliders, and a **statistically robust fitting methodology**
- **Extract Parton Distributions** at hadronic scales of **a few GeV,** where non-perturbative QCD sets in
- Use **perturbative evolution** to compute PDFs at high scales as **input to LHC predictions**



The NNPDF approach

A **novel approach to PDF determination**, improving the limitations of the traditional PDF fitting methods with the use of **advanced statistical techniques** such as **machine learning** and **multivariate analysis**

Non-perturbative PDF parametrization

- **Traditional approach**: based on **restrictive functional forms** leading to strong theoretical bias
- **NNPDF solution**: use **Artificial Neural Networks** as universal unbiased interpolants

PDF uncertainties and propagation to LHC calculations

- **Traditional approach**: Hessian method, limited to Gaussian/linear approximation
- **NNPDF solution**: based on the **Monte Carlo replica method** to create a probability distribution in the space of PDFs. Specially critical in **extrapolation regions** (**i.e.** high-x) for New Physics searches

Fitting technique

- **Traditional approach**: deterministic minimization of χ^2, **flat directions** problem
- **NNPDF solution**: **Genetic Algorithms** to explore efficiently the vast parameter space, with **cross-validation** to avoid fitting stat fluctuations

The Monte Carlo replica method

- Two main approaches to estimate **PDF uncertainties**: the **Hessian method** and the **Monte Carlo method**
- In the **Hessian method**, the χ^2 is expanded quadratically in the **fit parameters** $\{a_n\}$ around the best fit

$$H^n_{lm} \equiv \frac{1}{2}\frac{\partial \chi^2_n}{\partial a_l\,\partial a_m} = \sum_{i=1}^{N_{\rm pts.}} \frac{1}{(\sigma^{\rm uncorr.}_{n,i})^2 + \sum_k (\sigma^{\rm corr.}_{n,k,i})^2} \frac{\partial T_{n,i}(\{a\})/\mathcal{N}_n}{\partial a_l} \frac{\partial T_{n,i}(\{a\})/\mathcal{N}_n}{\partial a_m}$$

- The Hessian matrix is diagonalized, and PDF errors on **cross sections F** from linear error propagation

$$\Delta\chi^2_{\rm global} \equiv \chi^2_{\rm global} - \chi^2_{\rm min} = \sum_{i,j=1}^{n} H_{ij}(a_i - a_i^0)(a_j - a_j^0)$$

$$\Delta F = \frac{1}{2}\sqrt{\sum_{k=1}^{n}\left[F(S_k^+) - F(S_k^-)\right]^2},$$

- In the **Monte Carlo replica method,** pseudo-data replicas with same fluctuations as real data are generated, and then a PDF fit is performed **in each individual replica**
- Leads to **probability distribution in the space of PDFs**, without **linear/Gaussian approximations**

$$D_{m,i} \to \left(D_{m,i} + R^{\rm uncorr.}_{m,i}\,\sigma^{\rm uncorr.}_{m,i} + \sum_{k=1}^{N_{\rm corr.}} R^{\rm corr.}_{m,k}\,\sigma^{\rm corr.}_{m,k,i}\right)\cdot\left(1 + R^{\mathcal{N}}_m\,\sigma^{\mathcal{N}}_m\right)$$

Pseudo-data MC replica

Original data

$$\Delta F = \sqrt{\frac{N_{\rm rep}}{N_{\rm rep}-1}\left(\langle F^2\rangle - \langle F\rangle^2\right)}.$$

ANN for PDF parametrization

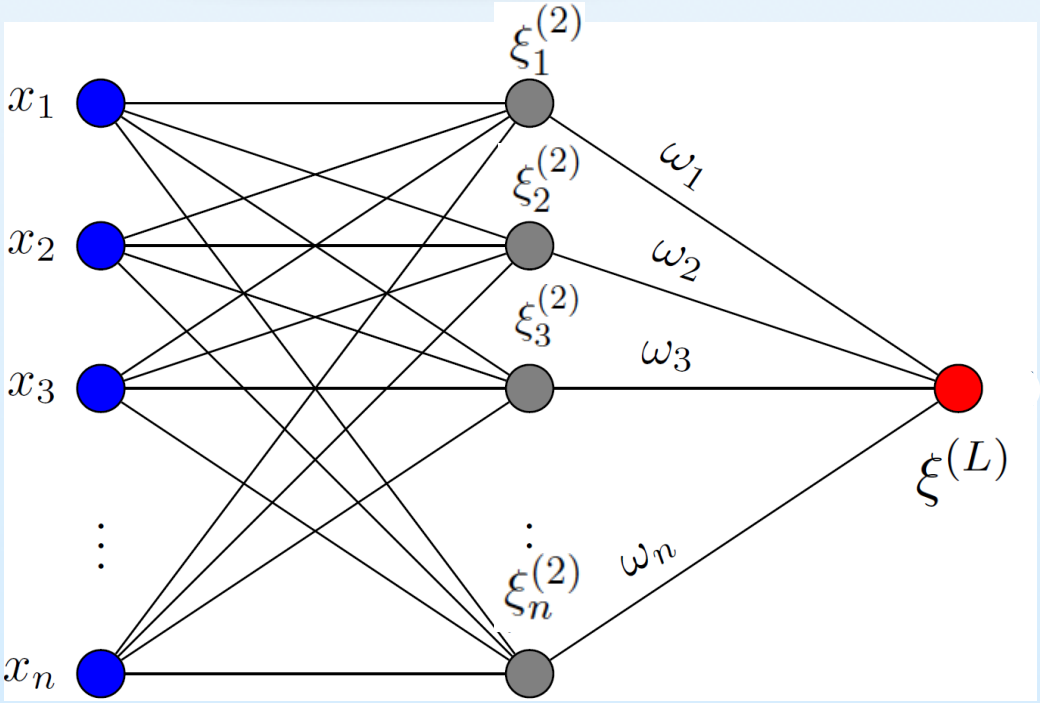
- ANNs are routinely exploited in **high-energy physics**, in most cases as **classifiers** to separate between interesting and more mundane events
- **ANNs also** provide **universal unbiased interpolants** to parametrize the non-perturbative dynamics that determines the **size and shape of the PDFs** from experimental data

Traditional approach

$$g(x,Q_0) = A_g(1-x)^{a_g}x^{-b_g}\left(1+c_g\sqrt{s}+d_g x+\ldots\right)$$

NNPDF approach

$$g(x,Q_0) = A_g \mathrm{ANN}_g(x)$$



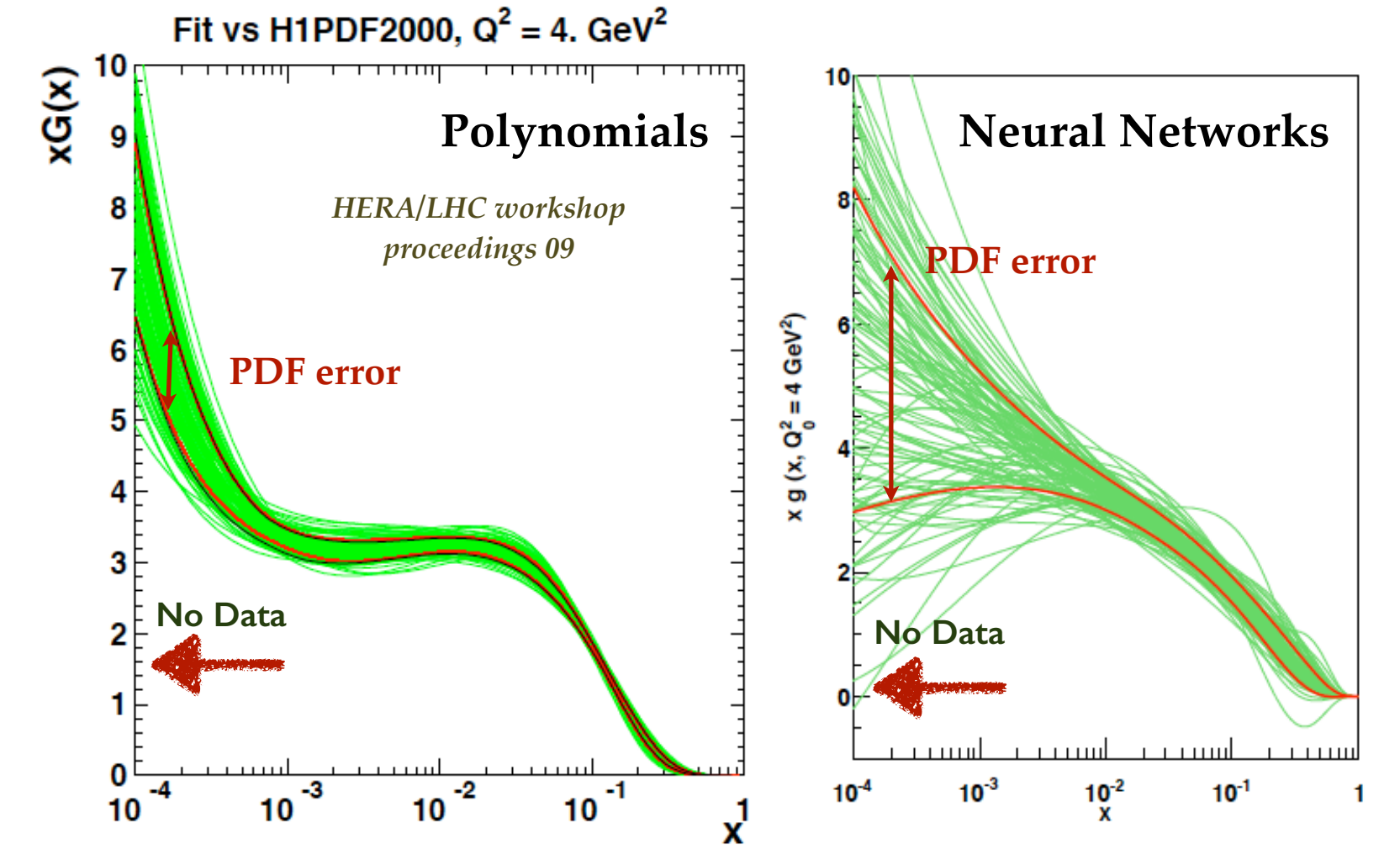
$$\mathrm{ANN}_g(x) = \xi^{(L)} = \mathcal{F}\left[\xi^{(1)}, \{\omega_{ij}^{(l)}\}, \{\theta_i^{(l)}\}\right]$$

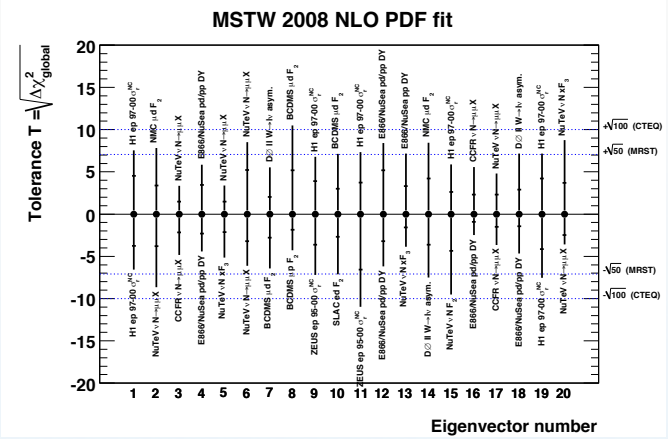
$$\xi_i^{(l)} = g\left(\sum_{j=1}^{n_{l-1}} \omega_{ij}^{(l-1)}\xi_j^{(l-1)} - \theta_i^{(l)}\right)$$

- ANNs eliminate **theory bias** introduced in PDF fits from choice of *ad-hoc* functional forms
- NNPDF fits used **O(400) free parameters**, to be compared with O(10-20) in traditional PDFs. Results stable if **O(4000) parameters used!**
- **Faithful extrapolation**: PDF uncertainties **blow up** in regions with scarce experimental data

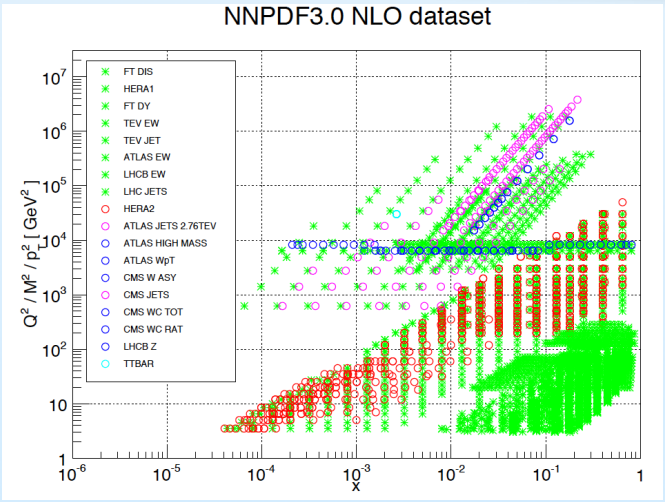
Artificial Neural Networks vs Polynomials

- Compare a **benchmark PDF analysis** where **the same dataset** is fitted with **Artificial Neural Networks** and with **standard polynomials,** other settings identical)
- **ANNs** avoid biasing the PDFs, **faithful extrapolation at small-x** (very few data, thus error blow up)





Combining Inconsistent Experiments in the PDF fit



Experimental data

- The global QCD analysis requires **combining different experiments** with disparate characteristics
  - **Type of high energy collision** (lepton-proton, proton-proton), **center-of-mass energy** of collision
  - Whether of not **experimental correlated systematics** are available, and if so, in **which format**
  - **Mutually inconsistent datasets** and datasets with **few points but large constraining power** vs datasets with **many points but moderate constraining power**

Lepton-Hadron collisions *NNPDF3.0 dataset* ***Hadron-Hadron collisions***

Experiment	Dataset	Ref.	Sys.	Unc.		$N_{\rm dat}$ no cuts (NLO/NNLO cuts)	Kinematics
NMC							
	NMC d/p	[19]	A	full		289 (132/132)	
	NMC $\sigma^{\rm NC,p}$	[20]	A	full		211 (224/224)	
SLAC							
	SLAC p	[23]	A	none	a	191 (37/37)	
	SLAC d	[23]	A	none	a	191 (37/37)	
BCDMS							
	BCDMS p	[21]	A	full	b	351 (333/333)	
	BCDMS d	[22]	A	full	b	254 (248/248)	
CHORUS							
	CHORUS ν	[35]	A	full	c	572 (431/431)	
	CHORUS $\bar{\nu}$	[35]	A	full	c	572 (431/431)	
NuTeV							
	NuTeV ν	[36,37]	A	none		45 (41/41)	
	NuTeV $\bar{\nu}$	[36,37]	A	none		44 (38/38)	
HERA-I							
	HERA-I NC e^+	[24]	M	full	d	434 (379/379)	
	HERA-I NC e^-	[24]	M	full	d	145 (145/145)	
	HERA-I CC e^+	[24]	M	full	d	34 (34/34)	
	HERA-I CC e^-	[24]	M	full	d	34 (34/34)	
ZEUS HERA-II							
	ZEUS-II NC e^-	[33]	M	full	e	90 (90/90)	
	ZEUS-II CC e^-	[34]	M	full	e	37 (37/37)	
	ZEUS-II NC e^+	[53]	M	full	f	90 (90/90)	$5\,10^{-3} \le x \le 0.40$ $200 \le Q^2 \le 3\,10^4$ GeV^2
	ZEUS-II CC e^+	[54]	M	full	f	35 (35/35)	$7.8\,10^{-3} \le x \le 0.42$ $280 \le Q^2 \le 3\,10^4$ GeV^2
H1 HERA-II							
	H1-II NC e^-	[51]	M	full	g	139 (139/139)	$2\,10^{-3} \le x \le 0.65$ $120 \le Q^2 \le 4\,10^4$ GeV^2
	H1-II NC e^+	[51]	M	full	g	138 (138/138)	$2\,10^{-3} \le x \le 0.65$ $120 \le Q^2 \le 4\,10^4$ GeV^2
	H1-II CC e^-	[51]	M	full	g	29 (29/29)	$8\,10^{-3} \le x \le 0.40$ $300 \le Q^2 \le 3\,10^4$ GeV^2
	H1-II CC e^+	[51]	M	full	g	29 (29/29)	$8\,10^{-3} \le x \le 0.40$ $300 \le Q^2 \le 3\,10^4$ GeV^2
	H1-II low Q^2	[52]	M	full		136 (124/124)	$2.8\,10^{-5} \le x \le 0.015$ $1.5 \le Q^2 \le 90$ GeV^2
	H1-II high y	[52]	M	full		55 (52/52)	$2.9\,10^{-5} \le x \le 5\,10^{-3}$ $2.5 \le Q^2 \le 90$ GeV^2
HERA $\sigma^c_{\rm NC}$		[55]	M	full		52 (47/47)	$3\,10^{-5} \le x \le 0.05$ $2.5 \le Q^2 \le 2\,10^3$ GeV^2

Experiment	Dataset	Ref.	Sys.	Unc.		$N_{\rm dat}$ no cuts (NLO/NNLO cuts)	Kinematics
DY E866							
	DY E866 d/p	[41]	M	none		15 (15/15)	
	DY E866 p	[39, 40]	M	none		184 (184/184)	
DY E605		[38]	M	none		119 (119/119)	
CDF							
	CDF Z rap	[43]	M	full	h	29 (29/29)	
	CDF Run-II k_t jets	[83]	M	full	h	76 (76/52)	
D0							
	D0 Z rap	[44]	M	full		28 (28/28)	
ATLAS							
	ATLAS W,Z 2010	[47]	M	full	i	30 (30/30)	
	ATLAS 7 TeV jets 2010	[50]	M	full	i,j	90 (90/9)	
	ATLAS 2.76 TeV jets	[63]	M	full	j	59 (59/3)	$20 \le p_T^{\rm jet} \le 200$ GeV $0 \le \lvert\eta^{\rm jet}\rvert \le 4.4$
	ATLAS high-mass DY	[56]	M	full		11 (5/5)	$116 \le M_{ll} \le 1500$ GeV
	ATLAS W p_T	[57]	M	full		11 (9/-)	$0 \le p_T^W \le 300$ GeV
CMS							
	CMS W electron asy	[48]	M	cov		11 (11/11)	
	CMS W muon asy	[58]	M	cov		11 (11/11)	$0 \le \lvert\eta_l\rvert \le 2.4$
	CMS jets 2011	[62]	M	full		133 (133/83)	$114 \le p_T^{\rm jet} \le 2116$ GeV $0 \le \lvert\eta^{\rm jet}\rvert \le 2.5$
	CMS $W+c$ total	[60]	M	cov		5 (5/5)	$0 \le \lvert\eta_l\rvert \le 2.1$
	CMS $W+c$ ratio	[60]	M	cov		5 (5/5)	$0 \le \lvert\eta_l\rvert \le 2.1$
	CMS 2D DY 2011	[59]	M	cov		124 (88/110)	$20 \le M_{ll} \le 1200$ GeV $0 \le \lvert\eta_{ll}\rvert \le 2.4$
LHCb							
	LHCb W rapidity	[49]	M	cov		10 (10/10)	
	LHCb Z rapidity	[61]	M	cov		9 (9/9)	$2.0 \le \eta_l \le 4.5$
$\sigma(t\bar{t})$							
	ATLAS $\sigma(t\bar{t})$	[65–67]	M	none		3 (3/3)	-
	CMS $\sigma(t\bar{t})$	[68–70]	M	none		3 (3/3)	-
Total						5179 (4276/4078)	

Experimental data

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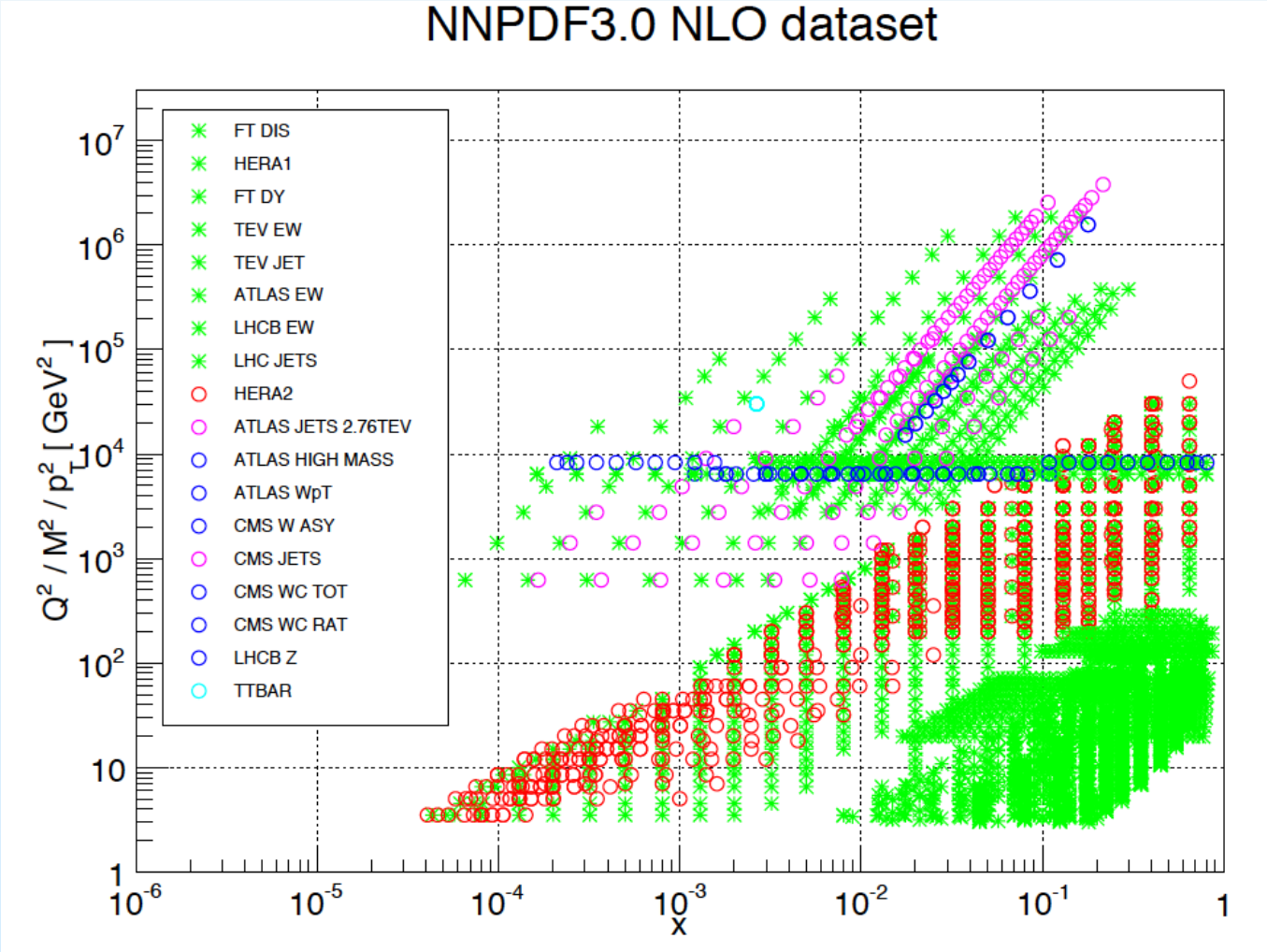
  - **Mutually inconsistent datasets** and datasets with **few points but large constraining power** vs datasets with **many points but moderate constraining power**



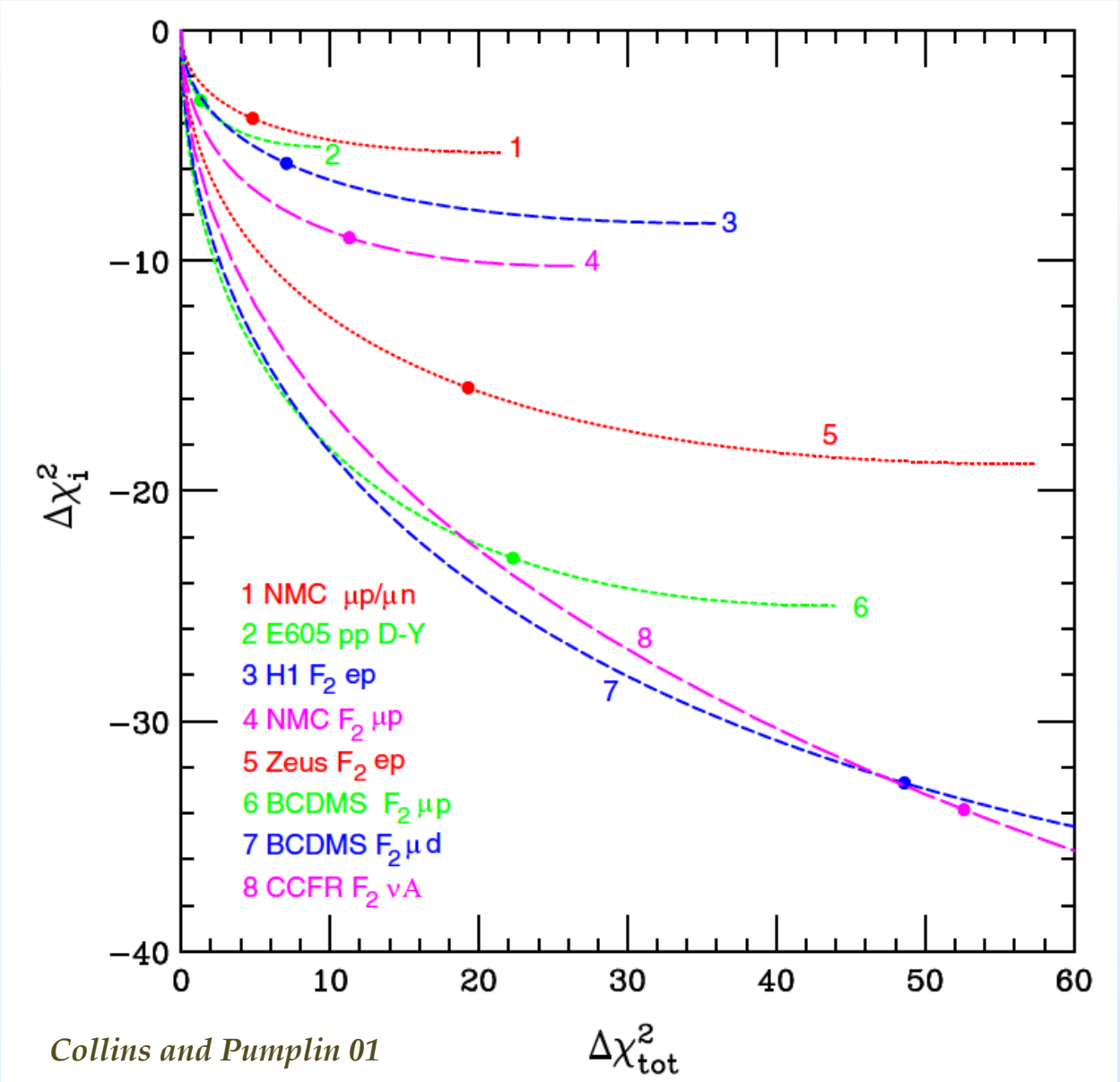
The kinematical coverage of the experiments included in NNPDF3.0 **span several orders** of magnitude both in x and Q^2

Inconsistent data

- What it is usually meant by ***inconsistent data***?
- Not a unique definition. Typically when one experiment that when added into a global fit leads to $\chi^2 >> N_{dat}$
- Many possible reasons for this:
  - ☑ **Underestimated systematic uncertainties**?
  - ☑ Incomplete/partial **theory calculations**?
  - ☑ **Methodological limitations**, ie, too restrictive PDF fitting functional forms?
  - ☑ **Genuine statistical pull** between different experiments in the global fit? (This is not inconsistency!)
- Dealing with **potentially inconsistent experiments in the global PDF fit** is very delicate. On the one hand, it is not advisable to bias *a priori* the fit with a **subjective selection of which experiments are more *reliable***. On the other hand, once wants to achieve statistically sound results, and in particular **PDF uncertainties that truly quantify our genuine lack of information**. So there are two complementary avenues:
  - ☑ Attempt to **understand how the inconsistencies arise**, and when possible fix them (for example using a better theory)
  - ☑ Devise a **fitting methodology that can deal with inconsistent experiments**, regardless of the origin of the inconsistency
- Note that some of the older fixed-target DIS experiments do not provide the **full breakdown of systematics**, but this is now a small weight in the global fit

Dealing with inconsistent data

- In the global PDF fit, **different experiments will prefer different solutions**, not always compatible
- Also, the number of datapoints (statistical weight) of each experiment can be quite different, and one wants to describe **both** exps with many data points and those with few
- Using textbook statistics, 68% CL uncertainties in the PDF fit parameters should be determined from the $\Delta\chi^2 = 1$ criteria
- However it has been shown that this criterion is not adequate in the global fits with many experiments
- So **global Hessian fits** use effectively an increased tolerance $\Delta\chi^2 >> 1$, to ensure that all fitted experiments are reasonably described

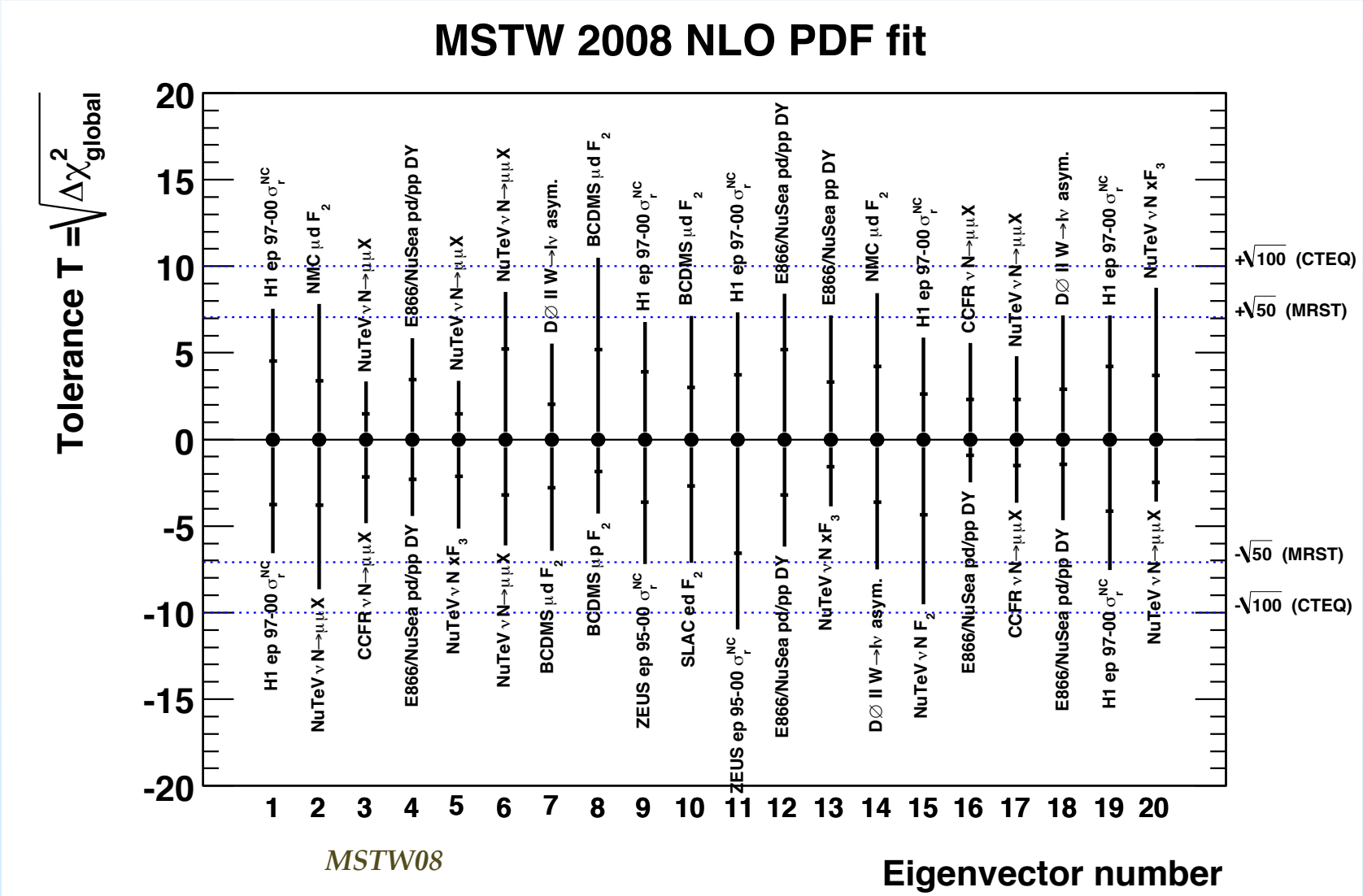


Collins and Pumplin 01

Dealing with inconsistent data

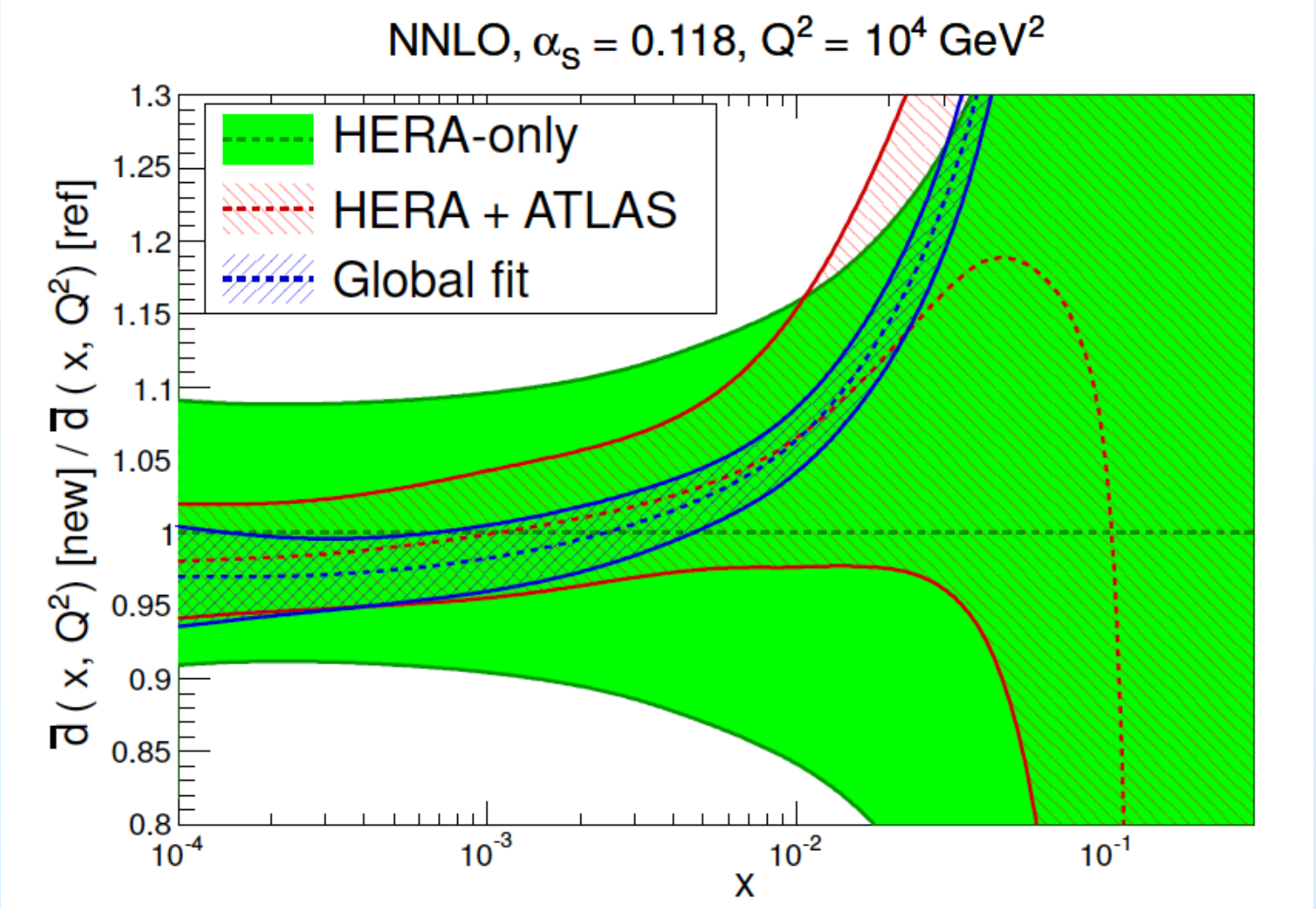
- In the MSTW approach, a **dynamical tolerance criterion** is used where different individual experiments determine the **allowed upper and lower variations of each eigenvectors**

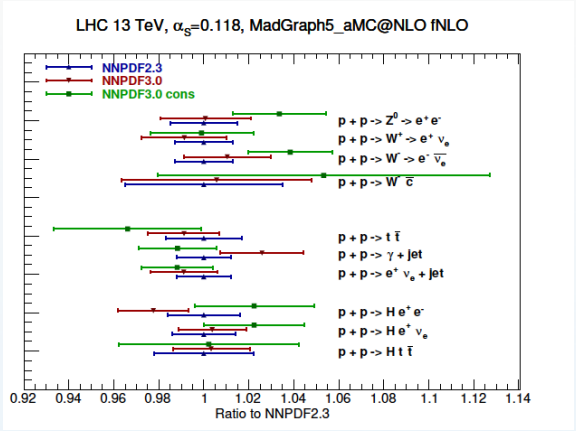
- This also indicates which datasets are more **sensitive to each eigenvector**



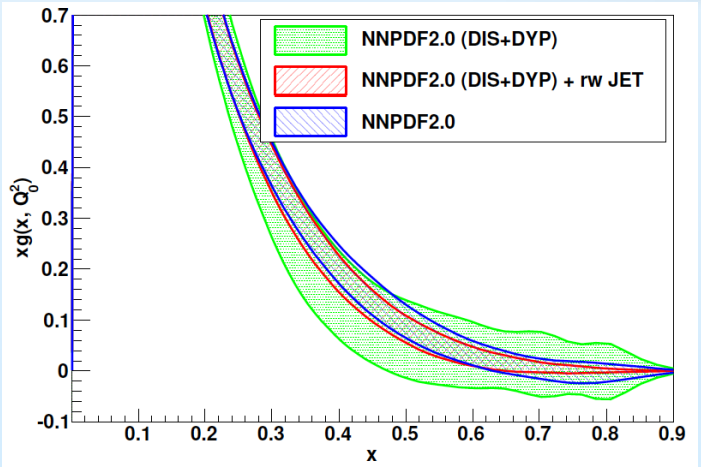
Dealing with inconsistent data

- In the NNPDF approach, **no need to modify the fitting methodology** in the presence of inconsistencies
- When new (compatible) experiments are added, then **PDF errors decrease**. If inconsistent experiments included, fit essentially unaffected and PDF errors not modified (since no new information added)
- Fitting methodology also unchanged even for **large variations of the fitted dataset**





Bayesian Reweighting



NNPDF, arXiv:1012.0836
NNPDF, arXiv:1108.1758

Reweighting as an alternative to fitting

When a **new dataset becomes available**, instead of updating the global fit, it is possible include this new information on a prior PDF set using **Bayes' Theorem**

The **weight (likelihood) in the presence of the new experiment** corresponding to each Monte Carlo replica *k* is given in terms of the χ^2_k **between data and the theory** computed with this replica

$$w_k = \frac{(\chi^2_k)^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi^2_k}}{\frac{1}{N}\sum_{k=1}^{N}(\chi^2_k)^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi^2_k}}$$

The Bayesian reweighting technique also allows to quantify the o**verall consistency of the new experiment** with those already included of the global fit by defining

$$\mathcal{P}(\alpha) \propto \frac{1}{\alpha}\sum_{k=1}^{N} w_k(\alpha).$$

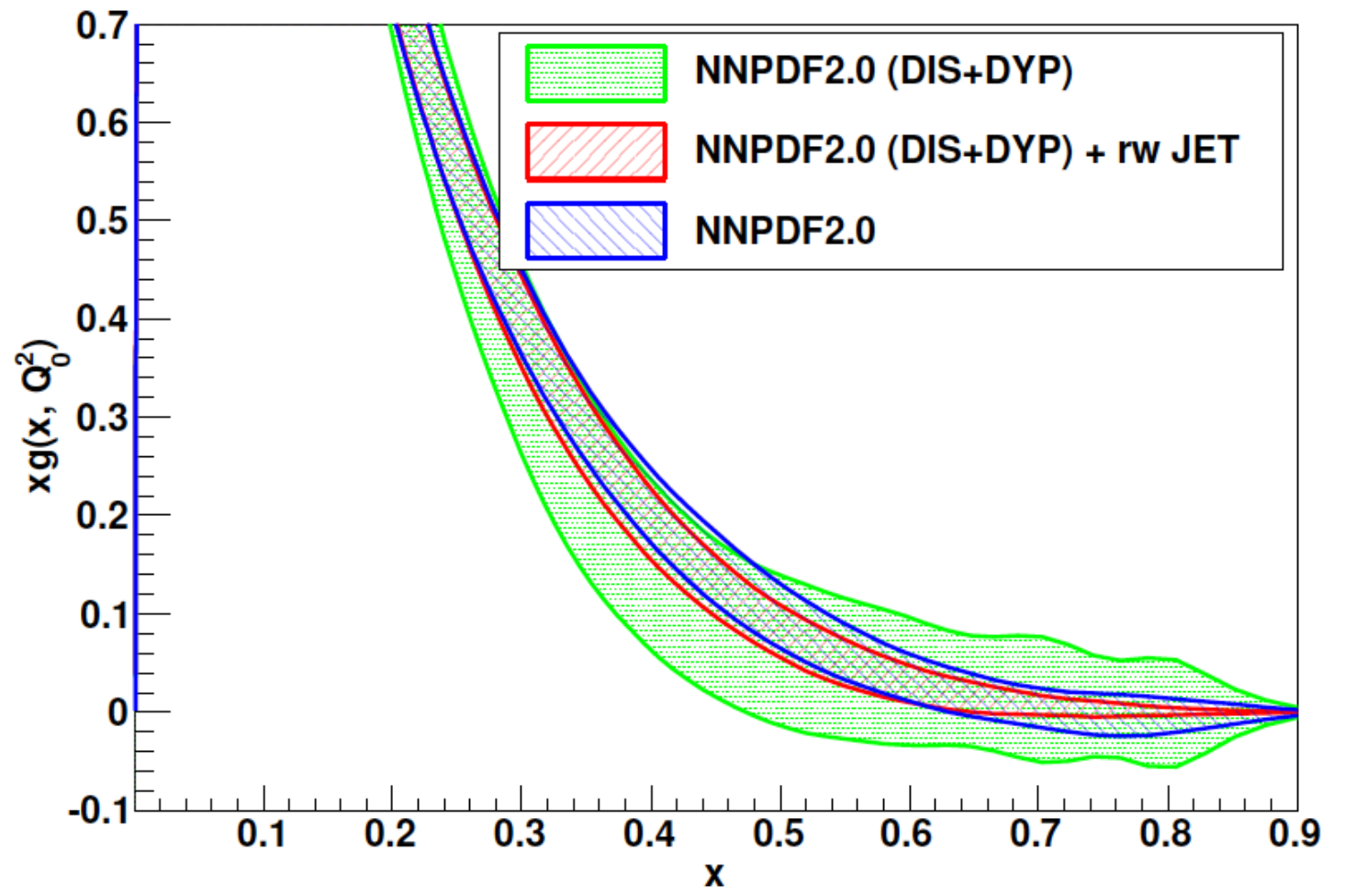
where $\omega_k(\alpha)$ are the **weights** ω_k now with χ^2 **rescaled as** χ^2/α, that is, they correspond to the case where the new experimental dataset has **uncertainties rescaled by a factor** $\alpha^{1/2}$

Any inconsistent experiment can be brought in agreement with the global fit by a **suitable rescaling of its uncertainties** (though this is not necessary in the NNPDF framework)

Reweighting as an alternative to fitting

Adding new experiments, in this case the **Tevatron inclusive jet data**, by reweighting leads to results that are **statistically consistent with refitting**

Main benefit of RW is that it can be performed using only public tools (PDF sets and codes for cross-section calculation) **without any input from the PDF fitters**

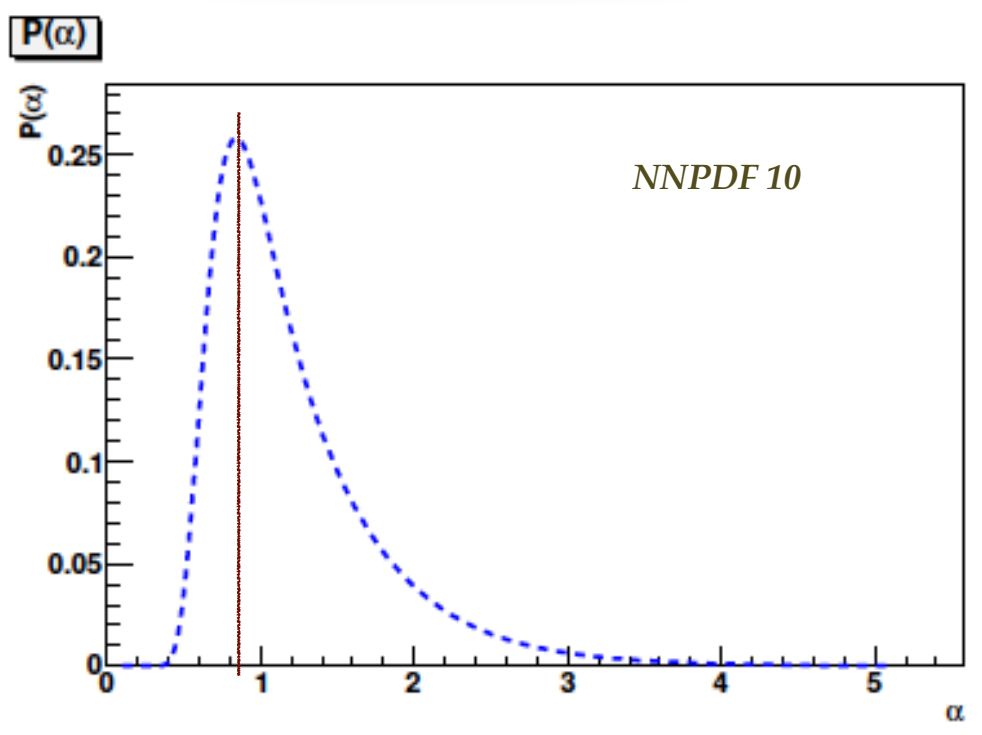


Reweighting as an alternative to fitting

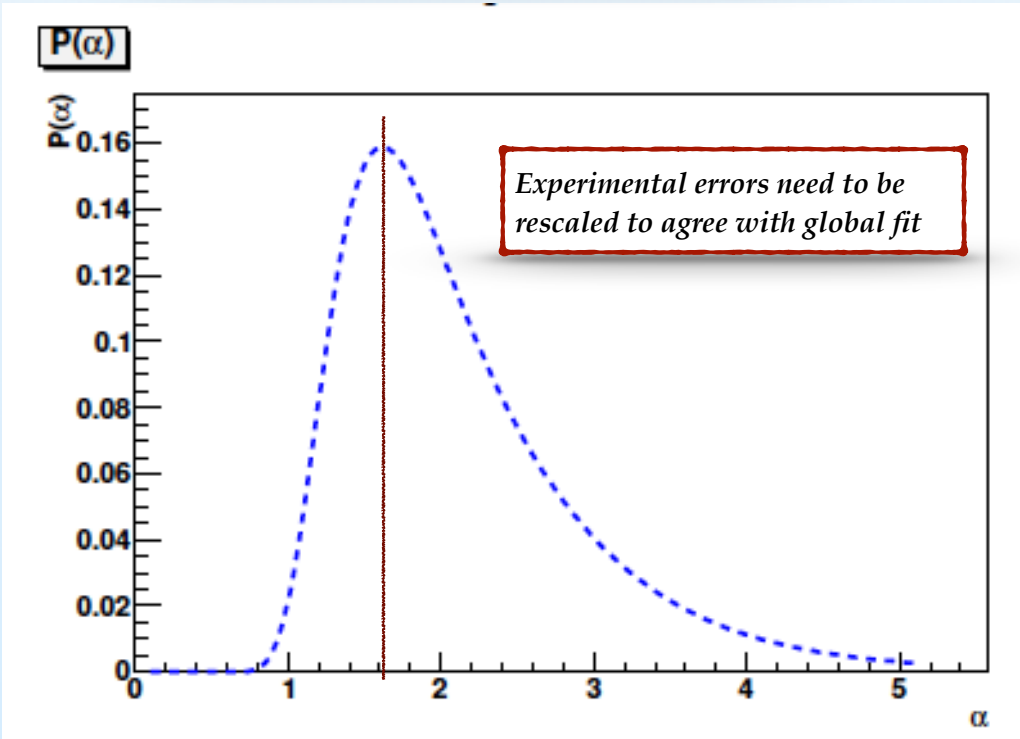
The distribution of the χ^2 rescaling parameter α allows to **quantify the level of (in)consistency of a new experiment** with those already included in the global fit

$$\mathcal{P}(\alpha) \propto \frac{1}{\alpha} \sum_{k=1}^{N} w_k(\alpha).$$

Consistent experiment



(mildly) Inconsistent experiment



Conservative Partons

- To study the robustness of the global fit results, it is possible to define **parton distributions based on a maximally consistent dataset**: the *conservative partons*

- Include in the conservative fit only those experiments which in the global fit have their **P(α) distribution peaked at $\alpha < \alpha_{\rm max}$**

- modifying this threshold allows to **tune the PDF fit to be more or less conservative**

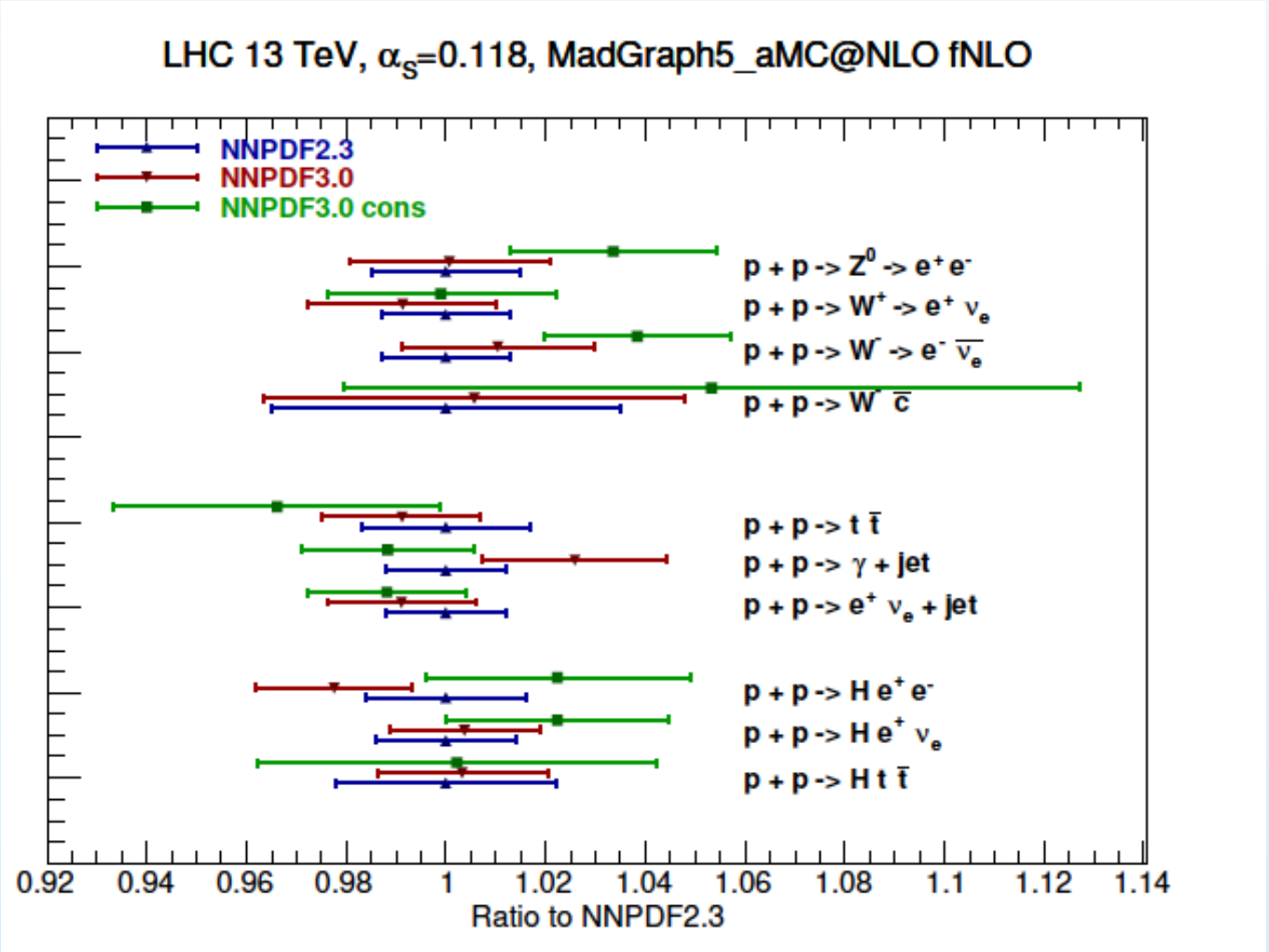
- Quantify impact of ***known*** **dataset inconsistencies** on the global fit PDFs

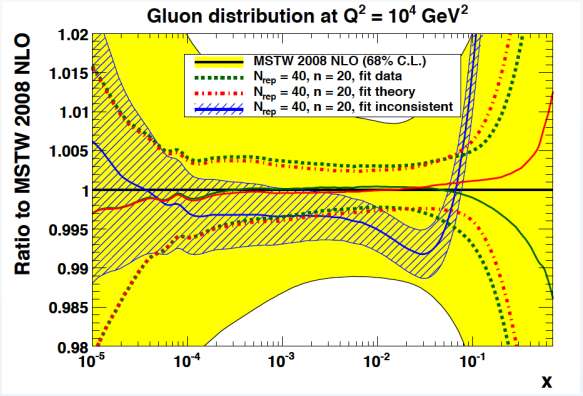
- This is not merely a conceptual detail: assessing robustness of PDF errors in LHC cross-section is central ***ie*** for the *characterisation of the Higgs boson*

	$\alpha_{\rm max}=1.1$		$\alpha_{\rm max}=1.2$		$\alpha_{\rm max}=1.3$		Global fit	
	$\chi^2_{\rm nlo}$	$\chi^2_{\rm nnlo}$	$\chi^2_{\rm nlo}$	$\chi^2_{\rm nnlo}$	$\chi^2_{\rm nlo}$	$\chi^2_{\rm nnlo}$	$\chi^2_{\rm nlo}$	$\chi^2_{\rm nnlo}$
Total	0.96	1.01	1.06	1.10	1.12	1.16	1.23	1.29
NMC d/p	0.91	0.91	0.89	0.89	0.88	0.89	0.92	0.93
NMC $\sigma^{\rm NC,p}$	-	-	-	-	-	-	1.63	1.52
SLAC	-	-	-	-	1.77	1.19	1.59	1.13
BCDMS	-	-	1.11	1.15	1.12	1.16	1.22	1.29
CHORUS	-	-	1.06	1.02	1.09	1.07	1.11	1.09
NuTeV	0.35	0.34	0.62	0.64	0.70	0.70	0.70	0.86
HERA-I	0.97	0.98	1.02	1.00	1.02	0.99	1.05	1.04
ZEUS HERA-II	-	-	-	-	1.41	1.48	1.40	1.48
H1 HERA-II	-	-	-	-	-	-	1.65	1.79
HERA $\sigma^c_{\rm NC}$	-	-	1.21	1.32	1.20	1.31	1.27	1.28
E886 d/p	0.30	0.30	0.43	0.40	0.44	0.46	0.53	0.48
E886 p	-	-	1.18	1.40	1.27	1.53	1.19	1.55
E605	1.04	1.10	0.74	0.83	0.75	0.88	0.78	0.90
CDF Z rapidity	-	-	-	-	-	-	1.33	1.53
CDF Run-II k_t jets	-	-	1.01	2.01	1.04	1.84	0.96	1.80
D0 Z rapidity	0.56	0.61	0.62	0.71	0.60	0.69	0.57	0.61
ATLAS W,Z 2010	-	-	1.19	1.13	1.19	1.17	1.19	1.23
ATLAS 7 TeV jets 2010	0.96	1.65	1.08	1.58	1.10	1.54	1.07	1.36
ATLAS 2.76 TeV jets	1.03	0.38	1.38	0.36	1.35	0.35	1.29	0.33
ATLAS high-mass DY	-	-	-	-	-	-	2.06	1.45
ATLAS W p_T	-	-	-	-	-	-	1.13	-
CMS W electron asy	0.98	0.84	0.82	0.72	0.85	0.73	0.87	0.73
CMS W muon asy	-	-	-	-	-	-	1.81	1.72
CMS jets 2011	0.90	2.09	0.96	2.09	0.99	2.10	0.96	1.90
CMS $W+c$ total	-	-	-	-	-	-	0.96	0.84
CMS $W+c$ ratio	-	-	-	-	-	-	2.02	1.77
CMS 2D DY 2011	-	-	-	-	1.20	1.30	1.23	1.36
LHCb W rapidity	-	-	0.69	0.65	0.74	0.69	0.71	0.72
LHCb Z rapidity	-	-	1.23	1.78	1.11	1.58	1.10	1.59
$\sigma(t\bar{t})$	-	-	-	-	-	-	1.43	0.66

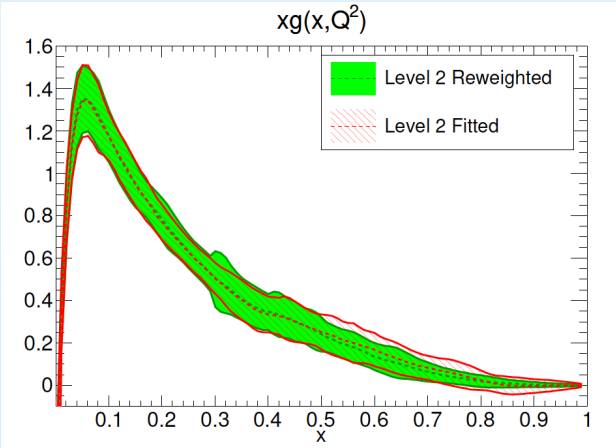
Conservative Partons

- At the level of LHC cross-sections, **conservative PDFs consistent with global fit PDFs** within uncertainties
- Conservative PDFs affected by **larger uncertainties due to reduced dataset**
- Non-trivial validation of the **robustness of the global fit results**



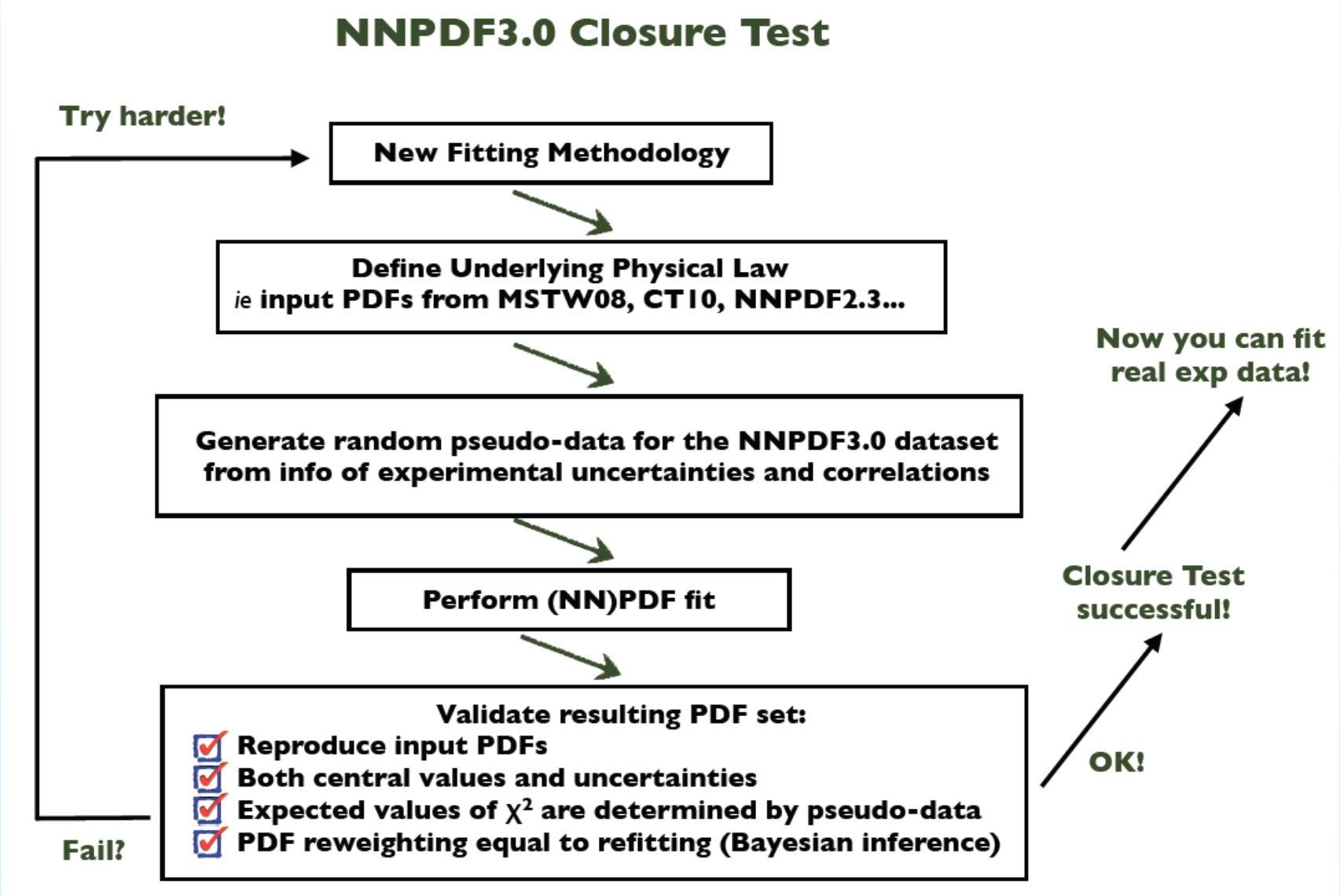


Statistical Methodology
Validation with Closure Testing



Closure Testing of Parton Distributions

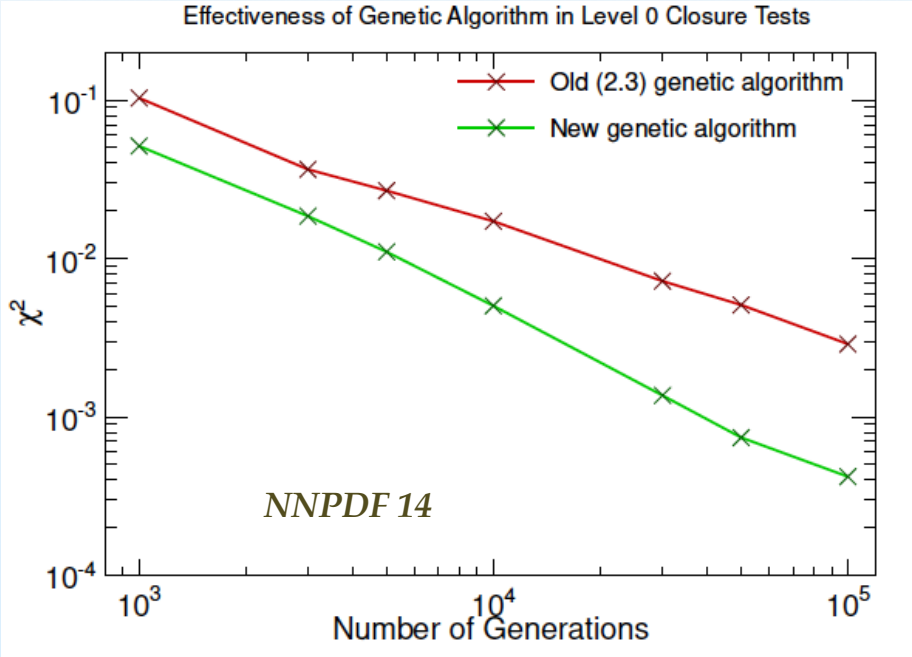
- **PDF uncertainties** have been often criticised by a **potential lack of statistical interpretation**
- Within NNPDF, we performed a systematic **closure tests analysis** based on pseudo-data, and verified that **PDF uncertainties** exhibit **a statistically robust behaviour**

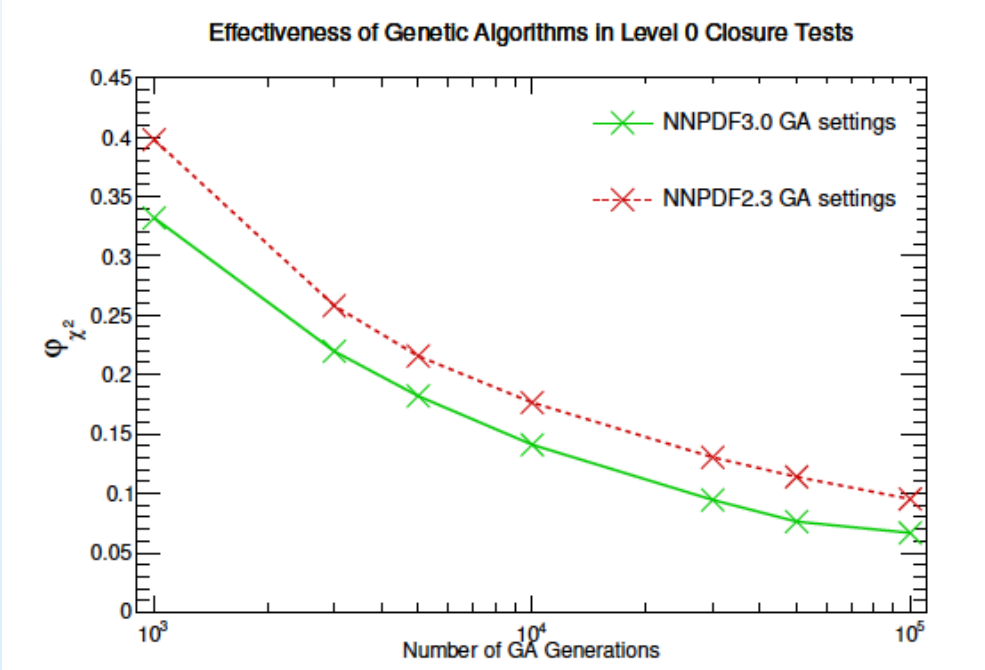


Closure Testing of Parton Distributions

For instance, if the **pseudo-data is generated without statistical fluctuations** (that is, identical to the input theory) then the agreement with theory by construction should become **arbitrarily good**

And indeed it does: as the minimization advances, the χ^2 **decreases monotonically,** and the PDF uncertainties as well are reduced, as the **fitted theory collapses to the underlying law**





$$\varphi_{\chi^2} \equiv \sqrt{\langle \chi^2[\mathcal{T}[f_{\rm fit}], \mathcal{D}_0] \rangle - \chi^2[\langle \mathcal{T}[f_{\rm fit}] \rangle, \mathcal{D}_0]} \,.$$

Measure of **PDF uncertainties** in units of **data uncertainties**

Closure Testing of Parton Distributions

Another important advantage of closure testing the global PDF analysis is that it allows to **disentangle the various components of the total PDF uncertainty**

Lvl0: fit pseudo-data **without fluctuations** (limit: χ^2 -> 0)

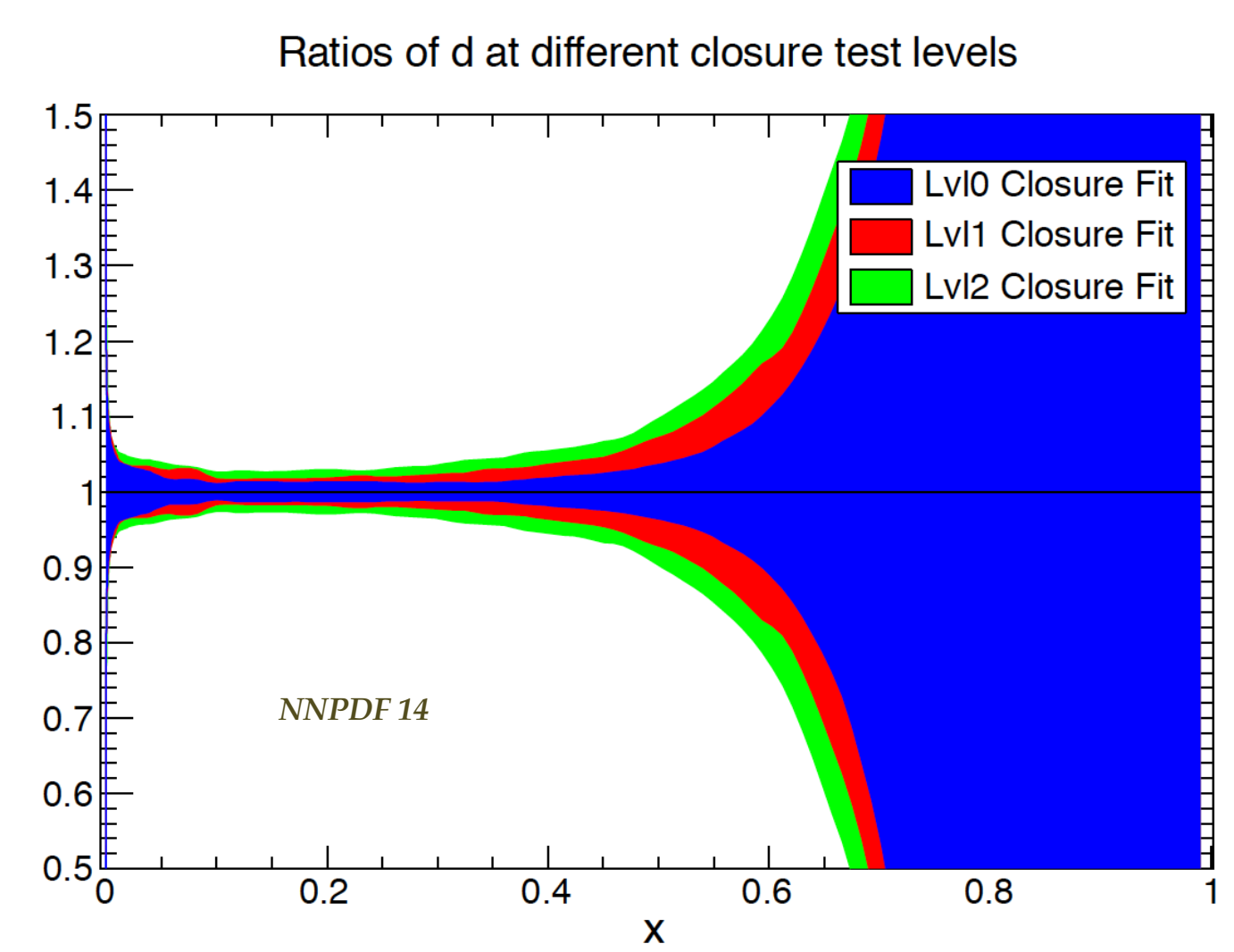
=> *Extrapolation Uncertainty*

Lvl1: fit pseudo-data **with fluctuations** (limit: χ^2 -> 1)

=> *Functional uncertainty*

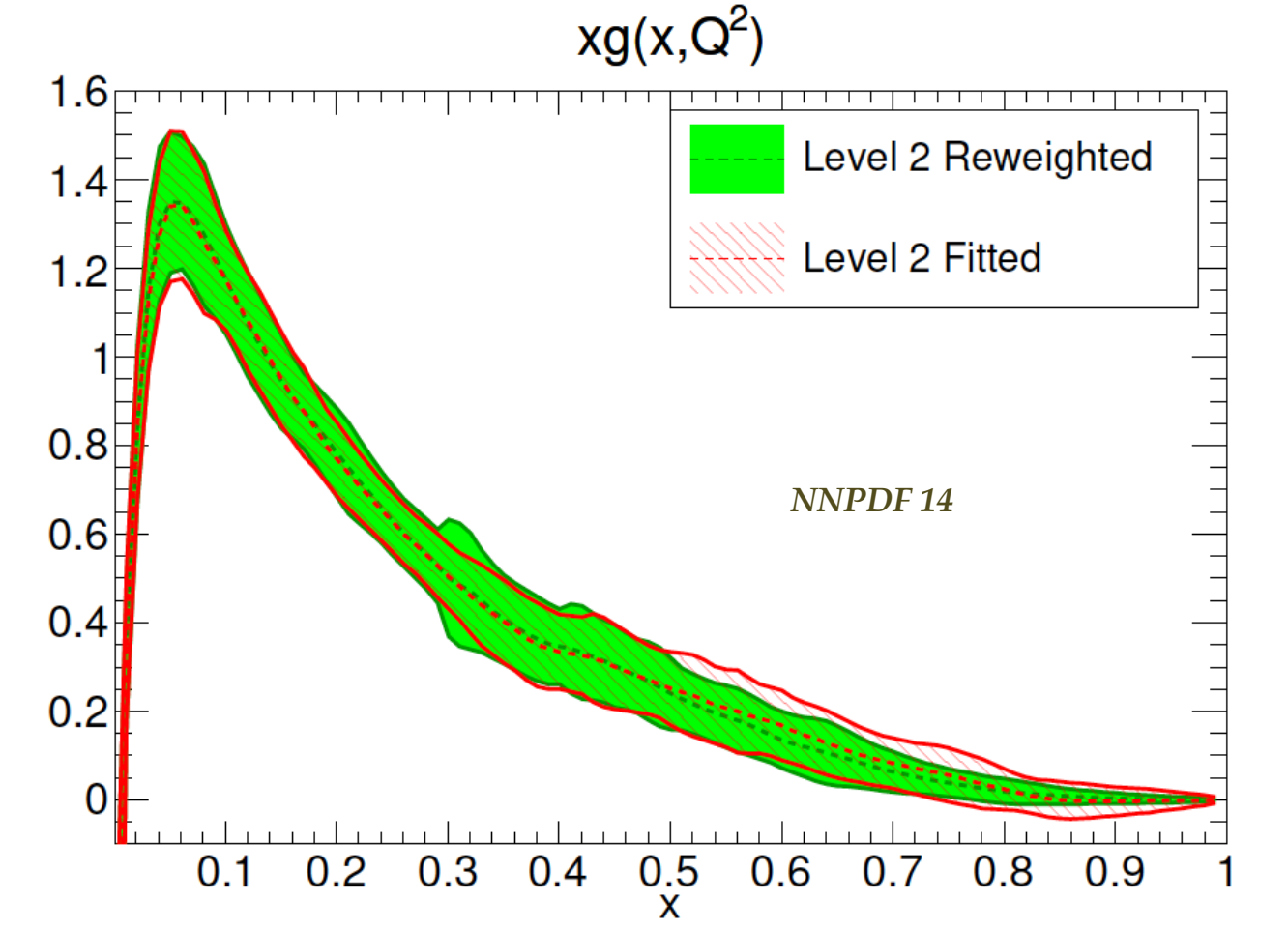
Lvl2: fit **Monte Carlo replicas of pseudo-data with fluctuations** (limit: χ^2->2)

=> *Data uncertainty*



Closure Testing of Parton Distributions

In the closure tests, it is possible to validate new techniques, such as the Bayesian reweighting, in a **clean environment where everything is under control** (free in particular of potential data inconsistencies)



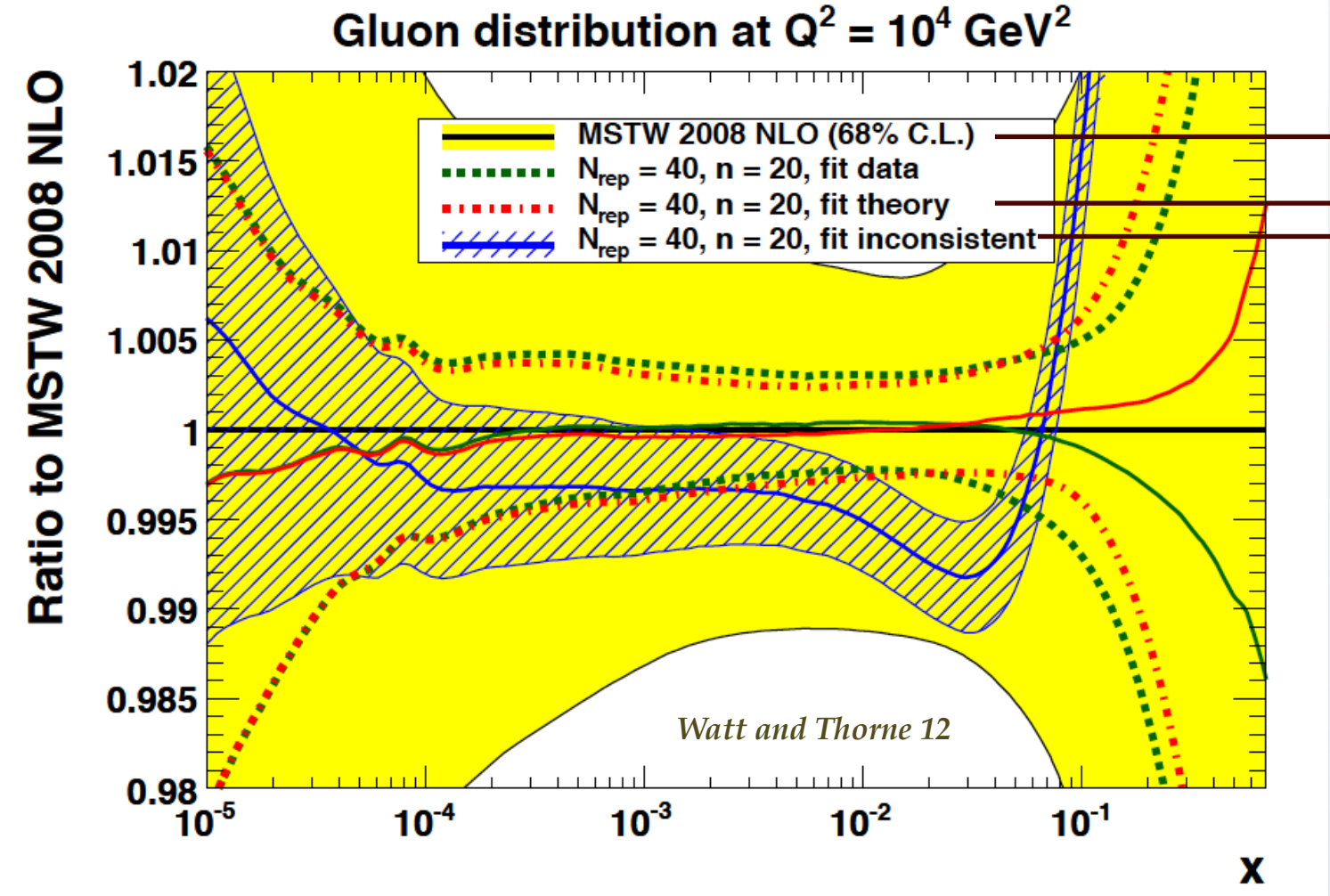
Closure testing the global fit allows disentangling **methodological issues of principle** (in an ideal world with perfectly consistent datasets, does my fitting methodology give the result it should?) with those of **practice** (how to deal with inconsistent experiments or with incomplete theory?)

Adding artificial inconsistencies

To test the fitting methodology in a realistic situation, it is possible to **generate pseudo-data adding artificial inconsistencies** and study how the resulting PDFs are modified.

In the MMHT approach, adding artificial inconsistencies in a closure test leads to **modified PDFs in most cases in agreement with the global fit PDF uncertainties**

This is only the case if their **dynamical tolerance criterion $\Delta\chi^2 >> 1$** is used, as opposed to $\Delta\chi^2 = 1$



Summary and outlook

- **Parton Distributions** are an essential ingredient for **LHC phenomenology**
- **At the LHC,** precision PDFs are required for many analysis from the characterisation of the Higgs sector to BSM searches and Monte Carlo event generators
- **The global QCD analysis** aims to extract parton distributions from a diverse experimental dataset using state-of-the-art theory and methodology
- This involves having to deal with several non-trivial statistical issues, in particular with **potential inconsistencies between fitted datasets**, that can arise from various sources: partial theory, limited fitted methodology or underestimated systematic uncertainties
- To deal with these problems, a number of techniques have been developed, which allow to validate the **robustness of our PDF uncertainty estimates for high-precision LHC phenomenology**