Neural Network Fits of Parton Distributions

Juan Rojo
Rudolf Peierls Center for Theoretical Physics
University of Oxford

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(unpolarized) Parton Distributions

The distribution of energy that **quarks and gluons carry inside the proton** is quantified by the **Parton Distribution Functions (PDFs)**

$$g(x,Q)$$

***Q*: Energy of the quark/gluon collision**
Inverse of the resolution length

g(x,Q)*: Probability** of **finding a gluon inside a proton**, carrying a fraction ***x of the proton momentum, when probed with energy ***Q***

x: Fraction of the proton's momentum

PDFs are determined by **non-perturbative QCD dynamics:** cannot be computed from first principles, and need to be **extracted from experimental data** with a **global analysis**

- **Energy conservation**

$$\int_0^1 dx \left(g(x,Q) + \sum_q q(x,Q) \right) = 1$$

- **Dependence with quark/gluon collision energy Q** determined in perturbation theory

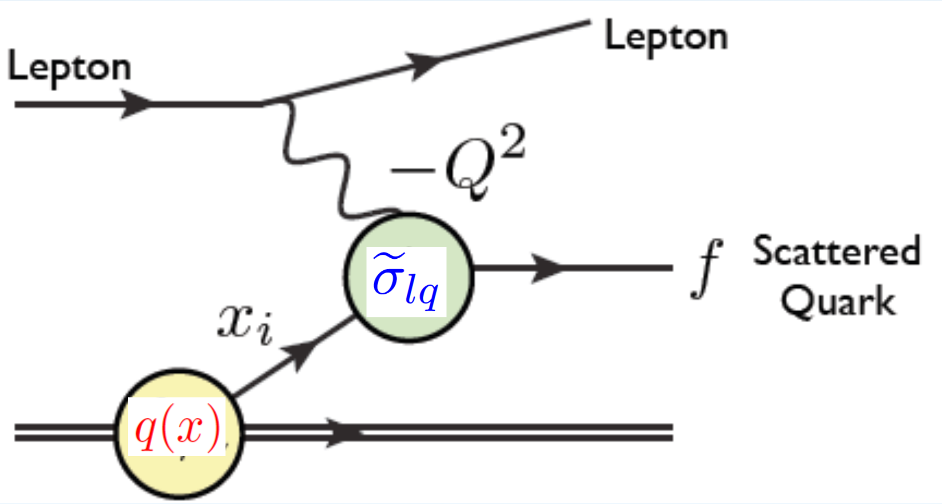
$$\frac{\partial g(x,Q)}{\partial \ln Q} = P_g\left(\alpha_s\right) \otimes g(x,Q) + P_q\left(\alpha_s\right) \otimes q(x,Q)$$

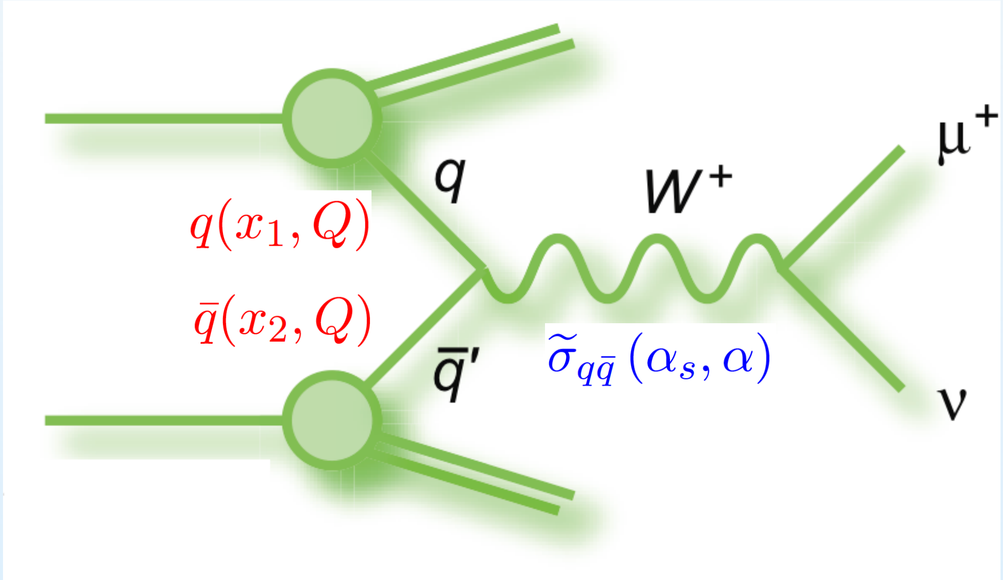
The Factorization Theorem

The **QCD Factorization Theorem** guarantees **PDF universality: extract them from a subset of process** and use them to provide pure predictions for **new processes**

$$\sigma_{lp} \simeq \widetilde{\sigma}_{lq}\,(\alpha_s, \alpha) \otimes q(x, Q)$$

$$\sigma_{pp} \simeq \widetilde{\sigma}_{q\bar{q}}\,(\alpha_s, \alpha) \otimes q(x_1, Q) \otimes \bar{q}(x_2, Q)$$





Determine PDFs in lepton-proton collisions ….

And use them to compute cross-sections in proton-proton collisions at the LHC

The NNPDF approach

A **novel approach to PDF determination**, improving the limitations of the traditional PDF fitting methods with the use of **advanced statistical techniques** such as **machine learning** and **multivariate analysis**

Non-perturbative PDF parametrization

- **Traditional approach**: based on **restrictive functional forms** leading to strong theoretical bias
- **NNPDF solution**: use **Artificial Neural Networks** as universal unbiased interpolants

PDF uncertainties and propagation to LHC calculations

- **Traditional approach**: limited to Gaussian/linear approximation
- **NNPDF solution**: based on the **Monte Carlo replica method** to create a probability distribution in the space of PDFs. Specially critical in **extrapolation regions** (**i.e.** high-x) for New Physics searches

Fitting technique

- **Traditional approach**: deterministic minimization of χ^2, **flat directions** problem
- **NNPDF solution**: **Genetic Algorithms** to explore efficiently the vast parameter space, with **cross-validation** to avoid fitting stat fluctuations

ANN as universal interpolators

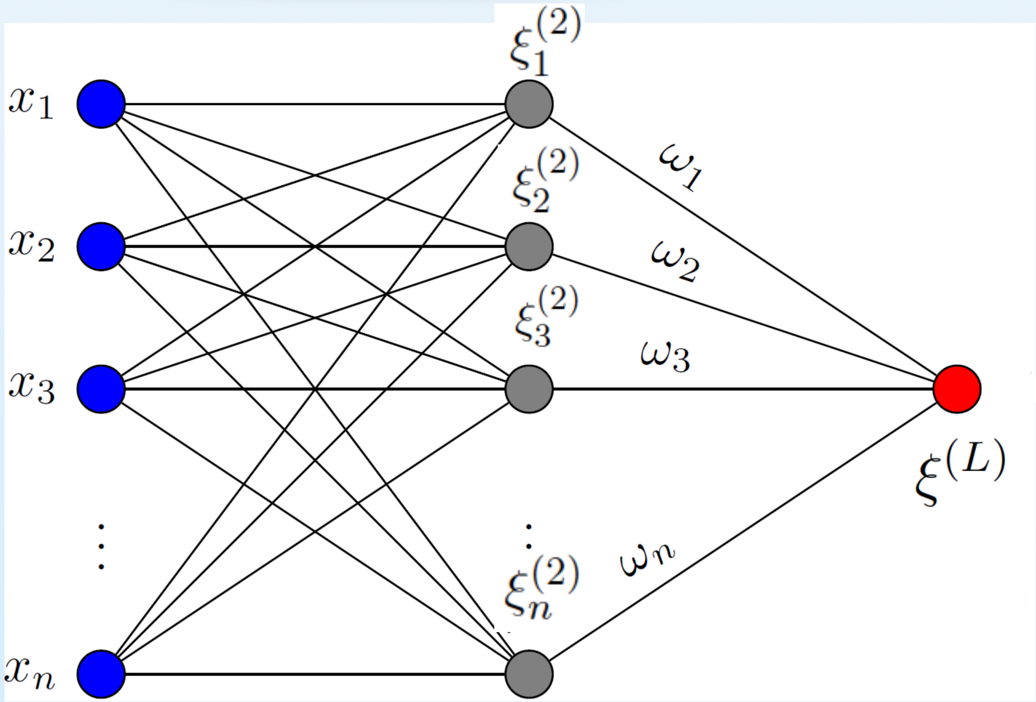
- ANNs are routinely exploited in **high-energy physics**, in most cases as **classifiers** to separate between interesting and more mundane events
- **ANNs also** provide **universal unbiased interpolants** to parametrize the non-perturbative dynamics that determines the **size and shape of the PDFs** from experimental data

Traditional approach

$$g(x,Q_0) = A_g(1-x)^{a_g}x^{-b_g}\left(1+c_g\sqrt{s}+d_gx+\ldots\right)$$

NNPDF approach

$$g(x,Q_0) = A_g \mathrm{ANN}_g(x)$$



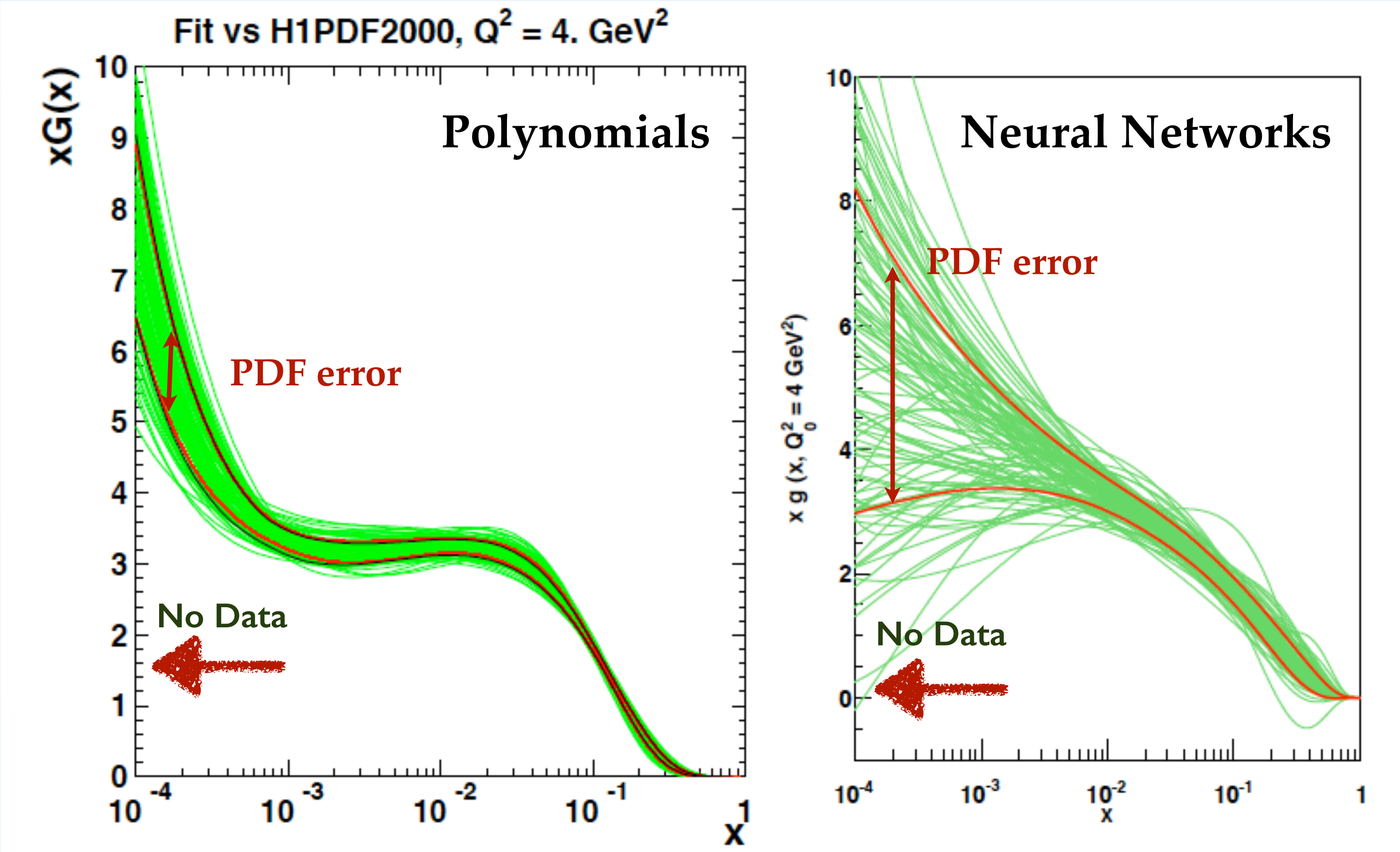
$$\mathrm{ANN}_g(x) = \xi^{(L)} = \mathcal{F}\left[\xi^{(1)},\{\omega_{ij}^{(l)}\},\{\theta_i^{(l)}\}\right]$$

$$\xi_i^{(l)} = g\left(\sum_{j=1}^{n_{l-1}}\omega_{ij}^{(l-1)}\xi_j^{(l-1)} - \theta_i^{(l)}\right)$$

- ANNs eliminate **theory bias** introduced in PDF fits from choice of *ad-hoc* functional forms
- NNPDF fits used **O(400) free parameters**, to be compared with O(10-20) in traditional PDFs. Results stable if **O(4000) parameters used!**
- **Faithful extrapolation**: PDF uncertainties **blow up** in regions with scarce experimental data

Artificial Neural Networks vs Polynomials

- Compare a **benchmark PDF analysis** where **the same dataset** is fitted with **Artificial Neural Networks** and with **standard polynomials,** other settings identical)
- **ANNs** avoid biasing the PDFs, **faithful extrapolation at small-x** (very few data, thus error blow up)



The Monte Carlo replica method

- Two main approaches to estimate **PDF uncertainties**: the **Hessian method** and the **Monte Carlo method**
- In the **Hessian method**, the χ^2 is expanded quadratically in the **fit parameters** $\{a_n\}$ around the best fit

$$H^n_{lm} \equiv \frac{1}{2}\frac{\partial \chi^2_n}{\partial a_l\, \partial a_m} = \sum_{i=1}^{N_{\rm pts.}} \frac{1}{(\sigma^{\rm uncorr.}_{n,i})^2 + \sum_k (\sigma^{\rm corr.}_{n,k,i})^2} \frac{\partial T_{n,i}(\{a\})/\mathcal{N}_n}{\partial a_l} \frac{\partial T_{n,i}(\{a\})/\mathcal{N}_n}{\partial a_m}$$

- The Hessian matrix is diagonalized, and PDF errors on **cross sections F** from linear error propagation

$$\Delta\chi^2_{\rm global} \equiv \chi^2_{\rm global} - \chi^2_{\rm min} = \sum_{i,j=1}^{n} H_{ij}(a_i - a_i^0)(a_j - a_j^0)$$

$$\Delta F = \frac{1}{2}\sqrt{\sum_{k=1}^{n}\left[F(S_k^+) - F(S_k^-)\right]^2},$$

- In the **Monte Carlo replica method,** pseudo-data replicas with same fluctuations as real data are generated, and then a PDF fit is performed **in each individual replica**
- Leads to **probability distribution in the space of PDFs**, without **linear/Gaussian approximations**

$$D_{m,i} \to \left(D_{m,i} + R^{\rm uncorr.}_{m,i}\,\sigma^{\rm uncorr.}_{m,i} + \sum_{k=1}^{N_{\rm corr.}} R^{\rm corr.}_{m,k}\,\sigma^{\rm corr.}_{m,k,i} \right) \cdot \left(1 + R^{\mathcal{N}}_m\,\sigma^{\mathcal{N}}_m\right)$$

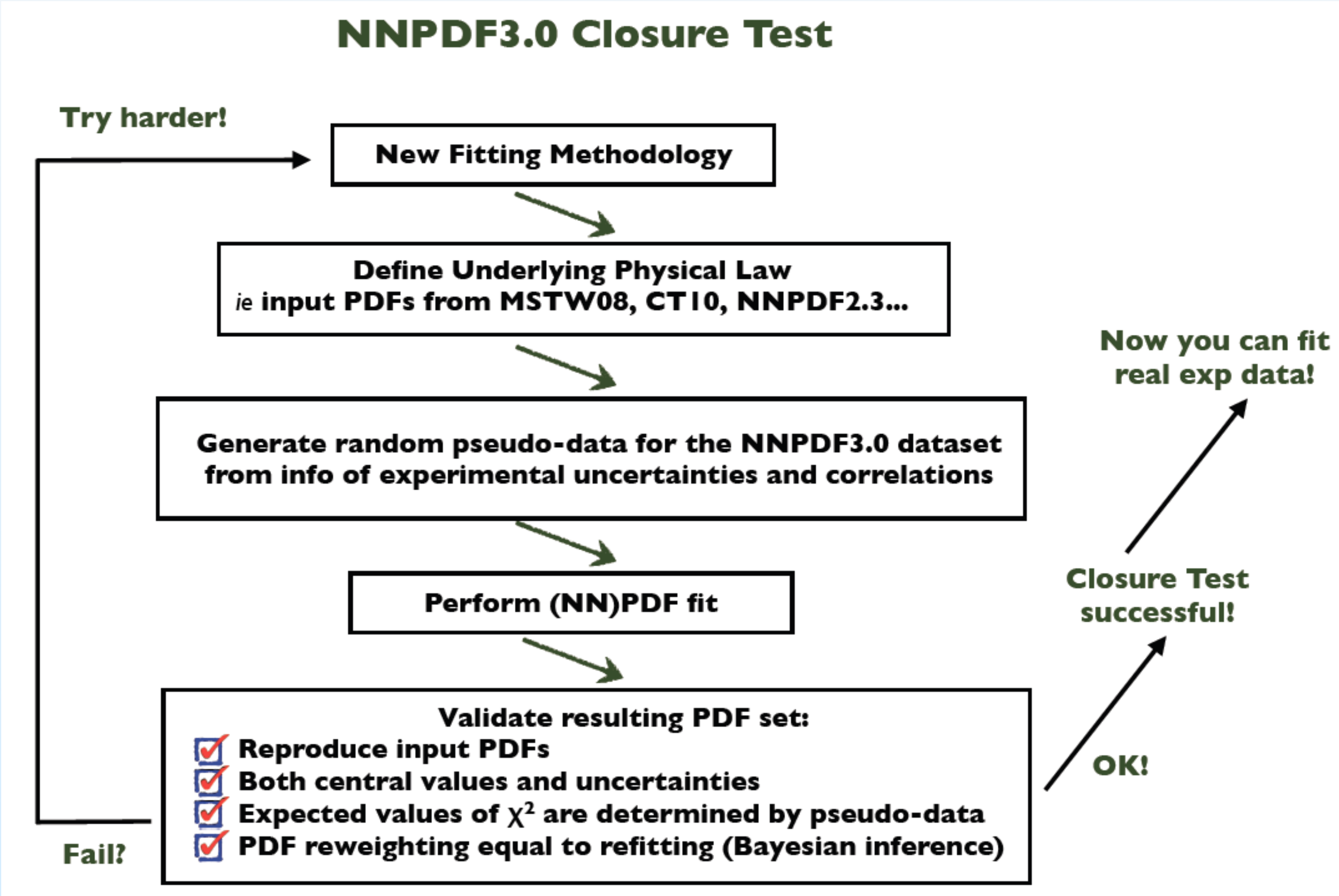
Pseudo-data MC replica

Original data

$$\Delta F = \sqrt{\frac{N_{\rm rep}}{N_{\rm rep}-1}\left(\langle F^2\rangle - \langle F\rangle^2\right)}.$$

Closure Testing of Parton Distributions

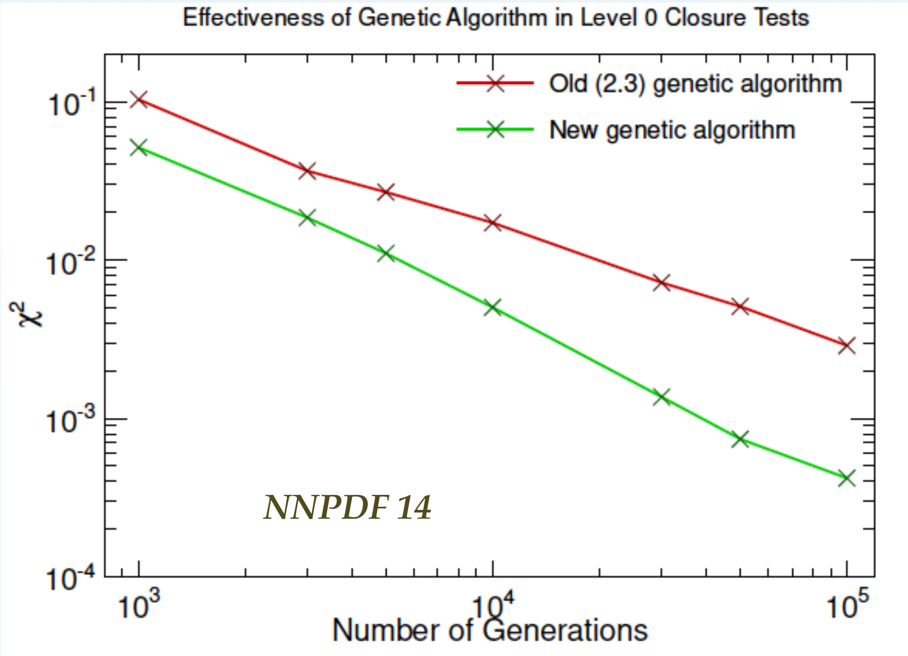
- **PDF uncertainties** have been often criticised by a **potential lack of statistical interpretation**
- NNPDF performed a systematic **closure tests analysis** based on pseudo-data, and verified that **PDF uncertainties** exhibit **a statistically robust behaviour**

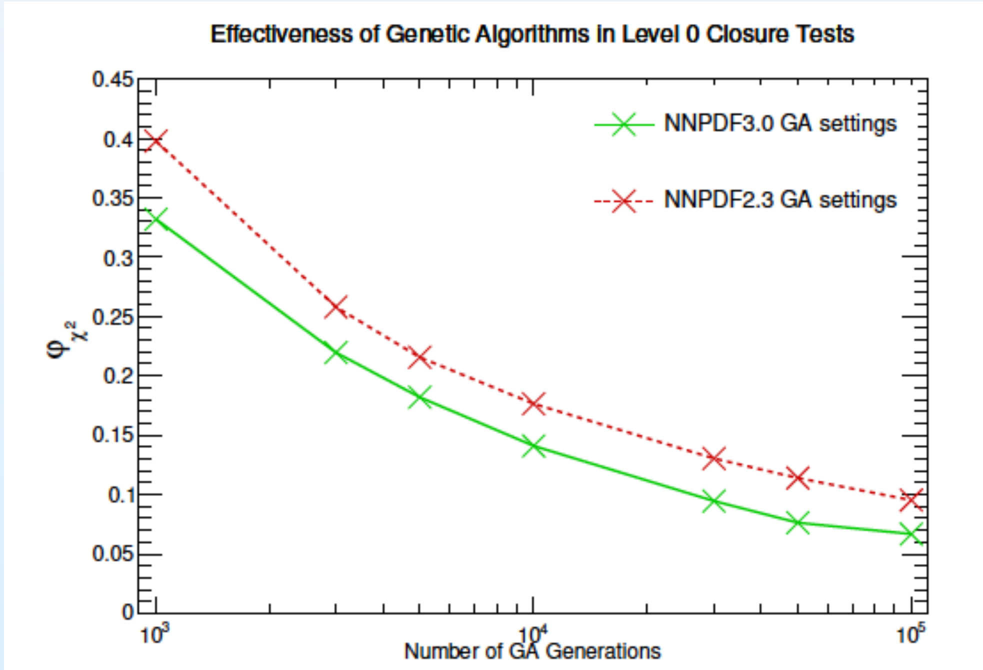


Closure Testing of Parton Distributions

- For instance, if the **pseudo-data is generated without statistical fluctuations** (that is, identical to the input theory) then the agreement with theory by construction should become **arbitrarily good**

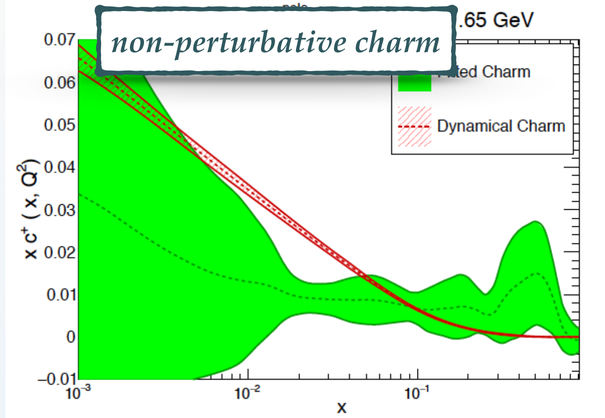
- And indeed it does: as the minimization advances, the χ^2 **decreases monotonically,** and the PDF uncertainties as well are reduced, as the **fitted theory collapses to the underlying law**



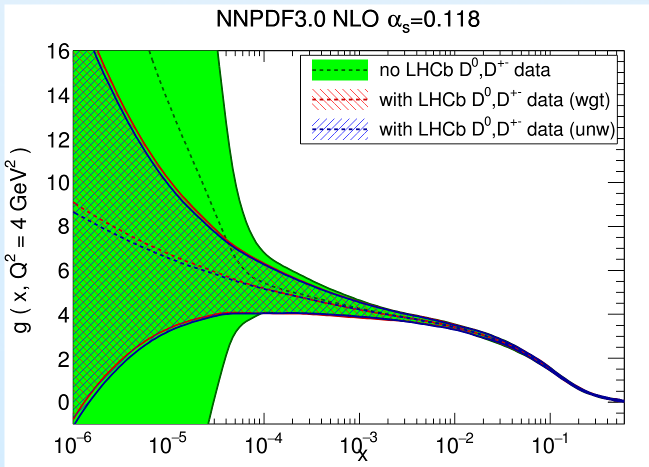


$$\varphi_{\chi^2} \equiv \sqrt{\langle \chi^2[\mathcal{T}[f_{\rm fit}], \mathcal{D}_0] \rangle - \chi^2[\langle \mathcal{T}[f_{\rm fit}] \rangle, \mathcal{D}_0]}\,.$$

Measure of **PDF uncertainties** in units of **data uncertainties**



Recent Results with Unpolarized NNPDFs



The NNPDF time-line

2016 NNPDF3IC: the charm content of the nucleon

2014 NNPDF3.0: the C++ generation, closure test validation

2013 NNPDF2.3QED: QCD+QED fits and photon PDF

2012 NNPDF2.3: PDFs with LHC data

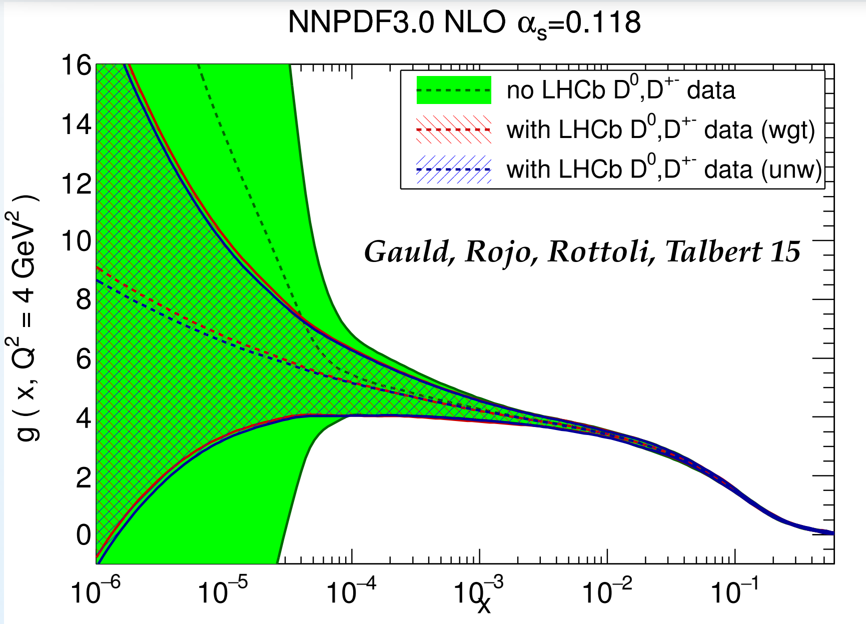
2011 NNPDF2.1: LO and NNLO, heavy quark mass effects

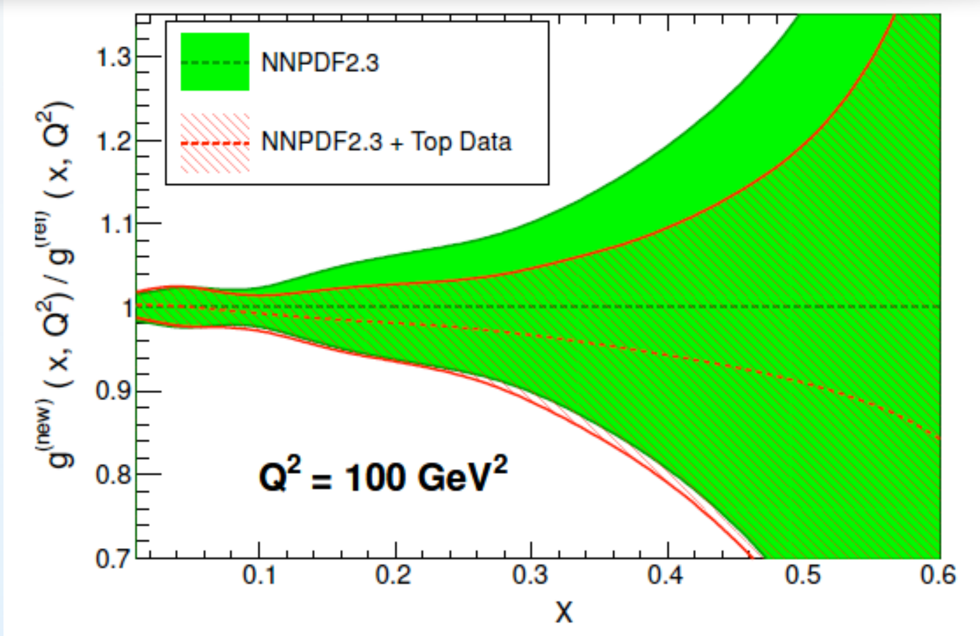
2010 NNPDF2.0: global fit with hadronic data
used in Higgs discovery papers

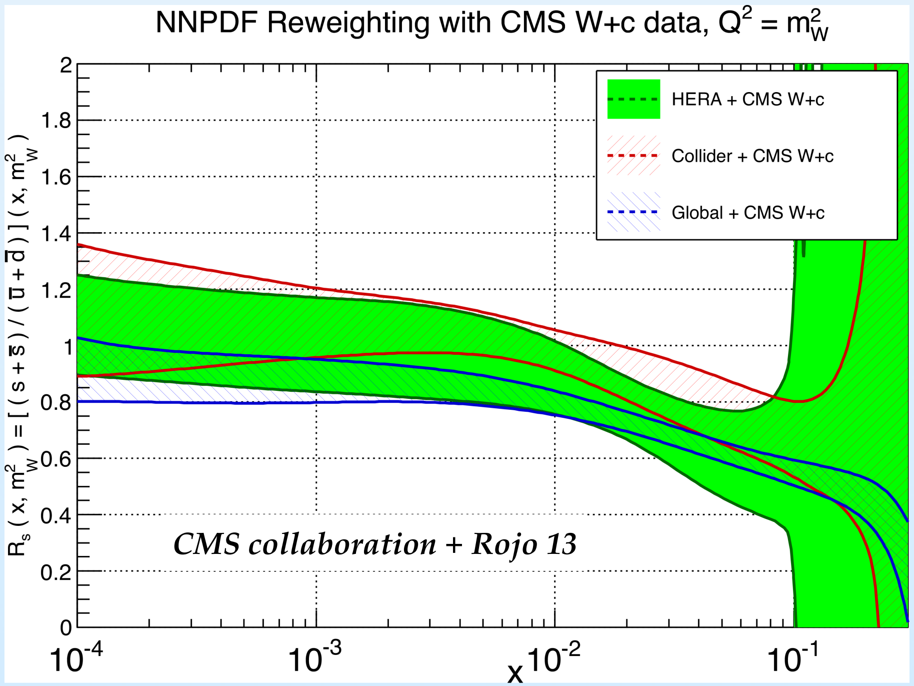
2008 NNPDF1.0: DIS-only PDFs

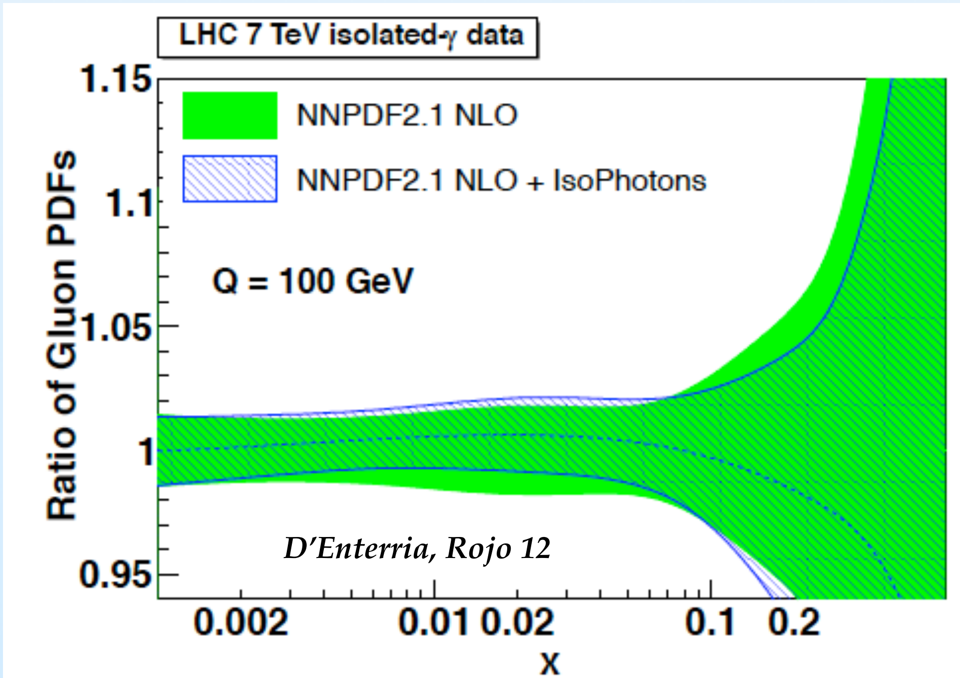
PDF constrains from LHC data

Exploitation of PDF-sensitive information from LHC data**: essential component of **global PDF fit program









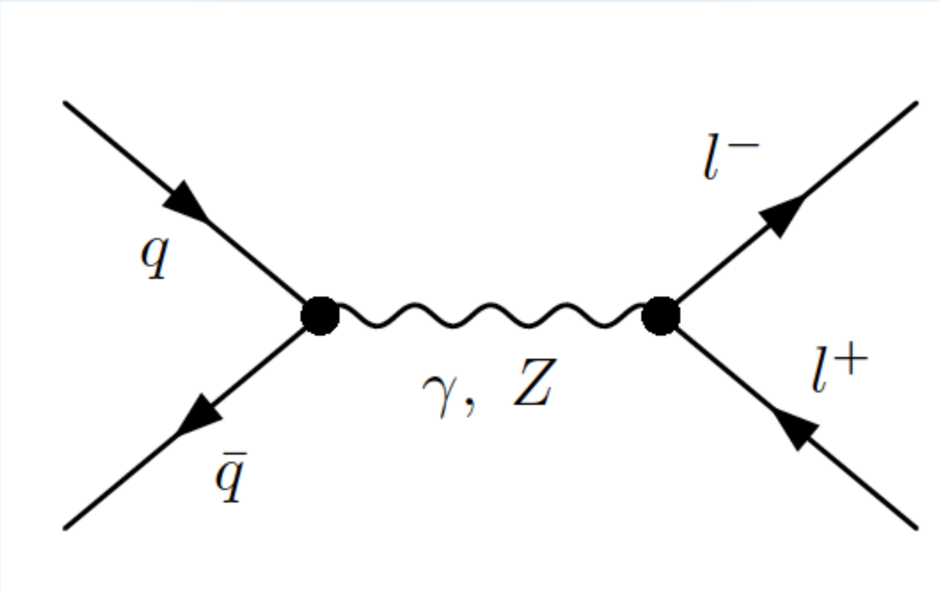
Parton Distributions with QED corrections

At the LHC, **electroweak corrections** can become comparable or larger than **QCD effects**

Precision physics thus requires **PDFs with QCD+QED evolution**, and a determination of the **photon PDF**

Drell-Yan process: high-mass lepton pair production

QCD-only, leading order



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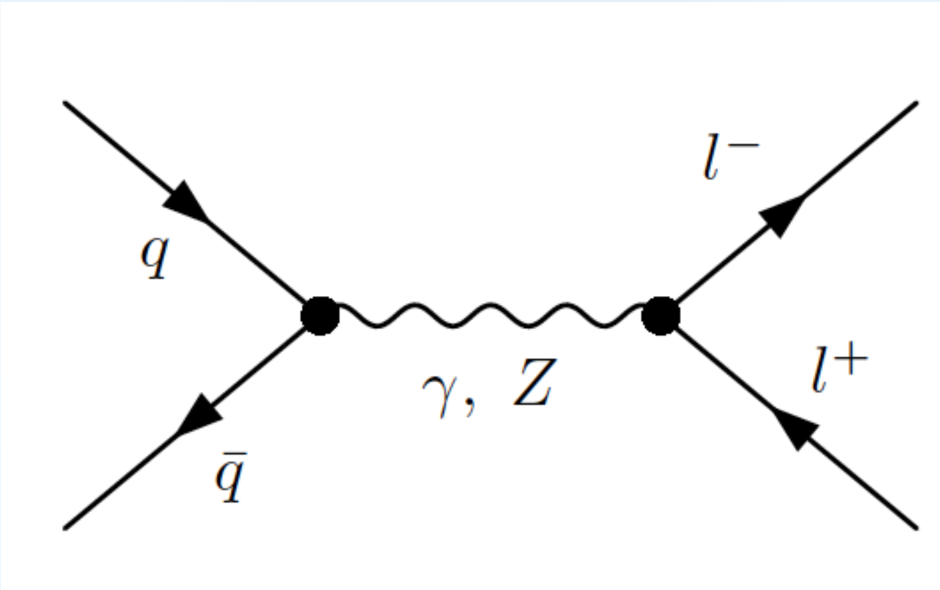
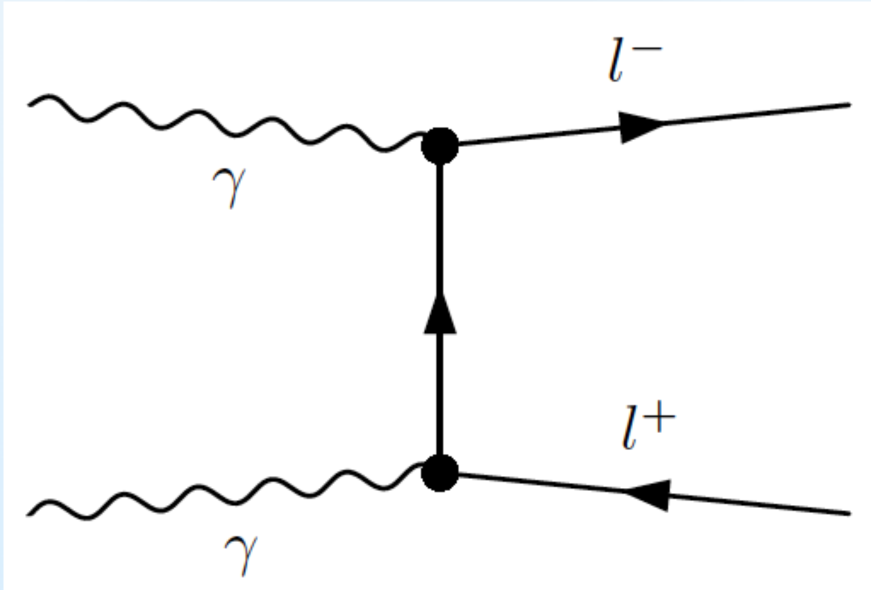


QCD+QED, leading order



For many processes, accurate determination of the **photon PDF** as important as that of **quark and gluon PDFs**

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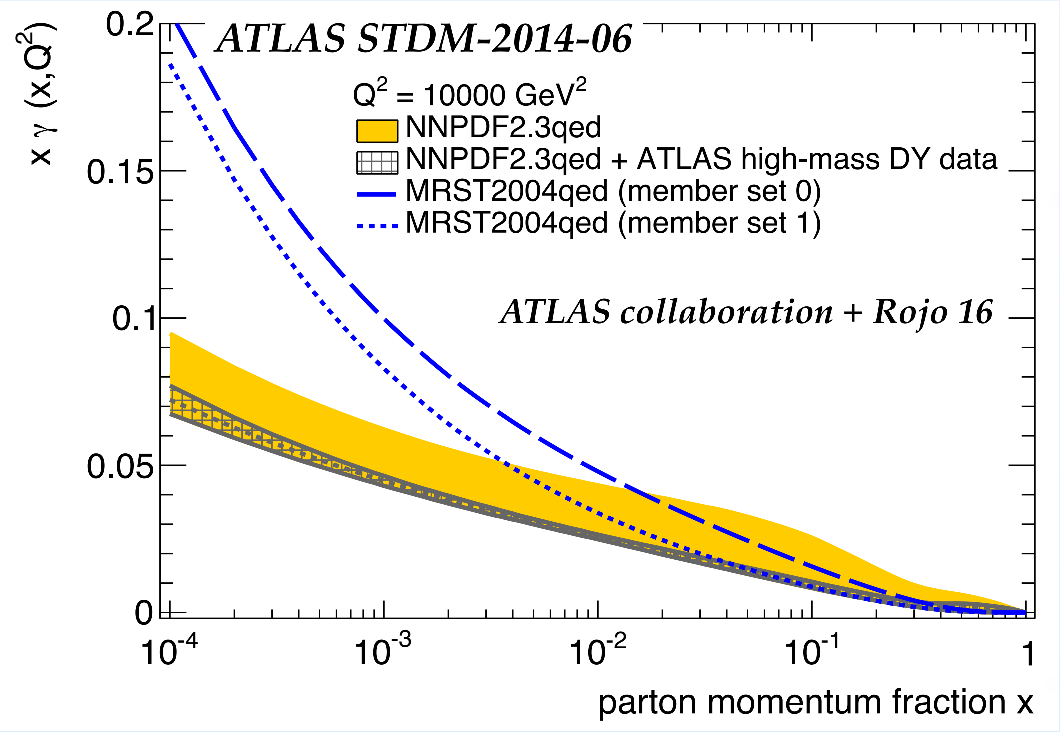
Precision physics thus requires **PDFs with QCD+QED evolution**, and a determination of the **photon PDF**

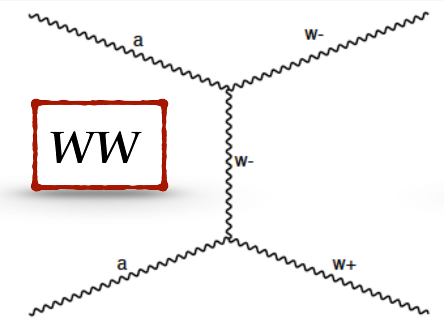
NNPDF2.3QED: *first (and only) QED PDF fit with* ***model-independent photon PDF determination ….***

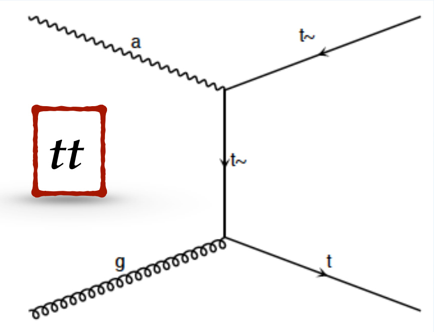
Ball, Bertone, Carrazza, Forte, Guffanti, Hartland, Rojo 13

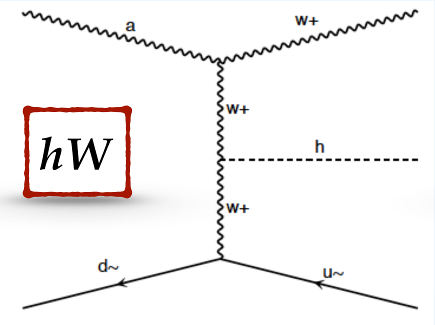


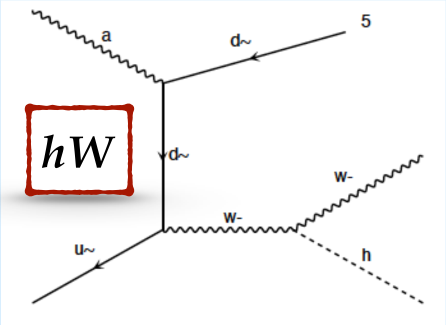
…. now being improved with high-statistics ***ATLAS high-mass Drell-Yan measurements***

Photon-induced processes *ubiquitous at LHC*



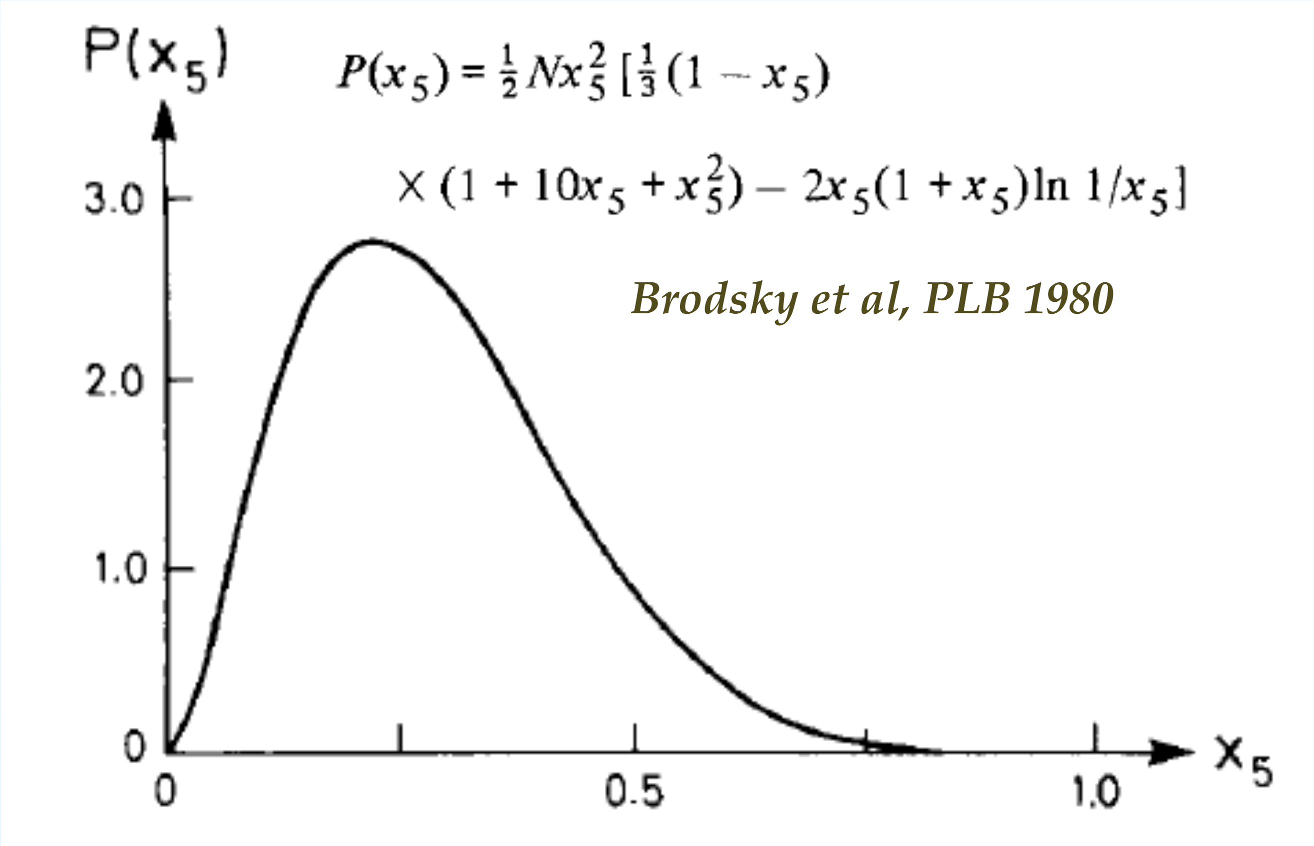






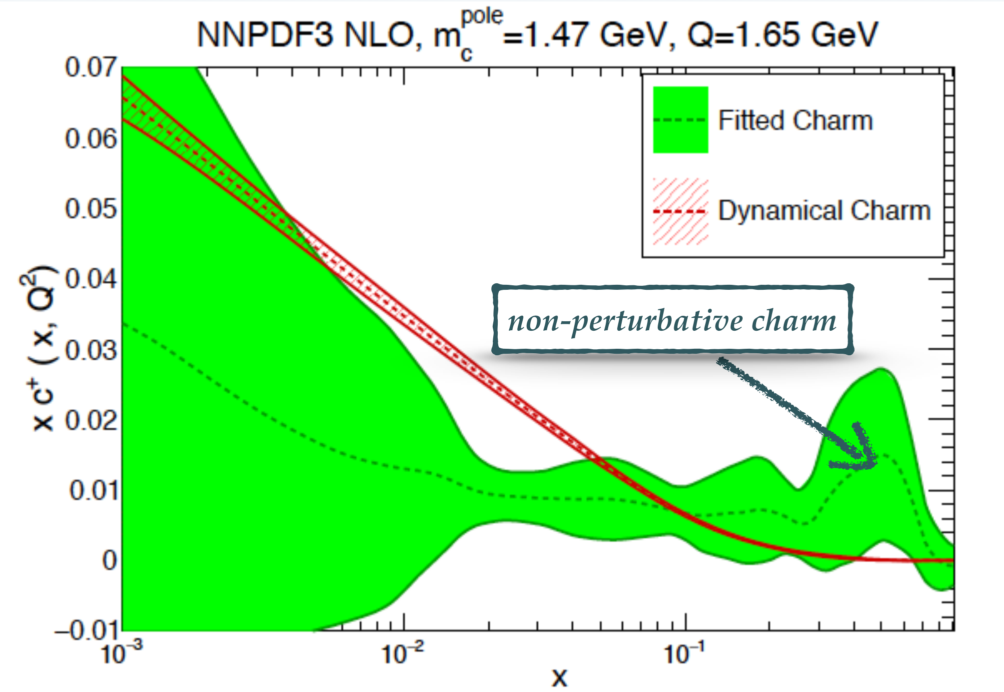
The charm content of the proton

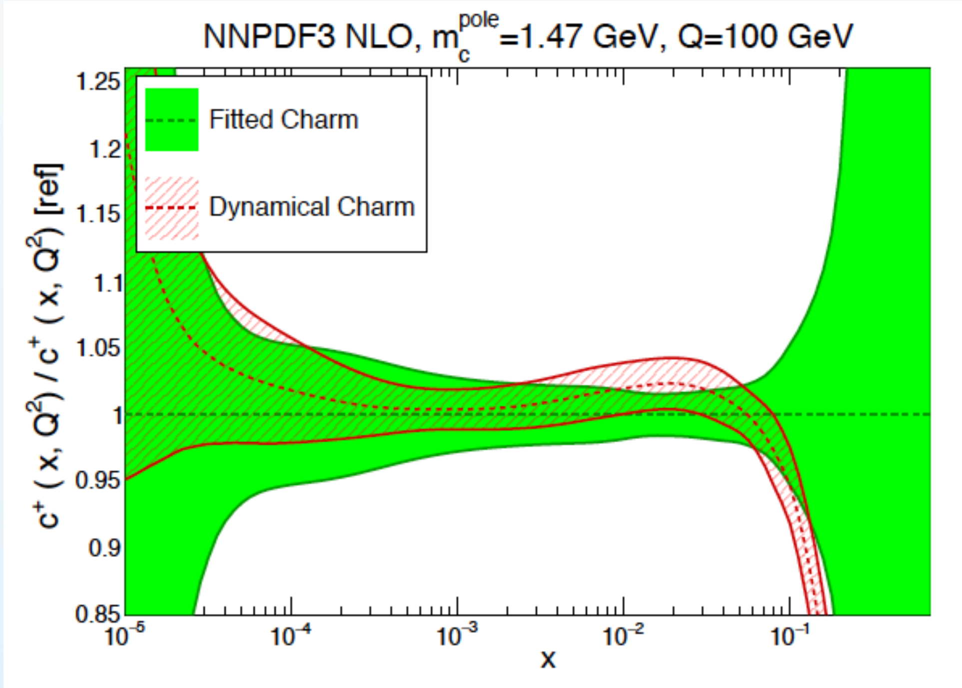
- The motivation **to fit a charm PDF** in a global analysis is **two-fold:**
  - ☑ Stabilise the dependence of **LHC calculations** with respect to **value of the charm mass**
  - ☑ Quantify the **non-perturbative charm component in the proton** and compare with models



A 30-years old conundrum of QCD!

Global QCD analysis with Intrinsic Charm





Ball, Bertone, Bonvini, Carrazza, Forte, Guffanti, Hartland, Rojo and Rottoli 16

At low scales, evidence for a **non-perturbative charm component**, but PDF errors still large

At LHC scales, **fitted and dynamical charm in good agreement** for $x < 0.01$

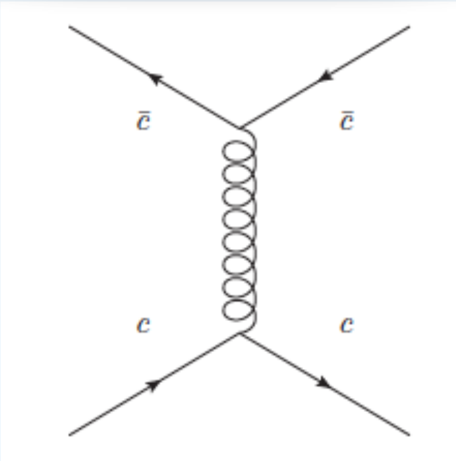
Charm can account up to **1% of the total proton momentum** at low scales

PDF set	Q	*Charm momentum fraction*
NNPDF3 dynamical charm	1.65 GeV	$(0.239 \pm 0.003)\%$
NNPDF3 fitted charm		$(0.7 \pm 0.3)\%$
NNPDF3 fitted charm (no EMC)		$(1.6 \pm 1.2)\%$

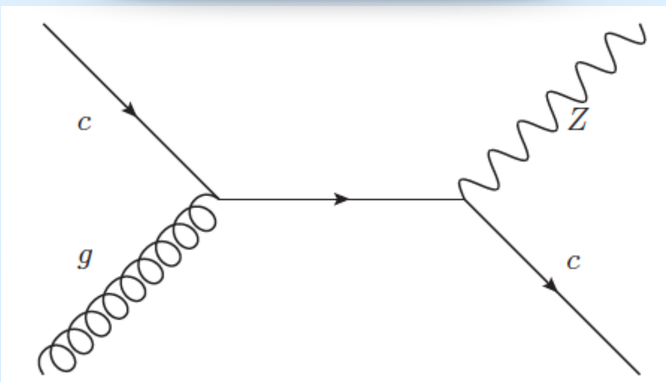
The charm PDF: implications for the LHC

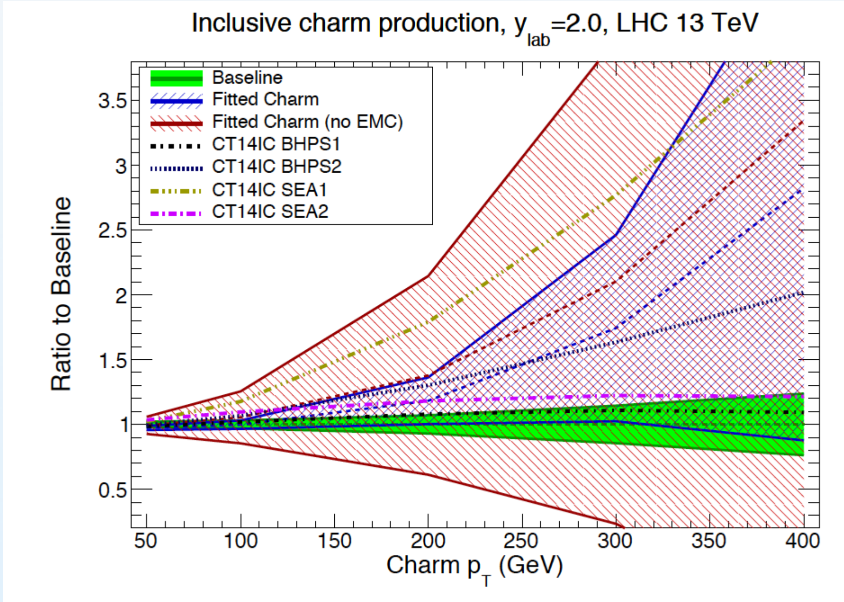
- A number of LHC processes are sensitive to the **charm content of the proton**
- Typically to probe **large-x charm** we need either **large pT** or **forward rapidities** production
- Within the **reach of the LHC** at Run II

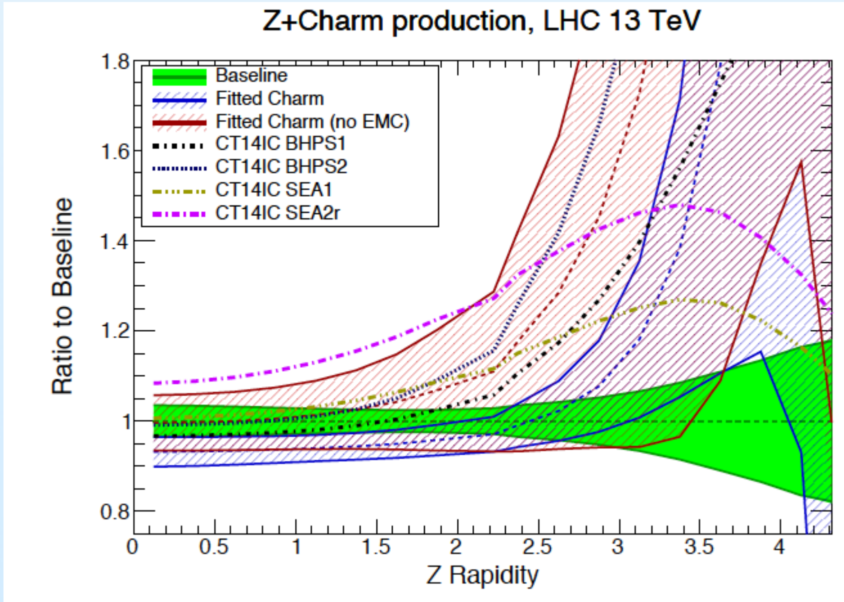
D meson production



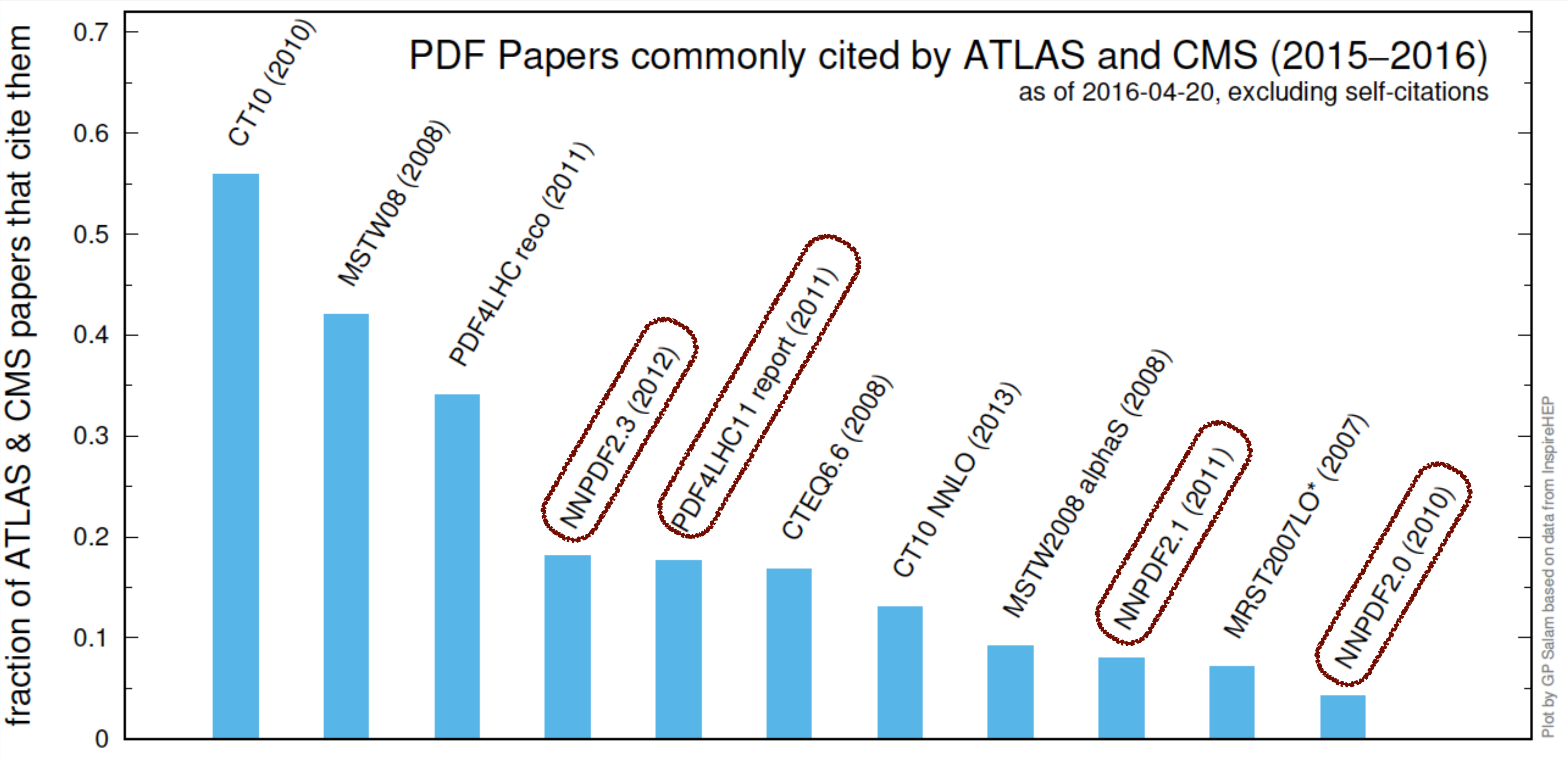
Z+charm production



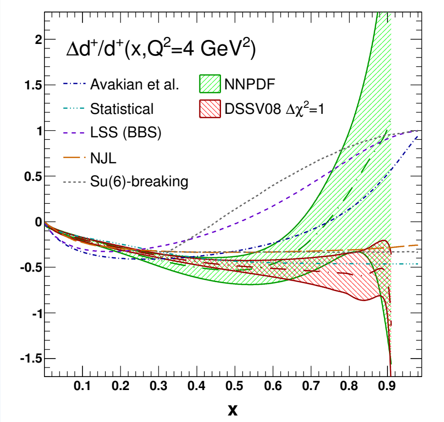




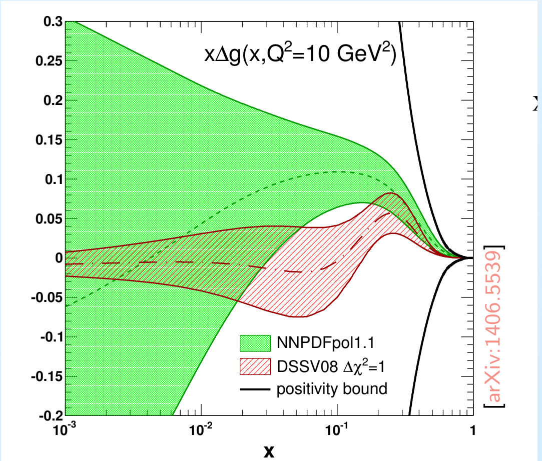
PDFs usage in ATLAS and CMS



Parton Distributions are an essential component of many LHC analyses



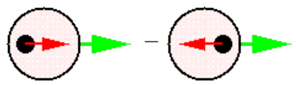
Recent Results with Polarized NNPDFs



NNPDF, arXiv:1303.7236
NNPDF, arXiv:1406.5539

Polarised Parton Distributions

- **Polarised PDFs** provide a unique windows on the **spin structure of the proton.**
- How the **proton spin is distributed among its constituents** is a crucial issue for our understanding of **non-perturbative QCD and confinement**



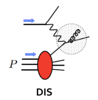
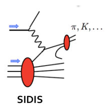
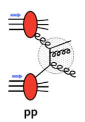
$$\Delta f(x, Q^2) = f^{\Rightarrow\rightarrow}(x, Q^2) - f^{\Rightarrow\leftarrow}(x, Q^2)$$

How do quarks (including sea quarks) and gluons carry the proton spin

$$\mathcal{S}(\mu) = \frac{1}{2} = \sum_f \left\langle P; S | \hat{J}_f^z(\mu) | P; S \right\rangle = \frac{1}{2}\int_0^1 dx \Delta\Sigma(x,\mu) + \int_0^1 dx \Delta g(x,\mu) + L_z$$

Quarks? **Gluons?** **Angular Mom?**

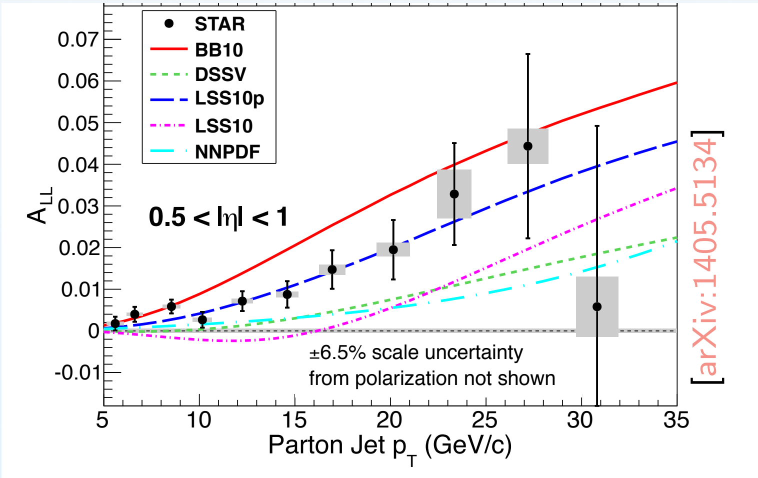
e.g. DIS $\quad d\Delta\sigma = \sum_{q,\bar{q},g} \Delta f(x, Q^2) \otimes d\Delta\hat{\sigma}_{\gamma^* f}(xP, \alpha_s(Q^2)) \quad d\Delta\hat{\sigma}_{\gamma^* f} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n d\Delta\hat{\sigma}^{(n)}_{\gamma^* f}$

	Reaction	Partonic subprocess	PDF probed	x	Q^2 [GeV^2]
	$\ell^\pm\{p,d,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$\Delta q + \Delta\bar{q}$ Δg	$0.003 \lesssim x \lesssim 0.8$	$1 \lesssim Q^2 \lesssim 70$
	$\ell^\pm\{p,d\} \to \ell^\pm hX$	$\gamma^* q \to q$	Δu $\Delta\bar{u}$ Δd $\Delta\bar{d}$ Δg	$0.005 \lesssim x \lesssim 0.5$	$1 \lesssim Q^2 \lesssim 60$
	$\ell^\pm\{p,d\} \to \ell^\pm DX$	$\gamma^* g \to c\bar{c}$	Δg	$0.06 \lesssim x \lesssim 0.2$	~ 10
	$\vec{p}\vec{p} \to jet(s)X$	$gg \to qg$ $qg \to qg$	Δg	$0.05 \lesssim x \lesssim 0.2$	$30 \lesssim p_T^2 \lesssim 800$
	$\vec{p}p \to W^\pm X$	$u_L\bar{d}_R \to W^+$ $d_L\bar{u}_R \to W^-$	Δu $\Delta\bar{u}$ Δd $\Delta\bar{d}$	$0.05 \lesssim x \lesssim 0.4$	$\sim M_W^2$
	$\vec{p}\vec{p} \to \pi X$	$gg \to qg$ $qg \to qg$	Δg	$0.05 \lesssim x \lesssim 0.4$	$1 \lesssim p_T^2 \lesssim 200$

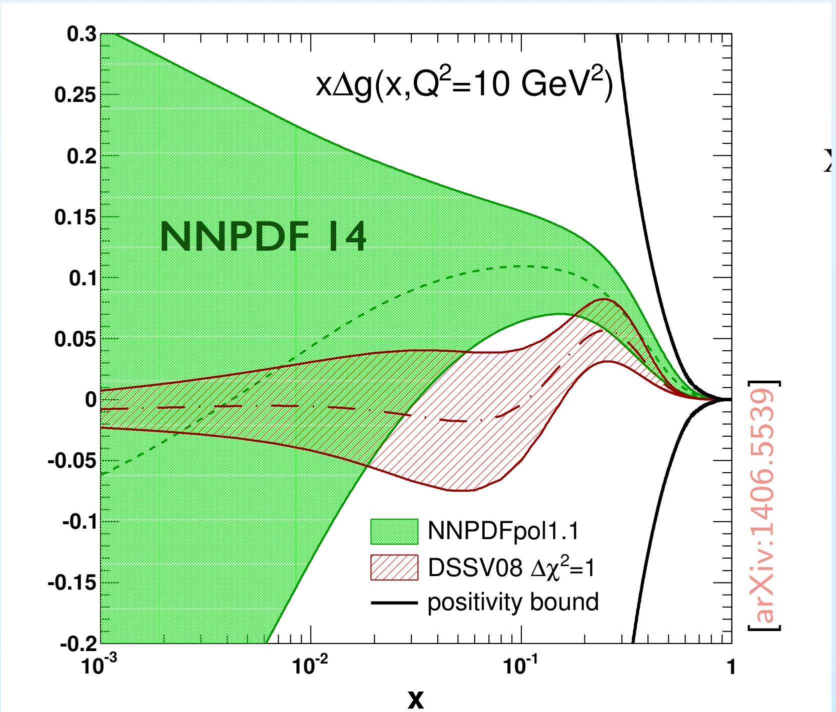
- First measurements with polarised DIS (80s) showed that **quark contribution much smaller than expected** (*proton spin crisis*)
- With the availability of polarised hadronic and semi-inclusive data, **global polarised PDF fits possible**
- The NNPDF framework has also been applied to the polarized case, with **NNPDFpol1.1** is the most updated set

Unraveling the gluon polarisation

- Contribution of **gluon polarisation to the proton spin** has been of the **big unknowns** in the last 30 years
- The analysis of **RHIC polarised jet data** in the **NNPDFpol1.1** and **DSSV** frameworks provides **first ever evidence for positive (non-zero) polarisation** of the gluon in the proton
- Importance of this important result recognised also in **media outlets**



$$A_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$



Proton Spin Mystery Gains a New Clue

Physicists long assumed a proton's spin came from its three constituent quarks. New measurements suggest particles called gluons make a significant contribution

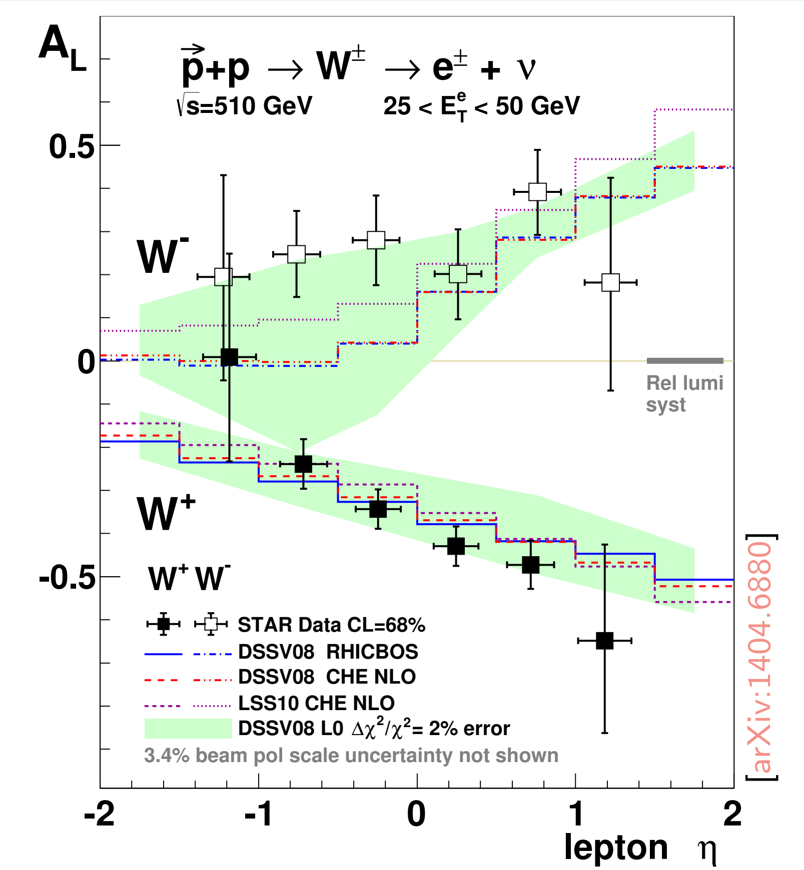
July 21, 2014 | By Clara Moskowitz

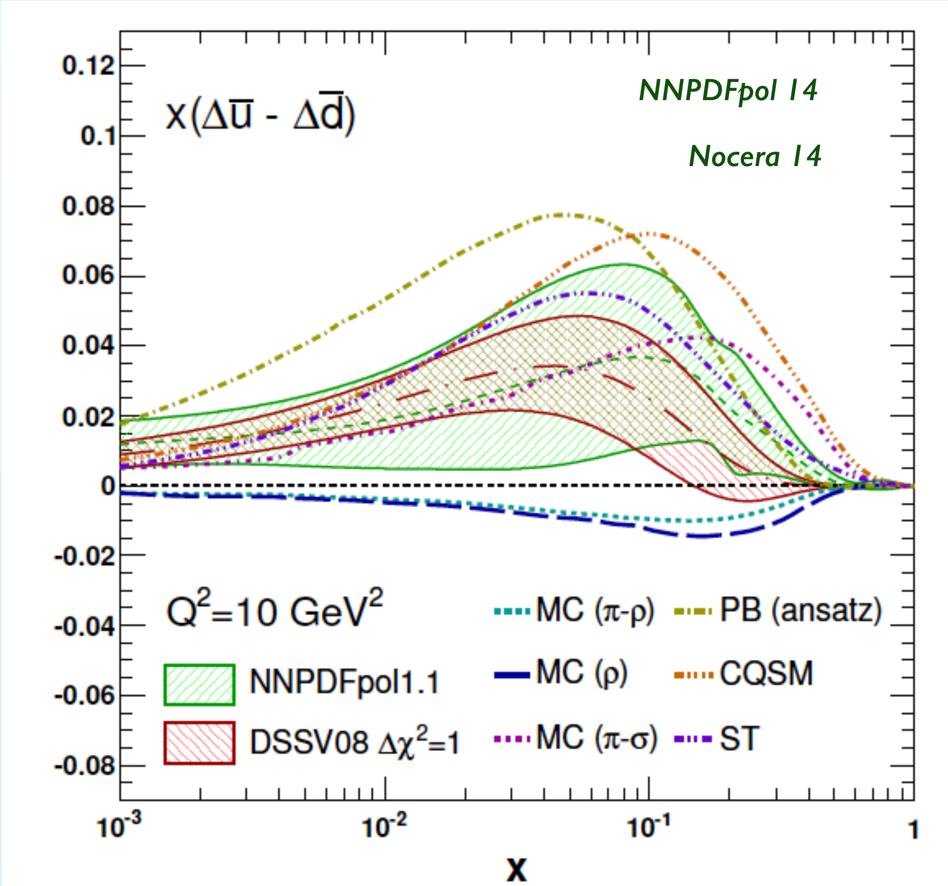
Protons have a constant spin that is an intrinsic particle property like mass or charge. Yet where this spin comes from is such a mystery it's dubbed the "proton spin crisis." Initially physicists thought a proton's spin was the sum of the spins of its three constituent quarks. But a 1987 experiment showed that quarks can account

Total contribution of gluons to proton spin still unknown since **large uncertainties at small-x** from lack of data: need an **Electron-Ion Collider**

The polarised quark sea

- **Inclusive DIS data** does not allow to separate polarised quarks from antiquarks
- Recent data on polarised **semi-inclusive DIS** and **hadronic *W* production** allow this separation
- Stringent constraints on **non-perturbative models of the polarized proton**

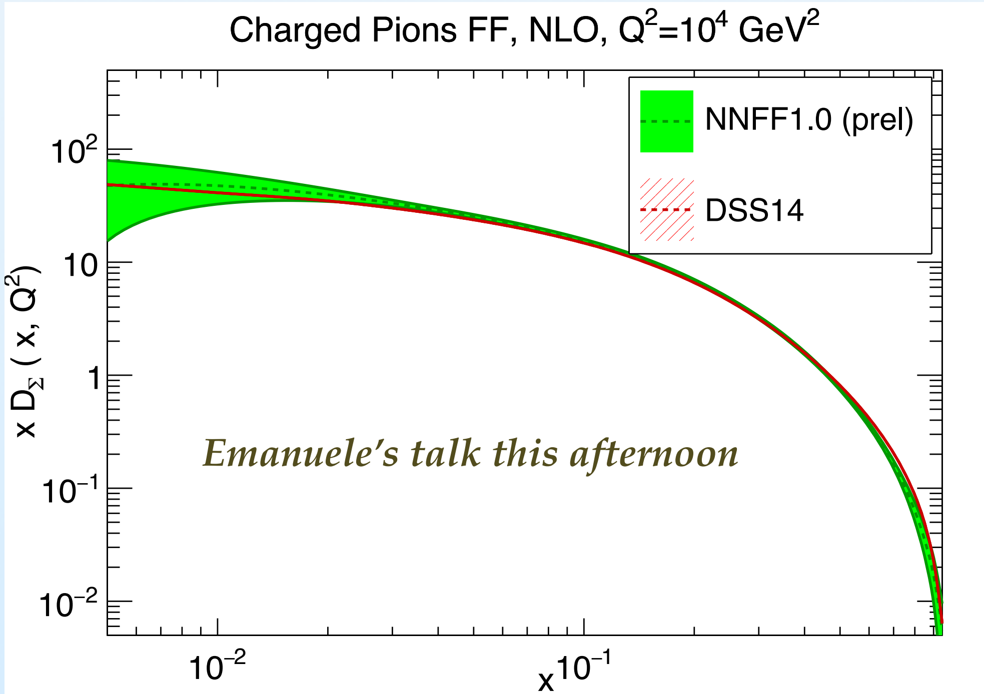


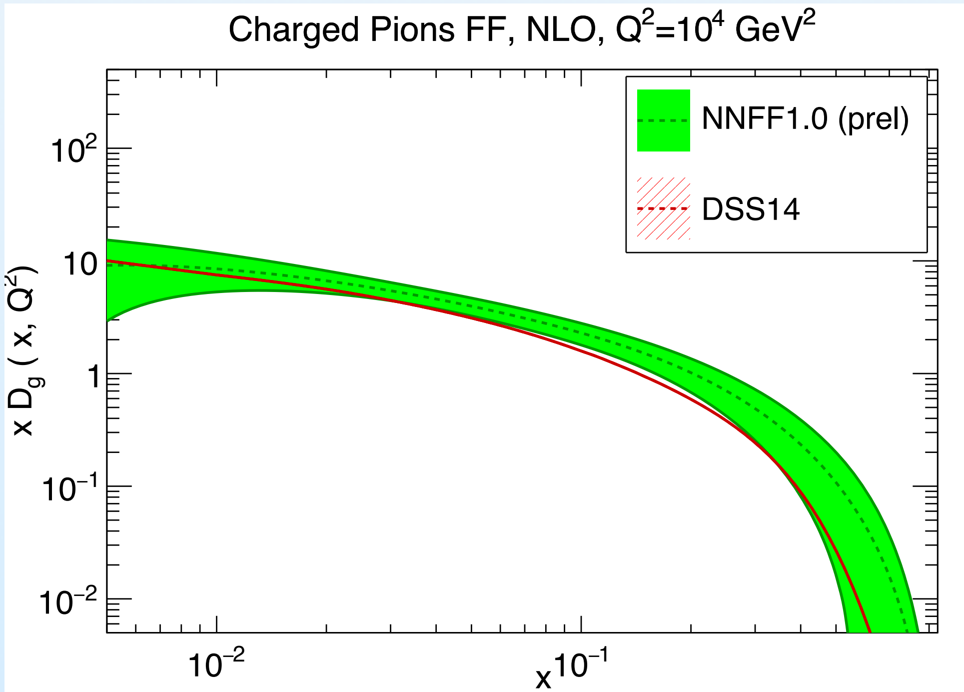


The Way Ahead

Neural Network Fragmentation Functions

- **Fragmentation functions (FFs)** parametrize the non-perturbative dynamics responsible for the **hadronization process** (*Rodolfo's talk*)
- As opposed to MC hadronization models, FFs can be used to compute hadron production to much **higher formal accuracy**: NNLO in e+e-, NLO in DIS and pp
- Crucial for our understanding of **non-perturbative QCD**, to obtain information on the nucleon structure from **semi-inclusive processes**, and for **LHC phenomenology**, *i.e.* inclusive hadron production as probe of the quark-gluon plasma
- Now working in NNFF1.0, the **first set of FFs using the NNPDF methodology**

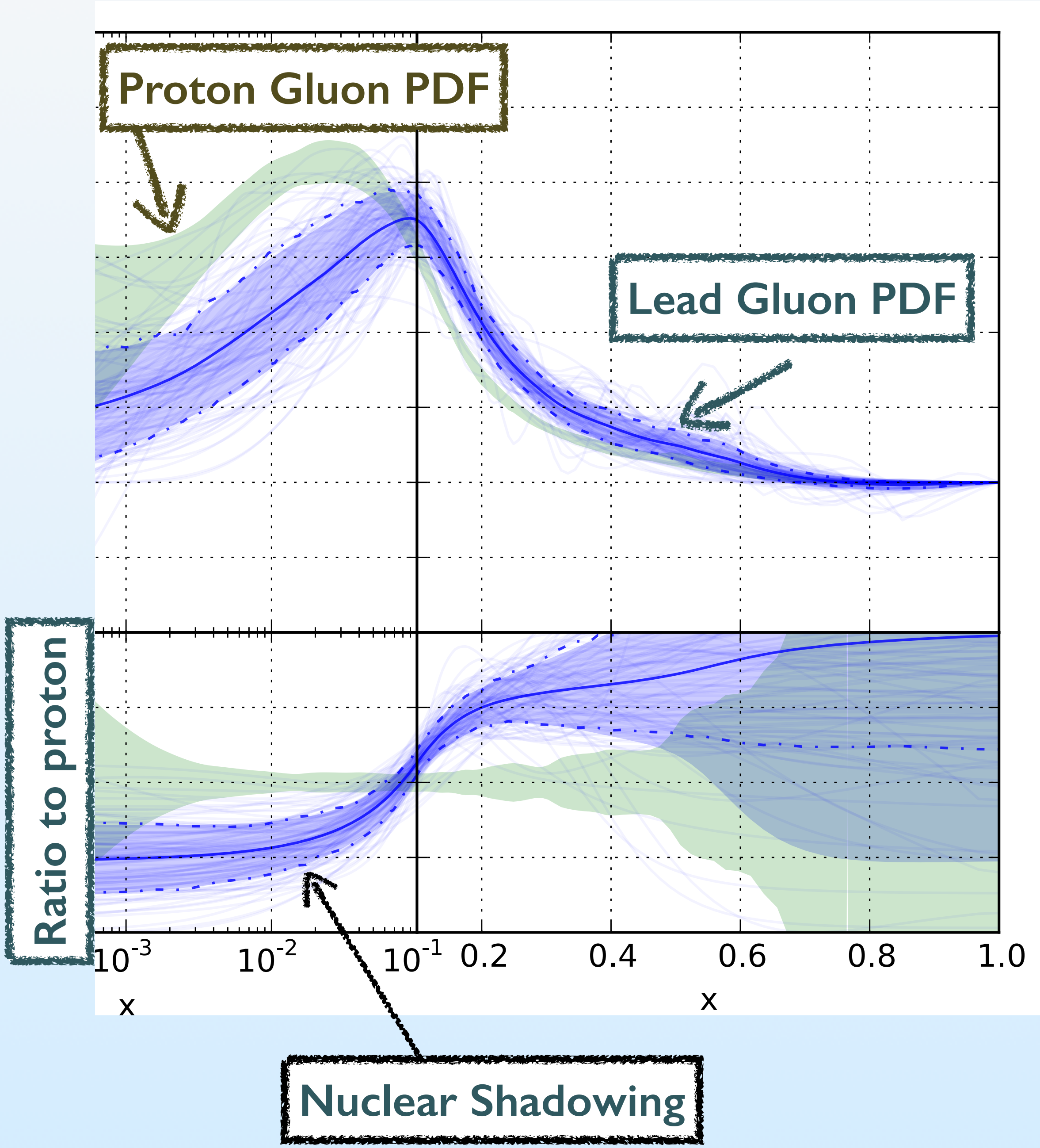




NNFF1.0: Bertone, Carrazza, Nocera and JR, in preparation

Neural Network Nuclear PDFs

- The **PDFs of nucleons in bound nuclei** are modified as compared to the free nucleon PDFs
- Nuclear PDFs parametrize a rich variety of **nuclear effects**: shadowing, the EMC effect, Fermi motion ….
- High-precision nuclear PDFs are a crucial ingredient of **the pPb and PbPb heavy ion program at the LHC**, providing the cold-nuclear matter benchmark for quark-gluon plasma characterisation
- Now working in N3PDF1.0, the **first set of nuclear PDFs using the NNPDF methodology**



The NNPDF Roadmap

Unpolarized
present: NNPDF3.0
future: NNPDF3.1/4.0
LHC Run II data, theory errors,
NNLO QCD+NLO EW ....

Nucleon Structure

The NNPDF Roadmap

Unpolarized
present: NNPDF3.0
future: NNPDF3.1/4.0
LHC Run II data, theory errors,
NNLO QCD+NLO EW ….

Polarized
present: NNPDFpol1.2
future: NNPDFpol2.0 (?)
Semi-inclusive data using NNFFs

Nucleon Structure

The NNPDF Roadmap

Unpolarized
present: NNPDF3.0
future: NNPDF3.1/4.0
LHC Run II data, theory errors,
NNLO QCD+NLO EW ….

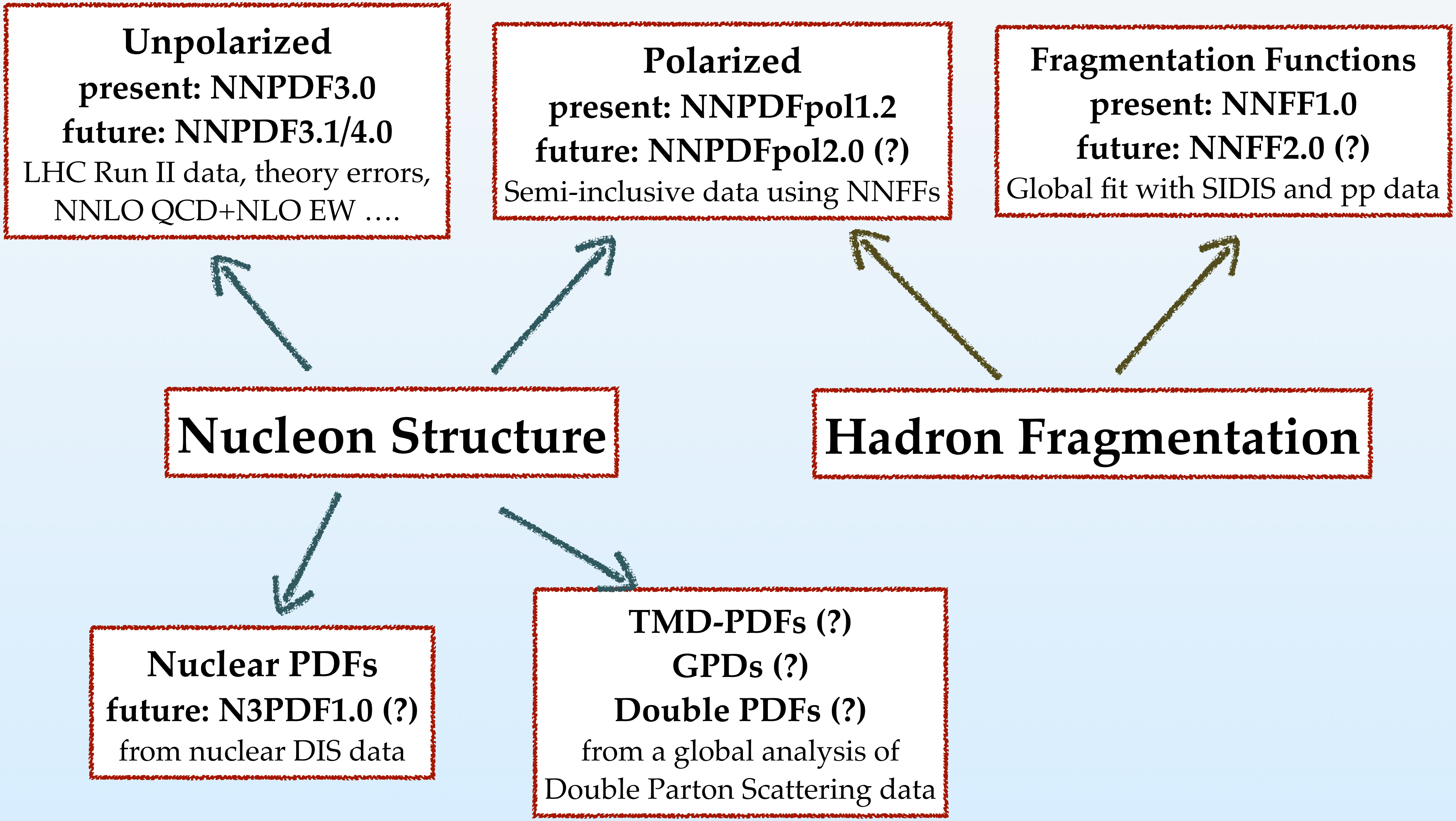
Polarized
present: NNPDFpol1.2
future: NNPDFpol2.0 (?)
Semi-inclusive data using NNFFs

Nucleon Structure

Nuclear PDFs
future: N3PDF1.0 (?)
from nuclear DIS data

TMD-PDFs (?)
GPDs (?)
Double PDFs (?)
from a global analysis of
Double Parton Scattering data

The NNPDF Roadmap



The NNPDF Roadmap

Unpolarized
present: NNPDF3.0
future: NNPDF3.1/4.0
LHC Run II data, theory errors,
NNLO QCD+NLO EW ….

Polarized
present: NNPDFpol1.2
future: NNPDFpol2.0 (?)
Semi-inclusive data using NNFFs

Fragmentation Functions
present: NNFF1.0
future: NNFF2.0 (?)
Global fit with SIDIS and pp data

Nucleon Structure

Fragmentation

Thanks for your attention!

Nucle DFs
future: N3PDF1.0 (?)
from nuclear DIS data

TMD-PDFs (?)
GPDs (?)
Double PDFs (?)
from a global analysis of
Double Parton Scattering data