Fragmentation Functions and Global QCD Fits

The 32nd Annual Hampton University Graduate Studies Program (HUGS2017)

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Table of contents

1. Lecture 1: Theoretical framework
    - foreword: why Fragmentation Functions?
    - the basics: recap on factorisation and evolution
    - single inclusive annihilation as a case study
    - single hadron production in semi-inclusive DIS and proton-proton collisions
2. Lecture 2: Methodological aspects of global QCD fits
    - determining the probability density in a space of functions
    - goodness of fit
    - parton parametrisation, uncertainty representation, parameter optimisation
    - fit validation
3. Lecture 3: Phenomenology of Fragmentation Functions
    - overview of recent determinations of Fragmentation Functions
    - higher-order corrections, heavy quark mass effects
    - applications of Fragmentation Functions

DISCLAIMER

These lectures contain a personal selection of topics and are certainly not exhaustive

Bibliography

1. General textbooks on perturbative QCD
    - R. K. Ellis, W. J. Stirling and B. R. Webber, *QCD and Collider Physics*, Cambridge (1996)
    - J. C. Collins, *Foundations of perturbative QCD*, Cambridge (2011)
2. Reviews on Fragmentation Functions
    - A. Metz and A. Vossen, Prog. Part. Nucl. Phys. 91 (2016) 136
    - S. Albino, Rev. Mod. Phys. 82 (2010) 2489
    - S. Albino *et al.* [arXiv:0804:2021]
3. Reviews on Parton Distribution Functions
    - S. Forte and G. Watt, Ann. Rev. Nucl. Part. Sci. 63 (2013) 291
    - P. Jimenez-Delgado, W. Melnitchouk and J. F. Owens, J. Phys. G40 (2013) 093102
    - S. Forte, Acta Phys. Polon. B41 (2010) 2859
4. Specific topics not addressed above
    - more journal references along the way as we proceed

DISCLAIMER

These lectures will focus on collinear leading-twist Fragmentation Functions
for single-inclusive unpolarised hadron production
Transverse-momentum-dependent/multi-hadron fragmentation not covered here

Fragmentation Functions and Global QCD Fits

Lecture 1: Theoretical framework

Outline

1. Foreword: why Fragmentation Functions?
   from partons in the initial state to partons in the final state
2. The basics
   definition of Fragmentation Functions, factorisation, evolution
   properties of splitting functions, theoretical constraints
3. SIA as a case study
   SIA cross sections/multiplicities: theoretical expectations and data sets
   flavours schemes: changing the number of active flavours
   higher-order QCD corrections
4. Other processes
   hadron production in SIDIS: multiplicities and data sets
   hadron production in pp collisions: cross section and data sets

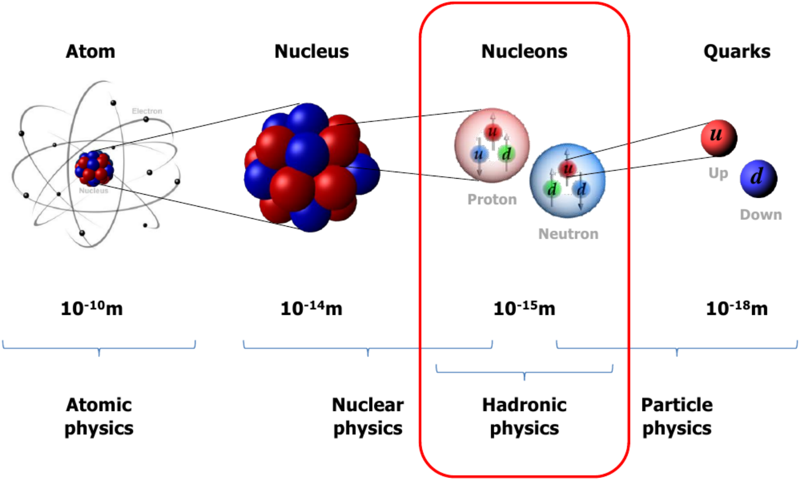
Focus the attention on single-inclusive annihilation
as a paradigm for the illustration of features/issues of fragmentation
in the cleanest theoretical framework

Try to minimise overlap with J. W. Qiu's lectures

1.1 Foreword: why Fragmentation Functions?

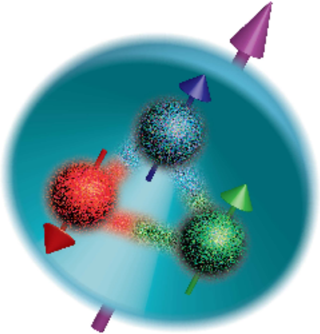
Hadron physics, or the quest for the nucleon structure

Nucleons make up all nuclei, and hence most of the visible matter in the Universe
They are bound states with internal structure and dynamics

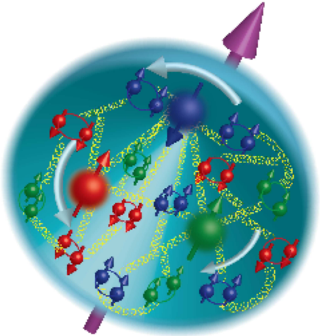


The QCD picture of the nucleon

naive picture



realistic picture



three non-relativistic quarks $\xleftrightarrow[\text{factorization,evolution}]{\text{QCD}}$ indefinite number of relativistic quarks and gluons

QCD, a powerful theoretical framework

1. What are the general features of QCD?
   KEYWORDS: renormalisation; asymptotic freedom; infrared safety; factorisation
   $\longrightarrow$ addressed in Jianwei Qiu's lecture on the first week
2. How reliable is a theoretical QCD calculation?
   KEYWORDS: scale dependence; higher-order corrections; resummation(s)
   $\longrightarrow$ addressed in Jianwei Qiu's lecture on the first week
3. How can we relate QCD to experiment?
   KEYWORDS: factorisation; evolution; distribution functions; global analysis
   $\longrightarrow$ addressed in these lectures (with a focus on Fragmentation Functions)
4. QCD has proven to be successful in the last 35+ years

GOALS

Qualitative QCD: discovery physics
Set up and check the framework

Quantitative QCD: precision physics
Understand and describe a large class of hard-scattering processes and phenomena

Precision QCD: new physics
Investigate possible deviations from the Standard Model

Lepton-hadron facilities

SLAC

CERN

DESY

JLab

Hadron-hadron facilities

Fermilab TeVatron [1987 → 2011]

$p\bar{p}$ collisions 0.63, 1.8, 1.96 TeV

top discovery, jet physics, ...
further established QCD

CERN $Sp\bar{p}S$ [1981 → 1990]

$p\bar{p}$ collisions 540, 630 GeV

W,Z discovery, jets, ...
early successes of QCD

CERN LHC [operating]

pp collisions 7, 14 TeV

a QCD machine, discoveries ?
also PbPb and pPb program

BNL RHIC [2000 → ...]

pp collisions up to 500 GeV

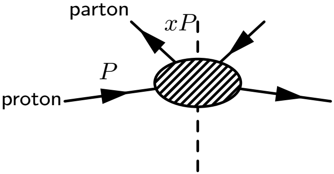
the World's first and only **polarized collider**
spin dep. phenomena, spin strct. of the nucleon
also versatile heavy ion program

Nucleons in the initial state: Parton Distribution Functions

1. The densites of partons $f = q, \bar{q}, g$ with momentum fraction x

$$f(x) \equiv f^{\uparrow}(x) + f^{\downarrow}(x) \qquad\qquad \Delta f(x) \equiv f^{\uparrow}(x) - f^{\downarrow}(x)$$

2. Allow for a proper field-theoretic definition as matrix elements of bilocal operators



collinear transition of a massles proton h
into a massless parton i
with fractional momentum x
local OPE $\Longrightarrow$ lattice formulation

[Rev.Mod.Phys. 67 (1995) 157]

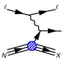
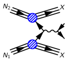
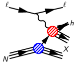
$$\begin{aligned} q(x) &= \frac{1}{4\pi}\int dy^- e^{-iy^- xP^+} \langle P|\bar{\psi}(0, y^-, \mathbf{0}_\perp)\gamma^+\psi(0)|P\rangle \\ \Delta q(x) &= \frac{1}{4\pi}\int dy^- e^{-iy^- xP^+} \langle P, S|\bar{\psi}(0, y^-, \mathbf{0}_\perp)\gamma^+\gamma^5\psi(0)|P, S\rangle \end{aligned}$$

with light-cone coordinates

$$y = (y^+, y^-, \mathbf{y}_\perp)\,, \quad y^+ = (y^0 + y^z)/\sqrt{2}\,, \quad y^- = (y^0 - y^z)/\sqrt{2}\,, \quad \mathbf{y}_\perp = (v^x, v^y)$$

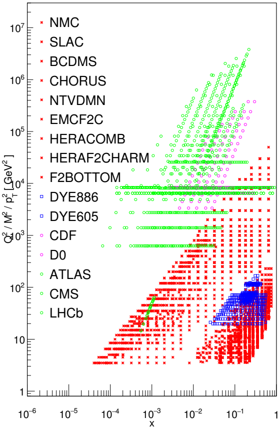
3. The definitions can be generalised to include a transverse-momentum dependence

Nucleons in the initial state: Parton Distribution Functions

Process	Reaction	Subprocess	PDFs probed	x
	$\ell^\pm\{p,n\}\to\ell^\pm+X$	$\gamma^*q\to q$	$q,\bar{q},g$	$x\gtrsim 0.01$
	$\ell^\pm n/p\to\ell^\pm+X$	$\gamma^*d/u\to d/u$	d/u	$x\gtrsim 0.01$
	$\nu(\bar\nu)N\to\mu^-(\mu^+)+X$	$W^*q\to q'$	$q,\bar{q}$	$0.01\lesssim x\lesssim 0.5$
	$\nu N\to\mu^-\mu^++X$	$W^*s\to c$	s	$0.01\lesssim x\lesssim 0.2$
	$\bar\nu N\to\mu^+\mu^-+X$	$W^*\bar{s}\to\bar{c}$	$\bar{s}$	$0.01\lesssim x\lesssim 0.2$
DIS	$e^\pm p\to e^\pm+X$	$\gamma^*q\to q$	$g,q,\bar{q}$	$0.0001\lesssim x\lesssim 0.1$
	$e^+p\to\bar\nu+X$	$W^+\{d,s\}\to\{u,c\}$	d,s	$x\gtrsim 0.01$
	$e^\pm p\to e^\pm c\bar{c}+X$	$\gamma^*c\to c,\gamma^*g\to c\bar{c}$	c,g	$0.0001\lesssim x\lesssim 0.1$
	$e^\pm p\to jet(s)+X$	$\gamma^*g\to q\bar{q}$	g	$0.01\lesssim x\lesssim 0.1$
	$\vec{\ell}^{\,\pm}\{\vec{p},\vec{d},\vec{n}\}\to\ell^\pm+X$	$\gamma^*q\to q$	$\Delta q+\Delta\bar{q},\,\Delta g$	$0.003\lesssim x\lesssim 0.8$
	$pp\to\mu^+\mu^-+X$	$u\bar{u},d\bar{d}\to\gamma^*$	$\bar{q}$	$0.015\lesssim x\lesssim 0.35$
	$pn/pp\to\mu^+\mu^-+X$	$(u\bar{d})/(u\bar{u})\to\gamma^*$	$\bar{d}/\bar{u}$	$0.015\lesssim x\lesssim 0.35$
	$p\bar{p}(pp)\to jet(s)+X$	$gg,qg,qq\to 2jets$	g,q	$0.005\lesssim x\lesssim 0.5$
	$p\bar{p}\to(W^\pm\to\ell^\pm\nu)+X$	$ud\to W^+,\,\bar{u}\bar{d}\to W^-$	$u,d,\bar{u},\bar{d}$	$x\gtrsim 0.05$
	$pp\to(W^\pm\to\ell^\pm\nu)+X$	$u\bar{d}\to W^+,\,d\bar{u}\to W^-$	$u,d,\bar{u},\bar{d},(g)$	$x\gtrsim 0.001$
pp	$p\bar{p}(pp)\to(Z\to\ell^+\ell^-)+X$	$uu,dd(u\bar{u},d\bar{d})\to Z$	$u,d(g)$	$x\gtrsim 0.001$
	$pp\to(W+c)+X$	$gs\to W^-c,\,g\bar{s}\to W^+\bar{c}$	$s,\bar{s}$	$x\sim 0.01$
	$pp\to t\bar{t}+X$	$gg\to t\bar{t}$	g	$x\sim 0.01$
	$\vec{p}p\to W^\pm+X$	$u_L\bar{d}_R\to W^+,\,d_L\bar{u}_R\to W^-$	$\Delta u\ \Delta\bar{u}\ \Delta d\ \Delta\bar{d}$	$0.05\lesssim x\lesssim 0.4$
	$\vec{p}\vec{p}\to\pi+X$	$gg\to qg,\,qg\to qg$	Δg	$0.05\lesssim x\lesssim 0.4$
	$\vec{\ell}^{\,\pm}\{\vec{p},\vec{d}\}\to\ell^\pm h+X$	$\gamma^*q\to q$	$\Delta u\ \Delta\bar{u}\ \Delta d\ \Delta\bar{d}$ Δg	$0.005\lesssim x\lesssim 0.5$
SIDIS	$\vec{\ell}^{\,\pm}\{\vec{p},\vec{d}\}\to\ell^\pm D+X$	$\gamma^*g\to c\bar{c}$	Δg	$0.06\lesssim x\lesssim 0.2$

Accessed kinematic coverage

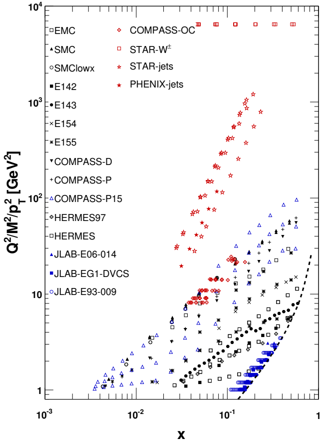
Unpolarised PDFs



$\mathcal{O}(4000)$ data points after cuts

$Q^2_{\rm cut} = 1$ GeV^2 $W^2_{\rm cut} = 3 - 12.5$ GeV^2

Polarised PDFs



$\mathcal{O}(400)$ data points after cuts

$Q^2_{\rm cut} = 1$ GeV^2 $W^2_{\rm cut} = 4 - 6.25$ GeV^2

kinematic cuts: $Q^2 \geq Q^2_{\rm cut}$ (pQCD) and $W^2 = m_p^2 + \frac{1-x}{x}Q^2 \geq W^2_{\rm cut}$ (no HT)

Unpolarised and polarised PDFs

Unpolarised PDFs

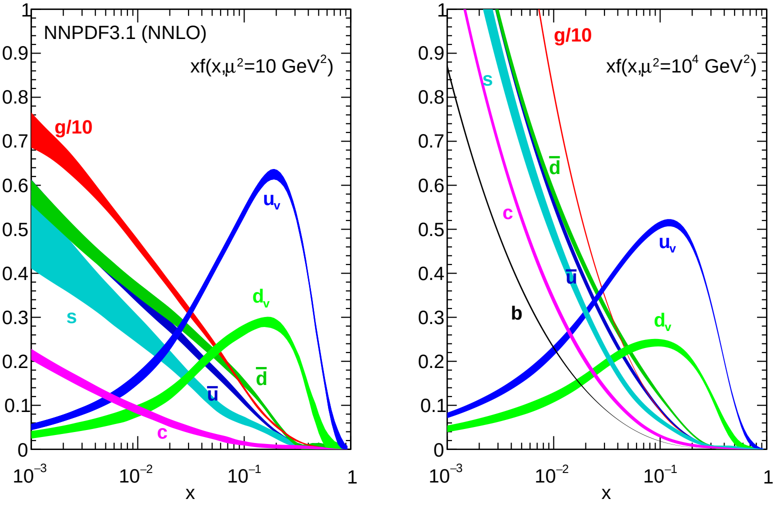
	CT14	MMHT14	NNPDF3.1	ABM16	HERAPDF2.0
fixed-target DIS	☑	☑	☑	☑	☒
HERA	☑	☑	☑	☑	☑
fixed-target DY	☑	☑	☑	☑	☒
Tevatron (W, Z)	☑	☑	☑	☑	☒
Tevatron (jets)	☑	☑	☑	☒	☒
LHC	☑	☑	☑	☑ (W, Z)	☒
latest update	PRD 89 (2014) 033009	EPJC 75 (2015) 204	arXiv:1706.00428	arXiv:1609.03327	EPJC 75 (2015) 580

Polarised PDFs

	DSSV	NNPDFpol1.1	JAM	LSS	BB
DIS	☑	☑	☑	☑	☑
SIDIS	☑	☒	☒	☑	☒
pp	☑ (jets, π^0)	☑ (jets, $W^\pm$)	☒	☒	☒
latest update	PRL 113 (2014) 012001	NPB 887 (2014) 276	PRD 93 (2016) 074005	PRD 82 (2010) 114018	NPB 841 (2010) 205

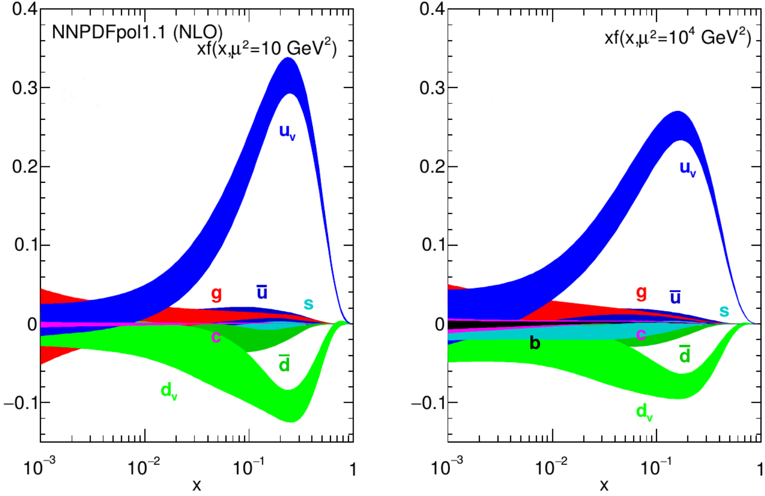
Most of them are publicly available through the LHAPDF interface [EPJ C75 (2015) 132]
https://lhapdf.hepforge.org/

Polarised and unpolarised PDFs



Largely benefit from HERA and LHC programs

Polarised and unpolarised PDFs

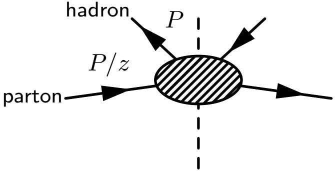


Are starting to benefit from JLAB and RHIC programs

1.2 Fragmentation Functions: the basics

Hadrons in the final state: Fragmentation Functions

FFs allow for a proper field-theoretic definition as matrix elements of bilocal operators



collinear transition
of a massless parton i
into a massless hadron h
with fractional momentum z
no local OPE $\Longrightarrow$ no lattice formulation
[Rev.Mod.Phys. 67 (1995) 157]

$$D_i^h(z) = \frac{1}{12\pi}\sum_X \int dy^- e^{-i\frac{P^+}{z}y^-}\,\mathrm{Tr}\,\left[\gamma^+\langle 0|\psi(y)\mathcal{P}|h(P)\,X\rangle\langle h(P)\,X|\mathcal{P}'\bar{\psi}(0)|0\rangle\right]$$

with light-cone coordinates and appropriate gauge links $\mathcal{P}$, $\mathcal{P}'$

$$y = (y^+, y^-, \mathbf{y}_\perp)\,,\qquad y^+ = (y^0 + y^z)/\sqrt{2}\,,\qquad y^- = (y^0 - y^z)/\sqrt{2}\,,\qquad \mathbf{y}_\perp = (v^x, v^y)$$

All these definitions have ultraviolet divergences which must be renormalized
in order to define finite PDFs to be used in the factorization formulas
(PDF/FFs are scheme dependent)

All these definitions can be generalised to include longitudinal/transverse polarizations

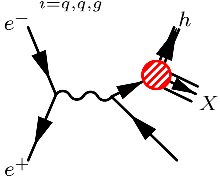
Factorisation of physical observables [Adv.Ser.Direct.HEP 5 (1988) 1]

1. A variety of sufficiently inclusive processes allow for a factorised description

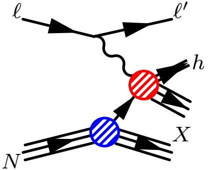
short-distance part		long-distance part
hard interaction of partons	factorisation $\longleftrightarrow$ scheme & scale μ	nucleon structure
process-dependent kernels		universal parton distributions

2. Physical observables are written as a convolution of coefficient functions and FFs

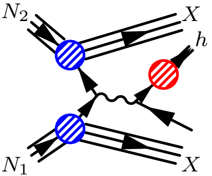
$$\mathcal{O}_I = \sum_{i=q,\bar{q},g} C_{Ii}(y,\alpha_s(\mu^2)) \otimes D_i(y,\mu^2) + \text{p.s. corrections} \qquad f \otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$



$e^+ + e^- \to h + X$
single-inclusive annihilation (SIA)



$\ell + N \to \ell' + h + X$
semi-inclusive deep-inelastic scattering (SIDIS)



$N_1 + N_2 \to h + X$
high-p_T hadron production in proton-proton collisions (pp)

3. Coefficient functions allow for a perturbative expansion

$$C_{If}(y,\alpha_s) = \sum_{k=0} a_s^k C_{If}^{(k)}(y), \qquad a_s = \alpha_s/(4\pi)$$

4. After factorization, all quantities (including FFs/PDFs) depend on μ

Evolution of FFs: DGLAP equations [NP B126 (1977) 298]

1. A set of $(2n_f + 1)$ integro-differential equations, n_f is the number of active flavors

$$\frac{\partial}{\partial \ln \mu^2} D_i(x, \mu^2) = \sum_j^{n_f} \int_x^1 \frac{dz}{z} P_{ji}\left(z, \alpha_s(\mu^2)\right) D_j\left(\frac{x}{z}, \mu^2\right)$$

2. Often written in a convenient basis of PDFs

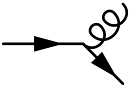
$$D_{\mathrm{NS};\pm} = (D_q \pm D_{\bar{q}}) - (D_{q'} \pm D_{\bar{q}'}) \qquad D_{\mathrm{NS;v}} = \sum_q^{n_f} (D_q - D_{\bar{q}}) \qquad D_\Sigma = \sum_q^{n_f} (D_q + D_{\bar{q}})$$

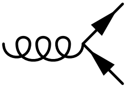
$$\frac{\partial}{\partial \ln \mu^2} D_{\mathrm{NS};\pm,v}(x, \mu^2) = P^{\pm,\mathrm{v}}(x, \mu_F^2) \otimes D_{\mathrm{NS};\pm,v}(x, \mu^2)$$

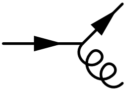
$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} D_\Sigma(x, \mu^2) \\ D_g(x, \mu^2) \end{pmatrix} = \begin{pmatrix} P^{qq} & 2n_f P^{gq} \\ \frac{1}{2n_f} P^{qg} & P^{gg} \end{pmatrix} \otimes \begin{pmatrix} D_\Sigma(x, \mu^2) \\ D_g(x, \mu^2) \end{pmatrix}$$

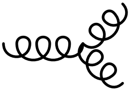
3. With perturbative computable (time-like) splitting functions

$$P_{ji}(z, \alpha_s) = \sum_{k=0} a_s^{k+1} P_{ji}^{(k)}(z), \qquad a_s = \alpha_s/(4\pi)$$

$P_{qq}^{(0)}$ 

$P_{gq}^{(0)}$ 

$P_{qg}^{(0)}$ 

$P_{gg}^{(0)}$ 

Splitting functions: LO and NLO

$$
\begin{aligned}
P^{(0)}_{\rm ns}(x) &= C_F\left(2p_{\rm qq}(x)+3\delta(1-x)\right)\\
P^{(0)}_{\rm ps}(x) &= 0\\
P^{(0)}_{\rm qg}(x) &= 2n_f\, p_{\rm qg}(x)\\
P^{(0)}_{\rm gq}(x) &= 2C_F\, p_{\rm gq}(x)\\
P^{(0)}_{\rm gg}(x) &= C_A\left(4p_{\rm gg}(x)+\frac{11}{3}\delta(1-x)\right)-\frac{2}{3}n_f\,\delta(1-x)
\end{aligned}
$$

LO: 1973

$$
\begin{aligned}
P^{(1)+}_{\rm ns}(x) &= 4C_AC_F\Big(p_{\rm qq}(x)\Big[\frac{67}{18}-\zeta_2+\frac{11}{6}H_0+H_{0,0}\Big]+p_{\rm qq}(-x)\Big[\zeta_2+2H_{-1,0}-H_{0,0}\Big]\\
&+\frac{14}{3}(1-x)+\delta(1-x)\Big[\frac{17}{24}+\frac{11}{3}\zeta_2-3\zeta_3\Big]\Big)-4C_Fn_f\Big(p_{\rm qq}(x)\Big[\frac{5}{9}+\frac{1}{3}H_0\Big]+\frac{2}{3}(1-x)\\
&+\delta(1-x)\Big[\frac{1}{12}+\frac{2}{3}\zeta_2\Big]\Big)+4C_F^{\,2}\Big(2p_{\rm qq}(x)\Big[H_{1,0}-\frac{3}{4}H_0+H_2\Big]-2p_{\rm qq}(-x)\Big[\zeta_2+2H_{-1,0}\\
&-H_{0,0}\Big]-(1-x)\Big[1-\frac{3}{2}H_0\Big]-H_0-(1+x)H_{0,0}+\delta(1-x)\Big[\frac{3}{8}-3\zeta_2+6\zeta_3\Big]\Big)\\
P^{(1)-}_{\rm ns}(x) &= P^{(1)+}_{\rm ns}(x)+16C_F\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[\zeta_2+2H_{-1,0}-H_{0,0}\Big]-2(1-x)\\
&-(1+x)H_0\Big)
\end{aligned}
$$

$$
P^{(1)}_{\rm ps}(x) = 4C_Fn_f\Big(\frac{20}{9}\frac{1}{x}-2+6x-4H_0+x^2\Big[\frac{8}{3}H_0-\frac{56}{9}\Big]+(1+x)\Big[5H_0-2H_{0,0}\Big]\Big)
$$

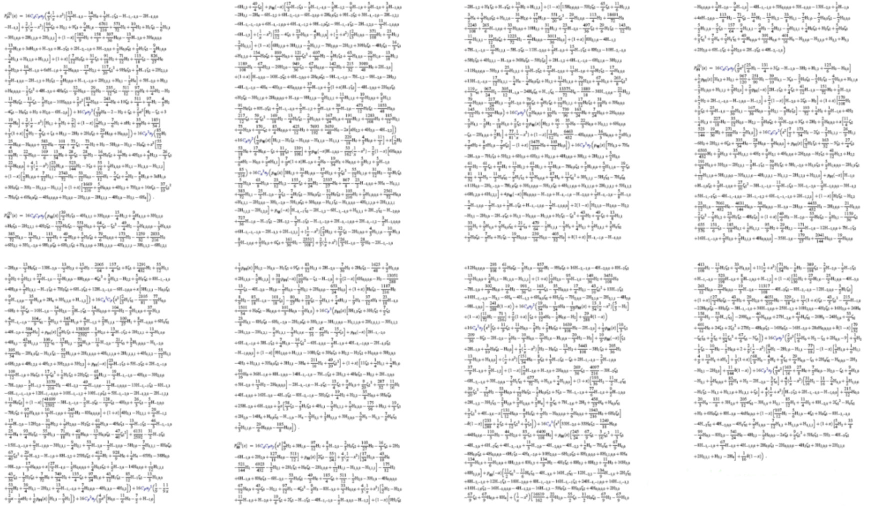
$$
\begin{aligned}
P^{(1)}_{\rm qg}(x) &= 4C_An_f\Big(\frac{20}{9}\frac{1}{x}-2+25x-2p_{\rm qg}(-x)H_{-1,0}-2p_{\rm qg}(x)H_{1,1}+x^2\Big[\frac{44}{3}H_0-\frac{218}{9}\Big]\\
&+4(1-x)\Big[H_{0,0}-2H_0+xH_1\Big]-4\zeta_2x-6H_{0,0}+9H_0\Big)+4C_Fn_f\Big(2p_{\rm qg}(x)\Big[H_{1,0}+H_{1,1}+H_2\\
&-\zeta_2\Big]+4x^2\Big[H_0+H_{0,0}+\frac{5}{2}\Big]+2(1-x)\Big[H_0+H_{0,0}-2xH_1+\frac{29}{4}\Big]-\frac{15}{2}-H_{0,0}-\frac{1}{2}H_0\Big)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\rm gq}(x) &= 4C_AC_F\Big(\frac{1}{x}+2p_{\rm gq}(x)\Big[H_{1,0}+H_{1,1}+H_2-\frac{11}{6}H_1\Big]-x^2\Big[\frac{8}{3}H_0-\frac{44}{9}\Big]+4\zeta_2-2\\
&-7H_0+2H_{0,0}-2H_1x+(1+x)\Big[2H_{0,0}-5H_0+\frac{37}{9}\Big]-2p_{\rm gq}(-x)H_{-1,0}\Big)-4C_Fn_f\Big(\frac{2}{3}x\\
&-p_{\rm gq}(x)\Big[\frac{2}{3}H_1-\frac{10}{9}\Big]\Big)+4C_F^{\,2}\Big(p_{\rm gq}(x)\Big[3H_1-2H_{1,1}\Big]+(1+x)\Big[H_{0,0}-\frac{7}{2}+\frac{7}{2}H_0\Big]-3H_{0,0}\\
&+1-\frac{3}{2}H_0+2H_1x\Big)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\rm gg}(x) &= 4C_An_f\Big(1-x-\frac{10}{9}p_{\rm gg}(x)-\frac{13}{9}\Big(\frac{1}{x}-x^2\Big)-\frac{2}{3}(1+x)H_0-\frac{2}{3}\delta(1-x)\Big)+4C_A^{\,2}\Big(27\\
&+(1+x)\Big[\frac{11}{3}H_0+8H_{0,0}-\frac{27}{2}\Big]+2p_{\rm gg}(-x)\Big[H_{0,0}-2H_{-1,0}-\zeta_2\Big]-\frac{67}{9}\Big(\frac{1}{x}-x^2\Big)-12H_0\\
&-\frac{44}{3}x^2H_0+2p_{\rm gg}(x)\Big[\frac{67}{18}-\zeta_2+H_{0,0}+2H_{1,0}+2H_2\Big]+\delta(1-x)\Big[\frac{8}{3}+3\zeta_3\Big]\Big)+4C_Fn_f\Big(2H_0\\
&+\frac{2}{3}\frac{1}{x}+\frac{10}{3}x^2-12+(1+x)\Big[4-5H_0-2H_{0,0}\Big]
\end{aligned}
$$

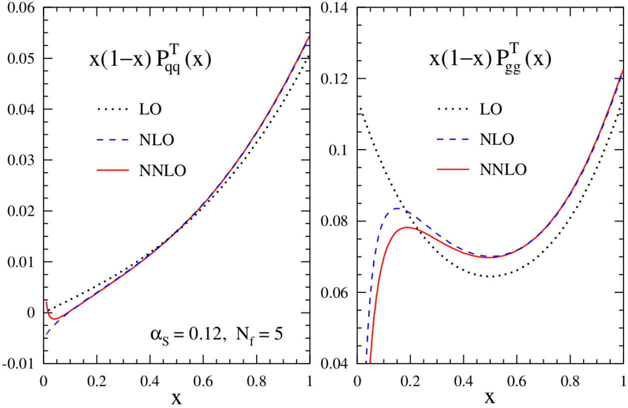
NLO: 1980

Splitting functions: NNLO



Properties of splitting functions

1. At LO:
   time-like and space-like splitting functions are equal, provided $P_{qg}^{S,(0)} \leftrightarrow P_{gq}^{T,(0)}$
2. At NLO [NPB 175 (1980) 27, PLB 97 (1980) 497, PRD 48 (1993) 116]
   time-like and space-like splitting functions are related by analytic continuation
3. at NNLO [PLB 638 (2006) 61, PLB 659 (2008) 290, NPB 854 (2012) 133]
   an uncertainty still remains on the exact form of $P_{\rm qg}^{(2)}$ (it does not affect its log behavior)



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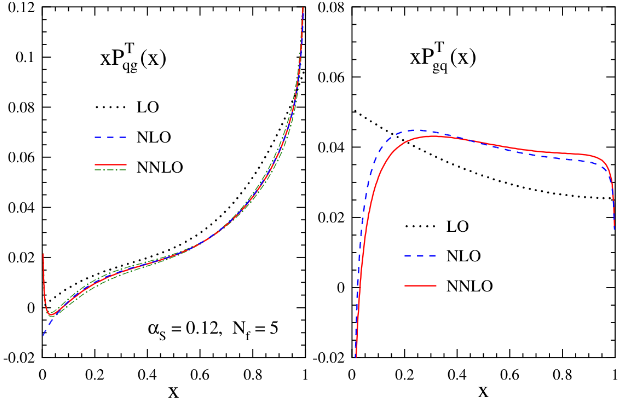


Properties of splitting functions

Must be careful with fixed-order splitting functions as $z \to 0$ $(m = 1, \ldots, 2k+1)$

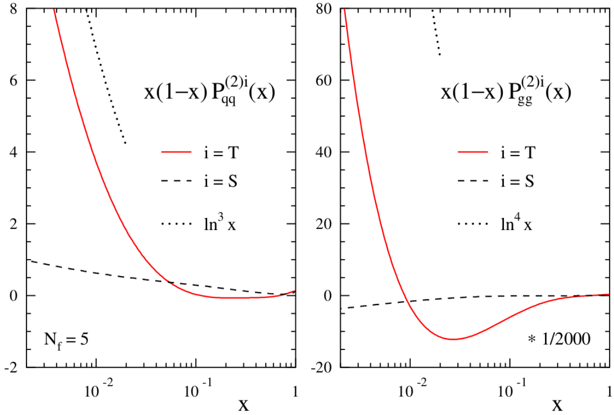
SPACE-LIKE CASE

$$P_{ji} \propto \frac{a_s^{k+1}}{x} \log^{k+1-m} \frac{1}{x}$$

TIME-LIKE CASE

$$P_{ji} \propto \frac{a_s^{k+1}}{z} \log^{2(k+1)-m-1} z$$

Soft gluon logarithms diverge more rapidly in the TL case than in the SL case: as z decreases, the unresummed SGLs spoil the convergence of the FO series for $P(z, a_s)$ if $\log \frac{1}{z} \geq \mathcal{O}\left(a_s^{-1/2}\right)$



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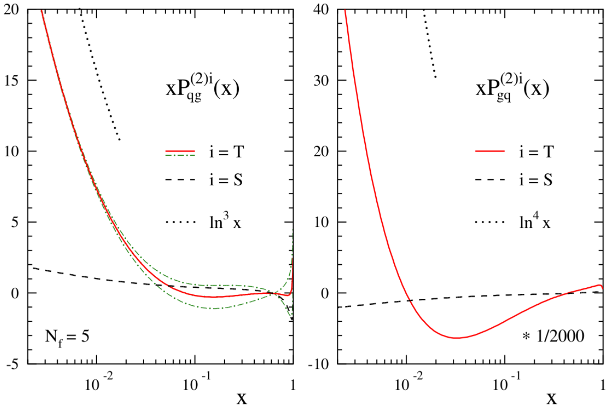


Solving DGLAP equations

1. Transform to Mellin space (convolutions become ordinary products)

$$\frac{\partial}{\partial \ln \mu^2} D_i(N,\mu^2) = \sum_j^{n_f} \gamma_{ji}\left(N, \alpha_s(\mu^2)\right) D_j\left(N,\mu^2\right) \qquad \gamma_{ji}(N) = \int_0^1 z z^{N-1} P_{ji}(z)$$

2. Solve evolution equations in Mellin space

$$D_i(N,\mu^2) = \sum_j \Gamma_{ij}\left(N, \alpha_s(\mu^2), \alpha_s(\mu_0^2)\right) D_j(N,\mu_0^2)$$

3. The evolution kernels $\Gamma_{ij}\left(N, \alpha_s(\mu^2), \alpha_s(\mu_0^2)\right)$ satisfy the evolution equations

$$\frac{\partial}{\partial \ln \mu^2} \Gamma_{ij}(N,\mu^2) = \sum_k^{n_f} \gamma_{ik}\left(N, \alpha_s(\mu^2)\right) \Gamma_{kj}\left(N,\mu^2\right) \qquad \frac{d\alpha_s}{d\ln Q^2} = \beta(\alpha_s) = -\sum_{n=0} \alpha_s^{n+2}\beta_n$$

$$\frac{\partial}{\partial \alpha_s} \Gamma_{ij}\left(N, \alpha_s(\mu^2), \alpha_s(\mu_0^2)\right) = -\sum_k R_{ik}(N,\alpha_s) \Gamma_{kj}\left(N, \alpha_s(\mu^2), \alpha_s(\mu_0^2)\right)$$

4. The perturbative solution of the matrix $R_{ij} = \mathbf{R} = \mathbf{R}^{(0)} + \alpha_s \mathbf{R}^{(1)} + \dots$ is

$$\mathbf{R}^{(0)} = \frac{\boldsymbol{\gamma}^{(0)}}{\beta_0} \qquad \mathbf{R}^{(k)} = \frac{\boldsymbol{\gamma}^{(k)}}{\beta_0} - \sum_{i=1}^{k} \frac{\beta_i}{\beta_0} \mathbf{R}^{(k-i)} \qquad \boldsymbol{\gamma} = \sum_{n=0} \alpha_s^{n+1} \boldsymbol{\gamma}^{(0)}$$

Solving DGLAP equations: LO example

1. For the flavour nonsinglet and valence quark FFs

$$\Gamma_{\rm NS,LO}\left(N,\alpha_s(\mu^2),\alpha_s(\mu_0^2)\right)=\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}^{-R^{(0)}_{\rm NS}}$$

2. For the flavour singlet, diagonalise the 2×2 $\mathbf{R}$ matrix

$$\mathbf{\Gamma}_{\rm S,LO}(N,\alpha_s(\mu^2),\alpha_s(\mu_0^2))=\mathbf{e}_+(N)\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{-\lambda_+(N)}+\mathbf{e}_-(N)\left(\frac{\alpha_s(\mu^2)}{\alpha_s9\mu_0^2)}\right)^{-\lambda_-(N)}$$

$$\lambda_\pm(N)=\frac{1}{2\beta_0}\left[\gamma_{qq}^{(0)}(N)+\gamma_{gg}^{(0)}(N)\pm\sqrt{\left(\gamma_{qq}^{(0)}(N)-\gamma_{gg}^{(0)}(N)\right)^2+4\gamma_{qg}^{(0)}(N)\gamma_{gq}^{(0)}(N)}\right]$$

$$\mathbf{e}_\pm(N)=\pm\frac{1}{\lambda_+(N)-\lambda_-(N)}(\mathbf{R}_{\rm S}^{(0)}(N)-\lambda_\mp(N)\mathbf{I})$$

3. Determine the evolution factors $\Gamma_{ij}\left(z,\alpha_s(\mu^2),\alpha_s(\mu_0^2)\right)$ by inverse Mellin transform

$$\Gamma_{ij}\left(z,\alpha_s(\mu^2),\alpha_s(\mu_0^2)\right)=\int_C\frac{dN}{2\pi i}x^{-N}\Gamma_{\rm NS}\left(N,\alpha_s(\mu^2),\alpha_s(\mu_0^2)\right)$$

4. Use public codes which solve the evolution equations numerically

`APFEL`[CPC 185 (2014) 1647] `MELA`[JHEP 1503 (2015) 046] `ffevol`[CPC 183 (2012) 1002] `QCDnum`[CPC 182 (2011) 490]

Theoretical constraints

1. Momentum sum rule

$$\sum_h \int_0^1 dz z\, D_i^h(z,\mu^2) = 1 \ \forall \ \text{parton } i$$

2. Charge sum rule

$$\sum_h \int_0^1 dz\, e_h\, D_i^h(z,\mu^2) = e_i \ \forall \ \text{parton } i$$

   where $e_{h(i)}$ is the electric charge of the hadron h (parton i)

3. Positivity of cross sections
   implies that FFs should be positive-definite at LO
4. Charge conjugation symmetry

$$D_{q\,(\bar{q})}^{h^+} = D_{\bar{q}\,(q)}^{h^-} \qquad D_g^{h^+} = D_g^{h^-}$$

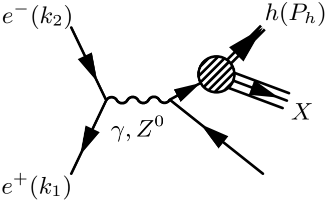
5. Isospin symmetry of the strong interaction

$$D_u^{\pi^+} = D_d^{\pi^-} \qquad D_d^{\pi^+} = D_u^{\pi^-}$$

   approximate, as $m_u \sim m_d$, but no phenomenological evidences of violation

1.3 SIA as a case study

SIA cross section: general structure



$$e^+(k_1) + e^-(k_2) \xrightarrow{\gamma, Z^0} h(P_h) + X$$

$$q = k_1 + k_2 \qquad q^2 = Q^2 > 0 \qquad z = \frac{2P_h \cdot q}{Q^2}$$

$$\boxed{\frac{d\sigma^h}{dz} = \mathcal{F}_T^h(z, Q^2) + \mathcal{F}_L^h(z, Q^2) = \mathcal{F}_2^h(x, Q^2)}$$

$$\boxed{\mathcal{F}^h_{k=T,L,2} = \frac{4\pi\alpha^2_{\rm em}}{Q^2}\langle e^2\rangle \left\{ D_\Sigma^h \otimes \mathcal{C}^{\rm S}_{k,q} + n_f D_g^h \otimes \mathcal{C}^{\rm S}_{k,g} + D^h_{\rm NS} \otimes \mathcal{C}^{\rm NS}_{k,q} \right\}}$$

$$\langle e^2 \rangle = \frac{1}{n_f}\sum_{p=1}^{n_f} \hat{e}_p^2 \qquad D_\Sigma^h = \sum_{p=1}^{n_f}\left(D_p^h + D_{\bar{p}}^h\right) \qquad D^h_{\rm NS} = \sum_{p=1}^{n_f}\left(\frac{\hat{e}_p^2}{\langle e^2\rangle} - 1\right)\left(D_p^h + D_{\bar{p}}^h\right)$$

$$\hat{e}_p^2 = e_p^2 - 2e_p\chi_1(Q^2)v_e v_p + \chi_2(Q^2)(1+v_e^2)(1+v_p^2)$$

$$\chi_1(s) = \frac{1}{16\sin^2\theta_W\cos^2\theta_W}\frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2}$$

$$\chi_2(s) = \frac{1}{256\sin^4\theta_W\cos^4\theta_W}\frac{s^2}{(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2}$$

SIA cross section: some remarks

1. Quark and antiquark FFs always appear throught the combination $D_{q^+} = D_q + D_{\bar{q}}$
   $\longrightarrow$ no separation between quark and antiquark FFs
2. The leading contribution to $C^{\rm S}_{2,g}$ is $\mathcal{O}(\alpha_s)$
   $\longrightarrow$ direct sensitivity to D_g only beyond LO
   $\longrightarrow$ indirect sensitivity to D_g vis DGLAP evolution
3. The effective electroweak charges $\hat{e}_q$ depend on the energy scale
   $\longrightarrow$ $\hat{e}_q^2/\langle e^2\rangle(M_Z) \sim 1$; $\hat{e}_q^2/\langle e^2\rangle(10\,\mathrm{GeV}) \gg 1$
   $\longrightarrow$ partial flavour flavour separation
4. Measurements are often multiplicities, *i.e.* cross sections normalised to $\sigma_{\rm tot}$

$$\sigma_{\rm tot}(Q) = \frac{4\pi\alpha^2(Q)}{Q^2}\left(\sum_q^{n_f}\hat{e}_q^2(Q)\right)\sum_{n=1}\left(\frac{\alpha_s(Q)}{\pi}C_n\left(\frac{s}{\mu^2}\right)\right)^n$$

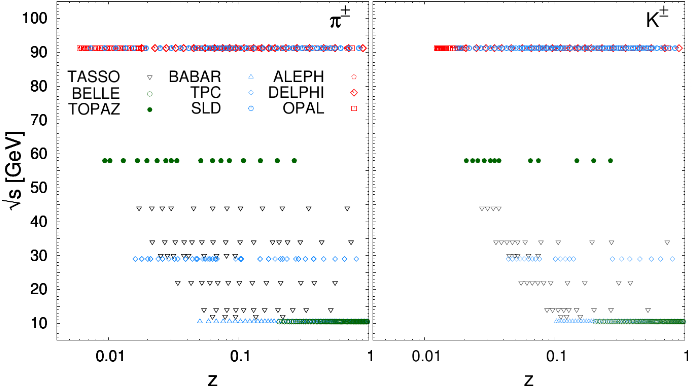
with coefficients

$$C_1(1) = 1$$

$$C_2(1) \simeq 1.986 - 0.115 n_f$$

$$C_3(1) \simeq -6.637 - 1.200 n_f - 0.005 n_f^2 - 1.240\left(\sum_q^{n_f}\hat{e}_q^2\right)^2 \Big/ \sum_q^{n_f}\hat{e}_q^2$$

Experimental data



CERN-LEP: ALEPH [ZP C66 (1995) 353] DELPHI [EPJ C18 (2000) 203] OPAL [ZP C63 (1994) 181]

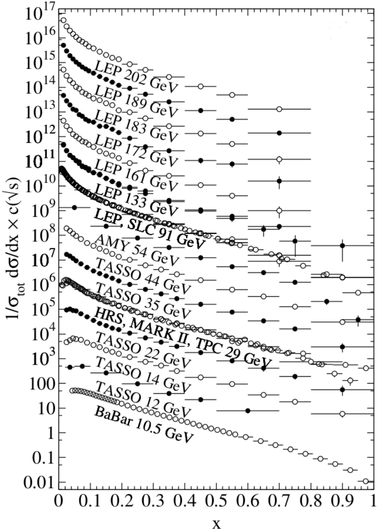
KEK: BELLE ($n_f = 4$) [PRL 111 (2013) 062002] TOPAZ [PL B345 (1995) 335]

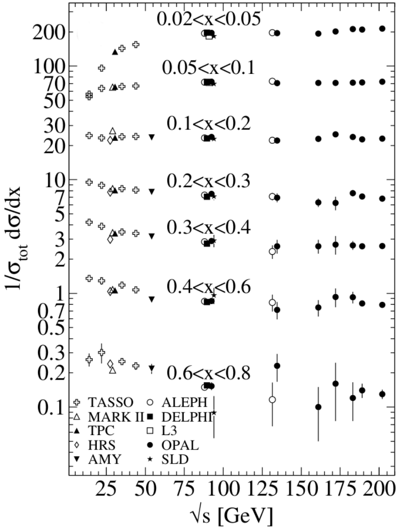
DESY-PETRA: TASSO [PL B94 (1980) 444, ZP C17 (1983) 5, ZP C42 (1989) 189]

SLAC: BABAR ($n_f = 4$) [PR D88 (2013) 032011] SLD [PR D58 (1999) 052001] TPC [PRL 61 (1988) 1263]

$$\frac{d\sigma^h}{dz} = \frac{4\pi\alpha^2(Q^2)}{Q^2} F_2^h(z, Q^2) \quad h = \pi^+ + \pi^-,\, K^+ + K^-; \quad \text{possibly normalised to } \sigma_{\rm tot}$$

Scaling violations





As the scale increases, multiplicites are shifted towards lower values

Flavour schemes

1. Assumption: the final-state hadron has no intrinsic heavy quark component
2. Fixed-flavour number sheme (FFNS)
   the only quarks treated as partons are the n_L light quarks

$$\mathcal{F}_2^{(n_L)}(z,Q^2,m_h^2) = \mathcal{F}_2^{L,(n_L)}(z,Q^2) + \mathcal{F}_2^{H,(n_L)}(z,Q^2,m_h^2)$$

$$\mathcal{F}_2^{L,(n_L)}(z,Q^2) = \sum_i^{n_L} C_{2,i}^{L,(n_L)}\left(z,\frac{Q^2}{\mu^2}\right) \otimes D_i^{(n_L)}(z,\mu^2)$$

$$\mathcal{F}_2^{H,(n_L)}(z,Q^2,m_h^2) = \sum_i^{n_L} C_{2,i}^{H,(n_L)}\left(z,\frac{Q^2}{m_h^2},\frac{\mu^2}{m_h^2}\frac{Q^2}{\mu^2}\right) \otimes D_i^{(n_L)}(z,\mu^2)$$

   is accurate in the quark mass threshold region and below
   suffers from unresummed $\ln(Q^2/m_h^2)$ in the Wilson coefficients, large for $Q^2 \gg m_h^2$
3. Zero-mass variable flavour number scheme (ZM-VFNS)
   treat heavy quarks as massless partons below their threshold
   introduce a heavy quark FF
   the remormalisation of the FF resums logs due to parton splitting via DGLAP

$$\mathcal{F}_2^{(n_L+1)}(z,Q^2) = \sum_i^{n_L+1} C_{2,i}^{(n_L+1)}\left(z,\frac{Q^2}{\mu^2}\right) \otimes D_i^{(n_L+1)}(z,\mu^2)$$

   the reliability of the ZM-VFNS is reduced where powers of m_h^2/Q^2 are significant

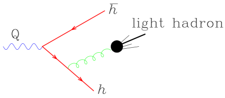
Changing the number of active flavours [JHEP 0510 (2005) 034]

1. All partons with mass $m < \mu = \sqrt{s}$ must be assigned as active
   $\longrightarrow$ even though they are not active at $\mu = \mu_0$
   $\longrightarrow$ the number of active partons depend on the scale of the process
2. Determine the *matching conditions* between a scheme with n_f and $n_f + 1$ flavours
3. Consider the production of a light hadron h

$$\frac{d\sigma}{dz} = \sum_{i \in \mathbb{I}_{n_L}} D_i^{(n_L)}(z, \mu^2) \otimes \frac{d\hat{\sigma}_i^{(n_L)}(z, \mu^2)}{dz} + D_g^{(n_L)}(z, \mu^2) \otimes \frac{d\sigma_{h\bar{h}g}(z)}{dz}$$

$\longrightarrow$ the first term involves the FF and the $\hat{\sigma}_i$ for all partons i excluding heavy flavours
$\longrightarrow$ the second term is the $\mathcal{O}(\alpha_s)$ heavy-flavour contribution

$$\frac{d\sigma_{h\bar{h}g}(z)}{dz} = \sigma_{h\bar{h}} \frac{\alpha_s}{2\pi} C_F \, 2 \frac{1 + (1-y)^2}{y} \left\{ \ln \frac{s}{m_h^2} + \ln(1-y) - 1 \right\}$$



4. In the $\overline{\text{MS}}$ scheme, treating all flavour as massless

$$\frac{d\sigma}{dz} = \sum_{i \in \mathbb{I}_n} D_i^{(n)}(z, \mu^2) \otimes \frac{d\hat{\sigma}_i^{(n)}(z, \mu^2)}{dz} \qquad \mathbb{I}_n = \mathbb{I}_{n_L} \cup \{h, \bar{h}\}$$

Changing the number of active flavours [JHEP 0510 (2005) 034]

The difference between the two cross sections should vanish

$$0 = \sum_{i\in\mathbb{I}_{n_L}, i\neq g} \left[D_i^{(n)}(z,\mu^2) - D_i^{(n_L)}(z,\mu^2)\right] \otimes \frac{d\hat{\sigma}_i(z,\mu^2)}{dx}$$

$$+ \left[D_g^{(n)}(z,\mu^2) \otimes \frac{d\hat{\sigma}^{(n)}(z,\mu^2)}{dz} - D_g^{(n_L)}(z,\mu^2) \otimes \frac{d\hat{\sigma}^{(n_L)}(z,\mu^2)}{dz}\right]$$

$$+ \sum_{i\in\{h,\bar{h}\}} D_i^{(n)}(z,\mu^2) \otimes \frac{d\hat{\sigma}(z,\mu^2)}{dz} - D_g^{(n_L)}(z,\mu^2) \otimes \frac{d\sigma_{h\bar{h}g(z)}}{dz}$$

$\hat{\sigma}_i^{(n)}$ and $\hat{\sigma}_i^{(n_L)}$ replaced with $\hat{\sigma}_i$, since these cross sections differ by terms $\mathcal{O}(\alpha_s^2)$

$\hat{\sigma}_g$ is $\mathcal{O}(\alpha_s)$, hence $D_g^{(n_L)} = D_g^{(n)}$ (they differ by $\mathcal{O}(\alpha_s)$)

$d\hat{\sigma}_g^{(n)}(z,\mu^2)/dz - d\hat{\sigma}_g^{(n_L)}(z,\mu^2)/dz = d\hat{\sigma}_{h\bar{h}g}(z,\mu^2)/dz$ (massless $\overline{\mathrm{MS}}$)

$D_{h/\bar{h}}^{(n)}$ are $\mathcal{O}(\alpha_s)$, so that only the Born hard cross section is needed

$$\frac{d\hat{\sigma}_{h/\bar{h}}(z,\mu^2)}{dz} - \sigma_{h\bar{h}}\delta(1-z) + \mathcal{O}(\alpha_s)$$

$$\frac{d\hat{\sigma}_{h\bar{h}g}(z,\mu^2)}{dz} = \sigma_{h\bar{h}} \frac{\alpha_s}{2\pi} C_F\, 2\frac{1+(1-z)^2}{z} \left\{2\ln z + \ln(1-z) + \ln\frac{Q^2}{\mu^2}\right\}$$

Changing the number of active flavours [JHEP 0510 (2005) 034]

1. The matching consitions for the light and heavy quarks are

$$D_i^{(n)}(z,\mu^2) = D_i^{(n_L)}(z,\mu^2) \qquad \text{for } i \in \mathbb{I}_{n_L},\, i \neq g$$

$$D_h^{(n)}(z,\mu^2) = D_{\bar{h}}^{(n)}(z,\mu^2) = D_g(z,\mu^2) \otimes \frac{\alpha_s}{2\pi} C_F \,\frac{1+(1-z)^2}{z}\left[\ln\frac{\mu^2}{m_h^2} - 1 - 2\ln z\right]$$

2. Similarly one can obtain the matching condition for the gluon

$$D_g^{(n)}(z,\mu^2) = D_g^{(n_L)}(z,\mu^2)\left(1 - \frac{T_F\alpha_s}{3\pi}\ln\frac{\mu^2}{m_h^2}\right)$$

3. Matching conditions
   - ⟶ for FFs are non-zero at NLO (at variance with PDFs)
   - ⟶ can be used to generate radiatively charm and bottom contributions to $\mathcal{F}_2^h$
   - ⟶ are required when evolving the perturbative charm FF through the bottom threshold

4. In practice
   - ⟶ the FF heavy-quark intrinsic component (non-vanishing if not active) is non-negligible
   - ⟶ one usually treats (discontinously) heavy-quark FFs as unknown functions
   - ⟶ inconsistency: heavy-quark FF effects should not depend on the flavour scheme
   - ⟶ alleviated when the scale of the process is above the heavy quark threshold

General-mass flavour number scheme

1. Combine a FFN scheme (at $Q \lesssim m_h$) with a ZM-VFN scheme (at $Q \gg m_h$)
   a GM-VFN scheme operates as a tower of FFN schemes
   increasing n_L as the scale increases over each quark mass threshold
2. Demand that physical observables are continuous across the thresholds

$$\mathcal{F}_2^{\rm GM}(z, m_h^2, \mu^2) = \sum_j^{n_L} C_{2,j}^{\rm GM\,(n_L)}(z, m_h^2, \mu^2) \otimes D_j^{(n_L)}(z, \mu^2)$$

$$= \sum_i^{n_L+1} C_{2,i}^{\rm GM\,(n_L+1)}(z, m_h^2, \mu^2) \otimes D_j^{(n_L+1)}(z, \mu^2)$$

3. Demand the matching conditions for the FFs

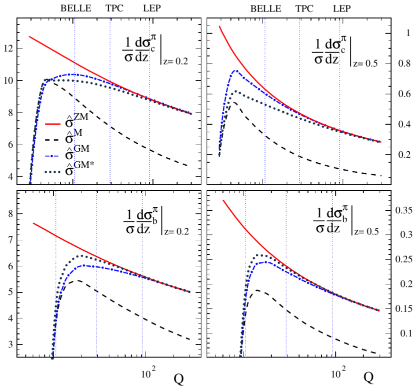
$$D_i^{(n_L+1)}(z, \mu^2) = \sum_j^{n_L} A_{ij}^{(n_L)}(\mu^2/m_h^2) \otimes D_j^{(n_L)}(z, \mu^2)$$

4. Combine the two to obtain the condition

$$C_{2,j}^{\rm GM\,(n_L)}(z, m_h^2, \mu^2) = \sum_i^{n_L+1} C_{i,2}^{\rm GM\,(n_L+1)}(z, m_h^2, \mu^2) \otimes A_{ij}^{(n_L)}(\mu^2/m_h^2)$$

5. Ensure that the condition above is satisfied at all orders

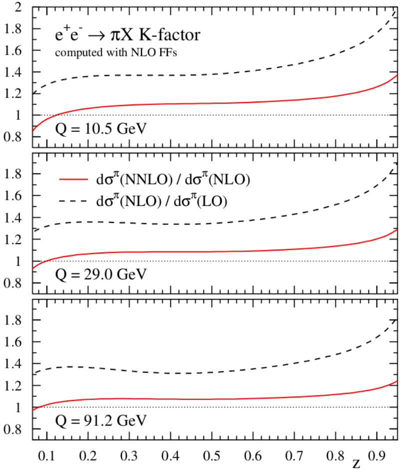
Comparing various flavour schemes [PRD 94 (2016) 034037]

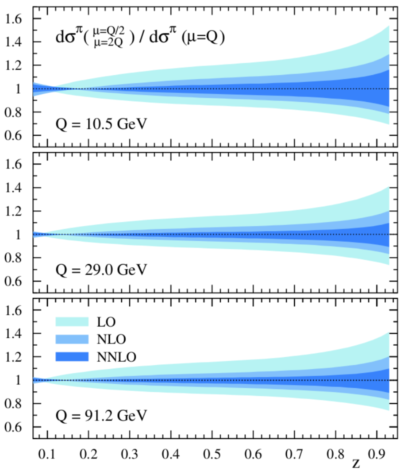


$$\frac{d\sigma^{\rm ZM}}{dz} = \sum_{i=q,g,h} \hat{\sigma}_i^{\rm ZM}(Q) \otimes D_i^{\rm ZM}(Q) \qquad \frac{d\sigma^{\rm GM}}{dz} = \sum_{i=q,g,h} \hat{\sigma}^{\rm GM}(Q,m_h) \otimes D_i^{\rm GM}(Q)$$

$$\frac{d\sigma^{\rm M}}{dz} = \sum_{i=q,g} \hat{\sigma}_i^{\rm M}(Q,m_h) \otimes D_i^{\rm M}(Q) + \hat{\sigma}_h^{\rm M}(Q,m_h) \otimes D_h^{\rm M}$$

Impact of higher-order QCD corrections [PRD 92 (2015) 114017]





K-factor smaller at NNLO/NLO than at NLO/LO

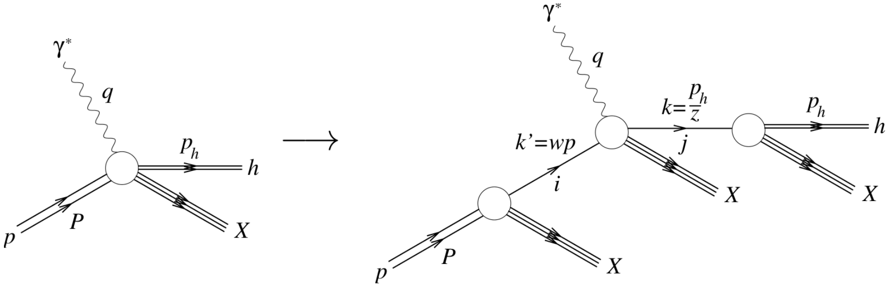
indication of large unresummed logarithms at small and large values of z

renormalization scale variations: estimate of missing higher-order corrections

at most $\sim 5\%$ at NNLO at the lowest Q theoretical uncertainties under control

1.4 Other processes

SIDIS cross section



$$\ell(m) + p(P) \xrightarrow{\gamma} \ell(m') + h(p_h) + X$$

$$\frac{d\sigma^h}{dxdydz} = \frac{2\pi\alpha_{em}^2}{Q^2}\left[\frac{1+(1-y)^2}{y}2F_1^h + \frac{2(1-y)}{y}F_L^h\right]$$

$$2F_1^h = \sum_{q,\bar{q}} e_q^2 \left\{ q \otimes D_q^h + \frac{\alpha_s}{2\pi}\left[q \otimes C_{qq}^1 \otimes D_q^h + q \otimes C_{gq}^1 \otimes D_g^h + g \otimes C_{qg}^1 \otimes D_q^h\right]\right\}$$

$$F_L^h = \frac{\alpha_s}{2\pi}\sum_{q,\bar{q}} e_q^2 \left[q \otimes C_{qq}^L \otimes D_q^h + q \otimes C_{gq}^L \otimes D_g^h + g \otimes C_{qg}^L \otimes D_q^h\right]$$

separate information on D_q and $D_{\bar{q}}$, no direct information on D_g
additional convolution with a PDF

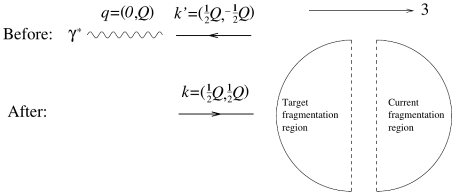
SIDIS multiplicities

1. The quantity usually measured by experiments is the SIDIS multiplicity

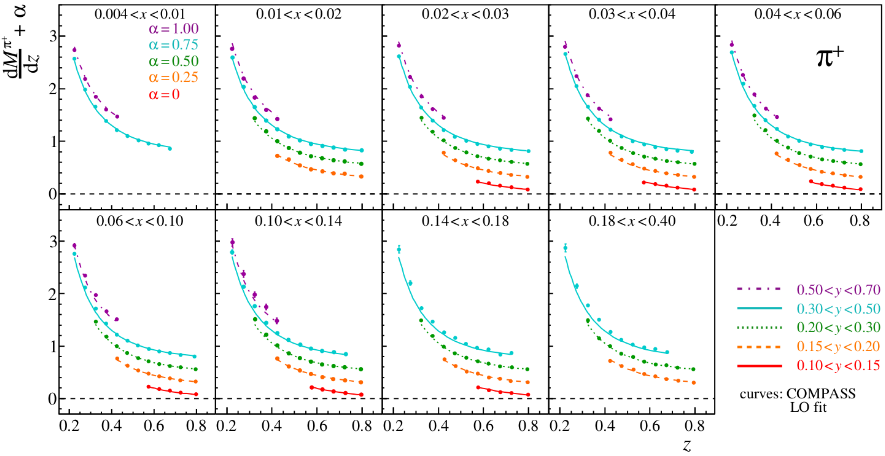
$$M^h(x,z,Q) = \frac{d\sigma^h/dxdydz}{d\sigma^h_{\rm DIS}/dxdy}$$

2. Define two regions in the Breit frame (the frame in which the virtual photon energy vanishes and its spatial momentum is antiparallel with the initial proton's momentum)
   current fragmentation region: $\theta < \pi/2$
   target fragmentation region: $\theta > \pi/2$
   (θ, angle between the spatial momentum of the detected hadron and the virtual photon)

3. *fracture functions* describe the partonic structure of the initial hadron after it has produced the detected hadron ($p \to h + i + X$)
   fracture functions do not contribute to the cross section when the direction of the detected hadron's momentum is within the current fragmentation region



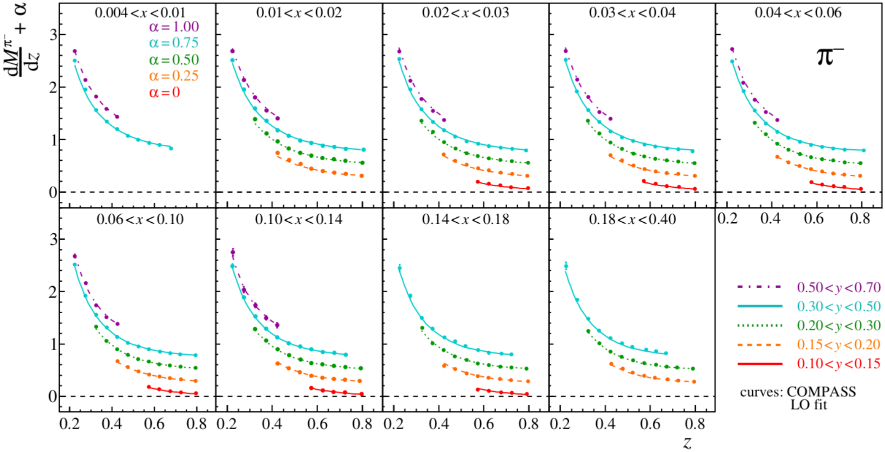
COMPASS multiplicities: pions [PLB 764 (2017) 1]



3-D kinematic binning in x, y, and z (317 kinematic bins)

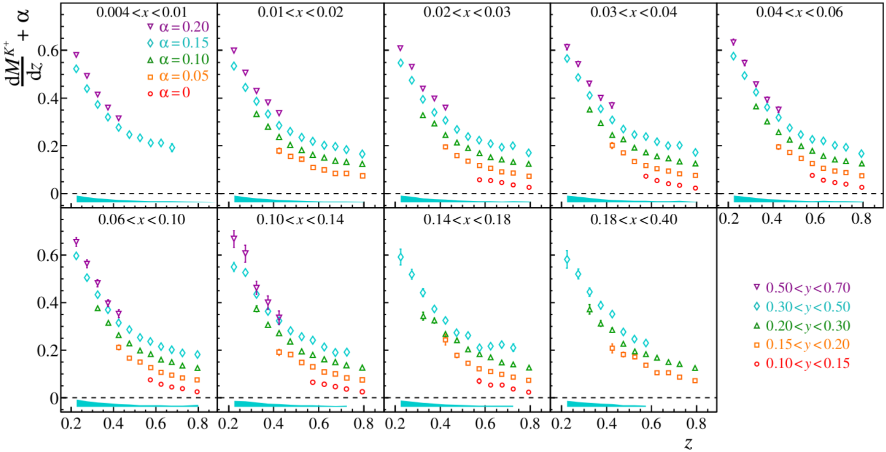
very weak y dependence, strong z dependence

COMPASS multiplicities: pions [PLB 764 (2017) 1]



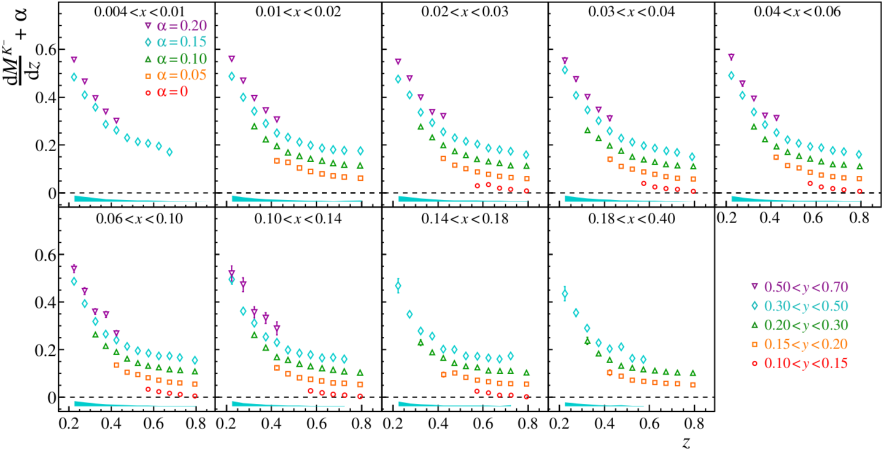
3-D kinematic binning in x, y, and z (317 kinematic bins)
very weak y dependence, strong z dependence

COMPASS multiplicities: kaons [PL B767 (2017) 133]



3-D kinematic binning in x, y, and z (317 kinematic bins)
very weak y dependence, strong z dependence

COMPASS multiplicities: kaons [PL B767 (2017) 133]



3-D kinematic binning in x, y, and z (317 kinematic bins)
very weak y dependence, strong z dependence

Hadron production in pp collisions

$$p(p_a) + p(p_b) \to h(p_h) + X$$

$$E_h \frac{d^3\sigma}{dp_h^3} = \sum_{a,b,c} f_a \otimes f_b \otimes \hat{\sigma}_{ab}^c \otimes D_c^h$$
$$= \sum_{i,j,k,l} \int \frac{dx_a}{x_a} \int \frac{dx_b}{x_b} \int \frac{dz}{z^2} f^{i/p_a}(x-a) f^{j/p_b} D^{h/k}(z) \hat{\sigma}^{ij\to kl} \delta(\hat{s}+\hat{t}+\hat{u})$$

can verify and improve constraints on the charge-sign and flavour separation of quark FFs provided by SIA and SIDIS

constrain gluon FFs significantly better than data from SIA and SIDIS, owing to the occurrence of the gluon FF at LO in the calculation

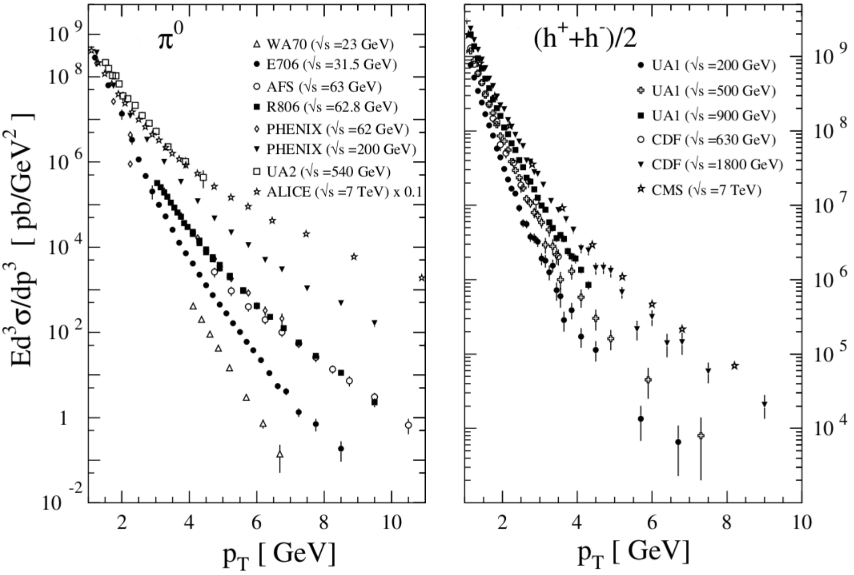
the detected hadron h will sometimes be a soft remnant from one of the initial state hadrons (as in SIDIS)

a current fragmentation region analogous to SIDIS is free of initial hadron remnants

depending on the kinematical region, uncertainties from the PDFs can be sizeable

available data sets are in a kinematical regime where higher order perturbative corrections (possibly requiring resummation) and/or higher twist effects are large

Hadron production in pp collisions



1.4 Summary of Lecture 1

Summary

1. Parton Distribution Functions are required in factorisation formulas
   - ⟶ if hadrons (typically nucleons) are prepared in the initial state
   - ⟶ unpolarised/polarised PDFs required depending on their polarisation
   - ⟶ unpolarised/polarised PDFs need to be determined from the data
2. Fragmentation Functions are required in factorisation formulas
   - ⟶ if hadrons are identified in the final state
   - ⟶ FFs are the time-like counterparts of PDFs
   - ⟶ inherit from PDFs evolution equations
   - ⟶ FFs need to be determined from the data
3. Variety of processes used to access FFs
   - ⟶ single hadron production in SIA, SIDIS and pp collisions
   - ⟶ SIA remains the theoretically cleanest process among the three
   - ⟶ SIA is also the less challenging, as it does not involve PDFs
4. Need to put together all this information in a global QCD analysis
   - ⟶ devise a suitable fitting methodology
   - ⟶ parametrisation, representation of uncertainties, fit validation
   - ⟶ fitting methodology valid for both PDFs and FFs

Lecture 2: Methodological aspects of global QCD fits