Parton distributions with theoretical uncertainties

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Cameron Voisey

UNIVERSITY OF CAMBRIDGE

NNPDF

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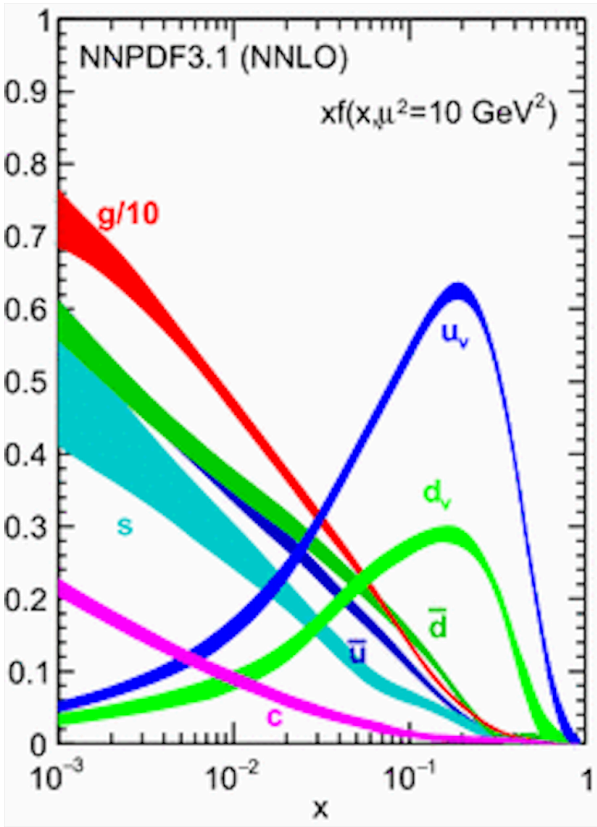
PDFs and their uncertainties

$$\sigma_{p_1 p_2 \to X} = \sum_{a,b \in \{q,\bar{q},g\}} \boxed{f_{p_1} \otimes f_{p_2}} \otimes \hat{\sigma}_{ab \to X}$$

PDFs



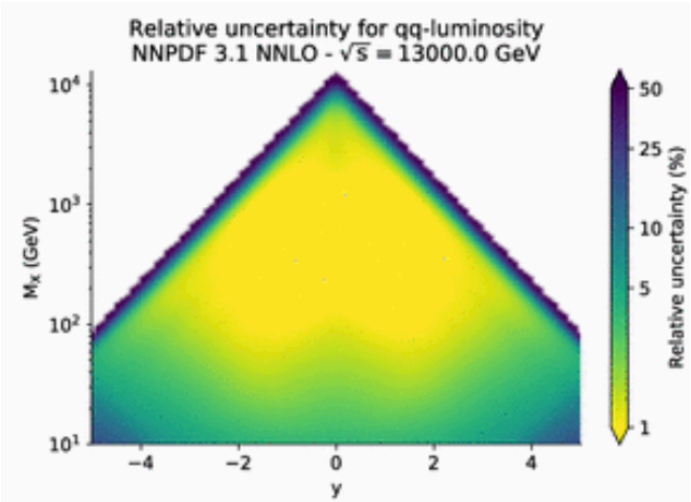
PDF uncertainties include:

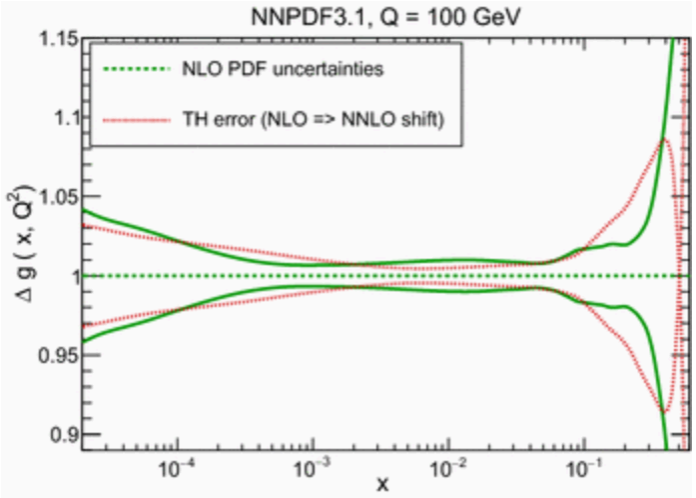
- Experimental uncertainties
- [Parametric uncertainties, e.g. α_s]

Theoretical uncertainties not included systematically

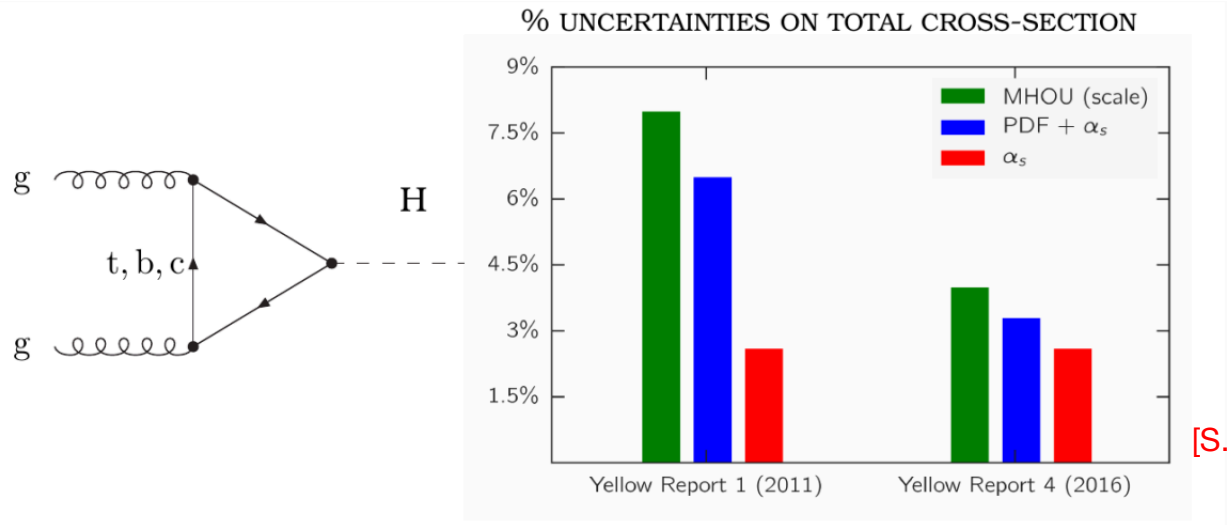
Goal: construct a framework for the systematic inclusion of theoretical uncertainties in PDFs

Why include theoretical uncertainties now?





Theoretical uncertainties at the LHC, e.g. ggF [credit: Z. Kassabov]

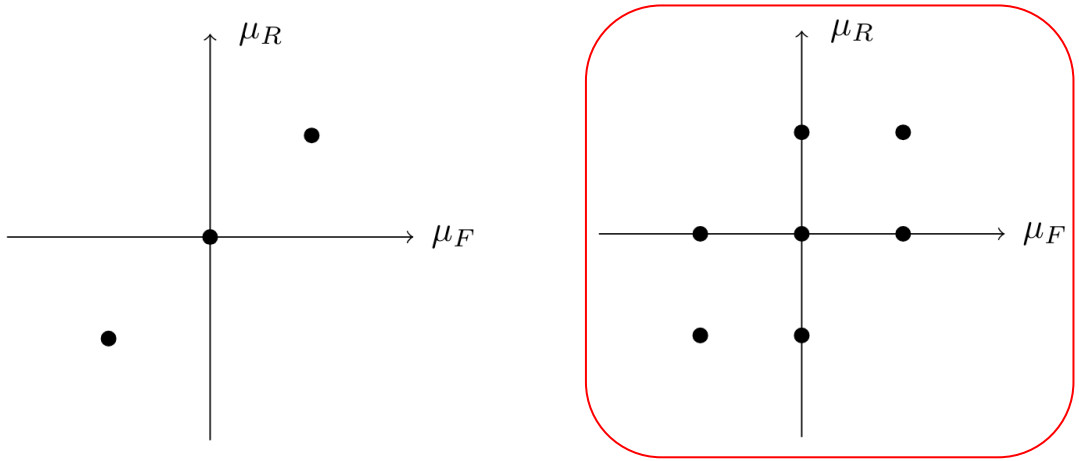


[S. Forte, Lattice 2017]

MHOUs and how we estimate them

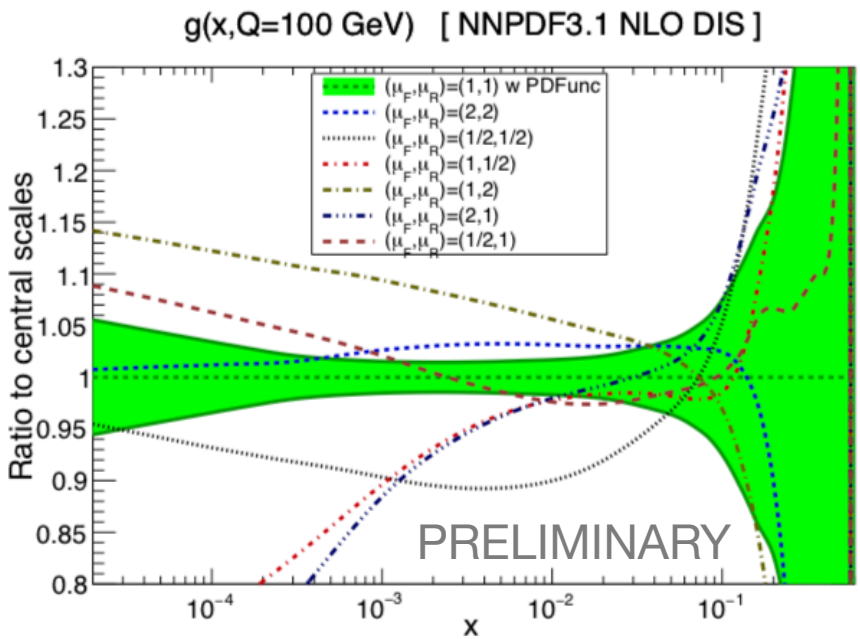
- Missing higher order uncertainties (MHOUs): arise due to truncation of perturbative expansions

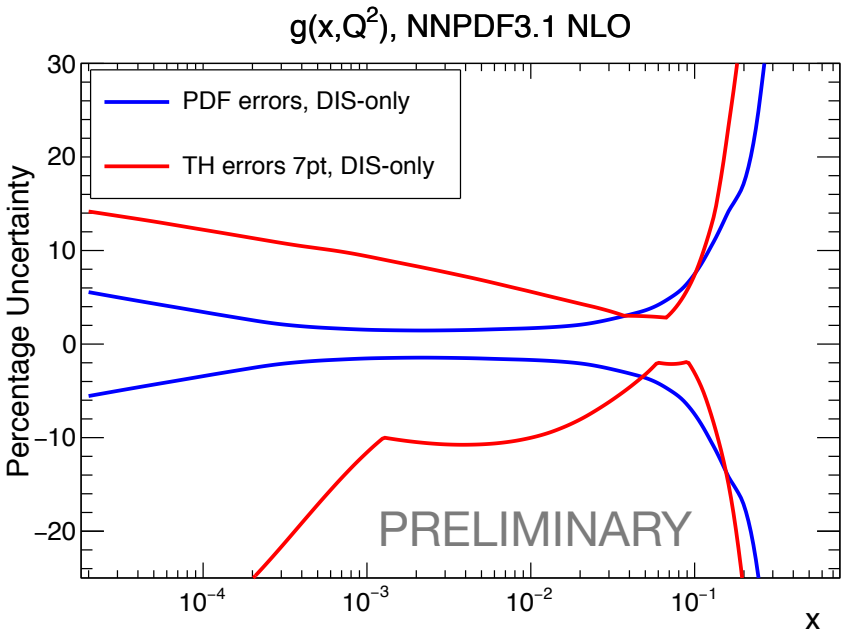
- Standard technique to estimate them: scale variations
- Convention: vary scales by 2 and ½
- Compute observables for different scale combinations and combine
- Standard choices are 3 or 7-point variations:



HXSWG recommendation

PDF fits with varied scales





Useful for estimating MHOUs in PDFs but want to include them in fitting methodology, and in a way that is also applicable to other theoretical uncertainties

The theoretical covariance matrix (I)

- $\mathrm{cov}_{\mathrm{exp}}$: used to propagate experimental uncertainties to PDFs

$$\chi^2 = (d-t)^T \mathrm{cov}_{\mathrm{exp}}^{-1} (d-t)$$

- Minimum $\chi^2 \Rightarrow$ maximum agreement between data and theory

- Define new χ^2 to minimise in fits by [R. D. Ball & A. Deshpande, arXiv:1801.04842]:
  1. Assuming theoretical uncertainties are Gaussian
  2. Taking Bayesian approach

$$\chi^2_{\mathrm{tot}} = (d-t)^T (\mathrm{cov}_{\mathrm{exp}} + \mathrm{cov}_{\mathrm{th}})^{-1} (d-t)$$

- Applicable to other types of theoretical uncertainty

The theoretical covariance matrix (II)

How should we compute $\mathrm{cov}_{\mathrm{th}}$ for MHOUs?

$$\mathrm{cov}_{\mathrm{th},ij} = \langle (t[\mu_R, \mu_F] - t[\mu_{R,0}, \mu_{F,0}])_i (t[\mu_R, \mu_F] - t[\mu_{R,0}, \mu_{F,0}])_j \rangle$$

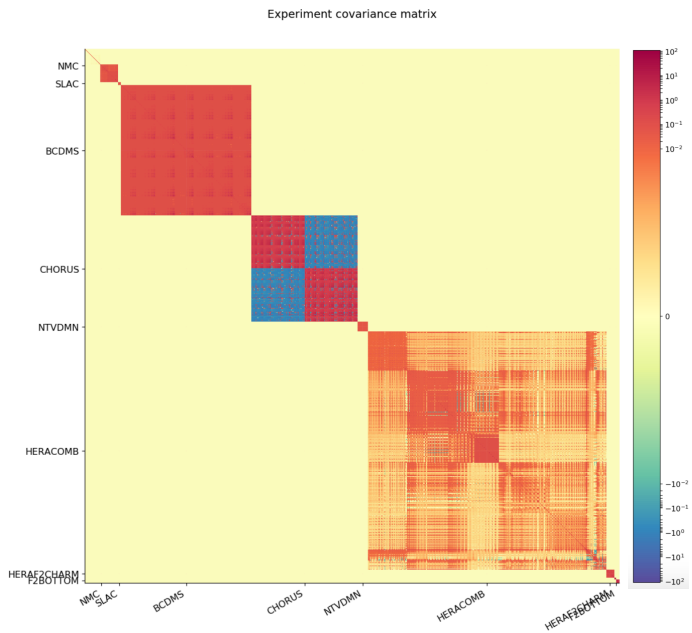
- μ_F variations are correlated across all processes (PDF evolution)
- μ_R variations are correlated by process (hard cross sections)

- Recipe:
  1. Vary scales in $\frac{1}{2} \leq \frac{\mu_F}{\mu_{F,0}}, \frac{\mu_R}{\mu_{R,0}} \leq 2$
  2. 3 or 7-point variations
  3. Average over flat distribution of points
  4. Consider different correlation treatments

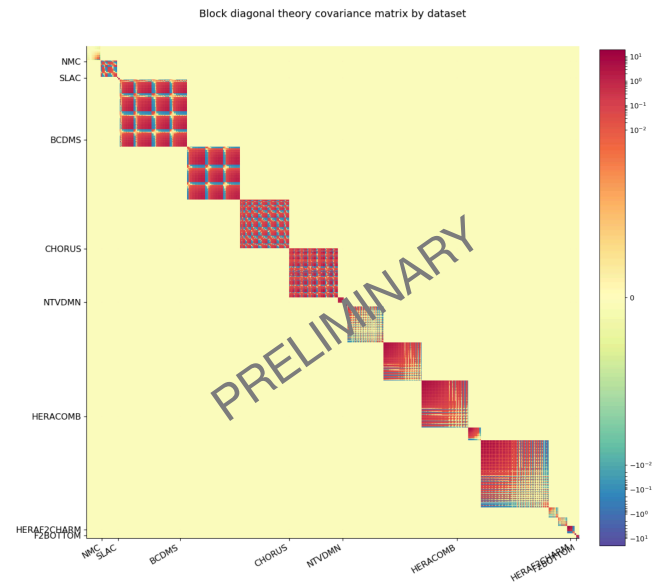
Results (I)

DIS-only dataset, 3-point variations: NLO vs NNLO

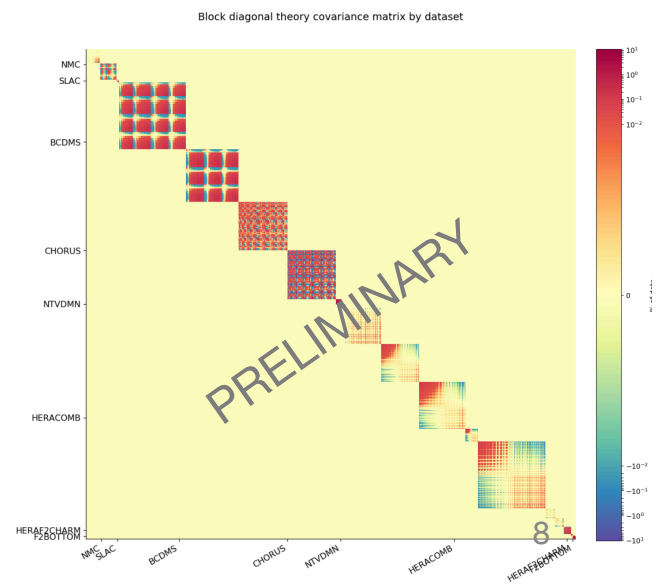
- Correlations within datasets, but not experiments



$$\Delta\chi^2_{\mathrm{NLO}} = -0.040$$



$$\Delta\chi^2_{\mathrm{NNLO}} = -0.016$$



Results (II)

DIS-only dataset, 3-point variations: NLO vs NNLO

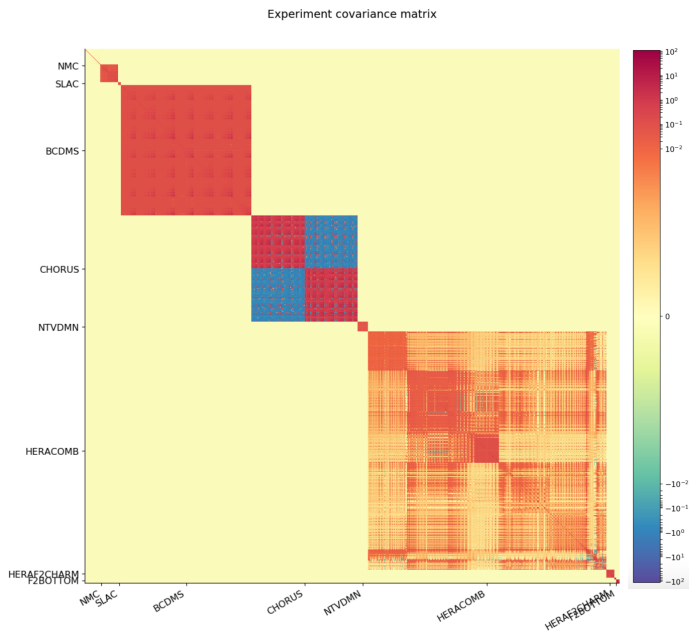
- Correlations within datasets, but not experiments

	χ^2/N_{data} using cov_{exp}		$\Delta(\chi^2/N_{\text{data}})$ using cov_{tot}	
Experiment	NLO	NNLO	NLO	NNLO
SLACP	0.64	0.59	−0.10	−0.04
SLACD	1.29	1.22	−0.94	−0.60
BCDMSP	1.07	1.07	−0.01	−0.01
BCDMSD	1.01	1.00	−0.01	−0.00

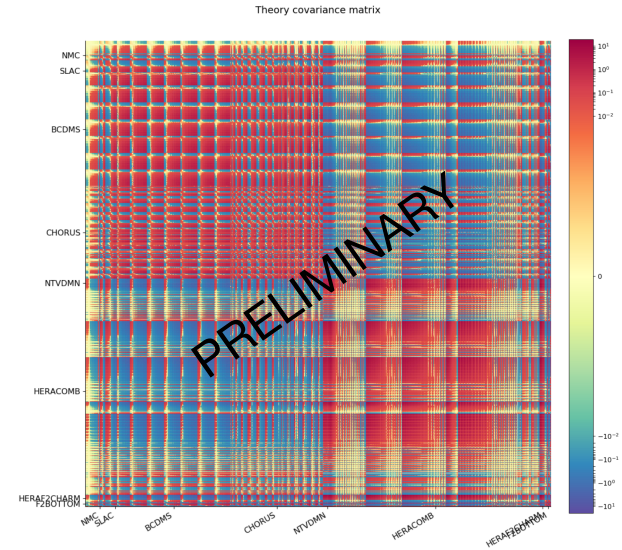
Results (III)

DIS-only dataset, 3-point variations: NLO vs NNLO

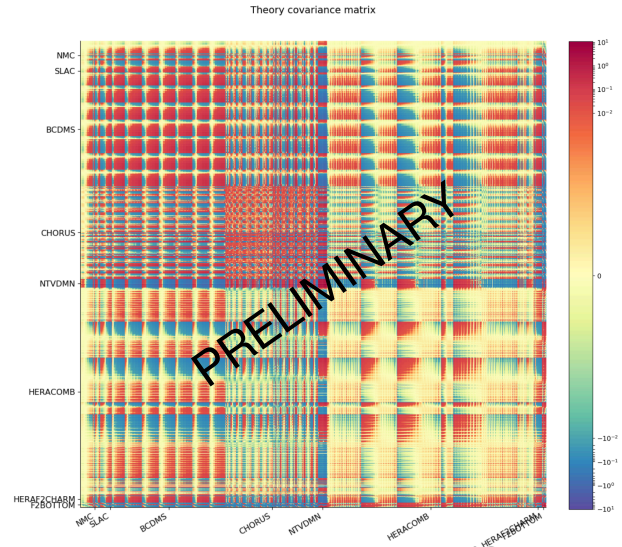
- Fully correlated



$$\Delta\chi^2_{\mathrm{NLO}} = -0.001$$



$$\Delta\chi^2_{\mathrm{NNLO}} = -0.002$$



Conclusion and outlook

- Developed a framework for the inclusion of MHOUs in PDF fits for the first time
- This framework is applicable to other sources of theoretical uncertainty
- Next step: run PDF fits with MHOUs, which will be the first fits of their kind

Thank you for listening!

Extra: theoretical covariance matrix

- Theory is perturbative expansion to some order : $t_p = \sum_{m=0}^{p} c_m$
- Standard case: $P(d|t_p) \propto \exp\left(-\frac{1}{2}\underline{(d-t_p)^T \mathrm{cov}_{\mathrm{exp}}^{-1}(d-t_p)}\right)$ χ^2_{exp}
- Bayes' theorem: $P(t_p|d) = \frac{P(d|t_p)P(t_p)}{P(d)} \propto P(d|t_p)P(t_p)$
- Assume Gaussian theory prior:

$$P(t_p) = \prod_{m=0}^{p} P(c_m) \quad \text{where} \quad P(c_m) \propto \exp\left(-\frac{1}{2}\underline{c_m^T \mathrm{cov}_{\mathrm{th},m}^{-1} c_m}\right)$$

χ^2_{th}

- Assume MHOUs due to $\mathcal{O}(\alpha^{p+1})$ terms only → marginalise these terms:

$$P(t_p|d) \propto \int dc_{p+1} P(d|c_{p+1})P(t_{p+1})$$

$$\propto \exp\left(-\frac{1}{2}\underline{(d-t_p)^T(\mathrm{cov}_{\mathrm{exp}} + \mathrm{cov}_{\mathrm{th}})^{-1}(d-t_p)}\right)$$

χ^2_{tot}

- Include higher order terms by induction