Towards a Neural Network determination of nuclear PDFs

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HP18

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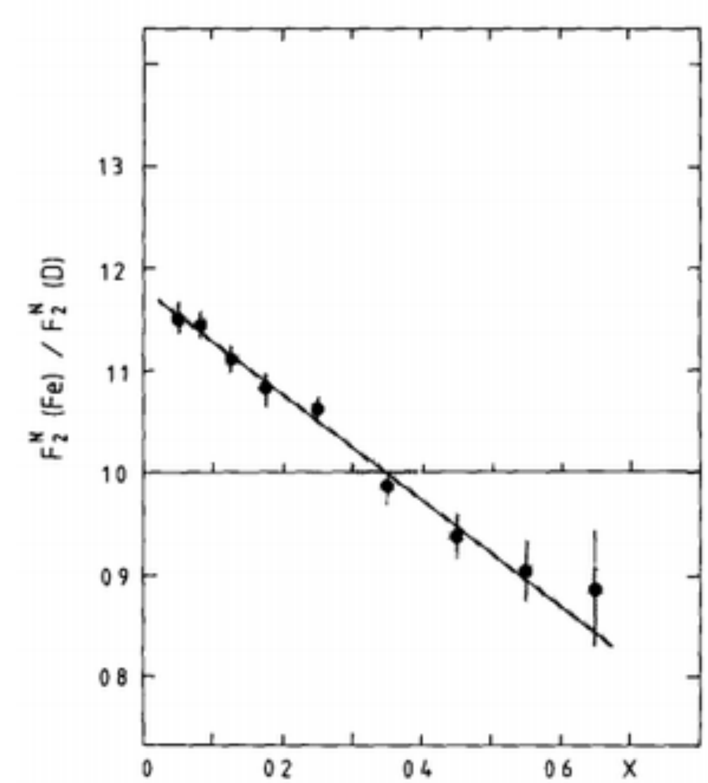
Physics Motivation

- **Simulations** of relativistic nucleus-nucleus reactions [LHC, RHIC, etc.]

- Understanding the **EMC effect.**



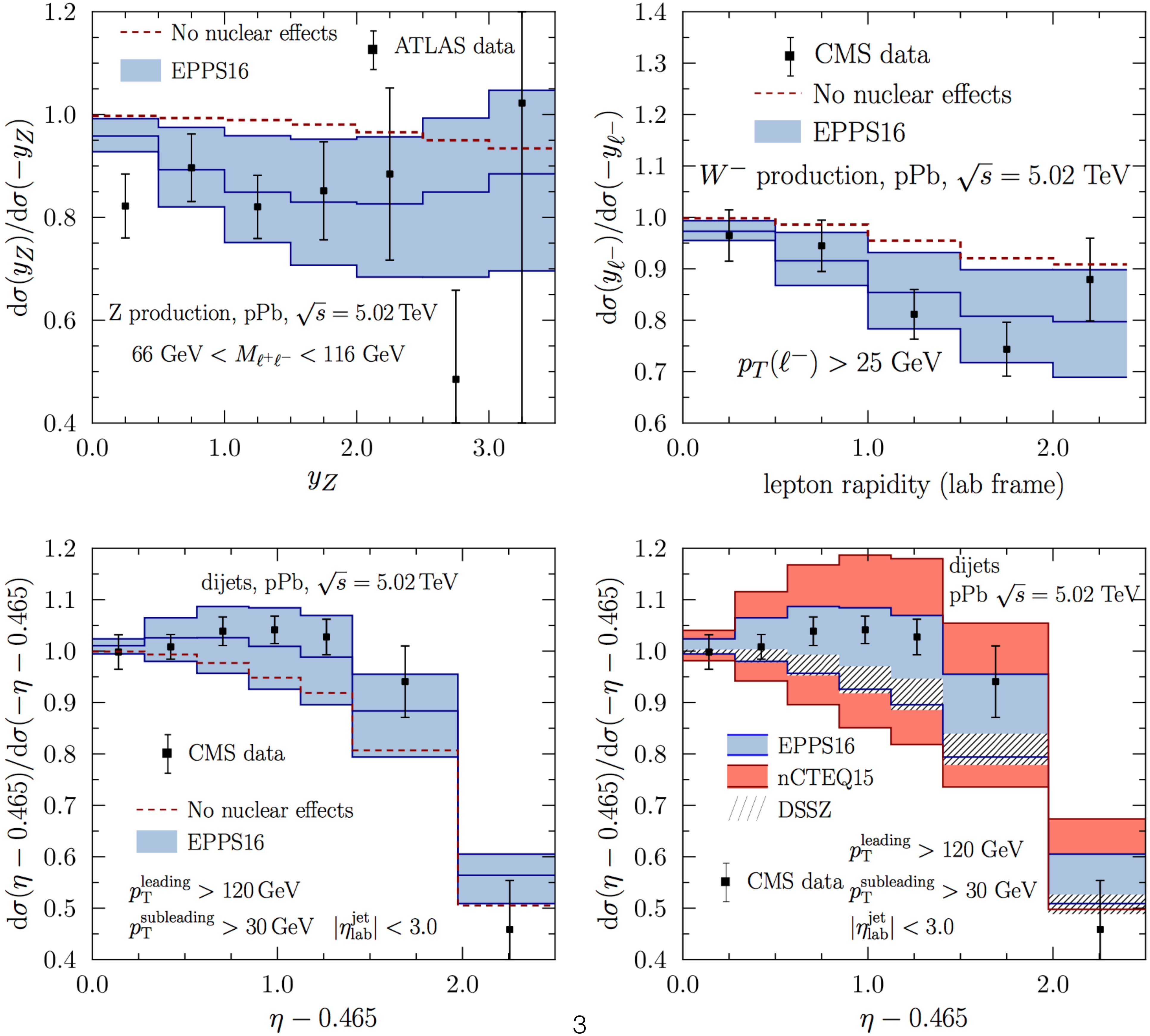
- **Lead, Xenon PDF** for the LHC heavy-ion program.

- Combining data across different nuclear targets allows testing **different nuclear models.**

- Heavy nuclear targets in ν-A DIS measurements provide important information on the quark **flavour separation** in proton PDF fits.

nPDF in LHC pPb observables

[1612.05741]



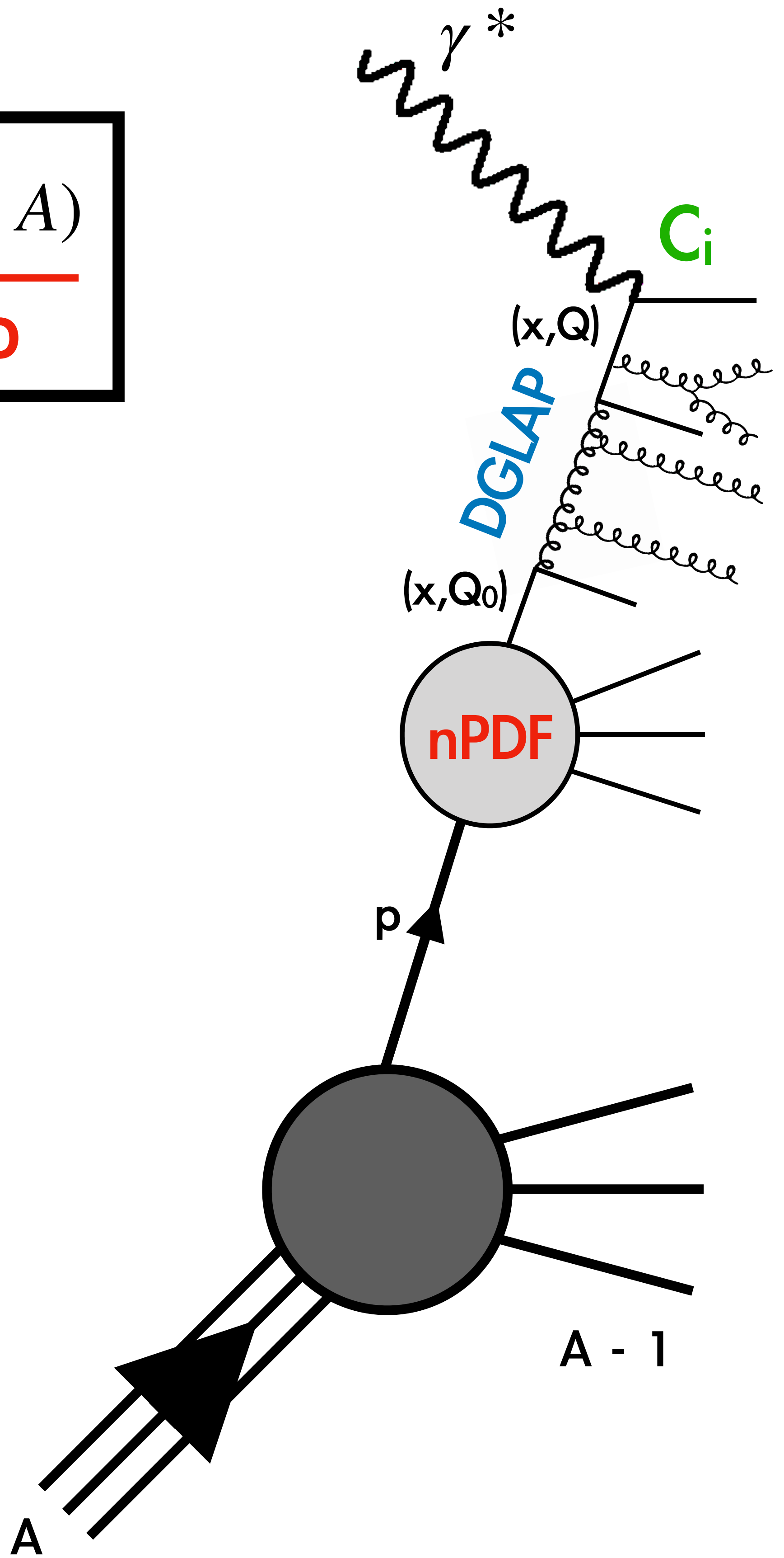
Data

	EPS09	DSSZ12	KA15	NCTEQ15	EPPS16	nNNPDF1.0
Order in α_s	LO & NLO	NLO	NNLO	NLO	NLO	NLO
Neutral current DIS ℓ+A/ℓ+d	✓	✓	✓	✓	✓	✓
Drell-Yan dilepton p+A/p+d	✓	✓	✓	✓	✓	
RHIC pions d+Au/p+p	✓	✓		✓	✓	
Neutrino-nucleus DIS		✓			✓	
Drell-Yan dilepton π+A					✓	
LHC p+Pb jet data					✓	
LHC p+Pb W, Z data					✓	
Q cut in DIS	1.3 GeV	1 GeV	1 GeV	2 GeV	1.3 GeV	1.3 GeV
datapoints	929	1579	1479	708	1811	605
free parameters	15	25	16	17	20	73
error analysis	Hessian	Hessian	Hessian	Hessian	Hessian	Monte
error tolerance $\Delta\chi^2$	50	30	not given	35	52	Carlo rep
Free proton baseline PDFs	CTEQ6.1	MSTW2008	JR09	CTEQ6M-like	CT14NLO	NNPDF3.1
Heavy-quark effects		✓		✓	✓	✓
Flavor separation				some	✓	
Reference	[JHEP 0904 065]	[PR D85 074028]	[PR D93, 014026]	[PR D93 085037]	[EPJ C77 163]	Preliminary

Nuclear NC Inclusive DIS

EM F_2 ($Q < M_Z$)

$$F_2(x, Q^2, A) = \sum_i^{n_f} \sum_j^{n_f} \underbrace{C_i(x, Q^2)}_{\text{pQCD}} \otimes \underbrace{\Gamma_{ij}(x, Q^2)}_{\text{DGLAP}} \otimes \underbrace{q_j(x, Q_0^2, A)}_{\text{npQCD}}$$



x

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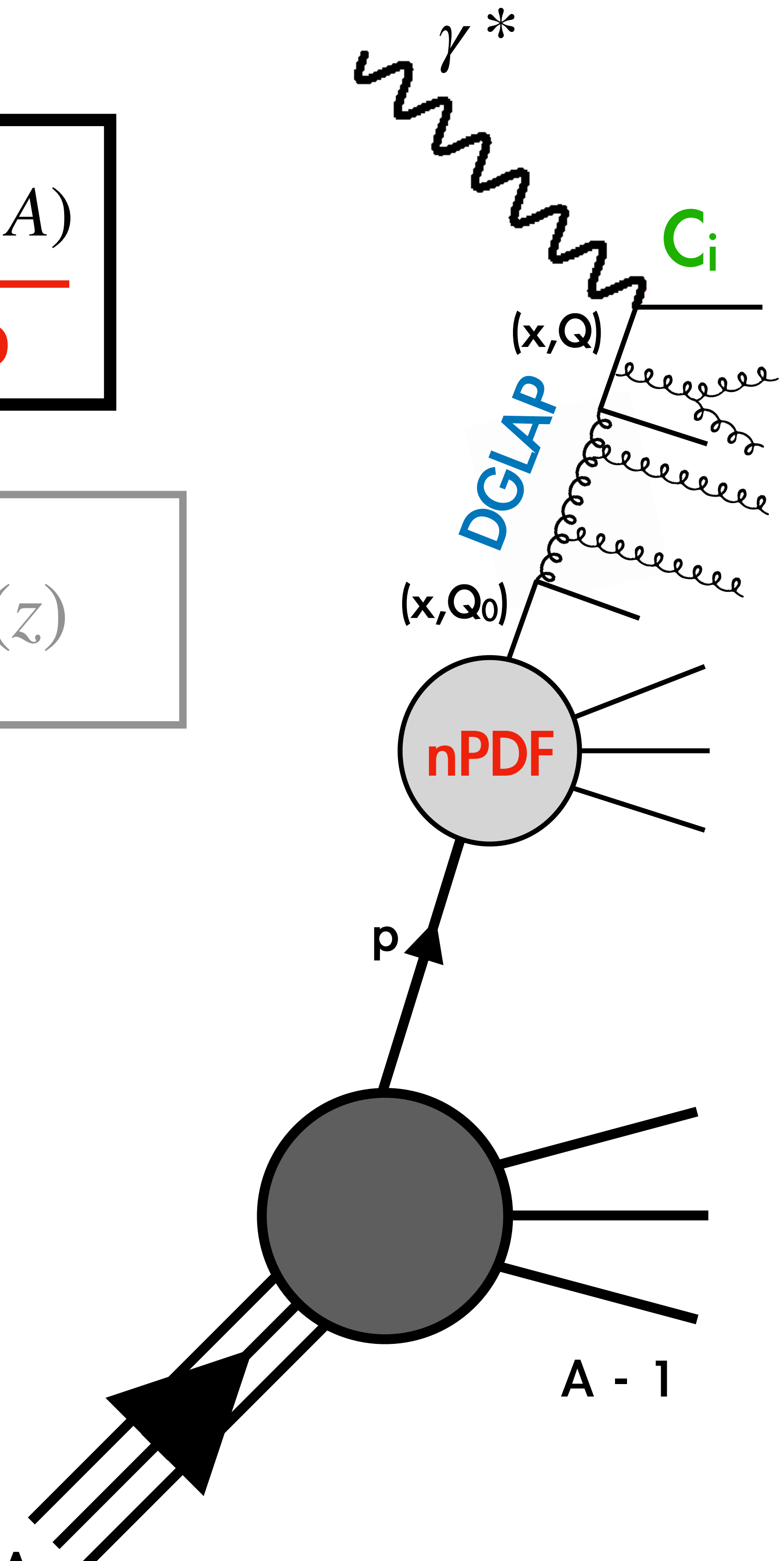
pQCD DGLAP npQCD

Convolution

$$f \otimes g = \int_x^1 \frac{dz}{z} f(\frac{x}{z}) g(z)$$



x

Nuclear NC Inclusive DIS

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pQCD DGLAP npQCD

Coefficient Functions

$$C_i = C_i^{(0)} + \frac{\alpha_s}{4\pi} C_i^{(1)} + O(\alpha_s^2)$$

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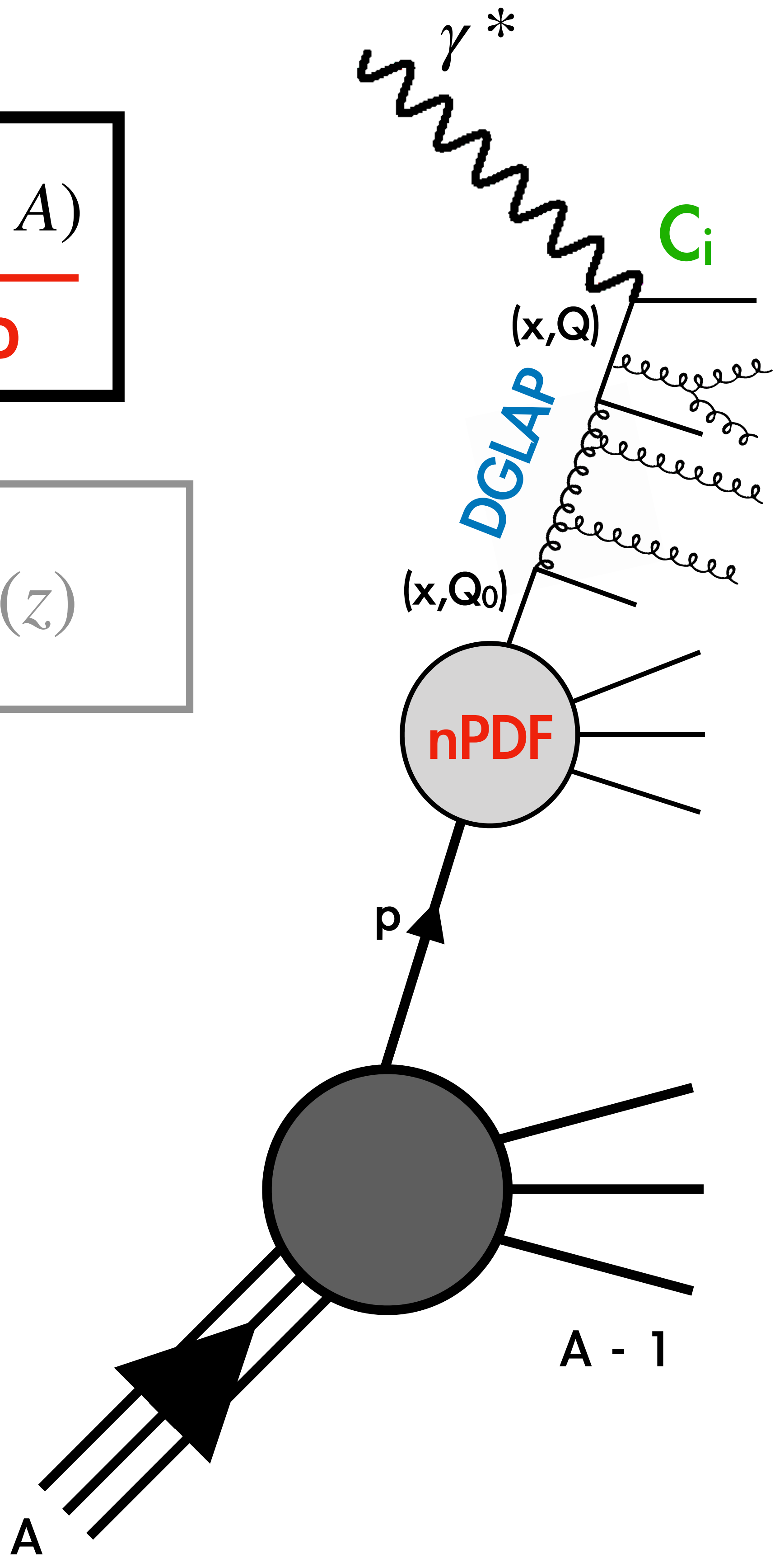

X

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pQCD — DGLAP — npQCD

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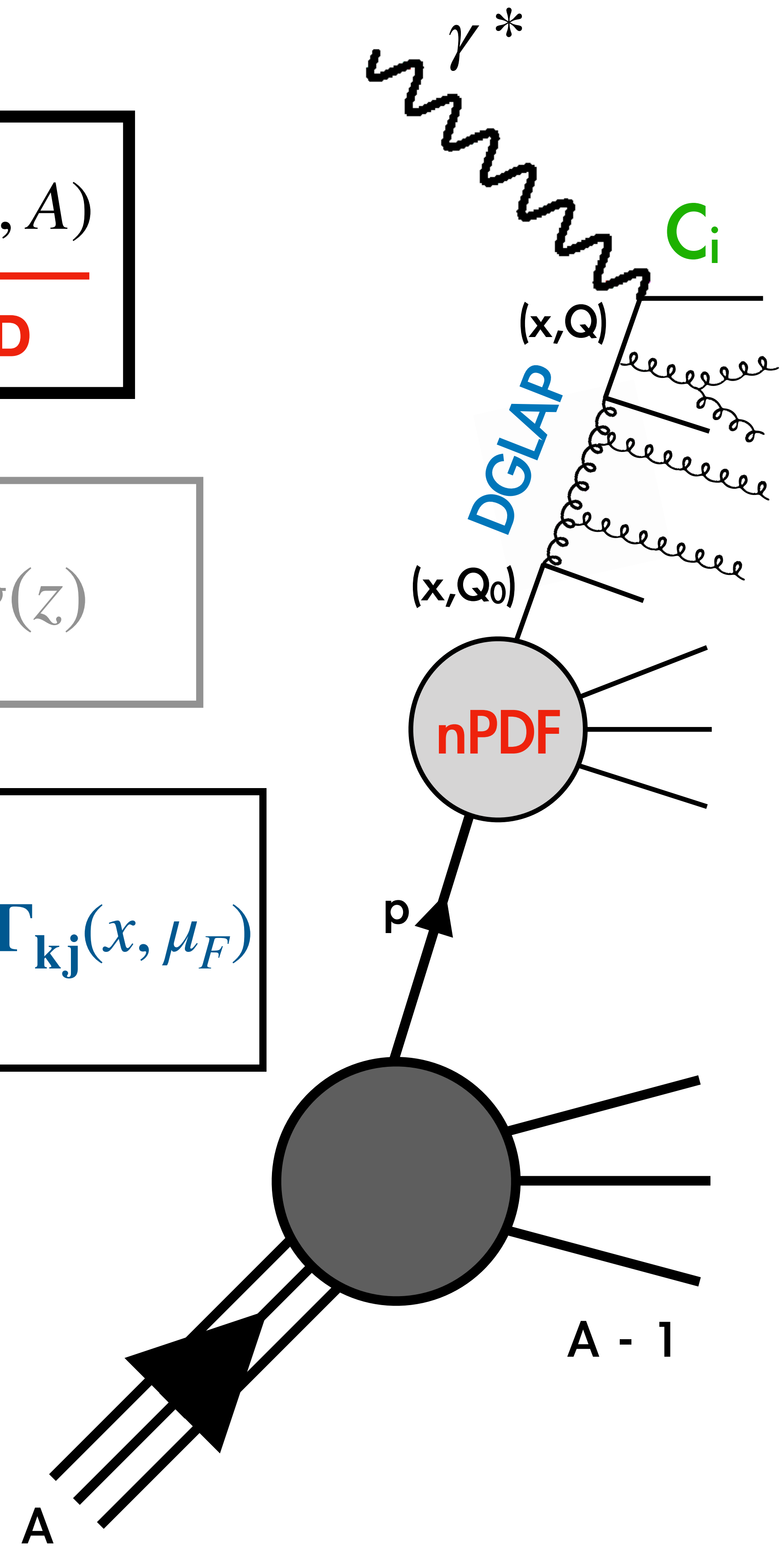
Convolution

$$f \otimes g = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) g(z)$$

DGLAP equation

$$\frac{\partial \Gamma_{ij}}{\partial \ln \mu_F^2} = \sum_k \frac{\alpha_s(\mu_F)}{4\pi} \left[P_{ik}^{(0)}(x) + \frac{\alpha_s(\mu_F)}{4\pi} P_{ik}^{(1)}(x) + \ldots \right] \otimes \Gamma_{kj}(x, \mu_F)$$



x

Nuclear NC Inclusive DIS

EM F2 (Q < MZ)

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pQCD DGLAP npQCD

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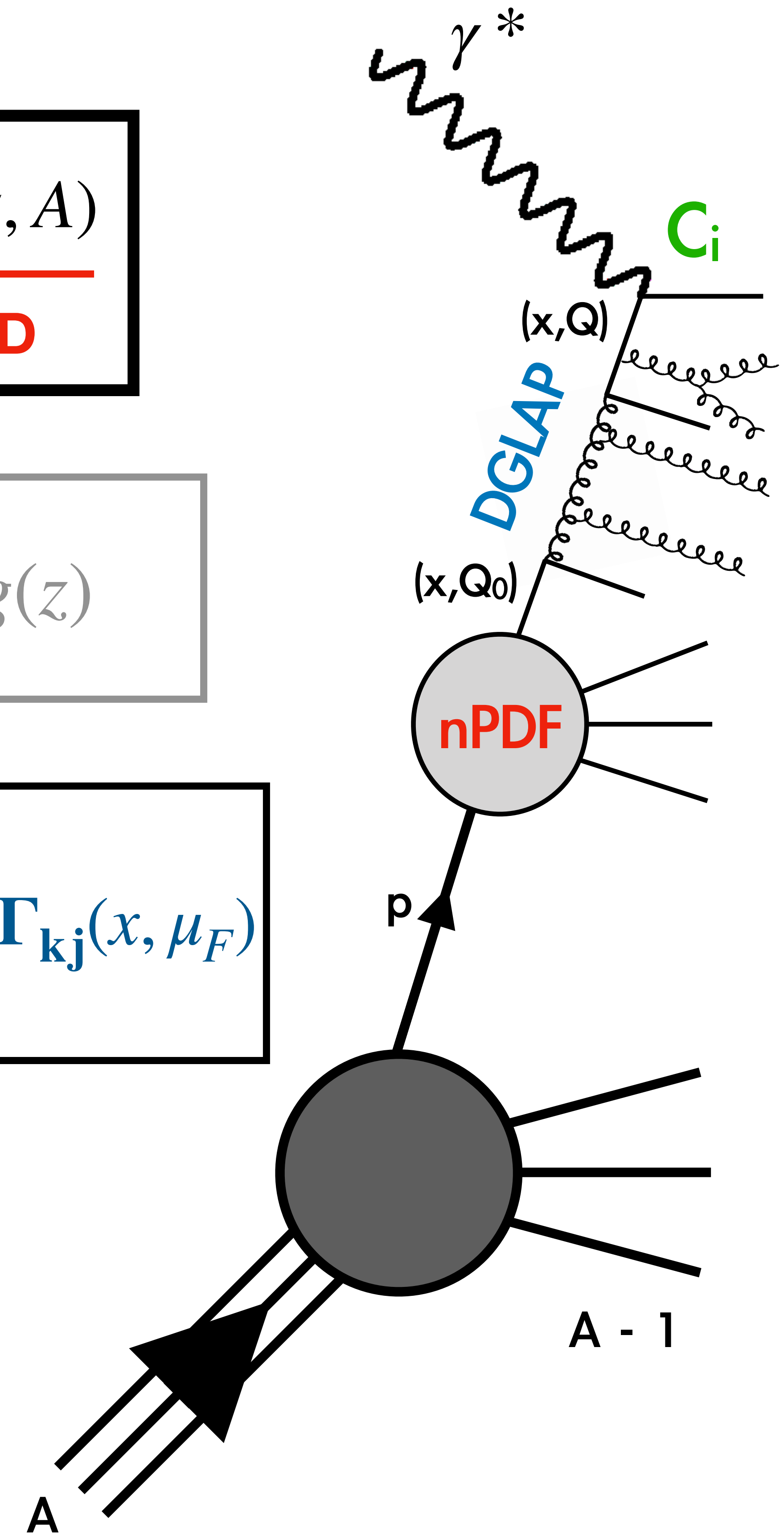
RG equation

$$\frac{\partial \alpha_s}{4\pi \cdot \partial \ln \mu_R^2} = -\left(\frac{\alpha_s(\mu_R)}{4\pi} \right)^2 \left[\beta_0 + \frac{\alpha_s(\mu_R)}{4\pi} \beta_1 + \ldots \right]$$



Nuclear NC Inclusive DIS

EM F2 (Q < MZ)

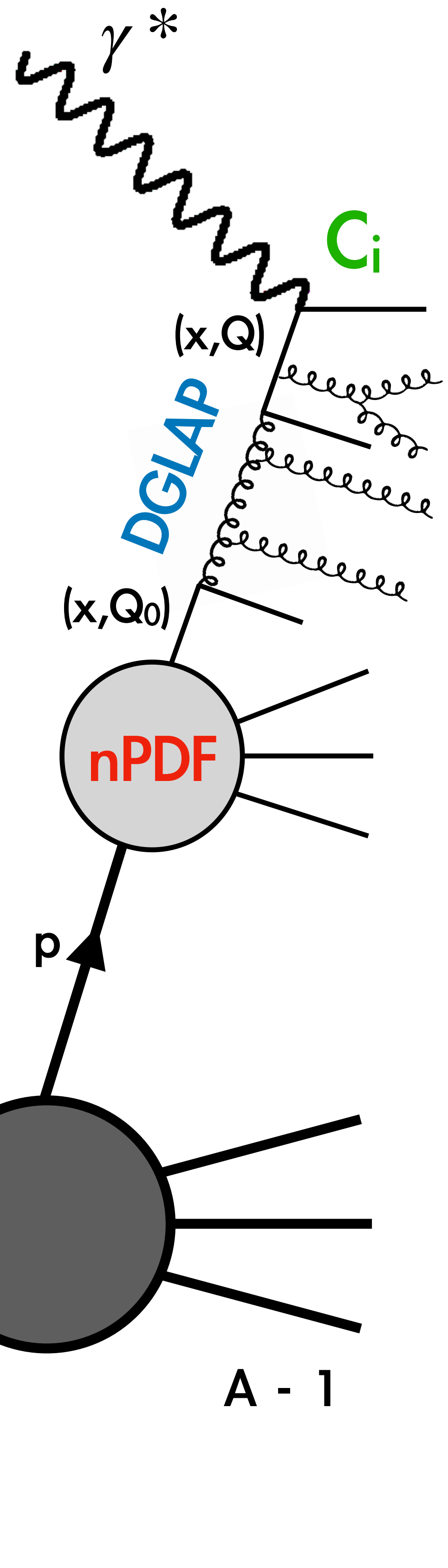
$$F_2(x,Q^2,A) = \sum_i^{n_f}\sum_j^{n_f} \underbrace{C_i(x,Q^2)}_{\text{pQCD}} \otimes \underbrace{\Gamma_{ij}(x,Q^2)}_{\text{DGLAP}} \otimes \underbrace{q_j(x,Q_0^2,A)}_{\text{npQCD}}$$

$$= C^S_{2,q}(x,\alpha_s(Q)) \otimes \Sigma(x,Q) \longrightarrow \text{Singlet}$$

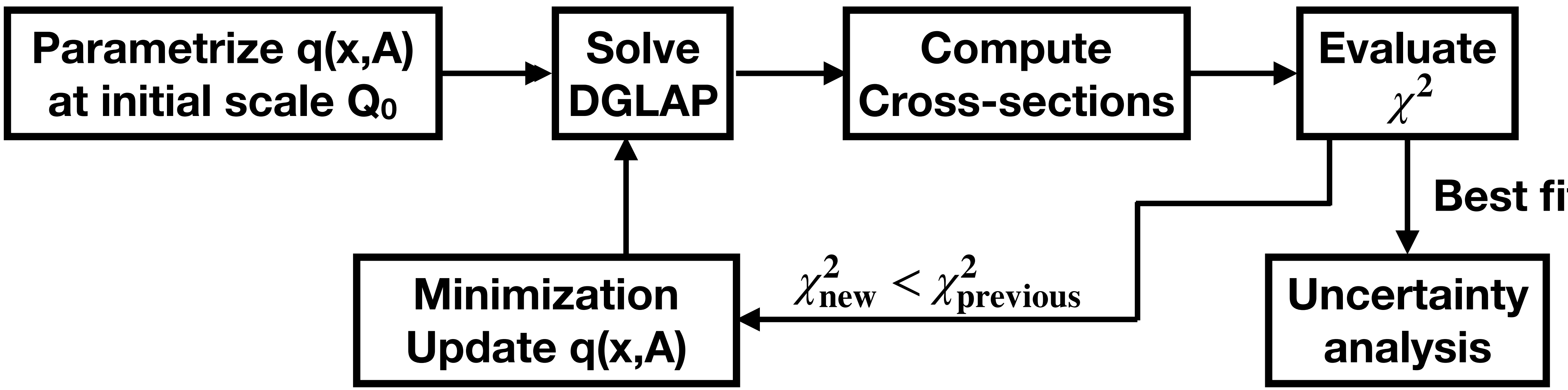
$$+ C^{NS}_{2,q}(x,\alpha_s(Q)) \otimes T(x,Q^2) \longrightarrow \text{Non-Singlet}$$

$$+ C^S_{2,g}(x,\alpha_s(Q)) \otimes g(x,Q^2) \longrightarrow \text{Gluon}$$

Theoretically, 3 Independent PDFs [backup]
Practically, depends on data kinematic range



Methodology



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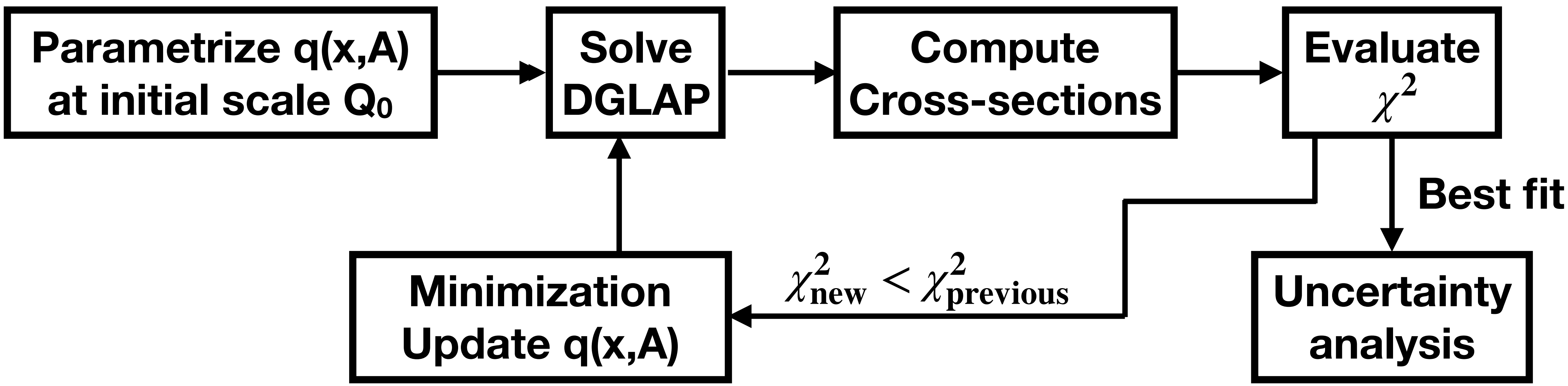
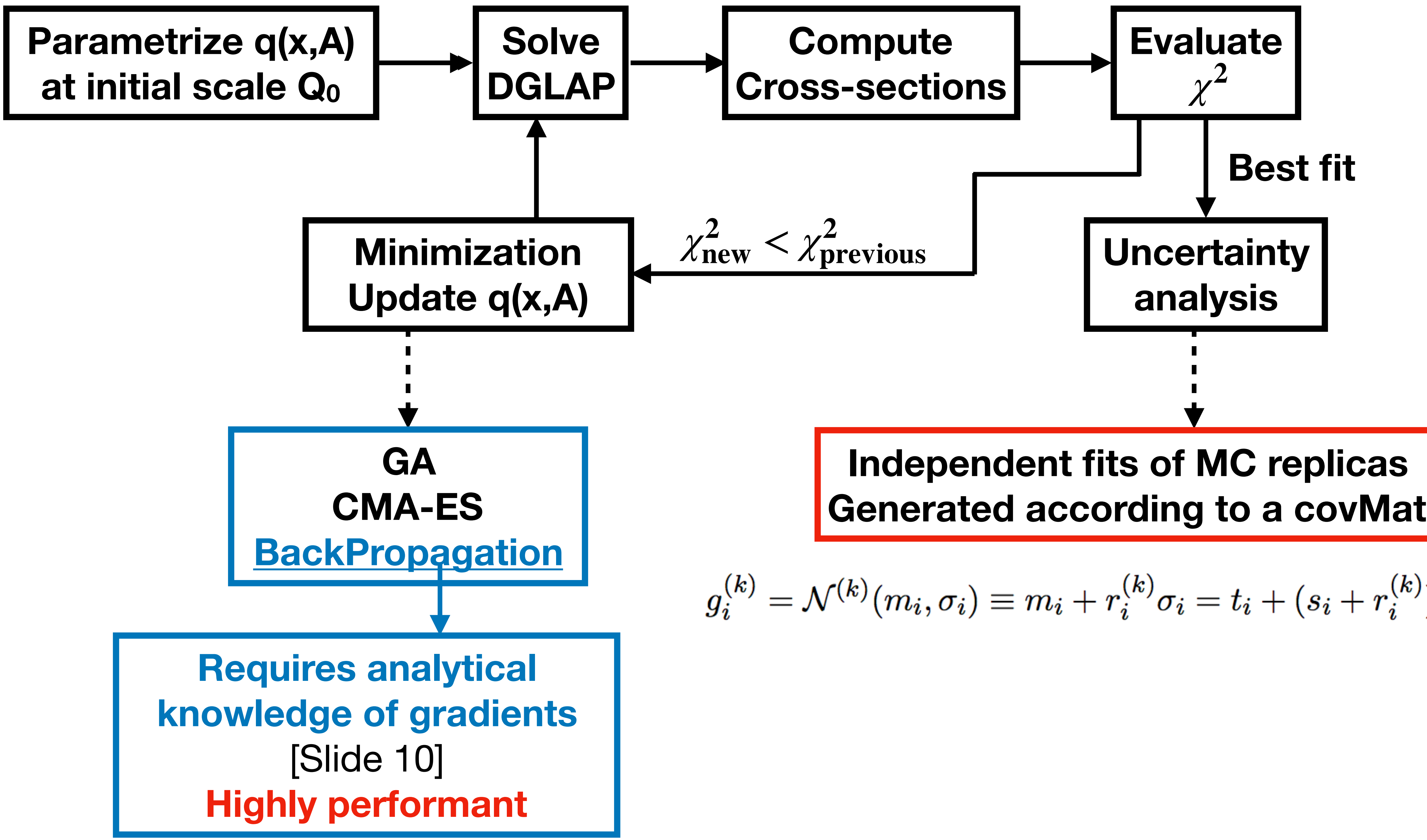


$$\chi^2 = \frac{1}{N_{data}} \sum_{n,m}^{N_{data}} (\hat{F}_2^{(n)} - F_2^{(n)})\, Cov_{nm}^{-1}\, (\hat{F}_2^{(m)} - F_2^{(m)})$$

Methodology



$$g_i^{(k)} = \mathcal{N}^{(k)}(m_i, \sigma_i) \equiv m_i + r_i^{(k)}\sigma_i = t_i + (s_i + r_i^{(k)})\sigma_i$$

Status of recent nPDFs

EPPS16 [1612.05741]

$$f_i^{p/A}(x,Q^2) = \underline{R_i^A}(x,Q^2) f_i^{p}(x,Q^2)$$

$$R_i^A(x,Q_0^2) = \begin{cases} a_0 + a_1(x-x_a)^2 & x \le x_a \\ b_0 + b_1 x^{\alpha} + b_2 x^{2\alpha} + b_3 x^{3\alpha} & x_a \le x \le x_e \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \le x \le 1, \end{cases}$$

nCTEQ15 [1509.00792]

$$x f_i^{p/A}(x,Q_0) = c_0\, x^{c_1} (1-x)^{c_2} e^{c_3 x} (1+e^{c_4} x)^{c_5},$$
$$\text{for} \quad i = u_v, d_v, g, \bar{u}+\bar{d}, s+\bar{s}, s-\bar{s},$$

$$\frac{\bar{d}(x,Q_0)}{\bar{u}(x,Q_0)} = c_0\, x^{c_1}(1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4}.$$

$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right),$$
$$k = \{1, \ldots, 5\}.$$

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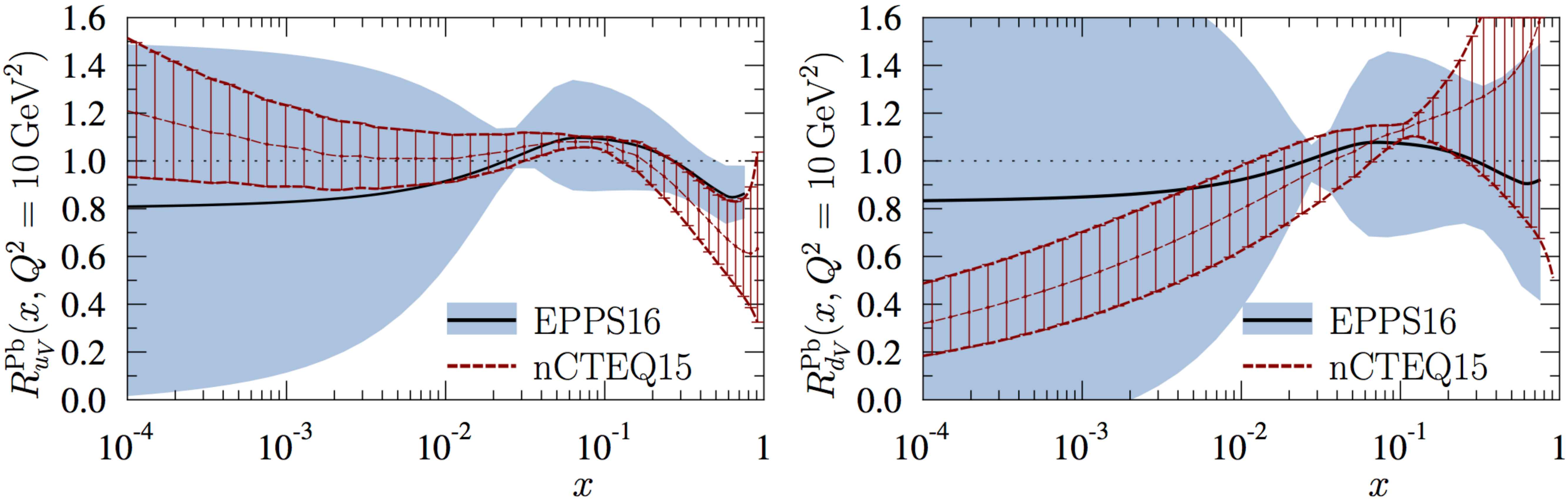

Parametrizations

EM F_2 ($Q < M_Z$)

$$F_2(x, Q^2, A) = \sum_i^{n_f} \sum_j^{n_f} C_i(x, Q^2) \otimes \Gamma_{ij}(x, Q^2) \otimes q_j(x, Q_0^2, A)$$

$$= \sum_i^{n_f} \widetilde{C}_i(x, Q^2) \otimes \mathbf{q_i(x, Q_0^2, A)}$$

X

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$$= \sum_i^{n_f} \widetilde{C}_i(x, Q^2) \otimes \mathbf{q_i(x, Q_0^2, A)}$$

$\widetilde{C}_i$ → **Pre-computed FK tables [NNPDF methodology]**

Parametrizations

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PDF - Proton case

$$q_i^p(x, Q_0^2) = xf_i(x, Q_0^2) \propto \mathbf{x}^\alpha(\mathbf{1 - x})^\beta \mathbf{NN(x)}$$

X

Parametrizations

EM F_2 ($Q < M_Z$)

$$F_2(x, Q^2, A) = \sum_i^{n_f} \sum_j^{n_f} C_i(x, Q^2) \otimes \Gamma_{ij}(x, Q^2) \otimes q_j(x, Q_0^2, A)$$

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PDF - Proton case

$$q_i^p(x, Q_0^2) = xf_i(x, Q_0^2) \propto \mathbf{x^\alpha(1 - x)^\beta NN(x)}$$

Option A

$$\mathbf{G(x, A, Q_0^2)} = A_G \cdot (1 - x)^{\alpha_G} \mathbf{NN_G(x, A)}$$

$$\mathbf{\Sigma(x, A, Q_0^2)} = (1 - x)^{\alpha_\Sigma} \cdot \mathbf{NN_\Sigma(x, A)}$$

$$\mathbf{T_8} = (1 - x)^{\alpha_{T8}} \cdot \mathbf{NN_{T8}(x, A)}$$

Where $$\mathrm{A_G} = \frac{1 - \int_0^1 \mathrm{x\Sigma(x, A)dx}}{\int_0^1 \mathrm{xG(x, A)dx}}$$

Parametrizations

Option B

$$q_i(x, A, Q_0^2) = \frac{1}{SR}[\mathbf{NN_i(x, A, Q_0^2)} - \mathbf{NN_i(x = 1, A, Q_0^2)}]$$

Where $\mathrm{SR} = \int_0^1 \mathrm{x[NN_G(x, A) + NN_\Sigma(x, A)]dx}$

Option A

$$\mathbf{G(x, A, Q_0^2)} = A_G \cdot (1 - x)^{\alpha_G} \mathbf{NN_G(x, A)}$$

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Where $\mathrm{A_G} = \frac{1 - \int_0^1 \mathrm{x\Sigma(x, A)dx}}{\int_0^1 \mathrm{xG(x, A)dx}}$

X

Parametrizations

Option B

$$q_i(x, A, Q_0^2) = \frac{1}{SR}[\mathbf{NN_i(x, A, Q_0^2) - NN_i(x = 1, A, Q_0^2)}]$$

$$\text{Where} \quad \mathrm{SR} = \int_0^1 \mathrm{x}[\mathrm{NN_G(x, A) + NN_\Sigma(x, A)}]\mathrm{dx}$$

Next Results are for:

Option A

$$\mathbf{G(x, A, Q_0^2)} = A_G \cdot (1 - x)^{\alpha_G} \mathbf{NN_G(x, A)}$$

$$\mathbf{\Sigma(x, A, Q_0^2)} = (1 - x)^{\alpha_\Sigma} \cdot \mathbf{NN_\Sigma(x, A)}$$

$$\mathbf{T_8} = (1 - x)^{\alpha_{T8}} \cdot \mathbf{NN_{T8}(x, A)}$$

$$\text{Where} \quad \mathrm{A_G} = \frac{1 - \int_0^1 \mathrm{x\Sigma(x, A)dx}}{\int_0^1 \mathrm{xG(x, A)dx}}$$

Neural Network

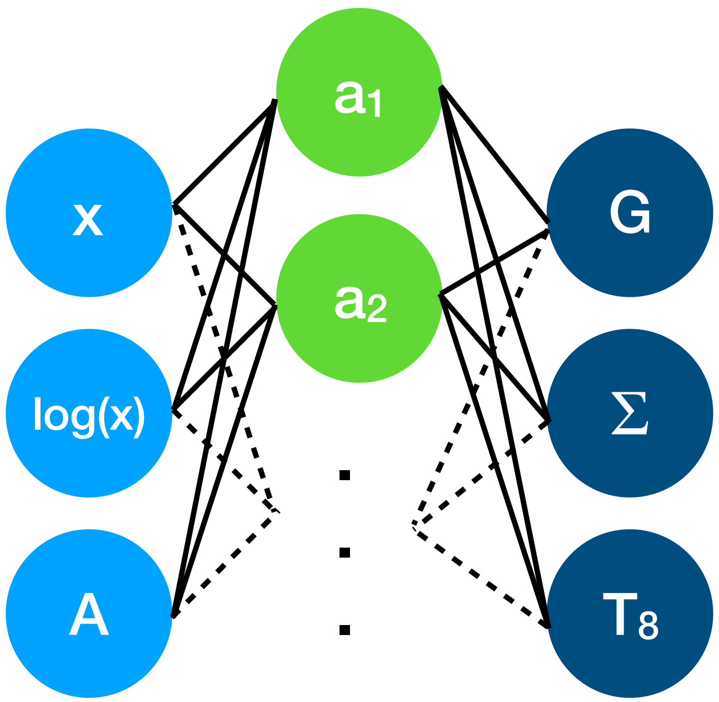
Option A

$$\mathbf{G}(\mathbf{x}, \mathbf{A}, \mathbf{Q_0^2}) = A_G \cdot (1 - x)^{\alpha_G} \mathbf{NN_G}(\mathbf{x}, \mathbf{A})$$

$$\mathbf{\Sigma}(\mathbf{x}, \mathbf{A}, \mathbf{Q_0^2}) = (1 - x)^{\alpha_\Sigma} \cdot \mathbf{NN_\Sigma}(\mathbf{x}, \mathbf{A})$$

$$\mathbf{T_8} = (1 - x)^{\alpha_{T8}} \cdot \mathbf{NN_{T8}}(\mathbf{x}, \mathbf{A})$$

$$\text{Where} \quad \mathrm{A_G} = \frac{1 - \int_0^1 \mathrm{x\Sigma(x, A)dx}}{\int_0^1 \mathrm{xG(x, A)dx}}$$



Neural Network

$$\mathbf{NN_i}(\mathbf{x}; \{\mathbf{w_{ij}^{(\ell)}}, \mathbf{b_i^{(\ell)}}\}) = \sigma_L \left(\sum_j^{N_{L-1}} w_{ij}^{(L)} a_j^{(L-1)} + b_i^{(L)} \right)$$

$$= \sigma_L \left(\sum_j^{N_{L-1}} w_{ij}^{(L)} \sigma_{L-1} \left(\sum_k^{N_{L-2}} w_{jk}^{(L-1)} a_k^{(L-2)} + b_j^{(L-1)} \right) + b_i^{(L)} \right) = \ldots$$

Backpropagation

Gradient Descent

$$w_{ij} = w_{ij} - \frac{\eta}{N_{data}} \sum_{n}^{N_{data}} \frac{\partial \chi^{2(n)}}{\partial w_{ij}}$$

$$b_i = b_i - \frac{\eta}{N_{data}} \sum_{n}^{N_{data}} \frac{\partial \chi^{2(n)}}{\partial b_i}$$

χ^2 Derivatives

$$\frac{\partial \chi^2}{\partial w_{mn}^{(\ell)}} = 2\left(\frac{\widetilde{\mathbf{C}} \otimes \mathbf{NN} - F}{\sigma^2}\right) \widetilde{\mathbf{C}} \otimes \frac{\partial \mathbf{NN}}{\partial w_{mn}^{(\ell)}}$$

$$\frac{\partial \chi^2}{\partial b_{m}^{(\ell)}} = 2\left(\frac{\widetilde{\mathbf{C}} \otimes \mathbf{NN} - F}{\sigma^2}\right) \widetilde{\mathbf{C}} \otimes \frac{\partial \mathbf{NN}}{\partial b_{m}^{(\ell)}}$$

NN Derivatives

$$\frac{\partial \mathbf{NN}}{\partial w_{ij}^{(\ell)}} = \left[\prod_{\alpha=L}^{\ell+1} \mathbf{W}^{(\alpha)}\right] \cdot \frac{\partial \mathbf{a}^{(\ell)}}{\partial \mathbf{w}_{\mathbf{ij}}^{(\ell)}}$$

$$\frac{\partial \mathbf{NN}}{\partial b_{i}^{(\ell)}} = \left[\prod_{\alpha=L}^{\ell+1} \mathbf{W}^{(\alpha)}\right] \cdot \frac{\partial \mathbf{a}^{(\ell)}}{\partial \mathbf{b}_{\mathbf{i}}^{(\ell)}}$$

Neurons Derivatives

$$\frac{\partial \mathbf{a}_{\mathbf{k}}^{(\ell)}}{\partial \mathbf{w}_{\mathbf{ij}}^{(\ell)}} = \sigma'_{\ell}\left(z_k^{(\ell)}\right) \delta_{ik} a_j^{(\ell-1)}$$

$$\frac{\partial \mathbf{a}_{\mathbf{k}}^{(\ell)}}{\partial \mathbf{b}_{\mathbf{i}}^{(\ell)}} = \sigma'_{\ell}\left(z_k^{(\ell)}\right) \delta_{ik}$$

Results: Theory Benchmark

FK tables computation

EM F_2 ($Q < M_Z$)

$$F_2(x, Q^2, A) = \sum_i^{n_f} \widetilde{C}_i(x, Q^2) \otimes \mathbf{q_i}(\mathbf{x}, \mathbf{Q_0^2}, \mathbf{A})$$

$\chi^2_{\mathbf{total}}$

DataSet	APFELcomb	EPPS16	Ndata
NMC (He/D)	17.76	18.0	16/18
NMC (Li/D)	19.64	18.4	15/24
NMC (C/D)	25.55	25.70	31/42
NMC (Ca/D)	28.49	27.6	15/18

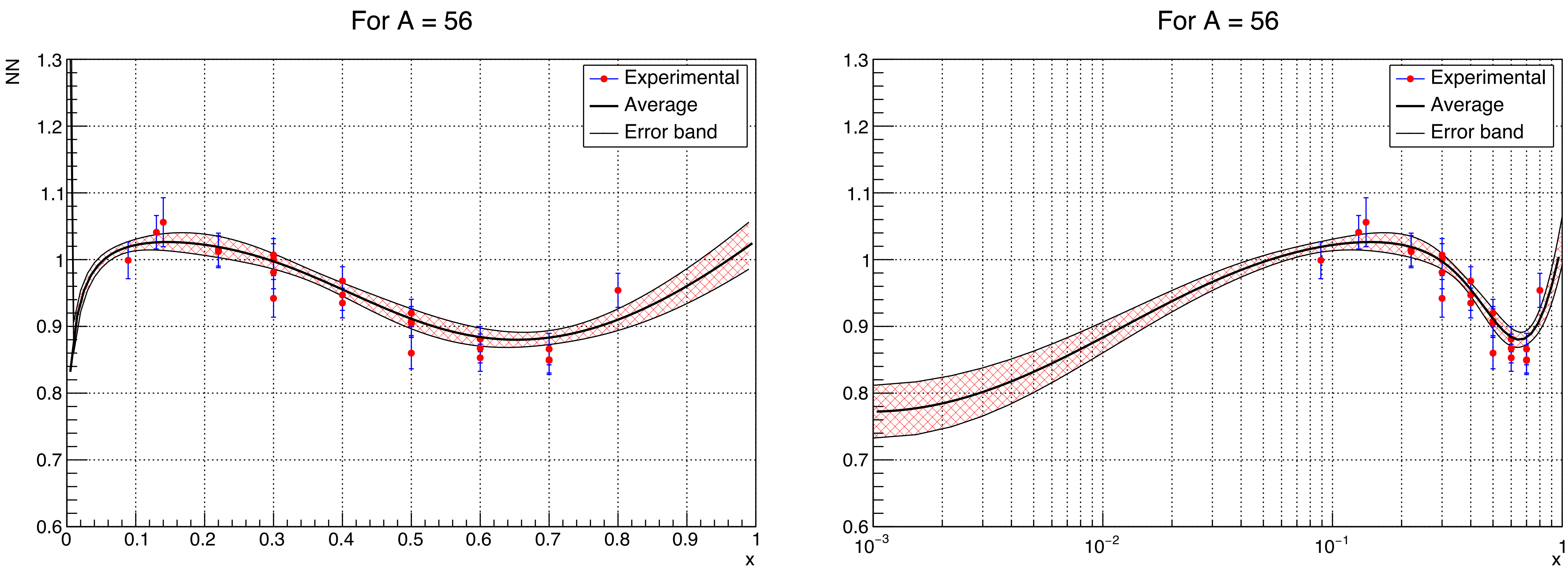
Results: WarmUp

F_2^A/F_2^D Ratio Fit

Analysis

$$NN(x, A) = \frac{F_2^A(x)}{F_2^D(x)}$$

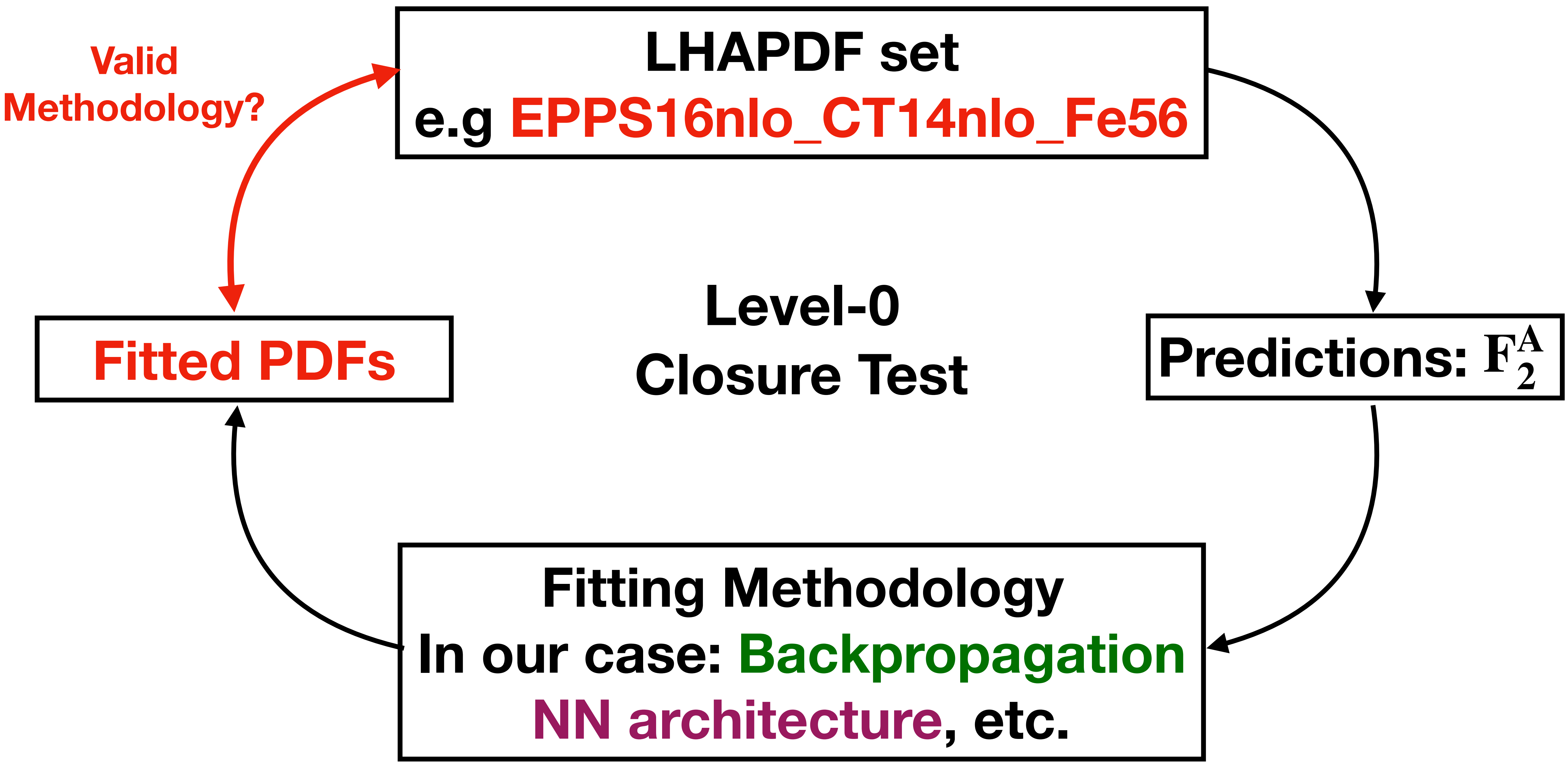
2D fit in (A, x) - Uncorrelated uncertainty



Results: Closure Test

NNPDF3.0 [1410.8849]

Concept



Results: Closure Test

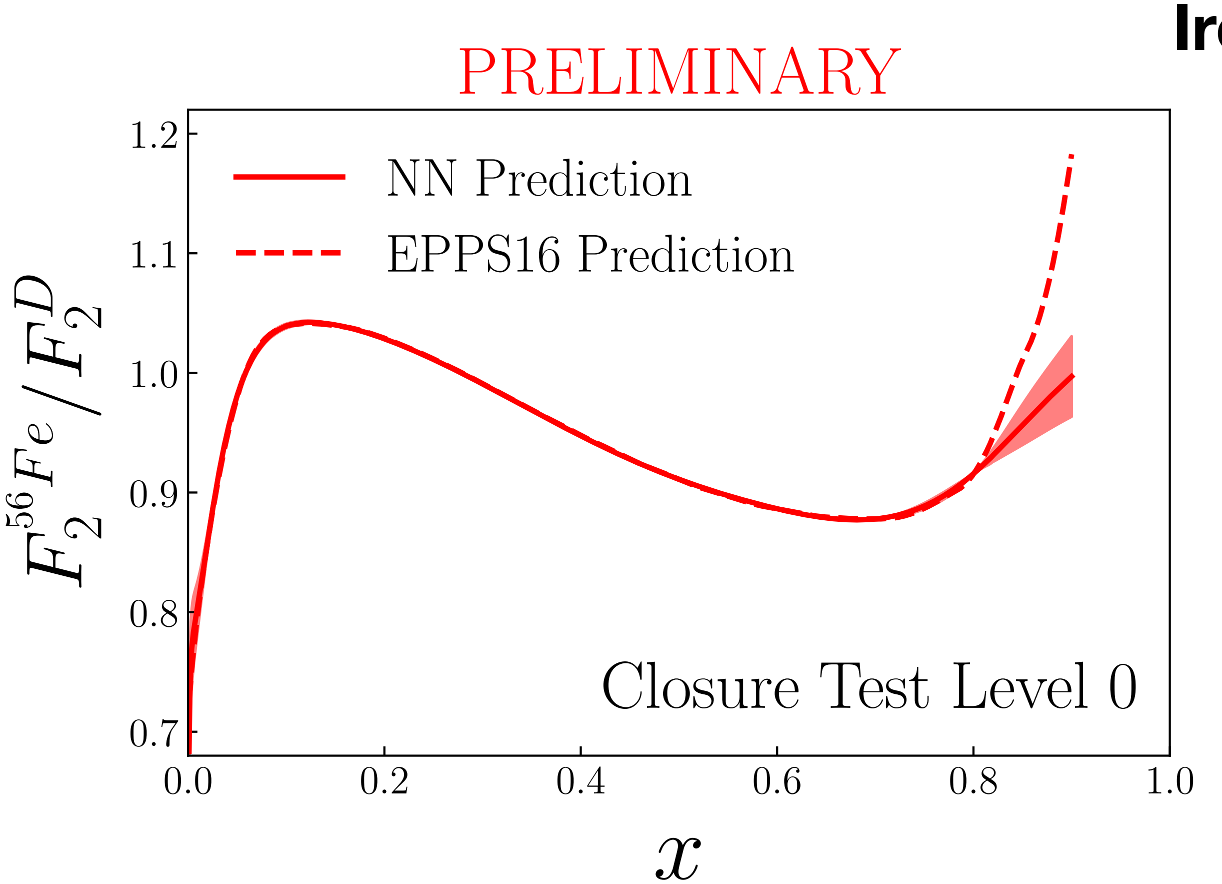
NNPDF3.0 [1410.8849]

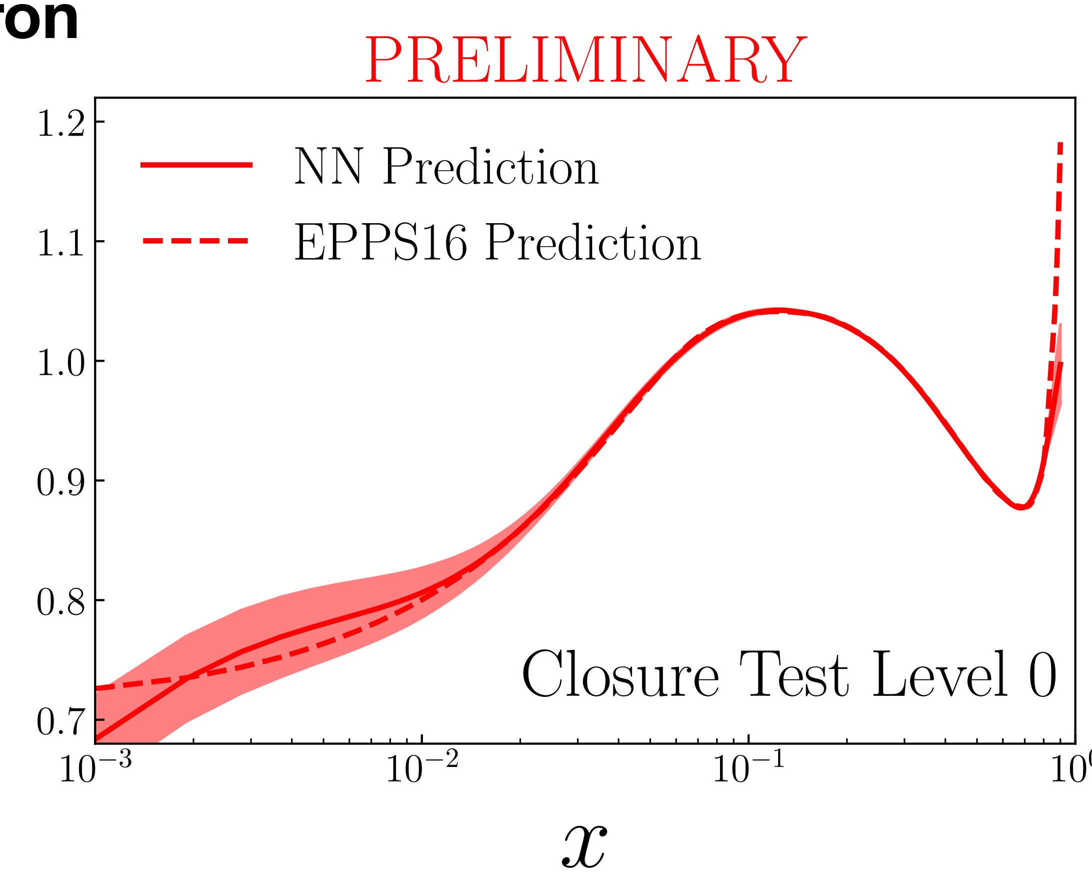
nPDF Fits: Observable Comparison

Analysis

Predictions $\frac{F_2^A}{F_2^D}$ → **EPPS16** / → **CT14nlo**	**Same theory** **Mass scheme, quarks mass…**
5 different Fits (TensorFlow + ADAM optimiser)	

Iron





Results: Closure Test

NNPDF3.0 [1410.8849]

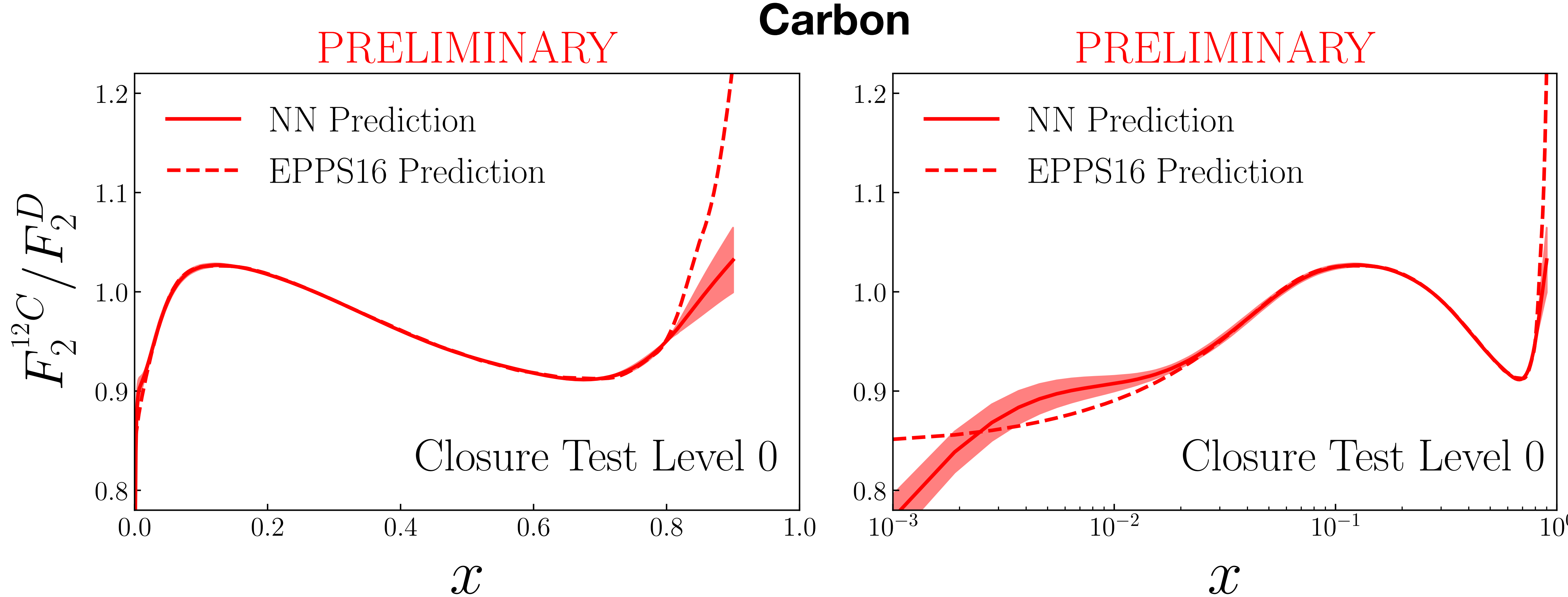
nPDF Fits: Observable Comparison

Analysis

Predictions $\frac{F_2^A}{F_2^D}$ F_2^A → **EPPS16**, F_2^D → **CT14nlo**	**Same theory** **Mass scheme, quarks mass…**

5 different Fits (TensorFlow + ADAM optimiser)

Carbon



Results: Closure Test

NNPDF3.0 [1410.8849]

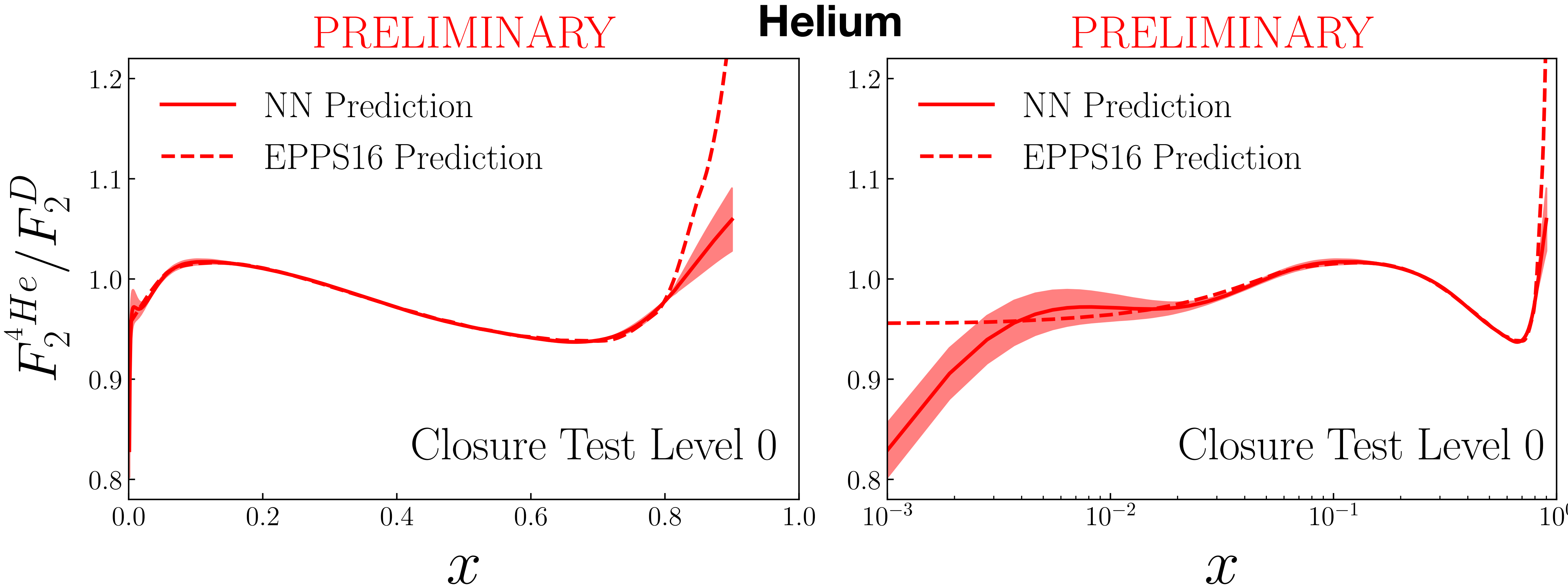
nPDF Fits: Observable Comparison

Analysis

Predictions $\frac{F_2^A}{F_2^D}$	Same theory
F_2^A → EPPS16 F_2^D → CT14nlo	Mass scheme, quarks mass…

5 different Fits (TensorFlow + ADAM optimiser)

Helium



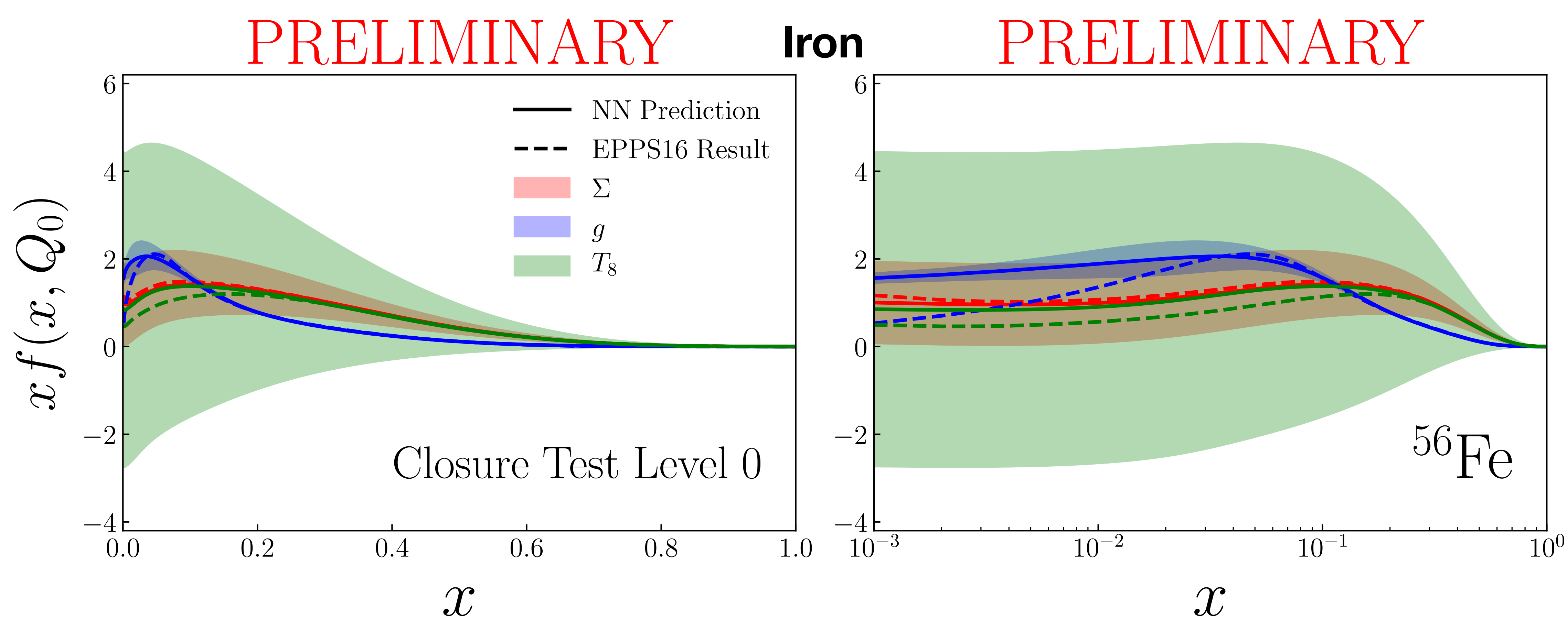
Results: Closure Test

NNPDF3.0 [1410.8849]

nPDF Fits: PDF Comparison

Analysis

Predictions $\frac{F_2^A}{F_2^D}$ → **EPPS16** / → **CT14nlo**	**Same theory** **Mass scheme, quarks mass…**
5 different Fits (TensorFlow + ADAM optimiser)	



Results: Closure Test

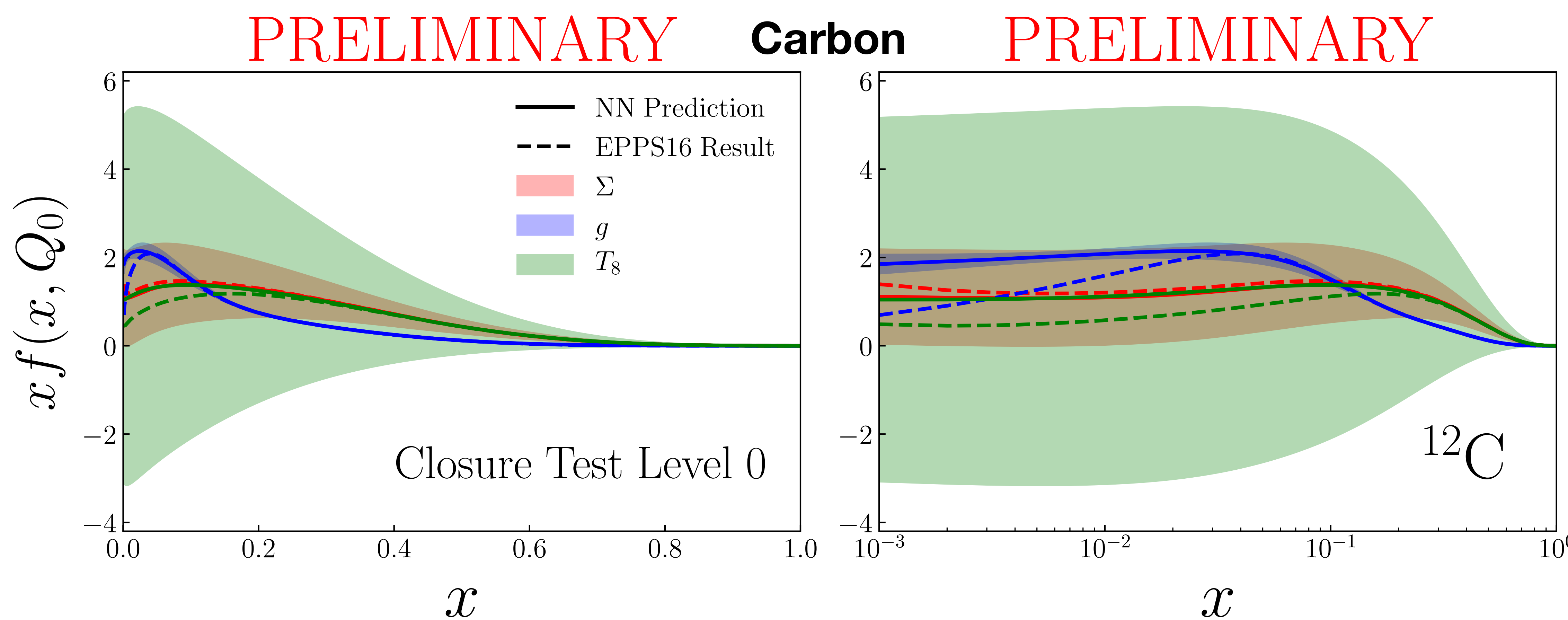
NNPDF3.0 [1410.8849]

nPDF Fits: PDF Comparison

Analysis

Predictions $\dfrac{F_2^A}{F_2^D}$	$F_2^A \rightarrow$ **EPPS16** $F_2^D \rightarrow$ **CT14nlo**	**Same theory** **Mass scheme, quarks mass…**

5 different Fits (TensorFlow + ADAM optimiser)



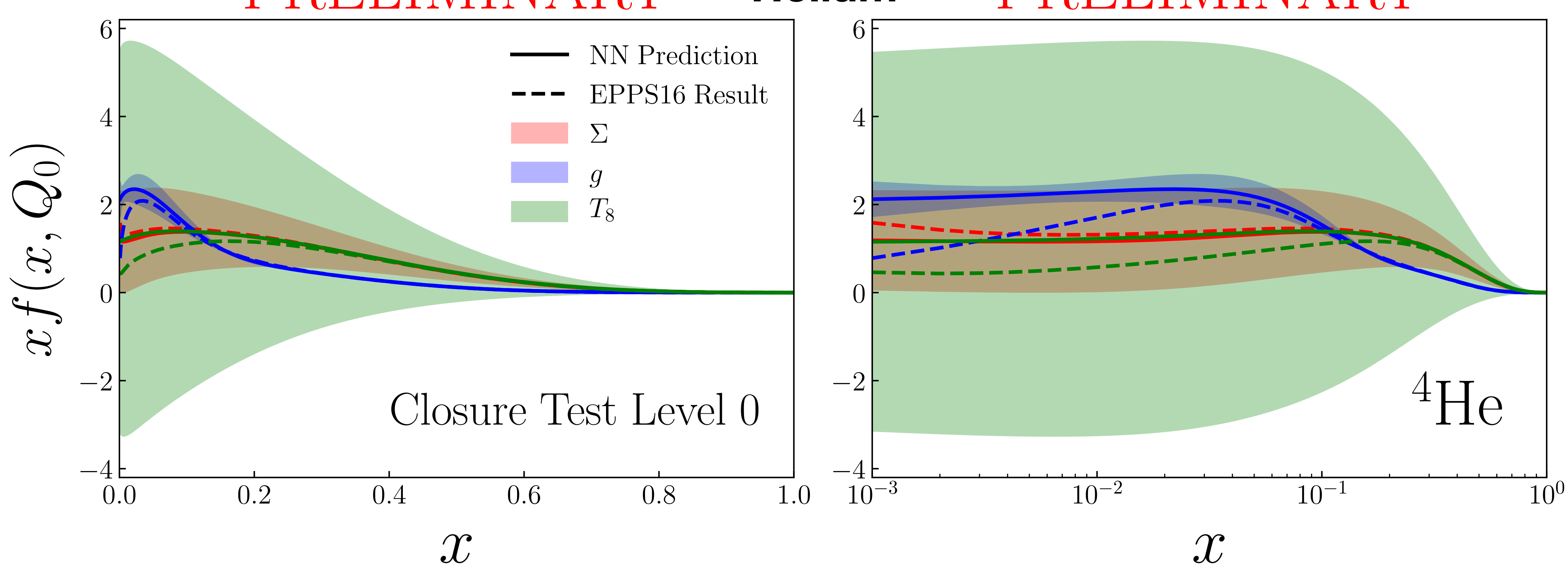
Results: Closure Test

NNPDF3.0 [1410.8849]

nPDF Fits: PDF Comparison

Analysis

Predictions $\frac{F_2^A}{F_2^D}$ F_2^A → **EPPS16**, F_2^D → **CT14nlo**	**Same theory** **Mass scheme, quarks mass…**
5 different Fits (TensorFlow + ADAM optimiser)	



Conclusion

- **Theory Benchmarked with EPPS16, On going for nCTEQ15.**

- **Constraints implemented on the level of the fit.**

- **Successful Level 0 closure tests using EPPS16**
  1. First use of Backpropagation to fit PDFs.
  2. Framework in place to perform DIS-only fits.

Outlook

- **Full-fledged Error analysis:**
  1. Correlated uncertainties.
  2. MC replica generation.

- **Better way of implementing constraints?**

- **Adding DY and additional processes to the fit.**

Backup

$$F_2(x,Q^2,A) = \sum_i^{n_f} C_i(x,Q^2) \otimes q_i(x,Q^2,A) \quad (2)$$

where $\otimes$ denotes the Mellin convolution defined as:

$$f(x) \otimes g(x) = \int_0^1 dy \int_0^1 dz f(y)g(z)\delta(x-yz) = \int_x^1 \frac{dy}{y} f(y) g(\frac{x}{y}) = \int_x^1 \frac{dz}{z} f(\frac{x}{z}) g(z) \quad (3)$$

We can factorize the nPDF dependance out of the convolution via expansion over a set of interpolating functions, spanning Q^2 and x such as:

$$q_i(x,Q^2,A) = \sum_\beta \sum_\tau q_{i,\beta\tau} I_\beta(x) I_\tau(Q^2) \quad (4)$$

where the nPDF $q_{i,\beta\tau}$ may be expressed as a product of nPDF at some initial fitting scale Q_0 and an evolution operator obtained by the solution of the DGLAP equation via the interpolation procedure as:

$$q_{i,\beta\tau} \equiv q_i(x_\beta, Q^2_\tau, A) = \sum_j \sum_\alpha \Gamma^\tau_{ij,\alpha\beta}\, q_j(x_\alpha, Q^2_0, A) \quad (5)$$

Finally:

$$F_2(x,Q^2,A) = \sum_i^{n_f} C_i(x,Q^2) \otimes [\sum_\alpha \sum_\tau \sum_j \sum_\beta \Gamma^\tau_{ij,\alpha\beta}\, q_j(x_\alpha, Q^2_0) I_\beta(x) I_\tau(Q^2)] \quad (6)$$

contracting the sum over τ, i and β then replacing j with i :

$$F_2(x,Q^2) = \sum_i^{n_f} \sum_\alpha^{n_x} FK_{i,\alpha}(x, x_\alpha, Q^2, Q^2_0) q_i(x_\alpha, Q^2_0) \quad (7)$$

where:

$$FK_{j,\alpha} = \sum_i C_i(x,Q^2) \otimes [\sum_\tau \sum_\beta \Gamma^\tau_{ij,\alpha\beta}\, I_\beta(x) I_\tau(Q^2)] \quad (8)$$

Backup

At LO, DGLAP basis

$$F_2^{\gamma}(x, Q^2, A) = \frac{1}{A}\left(Z\,\mathbf{F_2^p} + (A-Z)\,\mathbf{F_2^n}\right)$$

$$= \frac{5}{18}\Sigma - \left(\frac{\mathbf{Z}}{\mathbf{3A}} - \frac{1}{6}\right) T_3 + \frac{1}{18}(T_8 - T_{15}) + \frac{1}{30}(T_{24} - T_{35})$$

$$= \frac{2}{9}\Sigma - \left(\frac{\mathbf{Z}}{\mathbf{3A}} - \frac{1}{9}\right) T_3$$

Evolution basis

$$\Sigma = \sum_{i=1}^{n_f} q_i^+ \quad \textbf{where: } q^{\pm} = q \pm \bar{q}$$

$$T_3 = u^+ - d^+$$

$$T_8 = u^+ + d^+ - 2s^+$$

$$T_{15} = u^+ + d^+ + s^+ - 3c^+$$

$$T_{24} = u^+ + d^+ + s^+ + c^+ - 4b^+$$

$$T_{35} = u^+ + d^+ + s^+ + c^+ + b^+ - 5t^+$$

Evolution ($Q_0 < M_c$)

$$\Sigma(Q^2) = \Gamma_{qq}\,\Sigma(Q_0^2) + \Gamma_{qg}\,g(Q_0^2)$$

$$g(Q^2) = \Gamma_{gq}\,\Sigma(Q_0^2) + \Gamma_{gg}\,g(Q_0^2)$$

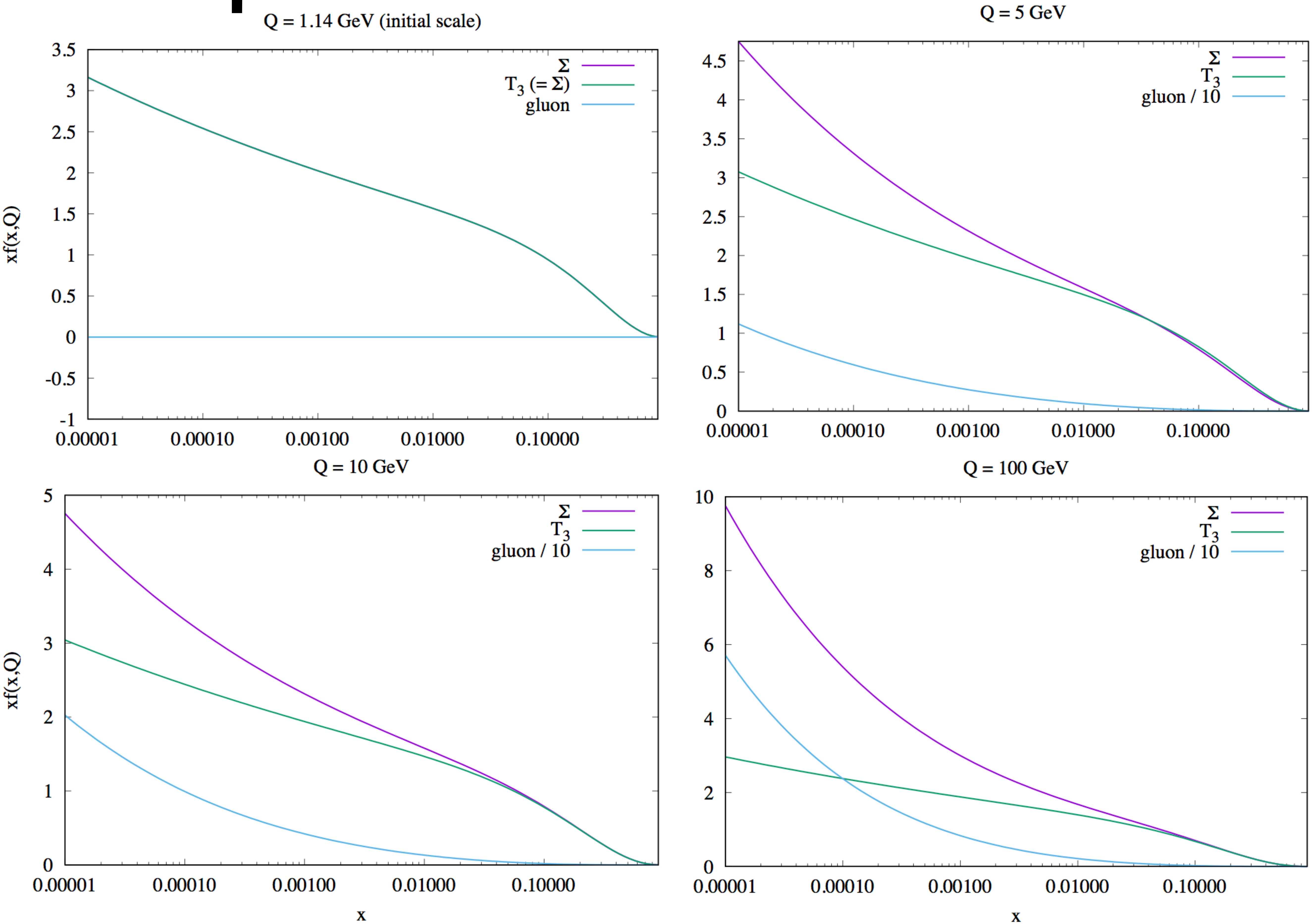
$$\Gamma^+ = \Gamma_{T_3} = \Gamma_{T_8}$$

$$T_{15}(Q_0^2) = T_{24}(Q_0^2) = T_{35}(Q_0^2) = \Sigma(Q_0^2)$$

3 independent distributions

$$\Sigma,\ g,\ T_3$$

Backup


Q = 1.14 GeV (initial scale)
Σ
T3 (= Σ)
gluon
xf(x,Q)
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0.00001
0.00010
0.00100
0.01000
0.10000
Q = 5 GeV
Σ
T3
gluon / 10
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Q = 10 GeV
5
4
3
2
1
0
x
Q = 100 GeV
10
8
6
4
2
0

Results (2) - WarmUp

Constraints on A-dependance

Testing the assumption $\mathrm{NN(x,A)} = A^n f(x)$ **via** $\dfrac{d\,ln(NN(x,A))}{d\,ln(A)} = n(x)$

For x = 0.121258