

STRUCTURE FUNCTIONS

& PARTON DISTRIBUTIONS

STEFANO FORTE
UNIVERSITÀ DI MILANO

BARYONS 2004

PARIS, OCTOBER 27, 2004

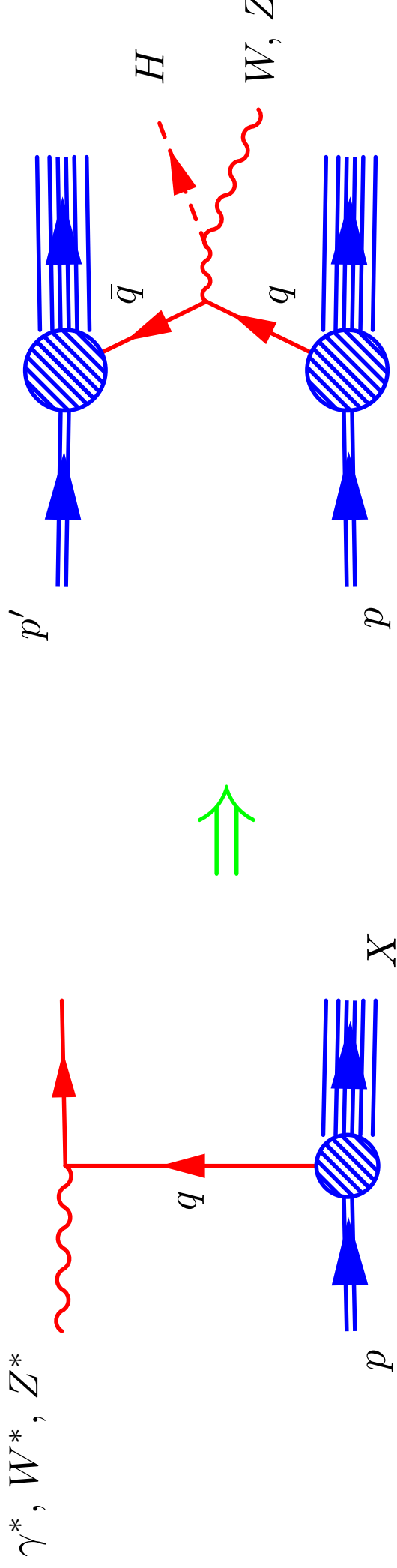
THE PARTON REVOLUTION:

THE ACCURATE COMPUTATION OF PHYSICAL PROCESS AT A HADRON COLLIDER
REQUIRES GOOD KNOWLEDGE OF PARTON DISTRIBUTIONS OF THE NUCLEON

THE PARTON REVOLUTION:

THE ACCURATE COMPUTATION OF PHYSICAL PROCESS AT A HADRON COLLIDER
REQUIRES GOOD KNOWLEDGE OF PARTON DISTRIBUTIONS OF THE NUCLEON

FACTORIZATION



IN ORDER TO EXTRACT THE RELEVANT PHYSICS SIGNAL,

WE NEED TO KNOW THE PARTON DISTRIBUTIONS AND THEIR UNCERTAINTY

PARTON DISTRIBUTIONS PROVIDE THE BRIDGE BETWEEN
HADRONIC PHYSICS AND PHYSICS AT THE ENERGY FRONTIER

AN ONGOING EFFORT...

HERA AND THE LHC

A workshop on the implications of HERA for LHC physics

March 2004 - January 2005

Parton density functions

Multijet final states
and energy flow

Heavy quarks

Diffraction

Monte Carlo tools



Startup Meeting
March 26-27 2004
Midterm Meeting
11-13 October 2004
CERN, Geneva

Final Meeting
January 2005
DESY, Hamburg

Organising Committee:

G. Altarelli (CERN), J. Blumlein (DESY),
M. Dole (JAXA), L. Motte-White (DESY),
A. Olschok (CERN) (chair), K. Eppert (CERN),
M. Jung (Lund/DESY) (chair), M. Sgarbi (CERN),
A. Marsh (CERN), P. Newman (Birmingham),
G. Passale (JLAB), O. Schneider (IPSA),
R. Yoshida (JAEA).

Advisory Committee:

J. Bartels (Hamburg), M. Goffin (DESY),
J. Ellis (CERN), J. Engelen (CERN),
G. Gustafson (Lund), G. Ingelman (Uppsala),
P. Jena (CERN), R. Klanner (DESY),
M. Klein (DESY), L. Motte-White (DESY),
T. Nakaya (CERN), D. Schmitt (CERN),
F. Schwenn (DESY), J. Sjostrand (CERN),
J. Stirling (Durham, UK), T. Torgersen (JLab),
A. Wiggles (DESY), R. Yndurain (JLab).

www.desy.de/~heralhc

heralhc.workshop@cern.ch

SUMMARY

- DETERMINING PARTON DISTRIBUTIONS:

- **PHENOMENOLOGY**: FROM DEEP-INELASTIC STRUCTURE FUNCTIONS TO

HADRON COLLIDERS

- **THEORY**: HIGHER ORDER CORRECTIONS AND RESUMMATIONS

- AN INTERLUDE: THE NUTeV ANOMALY

- **PRECISION PHYSICS** WITH PDFs COMES OF AGE

- THE UNCERTAINTY ON PARTON DISTRIBUTIONS

- **THE STATE OF THE ART**: PDFs WITH ERRORS AND THEIR LIMITATIONS

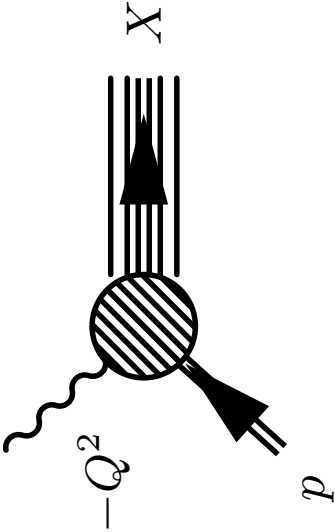
- **THE FUTURE?**: MONTECARLO PARTONS AND NEURAL PARTONS

DEEP-INELASTIC SCATTERING

STRUCTURE FUNCTIONS...

$$\gamma^*, W^*, Z^*$$

$$\text{Lepton fractional energy loss: } y = \frac{p \cdot q}{p \cdot k};$$



$$\text{Bjorken } x: x = \frac{Q^2}{2p \cdot q}$$

$$\text{lepton-nucleon CM energy: } s = \frac{Q^2}{xy};$$

$$\text{virtual boson-nucleon CM energy } W^2 = Q^2 \frac{1-x}{x};$$

$$\frac{d^2 \sigma^{\lambda_p \lambda_\ell}(x, y, Q^2)}{dx dy} = \frac{G_F^2}{2\pi(1 + Q^2/m_W^2)^2} \frac{Q^2}{xy} \left\{ \left[-\lambda_\ell y \left(1 - \frac{y}{2} \right) x F_3(x, Q^2) + (1-y) F_2(x, Q^2) \right] \right. \\ \left. + y^2 x F_1(x, Q^2) \right] - 2\lambda_p \left[-\lambda_\ell y(2-y)x g_1(x, Q^2) - (1-y) g_4(x, Q^2) - y^2 x g_5(x, Q^2) \right] \Bigg\}$$

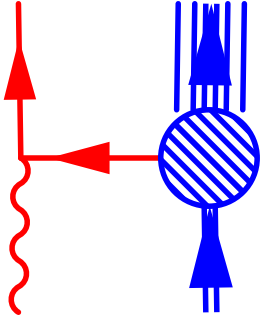
$$\lambda_l \rightarrow \text{lepton helicity}$$

$$\lambda_p \rightarrow \text{proton helicity}$$

	PARITY CONS.	PARITY VIOL.
UNPOL.	F_1, F_2	F_3
POL.	g_1	g_4, g_5

...AND PARTON DISTRIBUTIONS

$$\text{STRUCTURE FUNCTION}=\text{HARD COEFF.}\otimes\text{PARTON DISTN.}$$



$$F_2^{\text{NC}}(x,Q^2)=x\sum_{\text{flav. }i}e_i^2(q_i+\bar q_i)+\alpha_s\left[C_i[\alpha_s]\otimes(q_i+\bar q_i)+C_g[\alpha_s]\otimes g\right]$$

q_i quark, $\bar q_i$ antiquark, g gluon

LEADING PARTON CONTENT (up to $O[\alpha_s]$ corrections)

$$\begin{array}{lll} q_i \equiv q_i^{\uparrow\uparrow} + q_i^{\uparrow\downarrow} & \Delta q_i \equiv q_i^{\uparrow\uparrow} - q_i^{\uparrow\downarrow} & \\ \text{NC} \quad F_1^{\gamma,Z} = \sum_i e_i^2 (q_i + \bar q_i) & g_1^{\gamma,Z} = \sum_i e_i^2 (\Delta q_i + \Delta \bar q_i) & \\ \text{CC} \quad F_1^{W^+} = \bar u + d + s + \bar c & g_1^{W^+} = \Delta \bar u + \Delta d + \Delta s + \Delta \bar c & \\ \text{CC} \quad -F_3^{W^+} / 2 = \bar u - d - s + \bar c & g_5^{W^+} = \Delta \bar u - \Delta d - \Delta s + \Delta \bar c & \end{array}$$

$$F_2 = 2xF_1$$

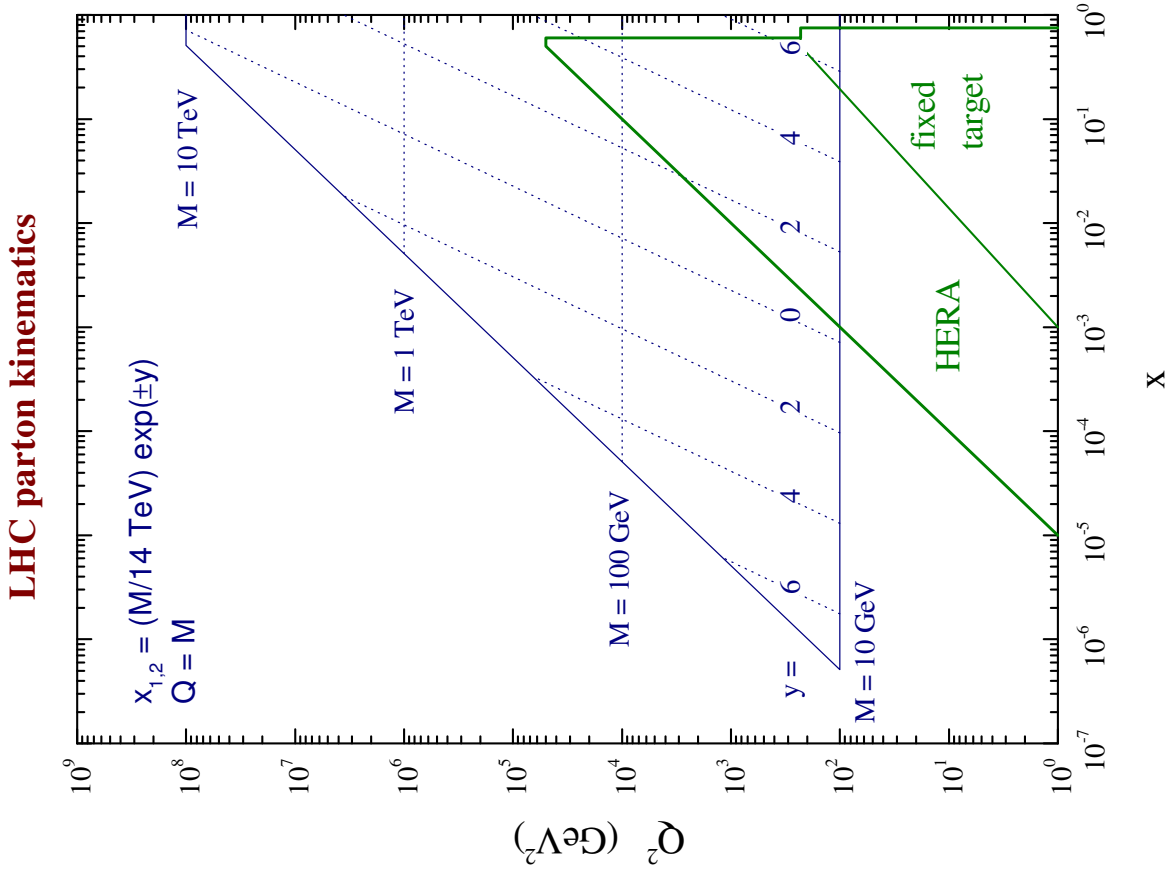
$$g_4 = 2xg_5$$

$$W^+ \rightarrow u \leftrightarrow d, \; c \leftrightarrow s; \text{ more combinations using Isospin: } p \rightarrow n \Rightarrow u \leftrightarrow d$$

FROM HERA TO LHC

AT A HADRON COLLIDER

- **SCALE Q DETERMINED BY MASS OF FINAL STATE**
- **MOMENTUM FRACTIONS x_1 x_2 DETERMINED BY MASS & RAPIDITY OF FINAL STATE**



FROM HERA TO LHC $\Rightarrow$ EVOLUTION

AT A HADRON COLLIDER

- **SCALE Q** DETERMINED BY MASS OF FINAL STATE
- **MOMENTUM FRACTIONS x_1 x_2** DETERMINED BY MASS & RAPIDITY OF FINAL STATE

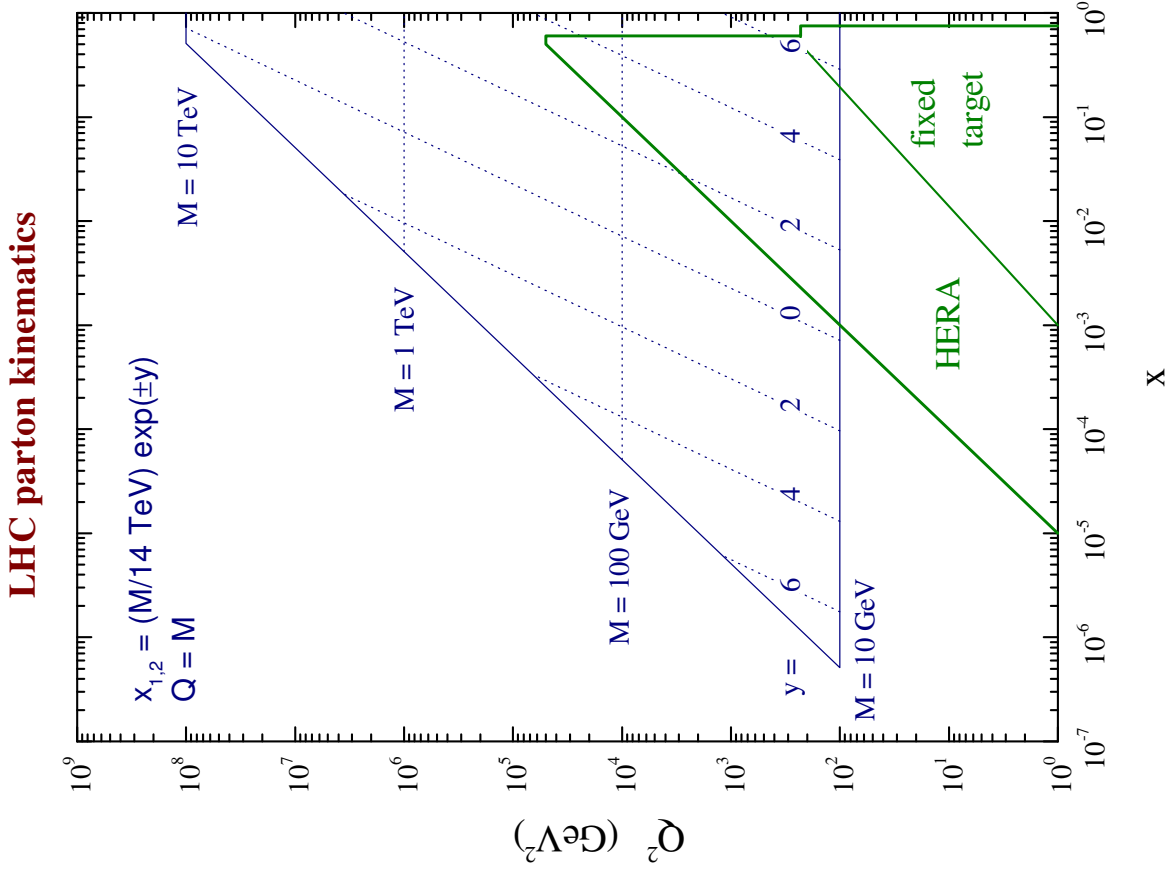
USE ALTARELLI-PARISI EQNS:

$$\frac{d}{dt} \Delta q_{NS} = \frac{\alpha_s(t)}{2\pi} P_{qq}^{NS} \otimes \Delta q_{NS},$$

$$\frac{d}{dt} \begin{pmatrix} \Delta \Sigma \\ \Delta g \end{pmatrix} = \frac{\alpha_s(t)}{2\pi} \begin{pmatrix} P_{qq}^S & 2n_f P_{qg}^S \\ P_{gq}^S & P_{gg}^S \end{pmatrix} \otimes \begin{pmatrix} \Delta \Sigma \\ \Delta g \end{pmatrix}$$

TO EVOLVE PARTONS

FROM DIS TO LHC KINEMATICS

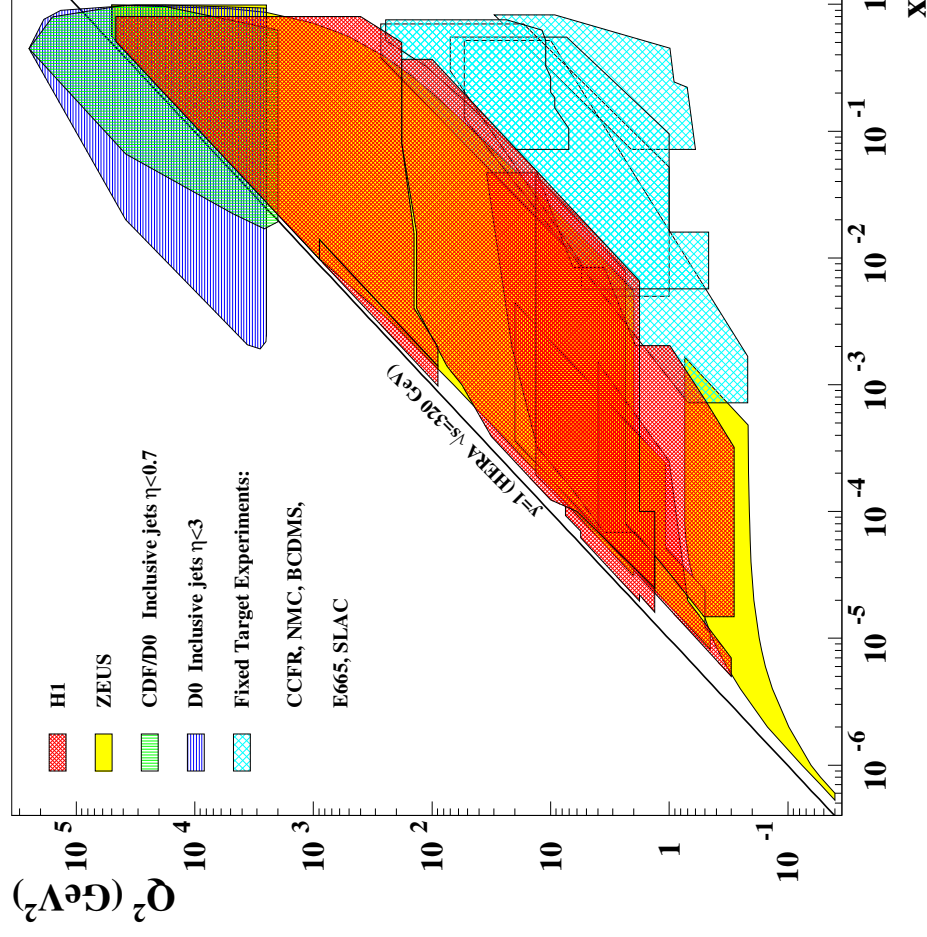


THE NAME OF THE GAME

DIS DATA $\rightarrow$ PARTON DISTRIBUTIONS

PROBLEMS:

- STRUCTURE FUNCTION (OR XSECT) IS A CON-VOLUTION OVER x OF PARTON DISTNS. AND PERTURBATIVE CROSS SECTION
 $\rightarrow$ MUST DECONVOLUTE

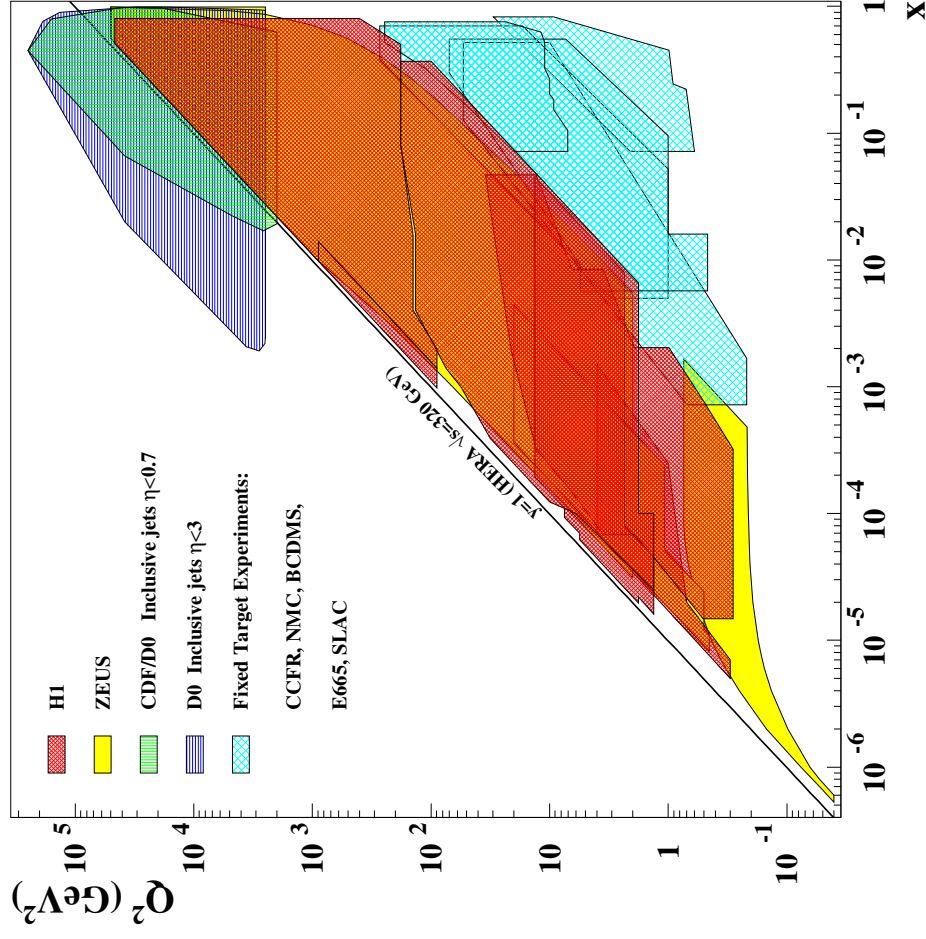


THE NAME OF THE GAME

DIS DATA $\rightarrow$ PARTON DISTRIBUTIONS

PROBLEMS:

- STRUCTURE FUNCTION (OR XSECT) IS A CON-VOLUTION OVER x OF PARTON DISTNS. AND PERTURBATIVE CROSS SECTION
 - $\rightarrow$ MUST DECONVOLUTE
- EACH STRUCTURE FUNCTION (OR XSECT) IS A LINEAR COMBINATION OF MANY PARTON DISTNS ($2N_f$ QUARKS + 1 GLUON)
 - $\rightarrow$ MUST COMBINE DIFFERENT PROCESSES

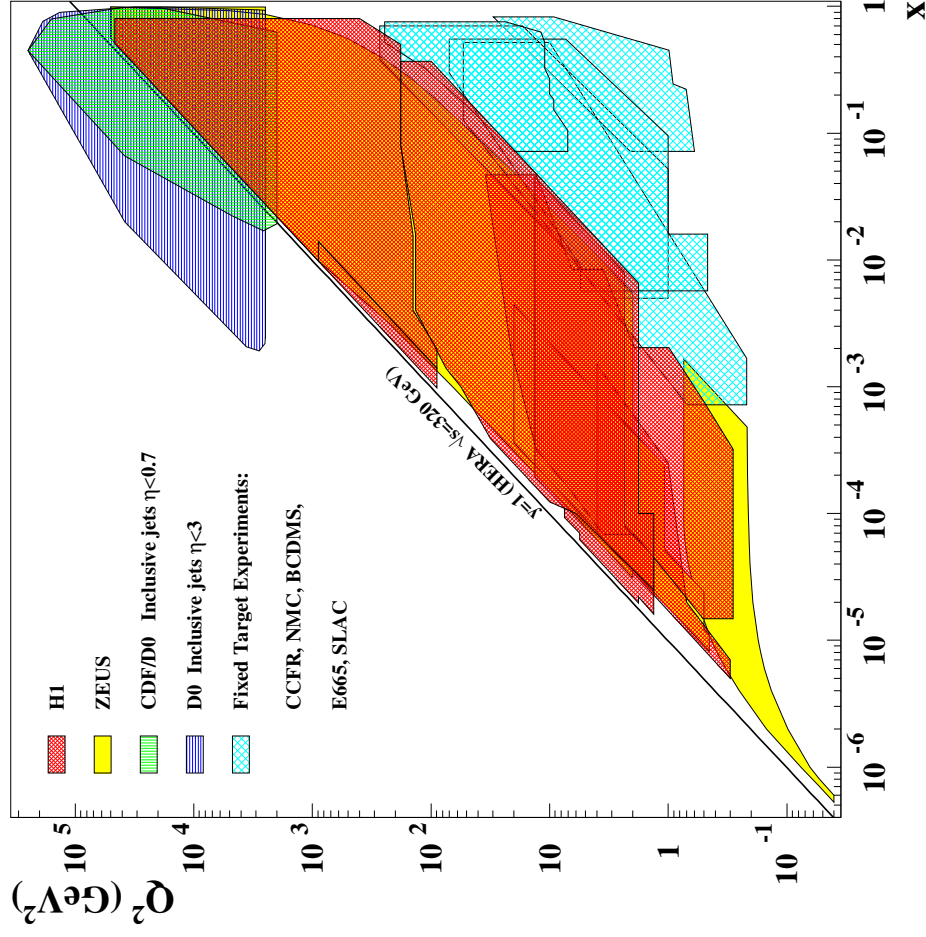


THE NAME OF THE GAME

DIS DATA $\rightarrow$ PARTON DISTRIBUTIONS

PROBLEMS:

- STRUCTURE FUNCTION (OR XSECT) IS A CON-
VOLUTION OVER x OF PARTON DISTNS. AND
PERTURBATIVE CROSS SECTION
 $\rightarrow$ MUST DECONVOLUTE
- EACH STRUCTURE FUNCTION (OR XSECT) IS
A LINEAR COMBINATION OF MANY PARTON
DISTNS ($2N_f$ QUARKS + 1 GLUON)
 $\rightarrow$ **MUST COMBINE DIFFERENT PROCESSES**
- DATA GIVEN AT VARIOUS SCALES, WANT PAR-
TON DISTNS. AS FCTN OF x AT COMMON
SCALE Q^2
 $\rightarrow$ **MUST EVOLVE**

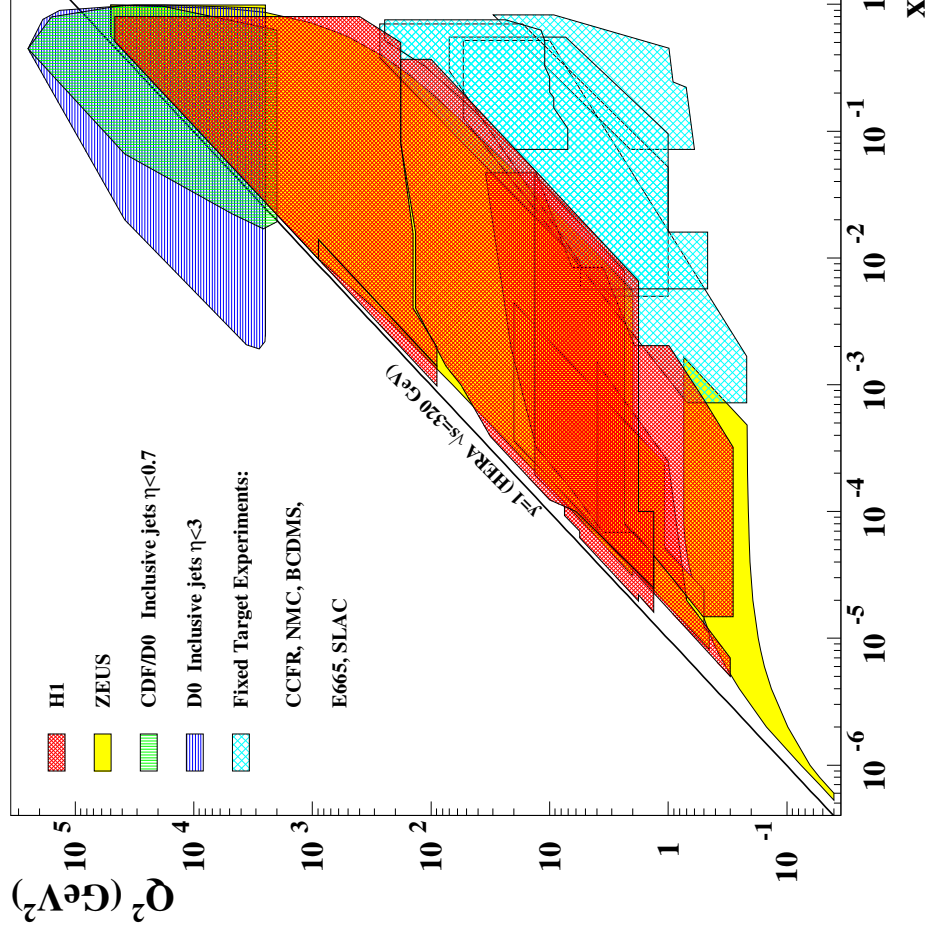


THE NAME OF THE GAME

DIS DATA $\rightarrow$ PARTON DISTRIBUTIONS

PROBLEMS:

- STRUCTURE FUNCTION (OR XSECT) IS A CON-
VOLUTION OVER x OF PARTON DISTNS. AND
PERTURBATIVE CROSS SECTION
 $\rightarrow$ MUST DECONVOLUTE
- EACH STRUCTURE FUNCTION (OR XSECT) IS
A LINEAR COMBINATION OF MANY PARTON
DISTNS ($2N_f$ QUARKS + 1 GLUON)
 $\rightarrow$ **MUST COMBINE DIFFERENT PROCESSES**
- DATA GIVEN AT VARIOUS SCALES, WANT PAR-
TON DISTNS. AS FCTN OF x AT COMMON
SCALE Q^2
 $\rightarrow$ **MUST EVOLVE**
- **TH UNCERTAINTIES: HIGHER PERTURBA-
TIVE ORDERS, RESUMMATION, NUCLEAR
CORRECTIONS, HIGHER TWIST, HEAVY
QUARK THRESHOLDS...**



PHENOMENOLOGY: DETERMINING THE GLUON

EVOLUTION:

SINGLET SCALING VIOLATIONS

$$\begin{aligned} \frac{d}{dt} F_2^s(N, Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \left[\gamma_{qq}(N) F_2^s + 2 n_f \gamma_{qg}(N) g(N, Q^2) \right] + O(\alpha_s^2) \\ F_2(N, Q^2) &\equiv \int_0^1 dx x^{N-1} F_2(x, Q^2); \quad \gamma_{ij}(N) \equiv \int_0^1 dx x^{N-1} P_{ij}(x, Q^2) \end{aligned}$$

PHENOMENOLOGY: DETERMINING THE GLUON

EVOLUTION:

SINGLET SCALING VIOLATIONS

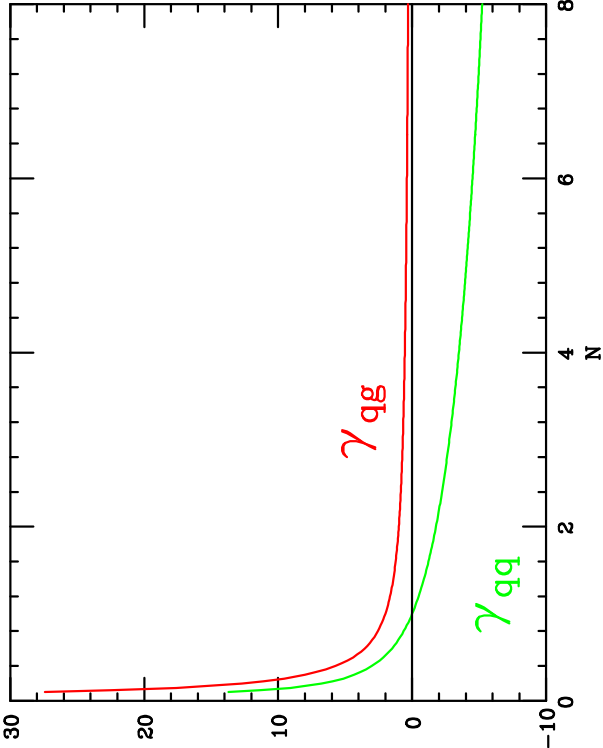
$$\frac{d}{dt} F_2^s(N, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \left[\gamma_{qq}(N) F_2^s + 2 n_f \gamma_{qg}(N) g(N, Q^2) \right] + O(\alpha_s^2)$$

$$F_2(N, Q^2) \equiv \int_0^1 dx \, x^{N-1} F_2(x, Q^2); \quad \gamma_{ij}(N) \equiv \int_0^1 dx \, x^{N-1} P_{ij}(x, Q^2)$$

$$\text{LARGE / SMALL } N \iff \text{LARGE / SMALL } N$$

AT LARGE N

$$\gamma_{qg} \ll \gamma_{qq}$$



PHENOMENOLOGY: DETERMINING THE GLUON

EVOLUTION:

SINGLET SCALING VIOLATIONS

$$\frac{d}{dt} F_2^s(N, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \left[\gamma_{qq}(N) F_2^s + 2 n_f \gamma_{qg}(N) g(N, Q^2) \right] + O(\alpha_s^2)$$

$$F_2(N, Q^2) \equiv \int_0^1 dx x^{N-1} F_2(x, Q^2); \quad \gamma_{ij}(N) \equiv \int_0^1 dx x^{N-1} P_{ij}(x, Q^2)$$

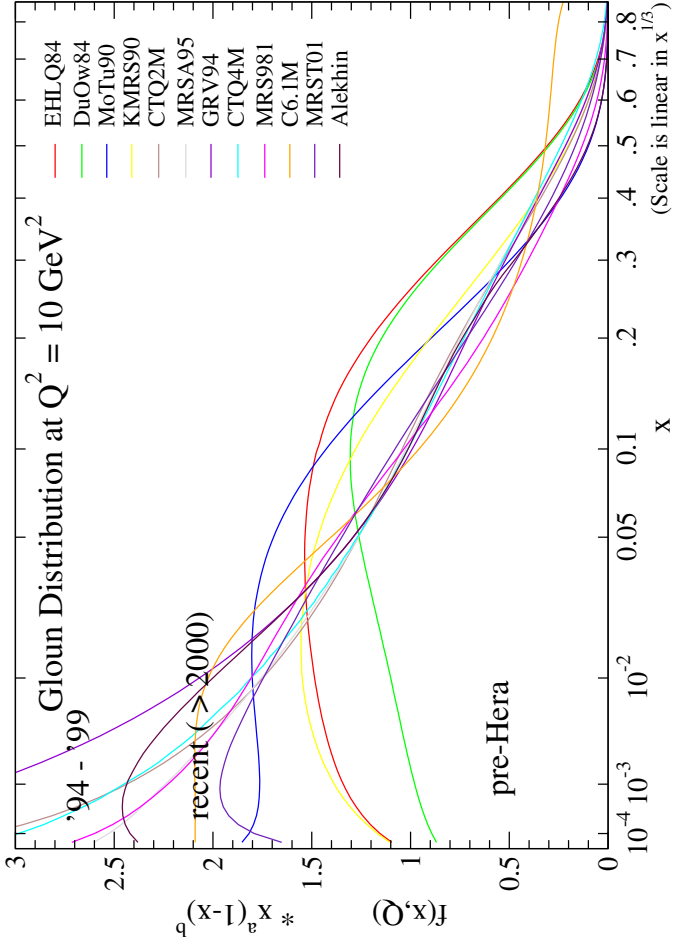
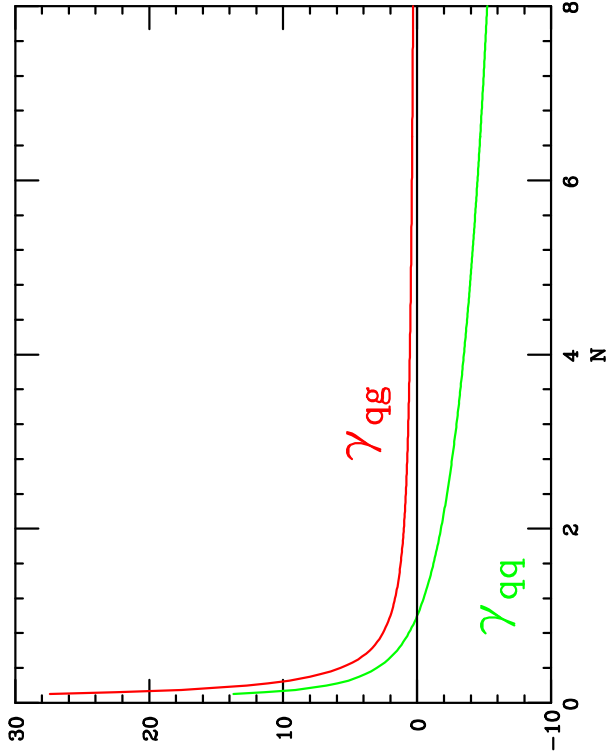
$$\text{LARGE / SMALL } x \Leftrightarrow \text{LARGE / SMALL } N$$

AT LARGE x

AT LARGE N

$\Rightarrow$ GLUON HARD TO DETERMINE

$$\gamma_{qg} \ll \gamma_{qq}$$

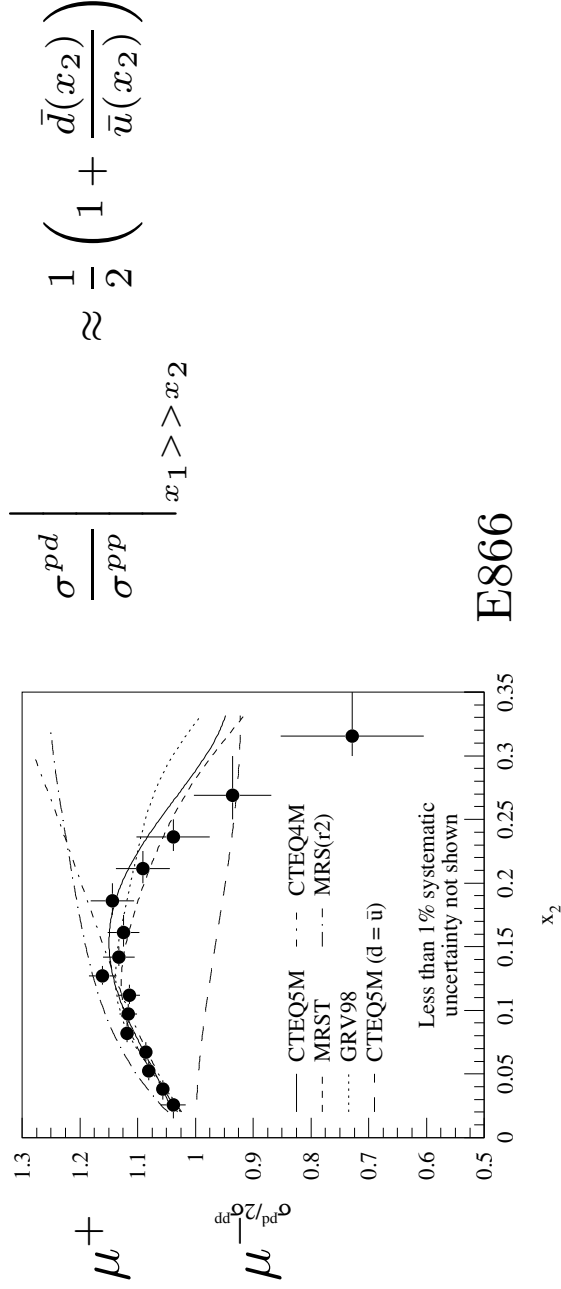
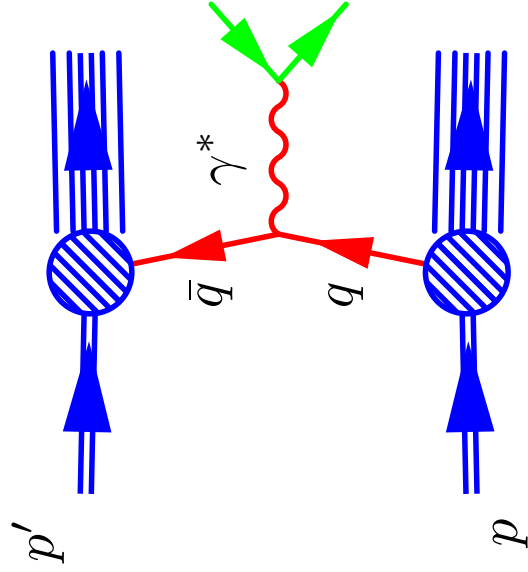


W.K. Tung, 2004

PHENOMENOLOGY: DISENTANGLING THE QUARK SEA

PHENOMENOLOGY: DISENTANGLING THE QUARK SEA

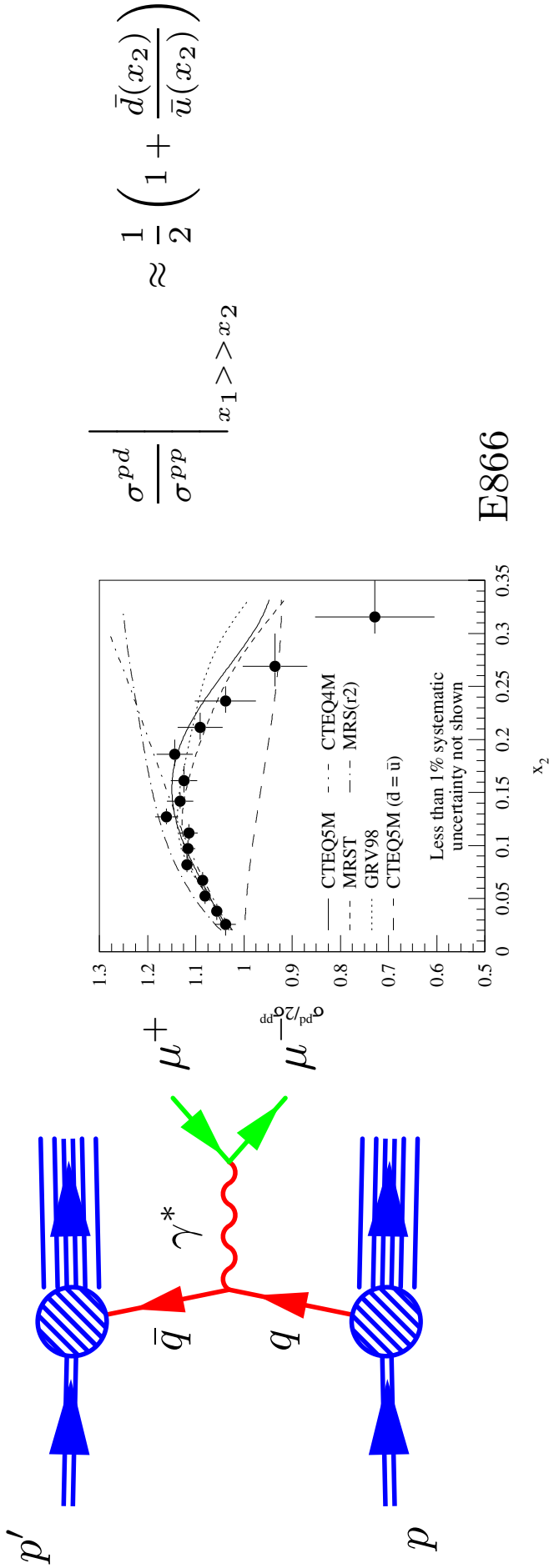
DRELL-YAN p/d ASYMMETRY



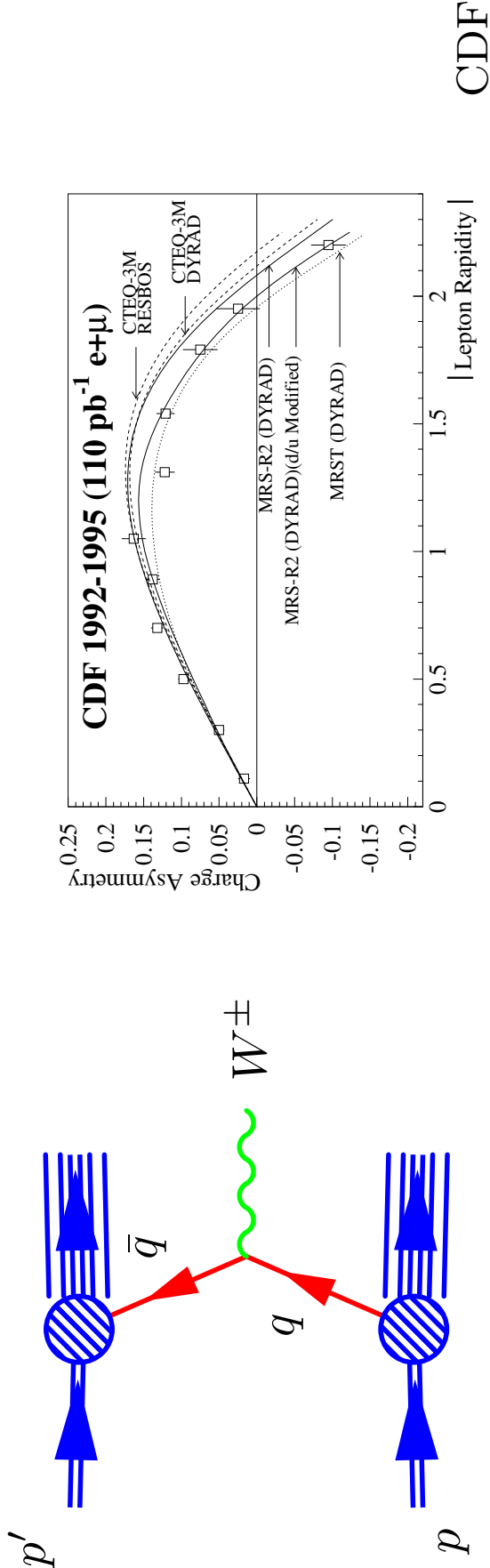
E866

PHENOMENOLOGY: DISENTANGLING THE QUARK SEA

DRELL-YAN p/d ASYMMETRY



$W^\pm$ ASYMMETRY



PHENOMENOLOGY: γ^* SCATTERING vs. $W^\pm$ SCATTERING: DISENTANGLING STRANGENESS

IN NC, CHARGED LEPTON DIS, ONLY MEASURE COMBINATION $\sum_i e_i^2 (q_i + \bar{q}_i)$

- CANNOT DETERMINE STRANGENESS
- CAN ONLY DETERMINE C-EVEN COMBINATION $q_i + \bar{q}_i$

PHENOMENOLOGY: γ^* SCATTERING vs. $W^\pm$ SCATTERING: DISENTANGLING STRANGENESS

IN NC, CHARGED LEPTON DIS, ONLY MEASURE COMBINATION $\sum_i e_i^2 (q_i + \bar{q}_i)$

- CANNOT DETERMINE STRANGENESS
- CAN ONLY DETERMINE C-EVEN COMBINATION $q_i + \bar{q}_i$

IN NEUTRINO DIS, CAN DISENTANGLE INDIVIDUAL PDFS BY LINEAR COMBINATION: AT LO

$$\begin{aligned}\frac{1}{2} \left(F_1^{W^-} + \frac{1}{2} F_3^{W^-} \right) &= u + \bar{c}; & \frac{1}{2} \left(F_1^{W^+} - \frac{1}{2} F_3^{W^+} \right) &= \bar{u} + \bar{c} \\ \frac{1}{2} \left(F_1^{W^+} + \frac{1}{2} F_3^{W^+} \right) &= d + \bar{s}; & \frac{1}{2} \left(F_1^{W^-} - \frac{1}{2} F_3^{W^-} \right) &= \bar{d} + \bar{s}\end{aligned}$$

$c, \bar{c}, s, \bar{s}$ only present above charm threshold

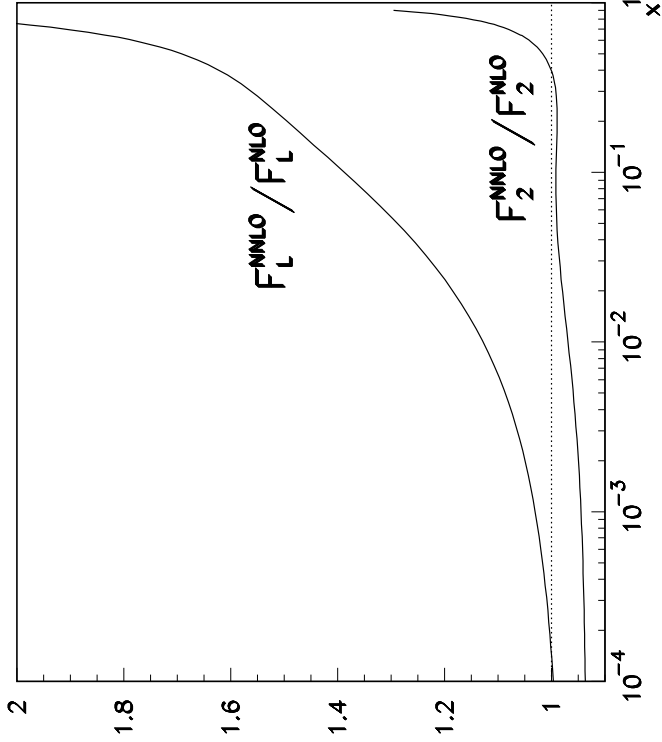
THEORETICAL ISSUES: NNLO CORRECTIONS

HOW BIG IS THE IMPACT OF HIGHER ORDER PERTURBATIVE CORRECTIONS?

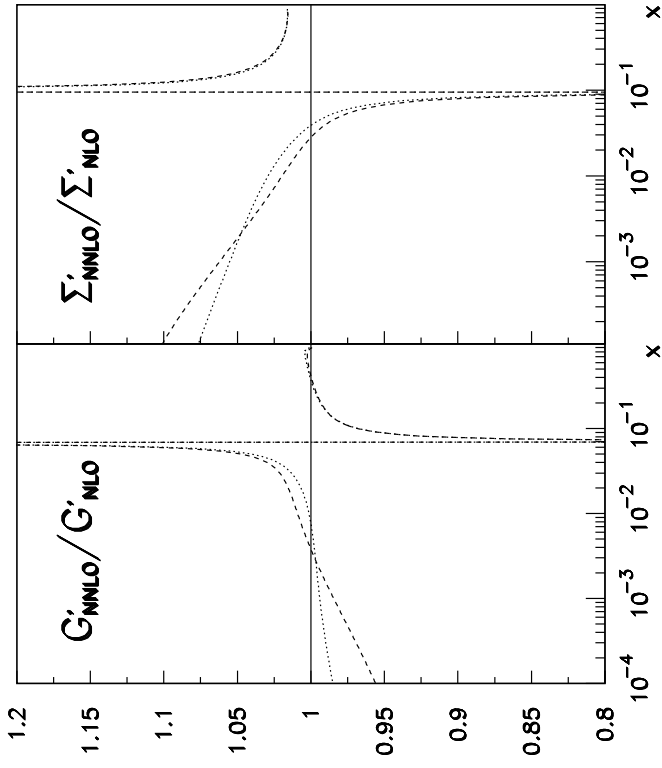
THEORETICAL ISSUES: NNLO CORRECTIONS

HOW BIG IS THE IMPACT OF HIGHER ORDER PERTURBATIVE CORRECTIONS?
FULL NNLO SPLITTING FUNCTIONS COMPUTED RECENTLY!
(MOCH, VERMASEREN AND VOGT, APRIL 2004)

PERTURBATIVE COEFFICIENTS



EVOLUTION



The Results

Anomalous dimensions in Mellin space

– One-loop : Gross, Wilczek '73

$$\gamma_{\text{ns}}^{(0)}(N) = C_F(2(\mathbf{N}_- + \mathbf{N}_+)S_1 - 3)$$

– Two-loop : Floratos, Ross, Sachrajda '79 ; Gonzalez-Arroyo, Lopez, Ynduráin '79

$$\begin{aligned} \gamma_{\text{ns}}^{(1)+}(N) &= 4C_A C_F \left(2\mathbf{N}_+ S_3 - \frac{17}{24} - 2S_{-3} - \frac{28}{3}S_1 + (\mathbf{N}_- + \mathbf{N}_+) \left[\frac{151}{18}S_1 + 2S_{1,-2} - \frac{1}{6} \right] \right. \\ &\quad \left. + 4C_F n_f \left(\frac{1}{12} + \frac{4}{3}S_1 - (\mathbf{N}_- + \mathbf{N}_+) \left[\frac{11}{9}S_1 - \frac{1}{3}S_2 \right] \right) + 4C_F^2 \left(4S_{-3} + 2S_1 + 2S_2 - \frac{3}{8} \right) \right. \\ &\quad \left. + \mathbf{N}_- \left[S_2 + 2S_3 \right] - (\mathbf{N}_- + \mathbf{N}_+) \left[S_1 + 4S_{1,-2} + 2S_{1,2} + 2S_{2,1} + S_3 \right] \right) \\ \gamma_{\text{ns}}^{(1)-}(N) &= \gamma_{\text{ns}}^{(1)+}(N) + 16C_F \left(C_F - \frac{C_A}{2} \right) \left((\mathbf{N}_- - \mathbf{N}_+) \left[S_2 - S_3 \right] - 2(\mathbf{N}_- + \mathbf{N}_+ - 2) \right) \end{aligned}$$

– Compact notation : $\mathbf{N}_{\pm} f(N) = f(N \pm 1)$, $\mathbf{N}_{\pm i} f(N) = f(N \pm i)$

The Results

Anomalous dimensions in Mellin space

- One-loop : Gross, Wilczek '73

THE ONE THEY

LIKE IN STOCKHOLM: $\Rightarrow$

$$\gamma_{\text{ns}}^{(0)}(N) = C_F(2(\mathbf{N}_- + \mathbf{N}_+)S_1 - 3)$$

- Two-loop : Floratos, Ross, Sachrajda '79 ; Gonzalez-Arroyo, Lopez, Ynduráin '79

$$\begin{aligned} \gamma_{\text{ns}}^{(1)+}(N) &= 4C_A C_F \left(2\mathbf{N}_+ S_3 - \frac{17}{24} - 2S_{-3} - \frac{28}{3}S_1 + (\mathbf{N}_- + \mathbf{N}_+) \left[\frac{151}{18}S_1 + 2S_{1,-2} - \frac{1}{6} \right] \right. \\ &\quad \left. + 4C_F n_f \left(\frac{1}{12} + \frac{4}{3}S_1 - (\mathbf{N}_- + \mathbf{N}_+) \left[\frac{11}{9}S_1 - \frac{1}{3}S_2 \right] \right) + 4C_F^2 \left(4S_{-3} + 2S_1 + 2S_2 - \frac{3}{8} \right) \right. \\ &\quad \left. + \mathbf{N}_- \left[S_2 + 2S_3 \right] - (\mathbf{N}_- + \mathbf{N}_+) \left[S_1 + 4S_{1,-2} + 2S_{1,2} + 2S_{2,1} + S_3 \right] \right) \\ \gamma_{\text{ns}}^{(1)-}(N) &= \gamma_{\text{ns}}^{(1)+}(N) + 16C_F \left(C_F - \frac{C_A}{2} \right) \left((\mathbf{N}_- - \mathbf{N}_+) \left[S_2 - S_3 \right] - 2(\mathbf{N}_- + \mathbf{N}_+ - 2) \right) \end{aligned}$$

- Compact notation : $\mathbf{N}_{\pm} f(N) = f(N \pm 1)$, $\mathbf{N}_{\pm i} f(N) = f(N \pm i)$

– Three-loop :

S.M., Vermaseren, Vogt '04

$$\begin{aligned}
\gamma_{\text{ns}}^{(2)+}(N) = & 16C_A C_F n_f \left(\frac{3}{2} \zeta_3 - \frac{5}{4} + \frac{10}{9} S_{-3} - \frac{10}{9} S_3 + \frac{4}{3} S_{1,-2} - \frac{2}{3} S_{-4} + 2S_{1,1} - \frac{25}{9} S_2 + \frac{257}{27} S_1 - \frac{2}{3} S_{-3,1} - \mathbf{N}_+ \left[S_{2,1} - \frac{2}{3} S_{3,1} \right. \right. \\
& - (\mathbf{N}_+ - 1) \left[\frac{23}{18} S_3 - S_2 \right] - (\mathbf{N}_- + \mathbf{N}_+) \left[S_{1,1} + \frac{1237}{216} S_1 + \frac{11}{18} S_3 - \frac{317}{108} S_2 + \frac{16}{9} S_{1,-2} - \frac{2}{3} S_{1,-2,1} - \frac{1}{3} S_{1,-3} - \frac{1}{2} S_{1,3} - \frac{1}{2} S_{2,1} \right. \\
& - \frac{1}{3} S_{2,-2} + S_1 \zeta_3 + \frac{1}{2} S_{3,1} \left. \right] + 16C_F C_A^2 \left(\frac{1657}{576} - \frac{15}{4} \zeta_3 + 2S_{-5} + \frac{31}{6} S_{-4} - 4S_{-4,1} - \frac{67}{9} S_{-3} + 2S_{-3,-2} + \frac{11}{3} S_{-3,1} + \frac{3}{2} S_{-2} \right. \\
& - 6S_{-2} \zeta_3 - 2S_{-2,-3} + 3S_{-2,-2} - 4S_{-2,-2,1} + 8S_{-2,1,-2} - \frac{1883}{54} S_1 - 10S_{1,-3} - \frac{16}{3} S_{1,-2} + 12S_{1,-2,1} + 4S_{1,3} - 4S_{2,-2} - \frac{5}{2} S_4 + \\
& + \frac{176}{9} S_2 + \frac{13}{3} S_3 + (\mathbf{N}_- + \mathbf{N}_+ - 2) \left[3S_1 \zeta_3 + 11S_{1,1} - 4S_{1,1,-2} \right] + (\mathbf{N}_- + \mathbf{N}_+) \left[\frac{9737}{432} S_1 - 3S_{1,-4} + \frac{19}{6} S_{1,-3} + 8S_{1,-3,1} + \frac{91}{9} S_{1,-3,1} \right. \\
& - 6S_{1,-2,-2} - \frac{29}{3} S_{1,-2,1} + 8S_{1,1,-3} - 16S_{1,1,-2,1} - 4S_{1,1,3} - \frac{19}{4} S_{1,3} + 4S_{1,3,1} + 3S_{1,4} + 8S_{2,-2,1} + 2S_{2,3} - S_{3,-2} + \frac{11}{12} S_{3,1} - S_4, \\
& + \frac{1}{6} S_{2,-2} - \frac{1967}{216} S_2 + \frac{121}{72} S_3 \left. \right] - (\mathbf{N}_- - \mathbf{N}_+) \left[3S_2 \zeta_3 + 7S_{2,1} - 3S_{2,1,-2} + 2S_{2,-2,1} - \frac{1}{4} S_{2,3} - \frac{3}{2} S_{3,-2} - \frac{29}{6} S_{3,1} + \frac{11}{4} S_{4,1} + \frac{1}{2} S_{2,-2,1} \right. \\
& + \mathbf{N}_+ \left[\frac{28}{9} S_3 - \frac{2376}{216} S_2 - \frac{8}{3} S_4 - \frac{5}{2} S_5 \left. \right] + 16C_F n_f^2 \left(\frac{17}{144} - \frac{13}{27} S_1 + \frac{2}{9} S_2 + (\mathbf{N}_- + \mathbf{N}_+) \left[\frac{2}{9} S_1 - \frac{11}{54} S_2 + \frac{1}{18} S_3 \right] + 16C_F^2 C_A \left(\frac{45}{4} \right. \right. \\
& - \frac{151}{64} - 10S_{-5} - \frac{89}{6} S_{-4} + 20S_{-4,1} + \frac{134}{9} S_{-3} - 2S_{-3,-2} - \frac{31}{3} S_{-3,1} + 2S_{-3,2} - \frac{9}{2} S_{-2} + 18S_{-2} \zeta_3 + 10S_{-2,-3} - 6S_{-2,-2} \\
& + 8S_{-2,-2,1} - 28S_{-2,1,-2} + 46S_{1,-3} + \frac{26}{3} S_{1,-2} - 48S_{1,-2,1} + \frac{28}{3} S_{1,2} - \frac{185}{6} S_3 - 8S_{1,3} + 2S_{3,-2} - 4S_5 - (\mathbf{N}_- + \mathbf{N}_+ - 2) \left[9S_1 \zeta_3 \right. \\
& + \frac{209}{6} S_{1,1} - 14S_{1,1,-2} - \frac{242}{18} S_2 + 9S_{2,-2} + \frac{33}{4} S_4 - 3S_{3,1} + \frac{14}{3} S_{2,1} \left. \right] + (\mathbf{N}_- + \mathbf{N}_+) \left[17S_{1,-4} - \frac{107}{6} S_{1,-3} - 32S_{1,-3,1} - \frac{173}{9} S_{1,-3,1}, \right.
\end{aligned}$$

$$\begin{aligned}
& +16S_{1,-2,-2} + \frac{103}{3}S_{1,-2,1} - 2S_{1,-2,2} - 36S_{1,1,-3} + 56S_{1,1,-2,1} + 8S_{1,1,3} - \frac{109}{9}S_{1,2} - 4S_{1,2,-2} + \frac{43}{3}S_{1,3} - 8S_{1,3,1} - 11S_{1,3,2} \\
& + 21S_{2,-3} - 30S_{2,-2,1} - 4S_{2,1,-2} - 5S_{2,3} - S_{4,1} + \frac{31}{6}S_{2,-2} - \frac{67}{9}S_{2,1} \Big] + (\mathbf{N}_- - \mathbf{N}_+) \Big[9S_2\zeta_3 + 2S_{2,-3} + 4S_{2,-2,1} - 12S_{2,1,1} \\
& + 13S_{4,1} + \frac{1}{2}S_{2,-2} + \frac{11}{2}S_4 - \frac{33}{2}S_{3,1} + \frac{59}{9}S_3 + \frac{127}{6}S_{2,1} - \frac{1153}{72}S_2 \Big] + \mathbf{N}_+ \Big[8S_{3,-2} + \frac{4}{3}S_{3,1} - 2S_{3,2} + 14S_5 + \frac{23}{6}S_4 + \frac{73}{3}S_3 \\
& + 16\mathbf{C}_F^2 n_f \Big(\frac{23}{16} - \frac{3}{2}\zeta_3 + \frac{4}{3}S_{-3,1} - \frac{59}{36}S_2 + \frac{4}{3}S_{-4} - \frac{20}{9}S_{-3} + \frac{20}{9}S_1 - \frac{8}{3}S_{1,-2} - \frac{8}{3}S_{1,1} - \frac{4}{3}S_{1,2} + \mathbf{N}_+ \Big[\frac{25}{9}S_3 - \frac{4}{3}S_{3,1} - \frac{1}{3}S_{3,2} \\
& - (\mathbf{N}_+ - 1) \Big[\frac{67}{36}S_2 - \frac{4}{3}S_{2,1} + \frac{4}{3}S_3 \Big] + (\mathbf{N}_- + \mathbf{N}_+) \Big[S_1\zeta_3 - \frac{325}{144}S_1 - \frac{2}{3}S_{1,-3} + \frac{32}{9}S_{1,-2} - \frac{4}{3}S_{1,-2,1} + \frac{4}{3}S_{1,1} + \frac{16}{9}S_{1,2} - \frac{4}{3}S_{1,2,1} \\
& + \frac{11}{18}S_2 - \frac{2}{3}S_{2,-2} + \frac{10}{9}S_{2,1} + \frac{1}{2}S_4 - \frac{2}{3}S_{2,2} - \frac{8}{9}S_3 \Big] \Big) + 16\mathbf{C}_F^3 \Big(12S_{-5} - \frac{29}{32} - \frac{15}{2}\zeta_3 + 9S_{-4} - 24S_{-4,1} - 4S_{-3,-2} + 6S_{-3,1} \\
& - 4S_{-3,2} + 3S_{-2} + 25S_3 - 12S_{-2}\zeta_3 - 12S_{-2,-3} + 24S_{-2,1,-2} - 52S_{1,-3} + 4S_{1,-2} + 48S_{1,-2,1} - 4S_{3,-2} + \frac{67}{2}S_2 - 17S_4 \\
& + (\mathbf{N}_- + \mathbf{N}_+ - 2) \Big[6S_1\zeta_3 - \frac{31}{8}S_1 + 35S_{1,1} - 12S_{1,1,-2} + S_{1,2} + 10S_{2,-2} + S_{2,1} + 2S_{2,2} - 2S_{3,1} - 3S_5 \Big] + (\mathbf{N}_- + \mathbf{N}_+) \Big[23S_{1,2} \\
& - 22S_{1,-4} + 32S_{1,-3,1} - 2S_{1,-2} - 8S_{1,-2,-2} - 30S_{1,-2,1} - 6S_{1,3} + 4S_{1,-2,2} + 40S_{1,1,-3} - 48S_{1,1,-2,1} + 8S_{1,2,-2} + 4S_{1,2,2} \\
& + 4S_{1,4} + 28S_{2,-2,1} + 4S_{2,1,2} + 4S_{2,2,1} + 4S_{3,1,1} - 4S_{3,2} + 8S_{2,1,-2} - 26S_{2,-3} - 2S_{2,3} - 4S_{3,-2} - 3S_{2,-2} - 3S_{2,2} + \frac{3}{2}S_4 \Big] \\
& + (\mathbf{N}_- - \mathbf{N}_+) \Big[12S_{2,1,-2} - 6S_2\zeta_3 - 2S_{2,-3} + 3S_{2,3} + 2S_{3,-2} - \frac{81}{4}S_{2,1} + 14S_{3,1} - 5S_{2,-2} - \frac{1}{2}S_{2,2} + \frac{15}{8}S_2 + \frac{1}{2}S_3 - 13S_{4,1} \\
& + \mathbf{N}_+ \Big[14S_4 - \frac{265}{8}S_2 - \frac{87}{4}S_3 - 4S_{4,1} - 4S_5 \Big] \Big)
\end{aligned}$$

THEORETICAL ISSUES: RESUMMATION

AT $O(\alpha_s^n)$, $O\left[ln(\frac{1}{x})^n\right]$ AND $O\left[(ln(1-x))^{2n-1}\right]$ CONTRIBUTIONS ARISE:
IS A FIXED-ORDER PERTURBATIVE CALCULATION SUFFICIENT?

$x_{\text{cut}} :$	0	0.0002	0.001	0.0025	0.005	0.01
# DATA POINTS	2097	2050	1961	1898	1826	1762
$\chi^2(x > 0)$	2267					
$\chi^2(x > 0.0002)$	2212	2203				
$\chi^2(x > 0.001)$	2134	2128	2119			
$\chi^2(x > 0.0025)$	2069	2064	2055	2040		
$\chi^2(x > 0.005)$	2024	2019	2012	1993	1973	
$\chi^2(x > 0.01)$	1965	1961	1953	1934	1917	1916
Δ_i^{i+1}	0.19 0.10 0.24 0.28 0.02					

DATA-THEORY AGREEMENT
FOR EVOLUTION OF F_2
IMPROVES IF SMALL x DATA
REMOVED (MRST 2003)
 χ^2 improves
with fixed # of pts
(same row)

THEORETICAL ISSUES: RESUMMATION

AT $O(\alpha_s^n)$, $O\left[\ln(\frac{1}{x})^n\right]$ AND $O\left[\left(\ln(1-x)\right)^{2n-1}\right]$ CONTRIBUTIONS ARISE:
IS A FIXED-ORDER PERTURBATIVE CALCULATION SUFFICIENT?

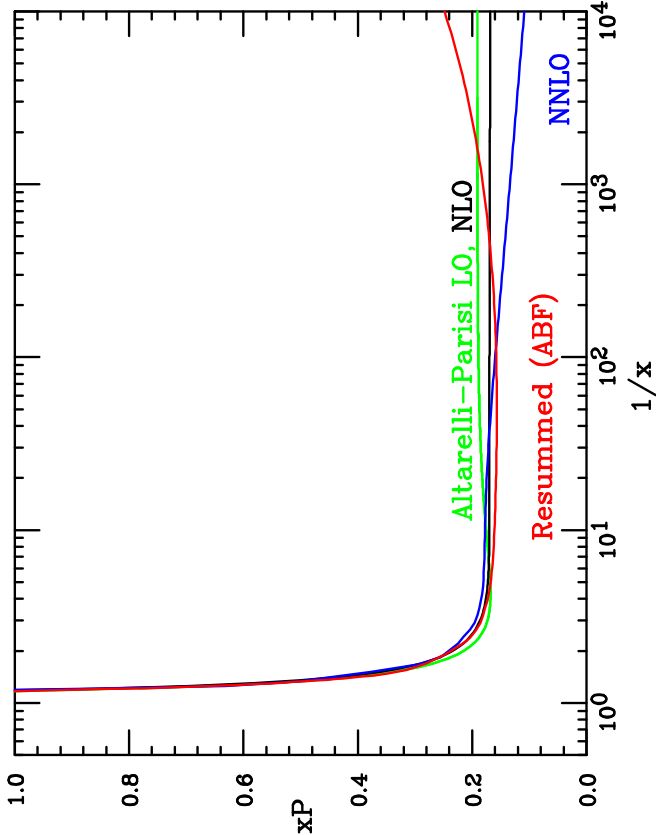
$x_{\text{cut}} :$	0	0.0002	0.001	0.0025	0.005	0.01
# DATA POINTS	2097	2050	1961	1898	1826	1762
$\chi^2(x > 0)$	2267					
$\chi^2(x > 0.0002)$	2212	2203				
$\chi^2(x > 0.001)$	2134	2128	2119			
$\chi^2(x > 0.0025)$	2069	2064	2055	2040		
$\chi^2(x > 0.005)$	2024	2019	2012	1993	1973	
$\chi^2(x > 0.01)$	1965	1961	1953	1934	1917	1916
Δ_i^{i+1}	0.19	0.10	0.24	0.28	0.02	

DATA-THEORY AGREEMENT
FOR EVOLUTION OF F_2
IMPROVES IF SMALL x DATA
REMOVED (MRST 2003)
 χ^2 improves
with fixed # of pts
(same row)

RESUMMED SPL. FCTN.
CLOSER TO LO THAN TO NNLO

Altarelli, Ball, S.F.

(see also Ciafaloni, Salam et al.)

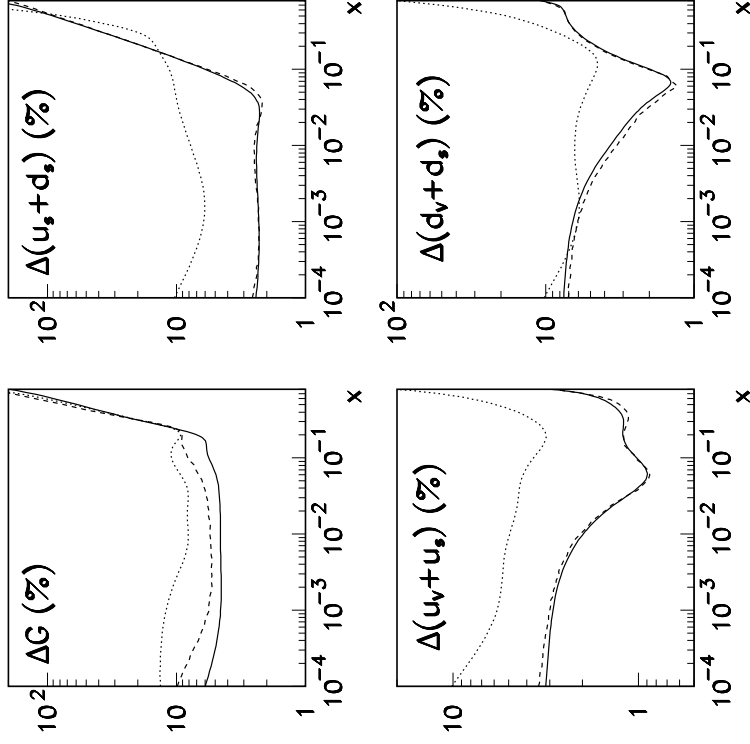


DOES IT ALL MATTER?

IMPACT OF HADR. COLL. DATA

PDF UNCERTAINTIES

$Q^2=9 \text{ GeV}^2$



top $\rightarrow$ DIS only

middle $\rightarrow$ DIS + Drell-Yan E866

bottom $\rightarrow$ DIS+DY (E866+LHC)

Alekhin 2004 prelim.

DOES IT ALL MATTER?

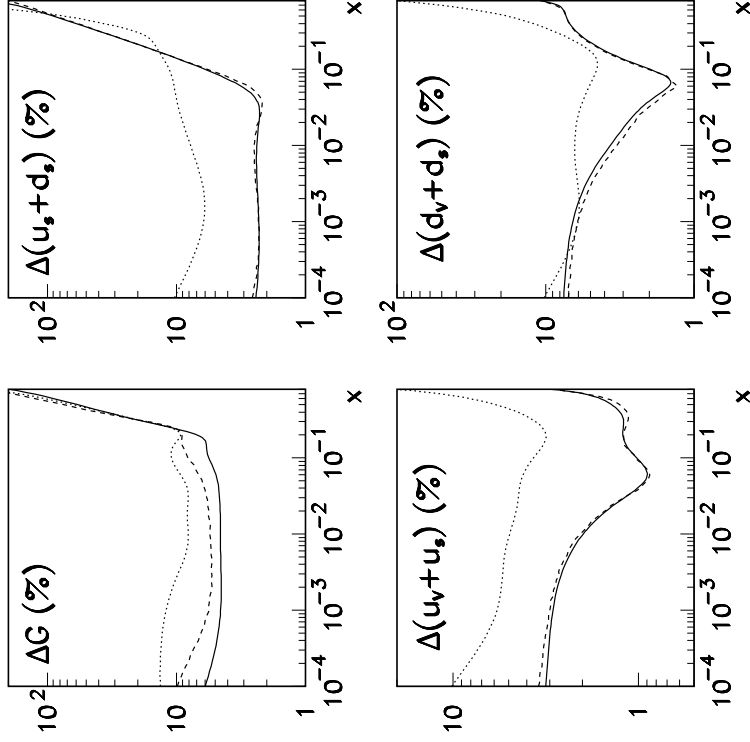
IMPACT OF HADR. COLL. DATA

IMPACT OF NNLO CORRECTIONS

PDF UNCERTAINTIES

GLUON DISTRIBUTION

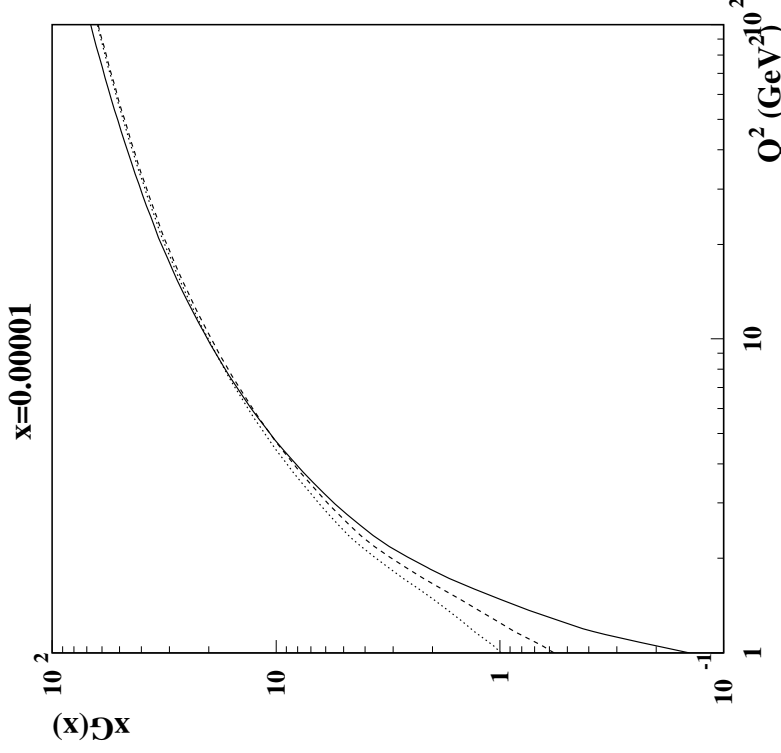
$Q^2 = 9 \text{ GeV}^2$



top $\rightarrow$ DIS only

middle $\rightarrow$ DIS + Drell-Yan E866

bottom $\rightarrow$ DIS+DY (E866+LHC)



bottom $\rightarrow$ NLO best fit

middle $\rightarrow$ NNLO best fit

top $\rightarrow$ NNLO evoln. of NLO best fit

NOTE LOG SCALE!

Alekhin 2004 prelim.

PDFS FOR PRECISION PHYSICS: THE NUTEV ANOMALY

PW RATIO: DATA...

NuTeV 2001 $\sin^2 \theta_W(\text{OS}) = 0.2272 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst}) \pm 0.0002(M_t, M_H)$
Global Fit 2003 $\sin^2 \theta_W(\text{OS}) = 0.2229 \pm 0.0004$

...VS. THEORY

$$\begin{aligned} R^- &= \frac{\sigma_{NC}(\nu) - \sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\nu) - \sigma_{CC}(\bar{\nu})} \\ &= \left(\frac{1}{2} - \sin^2 \theta_W \right) \end{aligned}$$

PDFS FOR PRECISION PHYSICS: THE NUTEV ANOMALY

PW RATIO: DATA...

NuTeV 2001 $\sin^2 \theta_W(\text{OS}) = 0.2272 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst}) \pm 0.0002(M_t, M_H)$
 Global Fit 2003 $\sin^2 \theta_W(\text{OS}) = 0.2229 \pm 0.0004$

...VS. THEORY

$$\begin{aligned}
 R^- &= \frac{\sigma_{NC}(\nu) - \sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\nu) - \sigma_{CC}(\bar{\nu})} \\
 &= \left(\frac{1}{2} - \sin^2 \theta_W\right) + 2 \left[\frac{(u - \bar{u}) - (d - \bar{d})}{u - \bar{u} + d - \bar{d}} \right] \times \left(\frac{1}{2} - \frac{7}{6} \sin^2 \theta_W \right) \\
 &\qquad\qquad\qquad + O(\delta(u - d)^2) \quad)
 \end{aligned}$$

U,D...DENOTE MOMENTUM FRACTIONS CARRIED BY CORRESP. QUARK FLAVORS

- ISOSPIN VIOLATION $\rightarrow$ corn. for non-isoscalar target included, but not $u^p \neq d^n$

PDFS FOR PRECISION PHYSICS: THE NUTeV ANOMALY

PW RATIO: DATA...

NuTeV 2001 $\sin^2 \theta_W(\text{OS}) = 0.2272 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst}) \pm 0.0002(M_t, M_H)$
Global Fit 2003 $\sin^2 \theta_W(\text{OS}) = 0.2229 \pm 0.0004$

...VS. THEORY

$$\begin{aligned}
 R^- &= \frac{\sigma_{NC}(\nu) - \sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\nu) - \sigma_{CC}(\bar{\nu})} \\
 &= \left(\frac{1}{2} - \sin^2 \theta_W \right) + 2 \left[\frac{(u - \bar{u}) - (d - \bar{d})}{u - \bar{u} + d - \bar{d}} - \frac{s - \bar{s}}{u - \bar{u} + d - \bar{d}} \right] \times \left(\frac{1}{2} - \frac{7}{6} \sin^2 \theta_W \right) \\
 &\quad + O(\delta(u - d)^2, \delta s^2)
 \end{aligned}$$

U,D...DENOTE MOMENTUM FRACTIONS CARRIED BY CORRESP. QUARK FLAVORS

- **ISOSPIN VIOLATION** $\rightarrow$ corn. for non-isoscalar target included, but not $u^p \neq d^n$
- **STRANGE ASYM.** $\rightarrow$ strangeness must vanish, but not valence mom. fract.!

PDFS FOR PRECISION PHYSICS: THE NUTeV ANOMALY

PW RATIO: DATA...

NuTeV 2001 $\sin^2 \theta_W(\text{OS}) = 0.2272 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst}) \pm 0.0002(M_t, M_H)$
Global Fit 2003 $\sin^2 \theta_W(\text{OS}) = 0.2229 \pm 0.0004$

...VS. THEORY

$$\begin{aligned}
 R^- &= \frac{\sigma_{NC}(\nu) - \sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\nu) - \sigma_{CC}(\bar{\nu})} \\
 &= \left(\frac{1}{2} - \sin^2 \theta_W \right) + 2 \left[\frac{(u - \bar{u}) - (d - \bar{d})}{u - \bar{u} + d - \bar{d}} - \frac{s - \bar{s}}{u - \bar{u} + d - \bar{d}} \right] \times \left[\left(\frac{1}{2} - \frac{7}{6} \sin^2 \theta_W \right) \right. \\
 &\quad \left. + \frac{4}{9} \frac{\alpha_s}{2\pi} \left(\frac{1}{2} - \sin^2 \theta_W \right) + O(\alpha_s^2) \right] + O(\delta(u - d)^2, \delta s^2)
 \end{aligned}$$

U,D...DENOTE MOMENTUM FRACTIONS CARRIED BY CORRESP. QUARK FLAVORS

- **ISOSPIN VIOLATION** → corn. for non-isoscalar target included, but not $u^p \neq d^n$
- **STRANGE ASYM.** → strangeness must vanish, but not valence mom. fract.!
- **QCD CORRECTIONS** → tiny (only enter through sym. violating terms)

PDFS FOR PRECISION PHYSICS: THE NUTeV ANOMALY

PW RATIO: DATA...

NuTeV 2001 $\sin^2 \theta_W(\text{OS}) = 0.2272 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst}) \pm 0.0002(M_t, M_H)$
 Global Fit 2003 $\sin^2 \theta_W(\text{OS}) = 0.2229 \pm 0.0004$

...VS. THEORY

$$\begin{aligned} R^- &= \frac{\sigma_{NC}(\nu) - \sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\nu) - \sigma_{CC}(\bar{\nu})} \\ &= \left(\frac{1}{2} - \sin^2 \theta_W \right) + 2 \left[\frac{(u - \bar{u}) - (d - \bar{d})}{u - \bar{u} + d - \bar{d}} - \frac{s - \bar{s}}{u - \bar{u} + d - \bar{d}} \right] \times \left[\left(\frac{1}{2} - \frac{7}{6} \sin^2 \theta_W \right) \right. \\ &\quad \left. + \frac{4}{9} \frac{\alpha_s}{2\pi} \left(\frac{1}{2} - \sin^2 \theta_W \right) + O(\alpha_s^2) \right] + O(\delta(u - d)^2, \delta s^2) \end{aligned}$$

U,D...DENOTE MOMENTUM FRACTIONS CARRIED BY CORRESP. QUARK FLAVORS

- ISOSPIN VIOLATION $\rightarrow$ corn. for non-isoscalar target included, but not $u^p \neq d^n$
- STRANGE ASYM. $\rightarrow$ strangeness must vanish, but not valence mom. fract.!
- QCD CORRECTIONS $\rightarrow$ tiny (only enter through sym. violating terms)

$s - \bar{s} \approx 0.004$ (OR 2% ISOSPIN VLN.) ENOUGH TO REMOVE ANOMALY: CAN WE TEST IT?

DISENTANGLING STRANGENESS

CHARM IS COPIOUSLY PRODUCED IN $W^+ + s \rightarrow c$

easily tagged through dimuon signal, 2nd muon from subsequent c decay

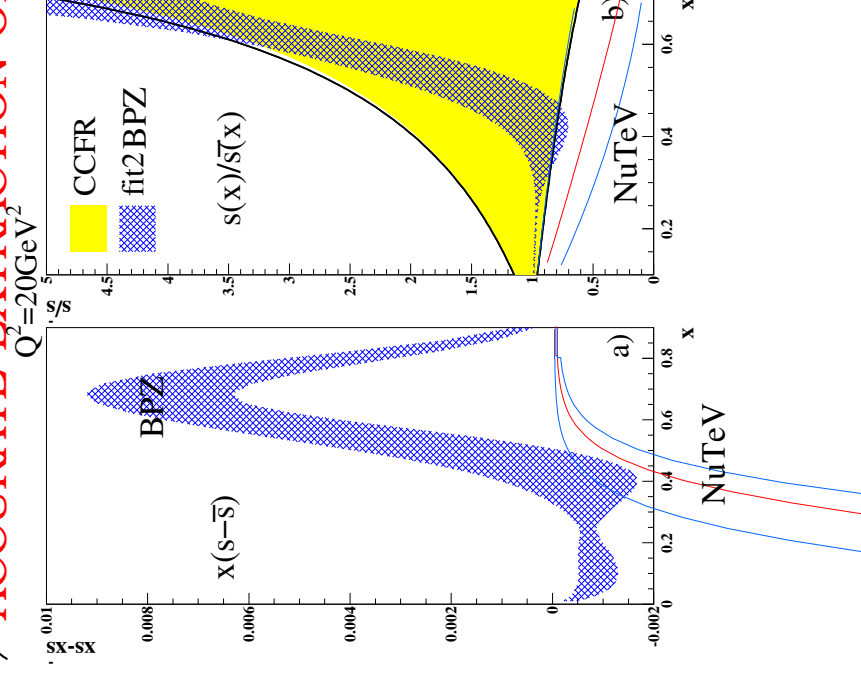
$\Rightarrow$ ACCURATE EXTRACTION OF THE STRANGE DISTRIBUTION

DISENTANGLING STRANGENESS

CHARM IS COPIOUSLY PRODUCED IN $W^+ + s \rightarrow c$

easily tagged through dimuon signal, 2nd muon from subsequent c decay

$\Rightarrow$ ACCURATE EXTRACTION OF THE STRANGE DISTRIBUTION



CCFR/NUTeV $s - \bar{s}$ DETERMINATION

5000 ν & 1500 $\bar{\nu}$ DIMUON EVENT SAMPLE:

ASSUMED PARM.: $s(x) = \kappa \frac{\bar{u}(x) + \bar{d}(x)}{2} (1 - x)^\alpha$

NEGATIVE $s - \bar{s}$ AT SMALL x

$\Rightarrow$ MOM. FRACT. $s - \bar{s} = -0.003 \pm 0.001$

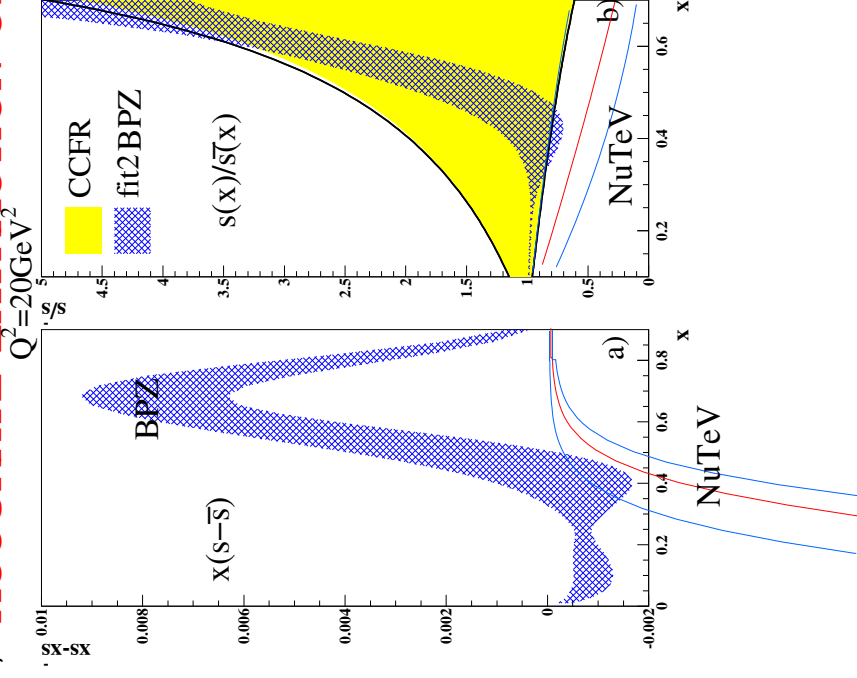
NUTeV ANOMALY WORSE!

DISENTANGLING STRANGENESS

CHARM IS COPIOUSLY PRODUCED IN $W^+ + s \rightarrow c$

easily tagged through dimuon signal, 2nd muon from subsequent c decay

$\Rightarrow$ **ACCURATE EXTRACTION OF THE STRANGE DISTRIBUTION**



CCFR/NUTeV $s - \bar{s}$ DETERMINATION

5000 ν & 1500 $\bar{\nu}$ DIMUON EVENT SAMPLE:

ASSUMED PARM.: $s(x) = \kappa \frac{\bar{u}(x) + \bar{d}(x)}{2} (1 - x)^\alpha$

NEGATIVE $s - \bar{s}$ AT SMALL x

$\Rightarrow$ **MOM. FRACT. $s - \bar{s} = -0.003 \pm 0.001$**

NUTeV ANOMALY WORSE!

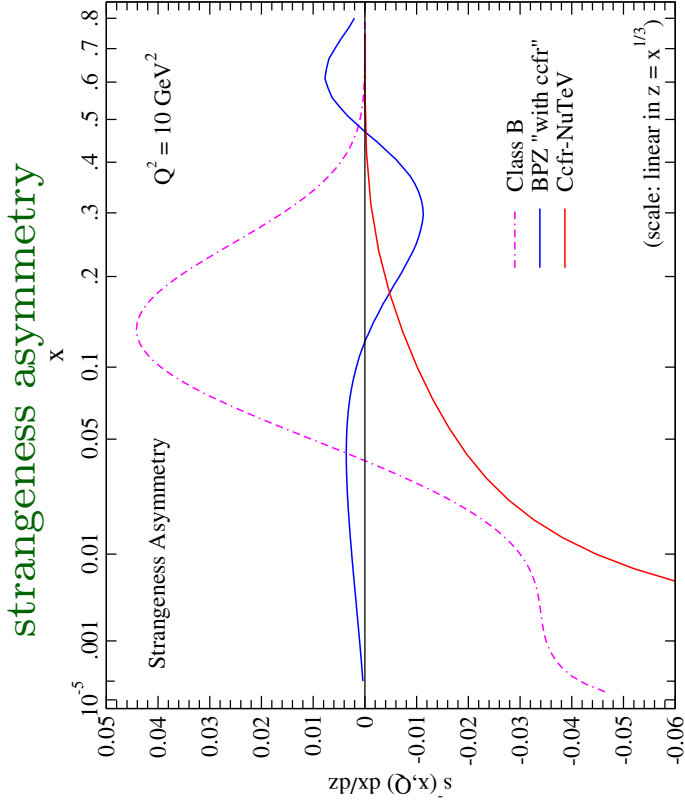
HOWEVER, BPZ GLOBAL FIT TO NEUTRINO INCLUSIVE DIS (Barone et al 2003) $\Rightarrow$

POSITIVE (TINY) ASYMMETRY

COMBINING INCLUSIVE AND EXCLUSIVE INFORMATION

CTEQ DEDICATED DIMUON ANALYSIS (April 2004)

- $\int_0^1 (s(x) - \bar{s}(x)) dx = 0$ IN PROTON
 $\Rightarrow$ EITHER $s(x) - \bar{s}(x)$ HAS A NODE OR IT VANISHES EVERYWHERE
- $[s(x) - \bar{s}(x)] < 0$ FOR SMALL $x \lesssim 0.05$ CONSTRAINED BY DIMUON

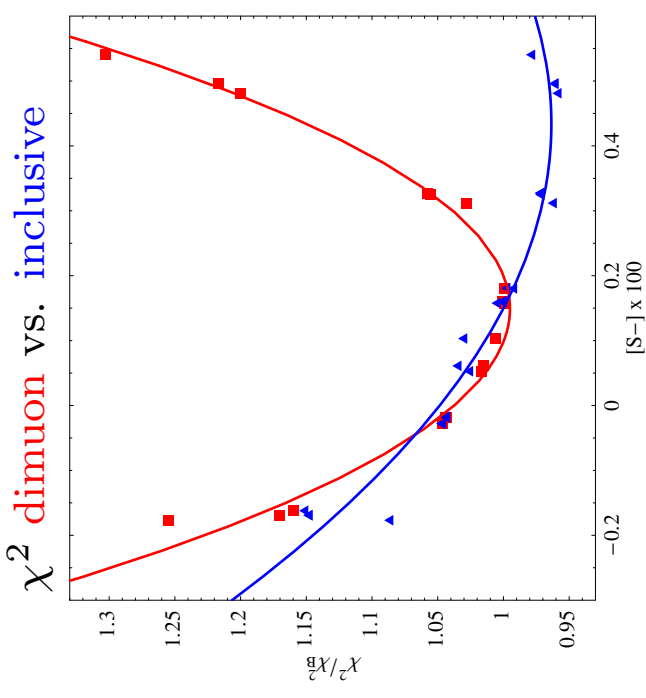
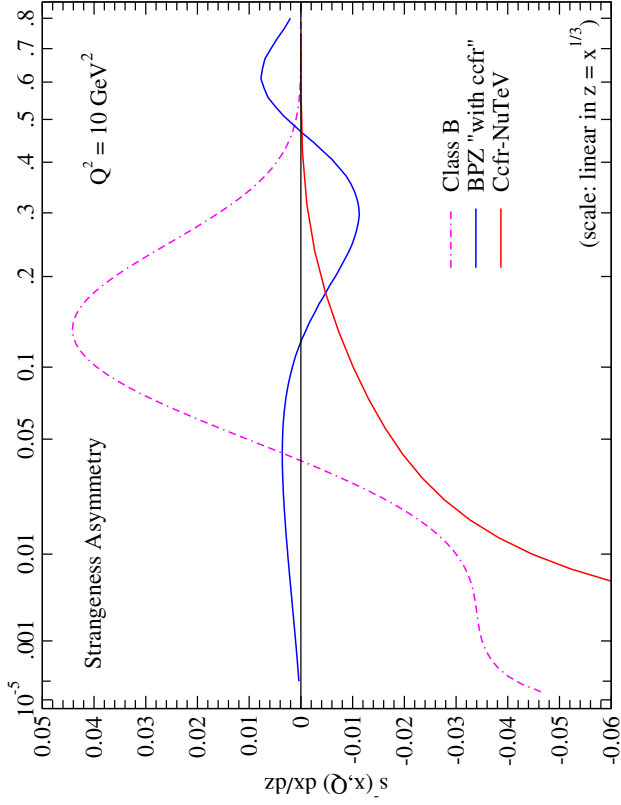


COMBINING INCLUSIVE AND EXCLUSIVE INFORMATION

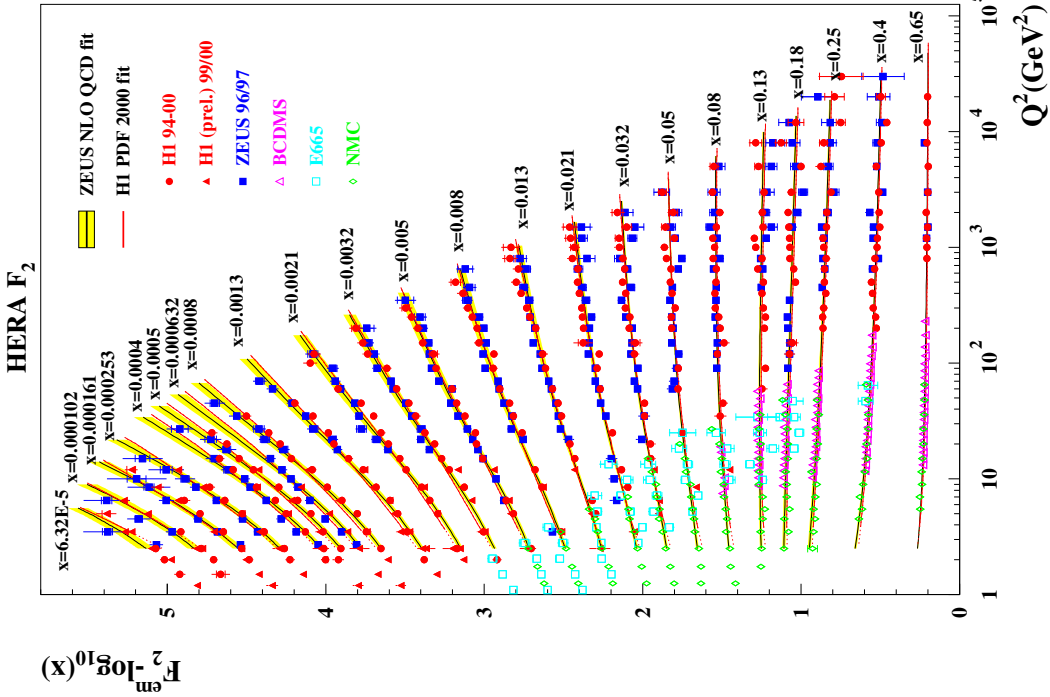
CTEQ DEDICATED DIMUON ANALYSIS (April 2004)

- $\int_0^1 (s(x) - \bar{s}(x)) dx = 0$ IN PROTON
 $\Rightarrow$ EITHER $s(x) - \bar{s}(x)$ HAS A NODE OR IT VANISHES EVERYWHERE
- $[s(x) - \bar{s}(x)] < 0$ FOR SMALL $x \lesssim 0.05$ CONSTRAINED BY DIMUON
- LARGE x REGION WEIGHS MORE IN MOMENTUM FRACTION
- **POSITIVE MOM. FRACTION $s - \bar{s} \approx 0.02$: THE END OF THE NUTEV ANOMALY**

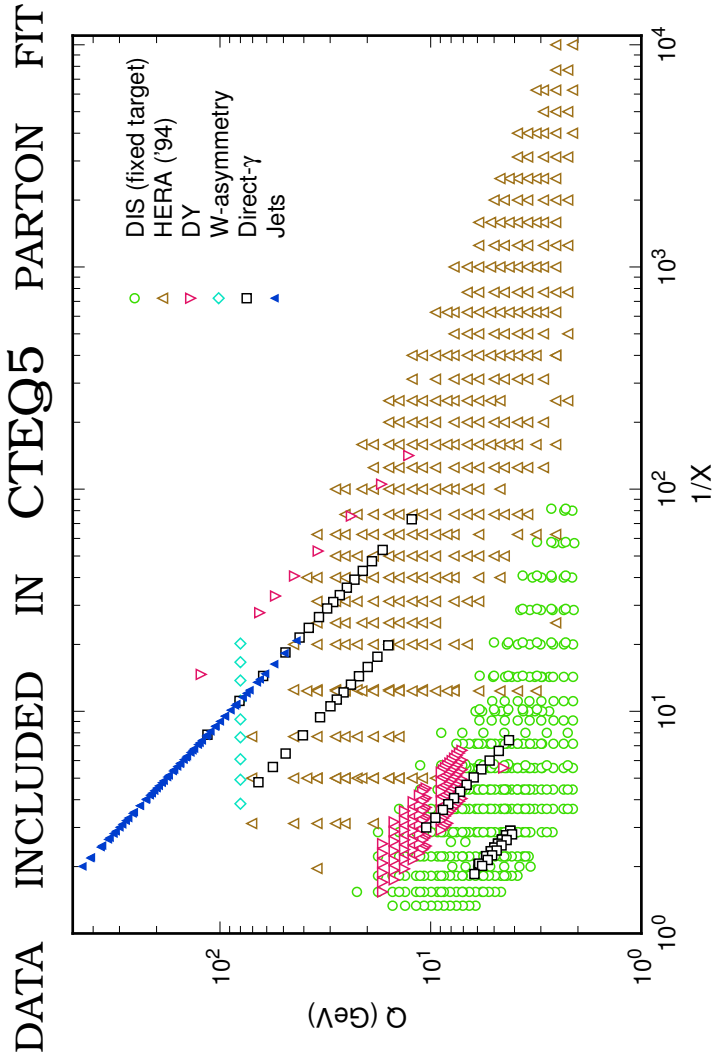
strangeness asymmetry



PARTONS WITH ERRORS



GIVEN A SET OF DATA POINTS
MUST DETERMINE A SET OF
FUNCTIONS WITH ERRORS



WHAT'S THE PROBLEM? D. Kosower, 1999

- FOR A SINGLE QUANTITY, WE QUOTE 1 SIGMA ERRORS: $\text{VALUE} \pm \text{ERROR}$
- FOR A PAIR OF NUMBERS, WE QUOTE A 1 SIGMA ELLIPSE
- FOR A FUNCTION, WE NEED AN “ERROR BAR” IN A SPACE OF FUNCTIONS

MUST DETERMINE THE PROBABILITY DENSITY (MEASURE) $\mathcal{P}[f_i(x)]$

IN THE SPACE OF PARTON DISTRIBUTION FUNCTIONS $f_i(x)$ (i =quark, antiquark, gluon)

WHAT'S THE PROBLEM? D. Kosower, 1999

- FOR A SINGLE QUANTITY, WE QUOTE 1 SIGMA ERRORS: VALUE $\pm$ ERROR
- FOR A PAIR OF NUMBERS, WE QUOTE A 1 SIGMA ELLIPSE
- FOR A FUNCTION, WE NEED AN “ERROR BAR” IN A SPACE OF FUNCTIONS

MUST DETERMINE THE PROBABILITY DENSITY (MEASURE) $\mathcal{P}[f_i(x)]$

IN THE SPACE OF PARTON DISTRIBUTION FUNCTIONS $f_i(x)$ (i =quark, antiquark, gluon)

EXPECTATION VALUE OF $\sigma[f_i(x)] \Rightarrow$ FUNCTIONAL INTEGRAL

$$\langle \sigma[f_i(x)] \rangle = \int \mathcal{D}f_i \sigma[f_i(x)] \mathcal{P}[f_i],$$

WHAT'S THE PROBLEM? D. Kosower, 1999

- FOR A SINGLE QUANTITY, WE QUOTE 1 SIGMA ERRORS: VALUE $\pm$ ERROR
- FOR A PAIR OF NUMBERS, WE QUOTE A 1 SIGMA ELLIPSE
- FOR A FUNCTION, WE NEED AN “ERROR BAR” IN A SPACE OF FUNCTIONS

MUST DETERMINE THE PROBABILITY DENSITY (MEASURE) $\mathcal{P}[f_i(x)]$

IN THE SPACE OF PARTON DISTRIBUTION FUNCTIONS $f_i(x)$ (i =quark, antiquark, gluon)

EXPECTATION VALUE OF $\sigma[f_i(x)] \Rightarrow$ FUNCTIONAL INTEGRAL

$$\langle \sigma[f_i(x)] \rangle = \int \mathcal{D}f_i \sigma[f_i(x)] \mathcal{P}[f_i],$$

MUST DETERMINE AN INFINITE-DIMENSIONAL OBJECT
FROM A FINITE SET OF DATA POINTS

THE STANDARD SOLUTION:

FUNCTIONAL PARTON FITTING

- CHOOSE A FIXED FUNCTIONAL FORM:

- MRST: 24 PARMS., SOME FIXED $\rightarrow$ 15 PARMS.

$$xq(x, Q_0^2) = A(1-x)^\eta(1+\epsilon x^{0.5} + \gamma x)x^\delta, \quad x[\bar{u} - \bar{d}](x, Q_0^2) = A(1-x)^\eta(1+\gamma x + \delta x^2)x^\delta.$$

$$xg(x, Q_0^2) = A_g(1-x)^{\eta_g}(1+\epsilon_g x^{0.5} + \gamma_g x)x^{\delta_g} - A_-(1-x)^{\eta_-}x^{-\delta_-},$$

- CTEQ: 20 PARMS.

$$xf(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} e^{A_3 x} (1+e^{A_4} x)^{A_5}$$

with independent params for combinations $u_v \equiv u - \bar{u}$, $d_v \equiv d - \bar{d}$, g , and $\bar{u} + \bar{d}$,
 $s = \bar{s} = 0.2(\bar{u} + \bar{d})$ at Q_0 ; NORM. FIXED BY SUM RULES

- ALEKHIN: 17 PARMS.

$$xu_V(x, Q_0) = \frac{2}{N_V^u} x^{a_u} (1-x)^{b_u} (1+\gamma_2^u x); \quad xu_S(x, Q_0) = \frac{A_S}{N_S} \eta_u x^{a_s} (1-x)^{b_{su}}$$

$$xd_V(x, Q_0) = \frac{1}{N_d^V} x^{a_d} (1-x)^{b_d}; \quad xd_S(x, Q_0) = \frac{A_S}{N_S} x^{a_s} (1-x)^{b_{sd}},$$

$$xs_S(x, Q_0) = \frac{A_S}{N_S} \eta_s x^{a_s} (1-x)^{(b_{su}+b_{sd})/2}; \quad xG(x, Q_0) = A_G x^{a_G} (1-x)^{b_G} (1+\gamma_1^G \sqrt{x} + \gamma_2^G x),$$

THE STANDARD SOLUTION:

FUNCTIONAL PARTON FITTING

- CHOOSE A FIXED FUNCTIONAL FORM:

- **MRST: 24 PARMS.**, SOME FIXED $\rightarrow$ **15 PARMS.**

$$xq(x, Q_0^2) = A(1-x)^\eta(1+\epsilon x^{0.5} + \gamma x)x^\delta, \quad x[\bar{u} - \bar{d}](x, Q_0^2) = A(1-x)^\eta(1+\gamma x + \delta x^2)x^\delta.$$

$$xg(x, Q_0^2) = A_g(1-x)^{\eta_g}(1+\epsilon_g x^{0.5} + \gamma_g x)x^{\delta_g} - A_-(1-x)^{\eta_-}x^{-\delta_-},$$

- **CTEQ: 20 PARMS.**

$$xf(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} e^{A_3 x} (1+e^{A_4} x)^{A_5}$$

with independent params for combinations $u_v \equiv u - \bar{u}$, $d_v \equiv d - \bar{d}$, g , and $\bar{u} + \bar{d}$,
 $s = \bar{s} = 0.2(\bar{u} + \bar{d})$ at Q_0 ; NORM. FIXED BY SUM RULES

- **ALEKHIN: 17 PARMS.**

$$xu_V(x, Q_0) = \frac{2}{N_V^u} x^{a_u} (1-x)^{b_u} (1+\gamma_2^u x); \quad xu_S(x, Q_0) = \frac{A_S}{N_S} \eta_u x^{a_s} (1-x)^{b_{su}}$$

$$xd_V(x, Q_0) = \frac{1}{N_d^V} x^{a_d} (1-x)^{b_d}; \quad xd_S(x, Q_0) = \frac{A_S}{N_S} x^{a_s} (1-x)^{b_{sd}},$$

$$xs_S(x, Q_0) = \frac{A_S}{N_S} \eta_s x^{a_s} (1-x)^{(b_{su}+b_{sd})/2}; \quad xG(x, Q_0) = A_G x^{a_G} (1-x)^{b_G} (1+\gamma_1^G \sqrt{x} + \gamma_2^G x),$$

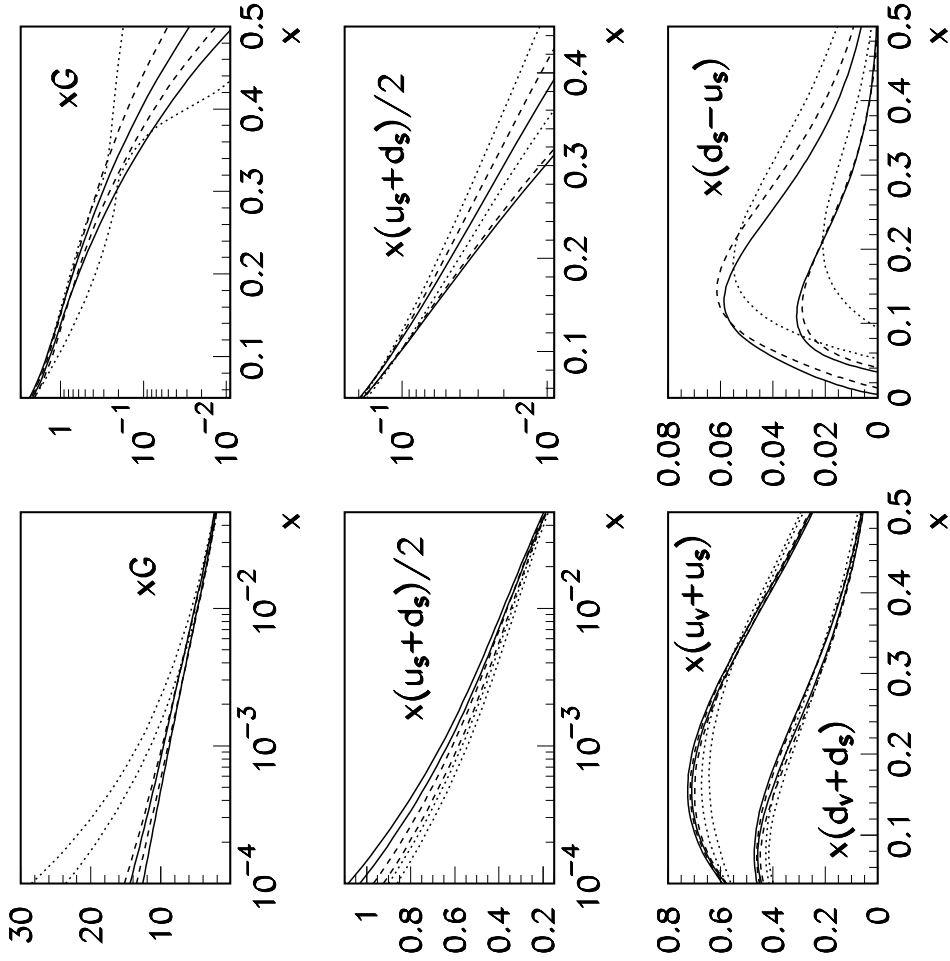
- EVOLVE TO DESIRED SCALE & COMPUTE PHYSICAL OBSERVABLES
- DETERMINE BEST-FIT VALUES OF PARAMETERS
- DETERMINE ERROR BY PROPAGATION OF ERROR ON PARMS (‘HESSIAN METHOD’) OR BY PARM. SCANS (‘LAGRANGE MULTIPLIER METHOD’)

PROBLEM PROJECTED ONTO THE FINITE-DIMENSIONAL SPACE OF PARAMETERS

HOW WELL DOES IT WORK?

ALEKHIN 2003 PARTONS (DIS only)

$Q^2 = 9 \text{ GeV}^2$



TOTAL ERROR BANDS FOR

LO (DOTS), NLO (DASHES),

NNLO (SOLID)

PARTON DISTRIBUTIONS

valence $u^v \equiv u - \bar{u}$, $d^v \equiv d - \bar{d}$,
 sea $u^s = \bar{u}^s = d^s = \bar{d}^s$

BUT CAN WE TRUST THESE ERRORS?

PDF ERRORS COMPARABLE TO OR EVEN LARGER THAN THEORY ERRORS

W PRODUCTION CROSS-SECTION TEVATRON

PDF SET	XSEC [NB]	PDF UNCERTAINTY
ALEKHIN	2.73	$\pm$ 0.05 (TOT)
MRST2002	2.59	$\pm$ 0.03 (EXPT)
CTEQ6	2.54	$\pm$ 0.10 (EXPT)

THORNE 2003

- ALEKHIN VS. MRST/CTEQ
→ W PRODUCTION XSECT AT
TEVATRON DO NOT AGREE
WITHIN RESPECTIVE ERRORS

BUT CAN WE TRUST THESE ERRORS?

PDF ERRORS COMPARABLE TO OR EVEN LARGER THAN THEORY ERRORS

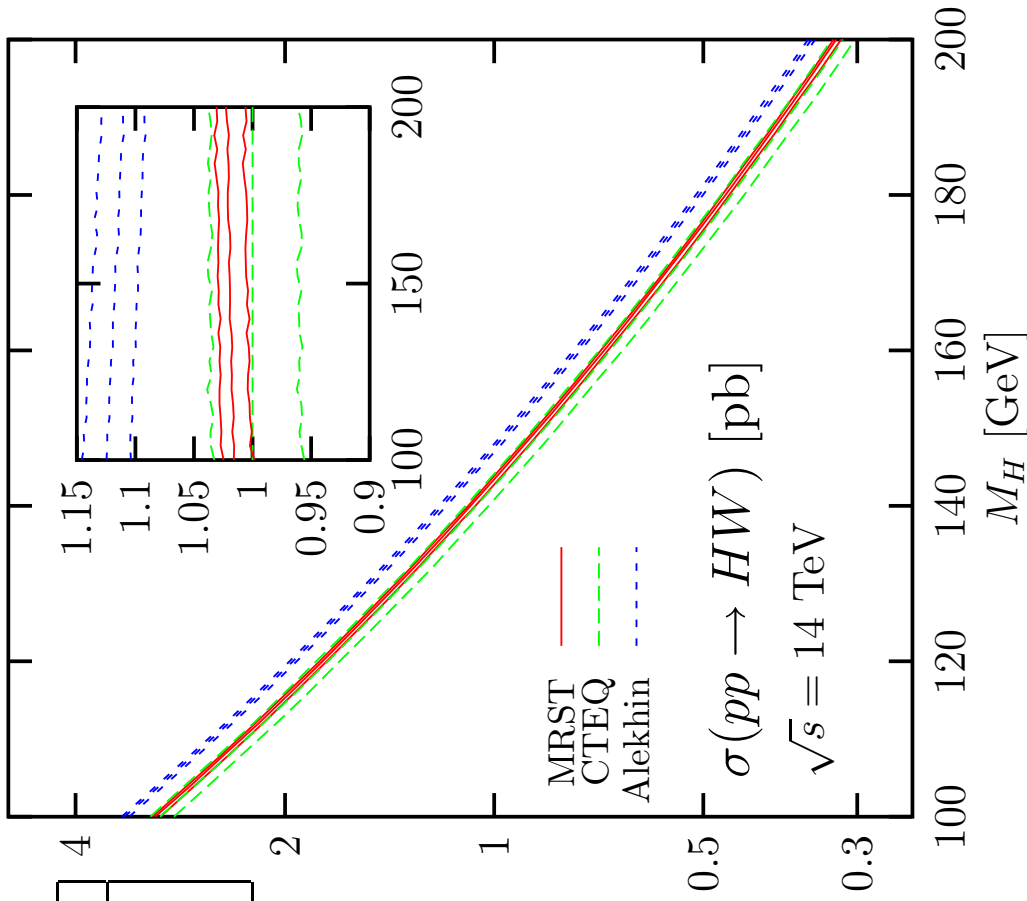
W PRODUCTION CROSS-SECTION
TEVATRON

PDF SET	XSEC [NB]	PDF UNCERTAINTY
ALEKHIN	2.73	$\pm$ 0.05 (TOT)
MRST2002	2.59	$\pm$ 0.03 (EXPT)
CTEQ6	2.54	$\pm$ 0.10 (EXPT)

THORNE 2003

- ALEKHIN VS. MRST/CTEQ
→ W PRODUCTION XSECT AT
TEVATRON DO NOT AGREE
WITHIN RESPECTIVE ERRORS
- ALEKHIN VS. MRST/CTEQ
→ PREDICTIONS FOR ASSO-
CIATE HIGGS W PRODUCTION
LHC DO NOT AGREE WITHIN
RESPECTIVE ERRORS

HIGGS PRODUCTION AT LHC



BUT CAN WE TRUST THESE ERRORS?

PDF ERRORS COMPARABLE TO OR EVEN LARGER THAN THEORY ERRORS

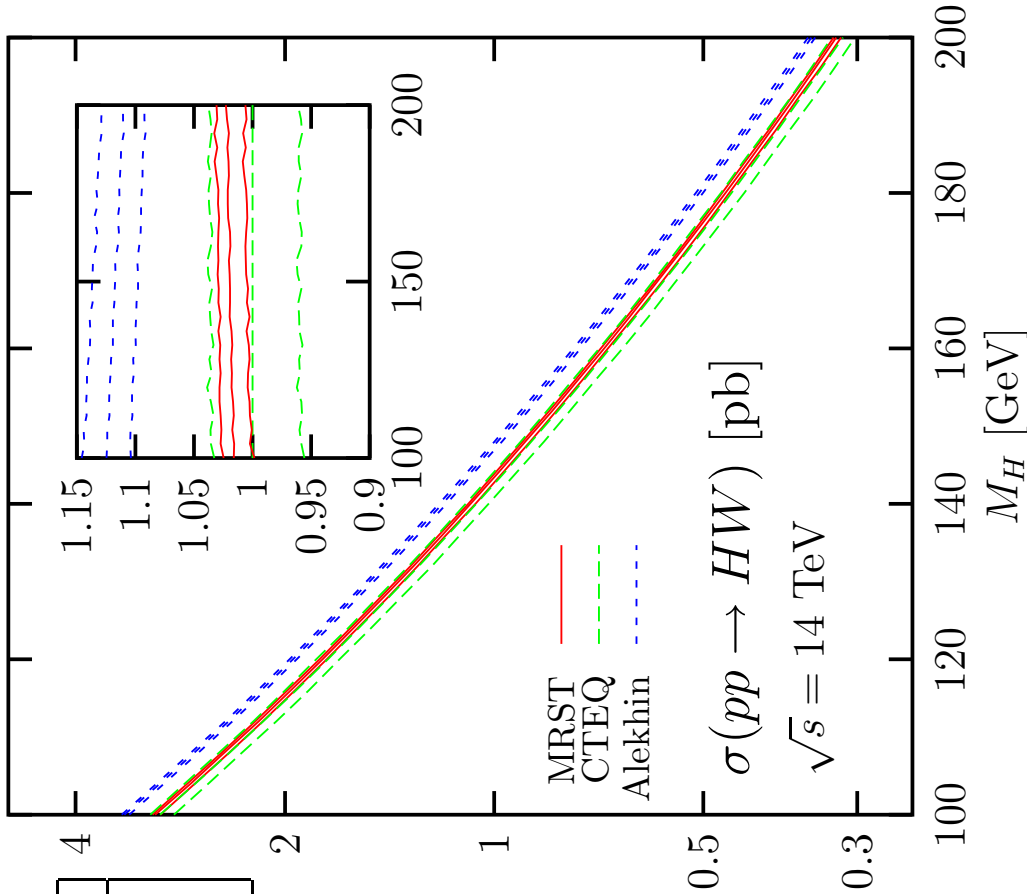
W PRODUCTION CROSS-SECTION
TEVATRON

PDF SET	XSEC [NB]	PDF UNCERTAINTY
ALEKHIN	2.73	$\pm$ 0.05 (TOT)
MRST2002	2.59	$\pm$ 0.03 (EXPT)
CTEQ6	2.54	$\pm$ 0.10 (EXPT)

THORNE 2003

- ALEKHIN VS. MRST/CTEQ
→ W PRODUCTION XSECT AT
TEVATRON DO NOT AGREE
WITHIN RESPECTIVE ERRORS
- ALEKHIN VS. MRST/CTEQ
→ PREDICTIONS FOR ASSO-
CIATE HIGGS W PRODUCTION
LHC DO NOT AGREE WITHIN
RESPECTIVE ERRORS

HIGGS PRODUCTION AT LHC



DJOUADI AND FERRAG, 2004

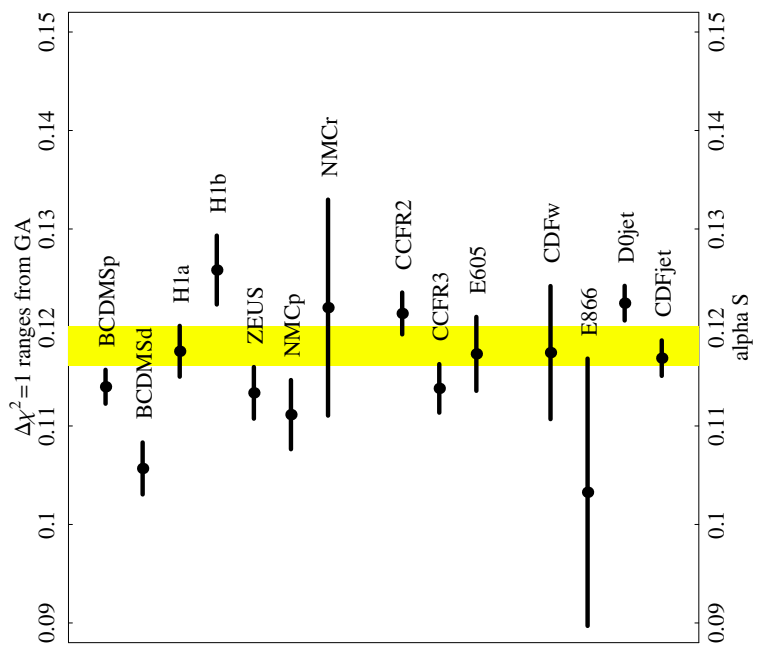
PARTON SETS DO NOT AGREE WITHIN RESPECTIVE ERRORS

HOW ARE PDF ERRORS DETERMINED?

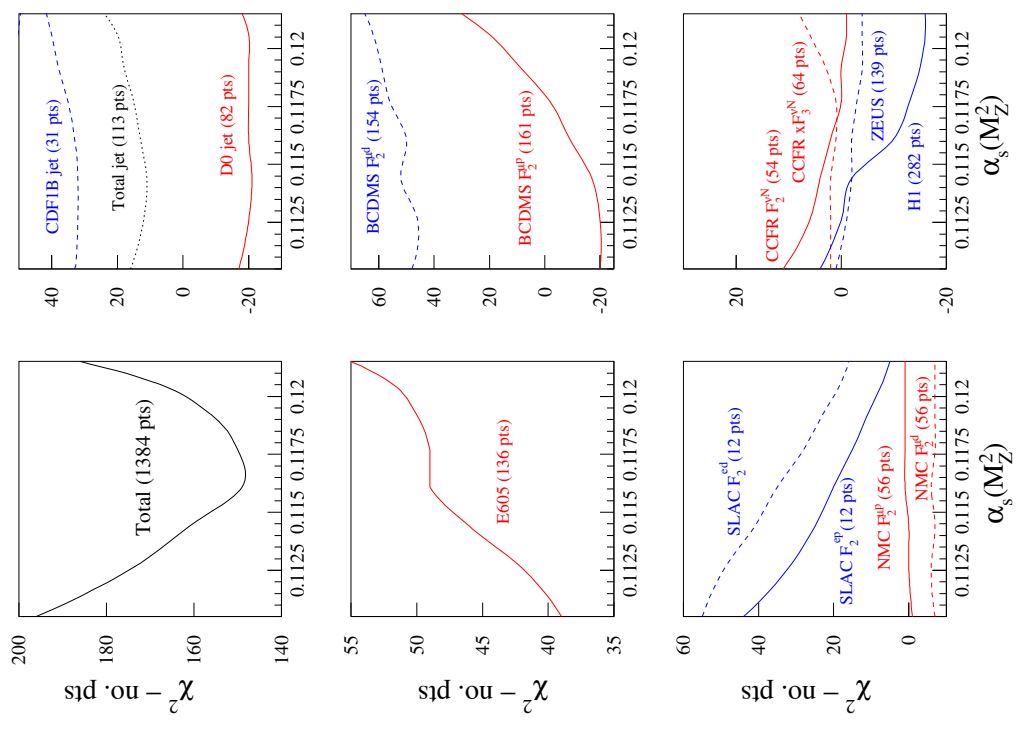
GLOBAL χ^2 MINIMUM DOES NOT
CORRESPOND TO LOCAL MINIMA

BEST-FIT OF α_s

$\Delta\chi^2 = 1$ INCONSISTENT
BETWEEN EXPERIMENTS



CTEQ 2003



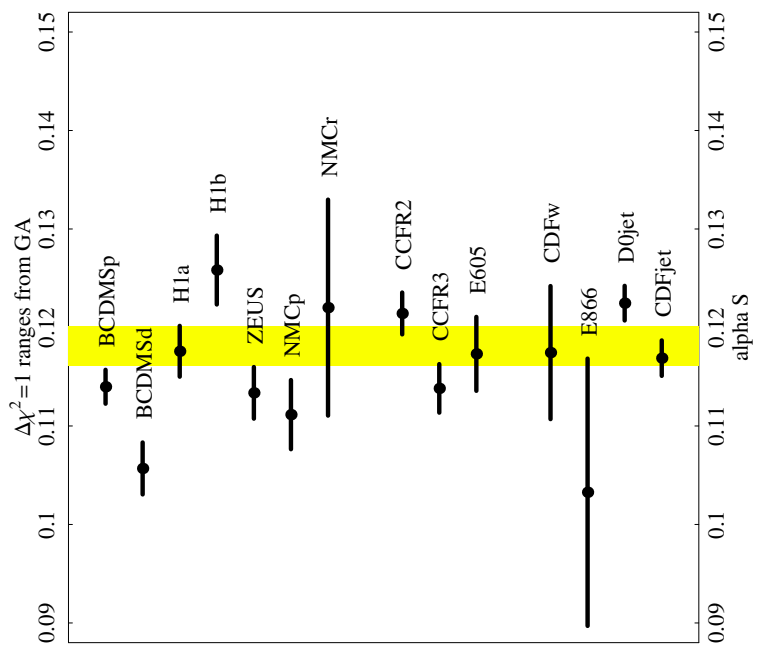
MRST 2003

HOW ARE PDF ERRORS DETERMINED?

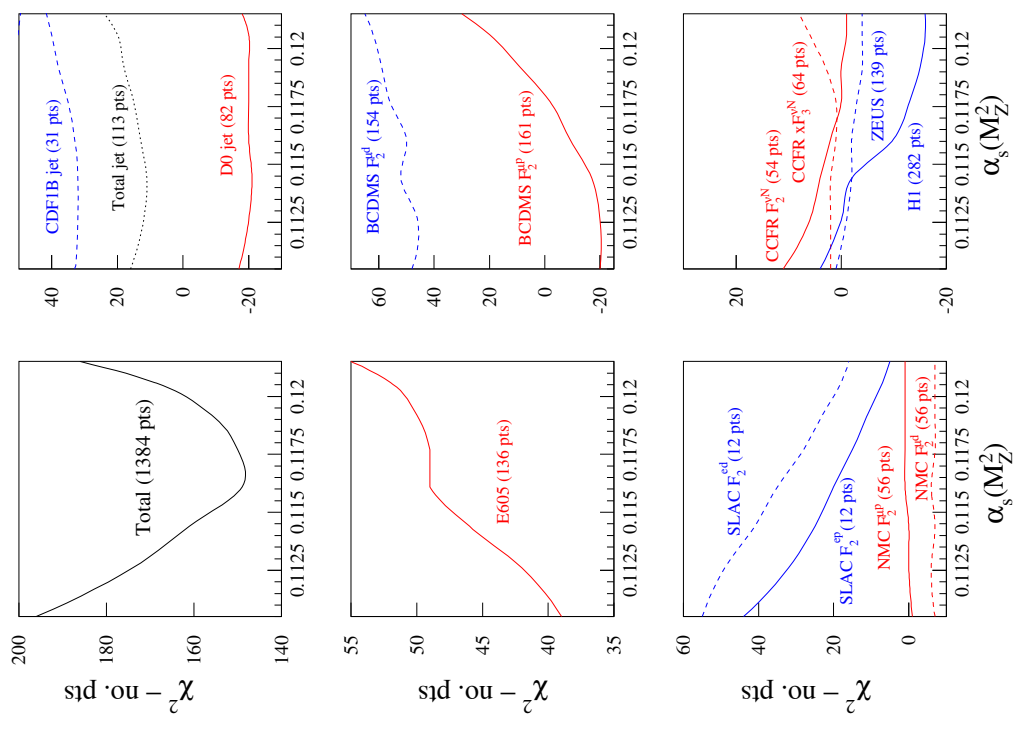
GLOBAL χ^2 MINIMUM DOES NOT
CORRESPOND TO LOCAL MINIMA

BEST-FIT OF α_s

$\Delta\chi^2 = 1$ INCONSISTENT
BETWEEN EXPERIMENTS



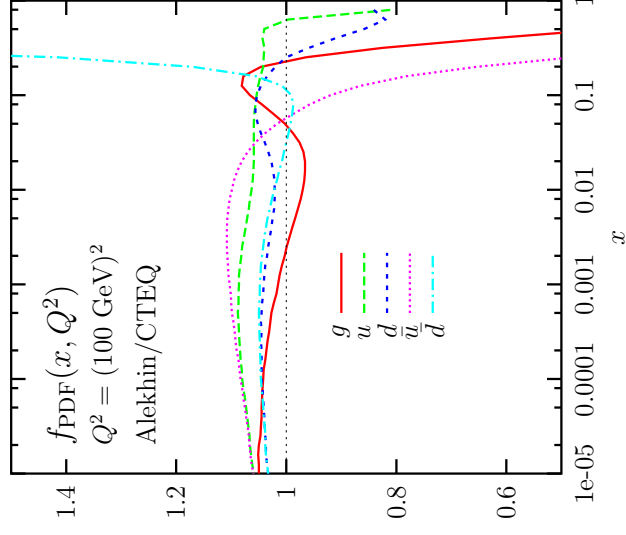
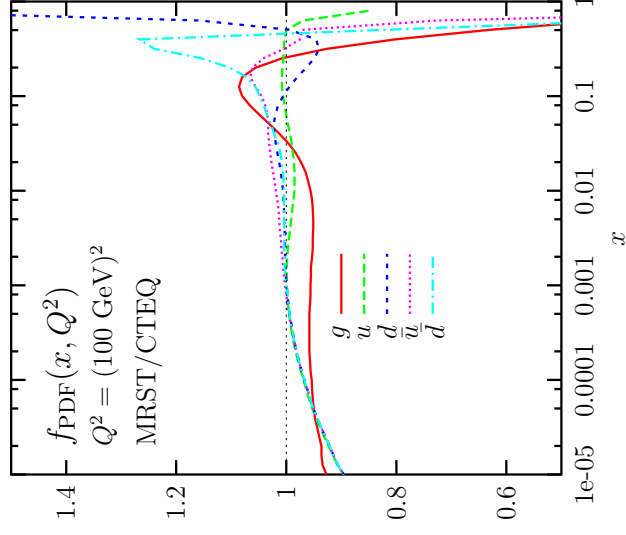
CTEQ 2003



MRST 2003

PROBLEM: INCONSISTENT EXPERIMENTS

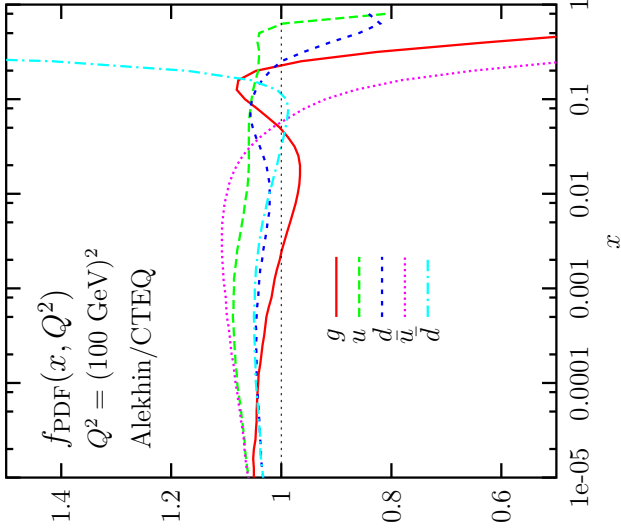
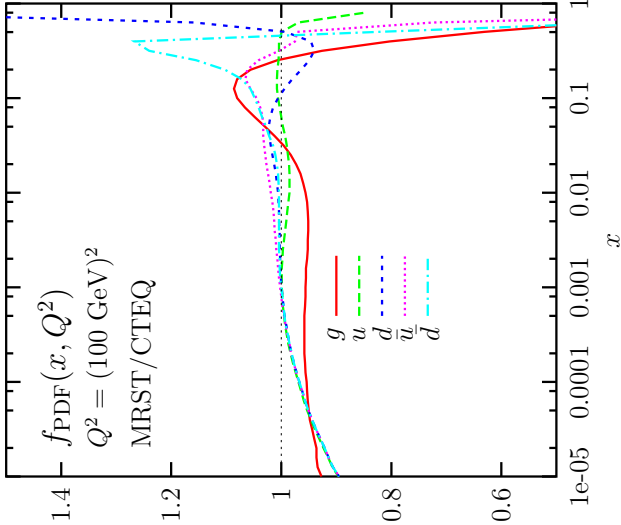
PARAMETRIZATION BIAS?



MRST & CTEQ
→ SIMILAR PARTONS

Djouadi and Ferrag 2003

PARAMETRIZATION BIAS?



MRST & CTEQ
→ SIMILAR PARTONS

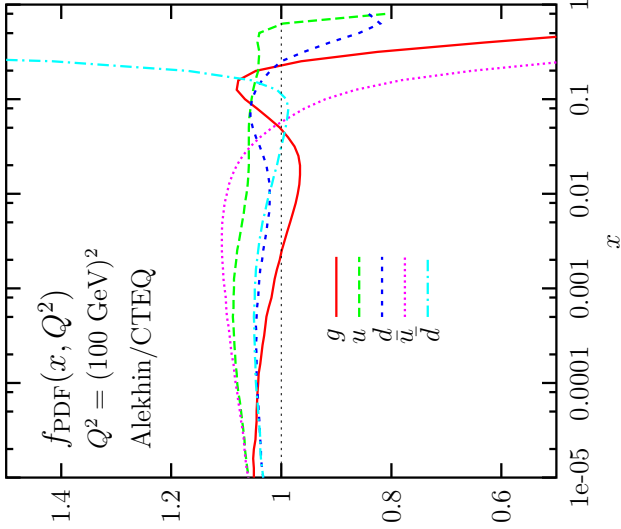
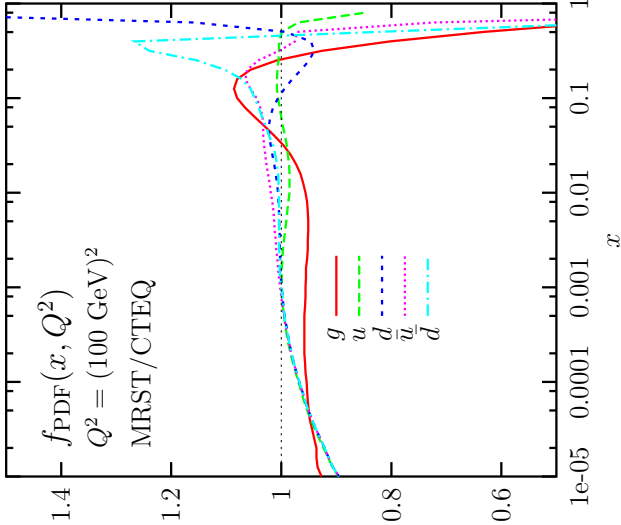
Djouadi and Ferrag 2003

THE W XSECT. AGAIN...

SIMILAR PAR-
TONS
→ SIMILAR
RESULTS

PDF SET	COMMENT	XSEC [NB]	PDF UNCERTAINTY
ALEKHIN	TEVATRON	2.73	± 0.05 (TOT)
MRST2002	TEVATRON	2.59	± 0.03 (EXPT)
CTEQ6	TEVATRON	2.54	± 0.10 (EXPT)
ALEKHIN	LHC	215	± 6 (TOT)
MRST2002	LHC	204	± 4 (EXPT)
CTEQ6	LHC	205	± 8 (EXPT)

PARAMETRIZATION BIAS?



MRST & CTEQ
→ SIMILAR PARTONS

Djouadi and Ferrag 2003

THE W XSECT. AGAIN...

SIMILAR PAR-
TONS
→ SIMILAR
RESULTS

PDF SET	COMMENT	XSEC [NB]	PDF UNCERTAINTY
ALEKHIN	TEVATRON	2.73	± 0.05 (TOT)
MRST2002	TEVATRON	2.59	± 0.03 (EXPT)
CTEQ6	TEVATRON	2.54	± 0.10 (EXPT)
ALEKHIN	LHC	215	± 6 (TOT)
MRST2002	LHC	204	± 4 (EXPT)
CTEQ6	LHC	205	± 8 (EXPT)

We do not seem to have the optimum parameterization for both finding the best fit and also investigating fluctuations about this best fit (...) This might then influence our error analysis...(MRST 2004)

CONSERVATIVE SOLUTIONS

CTEQ TOLERANCE CRITERION

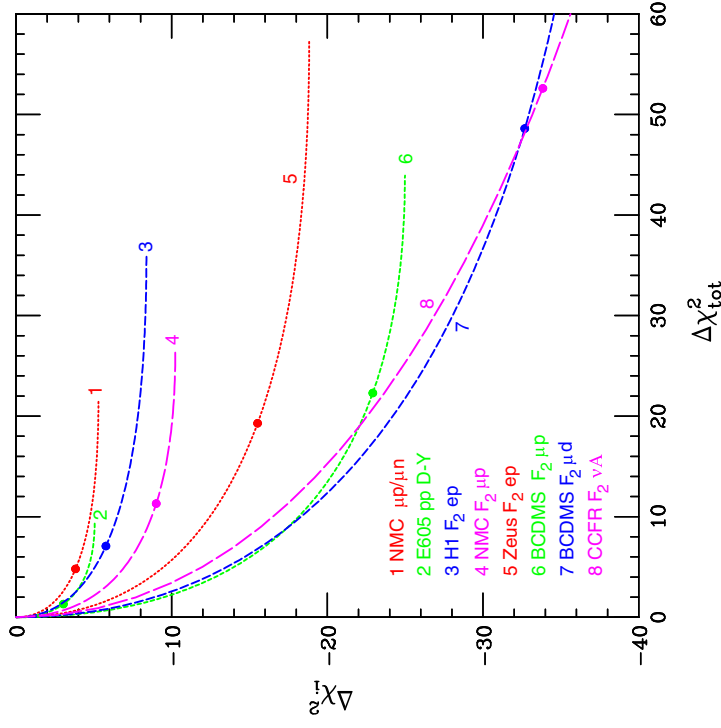
SINGLE OUT INCONSISTENT DATA

- how many parameters are significantly determined by each dataset?
- how consistent are the data from one set with the rest?

STUDY MINIMUM ALLOWED χ_i^2

FOR i -TH EXP. AS

GLOBAL χ^2 ALLOWED TO INCREASE



Collins, Pumplin 2001

CCFR, BCDMS

INCOMPATIBLE WITH THE REST

CONSERVATIVE SOLUTIONS

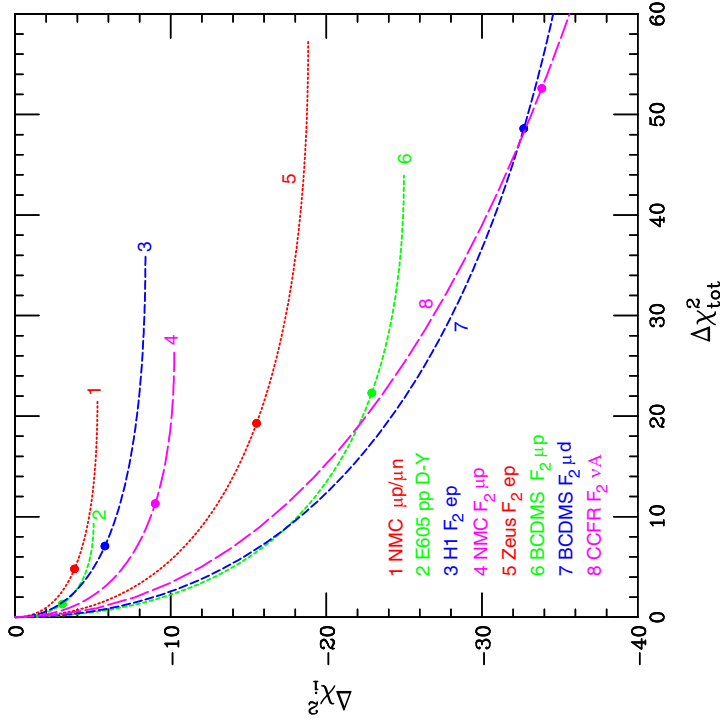
CTEQ TOLERANCE CRITERION

SINGLE OUT INCONSISTENT DATA

- how many parameters are significantly determined by each dataset?
- how consistent are the data from one set with the rest?

STUDY MINIMUM ALLOWED χ^2_i
FOR i -TH EXP. AS

GLOBAL χ^2 ALLOWED TO INCREASE



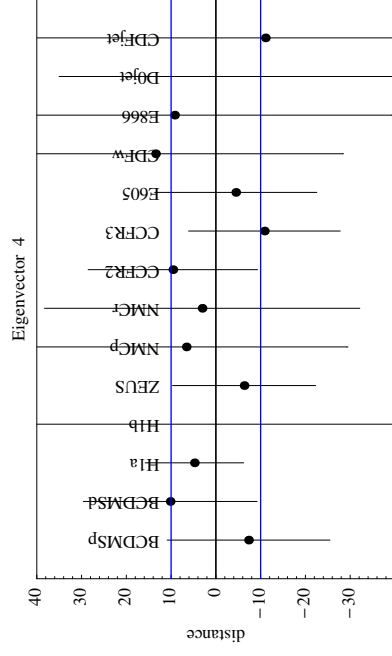
Collins, Pumplin 2001

CCFR, BCDMS

INCOMPATIBLE WITH THE REST

OPTIONS

- discard incompatible experiments
- reweight individual contributions
- INCORPORATE IN ERROR, TOLERATING FIXED MAX DEVIATION FOR EACH EXPERIMENT & EACH FIT PARAMETER



CONSERVATIVE SOLUTIONS

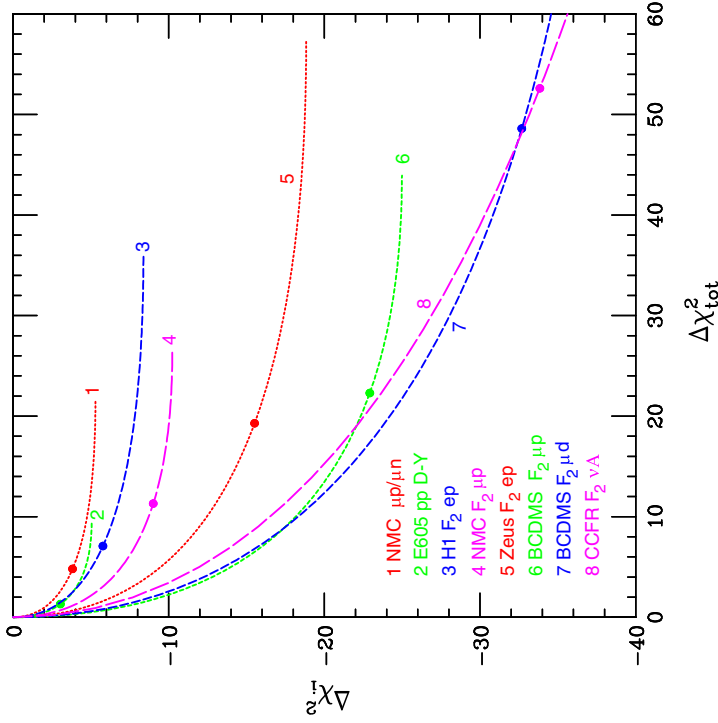
CTEQ TOLERANCE CRITERION

SINGLE OUT INCONSISTENT DATA

- how many parameters are significantly determined by each dataset?
- how consistent are the data from one set with the rest?

STUDY MINIMUM ALLOWED χ^2_i
FOR i -TH EXP. AS

GLOBAL χ^2 ALLOWED TO INCREASE

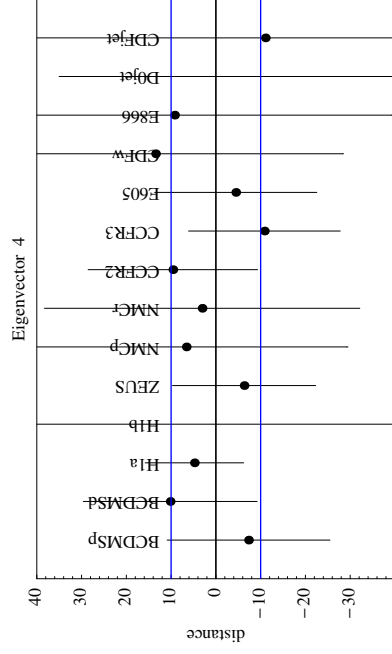


Collins, Pumplin 2001

CCFR, BCDMS
INCOMPATIBLE WITH THE REST

OPTIONS

- discard incompatible experiments
- reweight individual contributions
- INCORPORATE IN ERROR, TOLERATING FIXED MAX DEVIATION FOR EACH EXPERIMENT & EACH FIT PARAMETER



$$\Delta\chi^2 = 100 \text{ (CTEQ6)}$$

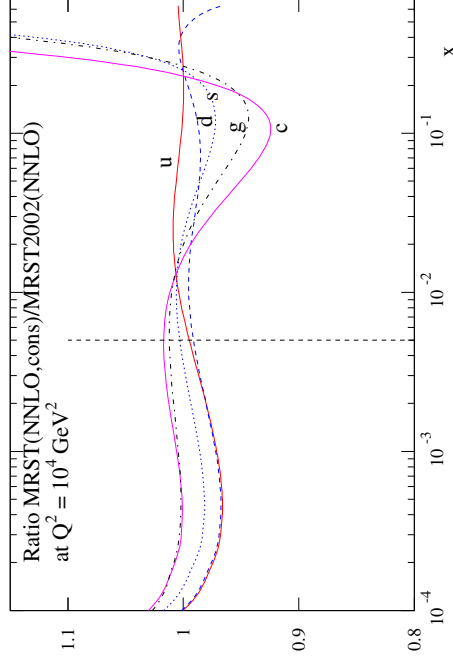
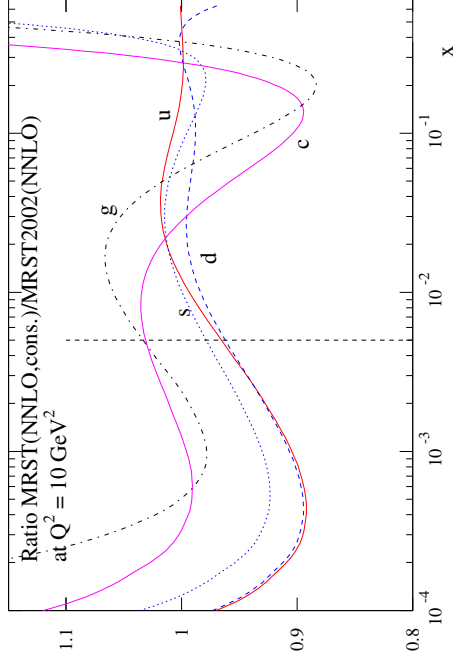
CONSERVATIVE SOLUTIONS

IMPOSE RESTRICTIVE KINEMATIC CUTS

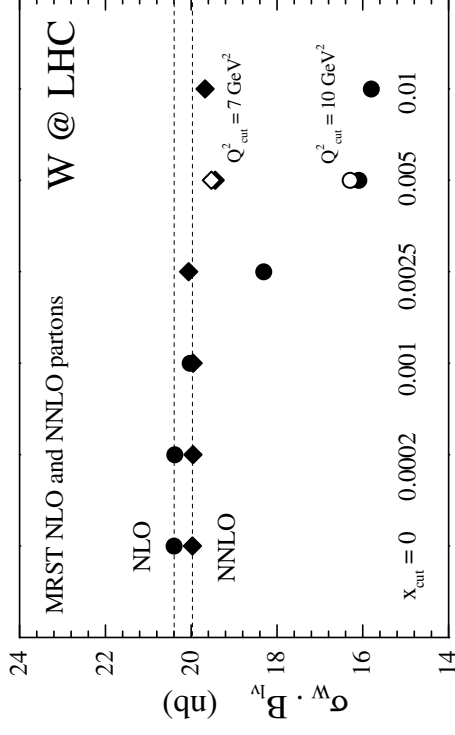
cut in Q^2 raised from 2 to 10 GeV²; cut off $x < 0.005$; cut in W^2 raised from 12.5 to 15 GeV²

MRST 2003 CONSERVATIVE PARTONS

- ABOUT 800 DATAPOINTS REMOVED OUT OF ABOUT 2000
- $\Delta\chi^2 = 5$ SUFFICIENT FOR REASONABLE 1- σ CONSERVATIVE/PLAIN



VARIATION OF THE W XSECT.



conservative partons unreliable
for total xsect.
because small x region important
partly stabilized by NNLO

CONSERVATIVE SOLUTIONS

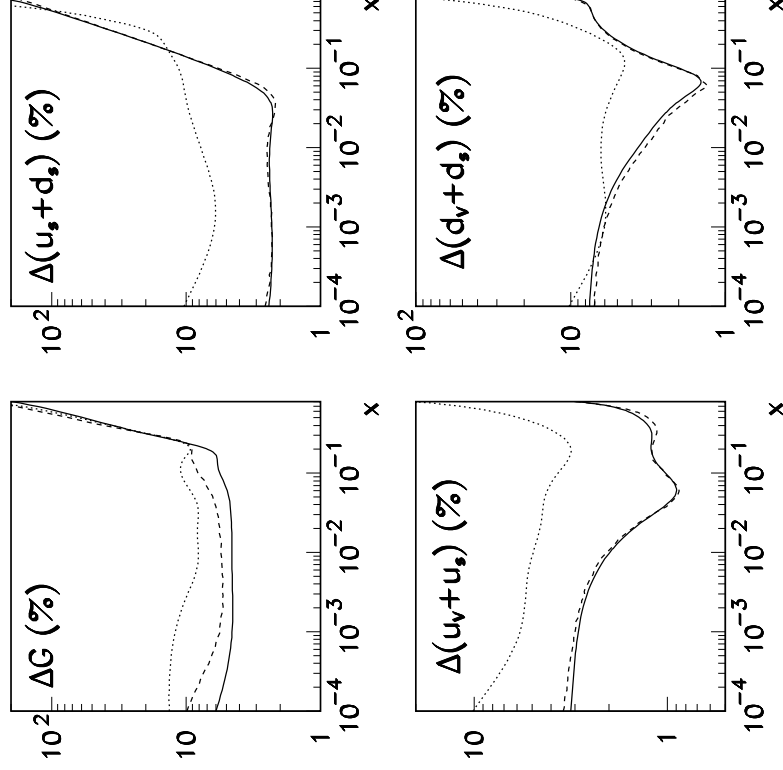
SELECT COHERENT SET OF DATA

ALEKHIN PARTONS

- ONLY DIS DATA INCLUDED
- $\Delta\chi^2 = 1$ PROVIDES GOOD 1- σ CURVES

PERCENTAGE ERRORS

$Q^2=9 \text{ GeV}^2$



- ALEKHIN 2003 PARTON UNCERTAINTIES (SOLID, DASHED) COMPARABLE TO CTEQ6 (DOTTED)
- CANNOT SEPARATE ACCURATELY ANTIQUARK DISTNS.
- ERROR ON σ_W COMPARABLE TO CTEQ, MRST

NEW SOLUTIONS

THE BAYESIAN MONTE CARLO APPROACH (GIELE, KOSOWER, KELLER 2001)

- generate a Monte-Carlo sample of fcts. with “reasonable” prior distn.
(e.g. an available parton set) $\rightarrow$ representation of probability functional $\mathcal{P}[f_i]$
- calculate observables with functional integral
- **update probability using Bayesian inference on MC sample:**
better agreement with data $\rightarrow$ more functions in sample
- **iterate until convergence achieved**

NEW SOLUTIONS

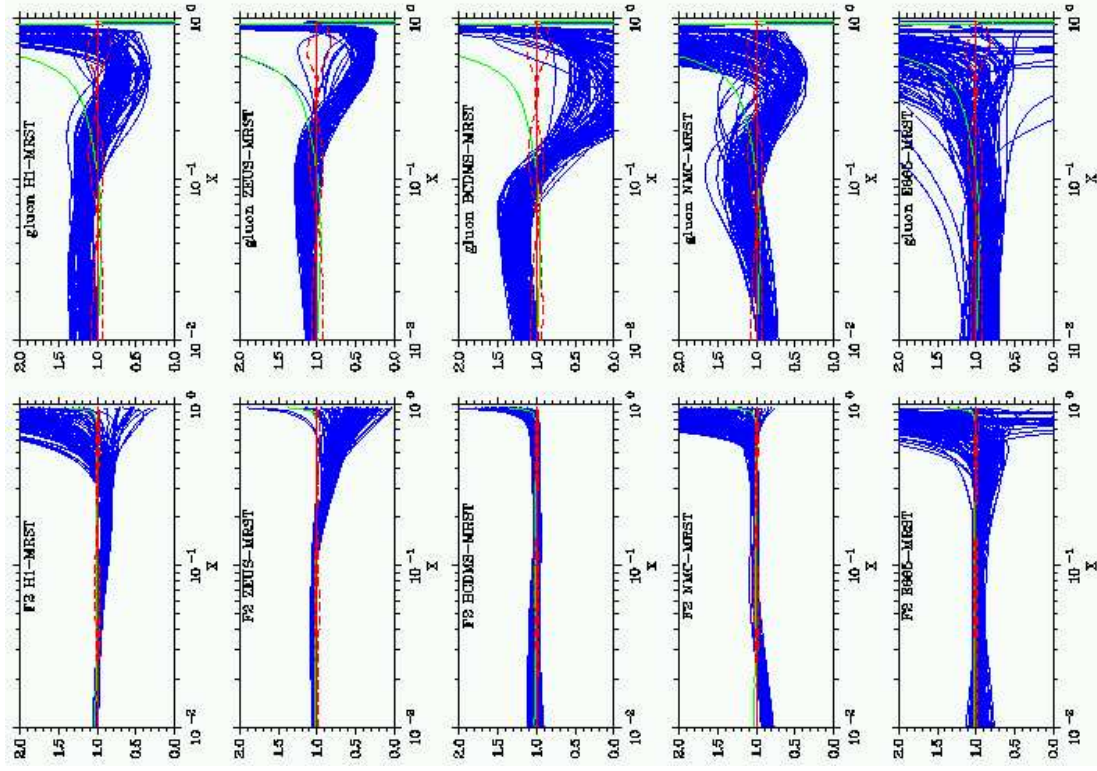
THE BAYESIAN MONTE CARLO APPROACH (GIELE, KOSOWER, KELLER 2001)

- generate a Monte-Carlo sample of fcts. with “reasonable” prior distn.
(e.g. an available parton set) $\rightarrow$ representation of probability functional $\mathcal{P}[f_i]$
- calculate observables with functional integral
- update probability using Bayesian inference on MC sample:
better agreement with data $\rightarrow$ more functions in sample
- iterate until convergence achieved

PROBLEM IS MADE FINITE-DIMENSIONAL BY THE CHOICE OF PRIOR, BUT
RESULT DO NOT DEPEND ON THE CHOICE IF SUFFICIENTLY GENERAL
HARD TO HANDLE “FLAT DIRECTIONS”

(Monte Carlo replicas which lead to same agreement with data);
COMPUTATIONALLY VERY INTENSIVE;
DIFFICULT TO ACHIEVE INDEP. FROM PRIOR

RESULT: FERMION PARTONS



F_2^{singlet} AND GLUON RATIOS FERMI/MRST

ONLY SUBSET OF DATA FITTED (H1, E665,
BCDMS DIS DATA)

GOOD AGREEMENT WITH TEVATRON W XSECT
TROUBLE WITH VALUE OF α_s

NEW SOLUTIONS

THE NEURAL MONTE CARLO APPROACH (S.F., GARRIDO, LATORRE, PICCIONE 2002)

BASIC IDEA: USE NEURAL NETWORKS AS UNIVERSAL UNBIASED INTERPOLANTS

NEW SOLUTIONS

THE NEURAL MONTE CARLO APPROACH (S.F., GARRIDO, LATORRE, PICCIONE 2002)

BASIC IDEA: USE NEURAL NETWORKS AS UNIVERSAL UNBIASED INTERPOLANTS

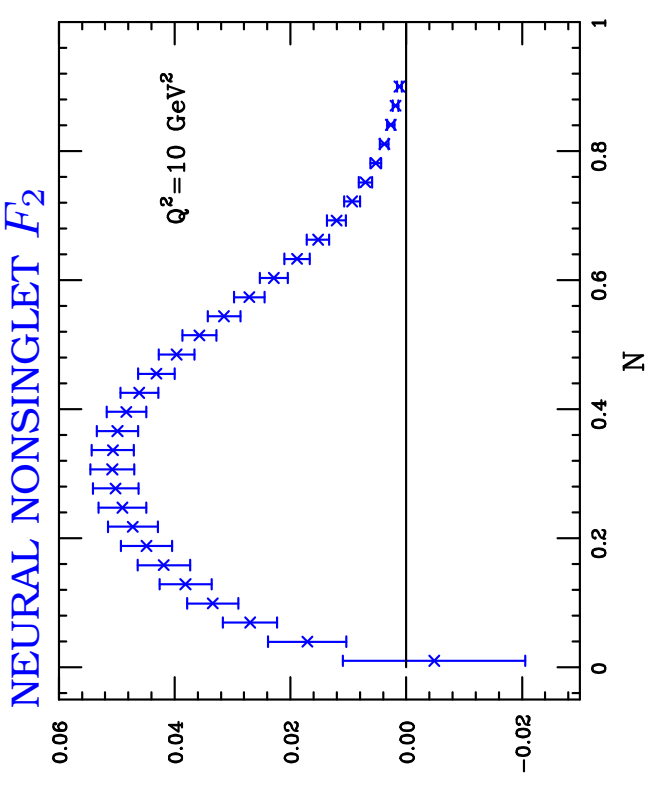
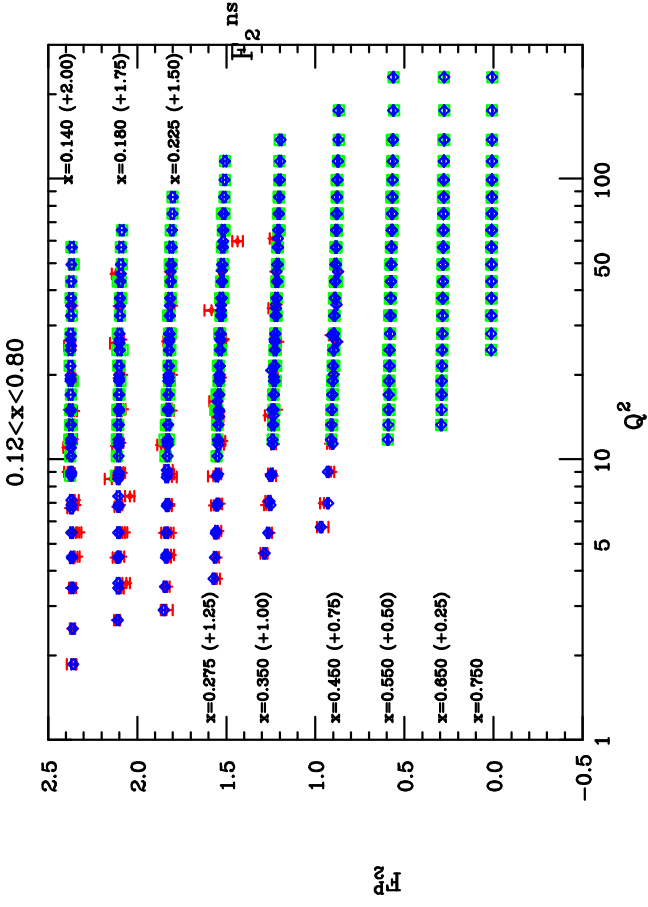
- GENERATE A SET OF MONTE CARLO REPLICAS $\sigma^{(k)}(p_i)$ OF THE ORIGINAL DATASET $\sigma^{(\text{data})}(p_i)$ (p_i values of the kin. variables where σ is measured)
 $\Rightarrow$ REPRESENTATION OF $\mathcal{P}[\sigma(p_i)]$ AT DISCRETE SET OF POINTS p_i
- TRAIN A NEURAL NET FOR EACH PDF ON EACH REPLICA, THUS OBTAINING A NEURAL REPRESENTATION OF THE PDFS $f_i^{(\text{net}), (k)}$
- THE SET OF NEURAL NETS IS A REPRESENTATION OF THE PROBABILITY DENSITY:

$$\left\langle \sigma[f_i] \right\rangle = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} \sigma[f_i^{(\text{net})(k)}]$$

- CHECK GOODNESS OF FIT THROUGH STATISTICAL INDICATORS (χ^2 , CORRELATION, ...)

PRELIMINARY RESULT: NEURAL STRUCTURE FUNCTIONS

NEURAL F_2 VS p DATA

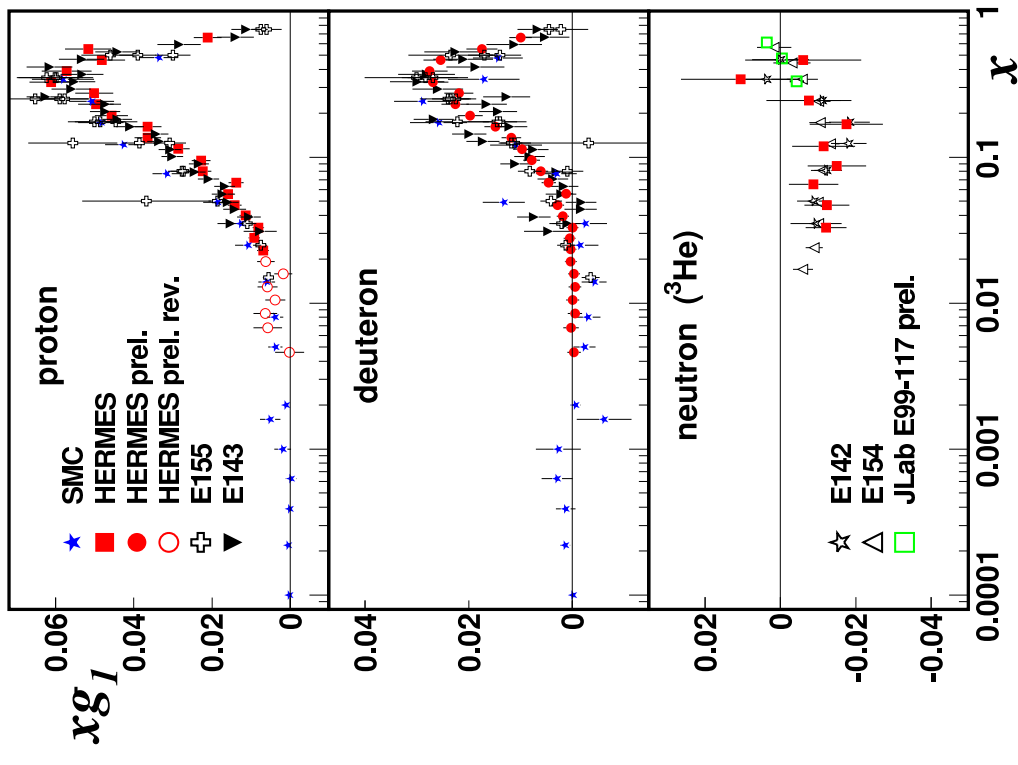


- FULL NEURAL FIT TO F_2 FOR PROTON, DEUTERON & NONSINGLET AVAILABLE
- ERRORS AND CORRELATIONS FAITHFULLY REPRODUCED, BUT STAT. UNCERTAINTIES OPTIMALLY COMBINED
 $\Rightarrow$ FIT CAN BE USED IN LIEU OF DATA, BUT BETTER THAN THEM (SEE F_2^{NS})
- FULL INTRICACIES OF THE PARTON FIT NOT IMPLEMENTED

THE POLARIZED CASE

- VIGOROUS EXPERIMENTAL PROGRAM AT CERN, SLAC, DESY DURING THE '90S, HERMES STILL RUNNING

POLARIZED STRUCTURE FUNCTION DATA

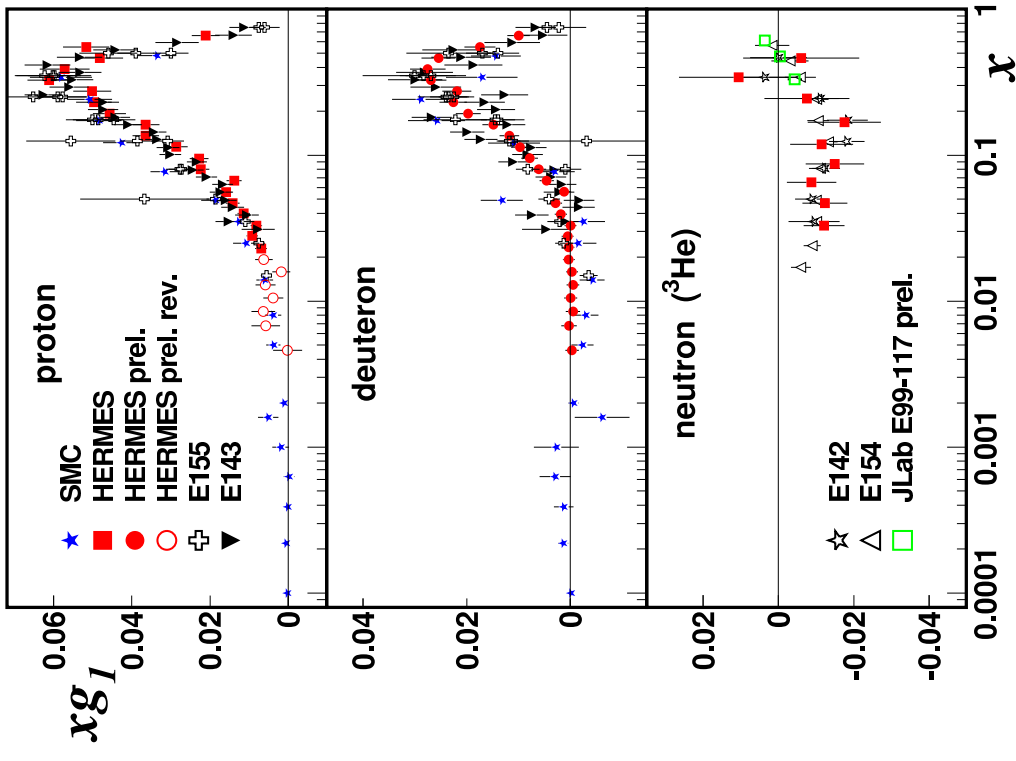


- SPIN SUM RULES $\Rightarrow$ SPIN CRISIS

THE POLARIZED CASE

- VIGOROUS EXPERIMENTAL PROGRAM AT CERN, SLAC, DESY DURING THE '90S, HERMES STILL RUNNING
- PROTON, DEUTERON AND ^3He DATA

POLARIZED STRUCTURE FUNCTION DATA



- SPIN SUM RULES $\Rightarrow$ SPIN CRISIS
- α_s FROM BJORKEN SR:
 $\alpha_s(M_z) = 0.120^{+0.010}_{-0.008}$ (ABFR 98);
 $\alpha_s(M_z) = 0.114^{+0.010}_{-0.009}$ (BB 2002)

THE POLARIZED CASE

- VIGOROUS EXPERIMENTAL PROGRAM AT CERN, SLAC, DESY DURING THE '90S, HERMES STILL RUNNING

- PROTON, DEUTERON AND ^3He DATA

- NO HADRON COLLIDER DATA YET

- SPIN SUM RULES $\Rightarrow$ SPIN CRISIS

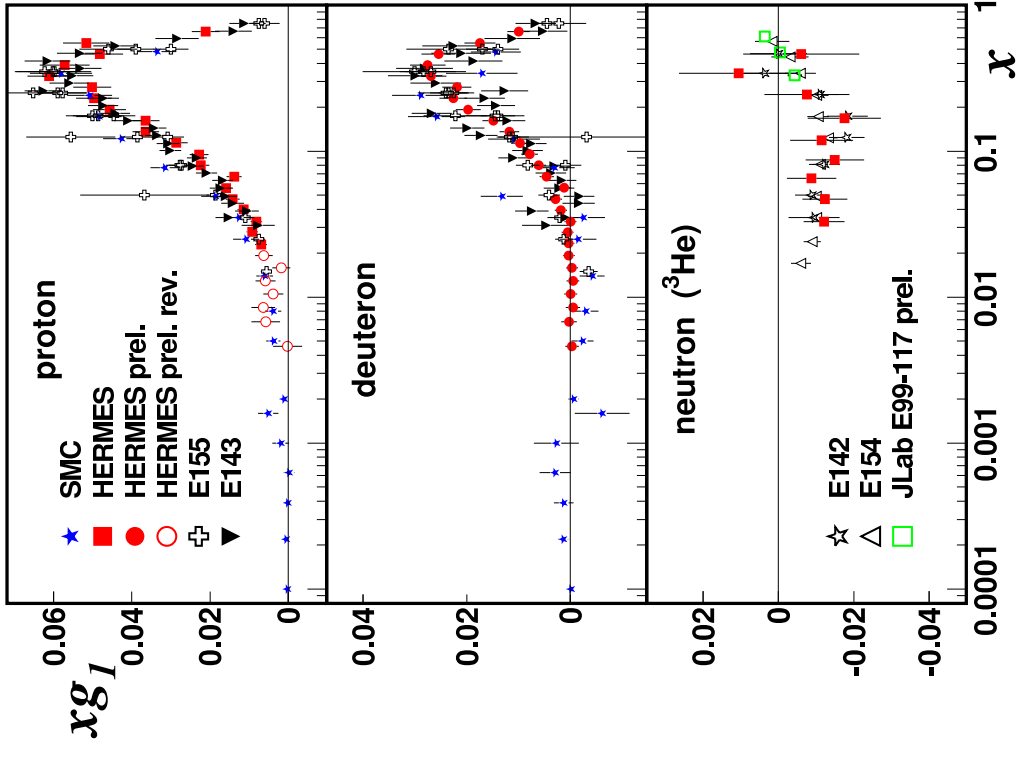
- α_s FROM BJORKEN SR:

$$\alpha_s(M_z) = 0.120^{+0.010}_{-0.008} \text{ (ABFR 98);}$$

$$\alpha_s(M_z) = 0.114^{+0.010}_{-0.009} \text{ (BB 2002)}$$

- $\Delta q, \Delta \bar{q}$ SEPARATION IMPOSSIBLE, Δg ONLY FROM SCALING VIOLATIONS

POLARIZED STRUCTURE FUNCTION DATA



THE POLARIZED CASE

- VIGOROUS EXPERIMENTAL PROGRAM AT CERN, SLAC, DESY DURING THE '90S, HERMES STILL RUNNING

- PROTON, DEUTERON AND ^3He DATA

- NO HADRON COLLIDER DATA YET

- NO CC CURRENT DATA

- SPIN SUM RULES $\Rightarrow$ SPIN CRISIS

- α_s FROM BJORKEN SR:

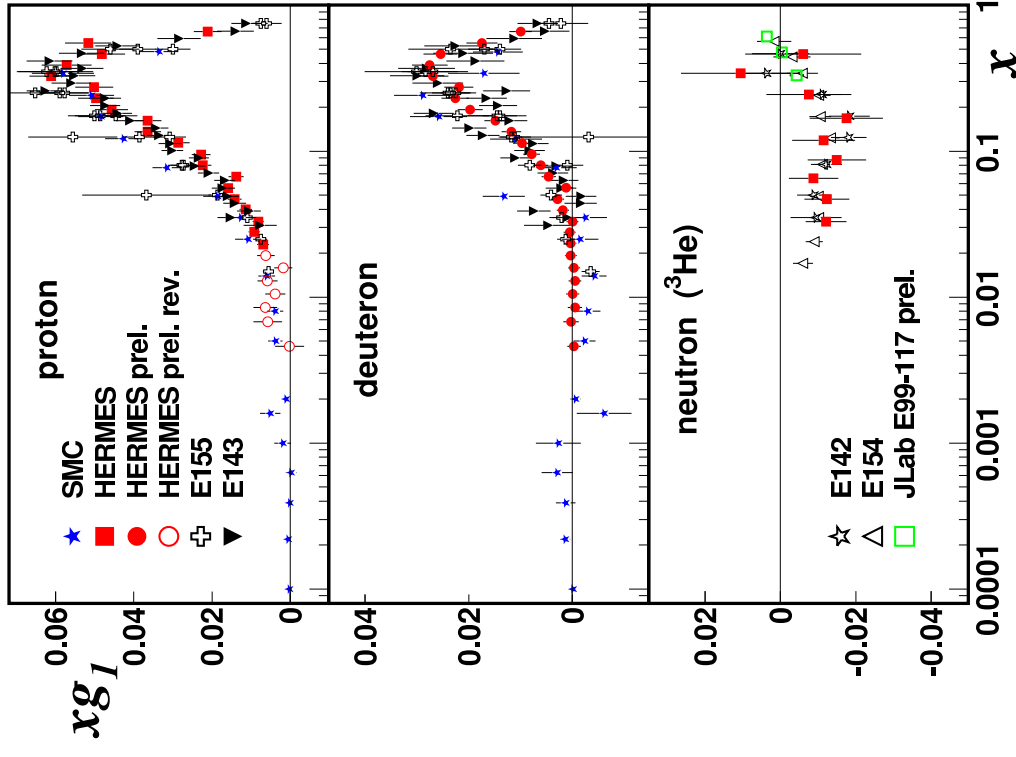
$$\alpha_s(M_z) = 0.120^{+0.010}_{-0.008} \text{ (ABFR 98);}$$

$$\alpha_s(M_z) = 0.114^{+0.010}_{-0.009} \text{ (BB 2002)}$$

- $\Delta q, \Delta \bar{q}$ SEPARATION IMPOSSIBLE, Δg ONLY FROM SCALING VIOLATIONS

- STRANGENESS SEPARATION HARD (ONLY 1ST MOM. POSSIBLE)

POLARIZED STRUCTURE FUNCTION DATA



POLARIZED PARTONS WITH ERRORS

FIRST MOMENTS AT NUCLEON SCALE

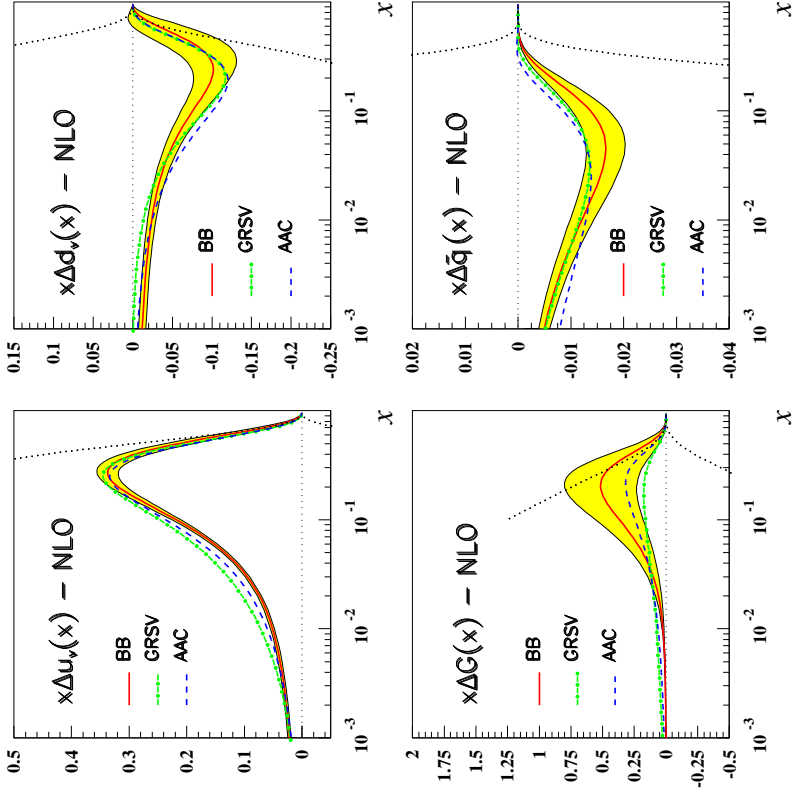
	Δg	a_0
ABFR	0.8 ± 0.4	$0.22^{+0.17}_{-0.05}$
BB	1.0 ± 0.5	0.14 ± 0.14
AAC	0.5 ± 1.9	0.24 ± 0.23

- FIRST MOMENT OF g_1 SMALL; FIRST MOMENT OF GLUON LARGE (?)

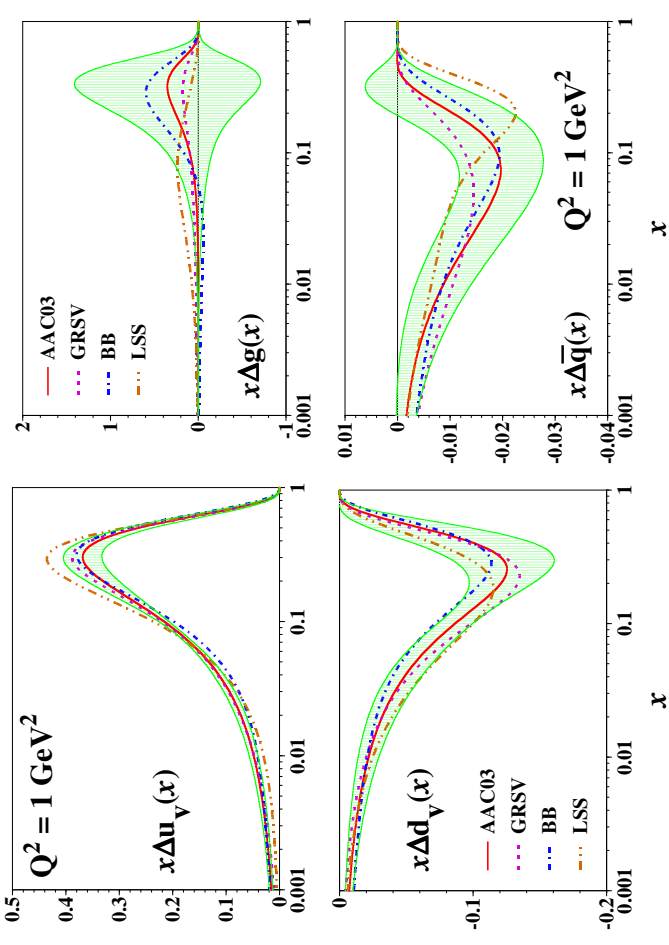
POLARIZED PARTONS WITH ERRORS

FIRST MOMENTS AT NUCLEON SCALE

	Δg	α_0
ABFR	0.8 ± 0.4	$0.22^{+0.17}_{-0.05}$
BB	1.0 ± 0.5	0.14 ± 0.14
AAC	0.5 ± 1.9	0.24 ± 0.23



BLÜMLEIN & BÖTTCHER, 2003



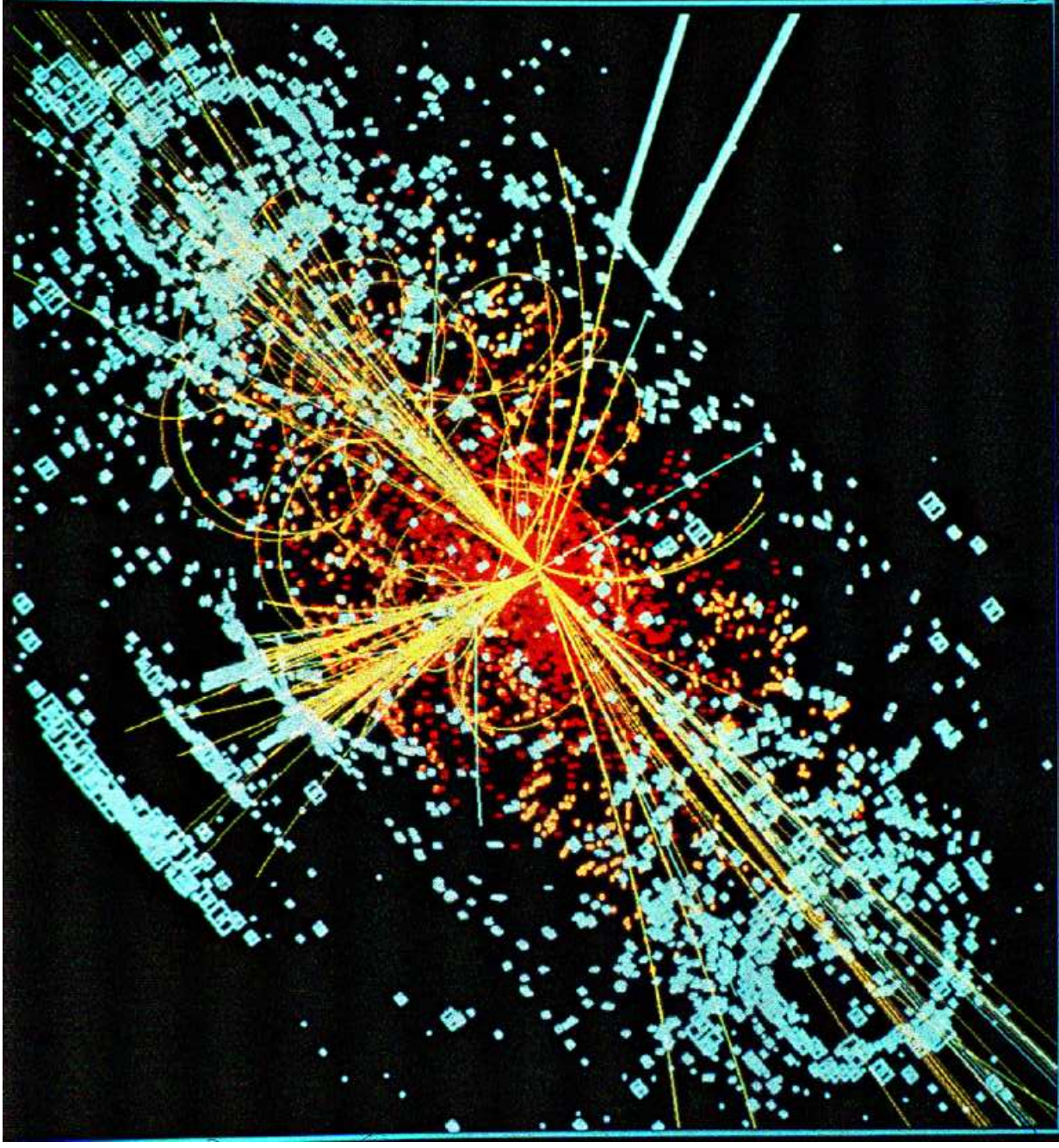
AA COLLN (JAPAN), 2004

- FIRST MOMENT OF g_1 **SMALL**; FIRST MOMENT OF GLUON LARGE (?)
- POOR DETERMINATION OF SHAPE;
GIVEN ERROR BANDS SUBJECT TO LARGE AND PARM. BIAS (GLUE) & THEOR. ASSUMPTIONS (SEA)

CONCLUSIONS

- THE NEEDS OF PRECISION PHENOMENOLOGY AT THE LHC HAS STIMULATED DRAMATIC PROGRESS IN THE PHYSICS OF STRUCTURE FUNCTIONS
- PARTON DISTRIBUTIONS WITH ERRORS ARE NOW BEING DEVELOPED THANKS TO THE AVAILABILITY OF A WIDE VARIETY OF DATA AND OF RAPID THEORETICAL PROGRESS IN PERTURBATIVE QCD
- “STANDARD” METHODS OF PDF DETERMINATION ARE STRETCHED TO THEIR LIMIT AND MOTIVATE THE DEVELOPMENT OF NEW TECHNIQUES

HIGGS PRODUCTION AT THE LHC



Higgs decay in $e^+e^- + 2 \text{ jets}$ at CMS