

**LES FONCTIONS DE  
DISTRIBUTION DES PARTONS:  
THÉORIE ET PHÉNOMÉNOLOGIE**

STEFANO FORTE  
UNIVERSITÀ DI MILANO

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# PARTONS FOR LHC:

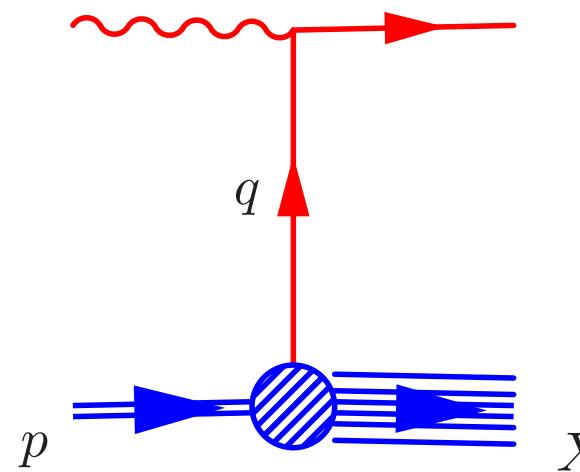
THE ACCURATE COMPUTATION OF PHYSICAL PROCESS AT A HADRON COLLIDER

REQUIRES GOOD KNOWLEDGE OF PARTON DISTRIBUTIONS OF THE NUCLEON

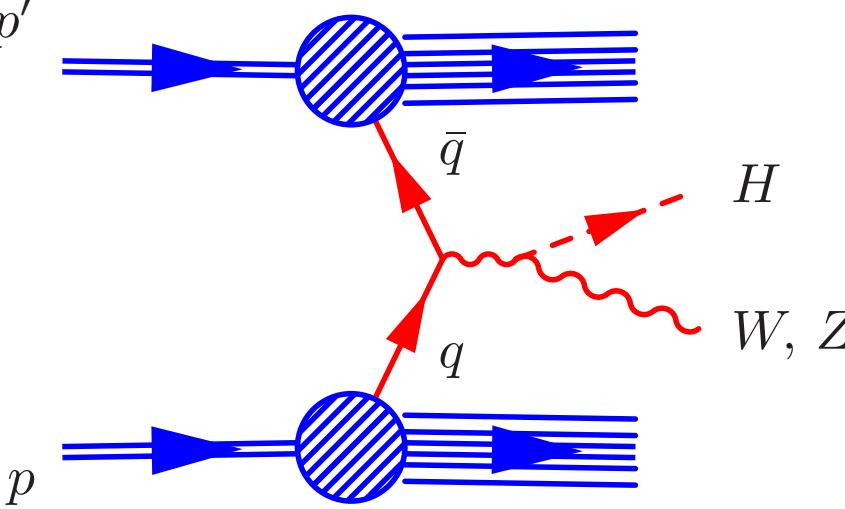
il faut maîtriser la physique standard pour extraire la nouvelle physique

## FACTORIZATION

$\gamma^*, W^*, Z^*$



$p'$



IN ORDER TO EXTRACT THE RELEVANT PHYSICS SIGNAL,

WE NEED TO KNOW THE PARTON DISTRIBUTIONS AND THEIR UNCERTAINTY

- IS THIS ASPECT OF LHC PHYSICS UNDER CONTROL?
- WILL LHC TEACH US SOMETHING ABOUT QCD TOO?

# SUMMARY

- **FACTORIZATION**

- factorization for DIS & hadronic processes
- evolution equations
- sum rules

- **DETERMINING PDFs:**

- valence, sea and isospin  $\Rightarrow$  case study: the NuTeV puzzle I
- strange quarks  $\Rightarrow$  case study: the NuTeV puzzle II
- scaling violations and the gluon  $\Rightarrow$  case study: large  $E_T$  jets

- **THEORETICAL ISSUES**

- higher order corrections
- resummation
- heavy quarks  $\Rightarrow$  case study:  $W$  production at LHC

- **PDFS WITH ERRORS**

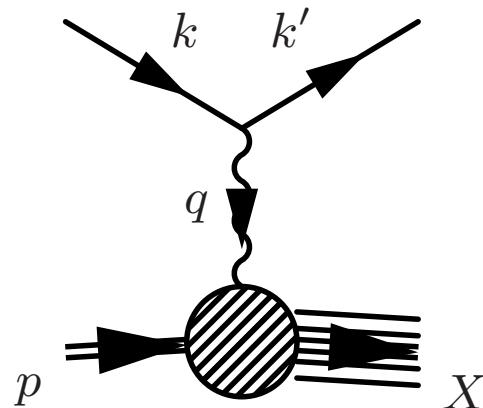
- the problem of PDF uncertainties
- the standard solution and its difficulties
- new ideas: Bayesian inference and neural networks

- **CONCLUSION: PDFS AT LHC**

# FACTORIZATION

# FACTORIZATION I: DEEP-INELASTIC SCATTERING

## STRUCTURE FUNCTIONS . . .



Lepton fractional energy loss:  $y = \frac{p \cdot q}{p \cdot k}$ ;  
 gauge boson virtuality:  $q^2 = -Q^2$  scale of proc.  
 Bjorken  $x$ :  $x = \frac{Q^2}{2p \cdot q}$  dimensionless var..  
 lepton-nucleon CM energy:  $s = \frac{Q^2}{xy}$ ;  
 virtual boson-nucleon CM energy  $W^2 = Q^2 \frac{1-x}{x}$ ;

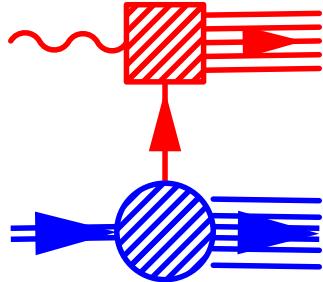
$$\frac{d^2\sigma^{\lambda_p \lambda_\ell}(x, y, Q^2)}{dxdy} = \frac{G_F^2}{2\pi(1 + Q^2/m_W^2)^2} \frac{Q^2}{xy} \left\{ \left[ -\lambda_\ell y \left(1 - \frac{y}{2}\right) x \textcolor{red}{F}_3(x, Q^2) + (1-y) \textcolor{blue}{F}_2(x, Q^2) \right. \right.$$

$$\left. \left. + y^2 x F_1(x, Q^2) \right] - 2\lambda_p \left[ -\lambda_\ell y(2-y)x \textcolor{red}{g}_1(x, Q^2) - (1-y) \textcolor{green}{g}_4(x, Q^2) - y^2 x \textcolor{green}{g}_5(x, Q^2) \right] \right\}$$

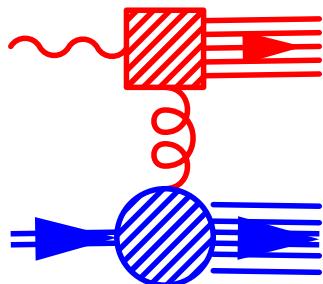
$\lambda_l$  → lepton helicity  
 $\lambda_p$  → proton helicity

	PARITY CONS.	PARITY VIOL.
UNPOL.	$F_1, F_2$	$F_3$
POL.	$g_1$	$g_4, g_5$

## ...AND PARTON DISTRIBUTIONS



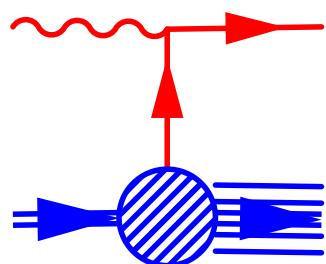
STRUCTURE FUNCTION=HARD COEFF. (PARTONIC STRUCTURE FUNCTION)  
 $\otimes$ PARTON DISTN.



$$F_2^{\text{NC}}(x, Q^2) = x \sum_{\text{flav. } i} e_i^2 (q_i + \bar{q}_i) + \alpha_s [C_i[\alpha_s] \otimes (q_i + \bar{q}_i) + C_g[\alpha_s] \otimes g]$$

$q_i$  quark,  $\bar{q}_i$  antiquark,  $g$  gluon

LEADING PARTON CONTENT ( $O[\alpha_s]$  corrections  $\Rightarrow$  gluons)



$$q_i \equiv q_i^{\uparrow\uparrow} + q_i^{\uparrow\downarrow}$$

$$\Delta q_i \equiv q_i^{\uparrow\uparrow} - q_i^{\uparrow\downarrow}$$

NC       $F_1^{\gamma, Z} = \sum_i e_i^2 (q_i + \bar{q}_i)$

$$g_1^{\gamma, Z} = \sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i)$$

CC       $F_1^{W^+} = \bar{u} + d + s + \bar{c}$

$$g_1^{W^+} = \Delta \bar{u} + \Delta d + \Delta s + \Delta \bar{c}$$

CC       $-F_3^{W^+}/2 = \bar{u} - d - s + \bar{c}$

$$g_5^{W^+} = \Delta \bar{u} - \Delta d - \Delta s + \Delta \bar{c}$$

$$F_2 = 2x F_1$$

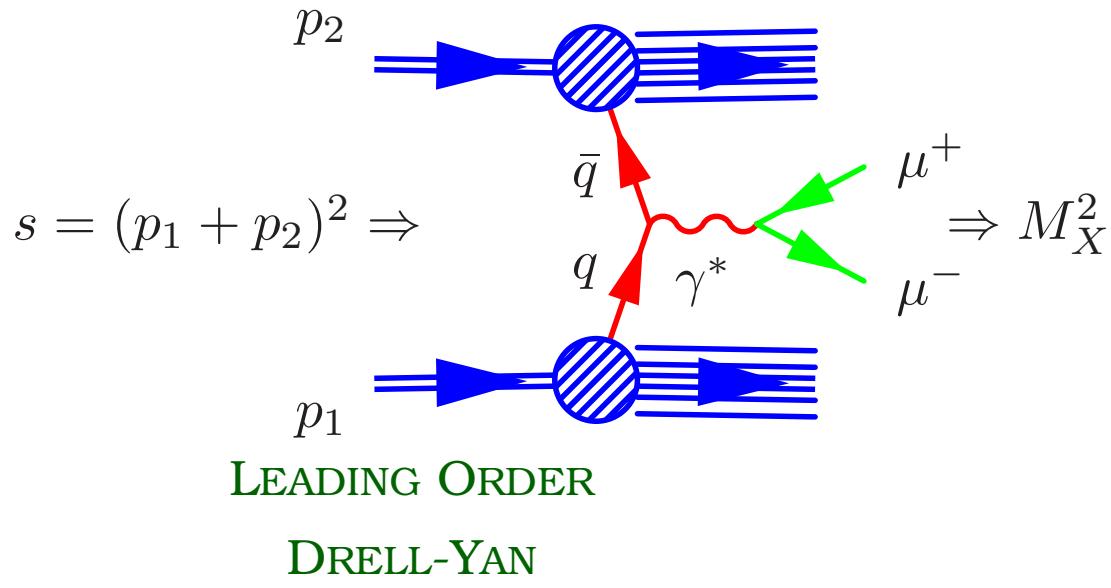
$$g_4 = 2x g_5$$

$W^+ \rightarrow W^- \Rightarrow u \leftrightarrow d, c \leftrightarrow s$ ; more combinations using Isospin:  $p \rightarrow n \Rightarrow u \leftrightarrow d$

# FACTORIZATION II: HADRONIC PROCESSES

$$\sigma_X(s, M_X^2) = \sum_{a,b} \int_{x_{\min}}^1 dx_1 dx_2 f_{a/h_1}(x_1) f_{b/h_2}(x_2) \hat{\sigma}_{q_a q_b \rightarrow X}(x_1 x_2 s, M_X^2)$$

LEAD. ORD.  $= \sigma_0 \sum_{a,b} \int_\tau^1 \frac{dx}{x} f_{a/h_1}(x) f_{b/h_2}(\tau/x) \equiv \sigma_0 \mathcal{L}(\tau) \Rightarrow \mathcal{L}$  PARTON LUMI



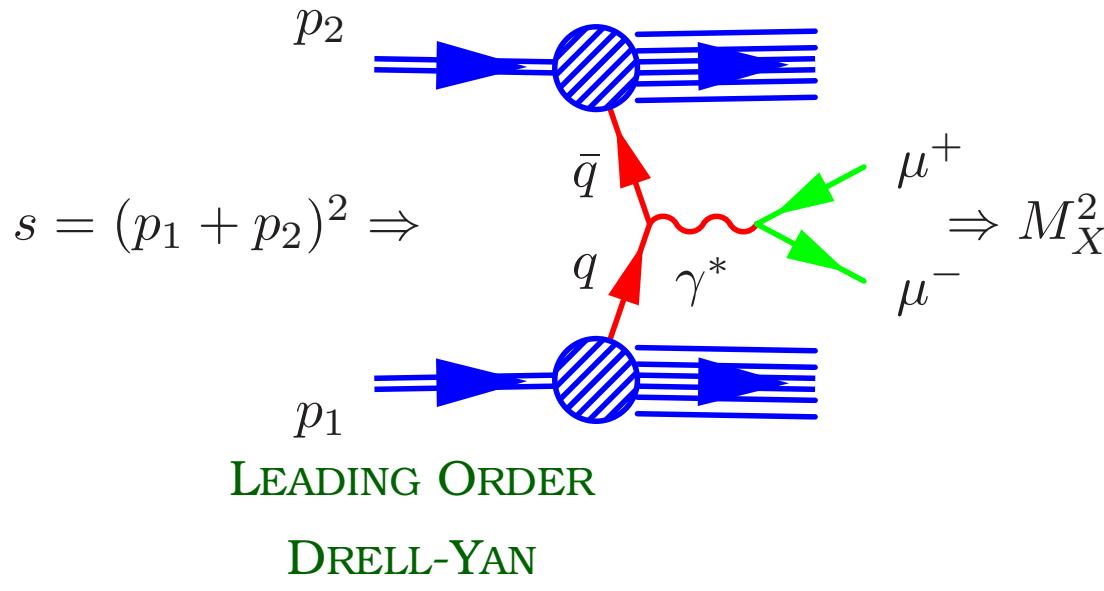
- Hadronic c.m. energy:  $s = (p_1 + p_2)^2$
- Momentum fractions  $x_{1,2} = \sqrt{\frac{\hat{s}}{s}} \exp \pm y$ ; Lead. Ord.  $\hat{s} = M^2$
- Partonic c.m. energy:  $\hat{s} = x_1 x_2 s$
- Invariant mass of final state  $X$  (dilepton, Higgs, . . . ):  $M_W^2 \Rightarrow$  scale of process
- Scaling variable  $\tau = \frac{M_X^2}{s}$

EXAMPLE: DRELL-YAN  $\sigma_X \rightarrow M^2 \frac{d\sigma}{dM^2}$ ;  $\sigma_0 = \frac{4}{9} \pi \alpha \frac{1}{s}$

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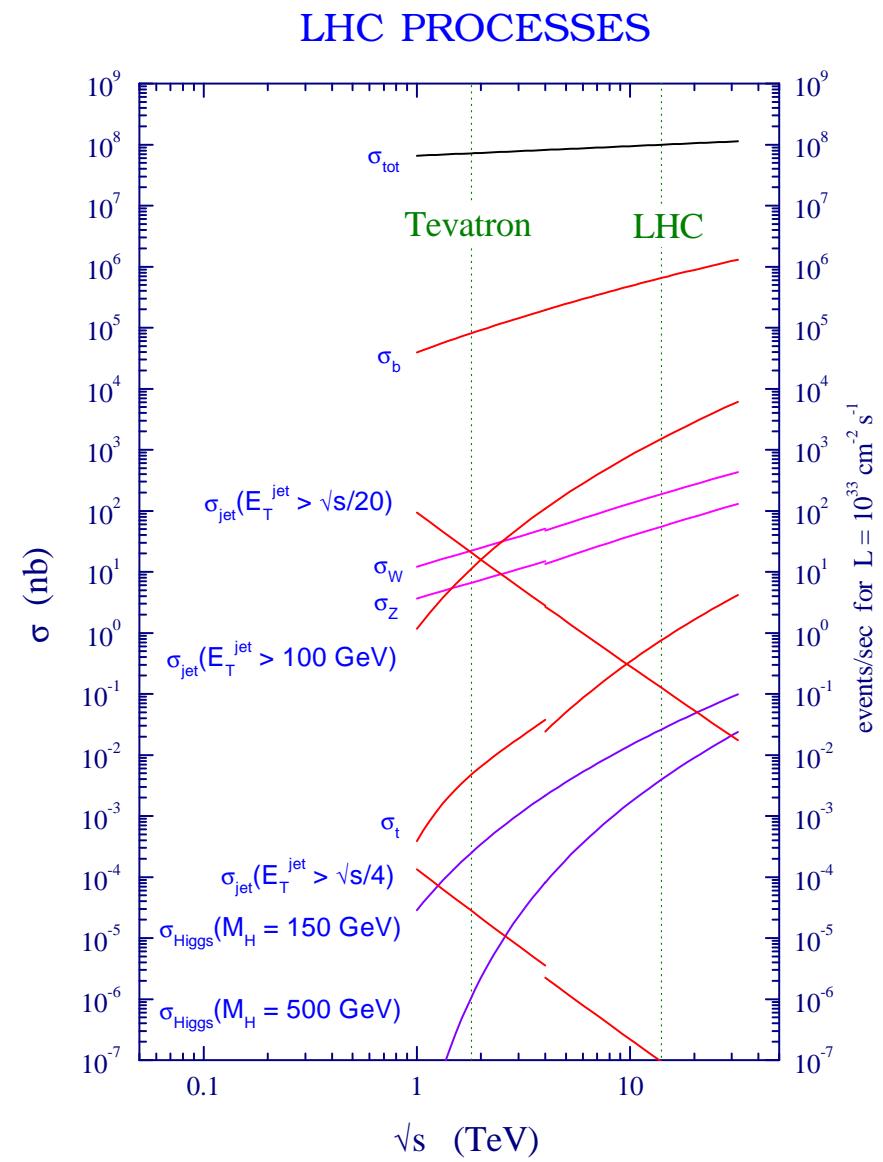
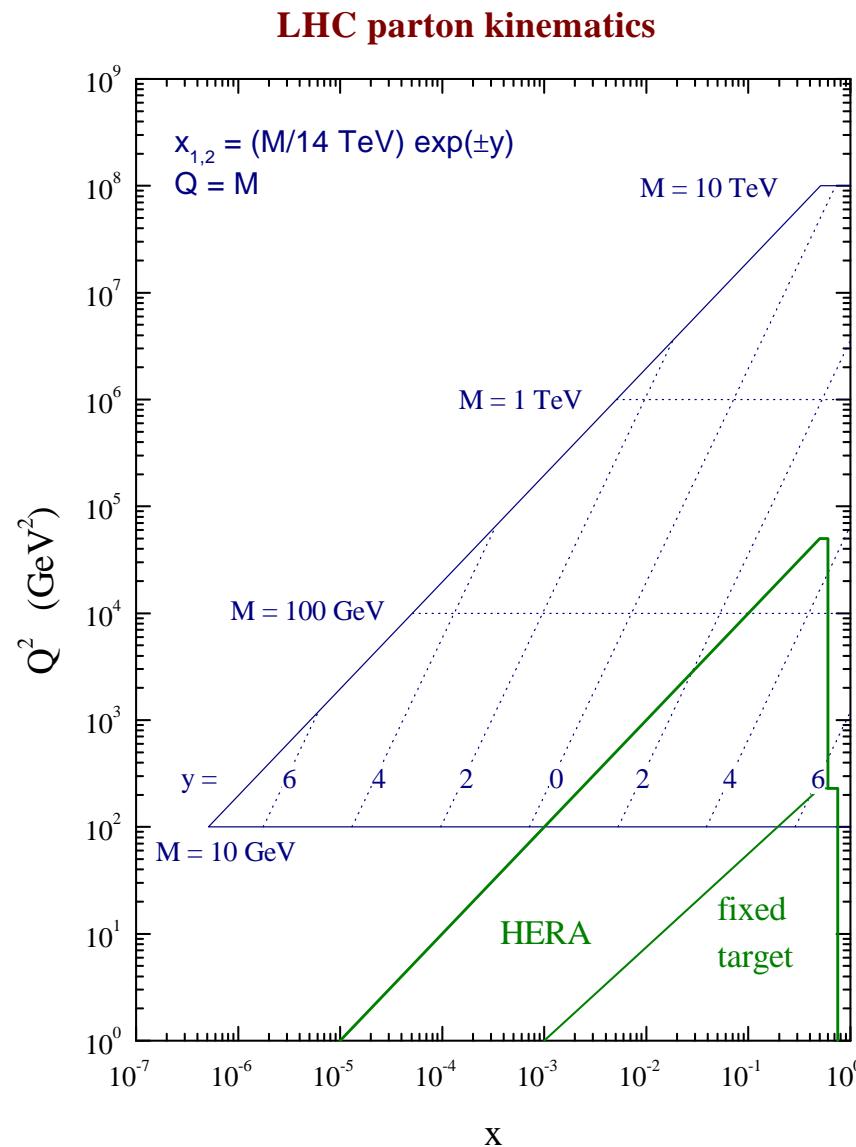


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- $\hat{\sigma}_{q_a q_b \rightarrow X} = \sigma_0 C(x, \alpha_s(M_H^2))$ ;  $C(x, \alpha_s(M_H^2)) = \delta(1-x) + O(\alpha_s)$
- $\sigma_X(s, M^2) = \sigma_0 \sum_{a,b} \int_{x_{\min}}^1 dx_1 dx_2 f_{a/h_1}(x_1) f_{b/h_2}(x_2) \delta(x_1 x_2 x - \tau) C\left(\frac{\tau}{x_1 x_2}, \alpha_s(M_H^2)\right)$   
 $= \sigma_0 \sum_{a,b} \int_{x_2}^1 \frac{dx_1}{x_1} \int_\tau^1 \frac{dx_2}{x_2} f_{a/h_1}(x_1) f_{b/h_2}(x_2) C\left(\frac{\tau}{x_1 x_2}, \alpha_s(M_H^2)\right)$

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# LHC: KINEMATICS AND PHYSICAL PROCESSES



# EVOLUTION EQUATIONS

MUST EVOLVE FROM HERA TO LHC → USE DGLAP EQUATIONS (RG EQNS)

- DEFINE MELLIN MOMENTS OF PARTON DISTRIBUTIONS

$$f(N, Q^2) \equiv \int_0^1 dx x^{N-1} f_2(x, Q^2)$$

NOTE LARGE/SMALL  $x \Leftrightarrow$  LARGE/SMALL  $N$

- DEFINE LOGARITHMIC SCALE  $t = \ln \frac{Q^2}{\Lambda^2}$ :

EVOLUTION GIVEN BY ORDINARY DIFFERENTIAL EQUATIONS (RG EQUATIONS)

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- ANOMALOUS DIMENSIONS RELATED TO DGLAP SPLITTING FUNCTIONS

$$\gamma(N, \alpha_s(t)) \equiv \int_0^1 dx x^{N-1} P(x, \alpha_s(t))$$

- COMPUTED IN PERTURBATION THEORY:

$$\gamma_i(N, \alpha_s(t)) = \gamma_i^{(0)}(N) + \alpha_s(t) \gamma_i^{(1)}(N) + \dots$$

$$\frac{d}{dt} \Delta q_{NS}(N, Q^2) = \frac{\alpha_s(t)}{2\pi} \gamma_{qq}^{NS}(N, \alpha_s(t)) \Delta q_{NS}(N, Q^2),$$

$$\frac{d}{dt} \begin{pmatrix} \Delta \Sigma(N, Q^2) \\ \Delta g(N, Q^2) \end{pmatrix} = \frac{\alpha_s(t)}{2\pi} \begin{pmatrix} \gamma_{qq}^S(N, \alpha_s(t)) & 2n_f \gamma_{qg}^S(N, \alpha_s(t)) \\ \gamma_{gq}^S(N, \alpha_s(t)) & \gamma_{gg}^S(N, \alpha_s(t)) \end{pmatrix} \otimes \begin{pmatrix} \Delta \Sigma(N, Q^2) \\ \Delta g(N, Q^2) \end{pmatrix},$$

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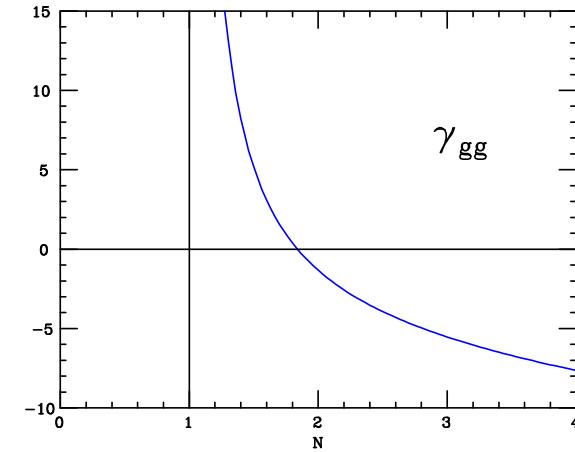
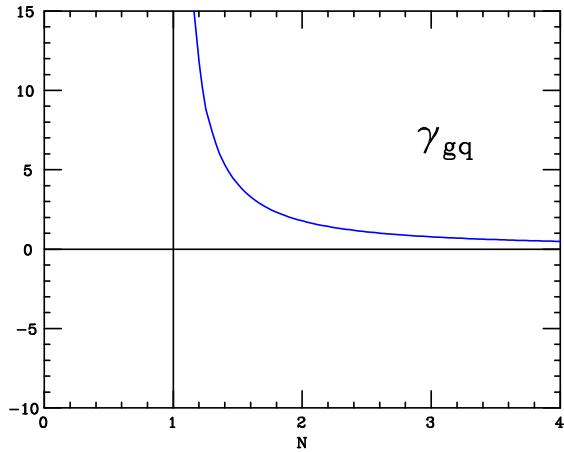
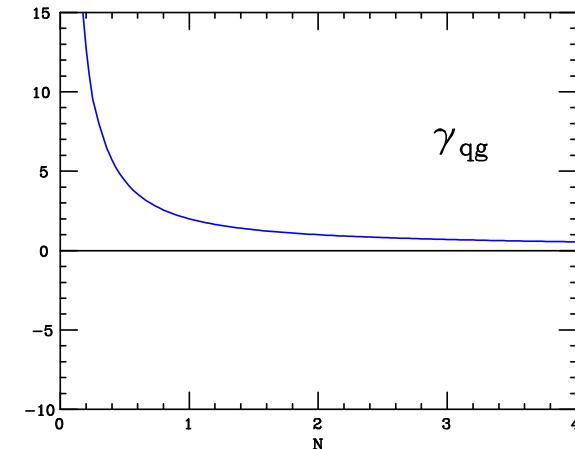
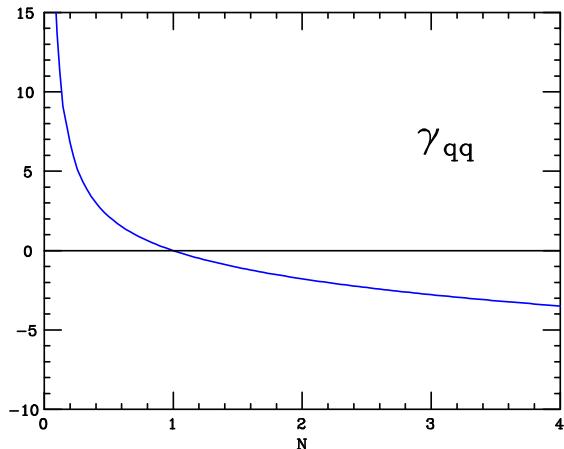
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- EVOLUTION OF SINGLET  $\Sigma(x, Q^2) = \sum_{i=1}^{n_f} (q_i(x, Q^2) + \bar{q}_i(x, Q^2))$  COUPLED TO GLUON

- ALL “NONSINGLET” QUARK COMBINATIONS  $q^{NS}(x, Q^2) = q_i(x, Q^2) - q_j(x, Q^2)$  EVOLVE INDEPENDENTLY

# PERTURBATIVE EVOLUTION

## THE LEADING ORDER ANOMALOUS DIMENSIONS



## QUALITATIVE FEATURES

recall large/small  $n \Leftrightarrow$  large/small  $x$

- AS  $Q^2$  INCREASES, PDFS DECREASE AT LARGE  $x$  & INCREASE AT SMALL  $x$  DUE TO RADIATION
- GLUON SECTOR SINGULAR AT  $N = 1 \Rightarrow$  GLUON GROWS MORE AT SMALL  $x$
- $\gamma_{qq}(1) = 0 \Rightarrow$  NUMBER OF QUARKS CONSERVED

# SUM RULES

CONSTRUCT CONSERVED QUANTUM NUMBERS CARRIED BY PARTON DISTRIBUTIONS:

- **BARYON NUMBER**  $\int_0^1 dx (u^p - \bar{u}^p) = 2 = 2 \int_0^1 dx (d^p - \bar{d}^p)$
- **MOMENTUM**  $\int_0^1 dx x \left[ \sum_{i=1}^{N_f} (q^i(x) + \bar{q}_i(x)) + g(x) \right] = 1$

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CANNOT DEPEND ON SCALE

- **BARYON NUMBER**  $\gamma_{qq}(1) - \gamma_{q\bar{q}}(1) = 0$ ; AT LO  $\gamma_{q\bar{q}}(1) = 0$  SO  $\gamma_{qq}(1) = 0$
- **MOMENTUM**  $\gamma_{qq}(2) + \gamma_{qg}(2) = 0$ ,  $\gamma_{gq}(2) + \gamma_{gg}(2) = 0$

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CAN EXTRACT FROM PHYSICAL OBSERVABLES: **BARYON NUMBER**

- GROSS-LLEWELLYN-SMITH SUM RULE  $\frac{1}{2} \int_0^1 dx (F_3^{\nu p}(x, Q^2) + F_3^{\nu n}(x, Q^2)) = C_{GLS}(Q^2) \int_0^1 dx [u(x, Q^2) - \bar{u}(x, Q^2) + d(x, Q^2) - \bar{d}(x, Q^2)]$
- BJORKEN (UNPOLARIZED) SUM RULE  $\frac{1}{2} \int_0^1 dx (F_1^{\nu p}(x, Q^2) - F_1^{\nu n}(x, Q^2)) = C_{BjU}(Q^2) \int_0^1 dx [u(x, Q^2) - \bar{u}(x, Q^2) - (d(x, Q^2) - \bar{d}(x, Q^2))]$

# FROM DATA TO PDFS

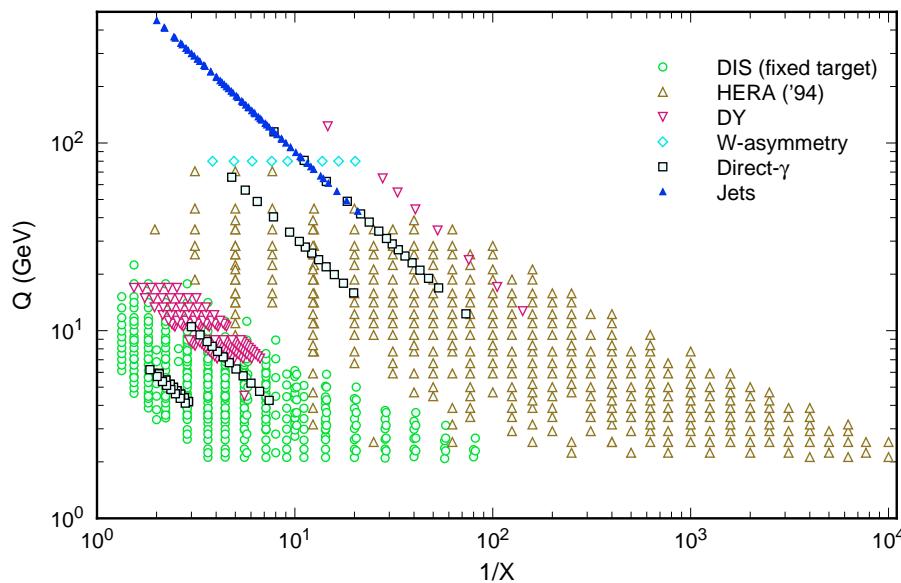
# PARTON FITS

DATA → PARTON DISTRIBUTIONS

## STRATEGY:

- CHOOSE SET OF OBSERVABLES (DIS, DRELL-YAN,  $W$  PRODUCTION...) & COMPUTE THEM IN PERT. THEORY
- CHOOSE A SET OF BASIS PARTON DISTRIBUTIONS (SINGLET, VALENCE, SEA...)
- FIT THE OBSERVABLES WITH THE PDFS AS FREE PARAMETERS

## DATA INCLUDED IN CTEQ5 PARTON FIT



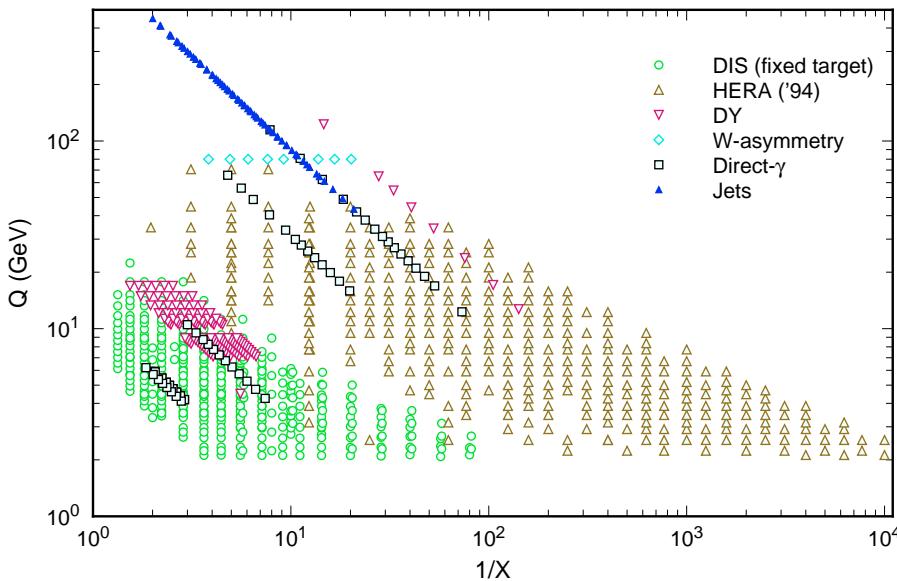
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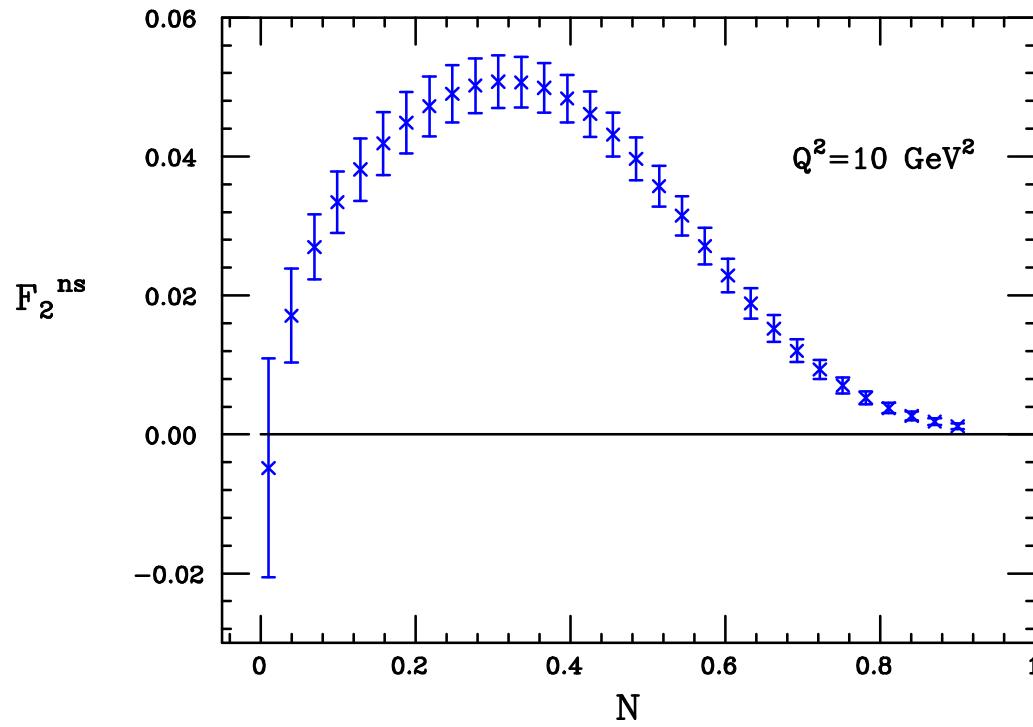
## TASKS:

- STRUCTURE FUNCTION (OR XSECT) IS A CONVOLUTION OVER  $x$  OF PARTON DISTNS. AND PERTURBATIVE CROSS SECTION  
→ MUST DECONVOLVE
- EACH STRUCTURE FUNCTION (OR XSECT) IS A LINEAR COMBINATION OF MANY PARTON DISTNS ( $2N_f$  QUARKS + 1 GLUON)  
→ MUST COMBINE DIFFERENT PROCESSES
- DATA GIVEN AT VARIOUS SCALES, WANT PARTON DISTNS. AS FCTN OF  $x$  AT COMMON SCALE  $Q^2$   
→ MUST EVOLVE
- TH UNCERTAINTIES: HIGHER ORDERS, RESUMMATION, HEAVY QUARK THRESHOLDS, NUCLEAR CORRECTIONS, HIGHER TWIST...

# DISENTANGLING QUARKS: UP VS. DOWN WITH THE HELP OF ISOSPIN SYMMETRY PROTON + NEUTRON $\Leftrightarrow$ UP + DOWN

$$u^p(x, Q^2) = d^n(x, Q^2); \quad d^p(x, Q^2) = u^n(x, Q^2)$$

$$F_2^p(x, Q^2) - F_2^d(x, Q^2) = \frac{1}{3} \left[ (u^p + \bar{u}^p) - (d^p + \bar{d}^p) \right] [1 + O(\alpha_s)]$$



# CASE STUDY I: THE NuTeV ANOMALY

## THE PASCHOS-WOLFENSTEIN RATIO: DATA...

NuTeV 2001     $\sin^2 \theta_W = 0.2272 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst})$

Global Fit 2003     $\sin^2 \theta_W = 0.2229 \pm 0.0004$

### ...VS. THEORY

$$\begin{aligned}
R^- &= \frac{\sigma_{NC}(\nu) - \sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\nu) - \sigma_{CC}(\bar{\nu})} \\
&= \left( \frac{1}{2} - \sin^2 \theta_W \right) + 2 \left[ \frac{(u - \bar{u}) - (d - \bar{d})}{u - \bar{u} + d - \bar{d}} - \frac{s - \bar{s}}{u - \bar{u} + d - \bar{d}} \right] \times \left[ \left( \frac{1}{2} - \frac{7}{6} \sin^2 \theta_W \right) \right. \\
&\quad \left. + \frac{4}{9} \frac{\alpha_s}{2\pi} \left( \frac{1}{2} - \sin^2 \theta_W \right) + O(\alpha_s^2) \right] + O(\delta(u - d)^2, \delta s^2)
\end{aligned}$$

u,d...denote momentum fractions carried by corresp. quark flavors

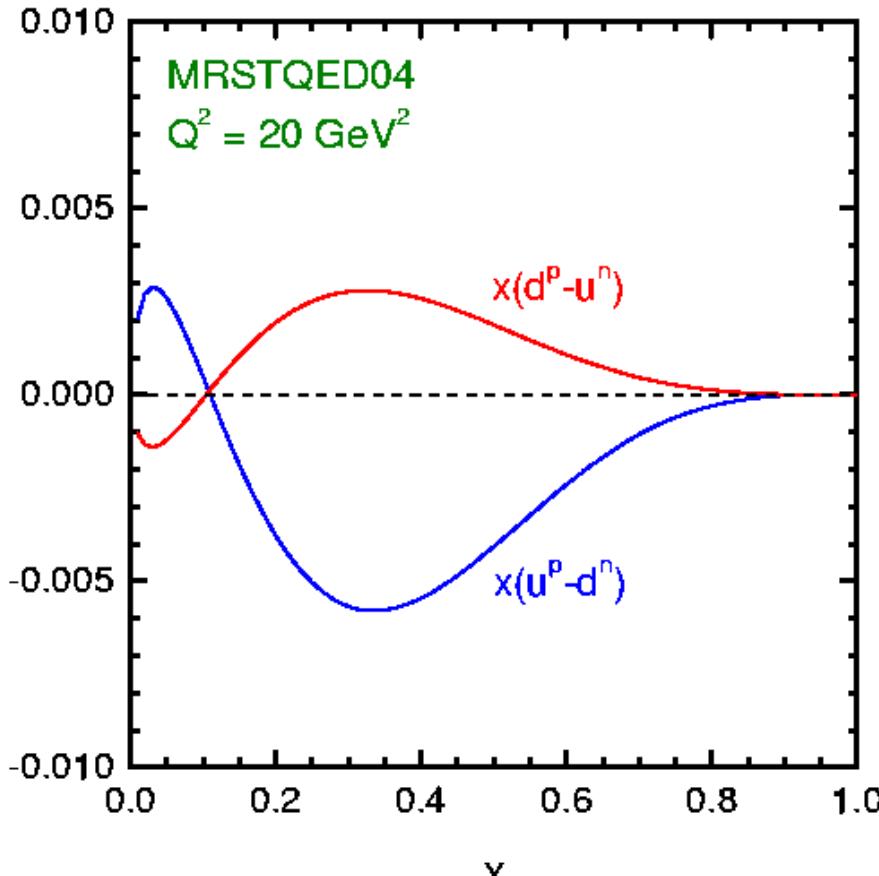
### NuTeV RESULT OBTAINED NEGLECTING:

- ISOSPIN VIOLATION → ISOSPIN KNOWN TO BE GOOD TO  $\sim .1\%$
- STRANGE ASYM. → EXPECT SEA TO BE FLAVOUR/ANTIFLAVOUR SYMMETRIC
- QCD CORRECTIONS → TINY (ONLY ENTER THROUGH SYM. VIOLATING TERMS)

# ISOSPIN VIOLATION

QED EFFECTS LEAD TO ISOSPIN VIOLATION:

$u - \bar{u}$  radiate more photons than  $d - \bar{d}$ :  $\frac{d}{dt}q_i \propto e_i^2 q_i$   
 $\Rightarrow$  MORE PHOTON MOMENTUM IN PROTON THAN NEUTRON  
 $\Rightarrow |u(x) - \bar{u}(x)| < |d(x) - \bar{d}(x)|$  AT LARGE  $x$



- SIGN OF EFFECT AS REQUIRED TO EXPLAIN NUTEV
- SIZE OF EFFECTS WITH REASONABLE ASSUMPTIONS ABOUT 1/2 OF NUTEV ANOMALY

MRST 2005: “QED” PARTON SET

# SYMMETRIES OF THE SEA

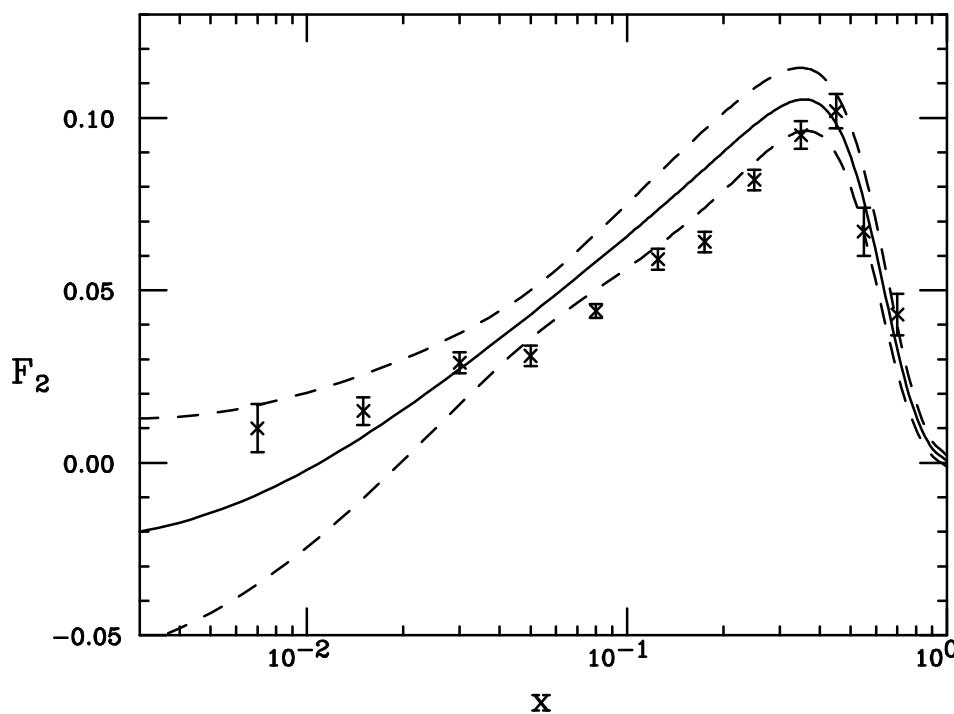
## THE GOTTFRIED SUM

SIMPLEST ASSUMPTION: LIGHT SEA IS SYMMETRIC:  $\bar{u}(x) = \bar{d}(x)$

LO EVOLUTION WOULD PRESERVE THIS SYMMETRY (BUT NOT NLO) ... THEN:

$$S_G(Q^2) \equiv \int_0^1 [F_2^p(x, Q^2) - F_2^d(x, Q^2)] = C(\alpha_s) \frac{1}{3} [(u^p + \bar{u}^p) - (d^p + \bar{d}^p)] \\ = C(\alpha_s) \frac{1}{3} [(u^p - \bar{u}^p) - (d^p - \bar{d}^p)] = \frac{1}{3} C(\alpha_s); C(\alpha_s) = 0.998 \text{ FOR } Q^2 = 3 \text{ GEV}^2$$

### THE NMC MEASUREMENT



- MEASURED VALUE (NMC 1997)  $S_G(Q^2) = 0.241 \pm 0.026$  AT  $Q^2 = 3 \text{ GEV}^2$ , **3.5 $\sigma$  AWAY FROM NAIVE PREDICTION**
- BUT: DATA ONLY FOR  $0.004 \leq x < 0.8$ , NEED TO EXTRAPOLATE
- REANALYSIS WITH NEURAL NETS:  $S_G(Q^2) = 0.244 \pm 0.045$ : **ERROR TWICE AS LARGE DUE TO LAST BIN!**

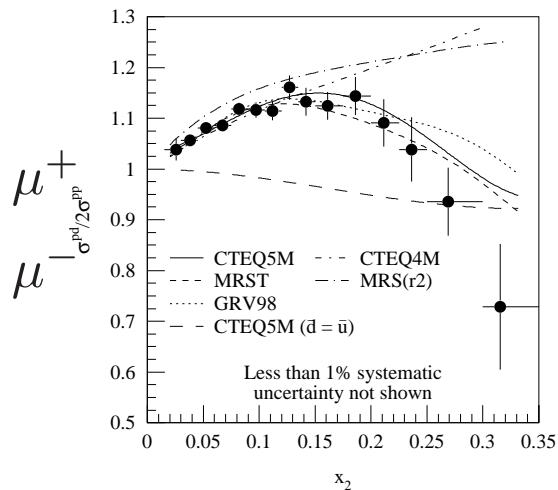
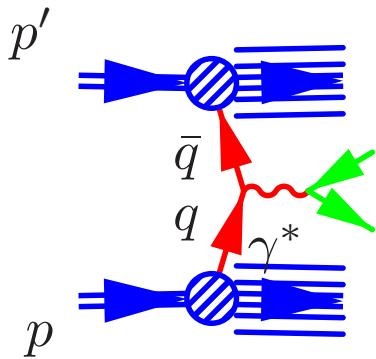
WEAK EVIDENCE FOR SYMMETRY VIOLATION,

STRONG EVIDENCE FOR UNRELIABILITY OF EXTRAPOLATION!

# DISENTANGLING QUARKS FROM ANTIQUARKS

$\gamma^*$  DIS ONLY MEASURES  $q + \bar{q}$  COMBINATION!

## DRELL-YAN $p/d$ ASYMMETRY

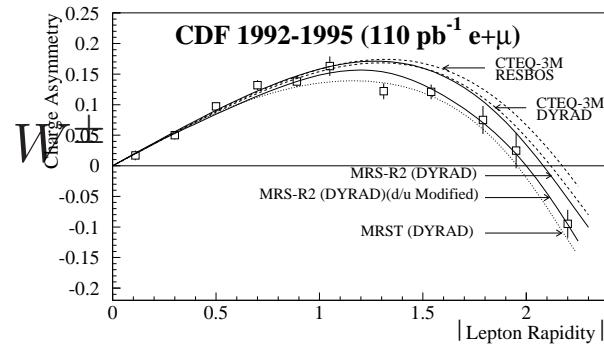
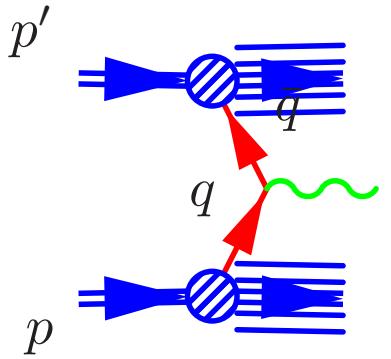


## LIGHT ANTIQUARK ASYMMETRY

$$\frac{\sigma^{pn}}{\sigma^{pp}} \sim \left. \frac{\frac{4}{9} u^p \bar{d}^p + \frac{1}{9} d^p \bar{u}^p}{\frac{4}{9} u^p \bar{u}^p + \frac{1}{9} d^p \bar{d}^p} \right|_{\text{large } x} \approx \frac{\bar{d}}{\bar{u}}$$

E866 (2001)

## $W^\pm$ ASYMMETRY



## LIGHT QUARK ASYMMETRY

$$\frac{\sigma^{p\bar{p}}}{\sigma^{p\bar{p}}} \sim \frac{u^p d^p}{d^p u^p} \quad (q^p = \bar{q}^p)$$

CDF (1998)

# DISENTANGLING STRANGENESS

## $\gamma^*$ SCATTERING VS. $W^\pm$ SCATTERING:

IN NC, CHARGED LEPTON DIS, ONLY MEASURE COMBINATION  $\sum_i e_i^2 (q_i + \bar{q}_i)$

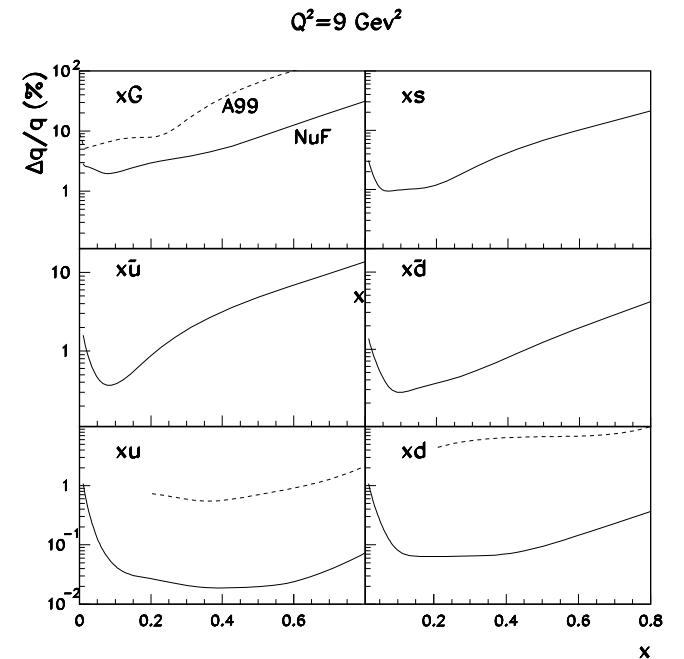
- ONLY C-EVEN  $q_i + \bar{q}_i$
- ONLY FIXED COMBINATION  $\frac{4}{9}(u + \bar{u}) + \frac{1}{9}(d + \bar{d}) + \frac{1}{9}(s + \bar{s})$

IN NEUTRINO DIS, CAN DISENTANGLE INDIVIDUAL PDFS BY LINEAR COMBINATION: AT LO

$$\begin{aligned} \frac{1}{2} \left( F_1^{W^-} + \frac{1}{2} F_3^{W^-} \right) &= u + c; & \frac{1}{2} \left( F_1^{W^+} - \frac{1}{2} F_3^{W^+} \right) &= \bar{u} + \bar{c} \\ \frac{1}{2} \left( F_1^{W^+} + \frac{1}{2} F_3^{W^+} \right) &= d + s; & \frac{1}{2} \left( F_1^{W^-} - \frac{1}{2} F_3^{W^-} \right) &= \bar{d} + \bar{s} \end{aligned}$$

$c, \bar{c}, s, \bar{s}$  only present above charm threshold

ERRORS ON PDFS AT A NUFACT COMPARED TO A PURE DIS FIT



## THE NUTEV ANOMALY, PART II THE PASCHOS-WOLFENSTEIN RATIO: DATA...

NuTeV 2001    $\sin^2 \theta_W = 0.2272 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst})$

Global Fit 2003    $\sin^2 \theta_W = 0.2229 \pm 0.0004$

### ...VS. THEORY

$$\begin{aligned}
 R^- &= \frac{\sigma_{NC}(\nu) - \sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\nu) - \sigma_{CC}(\bar{\nu})} \\
 &= \left( \frac{1}{2} - \sin^2 \theta_W \right) + 2 \left[ \frac{(u - \bar{u}) - (d - \bar{d})}{u - \bar{u} + d - \bar{d}} - \frac{s - \bar{s}}{u - \bar{u} + d - \bar{d}} \right] \times \left[ \left( \frac{1}{2} - \frac{7}{6} \sin^2 \theta_W \right) \right. \\
 &\quad \left. + \frac{4}{9} \frac{\alpha_s}{2\pi} \left( \frac{1}{2} - \sin^2 \theta_W \right) + O(\alpha_s^2) \right] + O(\delta(u - d)^2, \delta s^2)
 \end{aligned}$$

u,d...denote momentum fractions carried by corresp. quark flavors

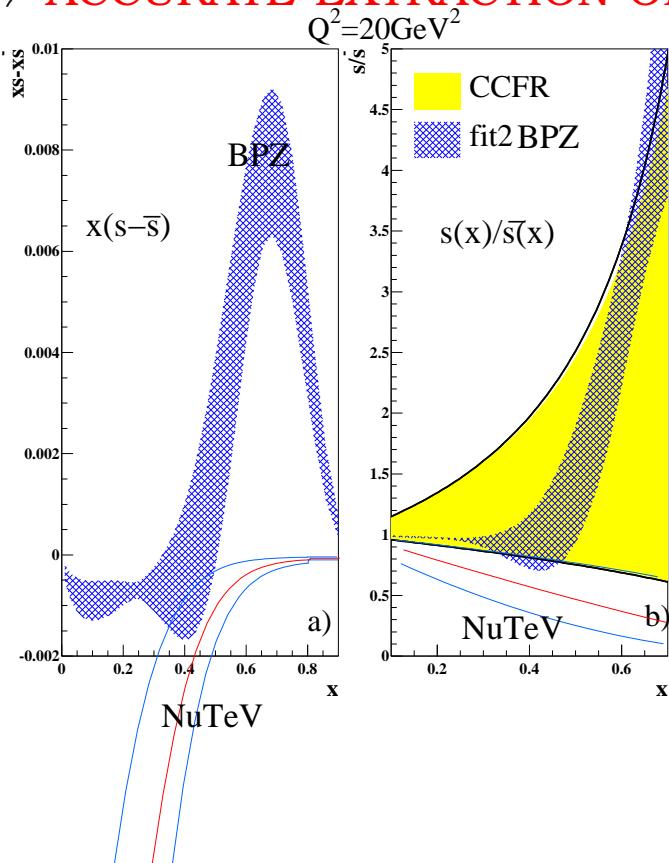
### NUTEV RESULT OBTAINED NEGLECTING:

- ISOSPIN VIOLATION → ISOSPIN KNOWN TO BE GOOD TO  $\sim .1\%$
- STRANGE ASYM. → EXPECT SEA TO BE FLAVOUR/ANTIFLAVOUR SYMMETRIC
- QCD CORRECTIONS → TINY (ONLY ENTER THROUGH SYM. VIOLATING TERMS)

# THE NUTEV ANOMALY, PART II

Q: ARE WE SURE THAT MOMENTUM FRACTION  $s - \bar{s} = 0$ ?

A: MEASURE IT!: CHARM IS COPIOUSLY PRODUCED IN  $W^+ + s \rightarrow c$   
easily tagged through dimuon signal, 2nd muon from subsequent  $c$  decay  
 $\Rightarrow$  ACCURATE EXTRACTION OF THE STRANGE DISTRIBUTION



CCFR/NUTEV  $s - \bar{s}$  DETERMINATION

5000  $\nu$  & 1500  $\bar{\nu}$  DIMUON EVENT SAMPLE:

ASSUMED PARM.:  $s(x) = \kappa \frac{\bar{u}(x) + \bar{d}(x)}{2} (1-x)^\alpha$

NEGATIVE  $s - \bar{s}$  AT SMALL  $x$

$\Rightarrow$  MOM. FRACT.  $s - \bar{s} = -0.003 \pm 0.001$

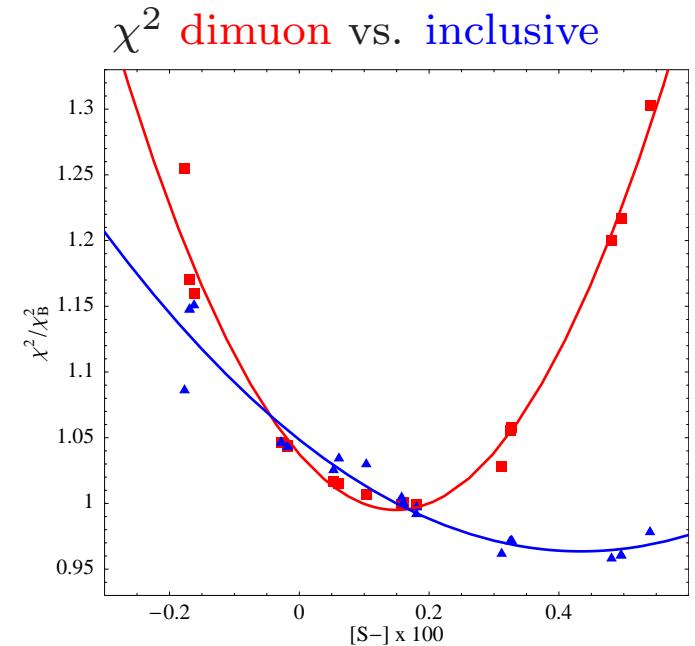
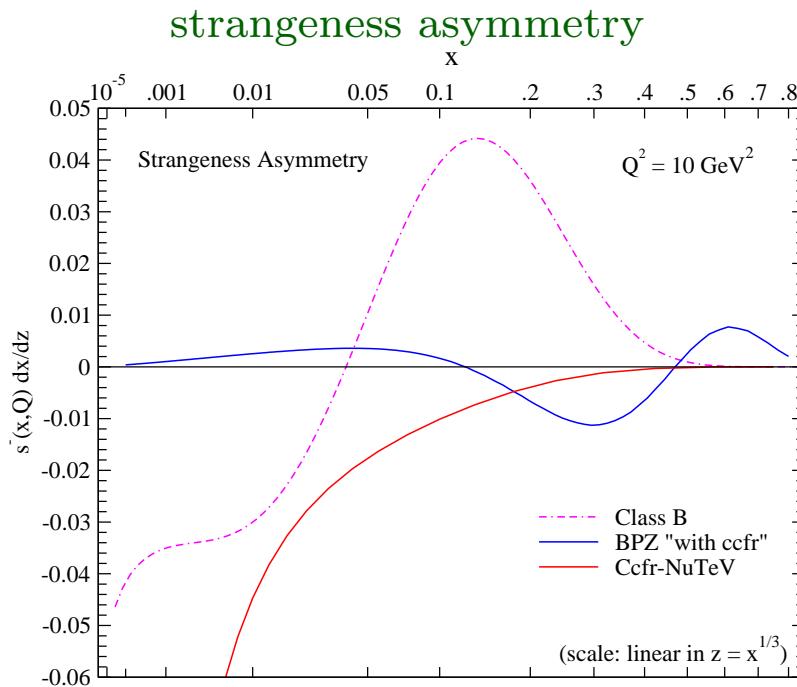
NUTEV ANOMALY WORSE!

HOWEVER, BPZ GLOBAL FIT TO NEUTRINO INCLUSIVE DIS (Barone et al 2003)  $\Rightarrow$   
POSITIVE (TINY) ASYMMETRY

# COMBINING INCLUSIVE AND EXCLUSIVE INFORMATION

## CTEQ DEDICATED DIMUON ANALYSIS (April 2004)

- $\int_0^1 (s(x) - \bar{s}(x)) dx = 0$  IN PROTON  
 $\Rightarrow$  EITHER  $s(x) - \bar{s}(x)$  HAS A NODE OR IT VANISHES EVERYWHERE
- $[s(x) - \bar{s}(x)] < 0$  FOR SMALL  $x \lesssim 0.05$  CONSTRAINED BY DIMUON
- LARGE  $x$  REGION WEIGHS MORE IN MOMENTUM FRACTION
- POSITIVE MOM. FRACTION  $s - \bar{s} \approx 0.02$ :



# DETERMINING THE GLUON

## EVOLUTION:

SINGLET SCALING VIOLATIONS

$$\frac{d}{dt} F_2^s(N, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} [\gamma_{qq}(N) F_2^s + 2 n_f \gamma_{qg}(N) g(N, Q^2)] + O(\alpha_s^2)$$

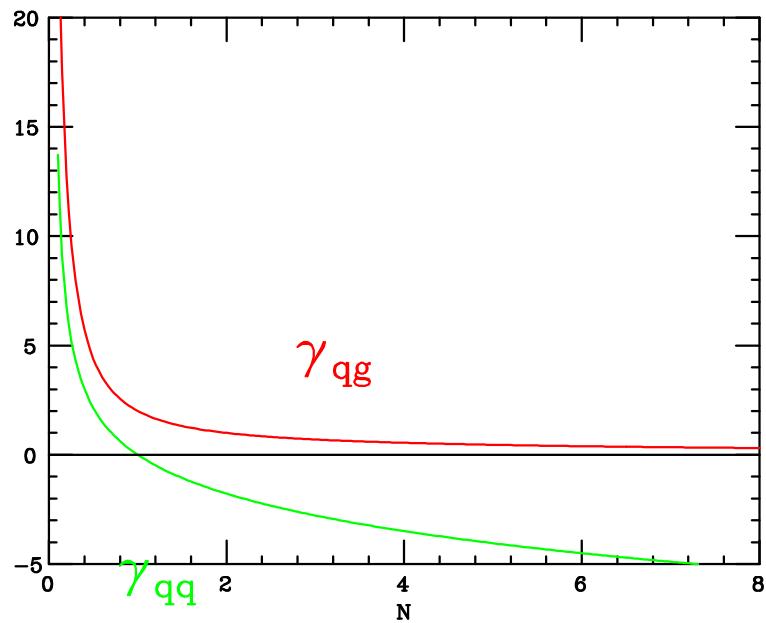
$$F_2(N, Q^2) \equiv \int_0^1 dx x^{N-1} F_2(x, Q^2); \quad \gamma_{ij}(N) \equiv \int_0^1 dx x^{N-1} P_{ij}(x, Q^2)$$

**LARGE / SMALL X  $\Leftrightarrow$  LARGE / SMALL N**

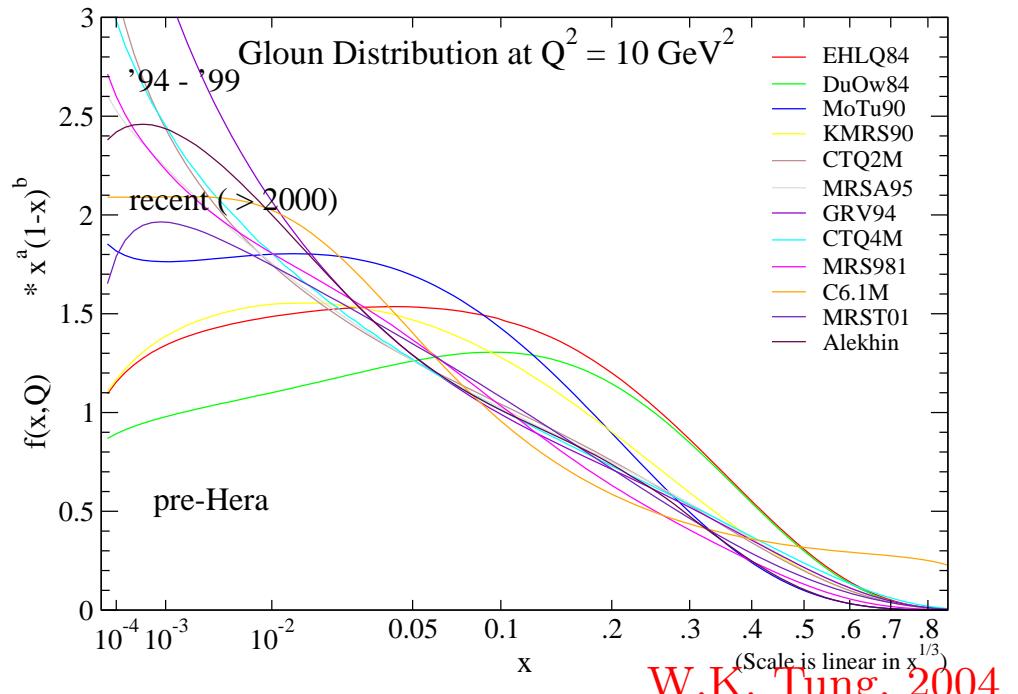
AT LARGE  $x$

**AT LARGE N**

$\gamma_{qg} \ll \gamma_{qq}$



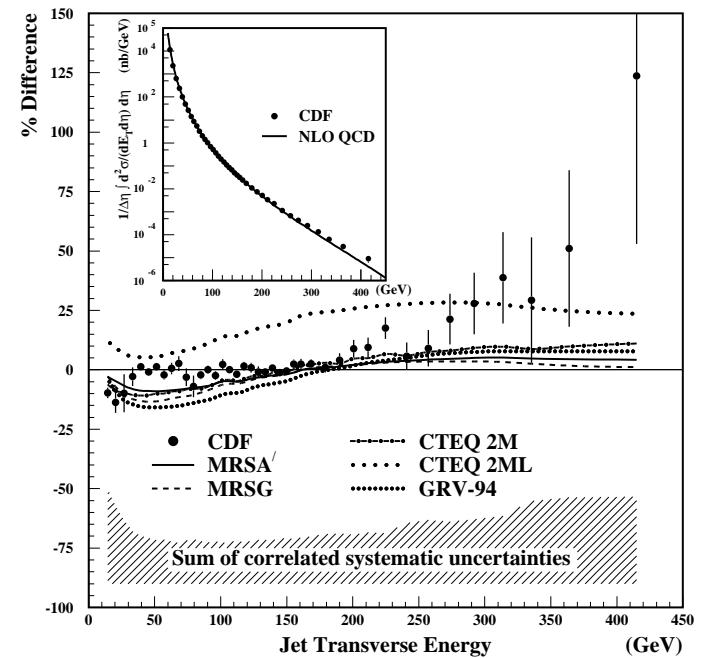
**⇒ GLUON HARD TO DETERMINE**



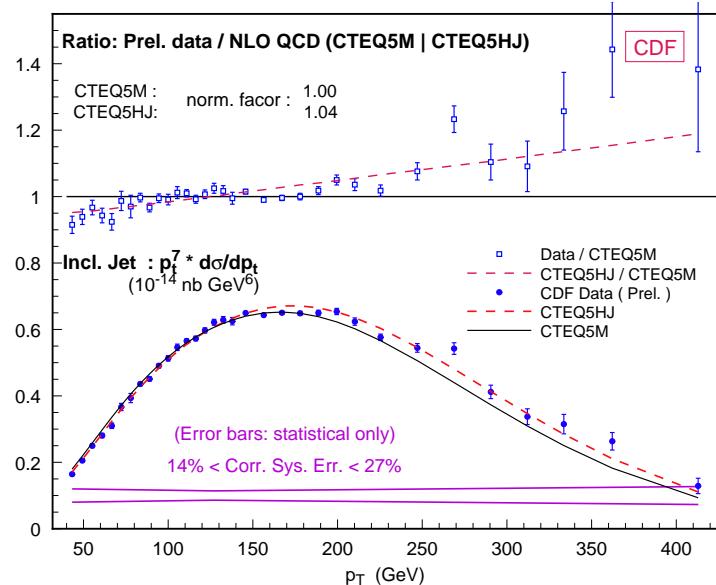
W.K. Tung, 2004

# CASE STUDY II: THE CDF LARGE $E_T$ JETS CDF 1995

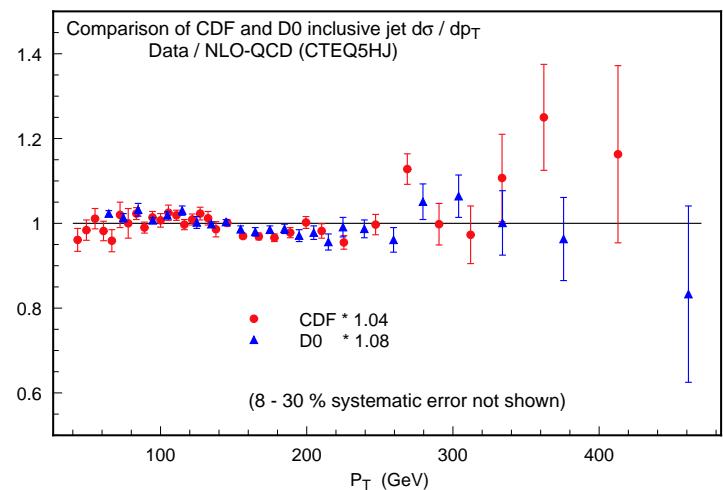
- DISCREPANCY BETWEEN QCD CALCULATION AND CDF JET DATA (1995)
- EVIDENCE FOR QUARK COMPOSITENESS?
- BUT NO INFO ON PARTON UNCERTAINTY  $\Rightarrow$   
RESULT STRONGLY DEPENDS ON  
GLUON AT  $x \gtrsim 0.1$



DISCREPANCY REMOVED IF JET DATA INCLUDED IN THE FIT  
NEW CTEQ FIT (1996)



FINAL CTEQ FIT (1998)



# THE STATE OF THE ART: FUNCTIONAL PARTON FITTING

- CHOOSE A FIXED FUNCTIONAL FORM:
  - MRST: 24 PARMS., SOME FIXED → 15 PARMS.

$$xq(x, Q_0^2) = A(1-x)^\eta(1+\epsilon x^{0.5} + \gamma x)x^\delta, \quad x[\bar{u} - \bar{d}](x, Q_0^2) = A(1-x)^\eta(1+\gamma x + \delta x^2)x^\delta.$$

$$xg(x, Q_0^2) = A_g(1-x)^{\eta_g}(1+\epsilon_g x^{0.5} + \gamma_g x)x^{\delta_g} - A_{-}(1-x)^{\eta_{-}}x^{-\delta_{-}},$$

- CTEQ: 20 PARMS.

$$x f(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} e^{A_3 x} (1+e^{A_4} x)^{A_5}$$

with independent params for combinations  $u_v \equiv u - \bar{u}$ ,  $d_v \equiv d - \bar{d}$ ,  $g$ , and  $\bar{u} + \bar{d}$ ,  $s = \bar{s} = 0.2(\bar{u} + \bar{d})$  at  $Q_0$ ; NORM. FIXED BY SUM RULES

- ALEKHIN: 17 PARMS.

$$xu_V(x, Q_0) = \frac{2}{N_u^V} x^{a_u} (1-x)^{b_u} (1+\gamma_2^u x); \quad xu_S(x, Q_0) = \frac{A_S}{N_S} \eta_u x^{a_s} (1-x)^{b_{su}}$$

$$xd_V(x, Q_0) = \frac{1}{N_d^V} x^{a_d} (1-x)^{b_d}; \quad xd_S(x, Q_0) = \frac{A_S}{N_S} x^{a_s} (1-x)^{b_{sd}},$$

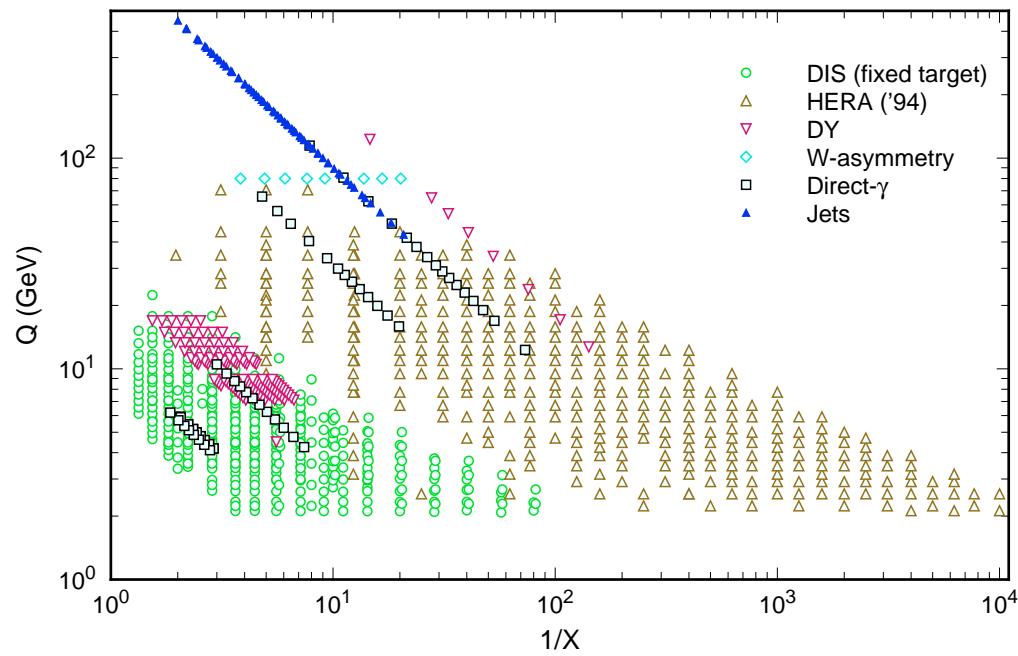
$$xs_S(x, Q_0) = \frac{A_S}{N_S} \eta_s x^{a_s} (1-x)^{(b_{su}+b_{sd})/2}; \quad xG(x, Q_0) = A_G x^{a_G} (1-x)^{b_G} (1+\gamma_1^G \sqrt{x} + \gamma_2^G x),$$

- EVOLVE TO DESIRED SCALE & COMPUTE PHYSICAL OBSERVABLES
- DETERMINE BEST-FIT VALUES OF PARAMETERS
- DETERMINE ERROR BY PROPAGATION OF ERROR ON PARMS ('HESSIAN METHOD') OR BY PARM. SCANS ('LAGRANGE MULTIPLIER METHOD')

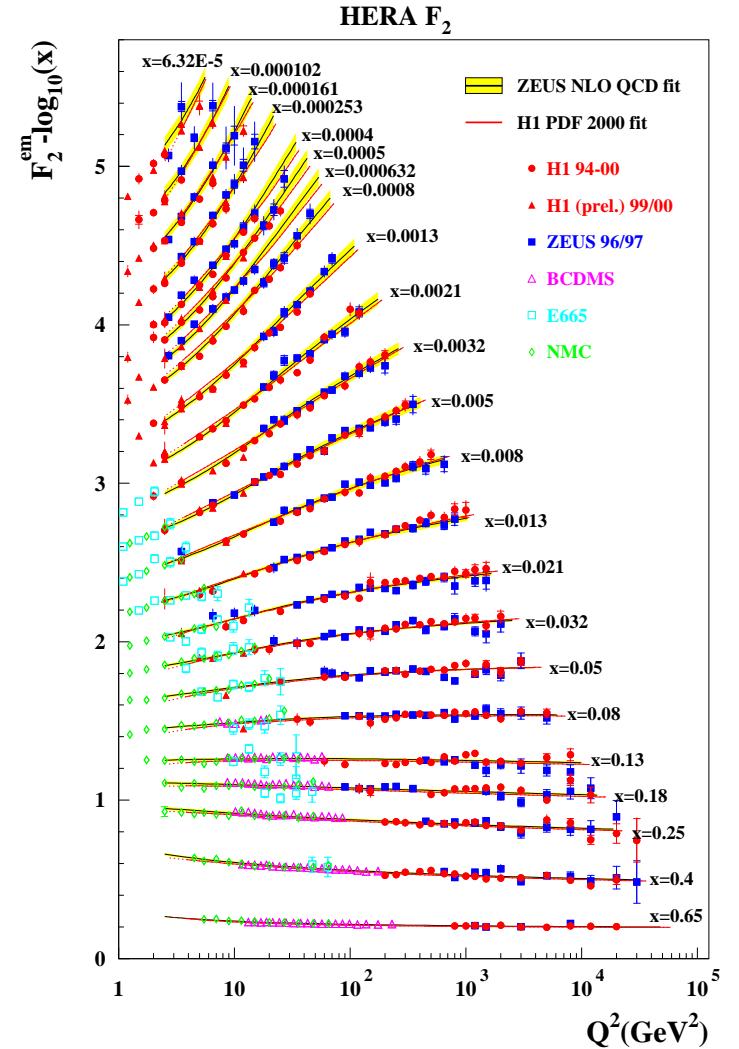
# HOW WELL DOES IT WORK?

RATHER WELL INDEED...

DATA INCLUDED IN CTEQ5 PARTON FIT



NOTE MOSTLY DIS DATA  $\Rightarrow$  HERA



# HOW WELL DOES IT WORK?

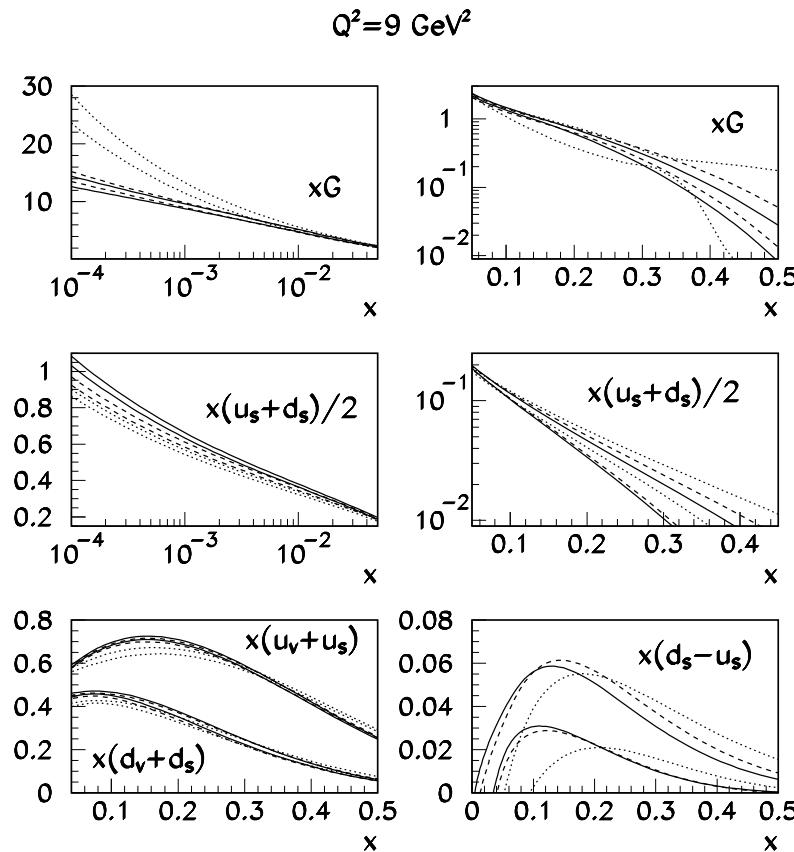
## DIS+DY ONLY (Alekhin 2003-2006)

DIS TOTAL ERROR BANDS FOR

LO (DOTS), NLO (DASHES), NNLO (SOLID)

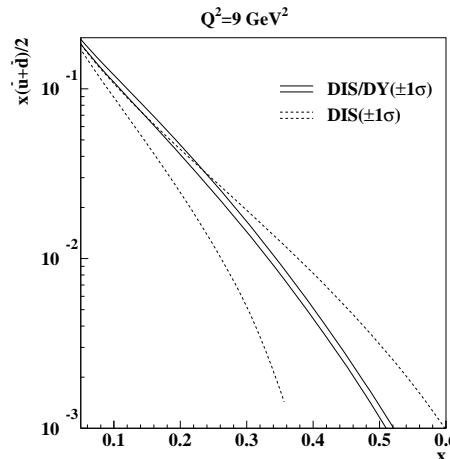
valence  $u^v \equiv u - \bar{u}$ ,  $d^v \equiv d - \bar{d}$ ,

sea  $u^s = \bar{u}^s = d^s = \bar{d}^s$

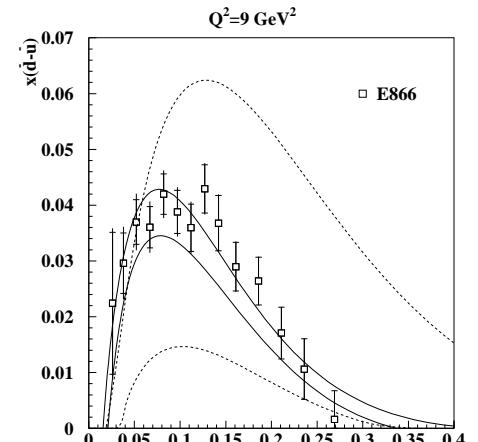


TOTAL ERROR BANDS:  
DIS (DOTS) vs. DIS+DY (SOLID)

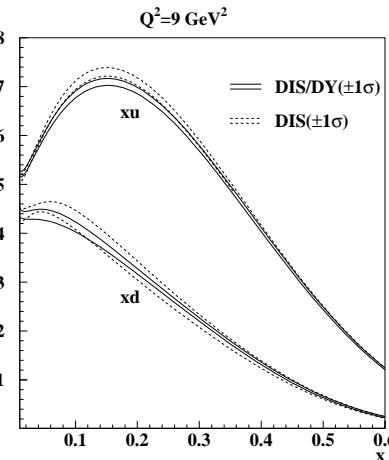
$\bar{u} + \bar{d}$



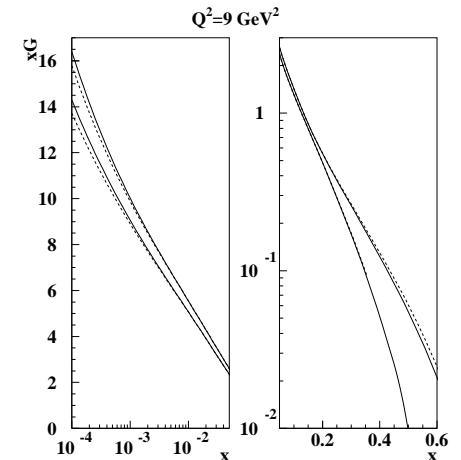
$\bar{u} - \bar{d}$



$u, d$



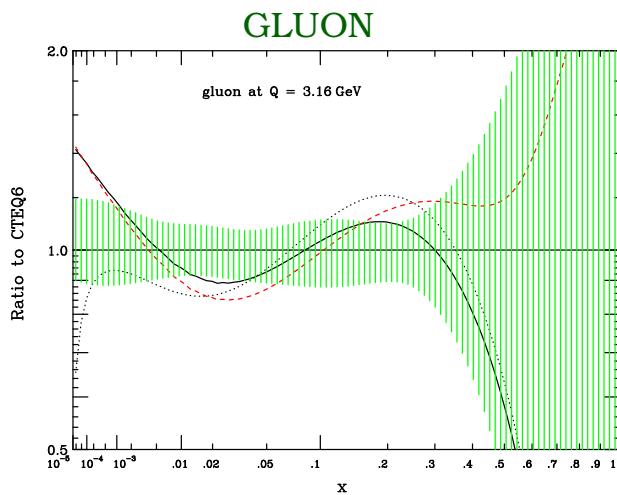
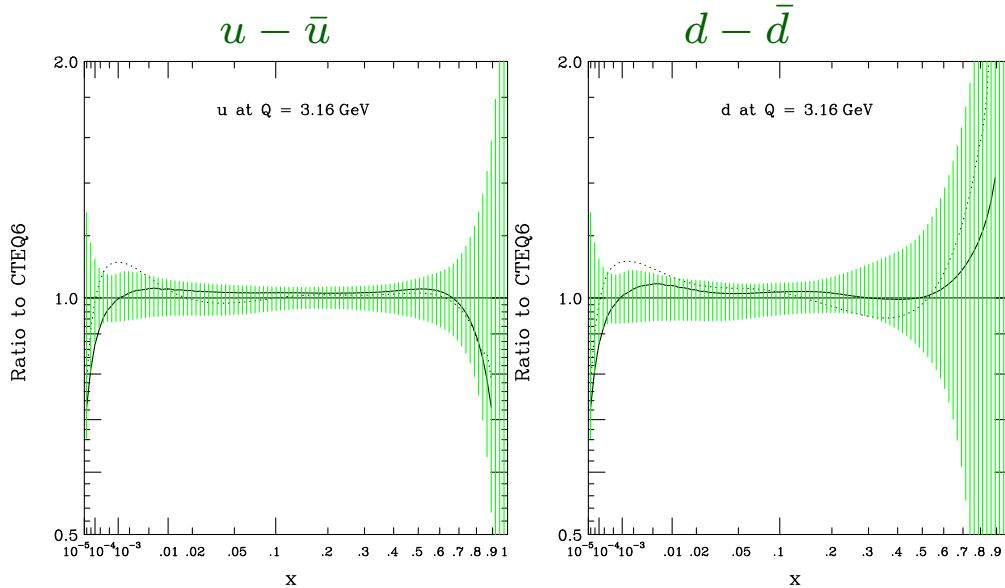
$g$



# HOW WELL DOES IT WORK?

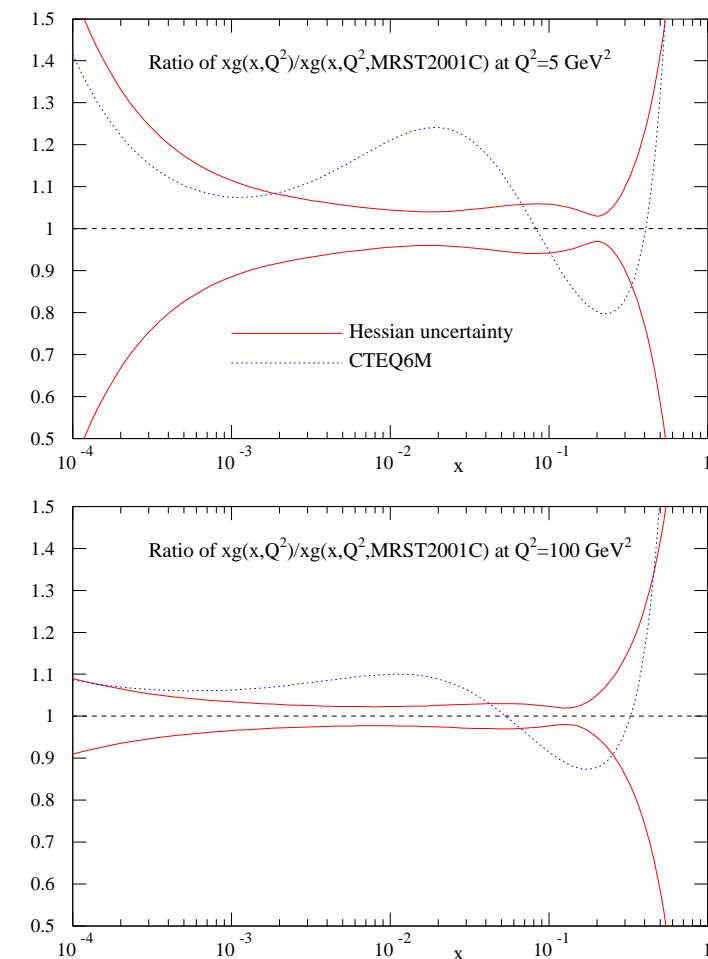
## GLOBAL FITS (MRST-CTEQ 2002-2006)

CTEQ ERROR BAND  
& MRST/CTEQ CURVE



MRST GLUON ERROR BAND  
& CTEQ/MRST CURVE

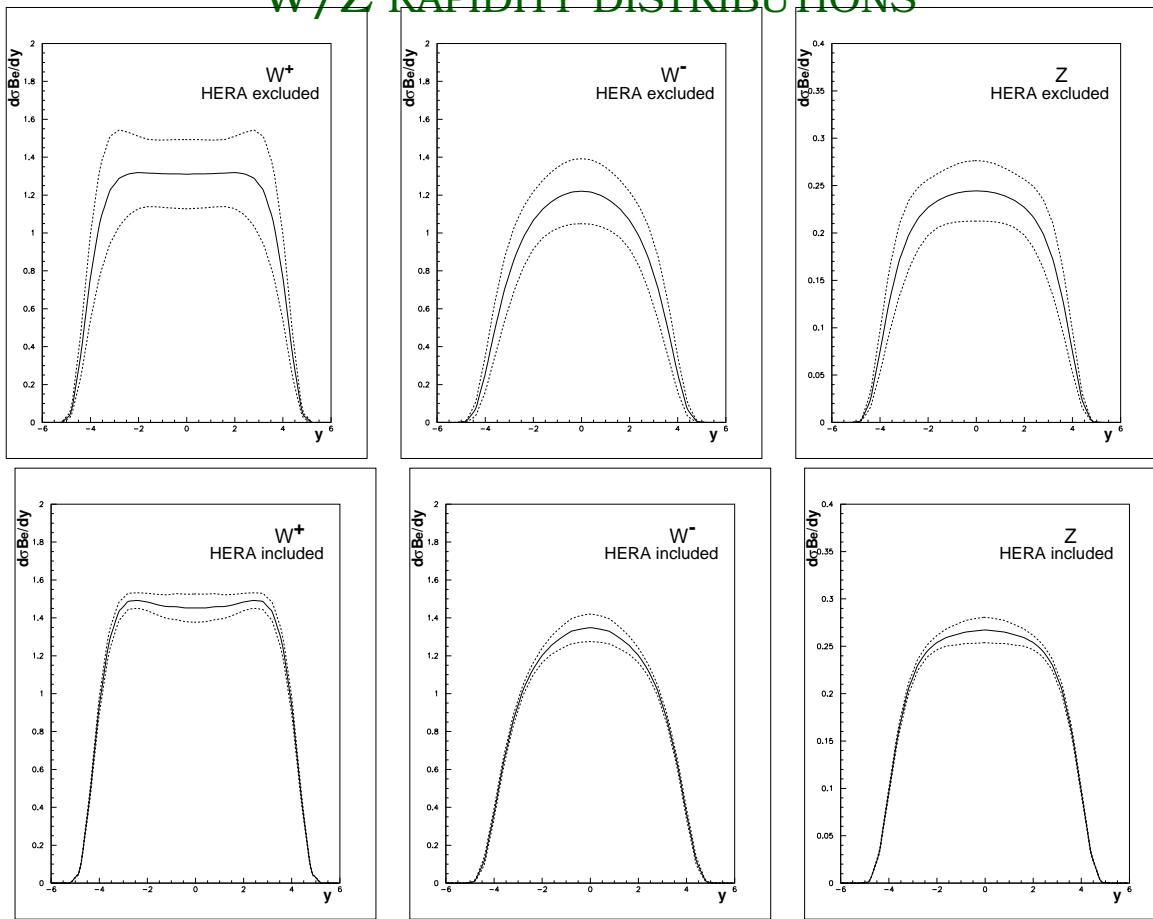
Uncertainty of gluon from Hessian method



-FEW PERCENT ERROR ON VALENCE & GLUE  
-OTHER PDFS:  
ERROR NOT WELL CONTROLLED

# CASE STUDY III: W PRODUCTION @ LHC

## W/Z RAPIDITY DISTRIBUTIONS



- W/Z RAPDITY SPECTRA & TOTAL CROSS SECTIONS:  
 $\sim 15\%$  PRE-HERA ACCURACY  
 $\sim 3 - 5\%$  POST-HERA ACCURACY
- GOOD AGREEMENT BETWEN DIFFERENT PDF SETS

PDF SET	$\sigma(W^+).B(W^+ \rightarrow l^+\nu_l)$	$\sigma(W^-).B(W^- \rightarrow l^-\bar{\nu}_l)$	$\sigma(Z).B(Z \rightarrow l^+l^-)$
ZEUS-S NO HERA	$10.63 \pm 1.73$ NB	$7.80 \pm 1.18$ NB	$1.69 \pm 0.23$ NB
ZEUS-S	$12.07 \pm 0.41$ NB	$8.76 \pm 0.30$ NB	$1.89 \pm 0.06$ NB
CTEQ6.1	$11.66 \pm 0.56$ NB	$8.58 \pm 0.43$ NB	$1.92 \pm 0.08$ NB
MRST01	$11.72 \pm 0.23$ NB	$8.72 \pm 0.16$ NB	$1.96 \pm 0.03$ NB

# THEORETICAL ISSUES

# NNLO CORRECTIONS

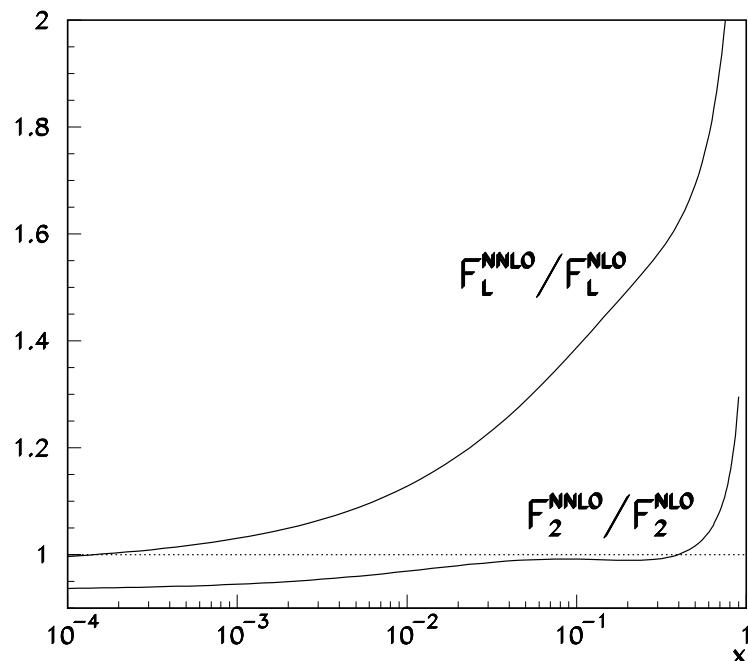
HOW BIG IS THE IMPACT OF HIGHER ORDER PERTURBATIVE CORRECTIONS?

NNLO SPLITTING FUNCTIONS AVAILABLE (Moch, Vermaseren and Vogt, 2004)

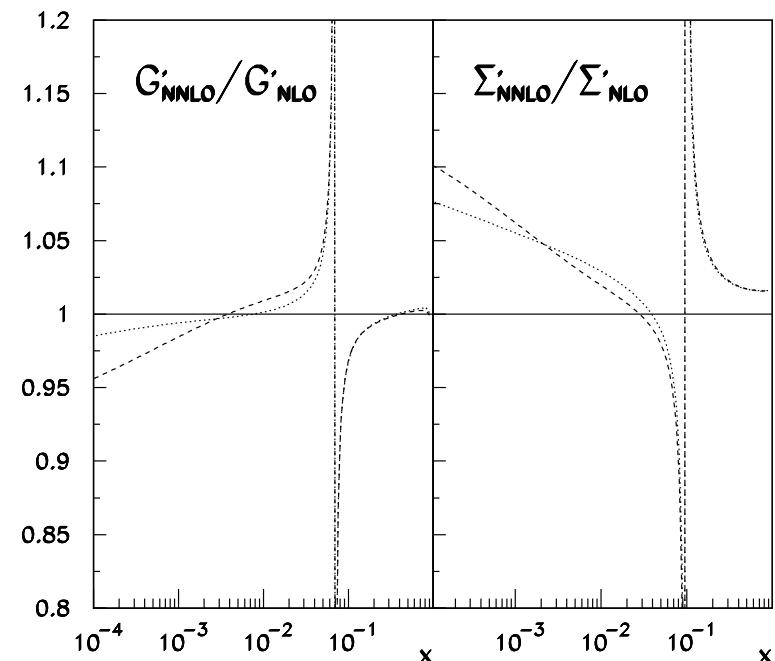
NNLO HARD XSECTS FOR DIS, DY, W AND HIGGS PRODUCTION (incl.,  $m_{top} \rightarrow \infty$ )

$\Rightarrow$  NNLO GLOBAL FITS AROUND THE CORNER

PERTURBATIVE COEFFICIENTS



EVOLUTION



Alekhin

- USUALLY ( $F_2$ , large  $x$  quark) (BUT NOT ALWAYS ( $F_L$ , very small  $x$  gluon))  
 $\Rightarrow$  MODERATE EFFECT
- MANDATORY TO ASSESS UNCERTAINTY



## The Results

### Anomalous dimensions in Mellin space

- One-loop : Gross, Wilczek '73

$$\gamma_{\text{ns}}^{(0)}(N) = C_F (2(\mathbf{N}_- + \mathbf{N}_+)S_1 - 3)$$

- Two-loop : Floratos, Ross, Sachrajda '79 ; Gonzalez-Arroyo, Lopez, Ynduráin '79

$$\begin{aligned} \gamma_{\text{ns}}^{(1)+}(N) &= 4C_A C_F \left( 2\mathbf{N}_+ S_3 - \frac{17}{24} - 2S_{-3} - \frac{28}{3}S_1 + (\mathbf{N}_- + \mathbf{N}_+) \left[ \frac{151}{18}S_1 + 2S_{1,-2} - \frac{11}{6}S_2 \right] \right) \\ &\quad + 4C_F n_f \left( \frac{1}{12} + \frac{4}{3}S_1 - (\mathbf{N}_- + \mathbf{N}_+) \left[ \frac{11}{9}S_1 - \frac{1}{3}S_2 \right] \right) + 4C_F^2 \left( 4S_{-3} + 2S_1 + 2S_2 - \frac{3}{8} \right. \\ &\quad \left. + \mathbf{N}_- \left[ S_2 + 2S_3 \right] - (\mathbf{N}_- + \mathbf{N}_+) \left[ S_1 + 4S_{1,-2} + 2S_{1,2} + 2S_{2,1} + S_3 \right] \right) \\ \gamma_{\text{ns}}^{(1)-}(N) &= \gamma_{\text{ns}}^{(1)+}(N) + 16C_F \left( C_F - \frac{C_A}{2} \right) \left( (\mathbf{N}_- - \mathbf{N}_+) \left[ S_2 - S_3 \right] - 2(\mathbf{N}_- + \mathbf{N}_+ - 2)S_1 \right) \end{aligned}$$

- Compact notation :  $\mathbf{N}_{\pm} f(N) = f(N \pm 1)$  ,  $\mathbf{N}_{\pm i} f(N) = f(N \pm i)$



– Three-loop :

S.M., Vermaseren, Vogt ‘04

$$\begin{aligned}
\gamma_{\text{ns}}^{(2)+}(N) = & 16C_A C_F n_f \left( \frac{3}{2}\zeta_3 - \frac{5}{4} + \frac{10}{9}S_{-3} - \frac{10}{9}S_3 + \frac{4}{3}S_{1,-2} - \frac{2}{3}S_{-4} + 2S_{1,1} - \frac{25}{9}S_2 + \frac{257}{27}S_1 - \frac{2}{3}S_{-3,1} - \mathbf{N}_+ \left[ S_{2,1} - \frac{2}{3}S_{3,1} - \frac{2}{3}S_4 \right] \right. \\
& - (\mathbf{N}_+ - 1) \left[ \frac{23}{18}S_3 - S_2 \right] - (\mathbf{N}_- + \mathbf{N}_+) \left[ S_{1,1} + \frac{1237}{216}S_1 + \frac{11}{18}S_3 - \frac{317}{108}S_2 + \frac{16}{9}S_{1,-2} - \frac{2}{3}S_{1,-2,1} - \frac{1}{3}S_{1,-3} - \frac{1}{2}S_{1,3} - \frac{1}{2}S_{2,1} \right. \\
& \left. \left. - \frac{1}{3}S_{2,-2} + S_1\zeta_3 + \frac{1}{2}S_{3,1} \right] \right) + 16C_F C_A^2 \left( \frac{1657}{576} - \frac{15}{4}\zeta_3 + 2S_{-5} + \frac{31}{6}S_{-4} - 4S_{-4,1} - \frac{67}{9}S_{-3} + 2S_{-3,-2} + \frac{11}{3}S_{-3,1} + \frac{3}{2}S_{-2} \right. \\
& - 6S_{-2}\zeta_3 - 2S_{-2,-3} + 3S_{-2,-2} - 4S_{-2,-2,1} + 8S_{-2,1,-2} - \frac{1883}{54}S_1 - 10S_{1,-3} - \frac{16}{3}S_{1,-2} + 12S_{1,-2,1} + 4S_{1,3} - 4S_{2,-2} - \frac{5}{2}S_4 + \frac{1}{2}S_5 \\
& + \frac{176}{9}S_2 + \frac{13}{3}S_3 + (\mathbf{N}_- + \mathbf{N}_+ - 2) \left[ 3S_1\zeta_3 + 11S_{1,1} - 4S_{1,1,-2} \right] + (\mathbf{N}_- + \mathbf{N}_+) \left[ \frac{9737}{432}S_1 - 3S_{1,-4} + \frac{19}{6}S_{1,-3} + 8S_{1,-3,1} + \frac{91}{9}S_{1,-2} \right. \\
& \left. - 6S_{1,-2,-2} - \frac{29}{3}S_{1,-2,1} + 8S_{1,1,-3} - 16S_{1,1,-2,1} - 4S_{1,1,3} - \frac{19}{4}S_{1,3} + 4S_{1,3,1} + 3S_{1,4} + 8S_{2,-2,1} + 2S_{2,3} - S_{3,-2} + \frac{11}{12}S_{3,1} - S_{4,1} - 4S_{2,-3} \right. \\
& \left. + \frac{1}{6}S_{2,-2} - \frac{1967}{216}S_2 + \frac{121}{72}S_3 \right] - (\mathbf{N}_- - \mathbf{N}_+) \left[ 3S_2\zeta_3 + 7S_{2,1} - 3S_{2,1,-2} + 2S_{2,-2,1} - \frac{1}{4}S_{2,3} - \frac{3}{2}S_{3,-2} - \frac{29}{6}S_{3,1} + \frac{11}{4}S_{4,1} + \frac{1}{2}S_{2,-3} - S_{2,-2} \right] \\
& + \mathbf{N}_+ \left[ \frac{28}{9}S_3 - \frac{2376}{216}S_2 - \frac{8}{3}S_4 - \frac{5}{2}S_5 \right] \right) + 16C_F n_f^2 \left( \frac{17}{144} - \frac{13}{27}S_1 + \frac{2}{9}S_2 + (\mathbf{N}_- + \mathbf{N}_+) \left[ \frac{2}{9}S_1 - \frac{11}{54}S_2 + \frac{1}{18}S_3 \right] \right) + 16C_F^2 C_A \left( \frac{45}{4}\zeta_3 \right. \\
& \left. - \frac{151}{64} - 10S_{-5} - \frac{89}{6}S_{-4} + 20S_{-4,1} + \frac{134}{9}S_{-3} - 2S_{-3,-2} - \frac{31}{3}S_{-3,1} + 2S_{-3,2} - \frac{9}{2}S_{-2} + 18S_{-2}\zeta_3 + 10S_{-2,-3} - 6S_{-2,-2} \right. \\
& \left. + 8S_{-2,-2,1} - 28S_{-2,1,-2} + 46S_{1,-3} + \frac{26}{3}S_{1,-2} - 48S_{1,-2,1} + \frac{28}{3}S_{1,2} - \frac{185}{6}S_3 - 8S_{1,3} + 2S_{3,-2} - 4S_5 - (\mathbf{N}_- + \mathbf{N}_+ - 2) \left[ 9S_1\zeta_3 - \frac{133}{36}S_1 \right. \right. \\
& \left. \left. + \frac{209}{6}S_{1,1} - 14S_{1,1,-2} - \frac{242}{18}S_2 + 9S_{2,-2} + \frac{33}{4}S_4 - 3S_{3,1} + \frac{14}{3}S_{2,1} \right] + (\mathbf{N}_- + \mathbf{N}_+) \left[ 17S_{1,-4} - \frac{107}{6}S_{1,-3} - 32S_{1,-3,1} - \frac{173}{9}S_{1,-2} \right. \right]
\end{aligned}$$



$$\begin{aligned}
& + 16S_{1,-2,-2} + \frac{103}{3}S_{1,-2,1} - 2S_{1,-2,2} - 36S_{1,1,-3} + 56S_{1,1,-2,1} + 8S_{1,1,3} - \frac{109}{9}S_{1,2} - 4S_{1,2,-2} + \frac{43}{3}S_{1,3} - 8S_{1,3,1} - 11S_{1,4} + \frac{11}{3}S_{2,2} \\
& + 21S_{2,-3} - 30S_{2,-2,1} - 4S_{2,1,-2} - 5S_{2,3} - S_{4,1} + \frac{31}{6}S_{2,-2} - \frac{67}{9}S_{2,1} \Big] + (\mathbf{N}_+ - \mathbf{N}_-) \Big[ 9S_2\zeta_3 + 2S_{2,-3} + 4S_{2,-2,1} - 12S_{2,1,-2} - 2S_{2,3} \\
& + 13S_{4,1} + \frac{1}{2}S_{2,-2} + \frac{11}{2}S_4 - \frac{33}{2}S_{3,1} + \frac{59}{9}S_3 + \frac{127}{6}S_{2,1} - \frac{1153}{72}S_2 \Big] + \mathbf{N}_+ \Big[ 8S_{3,-2} + \frac{4}{3}S_{3,1} - 2S_{3,2} + 14S_5 + \frac{23}{6}S_4 + \frac{73}{3}S_3 + \frac{151}{24}S_2 \Big] \\
& + 16C_F^2 n_f \Big( \frac{23}{16} - \frac{3}{2}\zeta_3 + \frac{4}{3}S_{-3,1} - \frac{59}{36}S_2 + \frac{4}{3}S_{-4} - \frac{20}{9}S_{-3} + \frac{20}{9}S_1 - \frac{8}{3}S_{1,-2} - \frac{8}{3}S_{1,1} - \frac{4}{3}S_{1,2} + \mathbf{N}_+ \Big[ \frac{25}{9}S_3 - \frac{4}{3}S_{3,1} - \frac{1}{3}S_4 \Big] \\
& - (\mathbf{N}_+ - 1) \Big[ \frac{67}{36}S_2 - \frac{4}{3}S_{2,1} + \frac{4}{3}S_3 \Big] + (\mathbf{N}_- + \mathbf{N}_+) \Big[ S_1\zeta_3 - \frac{325}{144}S_1 - \frac{2}{3}S_{1,-3} + \frac{32}{9}S_{1,-2} - \frac{4}{3}S_{1,-2,1} + \frac{4}{3}S_{1,1} + \frac{16}{9}S_{1,2} - \frac{4}{3}S_{1,3} \\
& + \frac{11}{18}S_2 - \frac{2}{3}S_{2,-2} + \frac{10}{9}S_{2,1} + \frac{1}{2}S_4 - \frac{2}{3}S_{2,2} - \frac{8}{9}S_3 \Big] \Big) + 16C_F^3 \Big( 12S_{-5} - \frac{29}{32} - \frac{15}{2}\zeta_3 + 9S_{-4} - 24S_{-4,1} - 4S_{-3,-2} + 6S_{-3,1} \\
& - 4S_{-3,2} + 3S_{-2} + 25S_3 - 12S_{-2}\zeta_3 - 12S_{-2,-3} + 24S_{-2,1,-2} - 52S_{1,-3} + 4S_{1,-2} + 48S_{1,-2,1} - 4S_{3,-2} + \frac{67}{2}S_2 - 17S_4 \\
& + (\mathbf{N}_- + \mathbf{N}_+ - 2) \Big[ 6S_1\zeta_3 - \frac{31}{8}S_1 + 35S_{1,1} - 12S_{1,1,-2} + S_{1,2} + 10S_{2,-2} + S_{2,1} + 2S_{2,2} - 2S_{3,1} - 3S_5 \Big] + (\mathbf{N}_- + \mathbf{N}_+) \Big[ 23S_{1,-3} \\
& - 22S_{1,-4} + 32S_{1,-3,1} - 2S_{1,-2} - 8S_{1,-2,-2} - 30S_{1,-2,1} - 6S_{1,3} + 4S_{1,-2,2} + 40S_{1,1,-3} - 48S_{1,1,-2,1} + 8S_{1,2,-2} + 4S_{1,2,2} + 8S_{1,3,1} \\
& + 4S_{1,4} + 28S_{2,-2,1} + 4S_{2,1,2} + 4S_{2,2,1} + 4S_{3,1,1} - 4S_{3,2} + 8S_{2,1,-2} - 26S_{2,-3} - 2S_{2,3} - 4S_{3,-2} - 3S_{2,-2} - 3S_{2,2} + \frac{3}{2}S_4 \Big] \\
& + (\mathbf{N}_- - \mathbf{N}_+) \Big[ 12S_{2,1,-2} - 6S_2\zeta_3 - 2S_{2,-3} + 3S_{2,3} + 2S_{3,-2} - \frac{81}{4}S_{2,1} + 14S_{3,1} - 5S_{2,-2} - \frac{1}{2}S_{2,2} + \frac{15}{8}S_2 + \frac{1}{2}S_3 - 13S_{4,1} + 4S_5 \Big] \\
& + \mathbf{N}_+ \Big[ 14S_4 - \frac{265}{8}S_2 - \frac{87}{4}S_3 - 4S_{4,1} - 4S_5 \Big] \Big)
\end{aligned}$$

# HIGH ENERGY (SMALL $x$ ) RESUMMATION

AT  $O(\alpha_s^n)$ ,  $O[\ln(\frac{1}{x})^n]$  CONTRIBUTIONS:

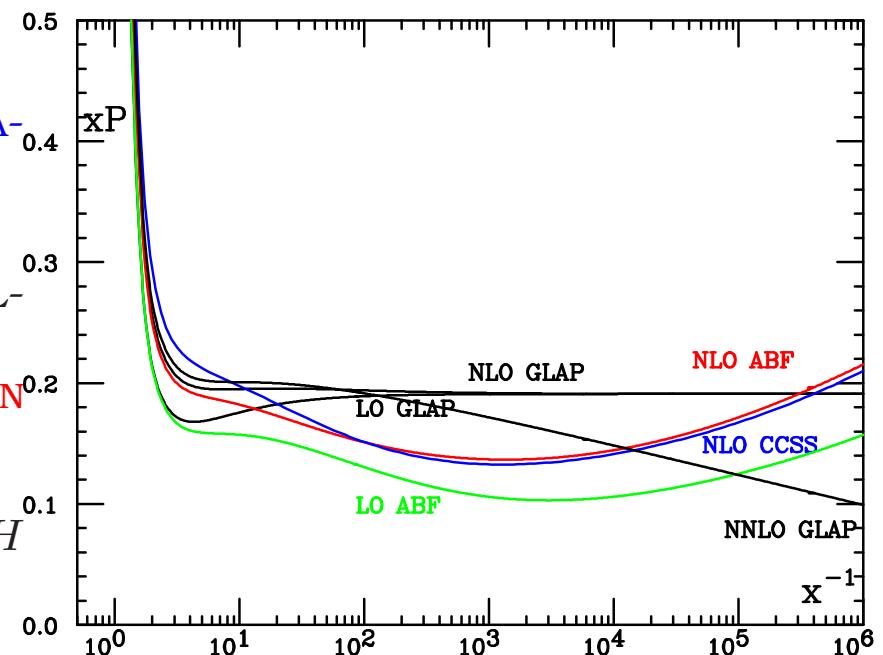
⇒ PERTURBATION THEORY UNSTABLE AT SMALL  $x$  (C.M. ENERGY >> FINAL STATE MASS)

$x_{\text{cut}}$ :	0	0.0002	0.001	0.0025	0.005	0.01
# DATA POINTS	2097	2050	1961	1898	1826	1762
$\chi^2(x > 0)$	2267					
$\chi^2(x > 0.0002)$	2212	2203				
$\chi^2(x > 0.001)$	2134	2128	2119			
$\chi^2(x > 0.0025)$	2069	2064	2055	2040		
$\chi^2(x > 0.005)$	2024	2019	2012	1993	1973	
$\chi^2(x > 0.01)$	1965	1961	1953	1934	1917	1916
$\Delta_i^{i+1}$	0.19	0.10	0.24	0.28	0.02	

DATA-THEORY AGREEMENT  
FOR EVOLUTION OF  $F_2$   
IMPROVES IF SMALL  $x$  DATA  
REMOVED (MRST 2003)

$\chi^2$  improves  
with fixed # of pts  
(same row)

- CONSIDERABLE PROGRESS IN FULL RESUMMATION OF SMALL  $x$  TERMS  
(Ciafaloni, Colferai, Salam, Stašto;  
Altarelli, Ball, S.F.)  
⇒ STABLE RESUMMED SPLITTING FCTNS AVAILABLE
- RESUMMED SPL. FCTN. CLOSER TO LO THAN TO NNLO
- NOT YET INCLUDED IN PARTON FITS  
NOTE: NLO SMALL  $x$  BEHAVIOUR OF  $gg \rightarrow H$   
INCORRECT IN LARGE  $m_t/m_H$  LIMIT



# THRESHOLD (LARGE $x$ ) RESUMMATION

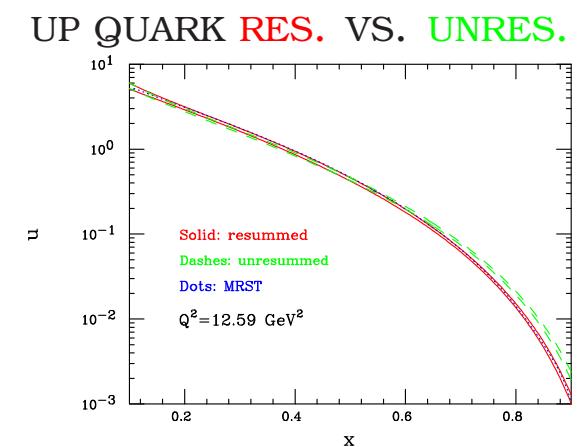
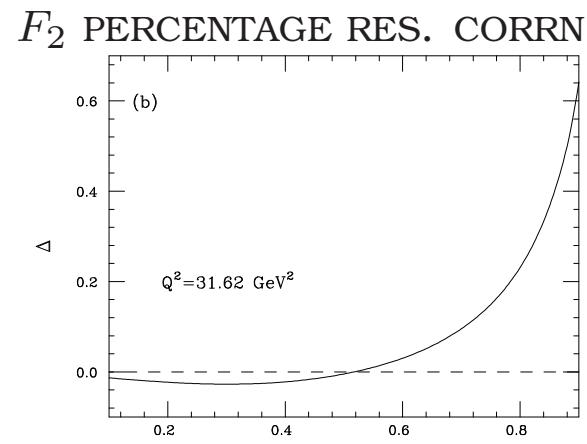
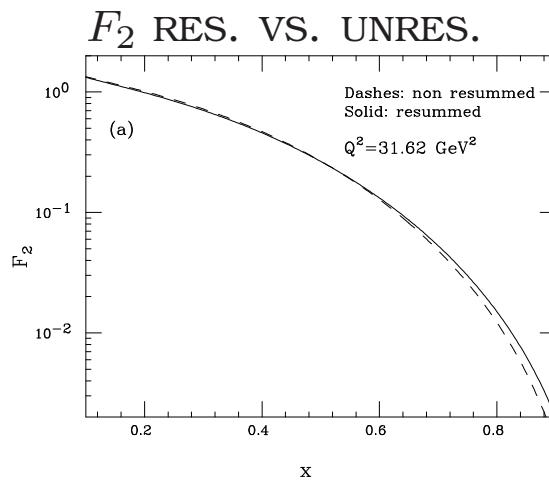
AT  $O(\alpha_s^n)$ ,  $O[\ln^{2m}(1-x)]$  CONTRIBUTIONS:

LEFTOVER OF KLN REAL/VIRTUAL CANCELLATION CLOSE TO THRESHOLD

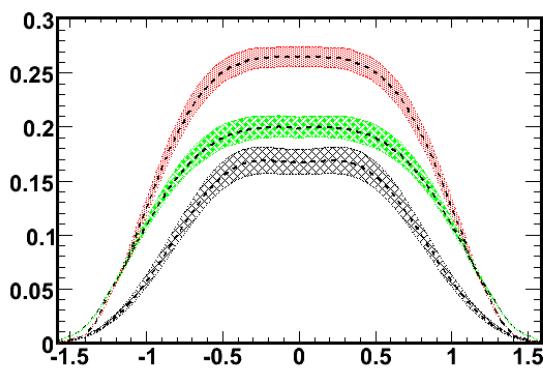
$\Rightarrow$  PERT. THEORY UNSTABLE AT LARGE  $x$  (C.M. ENERGY  $\sim$  FINAL STATE MASS)

THEORY KNOWN: CAN BE DONE UP TO THIRD LOG ORDER (Catani, Trentadue, Sterman 1987-89)

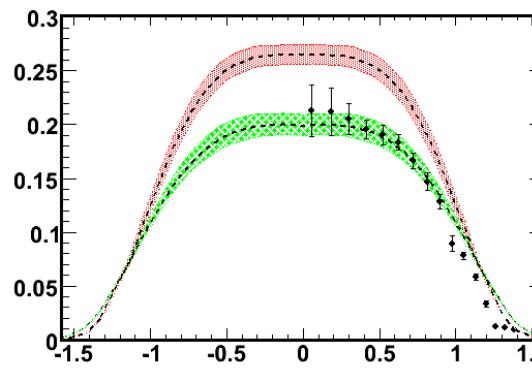
- DIS: SIZABLE ONLY @ VERY LARGE  $x$ , WHERE XSECT & PDF TINY (Corcella, Magnea 2005)
- DY: CAN HAVE SIZABLE EFFECTS, ESPECIALLY ON RAP. DISTN. (Bolzoni 2006)
- NOT INCLUDED IN CURRENT FITS



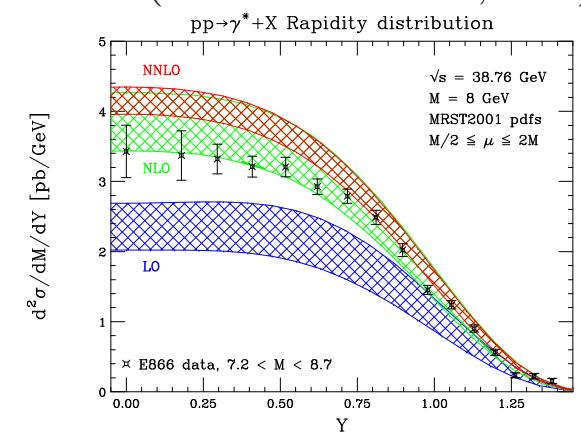
$d\sigma/dQ^2 dy$  VS.  $y$  LO NLO RES.



X  
DRELL YAN  
INCL. E866 DATA



NNLO (Anastasiou et al, 2003)



# HEAVY QUARKS

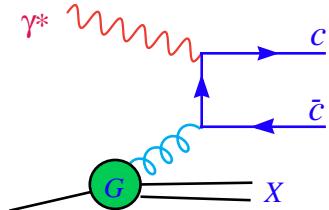
HOW CAN ONE ACCOUNT FOR HEAVY FLAVOURS (CHARM, BEAUTY...)?

SIMPLE OPTION: (CTEQ6, Alekhin) CHARM PDF VANISHES BELOW THRESHOLD, INCLUDED ALONG OTHER PDFS ABOVE THRESHOLD  $\Rightarrow$  EFFECTIVELY,  $m_c \approx 0$  FOR  $Q^2 > Q_{th}^2$ .

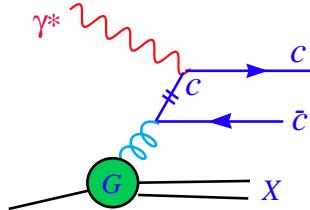
- HQ PDF GENERATED DYNAMICALLY BY PERTURBATIVE EVOLUTION (HQ PAIR-PRODUCED BY RADIATION FROM GLUONS)
- TREATMENT NOT ACCURATE IN  $Q^2 \approx Q_{th}^2$  REGION

MORE REFINED TREATMENT OF THRESHOLD: [Collins, Tung et al (ACOT) 1986-2006]

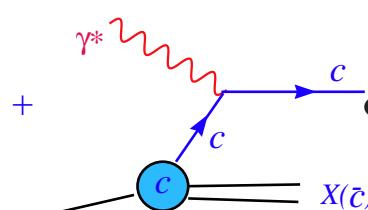
$m_c \neq 0$ , LO  
CHARM RADIATION



$m_c = 0$ , LO  
CHARM RADIATION

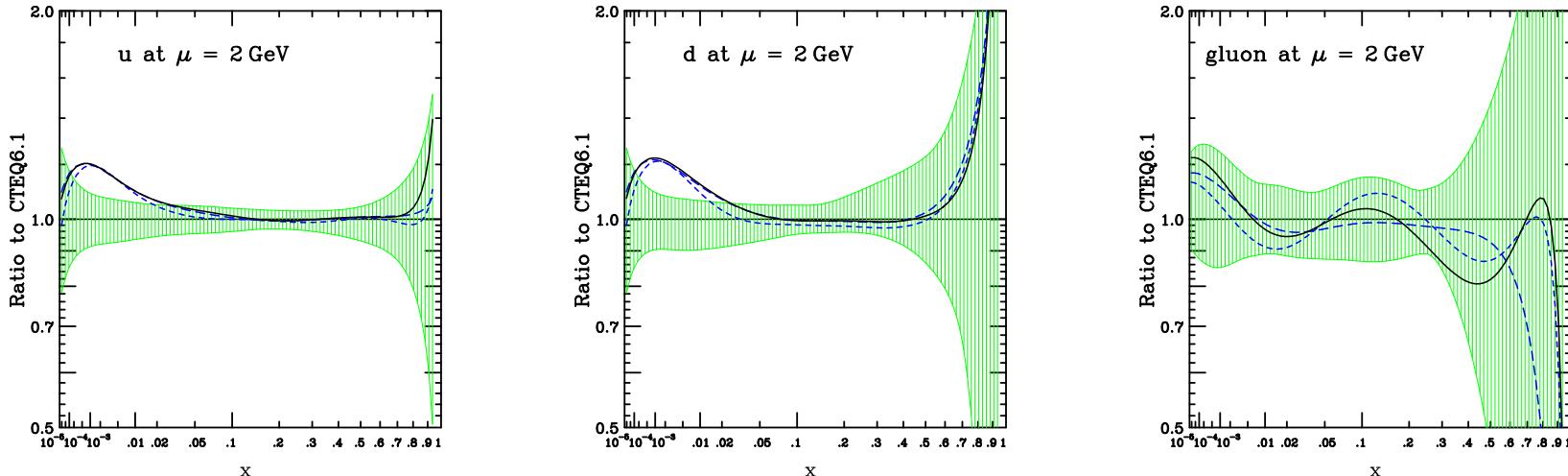


$m_c = 0$   
CHARM PDF

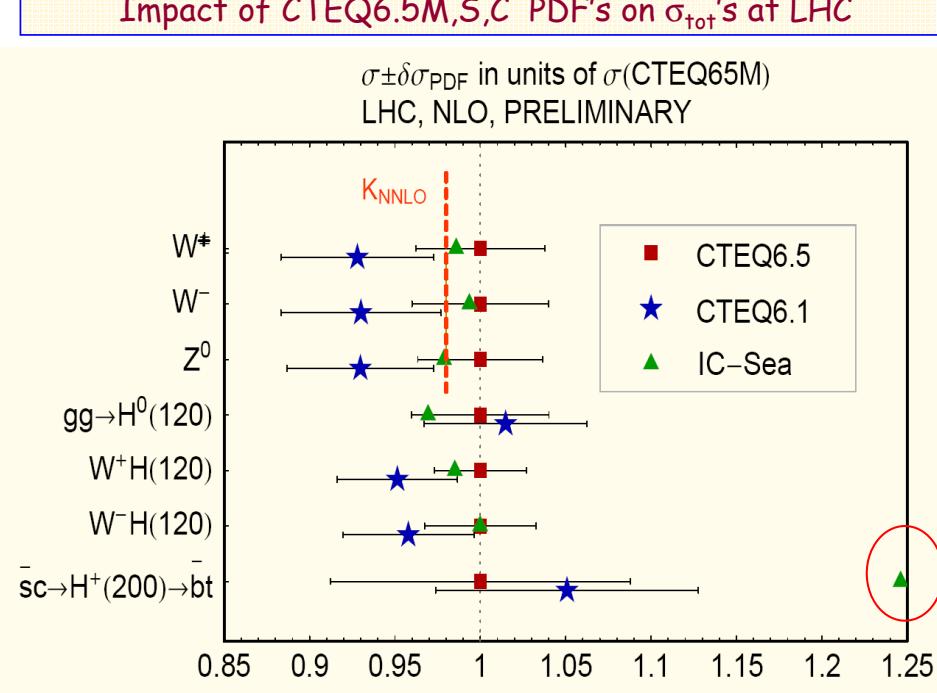


- $m_c \neq 0$  IN HARD XSECT  $\Rightarrow$  RELEVANT AROUND THRESHOLD
- $m_c = 0$  IN CHARM PDF  $\Rightarrow$  RELEVANT AT LARGE SCALE
- SUBTRACT DOUBLE COUNTING

# W PRODUCTION @ LHC PART II



## Impact of CTEQ6.5M,S,C PDF's on $\sigma_{\text{tot}}$ 's at LHC



- NEW (CTEQ6.5) PARTON SET INCLUDES HQ MATCHING
- EFFECT OF IMPROVED HW MASS FELT MOSTLY IN SMALL  $x$  QUARK SUPPRESSION OF CHARM  $\Rightarrow$  ENHANCEMENT OF LIGHT SEA
- IN COMPARISON TO PREVIOUS (CTEQ76.1), SIGNIFICANT CHANGE OF  $u, d$  QUARK DISTNS AT  $x \sim 0.01$
- $W, Z$  TOTAL XSECT NO LONGER AGREES WITH MRST THOUGH MRST INCLUDES HQ MATCHING!
- EFFECT OF INTRINSIC CHARM (IC) MINOR

# PDF UNCERTAINTIES

# WHAT'S THE PROBLEM?

- FOR A SINGLE QUANTITY, WE QUOTE 1 SIGMA ERRORS: VALUE  $\pm$  ERROR
- FOR A PAIR OF NUMBERS, WE QUOTE A 1 SIGMA ELLIPSE
- FOR A FUNCTION, WE NEED AN “ERROR BAR” IN A SPACE OF FUNCTIONS

MUST DETERMINE THE PROBABILITY DENSITY (MEASURE)  $\mathcal{P}[f_i(x)]$   
IN THE SPACE OF PARTON DISTRIBUTION FUNCTIONS  $f_i(x)$  ( $i = \text{quark, antiquark, gluon}$ )

EXPECTATION VALUE OF  $\sigma [f_i(x)] \Rightarrow$  FUNCTIONAL INTEGRAL

$$\left\langle \sigma [f_i(x)] \right\rangle = \int \mathcal{D}f_i \sigma [f_i(x)] \mathcal{P}[f_i],$$

# WHAT'S THE PROBLEM?

- FOR A SINGLE QUANTITY, WE QUOTE 1 SIGMA ERRORS: VALUE  $\pm$  ERROR
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MUST DETERMINE AN INFINITE-DIMENSIONAL OBJECT  
FROM A FINITE SET OF DATA POINTS

STANDARD SOLUTION: PROJECT ON FINITE-DIMENSIONAL SPACE OF  
PARAMETERS

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PARAMETERS

DOES THE RESULT DEPEND ON THE WAY ONE PROJECTS?

# CAN WE TRUST GLOBAL FITS?

PARTON SETS DO NOT AGREE WITHIN RESPECTIVE ERRORS!

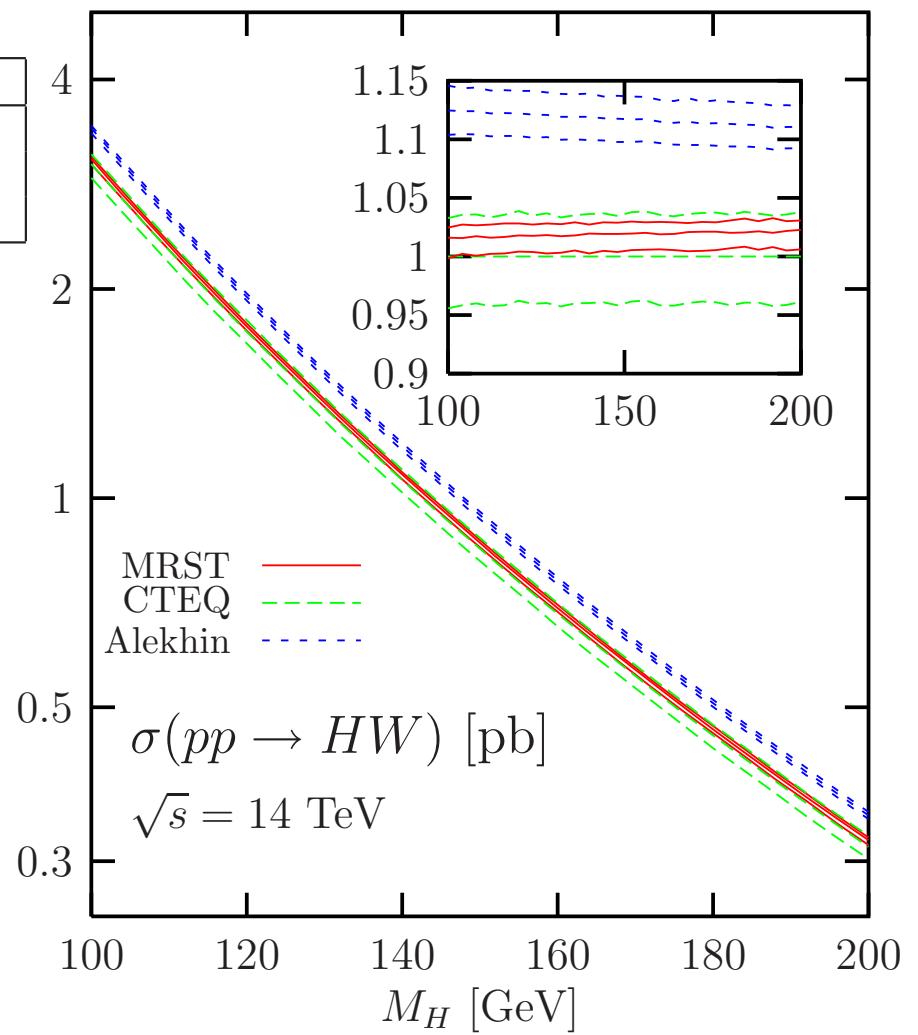
## $W$ PRODUCTION CROSS-SECTION TEVATRON

PDF SET	XSEC [NB]	PDF UNCERTAINTY
ALEKHIN	2.73	$\pm 0.05$ (TOT)
MRST2002	2.59	$\pm 0.03$ (EXPT)
CTEQ6	2.54	$\pm 0.10$ (EXPT)

THORNE 2003

- ALEKHIN VS. MRST/CTEQ  
 $\rightarrow$   $W$  PRODUCTION XSECT AT TEVATRON DO NOT AGREE WITHIN RESPECTIVE ERRORS
- ALEKHIN VS. MRST/CTEQ  
 $\rightarrow$  PREDICTIONS FOR ASSOCIATE HIGGS  $W$  PRODUCTION LHC DO NOT AGREE WITHIN RESPECTIVE ERRORS

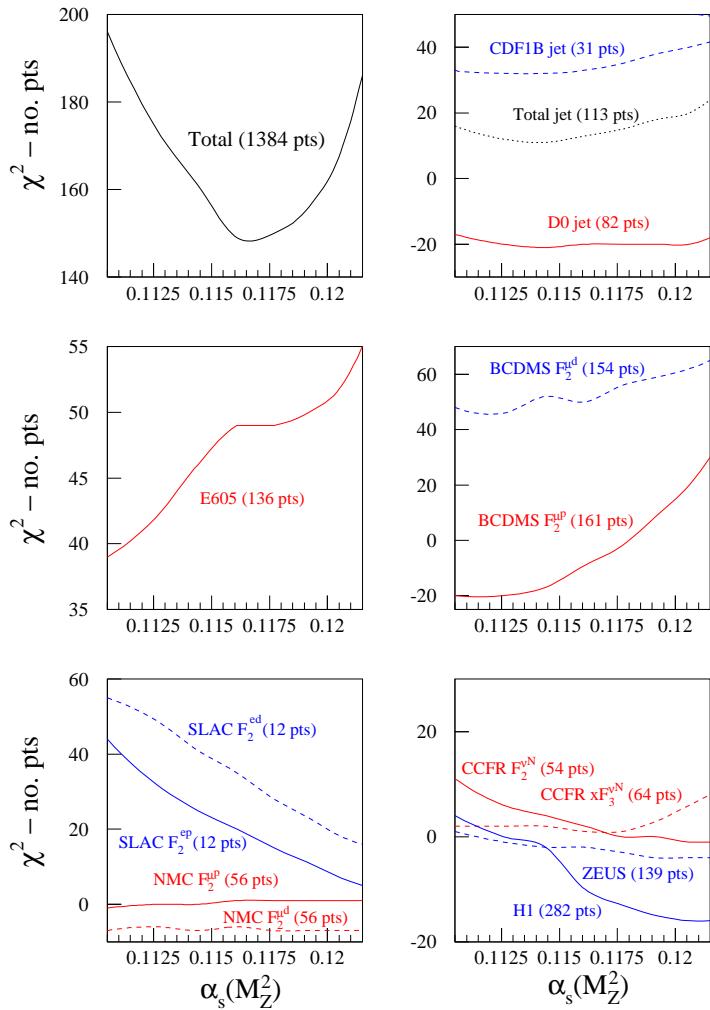
## HIGGS PRODUCTION AT LHC



DJOUADI AND FERRAG, 2004

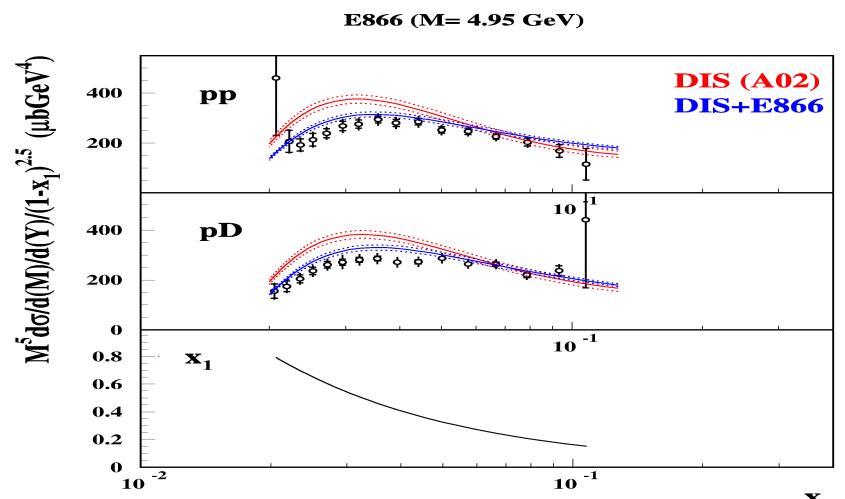
# INCOMPATIBLE DATA?

GLOBAL  $\chi^2$  MINIMUM MAY NOT CORRESPOND TO LOCAL MINIMA



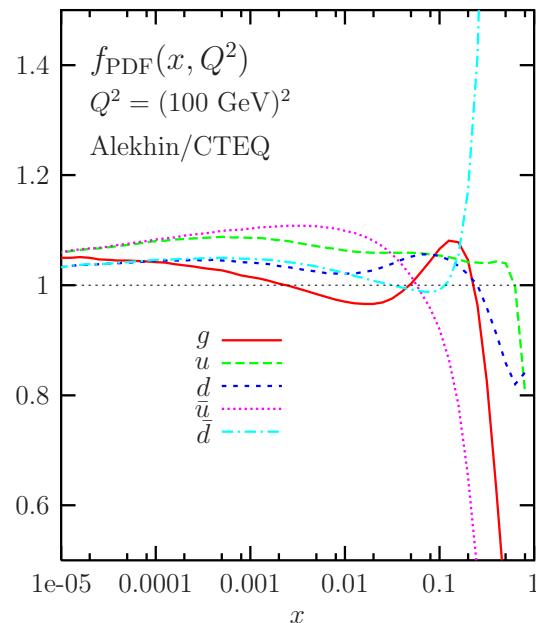
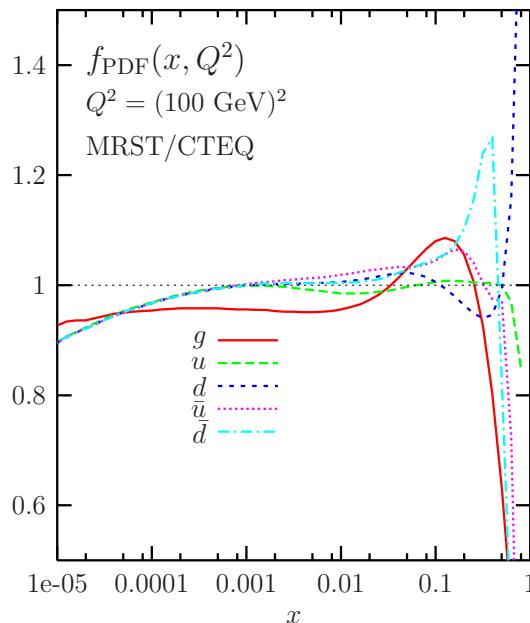
MRST 2003

E866 DY DATA DISAGREE WITH DIS DATA:  
 $\sigma_{DY} \sim q(x_1)q(x_2)$  DISAGREES WITH DIS QUARK AT SAME  $x$  AND  $Q^2$



ALEKHIN 2005

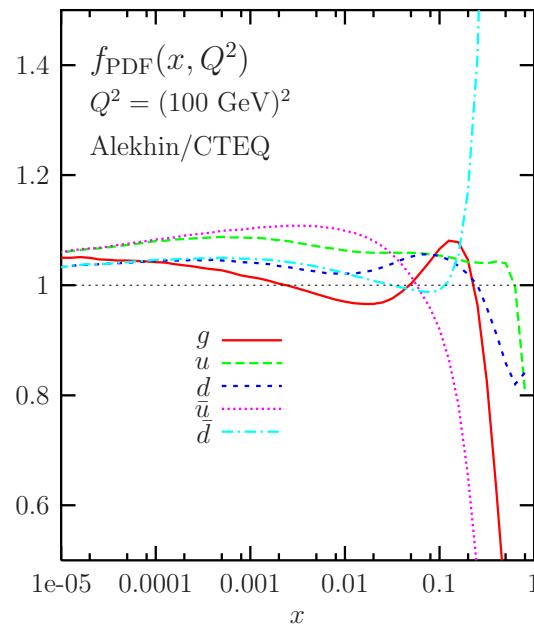
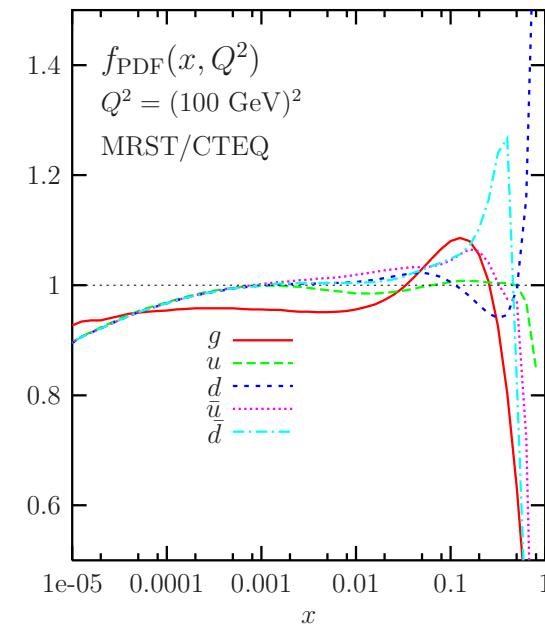
# PARAMETRIZATION BIAS?



MRST & CTEQ  
→ SIMILAR PARTONS

Djouadi and Ferrag 2003

# PARAMETRIZATION BIAS?



**MRST & CTEQ**  
→ SIMILAR PARTONS

Djouadi and Ferrag 2003

SIMILAR PAR-  
TONS  
→ SIMILAR  
RESULTS

PDF SET	COMMENT	XSEC [NB]	PDF UNCERTAINTY
ALEKHIN	TEVATRON	2.73	± 0.05 (TOT)
MRST2002	TEVATRON	2.59	± 0.03 (EXPT)
CTEQ6	TEVATRON	2.54	± 0.10 (EXPT)
ALEKHIN	LHC	215	± 6 (TOT)
MRST2002	LHC	204	± 4 (EXPT)
CTEQ6	LHC	205	± 8 (EXPT)

We do not seem to have the optimum parameterization for both finding the best fit and also investigating fluctuations about this best fit (...) This might then influence our error analysis... (MRST 2004)

# SOLUTIONS: CTEQ TOLERANCE CRITERION

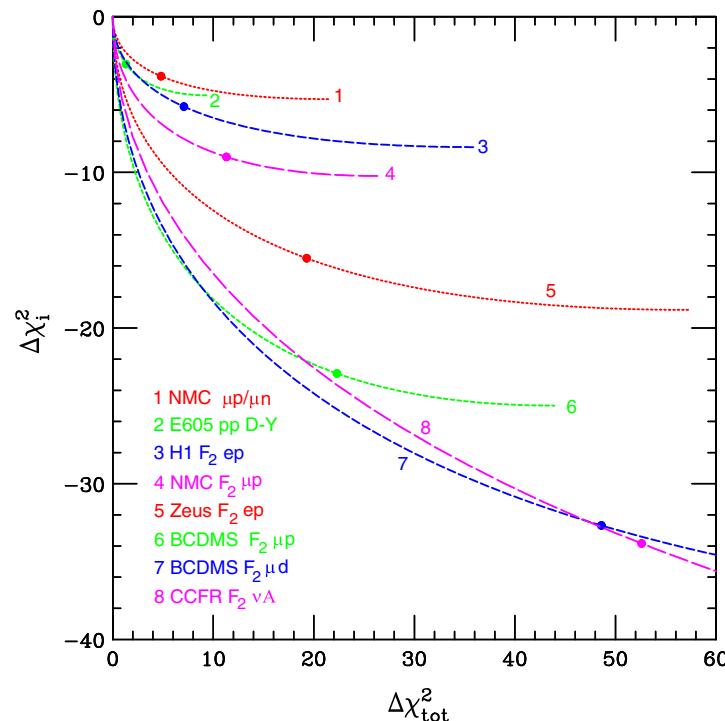
## SINGLE OUT INCONSISTENT DATA

- how many parameters are significantly determined by each dataset?
- how consistent are the data from one set with the rest?

STUDY MINIMUM ALLOWED  $\chi_i^2$

FOR  $i$ -TH EXP. AS

GLOBAL  $\chi^2$  ALLOWED TO INCREASE



Collins, Pumplin 2001

CCFR, BCDMS

INCOMPATIBLE WITH THE REST

# SOLUTIONS: CTEQ TOLERANCE CRITERION

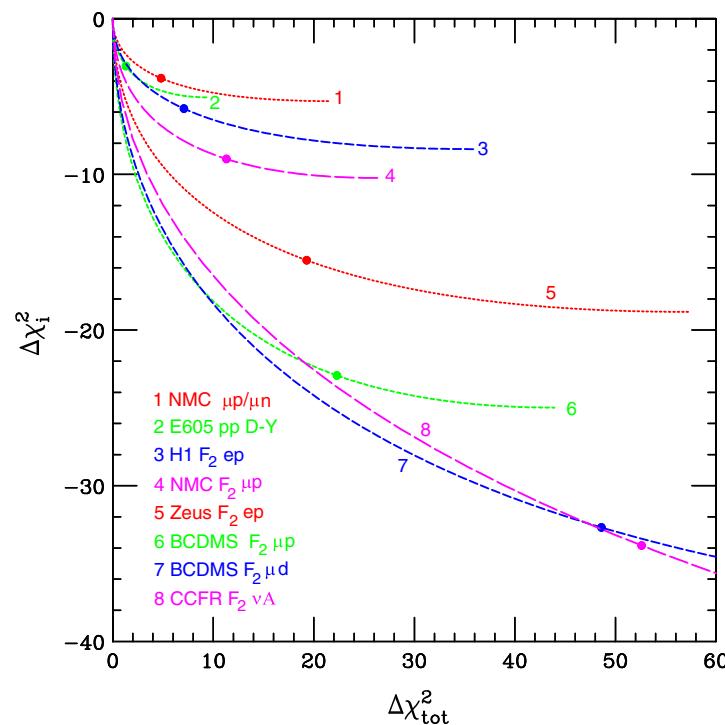
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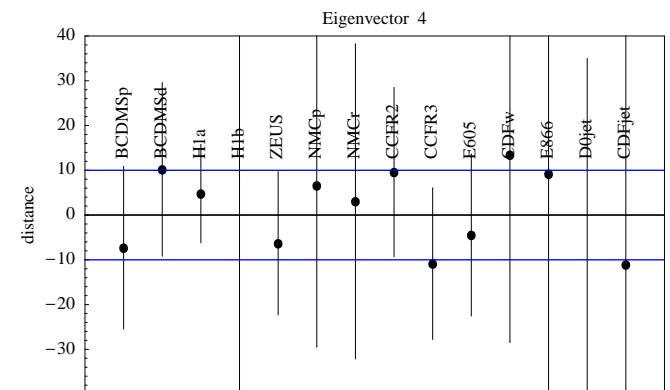
Collins, Pumplin 2001

CCFR, BCDMS

INCOMPATIBLE WITH THE REST

## OPTIONS

- discard incompatible experiments
- reweight individual contributions
- determine intersection of 90% confidence level bands for individual experiments  $\Rightarrow$  tolerance



# SOLUTIONS: CTEQ TOLERANCE CRITERION

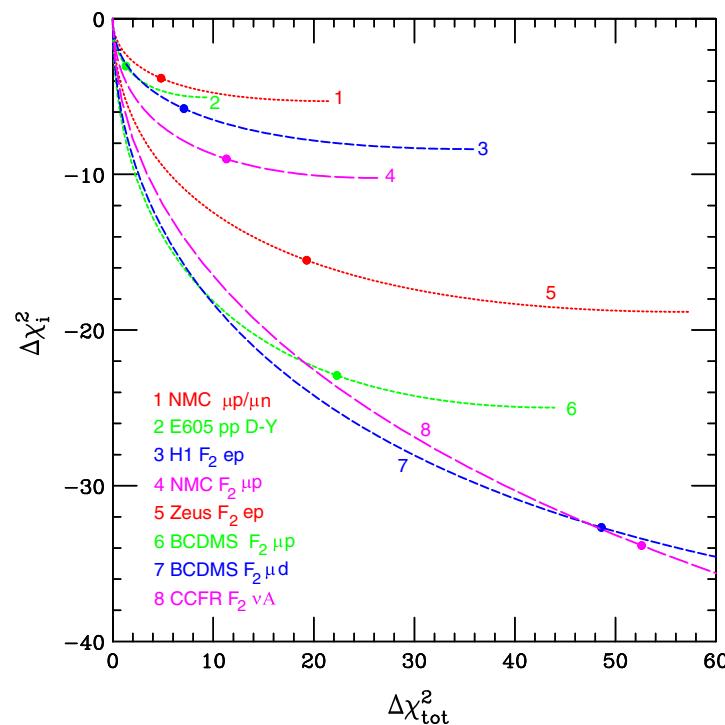
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FOR  $i$ -TH EXP. AS

GLOBAL  $\chi^2$  ALLOWED TO INCREASE



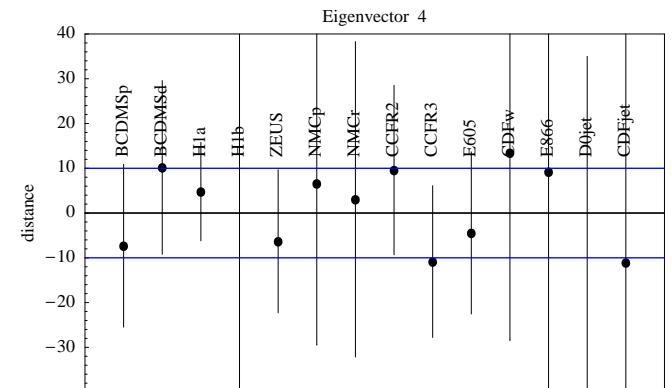
Collins, Pumplin 2001

CCFR, BCDMS

INCOMPATIBLE WITH THE REST

## OPTIONS

- discard incompatible experiments
- reweight individual contributions
- determine intersection of 90% confidence level bands for individual experiments  $\Rightarrow$  tolerance

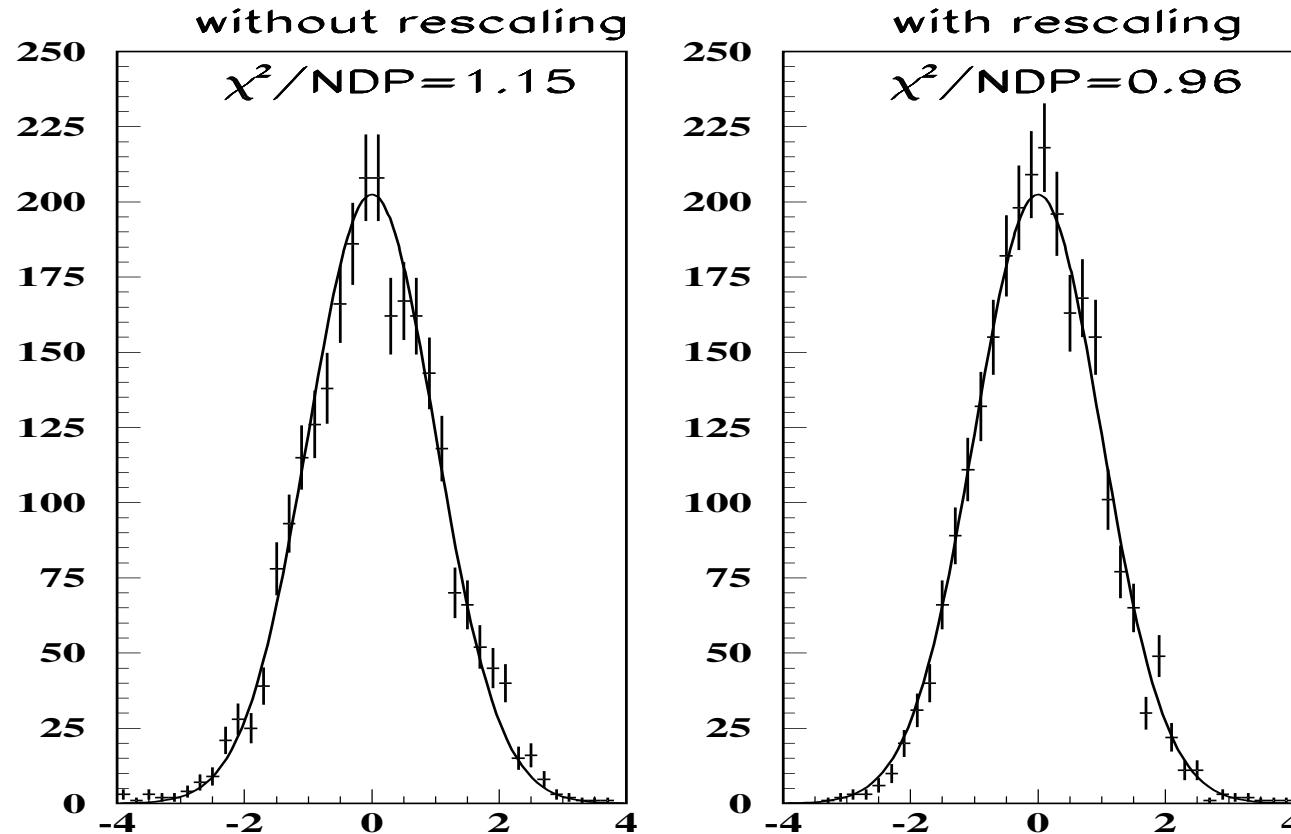


TOLERANCE  $\rightarrow \Delta\chi^2 = 100$  (CTEQ6)

## SOLUTIONS: ERROR RESCALING

HOW CAN DATA FROM INCONSISTENT SETS BE INCLUDED?

ASSUME INCONSISTENCY DUE TO UNDERESTIMATED (SYST.) ERROR:



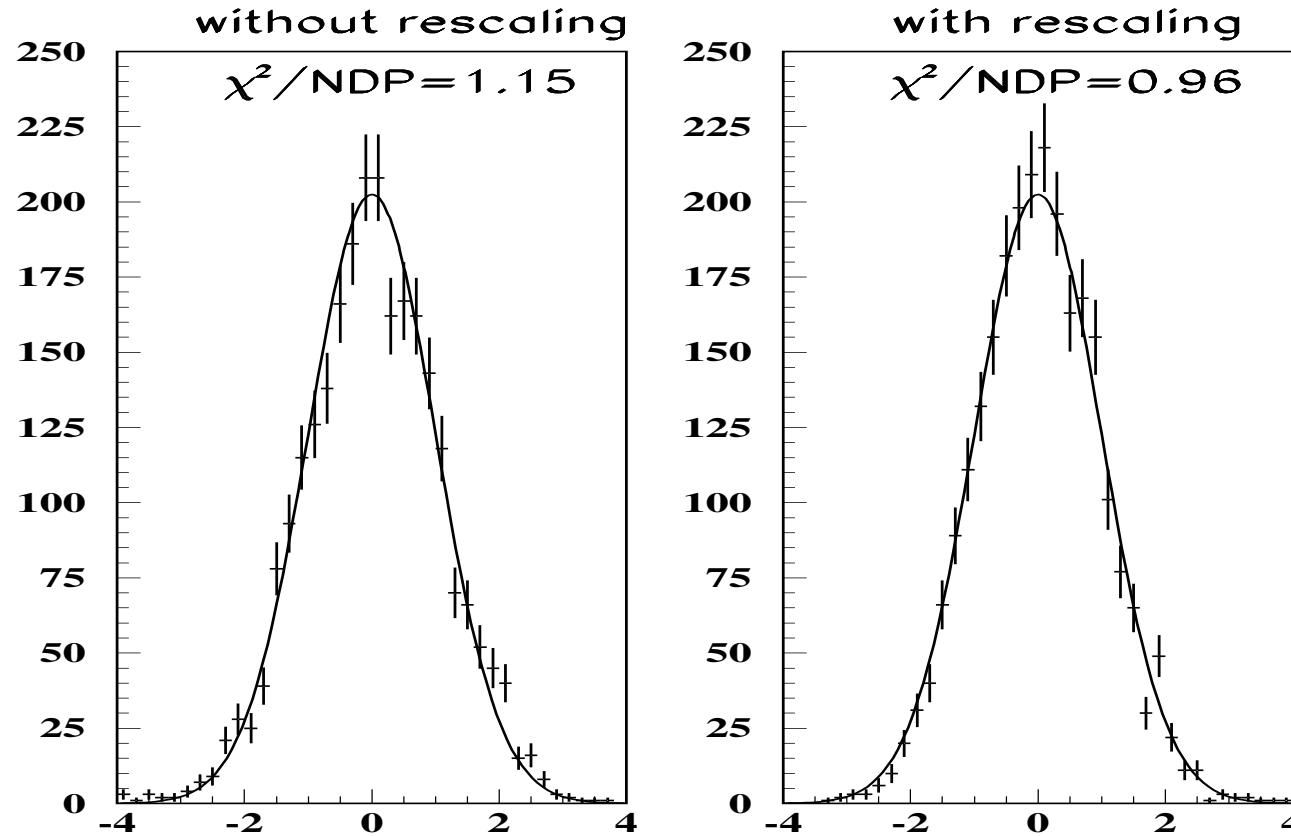
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ALEKHIN 2005

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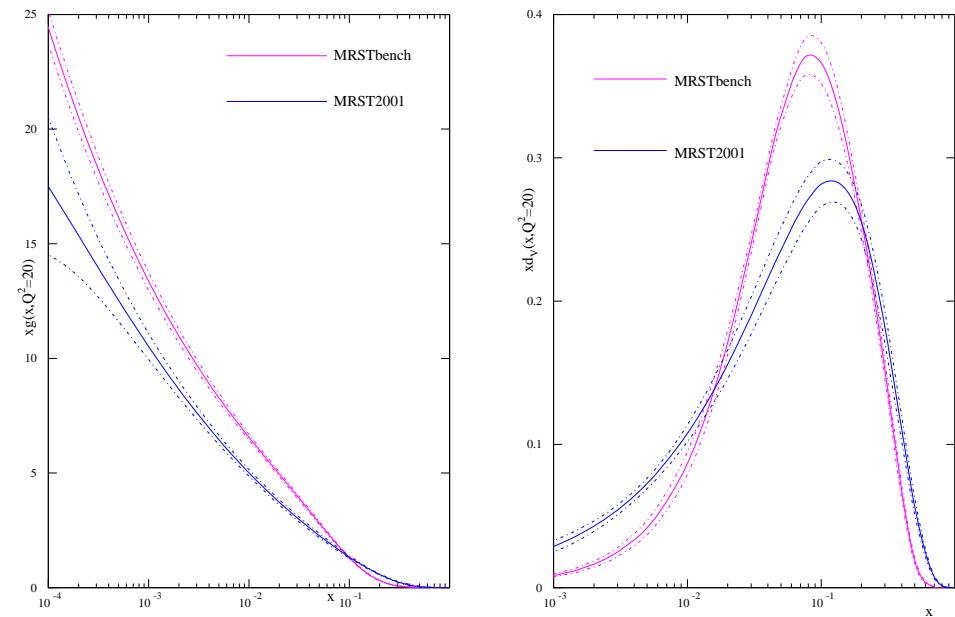
ALEKHIN 2005

ALTERNATIVE (ALEKHIN 2006): DISCARD INCONSISTENT DATA, RETAIN SUBSET

# THE HERA-LHC BENCHMARK: AN IMPASSE

HERA-LHC  
BENCHMARK PARTONS  
OBTAINED FROM NC DIS  
DATA ONLY,  $Q^2 > 9 \text{ GeV}^2$

GLUON AND  $d_V$ : MRST VS. BENCH

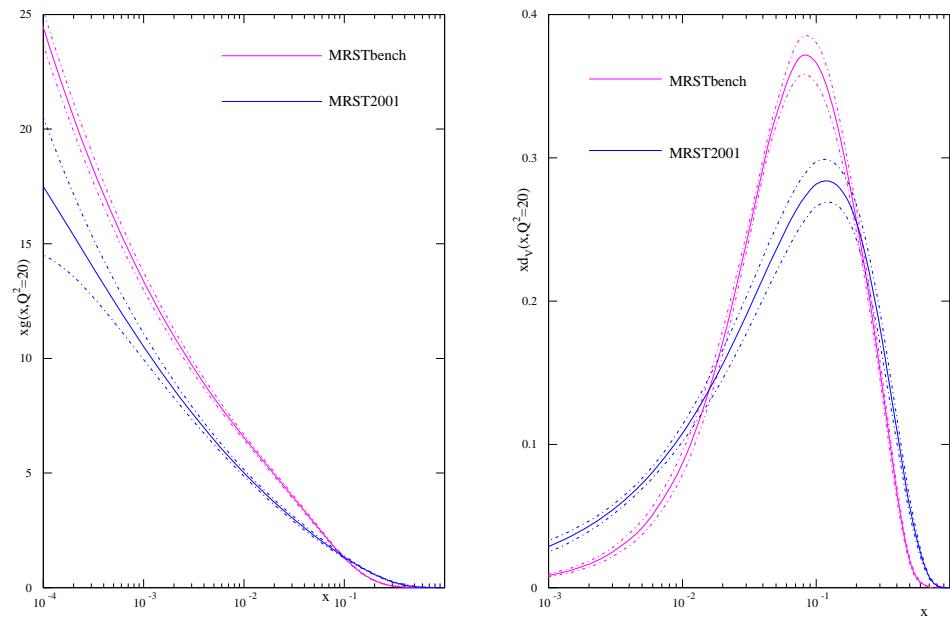


- IT IS UNSURPRIZING THAT CENTRAL VALUES DEPEND STRONGLY ON THE DATASET

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- IT IS UNSURPRIZING THAT CENTRAL VALUES DEPEND STRONGLY ON THE DATASET
- BUT IT IS VERY WORRISOME THAT THE RESULT WITH THE FULL DATA SET IS NOT WITHIN THE ERROR BAND OF THE RESULT FROM A DATA SUBSET

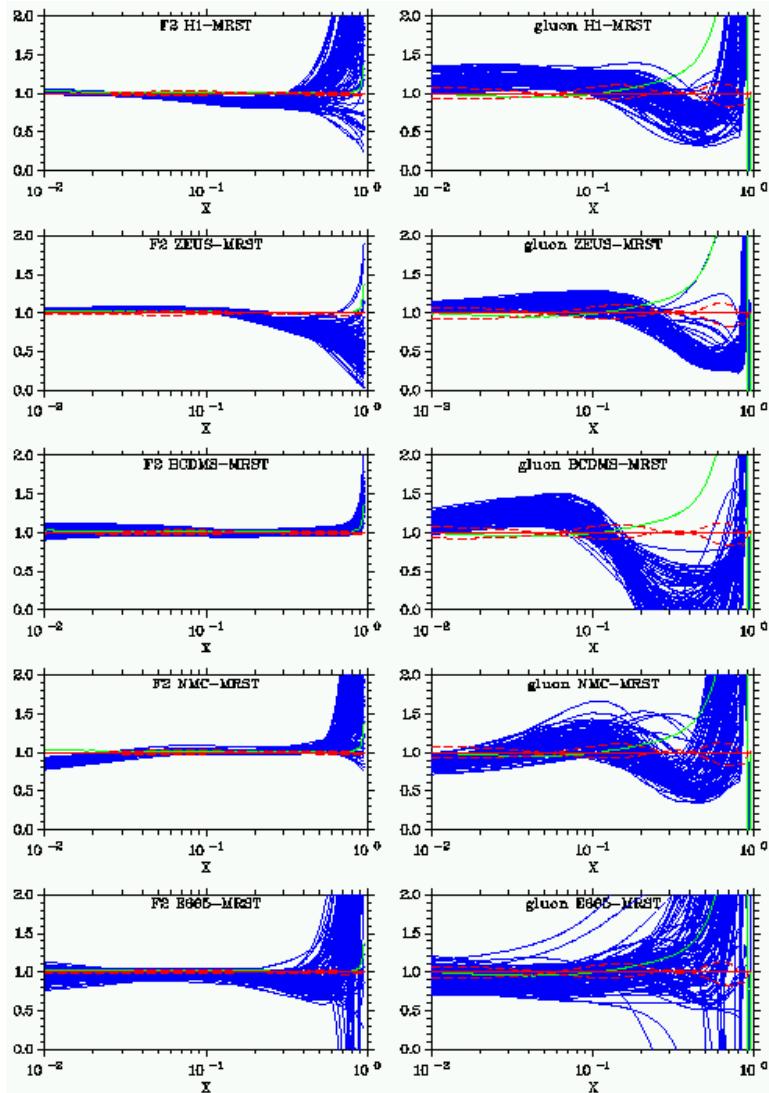
## THE BAYESIAN MONTE CARLO (GIELE, KOSOWER, KELLER 2001)

- generate a Monte-Carlo sample of fcts. with “reasonable” prior distn.  
(e.g. an available parton set) → representation of probability functional  $\mathcal{P}[f_i]$
- calculate observables with functional integral
- update probability using Bayesian inference on MC sample:  
better agreement with data → more functions in sample
- iterate until convergence achieved

PROBLEM IS MADE FINITE-DIMENSIONAL BY THE CHOICE OF PRIOR, BUT  
RESULT DO NOT DEPEND ON THE CHOICE IF SUFFICIENTLY GENERAL  
**HARD TO HANDLE “FLAT DIRECTIONS”**

(Monte Carlo replicas which lead to same agreement with data);  
**COMPUTATIONALLY VERY INTENSIVE;**  
**DIFFICULT TO ACHIEVE INDEP. FROM PRIOR**

# RESULT: FERMI PARTONS (2001)



$F_2^{\text{singlet}}$  AND GLUON RATIOS FERMI/MRST

ONLY SUBSET OF DATA FITTED (H1, E665, BCDMS DIS DATA)

GOOD AGREEMENT WITH TEVATRON  $W$  XSECT  
TROUBLE WITH VALUE OF  $\alpha_s$   
FINAL PARTON SET NEVER ACHIEVED

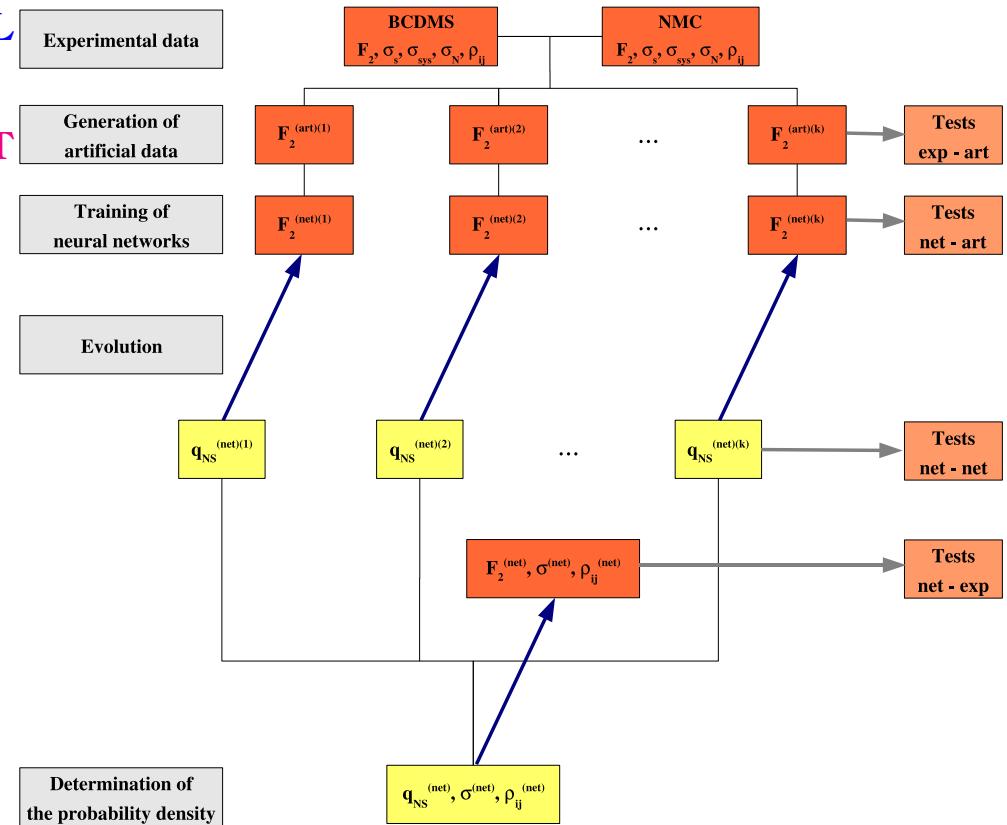
# THE NEURAL MONTE CARLO

## THE NNPDF COLLABORATION

(2004: Del Debbio, SF, Latorre, Piccione, Rojo; 2007: +Ball, Guffanti, Ubiali)

**BASIC IDEA:** USE NEURAL NETWORKS AS UNIVERSAL UNBIASED INTERPOLANTS

- GENERATE A SET OF MONTE CARLO REPLICAS  $\sigma^{(k)}(p_i)$  OF THE ORIGINAL DATASET  $\sigma^{(\text{data})}(p_i)$
- ⇒ REPRESENTATION OF  $\mathcal{P}[\sigma(p_i)]$  AT DISCRETE SET OF POINTS  $p_i$



# THE NEURAL MONTE CARLO

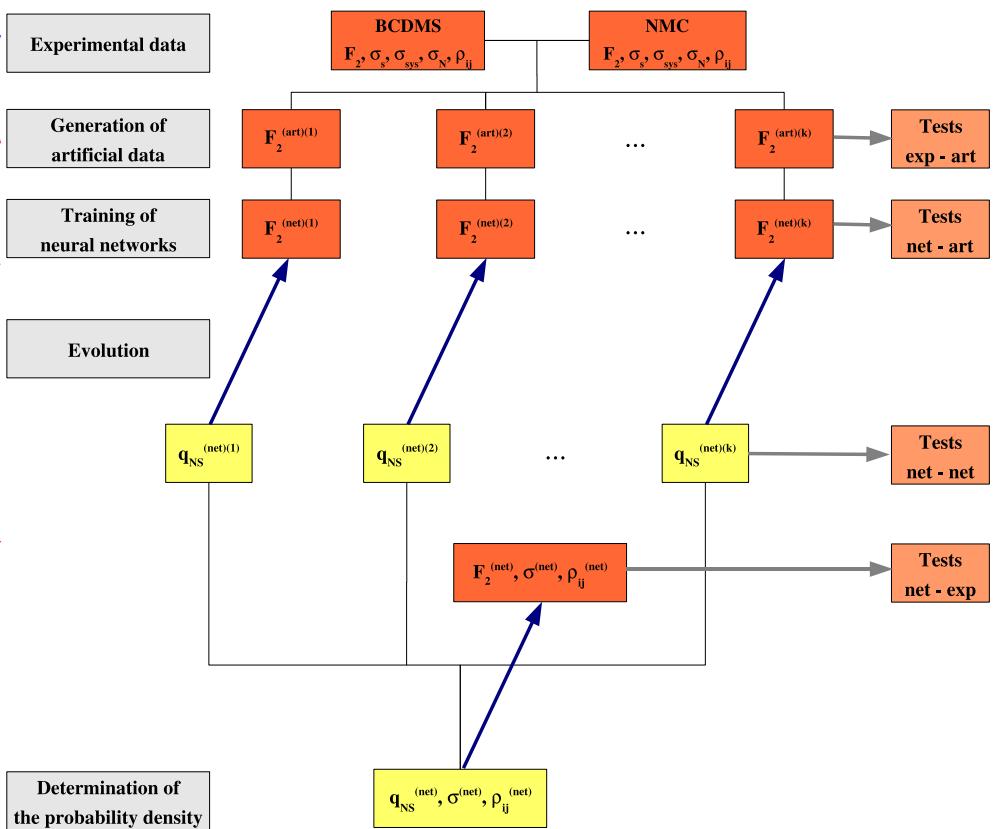
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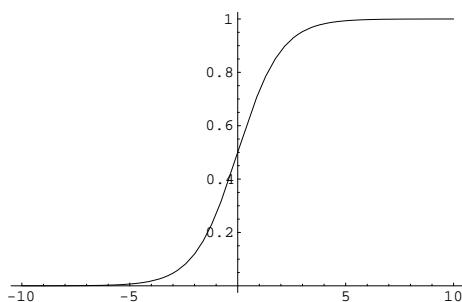
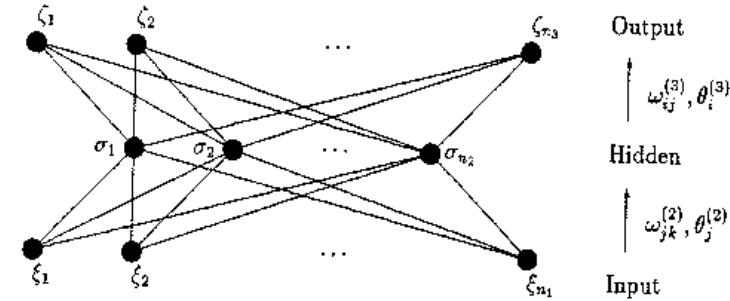
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 $\Rightarrow$  REPRESENTATION OF  $\mathcal{P}[\sigma(p_i)]$  AT DISCRETE SET OF POINTS  $p_i$
- TRAIN A NEURAL NET FOR EACH PDF ON EACH REPLICA, THUS OBTAINING A NEURAL REPRESENTATION OF THE PDFS  $f_i^{(\text{net}), (k)}$
- THE SET OF NEURAL NETS IS A REPRESENTATION OF THE PROBABILITY DENSITY:

$$\langle \sigma [f_i] \rangle = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} \sigma [f_i^{(\text{net})(k)}]$$



# WHAT ARE NEURAL NETWORKS?



## MULTILAYER FEED-FORWARD NETWORKS

- Each neuron receives input from neurons in preceding layer and feeds output to neurons in subsequent layer
- Activation determined by **weights** and **thresholds**

$$\xi_i = g \left( \sum_j \omega_{ij} \xi_j - \theta_i \right)$$

- Sigmoid activation function

$$g(x) = \frac{1}{1+e^{-\beta x}}$$

## JUST ANOTHER SET OF BASIS FUNCTIONS!

A 1-2-1 NN:  $\xi_1^{(3)}(\xi_1^{(1)}) = \frac{1}{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - \xi_1^{(1)} \omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - \xi_1^{(1)} \omega_{21}^{(1)}}}}$

THANKS TO NONLINEAR BEHAVIOUR,  
ANY FUNCTION CAN BE REPRESENTED BY A SUFFICIENTLY BIG NEURAL  
NETWORK

# WHAT ARE NEURAL NETWORKS GOOD FOR?

IN A **STANDARD** FIT, ONE LOOKS FOR MINIMUM  $\chi^2$  WITH GIVEN FINITE PARM.

- IF THE BASIS IS TOO LARGE, THE FIT NEVER CONVERGES
- IF THE BASIS IS TOO SMALL, THE FIT IS BIASED

**Q:** HOW CAN ONE BE SURE THAT THE COMPROMISE IS UNBIASED?

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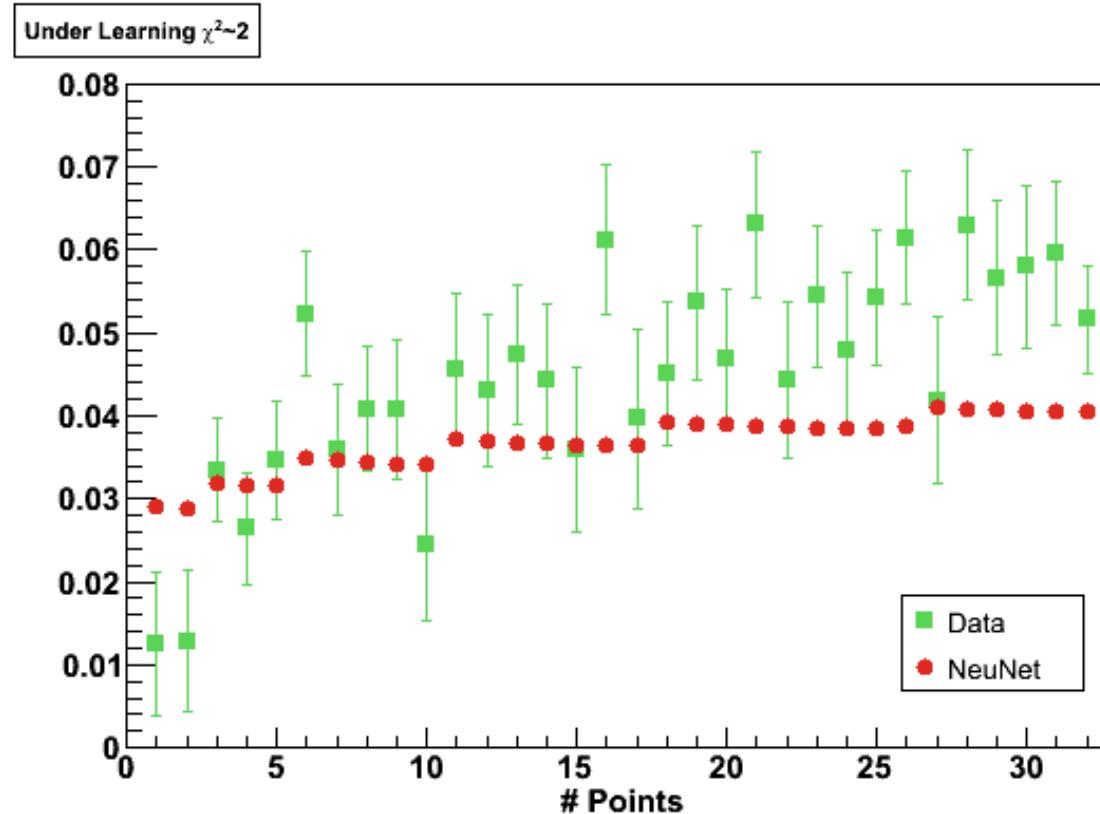
IN A STANDARD FIT, ONE LOOKS FOR MINIMUM  $\chi^2$  WITH GIVEN FINITE PARM.

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## UNDERLEARNING



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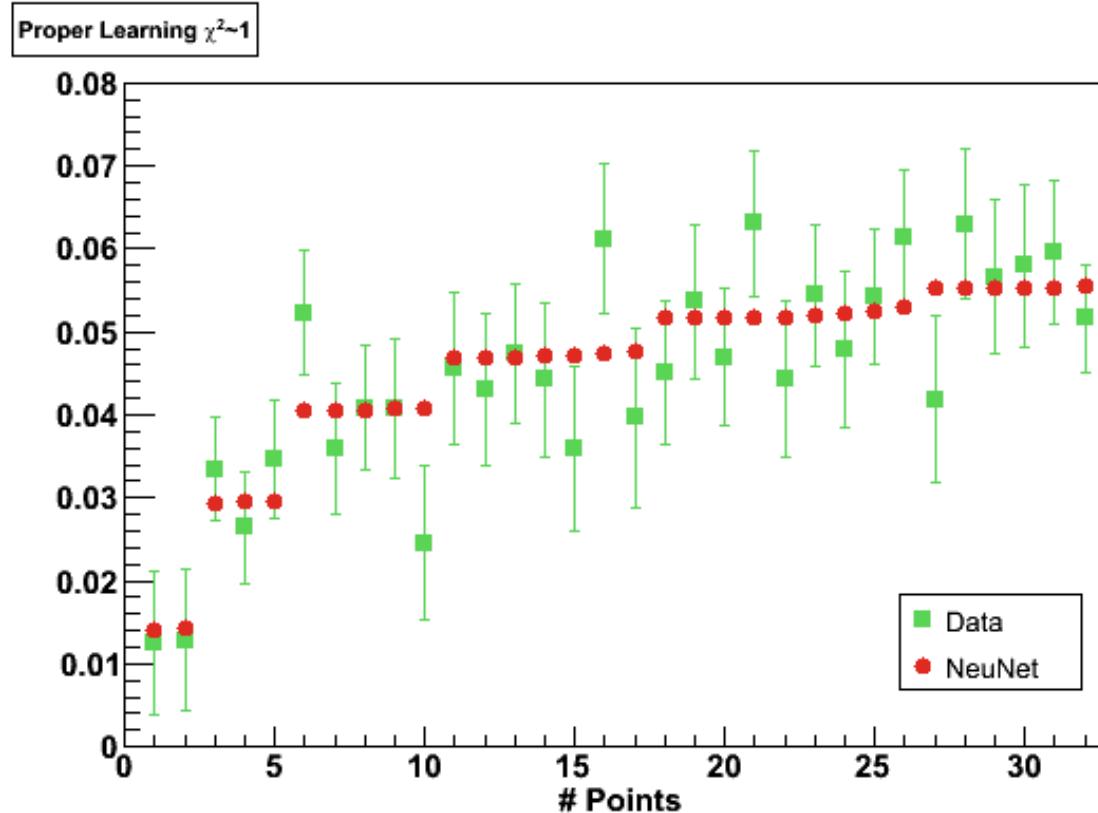
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## PROPER LEARNING



# WHAT ARE NEURAL NETWORKS GOOD FOR?

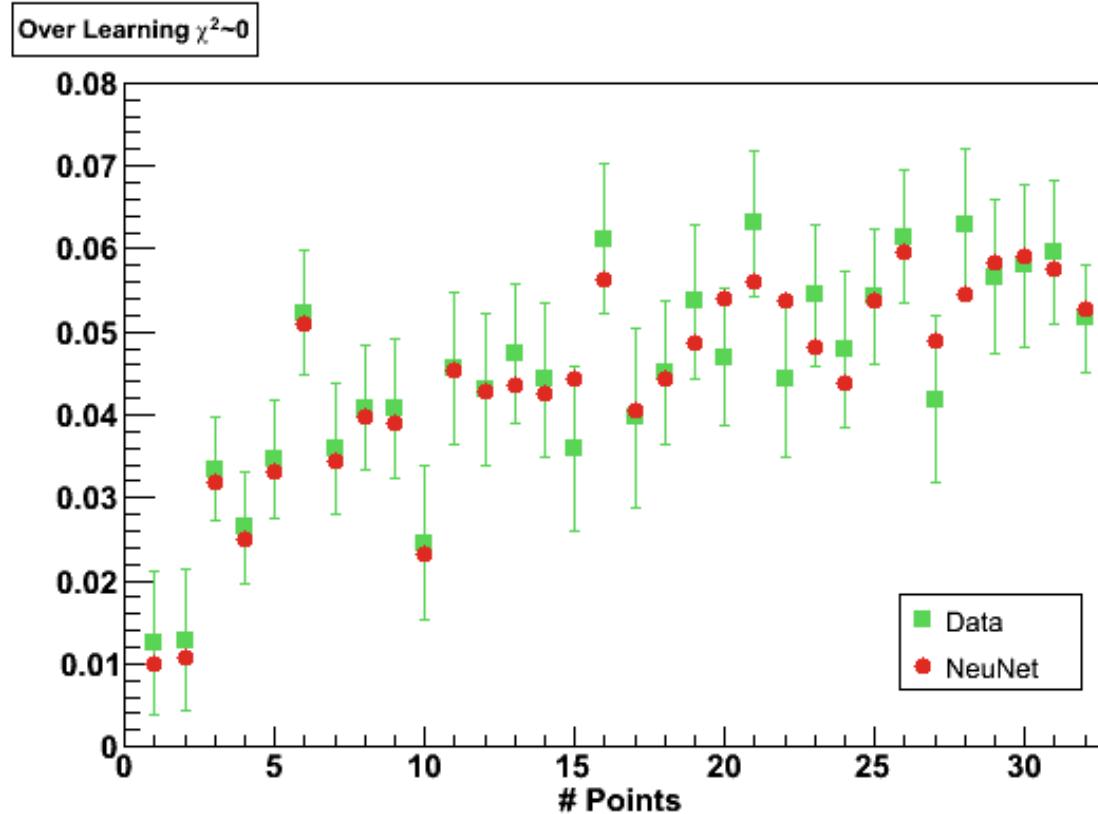
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## OVERLEARNING



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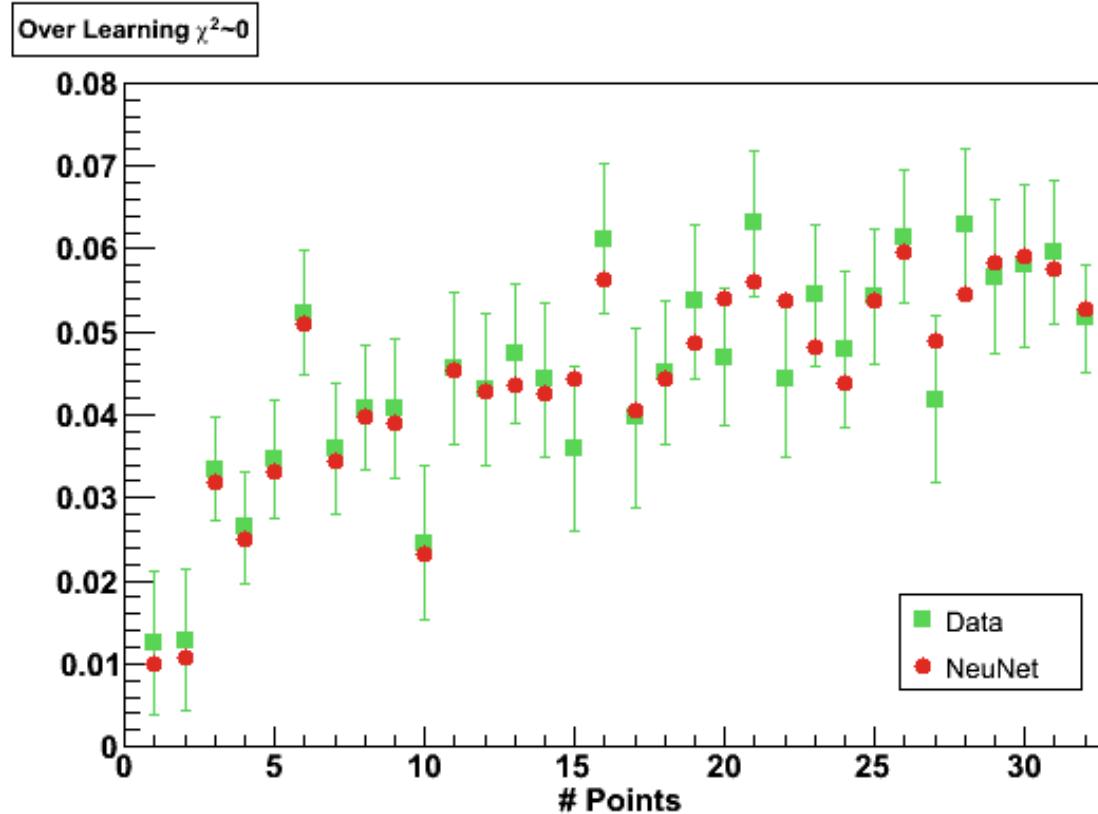
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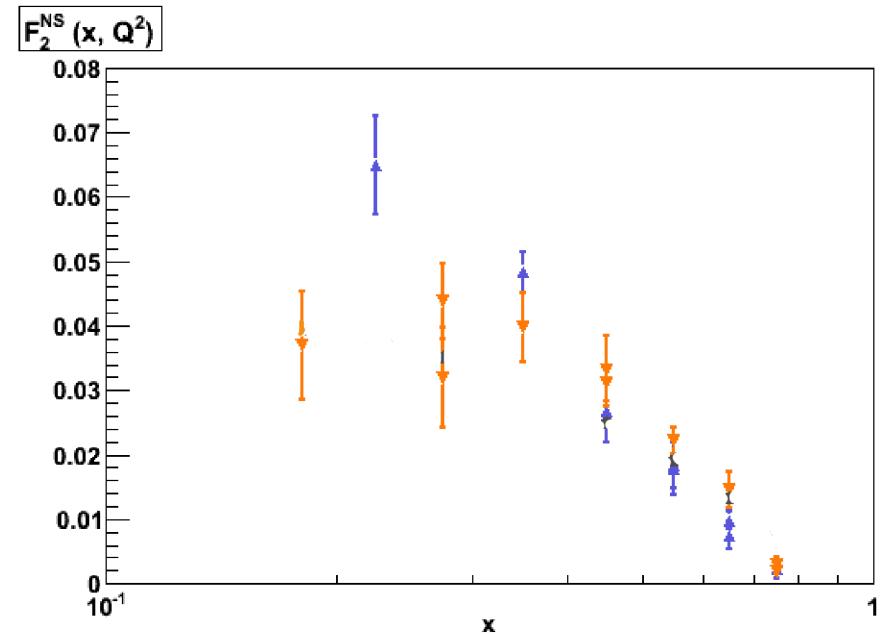
A: STOP THE FIT BEFORE OVERLEARNING SETS IN!

# THE STOPPING CRITERION

MINIMIZE BY GENETIC ALGORITHM:

AT EACH GENERATION, THE  $\chi^2$  EITHER UNCHANGED OR DECREASING

- DIVIDE THE DATA IN TWO SETS: TRAINING AND VALIDATION
- MINIMIZE THE  $\chi^2$  OF THE DATA IN THE TRAINING SET
- AT EACH ITERATION, COMPUTE THE  $\chi^2$  FOR THE DATA IN THE VALIDATION SET (NOT USED FOR FITTING)
- WHEN THE VALIDATION  $\chi^2$  STOPS DECREASING, STOP THE FIT



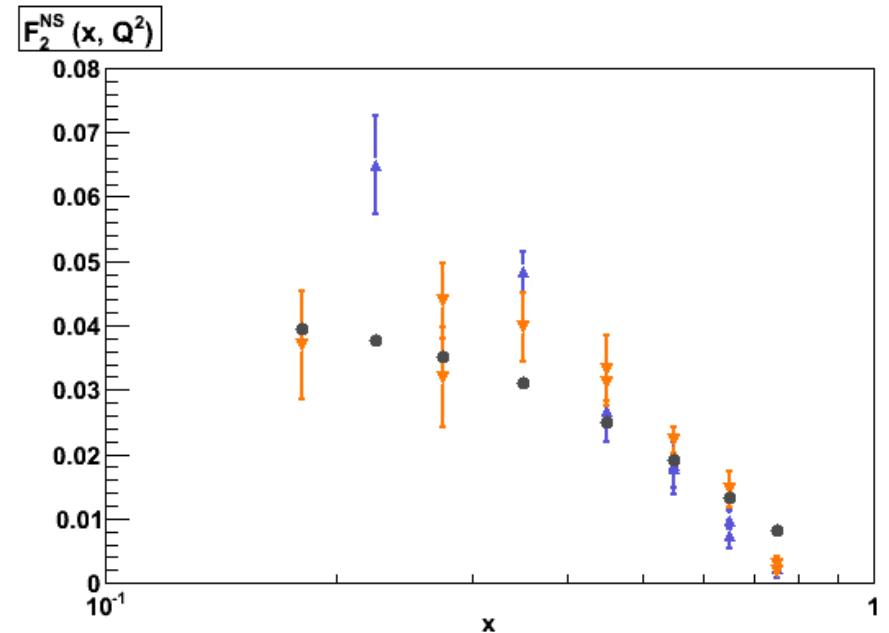
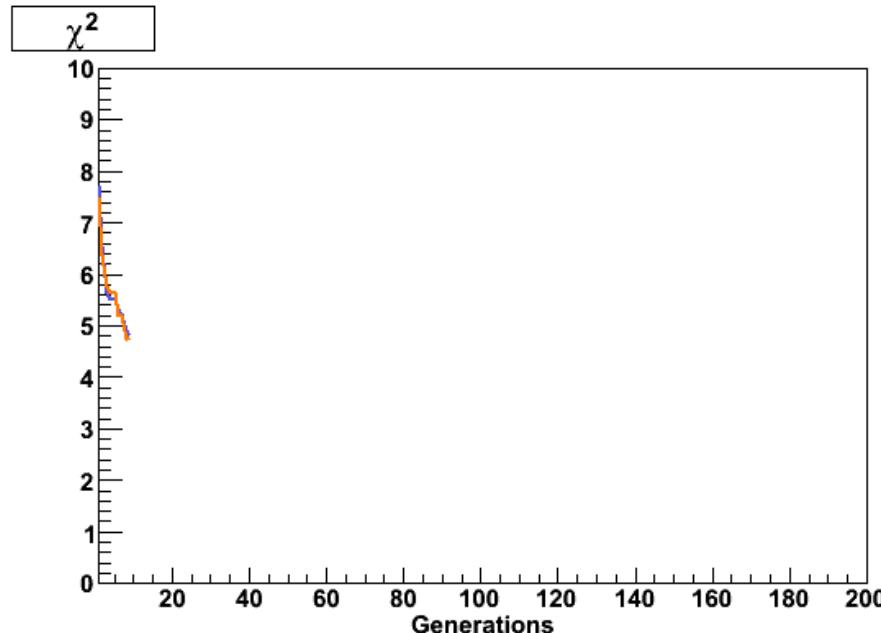
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GO!



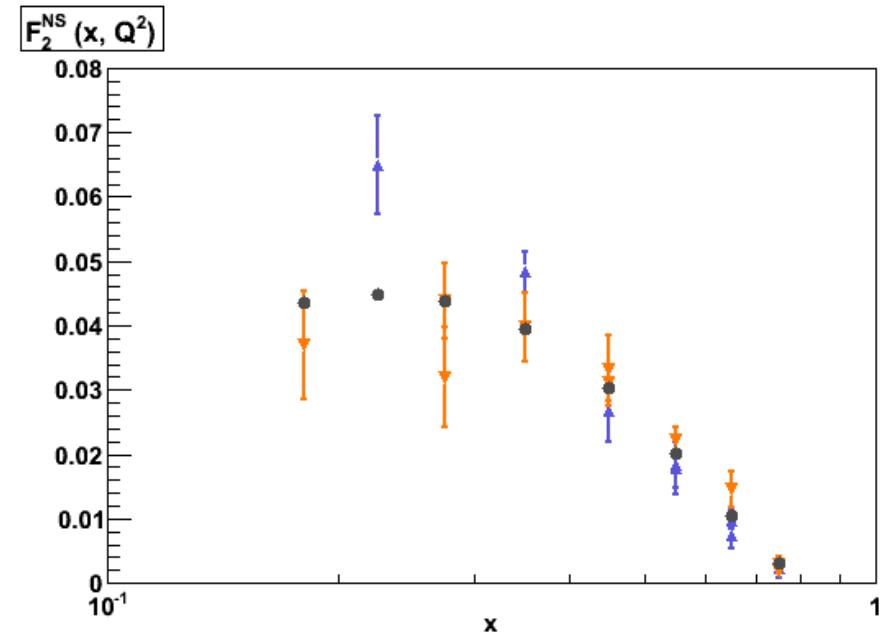
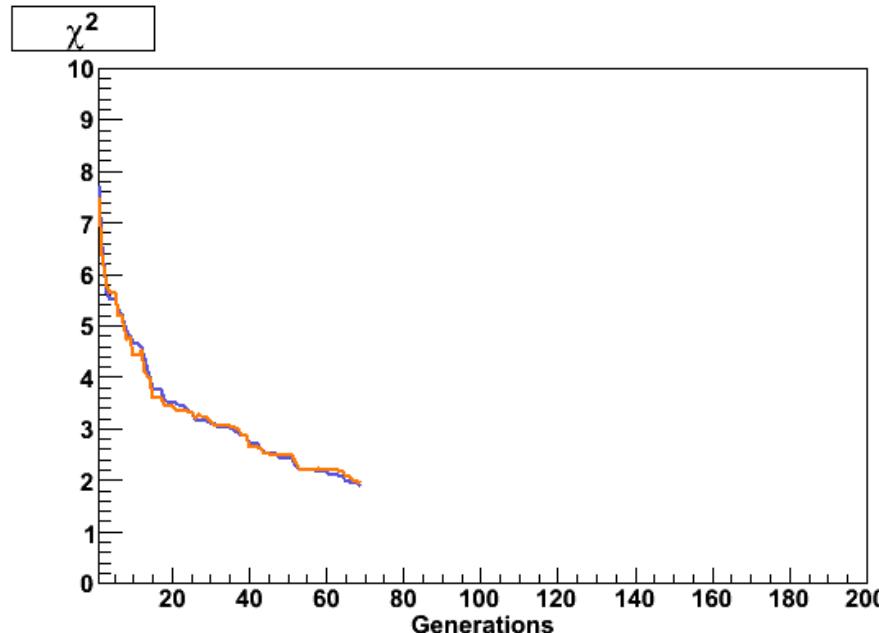
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STOP!



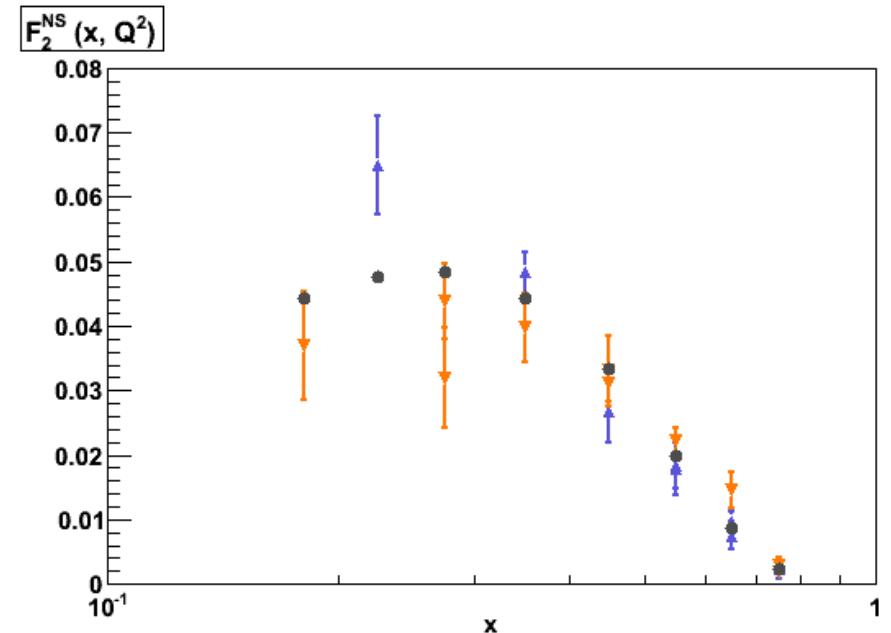
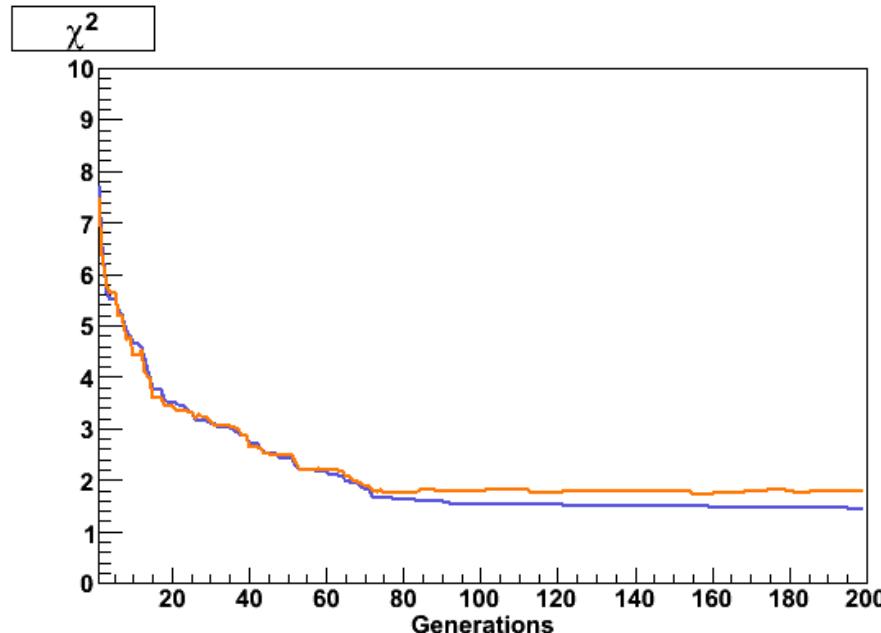
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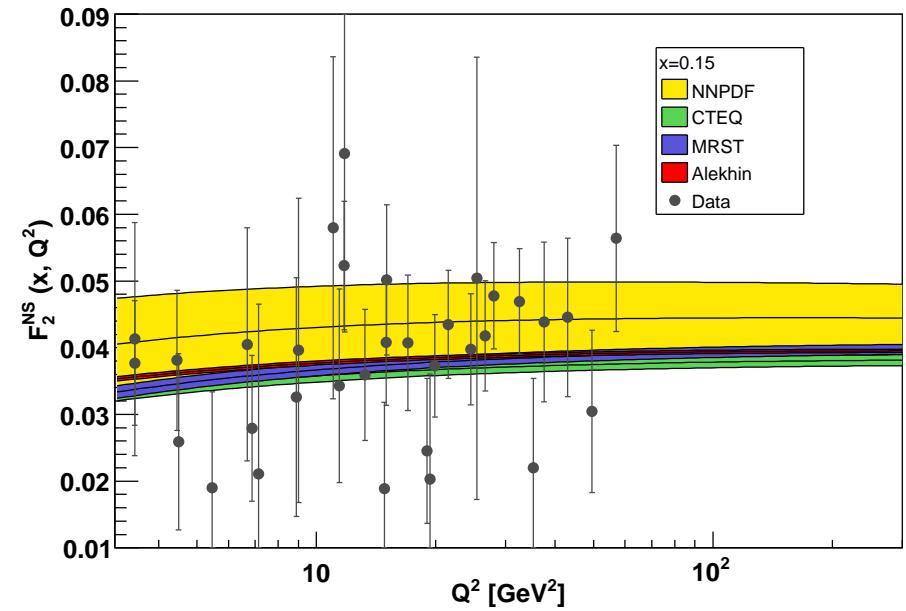
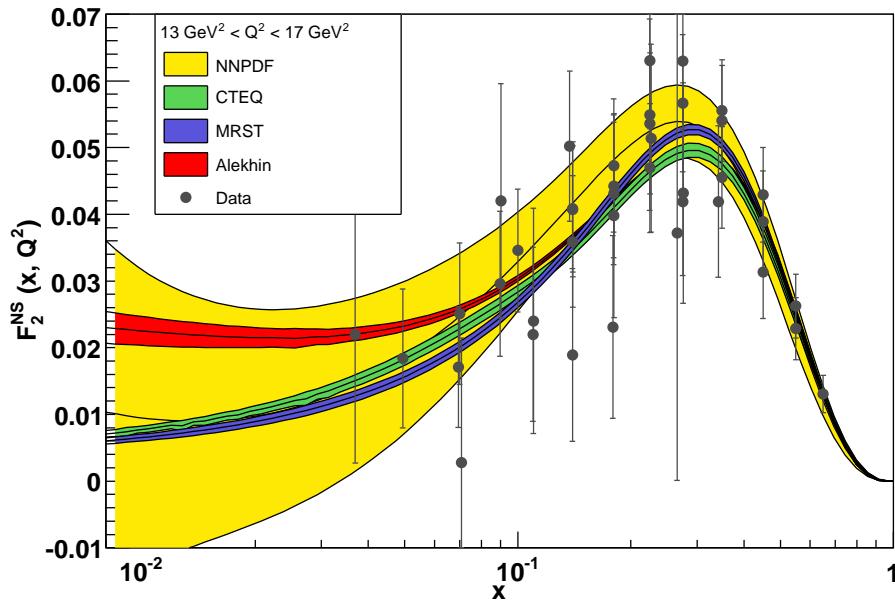
TOO LATE!



# A NEURAL NONSINGLET FIT<sub>(NNPDF 2007)</sub>

<http://sophia.ecm.ub.es/nqnns/>

NLO RESULTS: THE STRUCTURE FUNCTION  $F_2^{\text{NS}}(x, Q^2)$   
 VS  $x$  AT  $Q^2 = 15 \text{ GeV}^2$       VS  $Q^2$  AT  $x = 0.15$



- COMPATIBLE WITH EXISTING FITS WITHIN ERROR  
 (even when they disagree with each other)
- UNCERTAINTY MUCH LARGER IN EXTRAPOLATION BUT ALSO IN DATA REGION  
 (note no other global fit data constrain  $q_{\text{NS}}$ )
- CENTRAL FIT DISAGREES WITH EXISTING FITS IN VALENCE REGION  
 $0.1 \leq x \leq 0.3$

# STABILITY

CAN CHECK STABILITY BY COMPARING RESULTS IF THE WHOLE PROCEDURE IS REPEATED WITH A DIFFERENT SET OF REPLICAS

DEFINE R.M.S. DISTANCE  $\langle d[q] \rangle = \sqrt{\left\langle \frac{(\langle q_i \rangle_{(1)} - \langle q_i \rangle_{(2)})^2}{\sigma^2[q_i^{(1)}] + \sigma^2[q_i^{(2)}]} \right\rangle_{\text{dat}}}$

NOTE  $\sigma \Rightarrow$  ERROR ON AVERAGE = (ERROR ON  $q_i$ ) /  $\sqrt{N}$

$\Rightarrow$  TESTS BOTH ACCURACY OF CENTRAL VALUE & ERRORS

SELF-STABILITY:  
DIFFERENT SETS OF 100 REPLICAS

$\langle d[q] \rangle_{\text{dat}}$	0.96
$\langle d[q] \rangle_{\text{extra}}$	0.99
$\langle d[\sigma_q] \rangle_{\text{dat}}$	0.88
$\langle d[\sigma_q] \rangle_{\text{extra}}$	0.97

CHANGE OF ARCHITECTURE:  
2-4-3-1 VS. 2-5-3-1

$\langle d[q] \rangle_{\text{dat}}$	0.9
$\langle d[q] \rangle_{\text{extra}}$	0.9
$\langle d[\sigma_q] \rangle_{\text{dat}}$	0.9
$\langle d[\sigma_q] \rangle_{\text{extra}}$	1.4

DISTANCE COMPUTED FOR 14 POINTS LINEARLY SPACED IN THE DATA REGION ( $0.05 \leq x \leq 0.75$ )  
& 14 POINTS LOG SPACED IN THE EXTRAPOLATION REGION ( $10^{-3} \leq x \leq 10^{-2}$ )

# TOWARDS THE LHC

# WHERE DO WE STAND NOW?

## WHAT WE HAVE LEARNT

- LIGHT QUARK STRUCTURE IN  
“VALENCE” REGION  $0.1 \lesssim x \lesssim 0.5$  (old fixed target dis data)
- SINGLET AND GLUON AT SMALL  $x < 10^{-2}$  (HERA)
- SEA ASYMMETRY AT MEDIUM  $x \sim 0.1 \div 0.2$  (Drell-Yan)
- HINTS ON STRANGENESS (neutrinos)

## WHAT WE ARE STILL MISSING

- GLUONS AT LARGE  $x$  (large  $E_T$  jet problem)
- NONSINGLET & VALENCE AT SMALL  $x$
- DETAILED INFO ON STRANGENESS (NuTeV problem)
- INFO ON HEAVY QUARKS (small  $x$  W xsect problem)

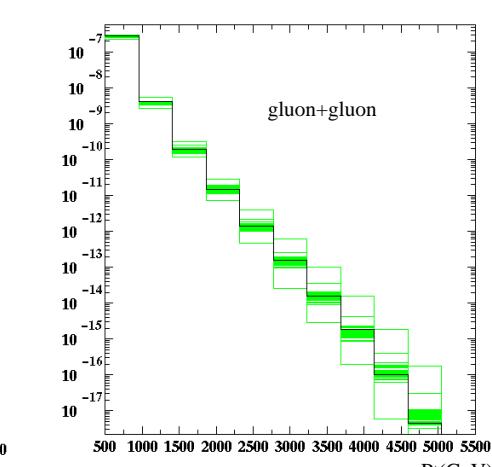
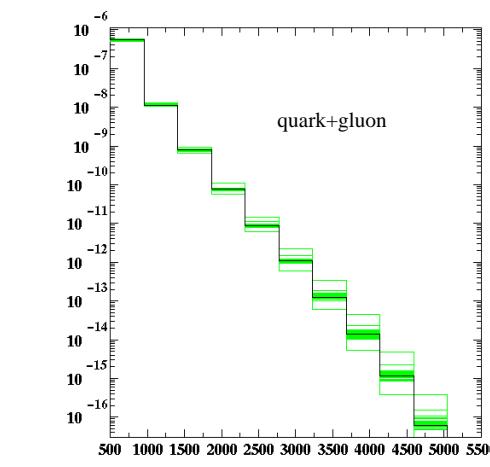
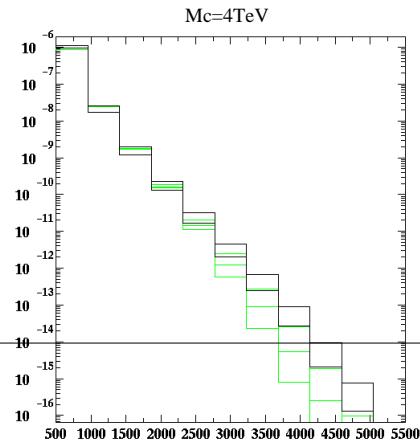
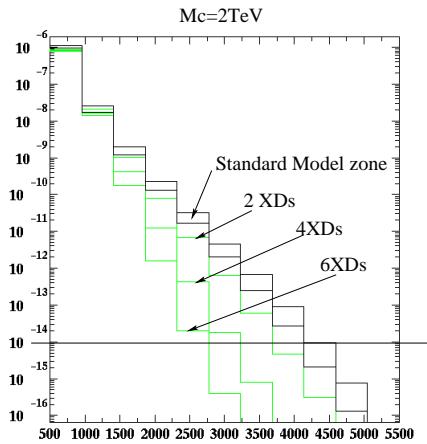
# IS IT A PROBLEM?

**EXAMPLE:** LACK OF KNOWLEDGE OF LARGE  $x$  GLUON LIMITS DISCOVERY POTENTIAL FOR EXTRA DIMENSIONS

UPPER LIMIT ON COMPACTIFICATION SCALE FROM DIJET CROSS SECTIONS  
FROM  $100 \text{ fb}^{-1}$  AT LHC Ferrag (ATLAS, 2006)

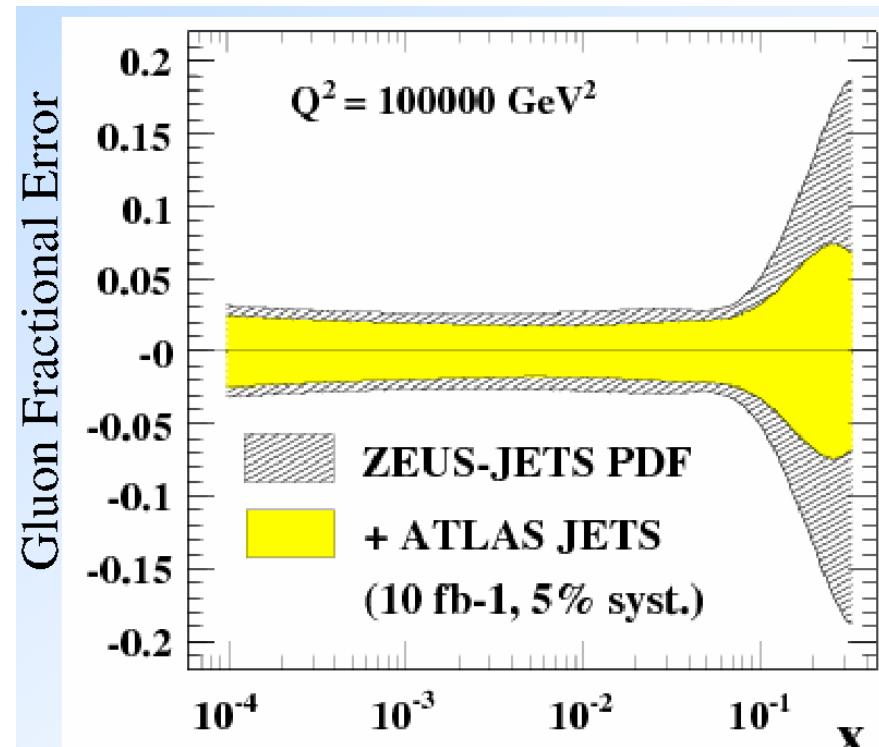
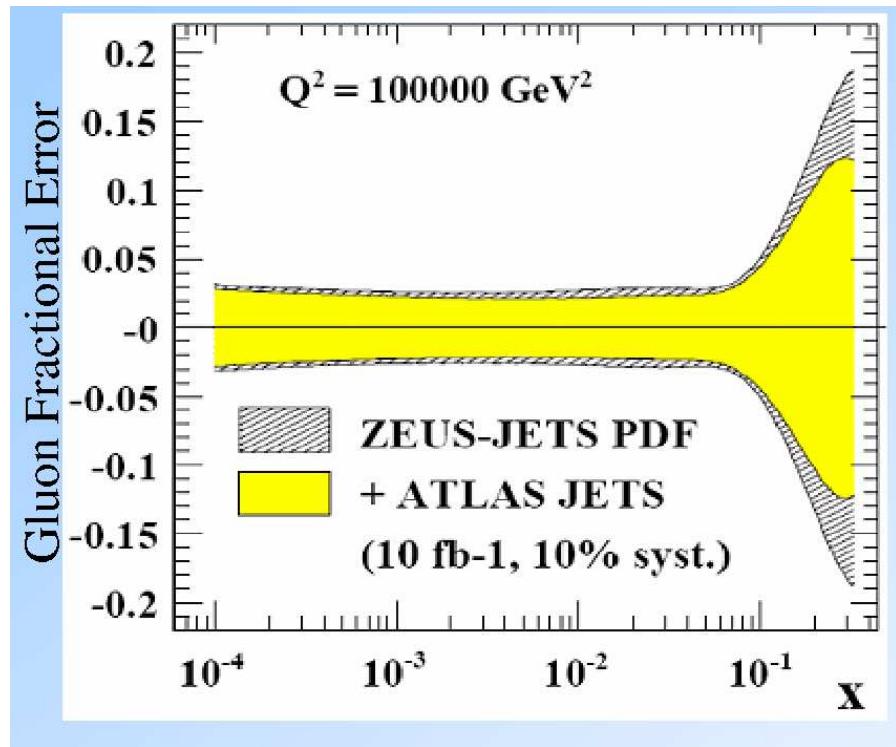
	2 extra dimensions	4 extra dimensions	6 extra dimensions
THEORETICALLY	5 TeV	5 TeV	5 TeV
INCLUDING PDF UNCERTAINTIES	< 2 TeV	< 3 TeV	< 4 TeV

CROSS-SECTION IN FIXED  $p_t$  BINS  
EXTRA DIMENSIONS VS STANDARD MODEL



# SOLUTIONS: LARGE $E_T$ JETS @ LHC DETERMINING THE GLUON AT LARGE $x$

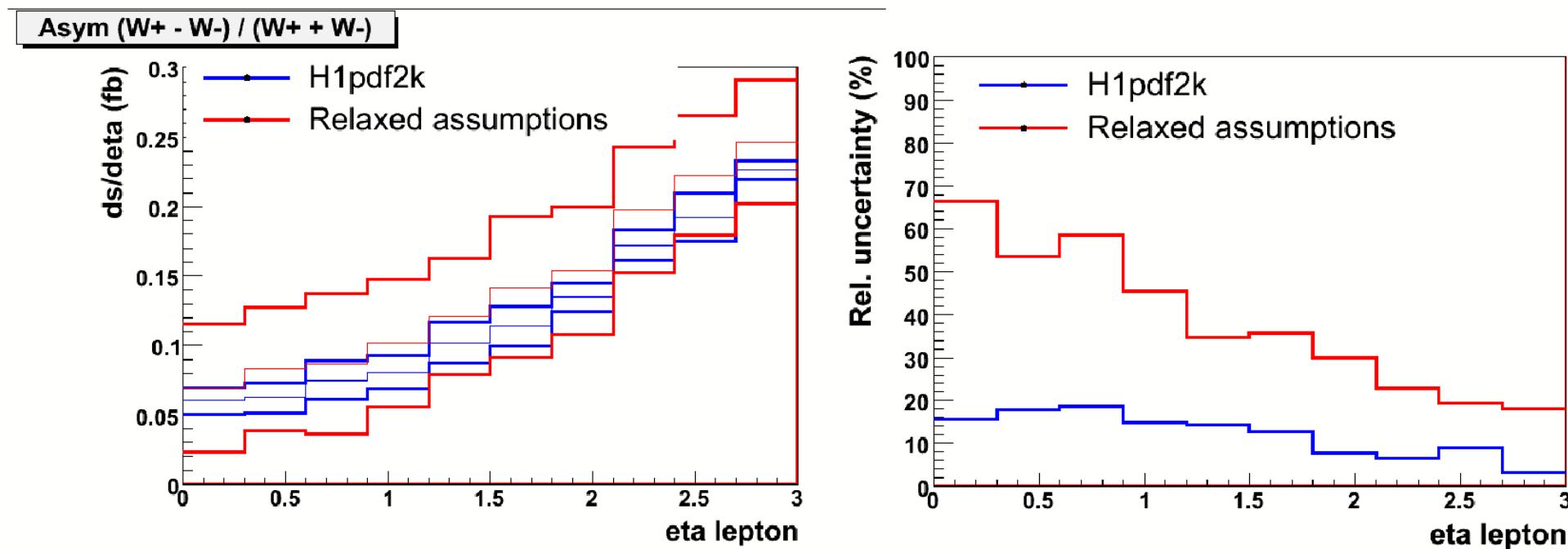
UNCERTAINTY IN THE GLUON GREATLY REDUCED  
PROVIDED SYSTEMATICS CAN BE KEPT AT FEW PERCENT LEVEL



D. Clements (Atlas 2006)

# SOLUTIONS: $W$ ASYMMETRY @ LHC DETERMINING QUARKS AT SMALL $x$

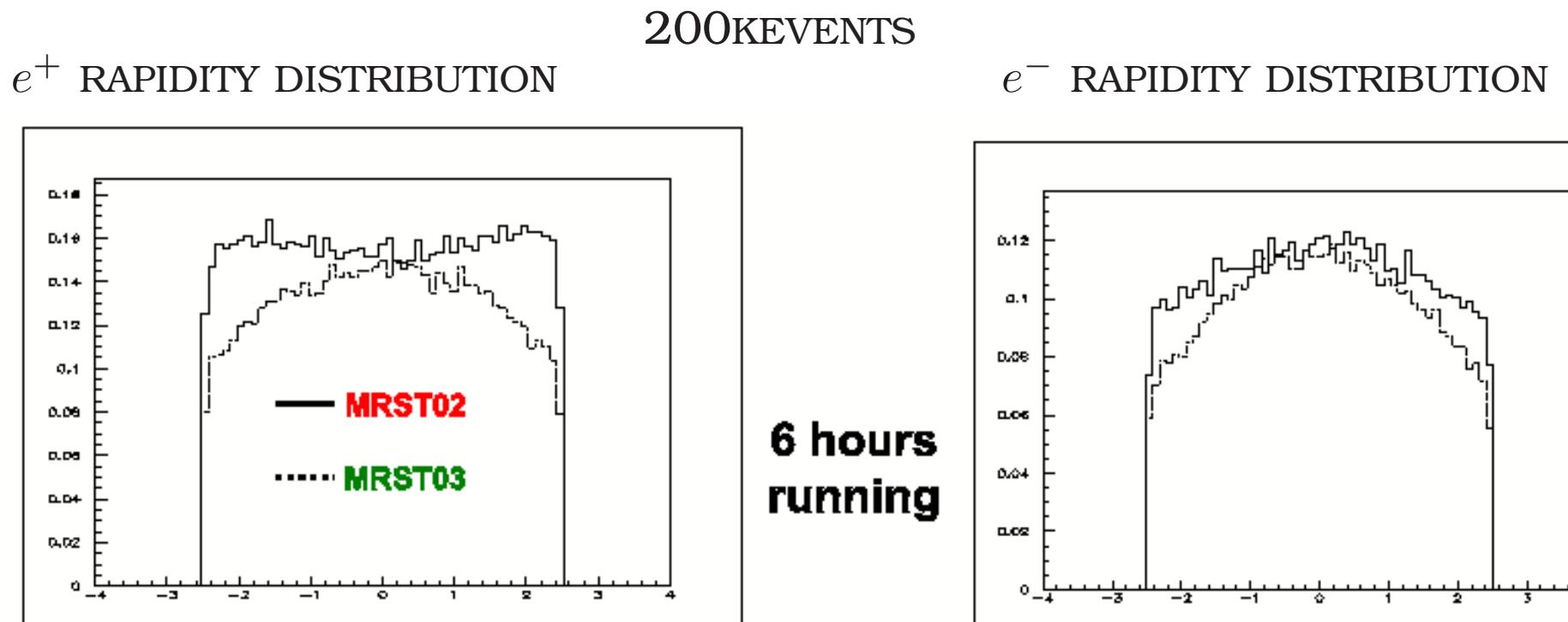
- $W$  PRODUCTION AT LHC PROBES  $x \sim 10^{-2}$
- $W^\pm$  ASYMMETRIES SENSITIVE TO  $\bar{u}/\bar{d}$
- $\Rightarrow$  IF SMALL  $x$  BEHAVIOUR IS NOT AS CURRENTLY ASSUMED (“REGGE”),  $W^\pm$  ASYMMETRY CHANGES BY UP TO FACTOR 5!



E. Perez (CMS 2006)

## SOLUTIONS: $W$ DISTRIBUTION @LHC PRECISION PHYSICS @ SMALL $x$

- MRST03  $\Rightarrow$  BEST FIT VS.  
MRST02  $\Rightarrow$  VERY SMALL  $x$  HERA DATA NOT INCLUDED
- DIFFERENCE IN ASYMETRY SEEN AFTER FEW HOURS OF RUNNING!



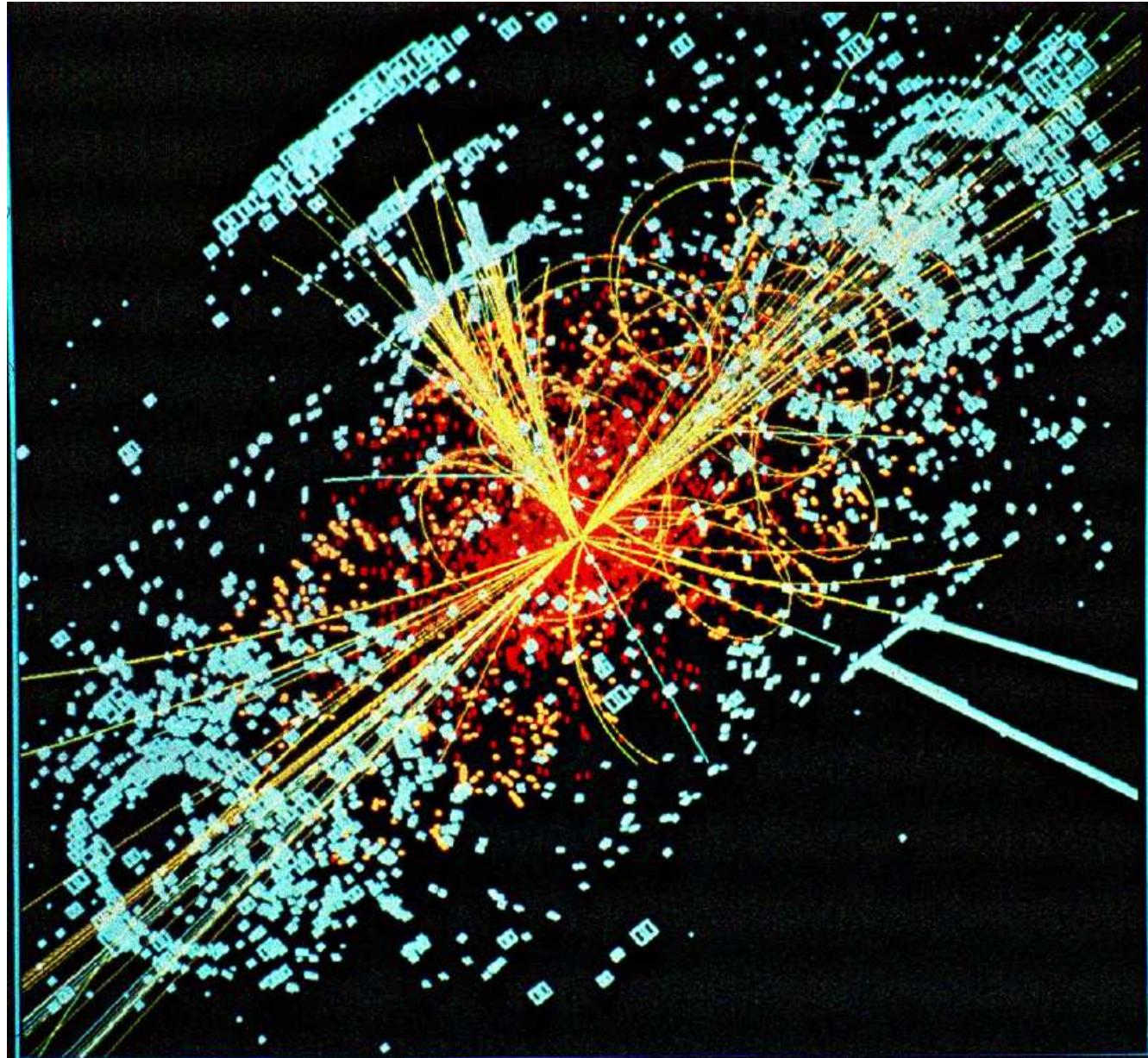
A. Cooper-Sarkar (Atlas 2006)

# CONCLUSIONS

AT LHC, WE NEED PRECISION PHYSICS

- SOME WE'VE GOT ALREADY (small  $x$  gluons, large  $x$  valence quarks, . . . )
- SOME WE'LL MEASURE AT LHC ITSELF (small  $x$  quarks, large  $x$  gluons, . . . )

FOR DISCOVERY PHYSICS!



Higgs decay in  $e^+e^- + 2$  jets at CMS