PARTON DISTRIBUTIONS FOR THE LHC

Stefano Forte
Università di Milano

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SUMMARY

- WHAT ARE PARTON DISTRIBUTIONS GOOD FOR?

  FACTORIZATION

- HOW DO WE DETERMINE THEM?

  DEEP-INELASTIC SCATTERING AND MORE

- ARE PARTON DISTRIBUTIONS AN ISSUE?

  LHC

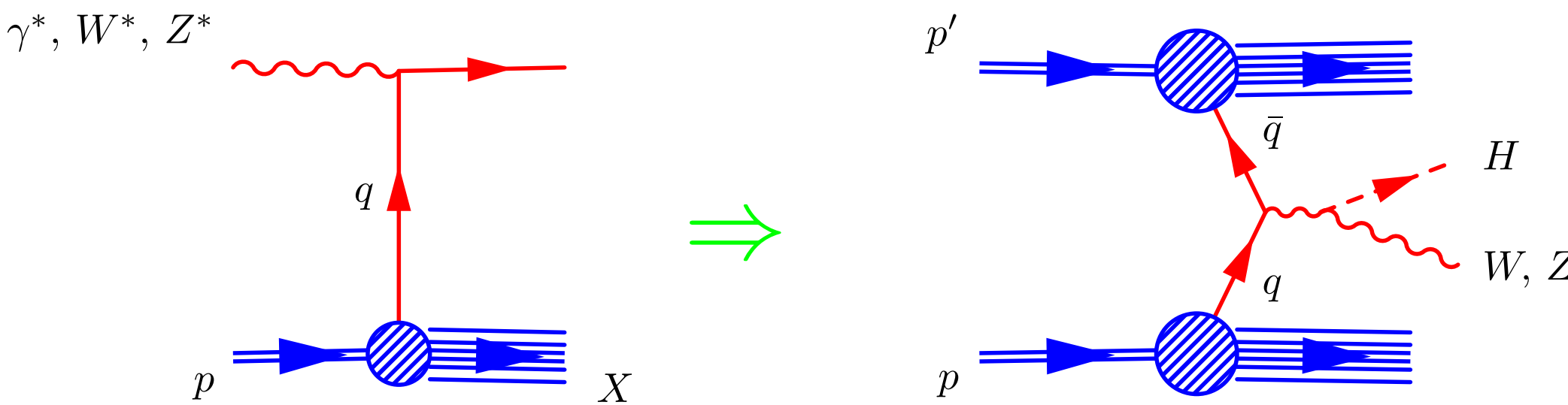
- HOW DO WE ADDRESS THIS ISSUE?

  THE STATE OF THE ART AND BEYOND

MOTIVATION:

THE ACCURATE COMPUTATION OF PHYSICAL PROCESS AT A HADRON COLLIDER REQUIRES GOOD KNOWLEDGE OF PARTON DISTRIBUTIONS OF THE NUCLEON

FACTORIZATION...



IN ORDER TO EXTRACT THE RELEVANT PHYSICS SIGNAL,

WE NEED TO KNOW THE PARTON DISTRIBUTIONS AND THEIR UNCERTAINTY

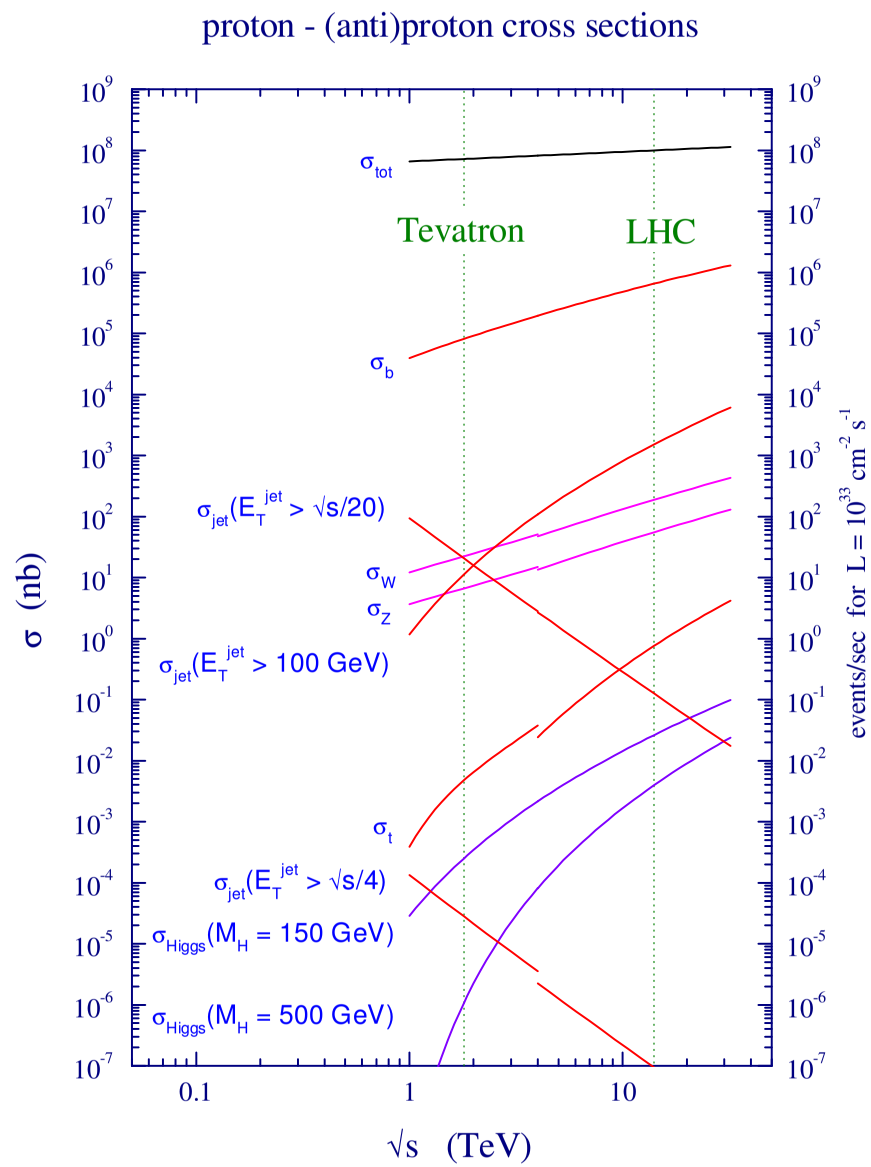
. . . FOR LHC PROCESSES

$$\sigma_X(p_1, p_2; Q, \{\ldots\}) =$$

$$= \sum_{a,b} \int_{x_{\min}}^{1} dx_1\, dx_2\, f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2)$$

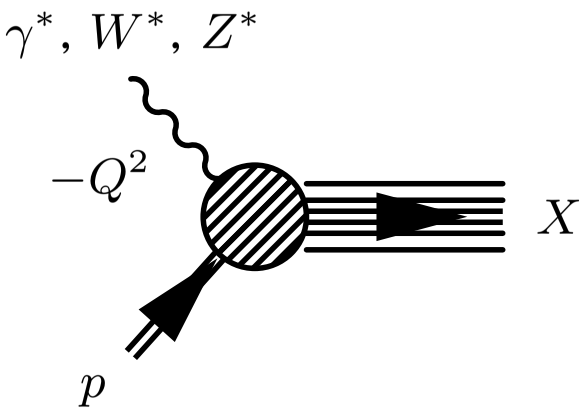
$$\times \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2; Q, \{\ldots\}; \mu_F^2)$$

- σ_X $\Rightarrow$ HADRONIC XSECT TO W, Z, H, JETS, . . .
- $\hat{\sigma}$ $\Rightarrow$ PERTURBATIVELY COMPUTABLE PARTON XSECT (NLO, RESUMMED, . . .)
- f_{a/h_1} $\Rightarrow$ DISTRIBUTION OF PARTON a IN HADRON h_1



DEEP-INELASTIC SCATTERING

STRUCTURE FUNCTIONS...



Lepton fractional energy loss: $y = \frac{p \cdot q}{p \cdot k}$;

Bjorken x: $x = \frac{Q^2}{2p \cdot q}$

lepton-nucleon CM energy: $s = \frac{Q^2}{xy}$;

virtual boson-nucleon CM energy $W^2 = Q^2 \frac{1-x}{x}$;

$$\frac{d^2\sigma^{\lambda_p \lambda_\ell}(x,y,Q^2)}{dxdy} = \frac{G_F^2}{2\pi(1+Q^2/m_W^2)^2}\frac{Q^2}{xy}\Bigg\{\Big[-\lambda_\ell\, y\left(1-\frac{y}{2}\right)xF_3(x,Q^2) + (1-y)F_2(x,Q^2)$$

$$+y^2xF_1(x,Q^2)\Big] - 2\lambda_p\left[-\lambda_\ell\, y(2-y)xg_1(x,Q^2) - (1-y)g_4(x,Q^2) - y^2xg_5(x,Q^2)\right]\Bigg\}$$

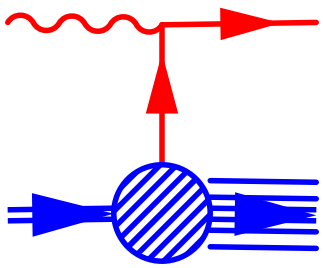
$\lambda_l \longrightarrow$ lepton helicity

$\lambda_p \longrightarrow$ proton helicity

	PARITY CONS.	PARITY VIOL.
UNPOL.	F_1, F_2	F_3
POL.	g_1	g_4, g_5

...and Parton Distributions

Structure Function=Hard coeff.$\otimes$parton distn.



$$F_2^{\rm NC}(x,Q^2) = x \sum_{\text{flav. } i} e_i^2 (q_i + \bar{q}_i) + \alpha_s \left[C_i[\alpha_s] \otimes (q_i + \bar{q}_i) + C_g[\alpha_s] \otimes g \right]$$

q_i quark, $\bar{q}_i$ antiquark, g gluon

Leading Parton Content (up to $O[\alpha_s]$ corrections)

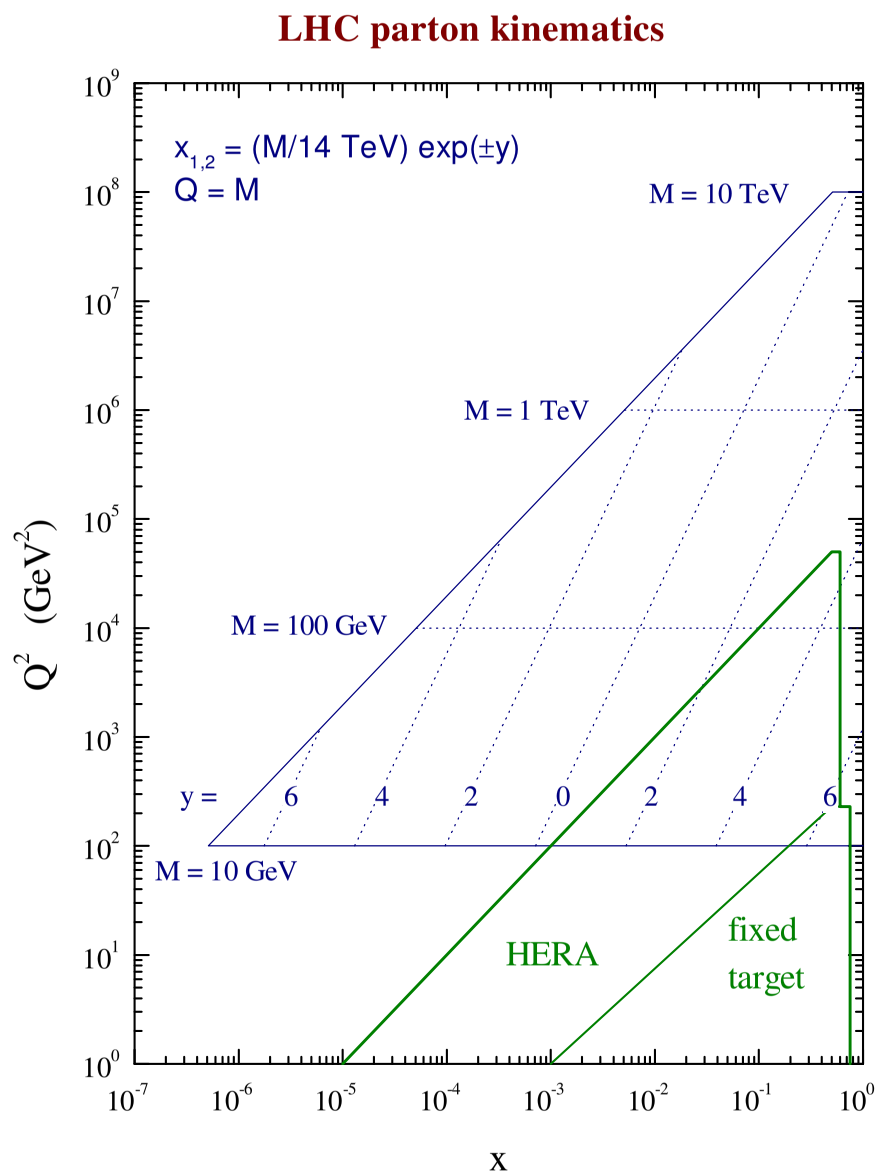
	$q_i \equiv q_i^{\uparrow\uparrow} + q_i^{\uparrow\downarrow}$	$\Delta q_i \equiv q_i^{\uparrow\uparrow} - q_i^{\uparrow\downarrow}$
NC	${F_1}^{\gamma,Z} = \sum_i e_i^2 (q_i + \bar{q}_i)$	${g_1}^{\gamma,Z} = \sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i)$
CC	$F_1^{W^+} = \bar{u} + d + s + \bar{c}$	$g_1^{W^+} = \Delta\bar{u} + \Delta d + \Delta s + \Delta\bar{c}$
CC	$-F_3^{W^+}/2 = \bar{u} - d - s + \bar{c}$	$g_5^{W^+} = \Delta\bar{u} - \Delta d - \Delta s + \Delta\bar{c}$
	$F_2 = 2xF_1$	$g_4 = 2xg_5$

$W^+ \to W^- \Rightarrow u \leftrightarrow d,\ c \leftrightarrow s$; more combinations using Isospin: $p \to n \Rightarrow u \leftrightarrow d$

LHC KINEMATICS

AT A HADRON COLLIDER

- SCALE Q DETERMINED BY MASS OF FINAL STATE
- MOMENTUM FRACTIONS x_1 x_2 DETERMINED BY MASS & RAPIDITY OF FINAL STATE



LHC KINEMATICS ⇒ EVOLUTION

AT A HADRON COLLIDER

- SCALE Q DETERMINED BY MASS OF FINAL STATE
- MOMENTUM FRACTIONS x_1 x_2 DETERMINED BY MASS & RAPIDITY OF FINAL STATE

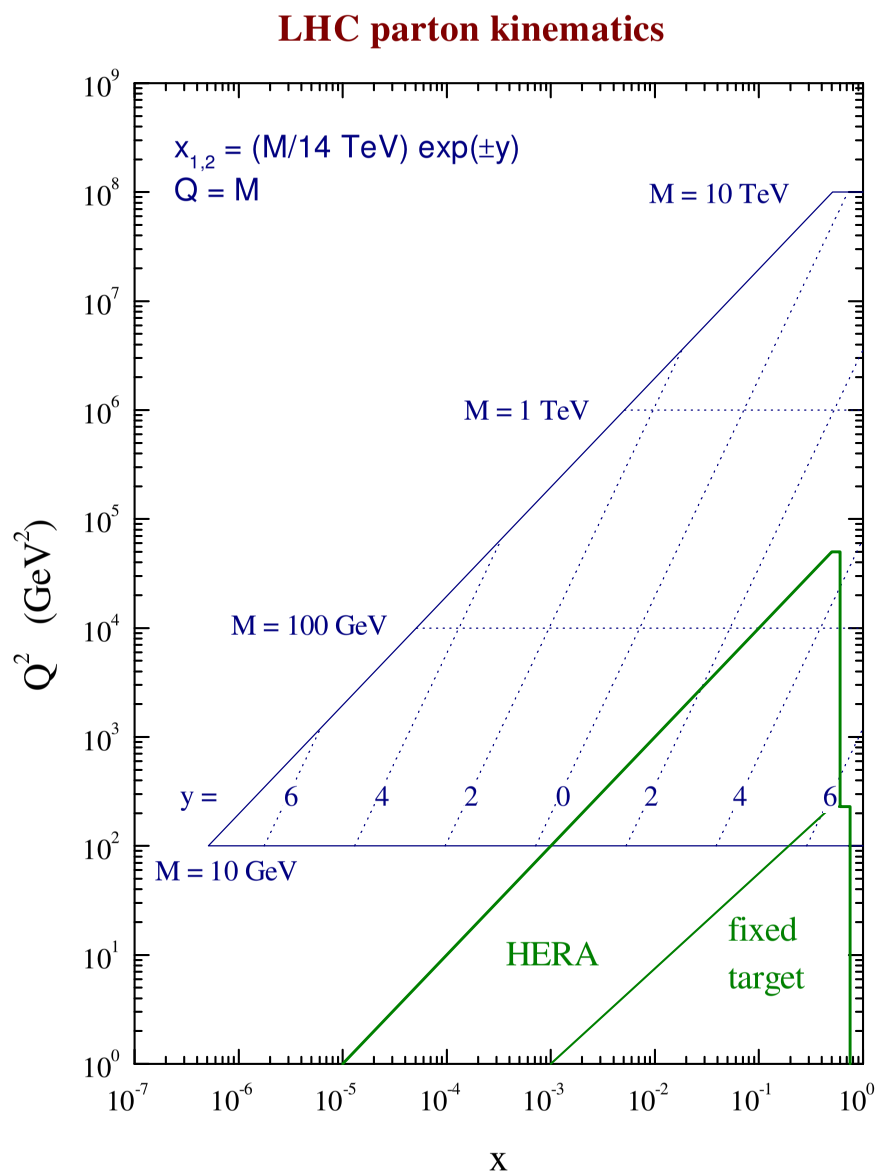
USE ALTARELLI-PARISI EQNS:

$$\frac{d}{dt}\Delta q_{NS} = \frac{\alpha_s(t)}{2\pi} P_{qq}^{NS} \otimes \Delta q_{NS},$$

$$\frac{d}{dt}\begin{pmatrix}\Delta\Sigma \\ \Delta g\end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix} P_{qq}^S & 2n_f P_{qg}^S \\ P_{gq}^S & P_{gg}^S\end{pmatrix} \otimes \begin{pmatrix}\Delta\Sigma \\ \Delta g\end{pmatrix}$$

TO EVOLVE PARTONS

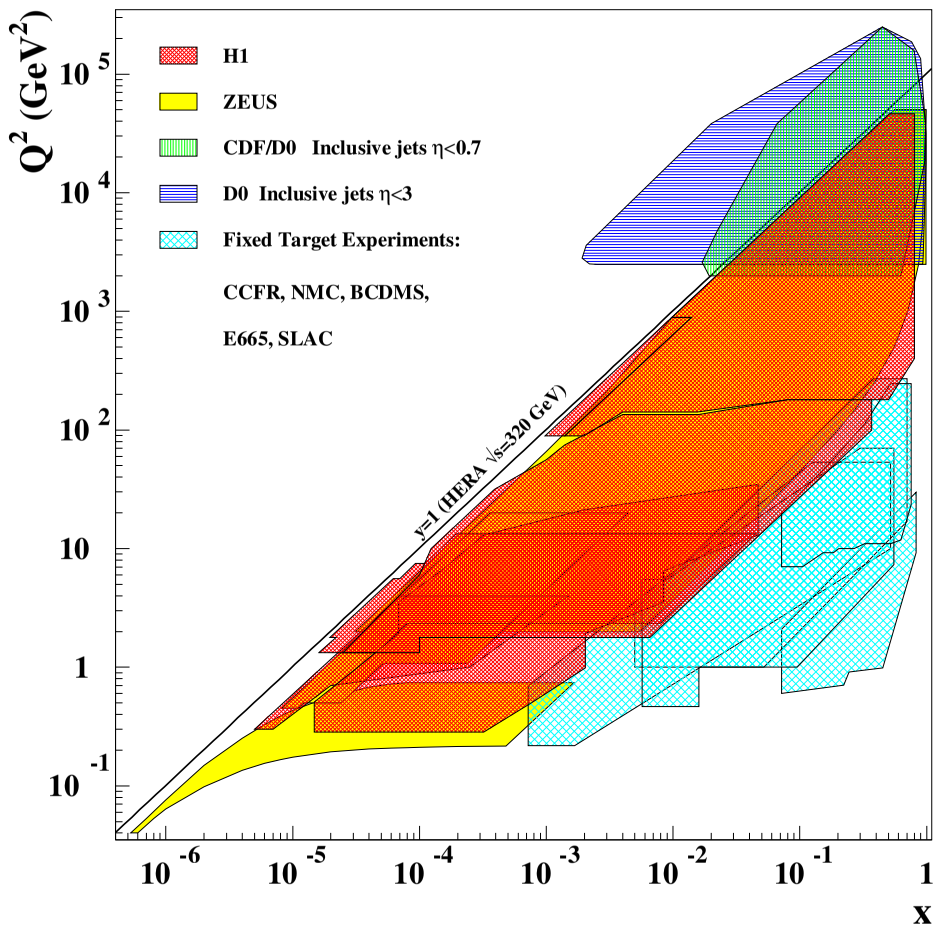
FROM DIS TO LHC KINEMATICS



THE NAME OF THE GAME

DIS DATA $\longrightarrow$ PARTON DISTRIBUTIONS

PROBLEMS:



- STRUCTURE FUNCTION (OR XSECT) IS A CONVOLUTION OVER x OF PARTON DISTNS. AND PERTURBATIVE CROSS SECTION
  $\longrightarrow$ MUST DECONVOLUTE

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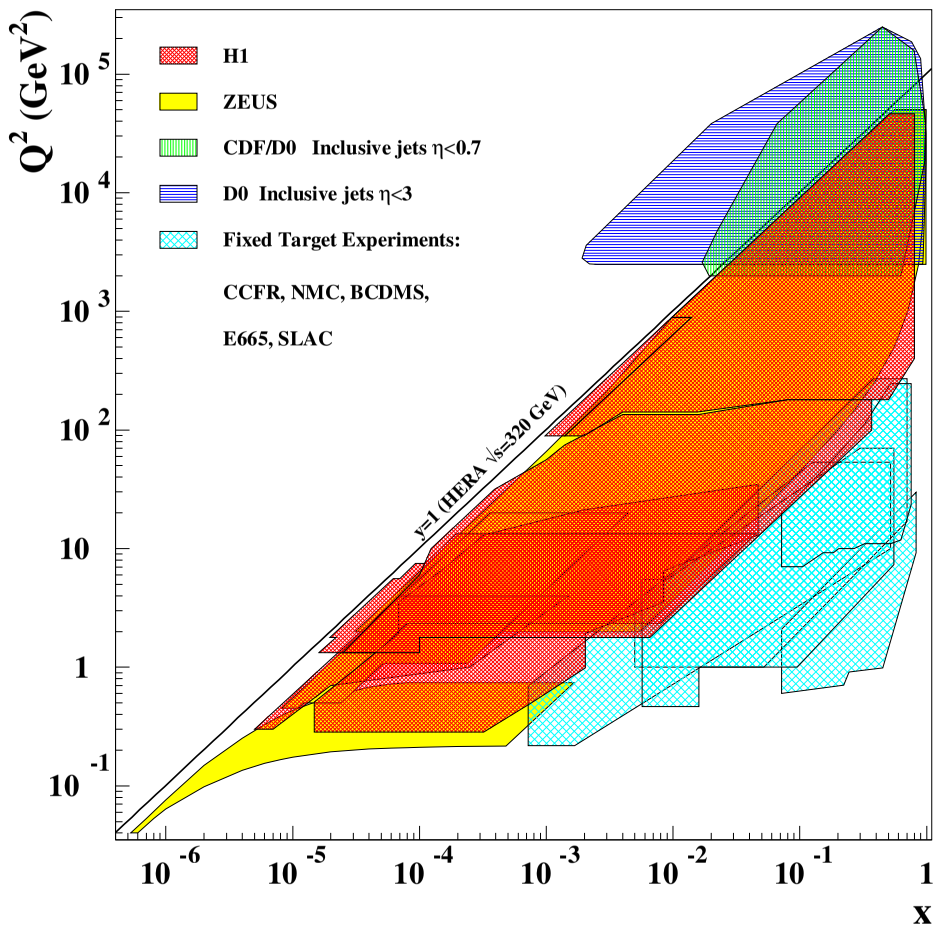

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- EACH STRUCTURE FUNCTION (OR XSECT) IS A LINEAR COMBINATION OF MANY PARTON DISTNS ($2N_f$ QUARKS + 1 GLUON)
  $\longrightarrow$ MUST COMBINE DIFFERENT PROCESSES

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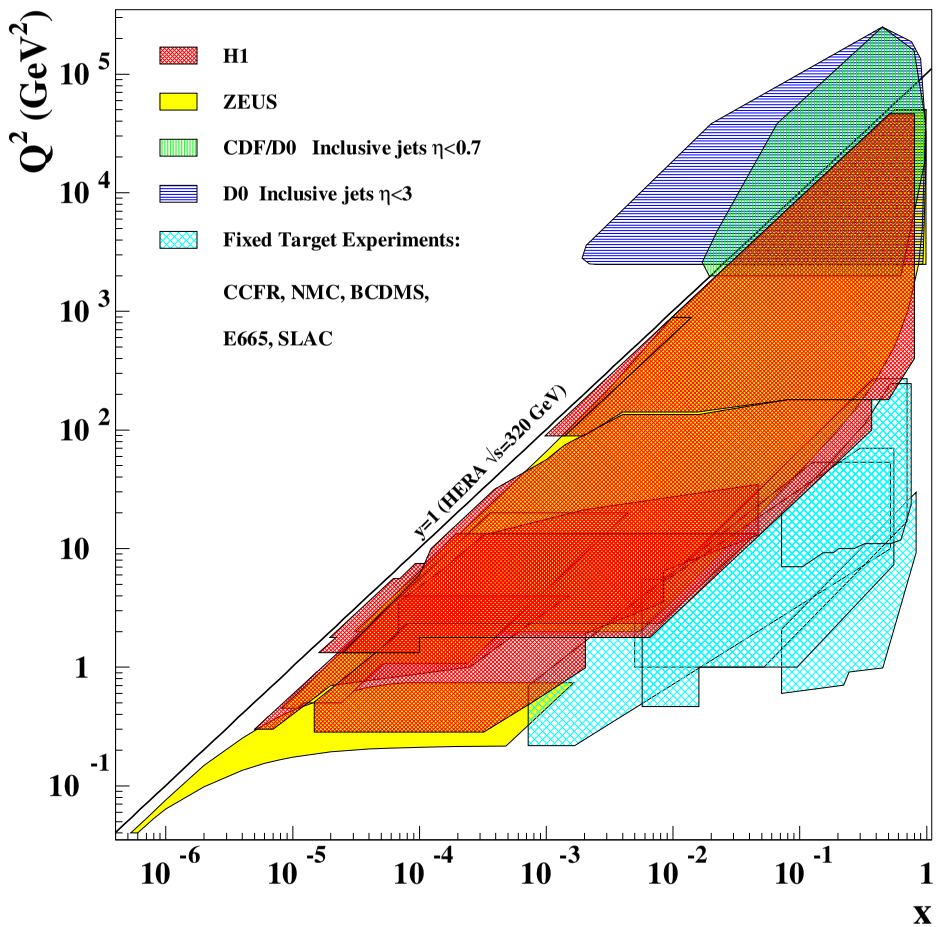

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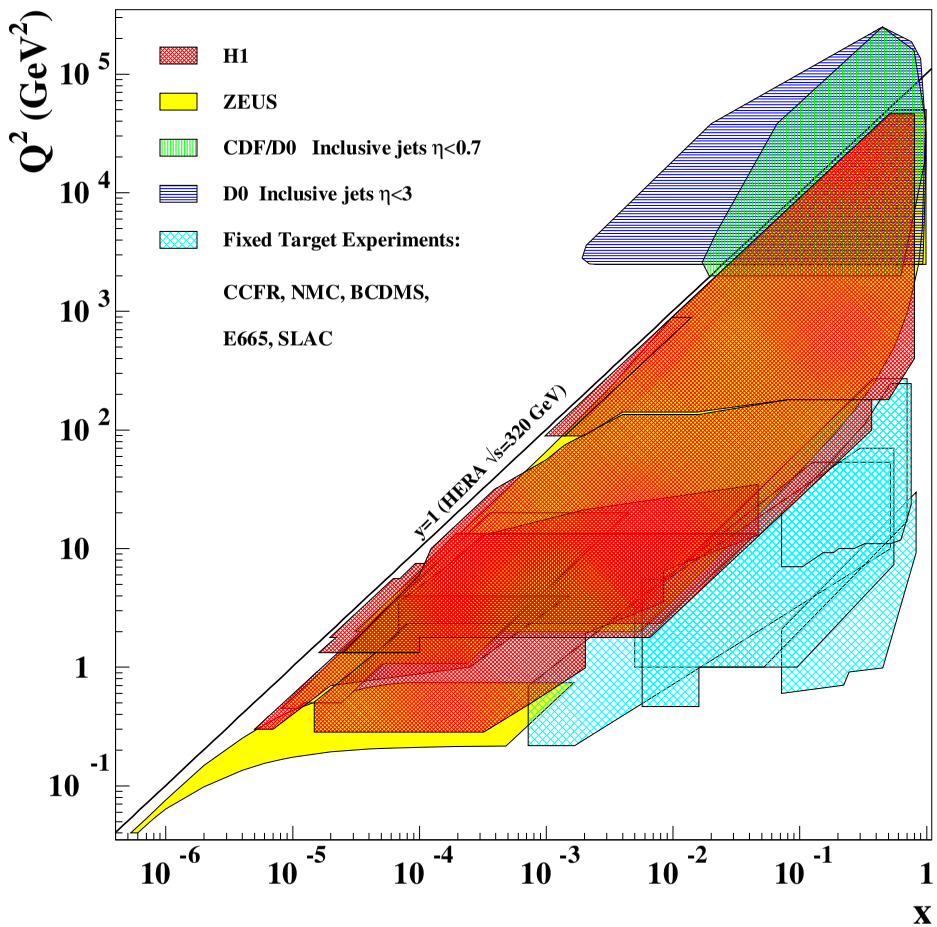

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  $\longrightarrow$ MUST EVOLVE
- TH UNCERTAINTIES: RESUMMATION, NUCLEAR CORRECTIONS, HIGHER TWIST, HEAVY QUARK THRESHOLDS. . .

DETERMINING THE GLUON

EVOLUTION:

SINGLET SCALING VIOLATIONS

$$\frac{d}{dt}F_2^s(N,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\left[\gamma_{qq}(N)F_2^s + 2\,n_f\gamma_{qg}(N)g(N,Q^2)\right] + O(\alpha_s^2)$$

$$F_2(N,Q^2) \equiv \int_0^1 dx\, x^{N-1}F_2(x,Q^2); \qquad \gamma_{ij}(N) \equiv \int_0^1 dx\, x^{N-1}P_{ij}(x,Q^2)$$

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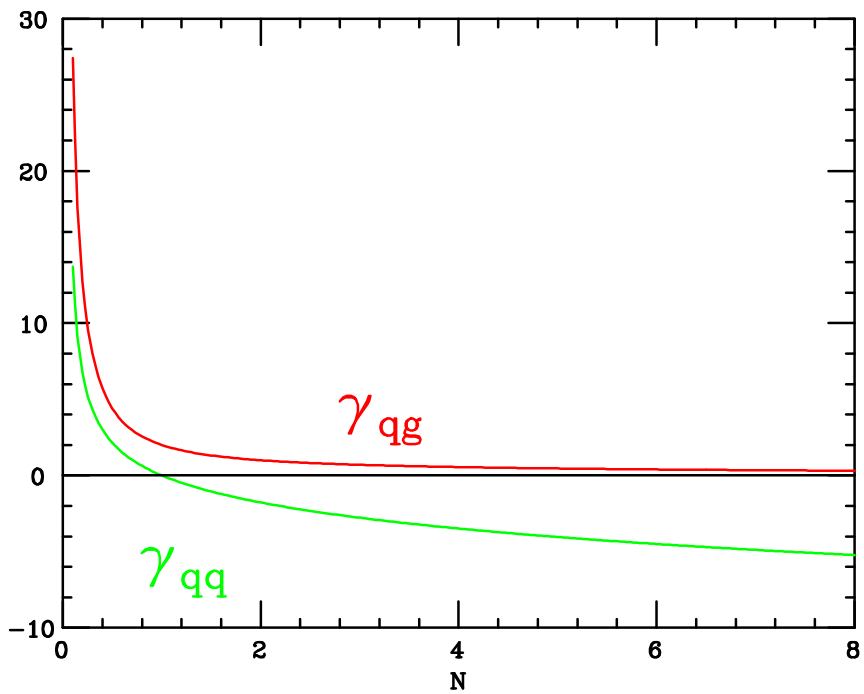
LARGE/SMALL X $\Leftrightarrow$ LARGE/SMALL N



AT SMALL N $\gamma_{qg} >> \gamma_{qq}$

AT SMALL x $\Rightarrow$ $F_2 \sim g$

DETERMINING THE GLUON

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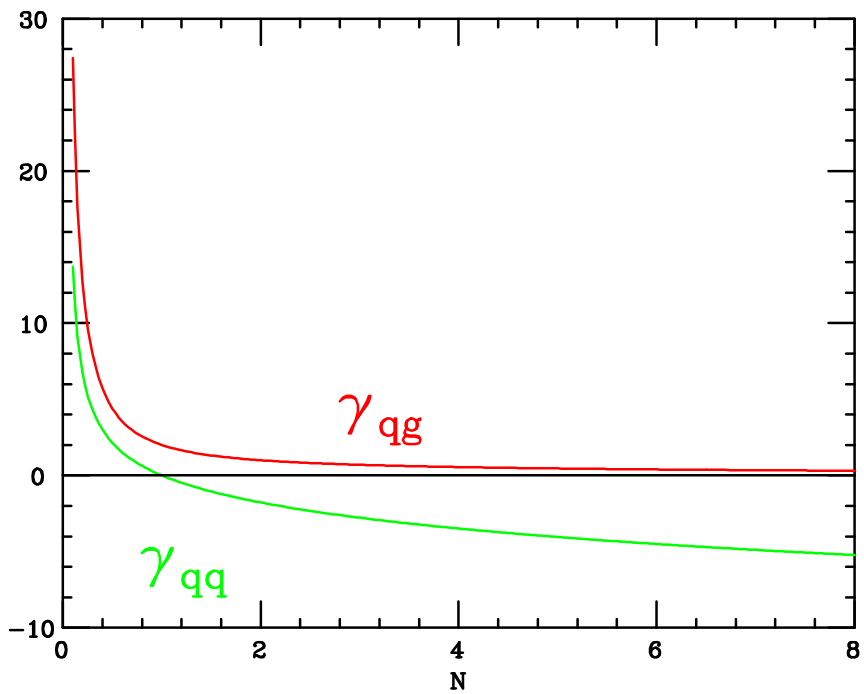


AT SMALL N $\gamma_{qg} >> \gamma_{qq}$

AT LARGE N $\gamma_{qg} << \gamma_{qq}$

AT SMALL x $\Rightarrow$ $F_2 \sim g$

AT LARGE x

$\Rightarrow$ GLUON HARD TO DETERMINE

DISENTANGLING QUARKS: UP VS. DOWN

SYMMETRIES:

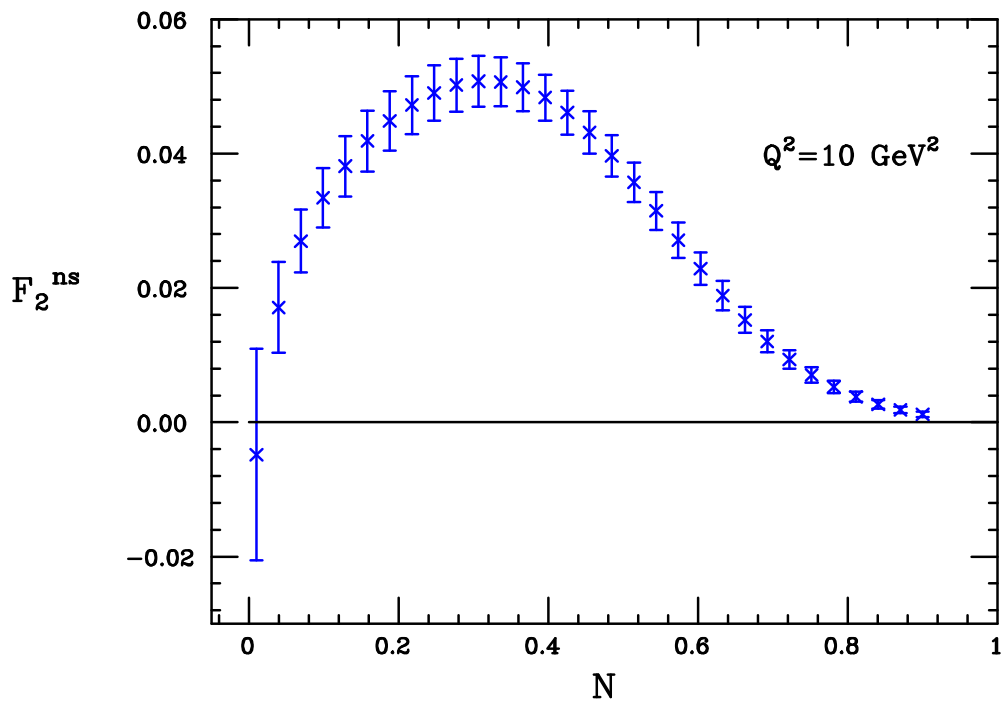
DISENTANGLING QUARKS: UP VS. DOWN

SYMMETRIES:

ISOSPIN

$$u^p(x,Q^2) = d^n(x,Q^2); \quad d^p(x,Q^2) = u^n(x,Q^2)$$

$$F_2^p(x,Q2) - F_2^d(x,Q^2) = \frac{1}{3}\left[(u^p + \bar{u}^p) - \left(d^p + \bar{d}^p\right)\right]\left[1 + O(\alpha_s)\right]$$



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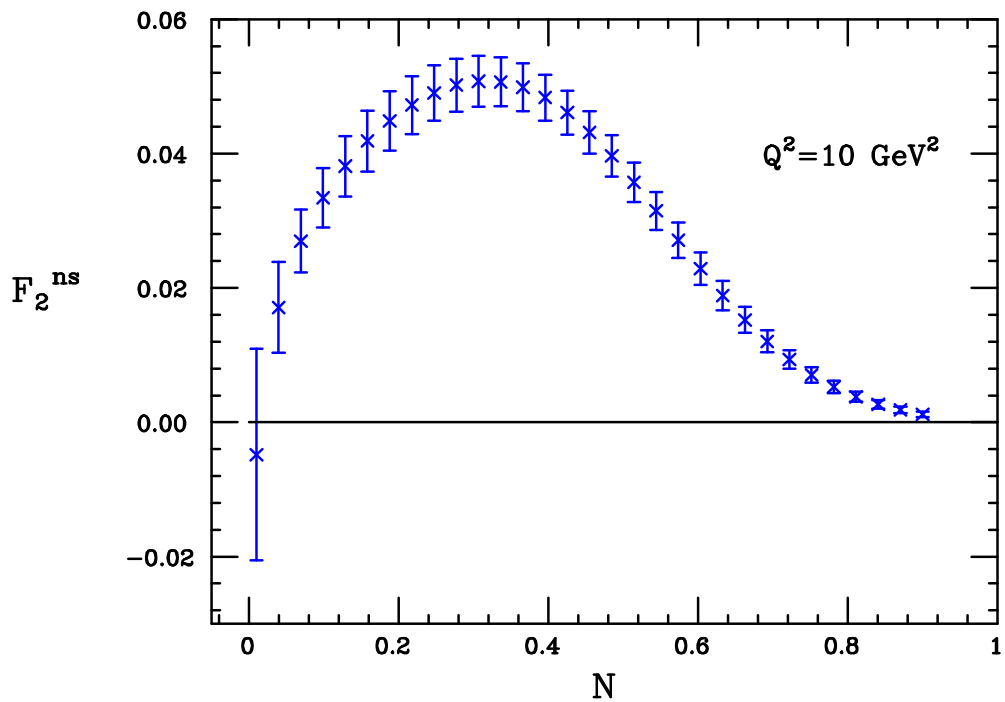

SUM RULES

BARYON NUMBER

$$\int_0^1 dx\,(u^p - \bar{u}^p) = 2 = 2\int_0^1 dx\left(d^p - \bar{d}^p\right)$$

MOMENTUM

$$\int_0^1 dx x\left[\sum_{i=1}^{N_f}\left(q^i(x) + \bar{q}_i(x)\right) + g(x)\right] = 1$$

DISENTANGLING QUARKS: STRANGENESS

γ^* SCATTERING VS. $W^\pm$ SCATTERING:

IN NC, CHARGED LEPTON DIS, ONLY MEASURE COMBINATION $\sum_i e_i^2 \left(q_i + \bar{q}_i\right)$

- CANNOT DETERMINE STRANGENESS
- CAN ONLY DETERMINE C-EVEN COMBINATION $q_i + \bar{q}_i$

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IN ν DIS, CAN DISENTANGLE INDIVIDUAL PDFS BY LINEAR COMBINATION: AT LO

$$\frac{1}{2}\left(F_1^{W^-} + \frac{1}{2}F_3^{W^-}\right) = u + c; \qquad \frac{1}{2}\left(F_1^{W^+} - \frac{1}{2}F_3^{W^+}\right) = \bar{u} + \bar{c}$$

$$\frac{1}{2}\left(F_1^{W^+} + \frac{1}{2}F_3^{W^+}\right) = d + s; \qquad \frac{1}{2}\left(F_1^{W^-} - \frac{1}{2}F_3^{W^-}\right) = \bar{d} + \bar{s}$$

c, $\bar{c}$, s, $\bar{s}$ only present above charm threshold

DISENTANGLING QUARKS FROM ANTIQUARKS

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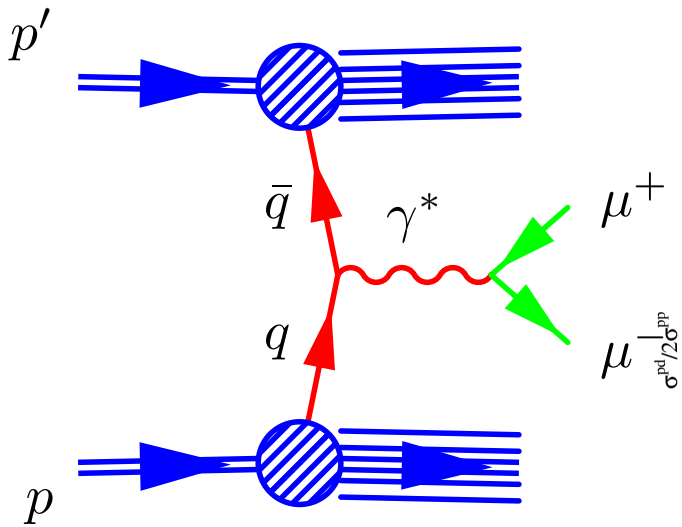
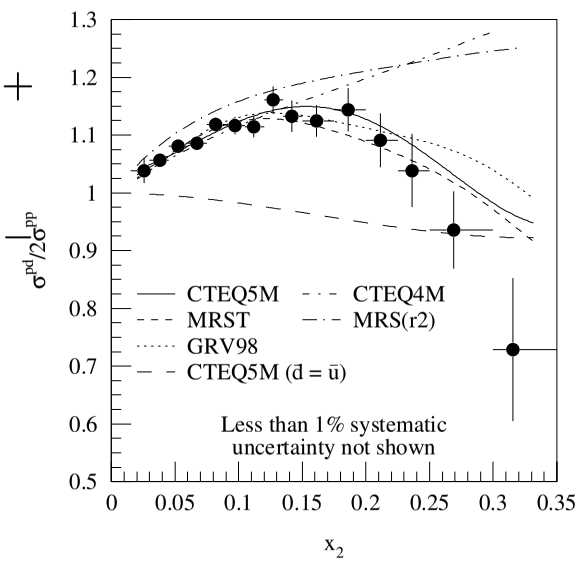
DRELL-YAN p/d ASYMMETRY



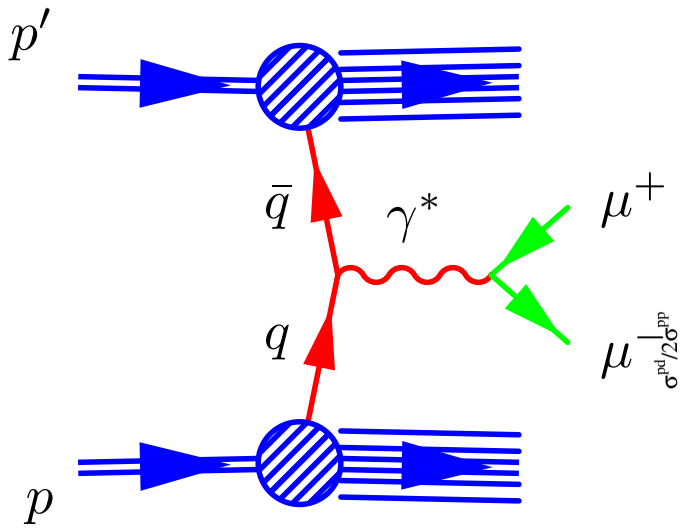


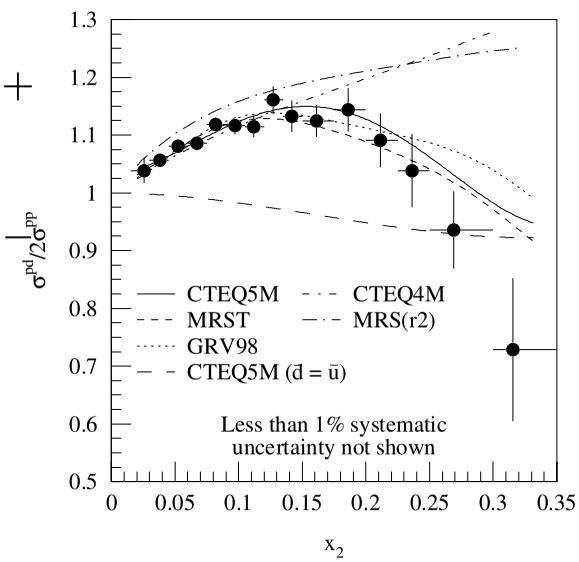
$$\left.\frac{\sigma^{pd}}{\sigma^{pp}}\right|_{x_1 >> x_2} \approx \frac{1}{2}\left(1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)}\right)$$

E866

DISENTANGLING QUARKS FROM ANTIQUARKS

DRELL-YAN p/d ASYMMETRY





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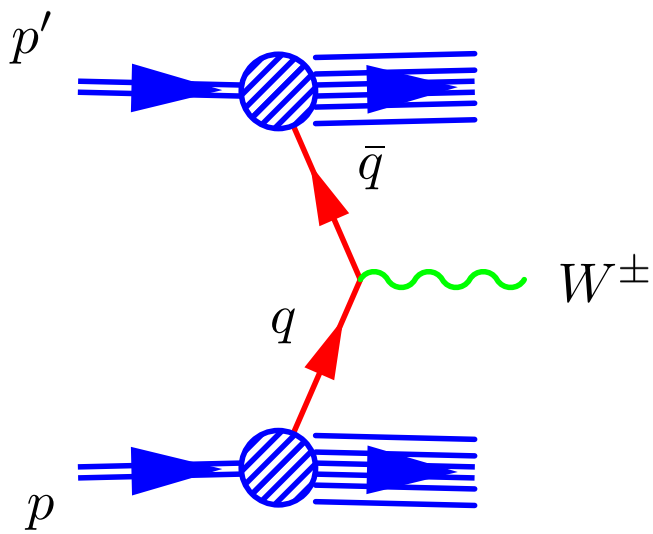

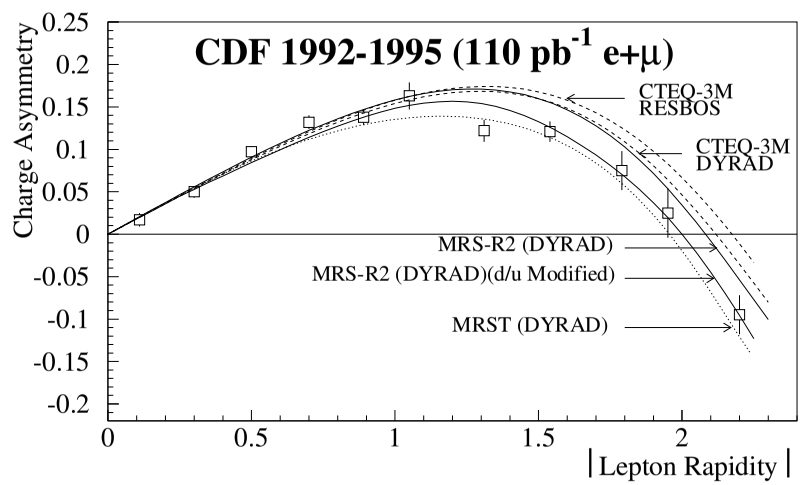

CDF

Theoretical Issues: NNLO corrections

how big is the impact of higher order perturbative corrections?

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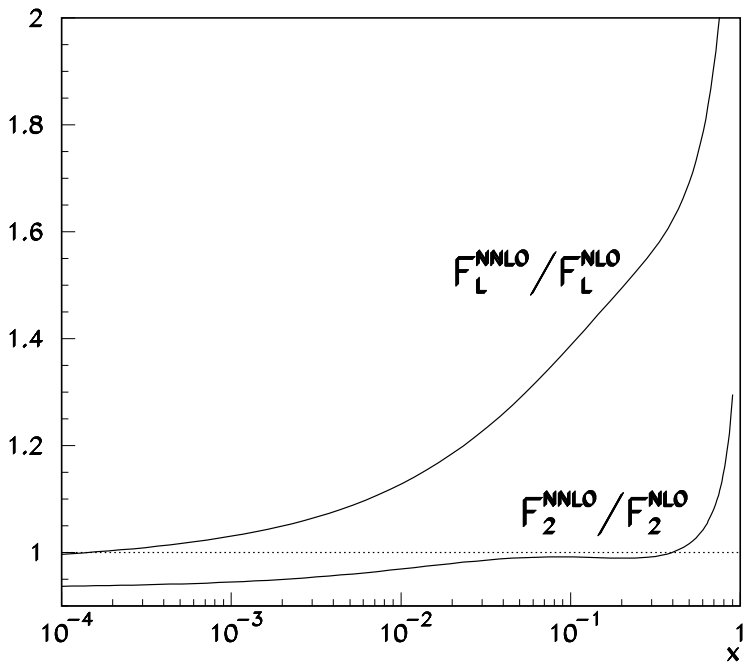
how big is the impact of higher order perturbative corrections?

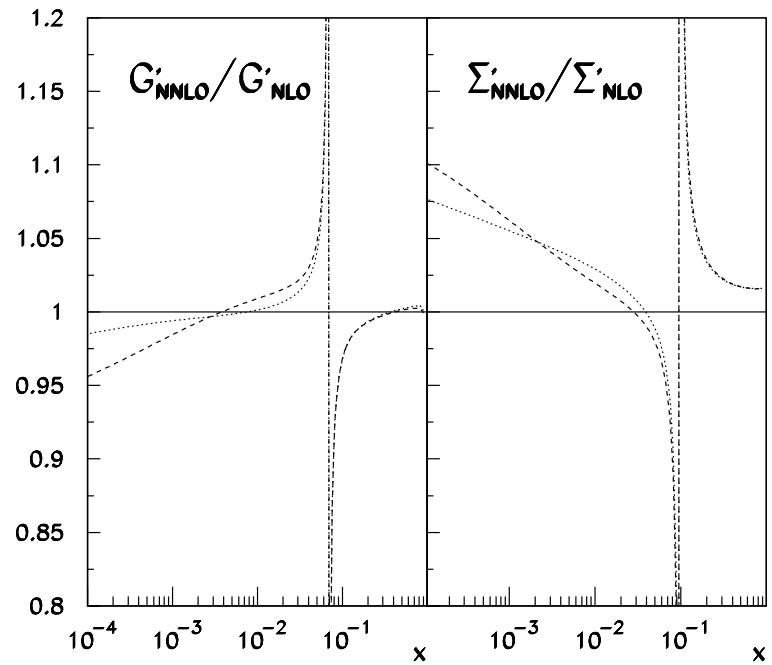
FULL NNLO splitting functions computed recently!

(Moch, Vermaseren and Vogt, April 2004)

perturbative coefficients



evolution



Alekhin

The Results

Anomalous dimensions in Mellin space

– One-loop : Gross, Wilczek '73

$$\gamma_{\rm ns}^{(0)}(N) \quad = \quad C_F\big(2(\mathbf{N}_- + \mathbf{N}_+)S_1 - 3\big)$$

– Two-loop : Floratos, Ross, Sachrajda '79 ; Gonzalez-Arroyo, Lopez, Ynduráin '79

$$\gamma_{\rm ns}^{(1)+}(N) \;=\; 4C_AC_F\Bigg(2\mathbf{N}_+S_3 - \frac{17}{24} - 2S_{-3} - \frac{28}{3}S_1 + (\mathbf{N}_- + \mathbf{N}_+)\Bigg[\frac{151}{18}S_1 + 2S_{1,-2} - \frac{1}{6}$$

$$+4C_Fn_f\Bigg(\frac{1}{12} + \frac{4}{3}S_1 - (\mathbf{N}_- + \mathbf{N}_+)\Bigg[\frac{11}{9}S_1 - \frac{1}{3}S_2\Bigg]\Bigg) + 4{C_F}^2\Bigg(4S_{-3} + 2S_1 + 2S_2 - \frac{3}{8}$$

$$+\mathbf{N}_-\Big[S_2 + 2S_3\Big] - (\mathbf{N}_- + \mathbf{N}_+)\Big[S_1 + 4S_{1,-2} + 2S_{1,2} + 2S_{2,1} + S_3\Big]\Bigg)$$

$$\gamma_{\rm ns}^{(1)-}(N) \;=\; \gamma_{\rm ns}^{(1)+}(N) + 16C_F\Bigg(C_F - \frac{C_A}{2}\Bigg)\Bigg((\mathbf{N}_- - \mathbf{N}_+)\Big[S_2 - S_3\Big] - 2(\mathbf{N}_- + \mathbf{N}_+ - 2)$$

– Compact notation : $\mathbf{N}_\pm f(N) \;=\; f(N \pm 1)\;, \qquad \mathbf{N}_{\pm\mathbf{i}} f(N) \;=\; f(N \pm i)$

– Three-loop :

S.M., Vermaseren, Vogt '04

$$\gamma_{\rm ns}^{(2)+}(N) \;=\; 16 C_A C_F n_f \Big(\frac{3}{2}\zeta_3 - \frac{5}{4} + \frac{10}{9}S_{-3} - \frac{10}{9}S_3 + \frac{4}{3}S_{1,-2} - \frac{2}{3}S_{-4} + 2S_{1,1} - \frac{25}{9}S_2 + \frac{257}{27}S_1 - \frac{2}{3}S_{-3,1} - \mathbf{N}_+\Big[S_{2,1} - \frac{2}{3}S_{3,1} -$$

$$-(\mathbf{N}_+ - 1)\Big[\frac{23}{18}S_3 - S_2\Big] - (\mathbf{N}_- + \mathbf{N}_+)\Big[S_{1,1} + \frac{1237}{216}S_1 + \frac{11}{18}S_3 - \frac{317}{108}S_2 + \frac{16}{9}S_{1,-2} - \frac{2}{3}S_{1,-2,1} - \frac{1}{3}S_{1,-3} - \frac{1}{2}S_{1,3} - \frac{1}{2}S_{2,1}$$

$$-\frac{1}{3}S_{2,-2} + S_1\zeta_3 + \frac{1}{2}S_{3,1}\Big]\Big) + 16 C_F C_A{}^2\Big(\frac{1657}{576} - \frac{15}{4}\zeta_3 + 2S_{-5} + \frac{31}{6}S_{-4} - 4S_{-4,1} - \frac{67}{9}S_{-3} + 2S_{-3,-2} + \frac{11}{3}S_{-3,1} + \frac{3}{2}S_{-2}$$

$$-6S_{-2}\zeta_3 - 2S_{-2,-3} + 3S_{-2,-2} - 4S_{-2,-2,1} + 8S_{-2,1,-2} - \frac{1883}{54}S_1 - 10S_{1,-3} - \frac{16}{3}S_{1,-2} + 12S_{1,-2,1} + 4S_{1,3} - 4S_{2,-2} - \frac{5}{2}S_4 +$$

$$+\frac{176}{9}S_2 + \frac{13}{3}S_3 + (\mathbf{N}_- + \mathbf{N}_+ - 2)\Big[3S_1\zeta_3 + 11S_{1,1} - 4S_{1,1,-2}\Big] + (\mathbf{N}_- + \mathbf{N}_+)\Big[\frac{9737}{432}S_1 - 3S_{1,-4} + \frac{19}{6}S_{1,-3} + 8S_{1,-3,1} + \frac{91}{9}S_{1,$$

$$-6S_{1,-2,-2} - \frac{29}{3}S_{1,-2,1} + 8S_{1,1,-3} - 16S_{1,1,-2,1} - 4S_{1,1,3} - \frac{19}{4}S_{1,3} + 4S_{1,3,1} + 3S_{1,4} + 8S_{2,-2,1} + 2S_{2,3} - S_{3,-2} + \frac{11}{12}S_{3,1} - S_{4,}$$

$$+\frac{1}{6}S_{2,-2} - \frac{1967}{216}S_2 + \frac{121}{72}S_3\Big] - (\mathbf{N}_- - \mathbf{N}_+)\Big[3S_2\zeta_3 + 7S_{2,1} - 3S_{2,1,-2} + 2S_{2,-2,1} - \frac{1}{4}S_{2,3} - \frac{3}{2}S_{3,-2} - \frac{29}{6}S_{3,1} + \frac{11}{4}S_{4,1} + \frac{1}{2}S_{2,}$$

$$+\mathbf{N}_+\Big[\frac{28}{9}S_3 - \frac{2376}{216}S_2 - \frac{8}{3}S_4 - \frac{5}{2}S_5\Big]\Big) + 16 C_F n_f^2\Big(\frac{17}{144} - \frac{13}{27}S_1 + \frac{2}{9}S_2 + (\mathbf{N}_- + \mathbf{N}_+)\Big[\frac{2}{9}S_1 - \frac{11}{54}S_2 + \frac{1}{18}S_3\Big]\Big) + 16 C_F{}^2 C_A\Big(\frac{45}{4}$$

$$-\frac{151}{64} - 10S_{-5} - \frac{89}{6}S_{-4} + 20S_{-4,1} + \frac{134}{9}S_{-3} - 2S_{-3,-2} - \frac{31}{3}S_{-3,1} + 2S_{-3,2} - \frac{9}{2}S_{-2} + 18S_{-2}\zeta_3 + 10S_{-2,-3} - 6S_{-2,-2}$$

$$+8S_{-2,-2,1} - 28S_{-2,1,-2} + 46S_{1,-3} + \frac{26}{3}S_{1,-2} - 48S_{1,-2,1} + \frac{28}{3}S_{1,2} - \frac{185}{6}S_3 - 8S_{1,3} + 2S_{3,-2} - 4S_5 - (\mathbf{N}_- + \mathbf{N}_+ - 2)\Big[9S_1\zeta_3$$

$$+\frac{209}{6}S_{1,1} - 14S_{1,1,-2} - \frac{242}{18}S_2 + 9S_{2,-2} + \frac{33}{4}S_4 - 3S_{3,1} + \frac{14}{3}S_{2,1}\Big] + (\mathbf{N}_- + \mathbf{N}_+)\Big[17S_{1,-4} - \frac{107}{6}S_{1,-3} - 32S_{1,-3,1} - \frac{173}{9}S_{1,-}$$

$$+16S_{1,-2,-2}+\frac{103}{3}S_{1,-2,1}-2S_{1,-2,2}-36S_{1,1,-3}+56S_{1,1,-2,1}+8S_{1,1,3}-\frac{109}{9}S_{1,2}-4S_{1,2,-2}+\frac{43}{3}S_{1,3}-8S_{1,3,1}-11S_{1}$$

$$+21S_{2,-3}-30S_{2,-2,1}-4S_{2,1,-2}-5S_{2,3}-S_{4,1}+\frac{31}{6}S_{2,-2}-\frac{67}{9}S_{2,1}\Big]+(\mathbf{N}_- -\mathbf{N}_+)\Big[9S_2\zeta_3+2S_{2,-3}+4S_{2,-2,1}-12S_{2,1,}$$

$$+13S_{4,1}+\frac{1}{2}S_{2,-2}+\frac{11}{2}S_4-\frac{33}{2}S_{3,1}+\frac{59}{9}S_3+\frac{127}{6}S_{2,1}-\frac{1153}{72}S_2\Big]+\mathbf{N}_+\Big[8S_{3,-2}+\frac{4}{3}S_{3,1}-2S_{3,2}+14S_5+\frac{23}{6}S_4+\frac{73}{3}S_3$$

$$+16C_F{}^2n_f\Big(\frac{23}{16}-\frac{3}{2}\zeta_3+\frac{4}{3}S_{-3,1}-\frac{59}{36}S_2+\frac{4}{3}S_{-4}-\frac{20}{9}S_{-3}+\frac{20}{9}S_1-\frac{8}{3}S_{1,-2}-\frac{8}{3}S_{1,1}-\frac{4}{3}S_{1,2}+\mathbf{N}_+\Big[\frac{25}{9}S_3-\frac{4}{3}S_{3,1}-\frac{1}{3}S$$

$$-(\mathbf{N}_+-1)\Big[\frac{67}{36}S_2-\frac{4}{3}S_{2,1}+\frac{4}{3}S_3\Big]+(\mathbf{N}_-+\mathbf{N}_+)\Big[S_1\zeta_3-\frac{325}{144}S_1-\frac{2}{3}S_{1,-3}+\frac{32}{9}S_{1,-2}-\frac{4}{3}S_{1,-2,1}+\frac{4}{3}S_{1,1}+\frac{16}{9}S_{1,2}-\frac{4}{3}S$$

$$+\frac{11}{18}S_2-\frac{2}{3}S_{2,-2}+\frac{10}{9}S_{2,1}+\frac{1}{2}S_4-\frac{2}{3}S_{2,2}-\frac{8}{9}S_3\Big]\Big)+16C_F{}^3\Big(12S_{-5}-\frac{29}{32}-\frac{15}{2}\zeta_3+9S_{-4}-24S_{-4,1}-4S_{-3,-2}+6S_{-3}$$

$$-4S_{-3,2}+3S_{-2}+25S_3-12S_{-2}\zeta_3-12S_{-2,-3}+24S_{-2,1,-2}-52S_{1,-3}+4S_{1,-2}+48S_{1,-2,1}-4S_{3,-2}+\frac{67}{2}S_2-17S_4$$

$$+(\mathbf{N}_-+\mathbf{N}_+-2)\Big[6S_1\zeta_3-\frac{31}{8}S_1+35S_{1,1}-12S_{1,1,-2}+S_{1,2}+10S_{2,-2}+S_{2,1}+2S_{2,2}-2S_{3,1}-3S_5\Big]+(\mathbf{N}_-+\mathbf{N}_+)\Big[23S$$

$$-22S_{1,-4}+32S_{1,-3,1}-2S_{1,-2}-8S_{1,-2,-2}-30S_{1,-2,1}-6S_{1,3}+4S_{1,-2,2}+40S_{1,1,-3}-48S_{1,1,-2,1}+8S_{1,2,-2}+4S_{1,2,2}$$

$$+4S_{1,4}+28S_{2,-2,1}+4S_{2,1,2}+4S_{2,2,1}+4S_{3,1,1}-4S_{3,2}+8S_{2,1,-2}-26S_{2,-3}-2S_{2,3}-4S_{3,-2}-3S_{2,-2}-3S_{2,2}+\frac{3}{2}S_4\Big]$$

$$+(\mathbf{N}_--\mathbf{N}_+)\Big[12S_{2,1,-2}-6S_2\zeta_3-2S_{2,-3}+3S_{2,3}+2S_{3,-2}-\frac{81}{4}S_{2,1}+14S_{3,1}-5S_{2,-2}-\frac{1}{2}S_{2,2}+\frac{15}{8}S_2+\frac{1}{2}S_3-13S_{4,1}$$

$$+\mathbf{N}_+\Big[14S_4-\frac{265}{8}S_2-\frac{87}{4}S_3-4S_{4,1}-4S_5\Big]\Big)$$

THEORETICAL ISSUES: RESUMMATION

AT $O(\alpha_s^n)$, $O\left[ln(\frac{1}{x})^n\right]$ AND $O\left[(ln(1-x))^{2n-1}\right]$ CONTRIBUTIONS ARISE:

IS A FIXED-ORDER PERTURBATIVE CALCULATION SUFFICIENT?

$x_{\rm cut}$:	0	0.0002	0.001	0.0025	0.005	0.01
# DATA POINTS	2097	2050	1961	1898	1826	1762
$\chi^2(x > 0)$	2267					
$\chi^2(x > 0.0002)$	2212	2203				
$\chi^2(x > 0.001)$	2134	2128	2119			
$\chi^2(x > 0.0025)$	2069	2064	2055	2040		
$\chi^2(x > 0.005)$	2024	2019	2012	1993	1973	
$\chi^2(x > 0.01)$	1965	1961	1953	1934	1917	1916
Δ_i^{i+1}	0.19	0.10	0.24	0.28	0.02	

DATA-THEORY AGREEMENT FOR EVOLUTION OF F_2 IMPROVES IF SMALL x DATA REMOVED (MRST 2003)

χ^2 improves with fixed # of pts (same row)

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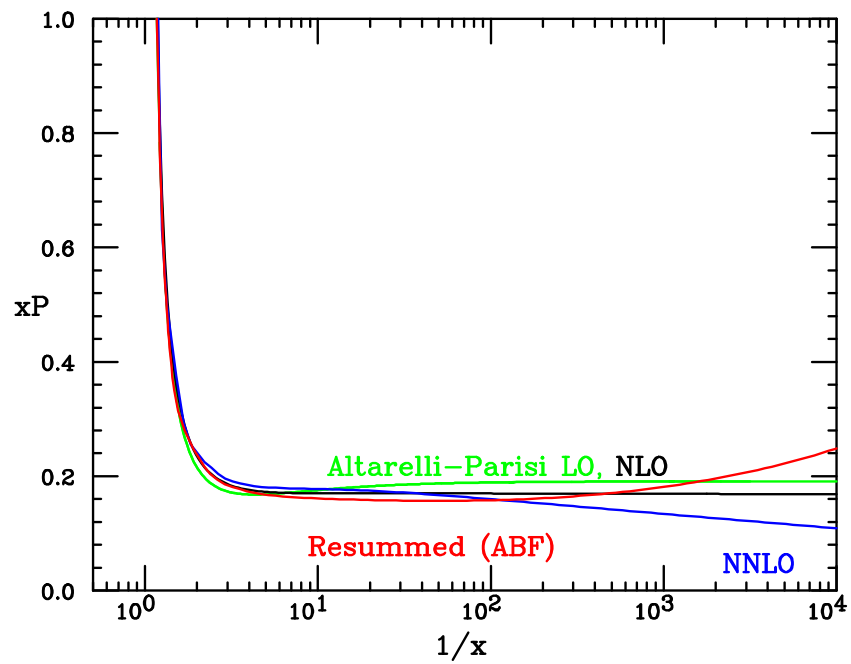
RESUMMED SPL. FCTN. CLOSER TO LO THAN TO NNLO

(ALTARELLI, BALL, S.F.)



DOES IT ALL MATTER?

W PRODUCTION CROSS-SECTION

PDF SET	COMMENT	XSEC [NB]	PDF UNCERTAINTY
ALEKHIN	TEVATRON	2.73	$\pm$ 0.05 (TOT)
MRST2002	TEVATRON	2.59	$\pm$ 0.03 (EXPT)
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Thorne 2003

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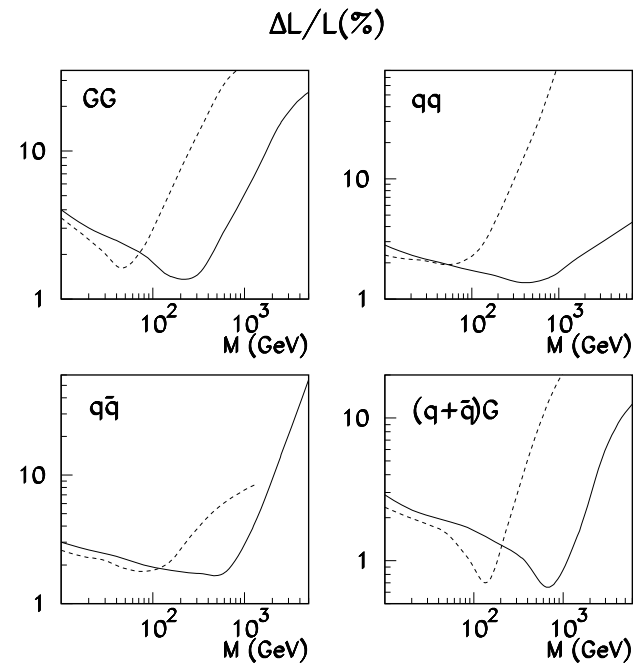
Thorne 2003

PARTON LUMINOSITY UNCERTAINTIES FOR TEVATRON & LHC

NOTE LOG SCALE ON Y AXIS!!! →



Alekhin 2003

An example: the NuTeV anomaly

PW Ratio: Data...

NuTeV 2001 $\sin^2\theta_W(\mathrm{OS}) = 0.2272 \pm 0.0013(\mathrm{stat}) \pm 0.0009(\mathrm{syst}) \pm 0.0002(M_t, M_H)$

Global Fit 2003 $\sin^2\theta_W(\mathrm{OS}) = 0.2229 \pm 0.0004$

...vs. Theory

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\begin{aligned}
R^- &= \frac{\sigma_{NC}(\nu_\mu) - \sigma_{NC}(\bar{\nu}_\mu)}{\sigma_{CC}(\nu_\mu) - \sigma_{CC}(\bar{\nu}_\mu)} \\
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&\qquad + O(\delta(u-d)^2 \qquad)
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U,D...DENOTE MOMENTUM FRACTIONS CARRIED BY CORRESP. QUARK FLAVORS

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&\qquad \left. + \frac{4}{9}\frac{\alpha_s}{2\pi}\left(\frac{1}{2} - \sin^2\theta_W\right) + O(\alpha_s^2)\right] + O(\delta(u-d)^2,\ \delta s^2)
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$s - \bar s \approx 0.004$ (or 2% isospin vln.) enough to remove anomaly: can we test it?

Disentangling strangeness

Charm is copiously produced in $W^+ + s \to c$

easily tagged through dimuon signal, 2nd muon from subsequent c decay

- branching ratios and decay const. of charmed mesons (e.g. D_s)
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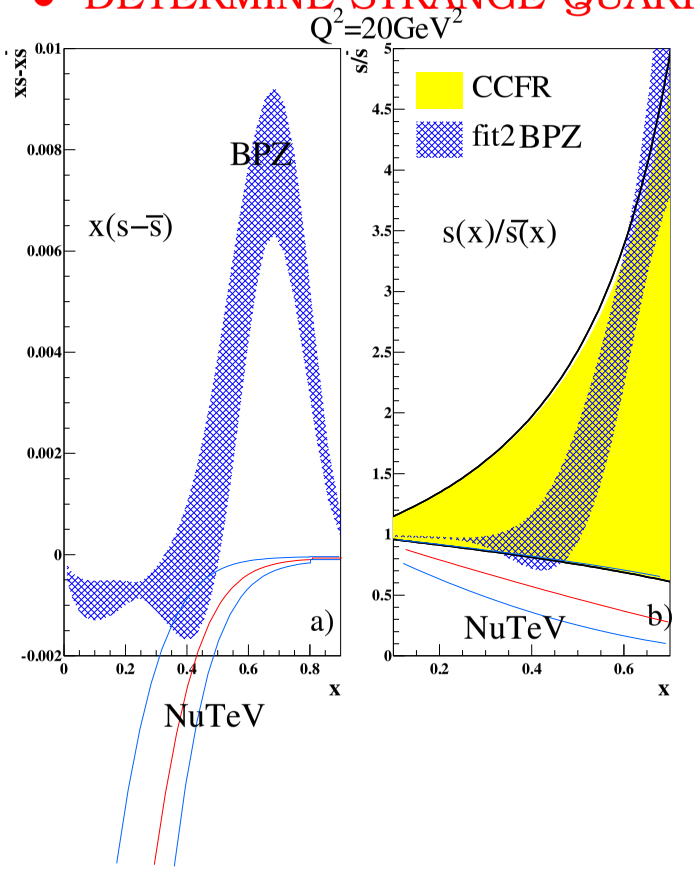
- determine strange quark distn. accurately



CCFR/NuTeV $s - \bar{s}$ determination

5000 ν & 1500 $\bar{\nu}$ dimuon event sample:

assumed parm.: $s(x) = \kappa \frac{\bar{u}(x)+\bar{d}(x)}{2}(1-x)^\alpha$

negative $s - \bar{s}$ at small x

$\Rightarrow$ mom. fract. $s - \bar{s} = -0.003 \pm 0.001$

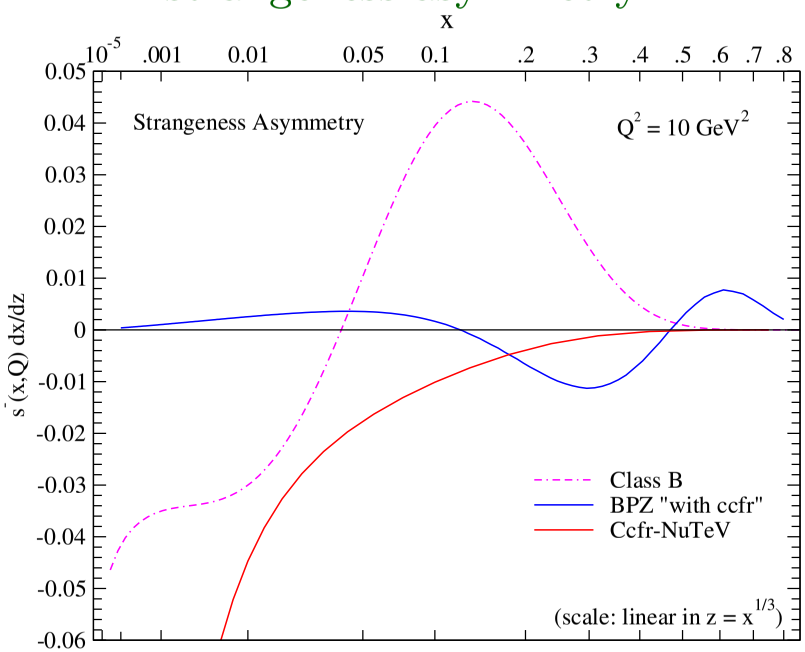
NuTeV anomaly worse!

COMBINING INCLUSIVE AND EXCLUSIVE INFORMATION

CTEQ DEDICATED DIMUON ANALYSIS (April 2004)

- $\int_0^1 (s(x) - \bar{s}(x))dx = 0$ IN PROTON
  $\Rightarrow$ EITHER $s(x) - \bar{s}(x)$ HAS A NODE OR IT VANISHES EVERYWHERE
- $[s(x) - \bar{s}(x)] < 0$ FOR SMALL $x \lesssim 0.05$ CONSTRAINED BY DIMUON

strangeness asymmetry



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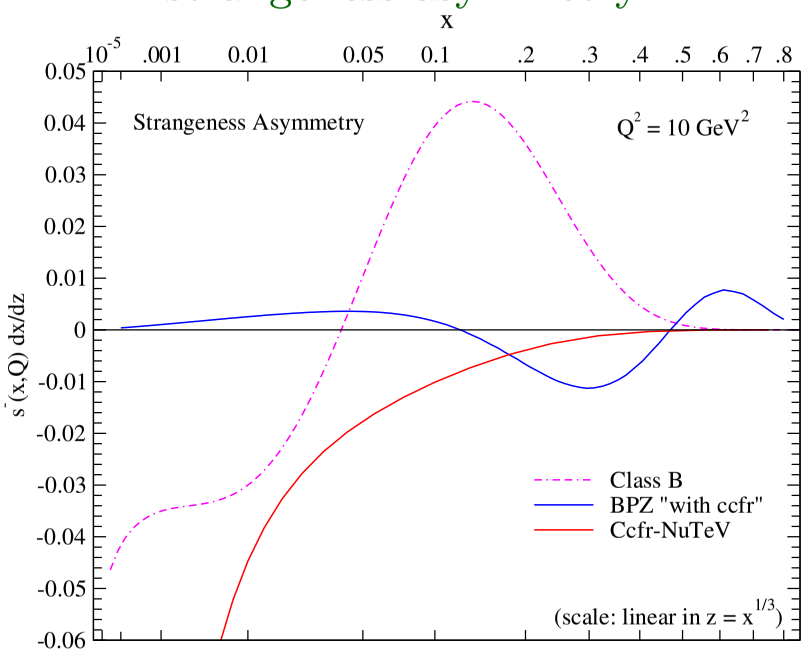
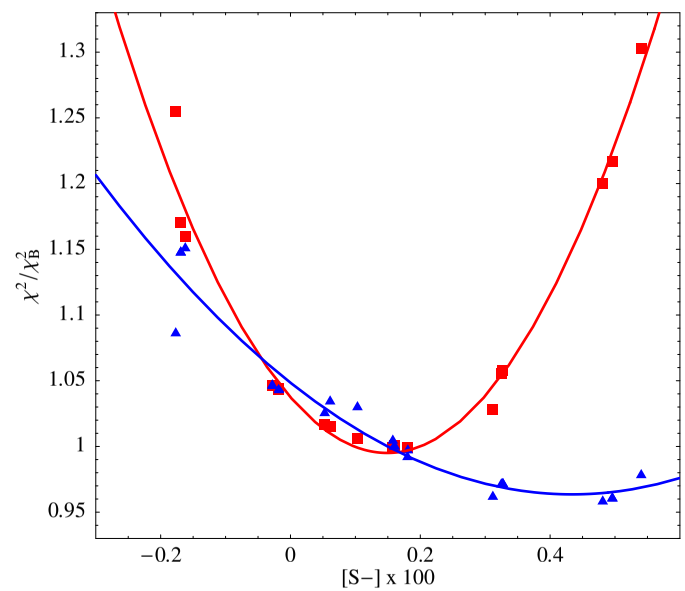
- LARGE x REGION WEIGHS MORE IN MOMENTUM FRACTION
- POSITIVE MOM. FRACTION $s - \bar{s} \approx 0.02$: THE END OF THE NUTEV ANOMALY

strangeness asymmetry



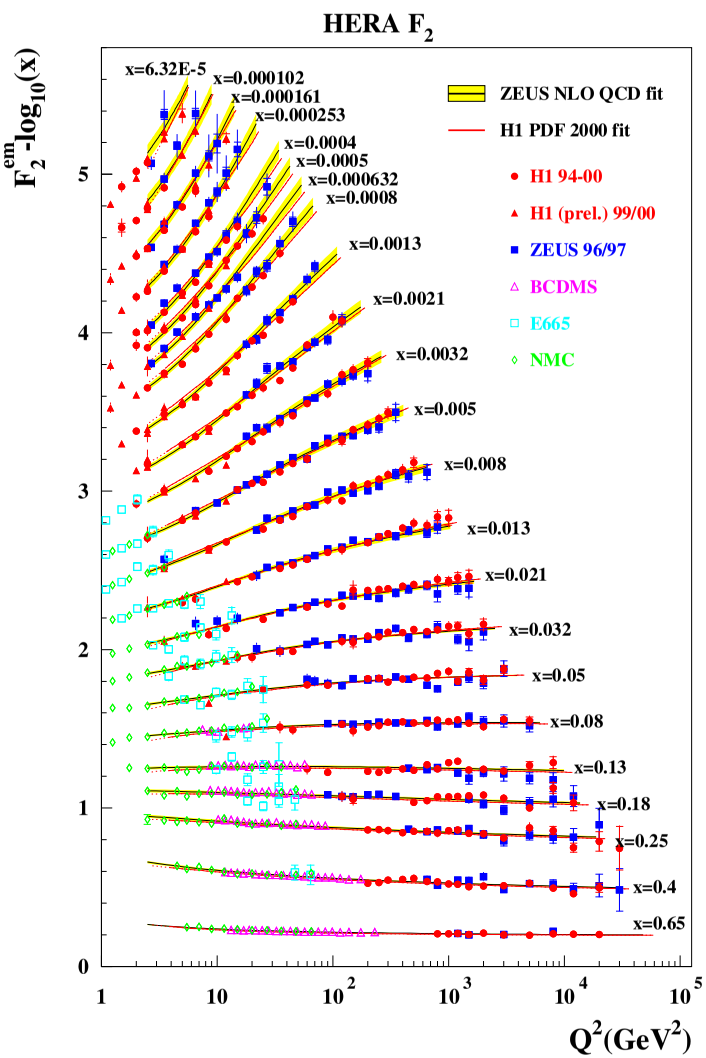
χ^2 dimuon vs. inclusive



DIS DATA → PARTON DISTRIBUTIONS

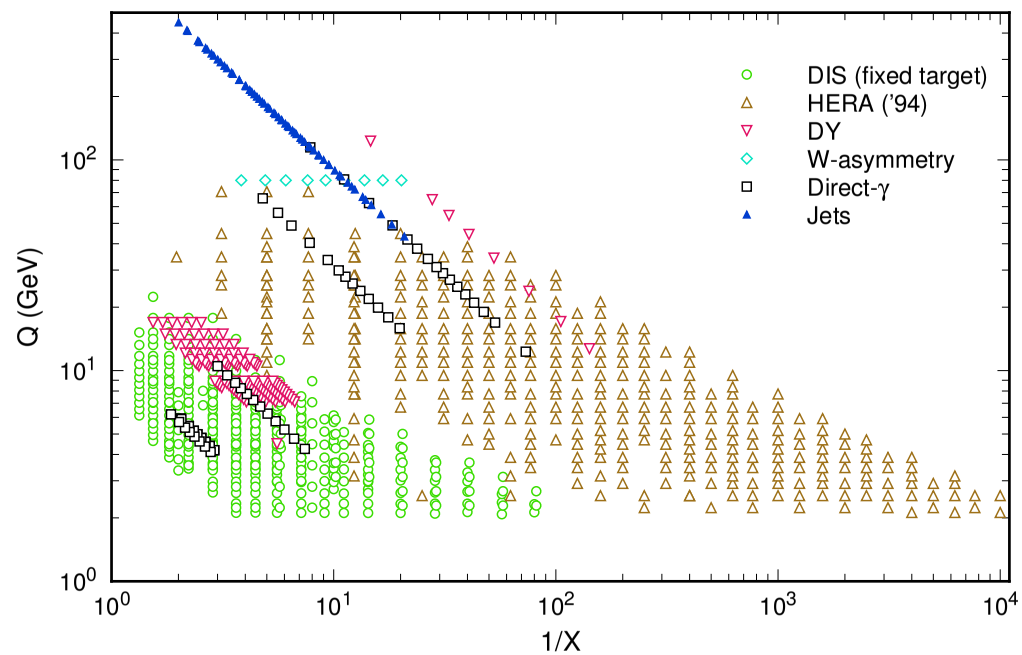
PROBLEMS:

PHENOMENOLOGY



GIVEN A SET OF DATA POINTS
MUST DETERMINE A SET OF
FUNCTIONS WITH ERRORS

DATA INCLUDED IN CTEQ5 PARTON FIT



WHAT'S THE PROBLEM? D. Kosower, 1999

- FOR A SINGLE QUANTITY, WE QUOTE 1 SIGMA ERRORS: VALUE$\pm$ ERROR
- FOR A PAIR OF NUMBERS, WE QUOTE A 1 SIGMA ELLIPSE
- FOR A FUNCTION, WE NEED AN "ERROR BAR" IN A SPACE OF FUNCTIONS

MUST DETERMINE THE PROBABILITY DENSITY (MEASURE) $\mathcal{P}[f_i(x)]$
IN THE SPACE OF PARTON DISTRIBUTION FUNCTIONS $f_i(x)$ (i=quark, antiquark, gluon)

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MUST DETERMINE AN INFINITE–DIMENSIONAL OBJECT FROM A FINITE SET OF DATA POINTS

THE STANDARD SOLUTION:

FUNCTIONAL PARTON FITTING

- CHOOSE A FIXED FUNCTIONAL FORM:
  - MRST: 24 PARMS., SOME FIXED $\longrightarrow$ 15 PARMS.

$$xq(x,Q_0^2) = A(1-x)^{\eta}(1+\epsilon x^{0.5}+\gamma x)x^{\delta}, \quad x[\bar{u}-\bar{d}](x,Q_0^2) = A(1-x)^{\eta}(1+\gamma x+\delta x^2)x^{\delta}.$$

$$xg(x,Q_0^2) = A_g(1-x)^{\eta_g}(1+\epsilon_g x^{0.5}+\gamma_g x)x^{\delta_g} - A_-(1-x)^{\eta_-}x^{-\delta_-},$$

  - CTEQ: 20 PARMS.

$$x\,f(x,Q_0) = A_0\,x^{A_1}\,(1-x)^{A_2}\,e^{A_3 x}\,(1+e^{A_4}\,x)^{A_5}$$

with independent params for combinations $u_v \equiv u-\bar{u}$, $d_v \equiv d-\bar{d}$, g, and $\bar{u}+\bar{d}$, $s=\bar{s}=0.2\,(\bar{u}+\bar{d})$ at Q_0; NORM. FIXED BY SUM RULES

  - ALEKHIN: 17 PARMS.

$$xu_{\rm V}(x,Q_0) = \frac{2}{N_{\rm u}^{\rm V}}x^{a_{\rm u}}(1-x)^{b_{\rm u}}(1+\gamma_2^{\rm u}x); \quad xu_{\rm S}(x,Q_0) = \frac{A_{\rm S}}{N_{\rm S}}\eta_{\rm u}x^{a_{\rm s}}(1-x)^{b_{\rm su}}$$

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THE STANDARD SOLUTION:

FUNCTIONAL PARTON FITTING

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- EVOLVE TO DESIRED SCALE & COMPUTE PHYSICAL OBSERVABLES
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- DETERMINE ERROR BY PROPAGATION OF ERROR ON PARMS ('HESSIAN METHOD') OR BY PARM. SCANS ('LAGRANGE MULTIPLIER METHOD')

PROBLEM PROJECTED ONTO THE FINITE–DIMENSIONAL SPACE OF PARAMETERS

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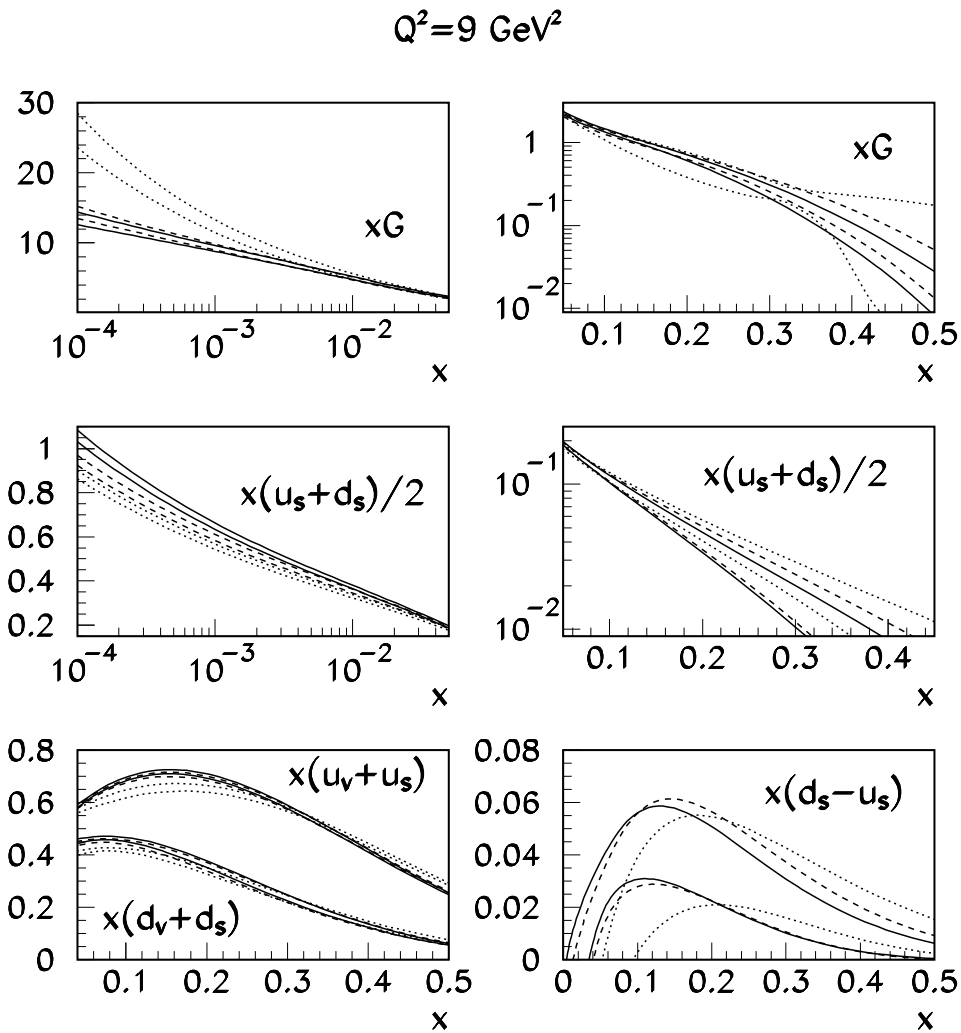
PROBLEM PROJECTED ONTO THE FINITE–DIMENSIONAL SPACE OF PARAMETERS

WHAT IS THE BIAS (THEOR. ERROR) DUE TO THE CHOICE OF FUNCTIONAL FORM?

HOW WELL DOES IT WORK?

ALEKHIN 2003 PARTONS



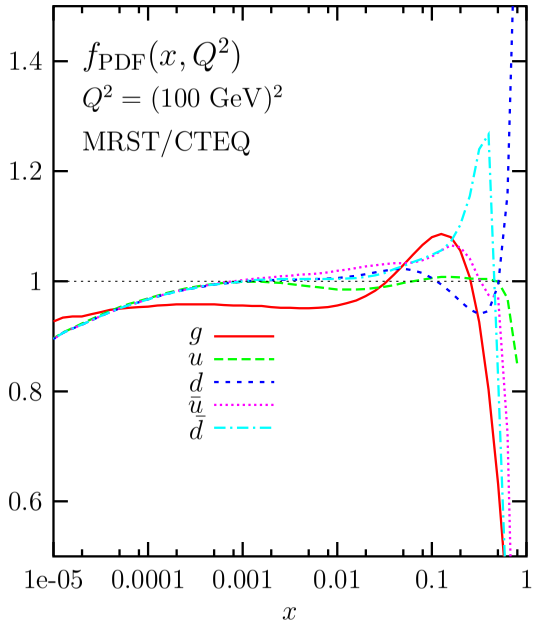
TOTAL ERROR BANDS FOR

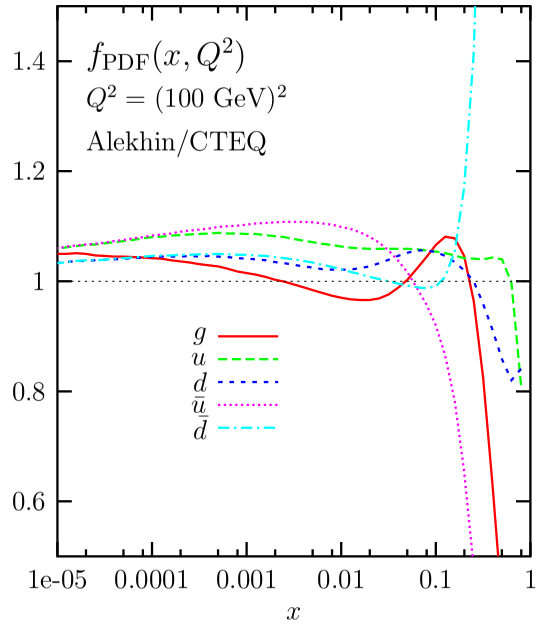
LO (DOTS), NLO (DASHES), NNLO (SOLID)

PARTON DISTRIBUTIONS

valence $u^v \equiv u - \bar{u}$, $d^v \equiv d - \bar{d}$,
sea $u^s = \bar{u}^s = d^s = \bar{d}^s$

PARAMETRIZATION BIAS?



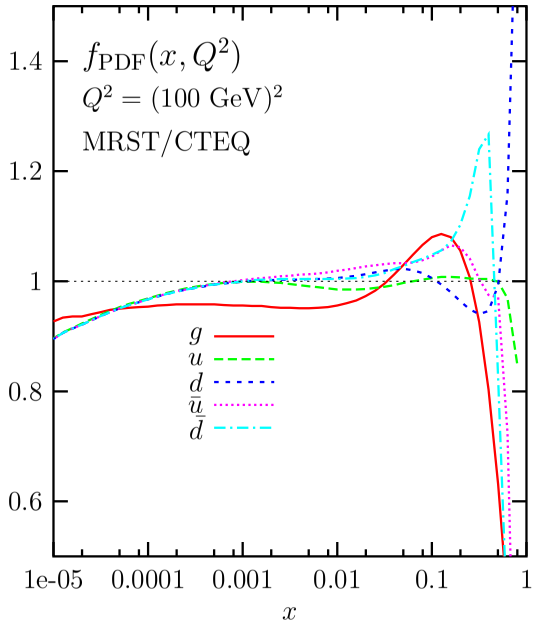


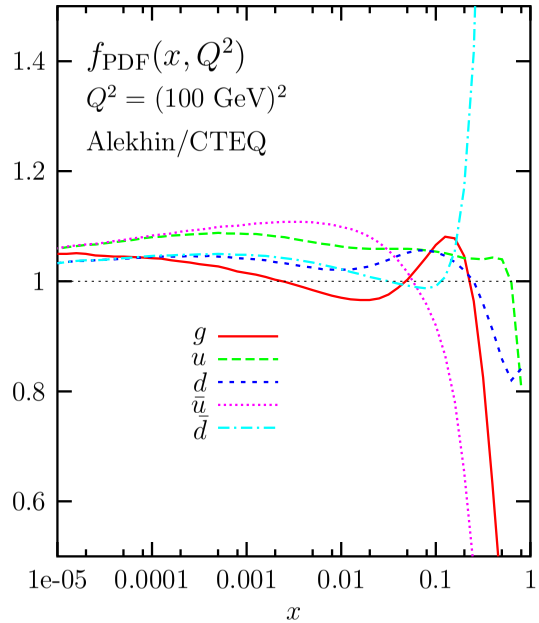
MRST & CTEQ

$\longrightarrow$ SIMILAR PARTONS

Djouadi and Ferrag 2003

PARAMETRIZATION BIAS?





MRST & CTEQ
$\longrightarrow$ SIMILAR PARTONS

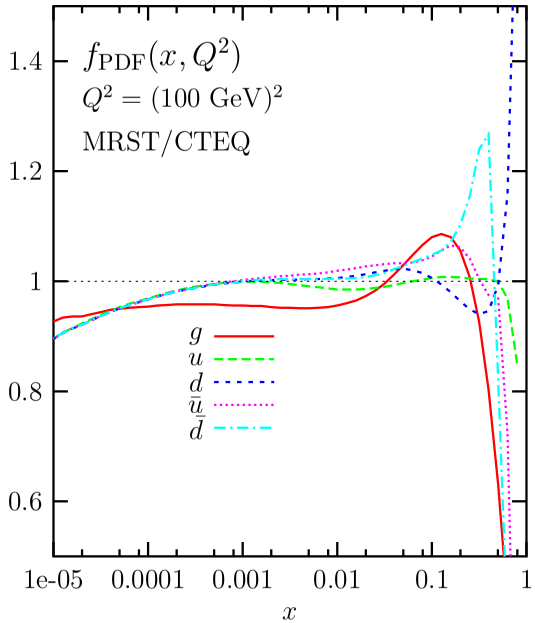
Djouadi and Ferrag 2003

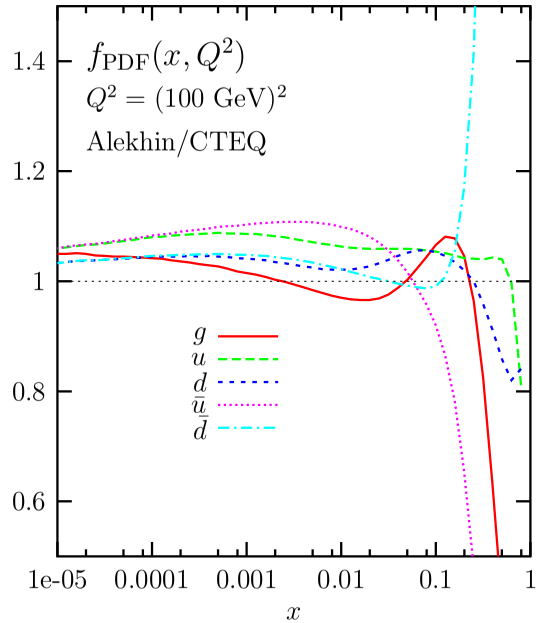
SIMILAR PARTONS $\longrightarrow$ SIMILAR RESULTS

THE W XSECT. AGAIN...

PDF SET	COMMENT	XSEC [NB]	PDF UNCERTAINTY
ALEKHIN	TEVATRON	2.73	$\pm$ 0.05 (TOT)
MRST2002	TEVATRON	2.59	$\pm$ 0.03 (EXPT)
CTEQ6	TEVATRON	2.54	$\pm$ 0.10 (EXPT)
ALEKHIN	LHC	215	$\pm$ 6 (TOT)
MRST2002	LHC	204	$\pm$ 4 (EXPT)
CTEQ6	LHC	205	$\pm$ 8 (EXPT)

PARAMETRIZATION BIAS?





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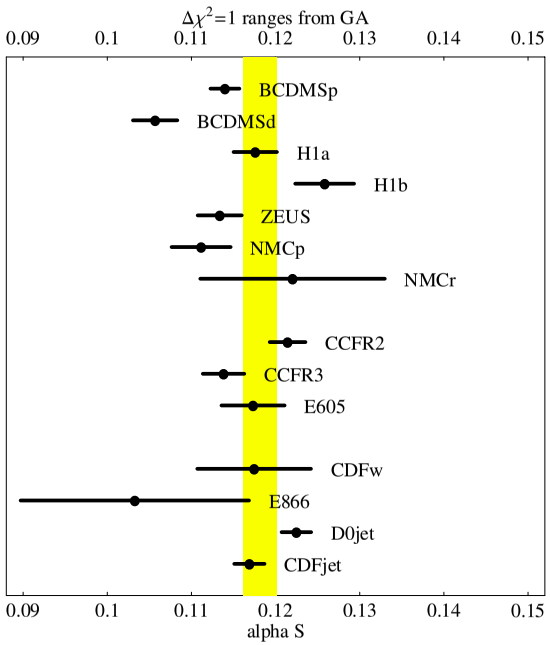
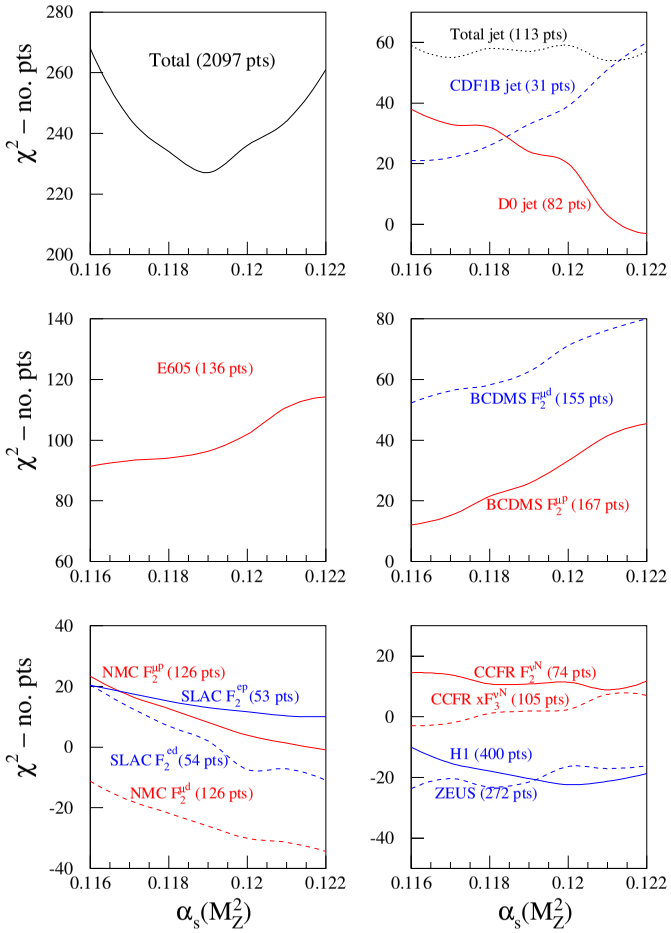
We do not seem to have the optimum parameterization for both finding the best fit and also investigating fluctuations about this best fit (...) This might then influence our error analysis...(MRST 2004)

CAN WE TRUST THE ERRORS?

THORNE, APR. 2004:

In full **global** fit art in choosing "correct" $\Delta\chi^2$ given complication of errors. Ideally $\Delta\chi^2 = 1$, but unrealistic.





Many approaches use $\Delta\chi^2 \sim 1$. CTEQ choose $\Delta\chi^2 \sim 100$ (conservative?). MRST choose $\Delta\chi^2 \sim 20$ for $1-\sigma$ error.

CONSERVATIVE SOLUTIONS

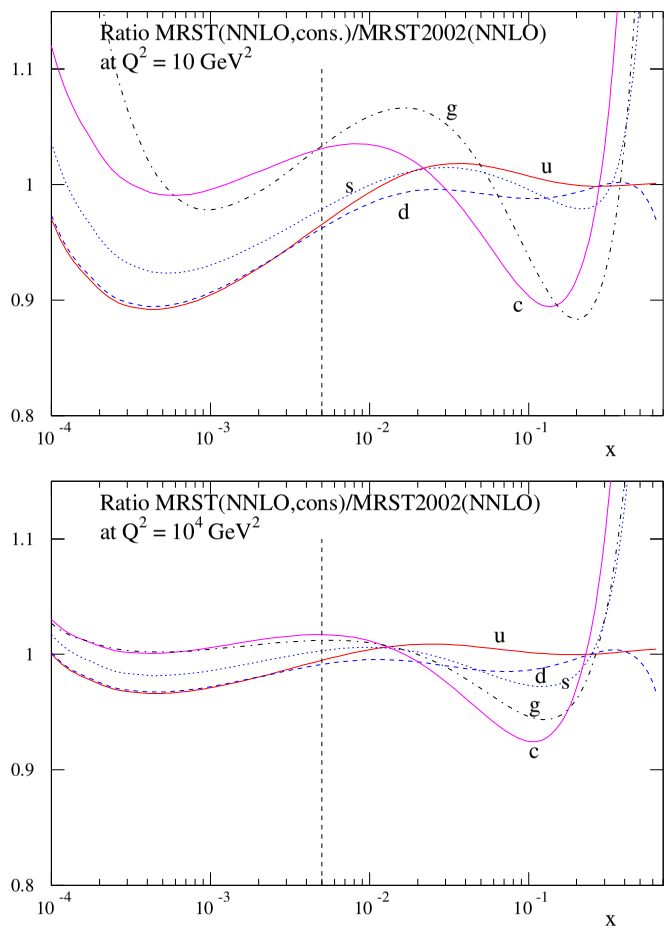
IMPOSE RESTRICTIVE KINEMATIC CUTS

cut in Q^2 raised from 2 to 10 GeV^2; cut off $x < 0.005$; cut in W^2 raised from 12.5 to 15 GeV^2

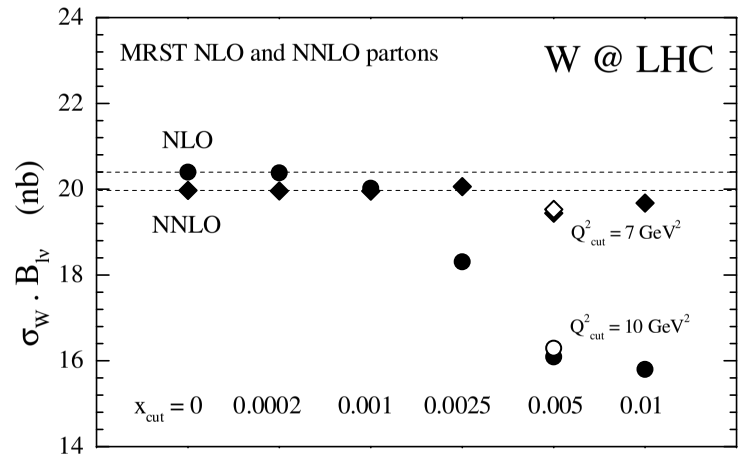
MRST 2003 CONSERVATIVE PARTONS

- ABOUT 800 DATAPOINTS REMOVED OUT OF ABOUT 2000
- $\Delta\chi^2 = 5$ SUFFICIENT FOR REASONABLE 1-σ

CONSERVATIVE/PLAIN



VARIATION OF THE W XSECT.



conservative partons unreliable for total xsect. because small x region important

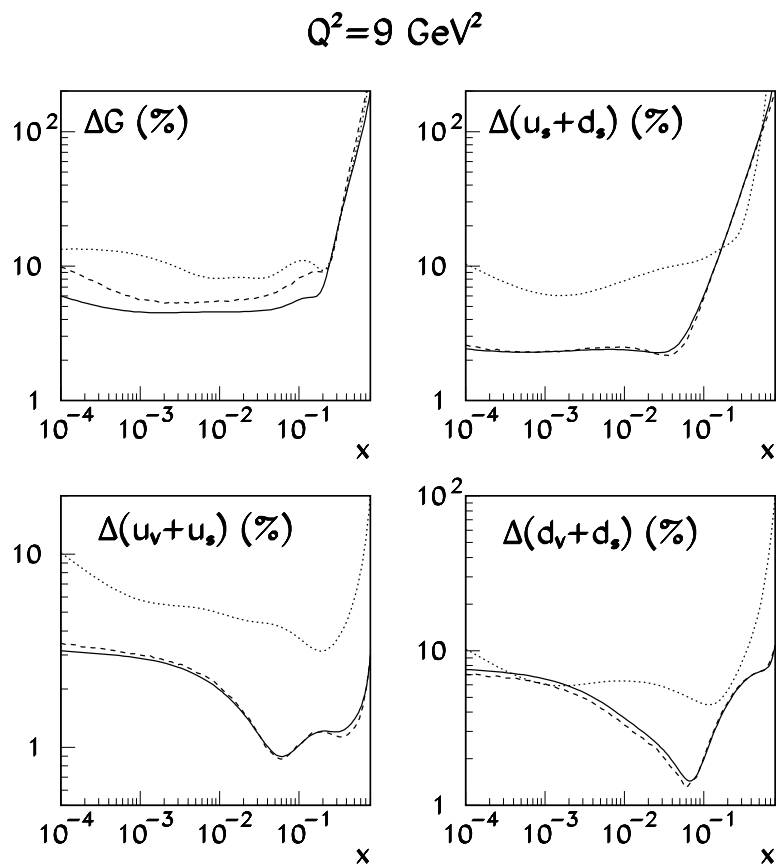
CONSERVATIVE SOLUTIONS

SELECT COHERENT SET OF DATA

ALEKHIN PARTONS

- ONLY DIS DATA INCLUDED
- $\Delta\chi^2 = 1$ PROVIDES GOOD 1-σ CURVES

PERCENTAGE ERRORS

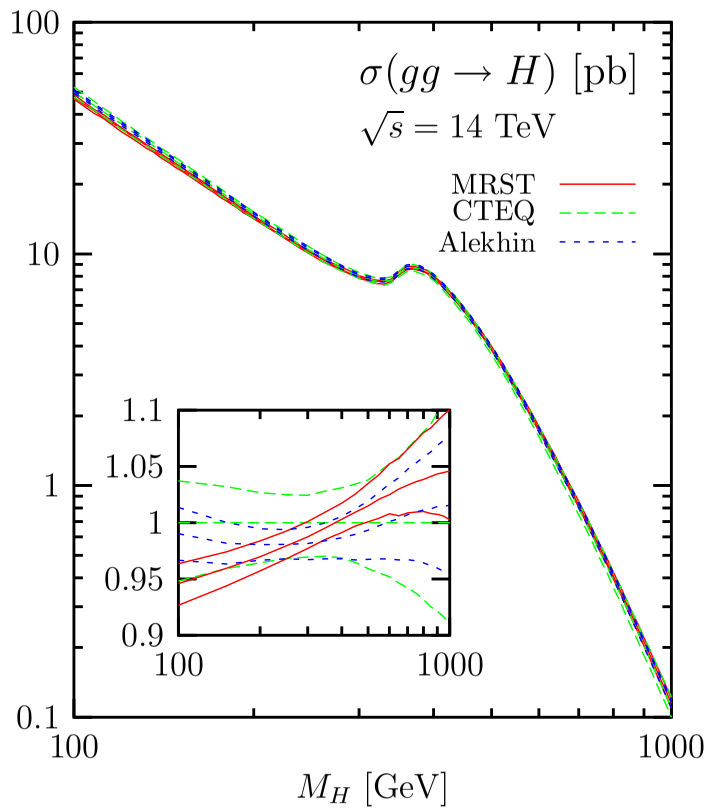


- ALEKHIN 2003 PARTON UNCERTAINTIES (SOLID, DASHED) COMPARABLE TO CTEQ6 (DOTTED)
- CANNOT SEPARATE ACCURATELY ANTIQUARK DISTNS.
- ERROR ON σ_W COMPARABLE TO CTEQ, MRST

TROUBLE...

PREDICTIONS FROM DIFFERENT SETS IN REASONABLE AGREEMENT,

HIGGS PRODUCTION AT LHC



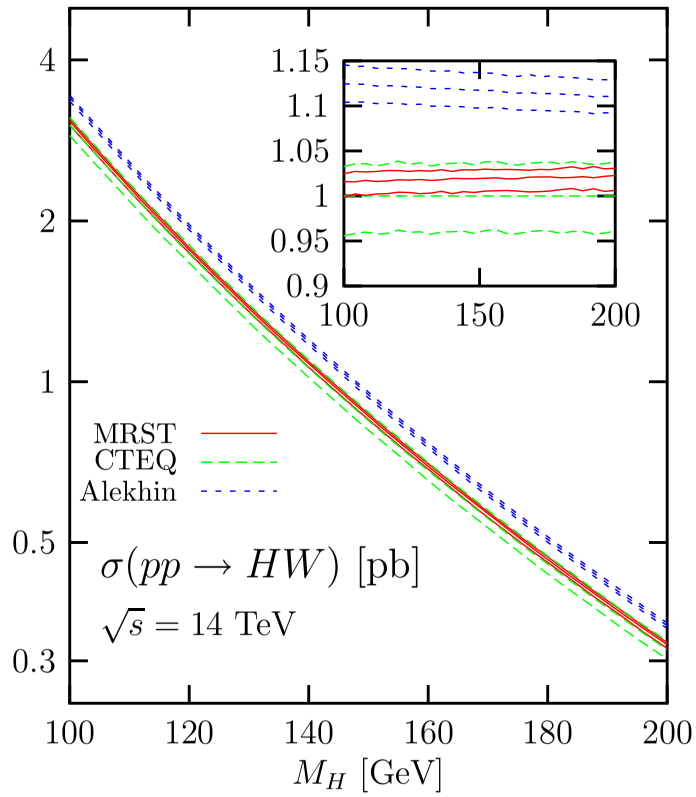
DJOUADI AND FERRAG, 2004

Alekhin gluon agrees with MRST/CTEQ → good agreement for Higgs production in gluon fusion

TROUBLE...

PREDICTIONS FROM DIFFERENT SETS IN REASONABLE AGREEMENT, BUT DO NOT ALWAYS AGREE WITHIN STATED ERRORS

HIGGS PRODUCTION AT LHC



DJOUADI AND FERRAG, 2004

Alekhin quark differs from MRST/CTEQ → difference visible in associate Higgs W production

NEW SOLUTIONS

THE BAYESIAN MONTE CARLO APPROACH (GIELE, KOSOWER, KELLER 2001)

- generate a Monte-Carlo sample of fcts. with "reasonable" prior distn. (e.g. an available parton set) $\longrightarrow$ representation of probability functional $\mathcal{P}[f_i]$
- calculate observables with functional integral
- update probability using Bayesian inference on MC sample: better agreement with data $\longrightarrow$ more functions in sample
- iterate until convergence achieved

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PROBLEM IS MADE FINITE-DIMENSIONAL BY THE CHOICE OF PRIOR, BUT RESULT DO NOT DEPEND ON THE CHOICE IF SUFFICIENTLY GENERAL

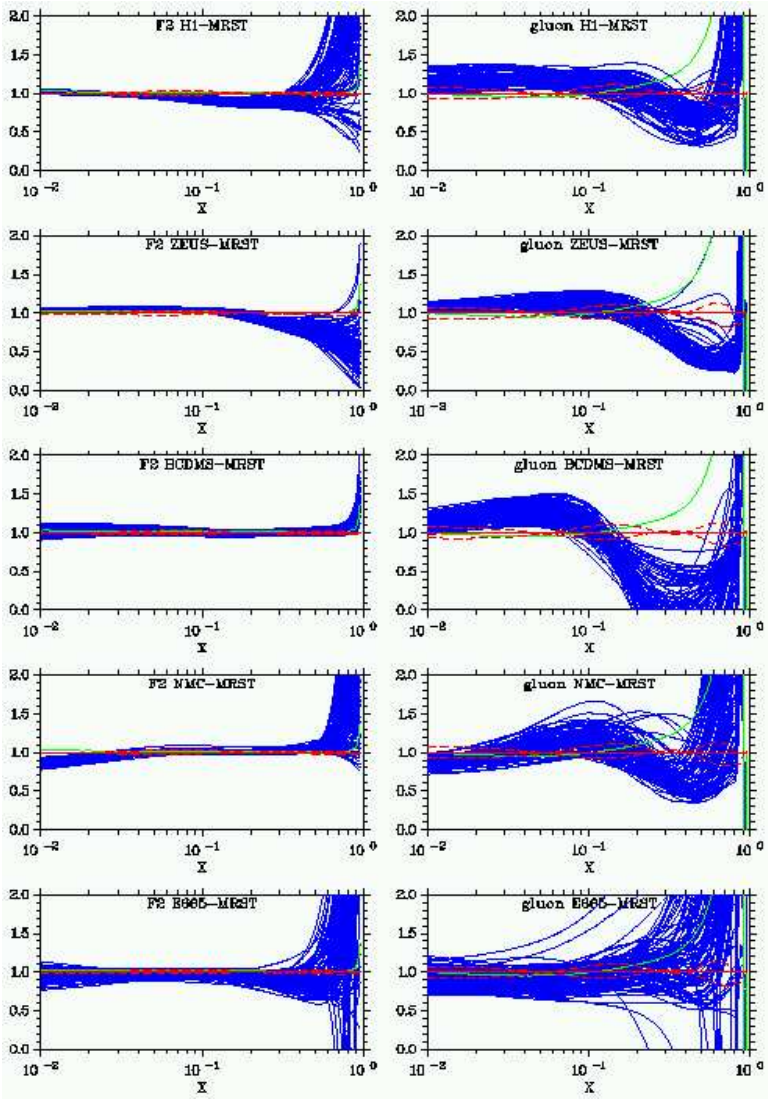
HARD TO HANDLE "FLAT DIRECTIONS"

(Monte Carlo replicas which lead to same agreement with data);

COMPUTATIONALLY VERY INTENSIVE;

DIFFICULT TO ACHIEVE INDEP. FROM PRIOR

Result: Fermi partons



F_2^{singlet} and gluon ratios Fermi/MRST

only subset of data fitted (H1, E665, BCDMS DIS data)

good agreement with Tevatron W xsect

trouble with value of α_s

NEW SOLUTIONS

THE NEURAL MONTE CARLO APPROACH (S.F., GARRIDO, LATORRE, PICCIONE 2002)

BASIC IDEA: USE NEURAL NETWORKS AS UNIVERSAL UNBIASED INTERPOLANTS

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THE NEURAL MONTE CARLO APPROACH (S.F., GARRIDO, LATORRE, PICCIONE 2002)

BASIC IDEA: USE NEURAL NETWORKS AS UNIVERSAL UNBIASED INTERPOLANTS

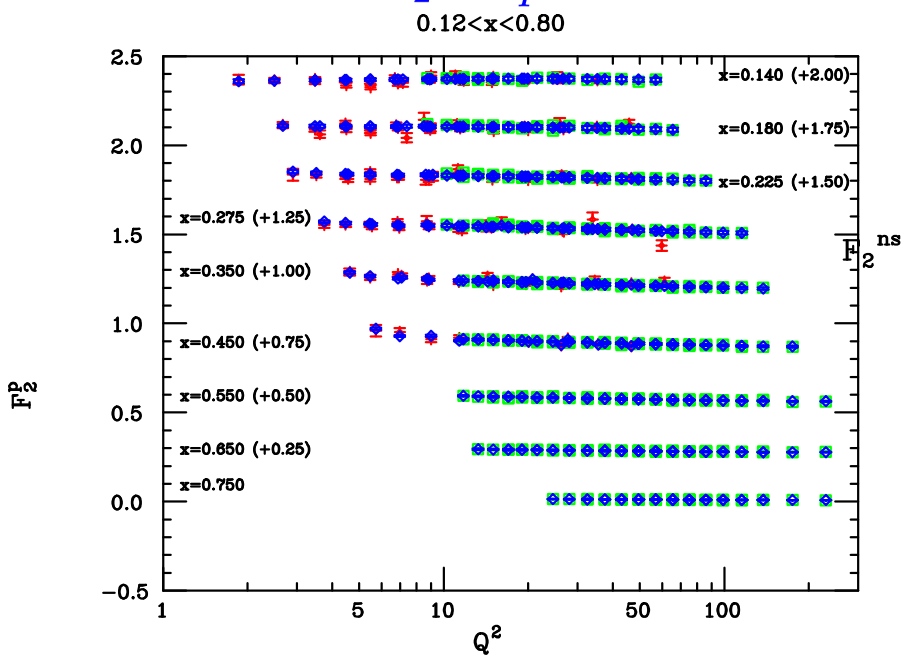
- GENERATE A SET OF MONTE CARLO REPLICAS $\sigma^{(k)}(p_i)$ OF THE ORIGINAL DATASET $\sigma^{(\mathrm{data})}(p_i)$ (p_i values of the kin. variables where σ is measured)
  $\Rightarrow$ REPRESENTATION OF $\mathcal{P}[\sigma(p_i)]$ AT DISCRETE SET OF POINTS p_i
- TRAIN A NEURAL NET FOR EACH PDF ON EACH REPLICA, THUS OBTAINING A NEURAL REPRESENTATION OF THE PDFS $f_i^{(net),(k)}$
- THE SET OF NEURAL NETS IS A REPRESENTATION OF THE PROBABILITY DENSITY:

$$\left\langle \sigma\left[f_i\right] \right\rangle = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} \sigma\left[{f_i}^{(net)(k)}\right]$$

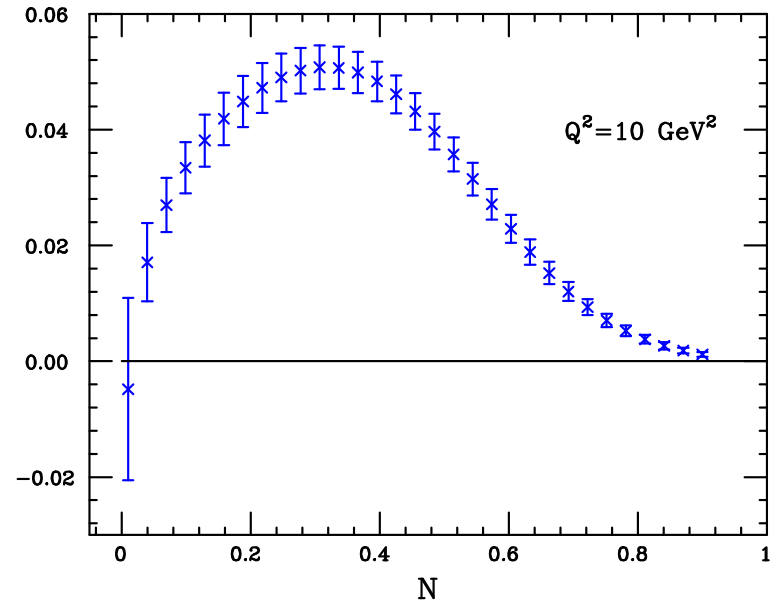
- CHECK GOODNESS OF FIT THROUGH STATISTICAL INDICATORS
  (χ^2, CORRELATION,. . .)

PRELIMINARY RESULT: NEURAL STRUCTURE FUNCTIONS

NEURAL F_2 VS p DATA



NEURAL NONSINGLET F_2



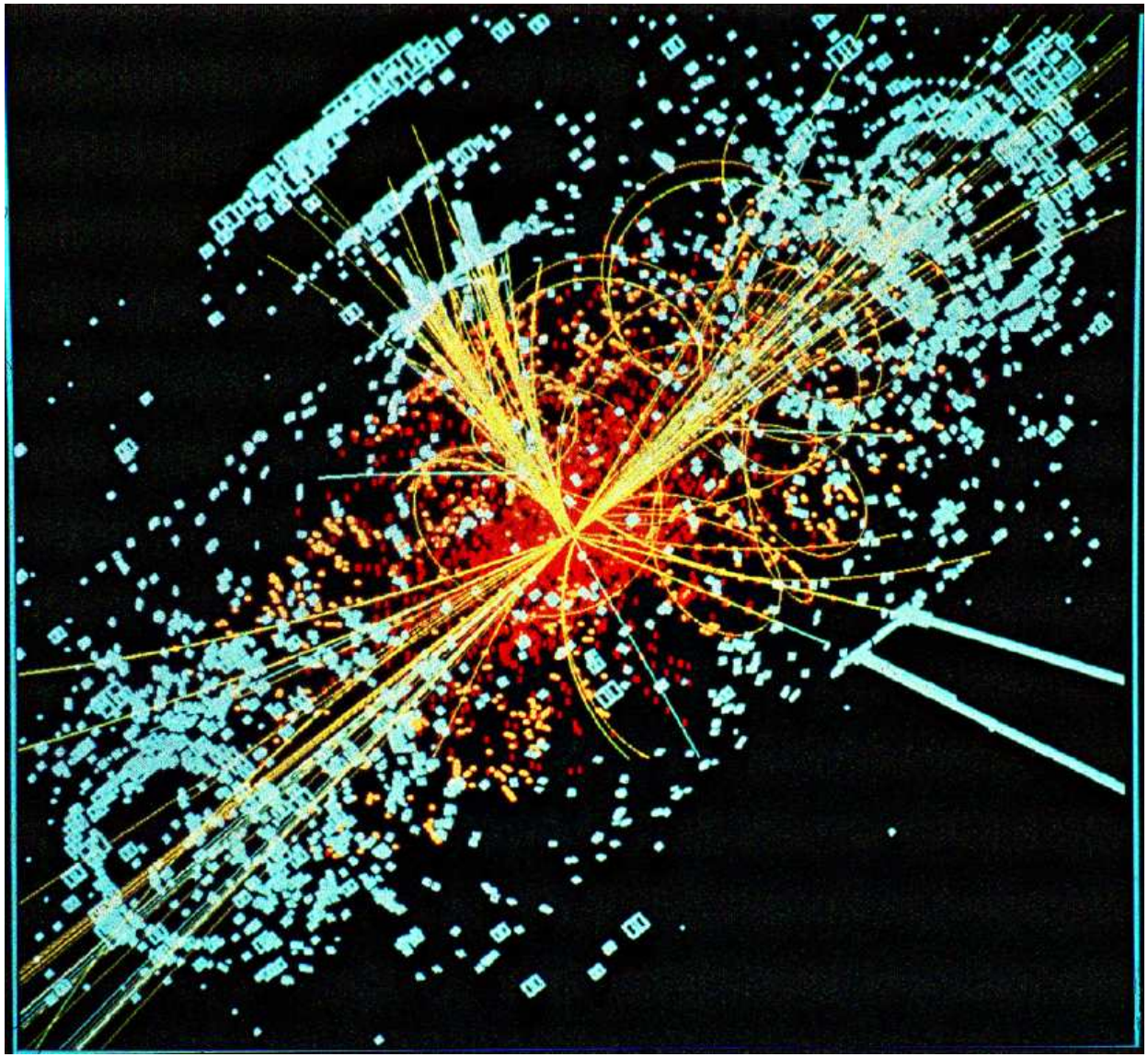
- FULL NEURAL FIT TO F_2 FOR PROTON, DEUTERON & NONSINGLET AVAILABLE
- ERRORS AND CORRELATIONS FAITHFULLY REPRODUCED, BUT STAT. UNCERTAINTIES OPTIMALLY COMBINED
  $\Rightarrow$ FIT CAN BE USED IN LIEU OF DATA, BUT BETTER THAN THEM (SEE F_2^{NS})
- FULL INTRICACIES OF THE PARTON FIT NOT IMPLEMENTED

OUTLOOK

WE NEED GOOD PDFS

OUTLOOK

WE NEED GOOD PDFS TO SEE IT CLEARLY!



Higgs decay in $e^+e^- +$ 2 jets at CMS