

Proton Structure and PDFs

Juan Rojo

VU Amsterdam & Theory group, Nikhef

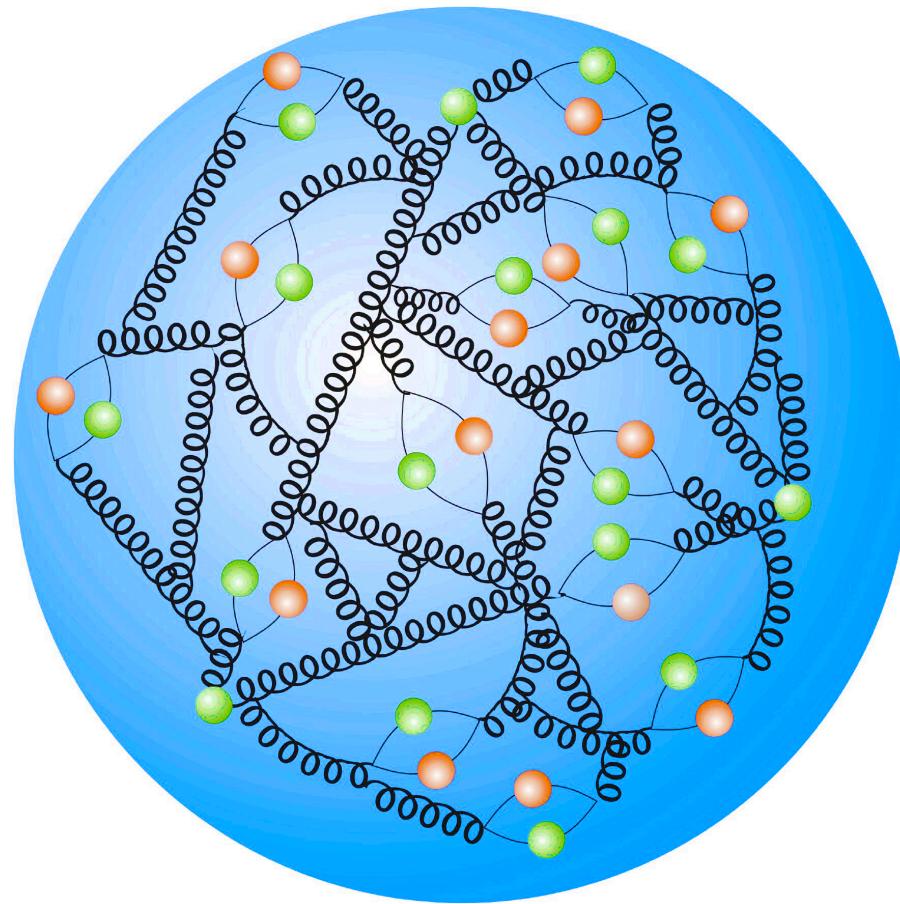
***XXVII International Workshop on Deep Inelastic
Scattering and Related Subjects (DIS2019)***



The inner life of protons

The many faces of the proton

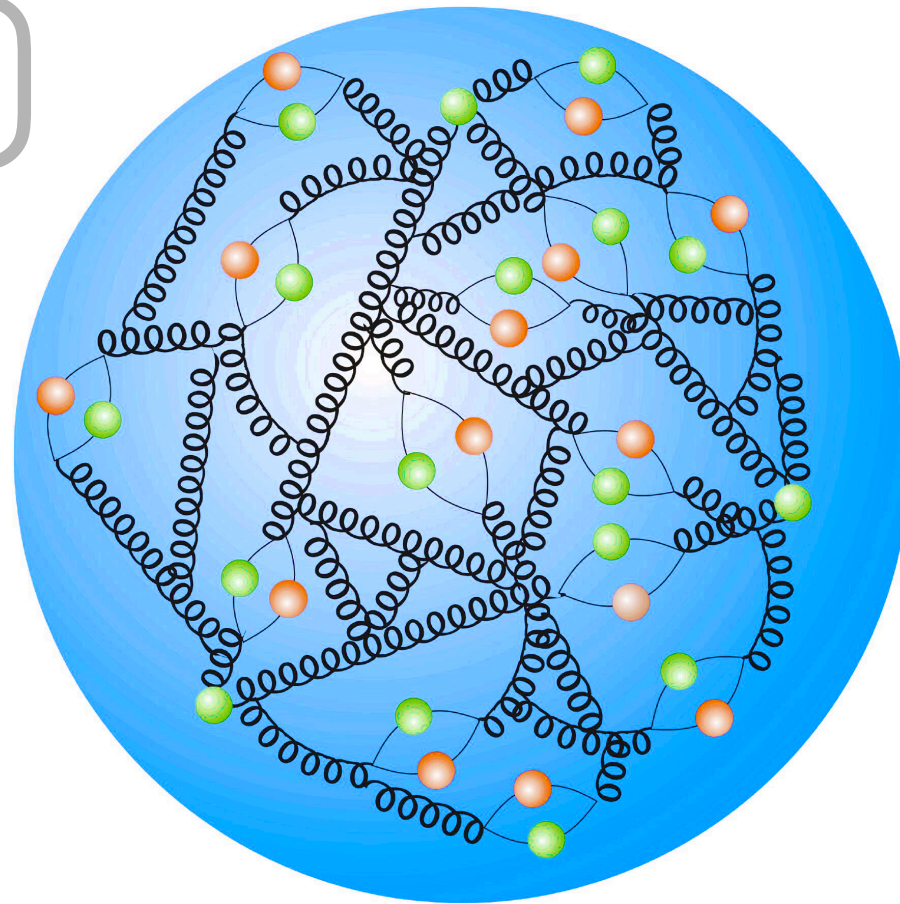
QCD bound state of **quarks** and **gluons**



The many faces of the proton

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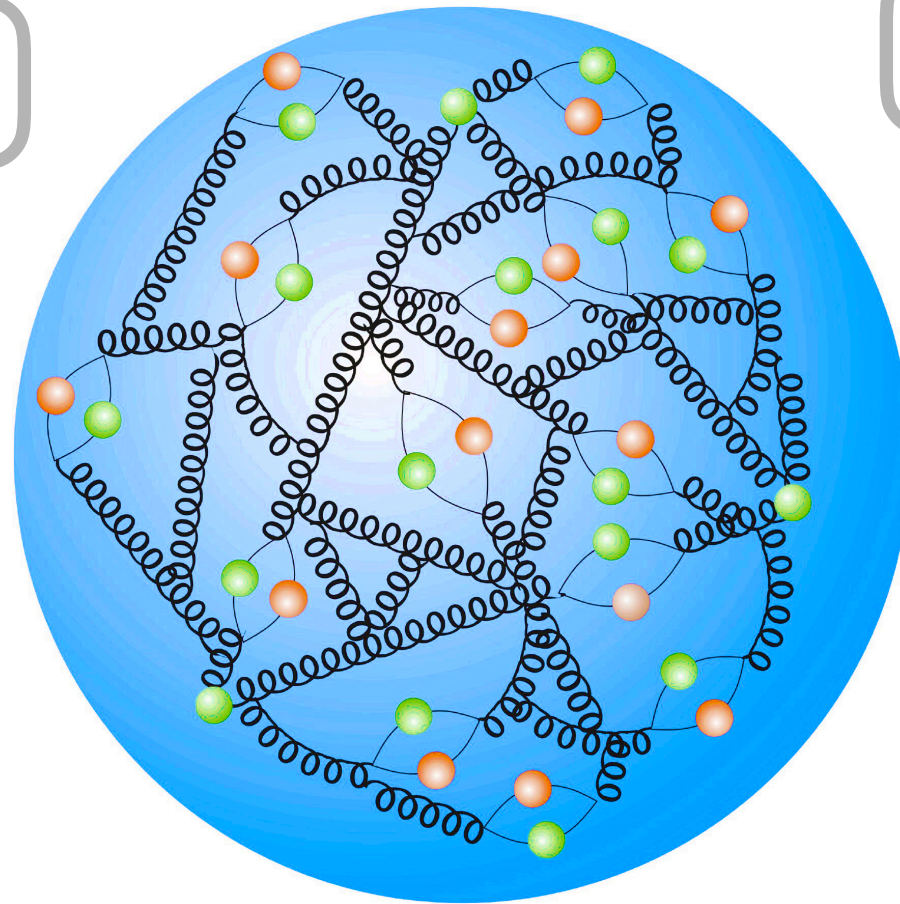
Origin of mass?



The many faces of the proton

QCD bound state of **quarks** and **gluons**

Origin of mass?



Origin of spin?

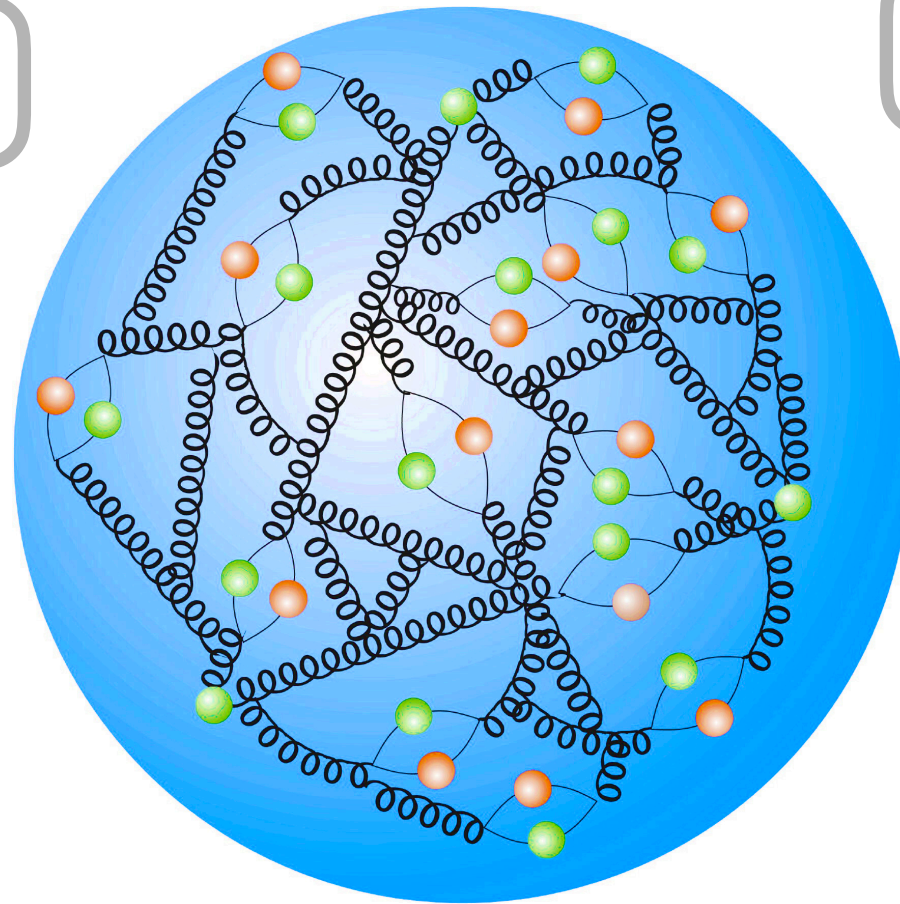
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Origin of mass?

Origin of spin?

*Gluon-dominated
matter?*



The many faces of the proton

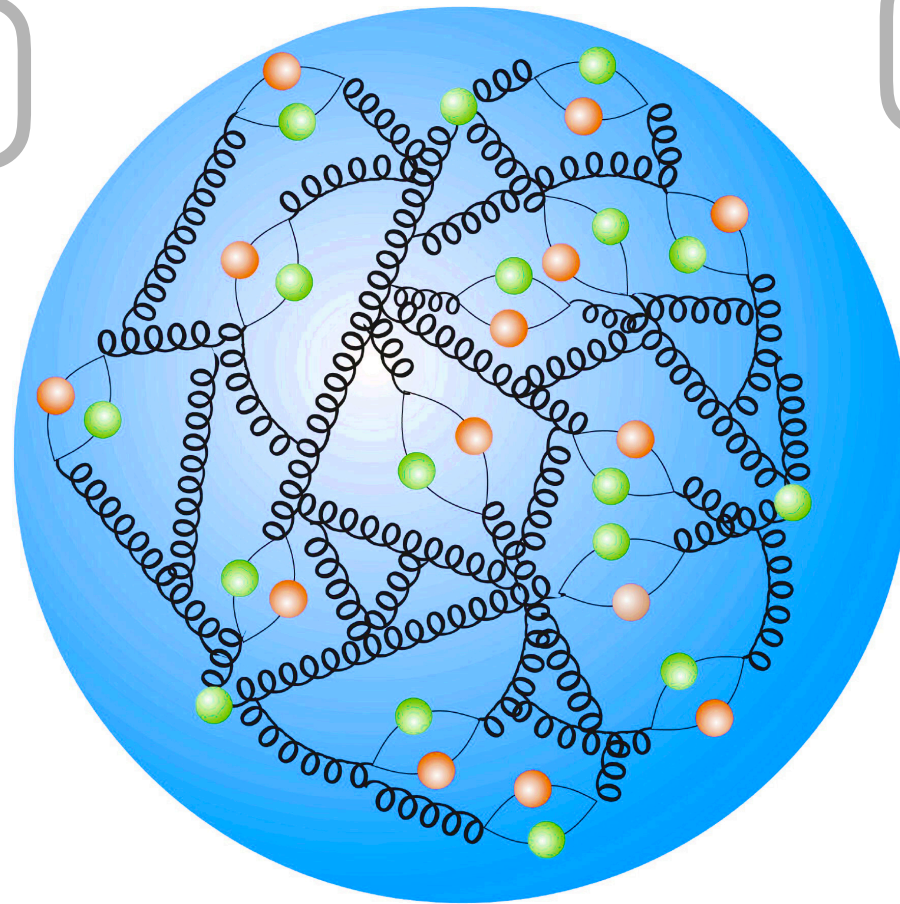
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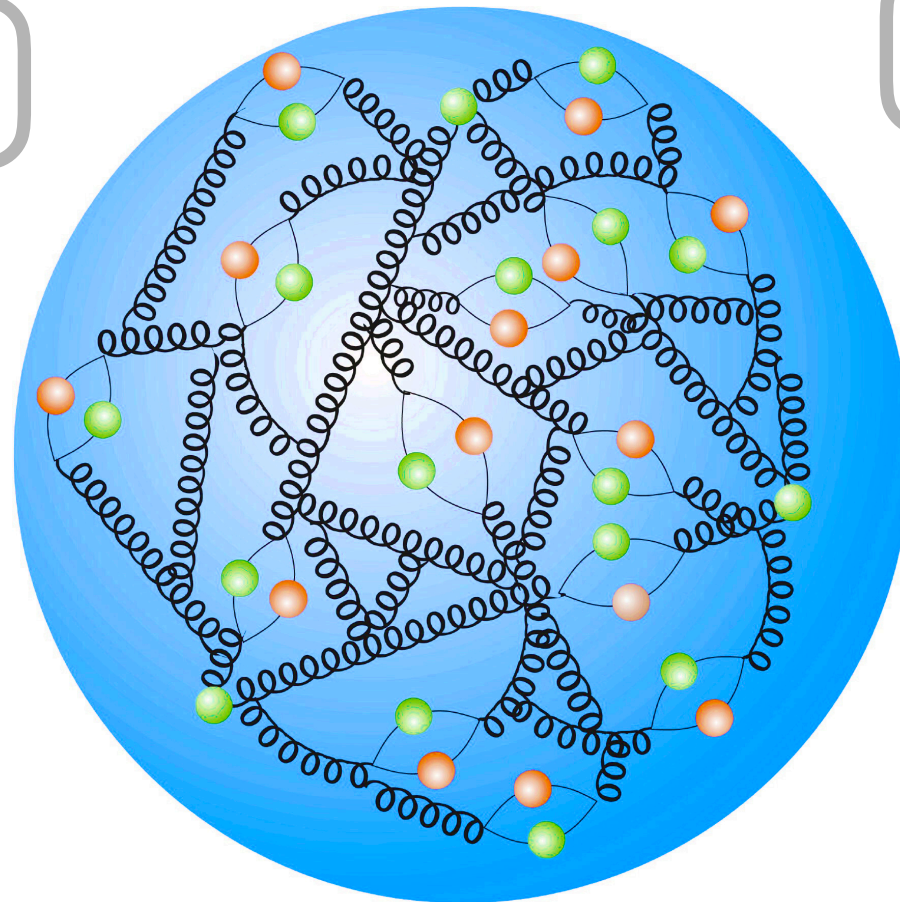
Origin of mass?

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3D imaging?

Heavy quark content?



The many faces of the proton

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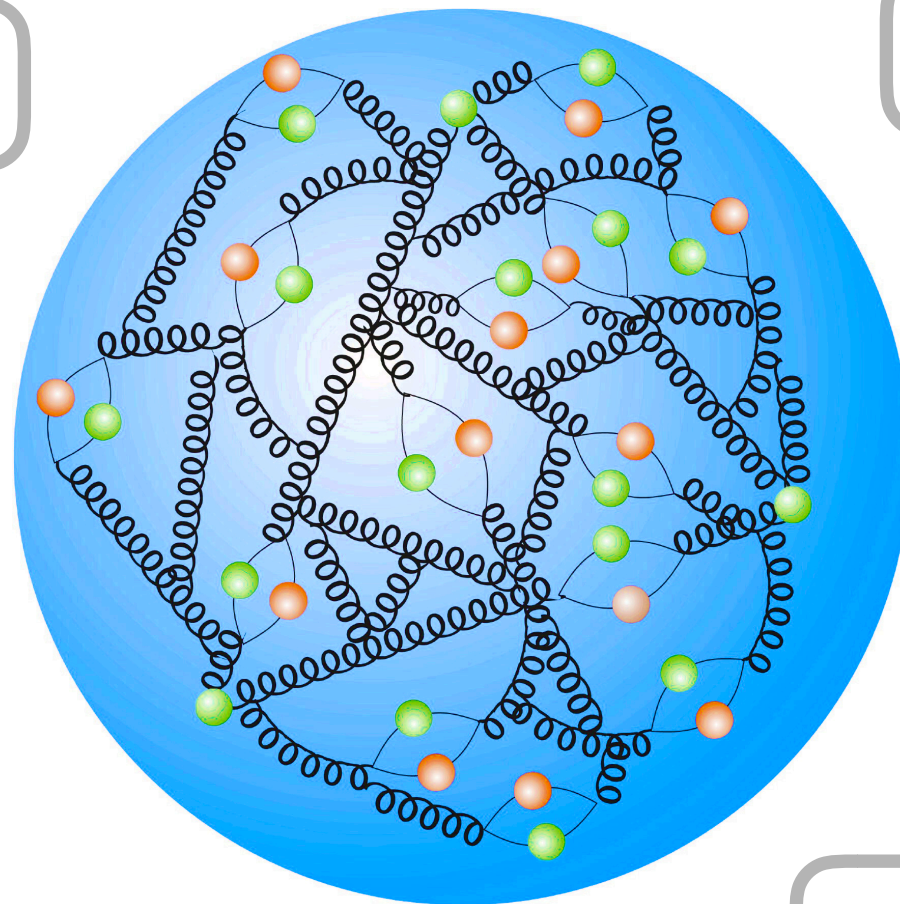
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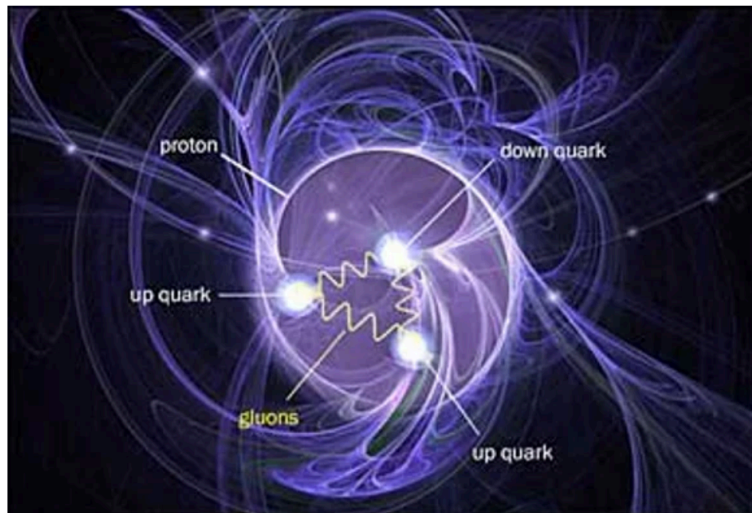
Nuclear modifications?



The proton in the spotlight

THE SCIENCES

Proton Spin Mystery Gains a New Clue



Non-zero gluon polarisation

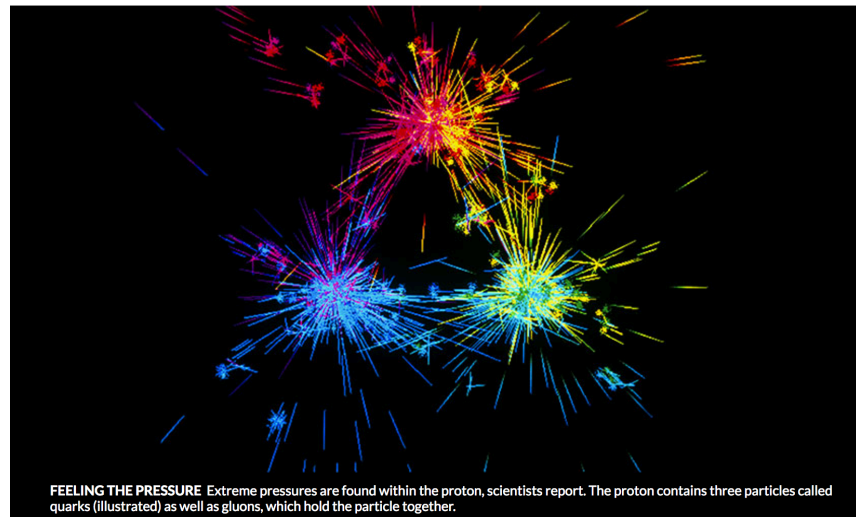
Scientific American (2014)

Nucleon pressure

NEWS PARTICLE PHYSICS

The inside of a proton endures more pressure than anything else we've seen

For the first time, scientists used experimental data to estimate the pressure inside a proton
BY EMILY CONOVER 1:18PM, MAY 16, 2018



Science News (2018)

After 40 years of studying the strong nuclear force, a revelation

This was the year that analysis of data finally backed up a prediction, made in the mid 1970s, of a surprising emergent behaviour in the strong nuclear force

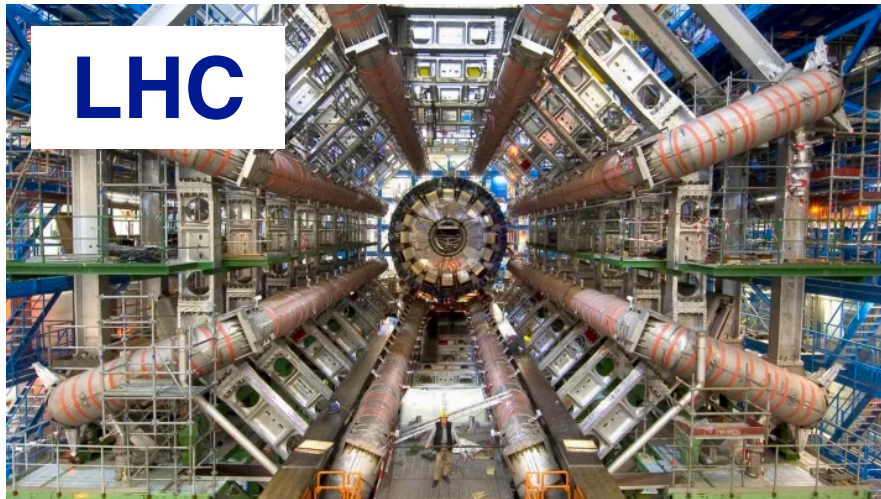


BFKL dynamics

The Guardian (2017)

The proton keeps surprising us as an endless source of **fundamental discoveries**

From colliders to the cosmos

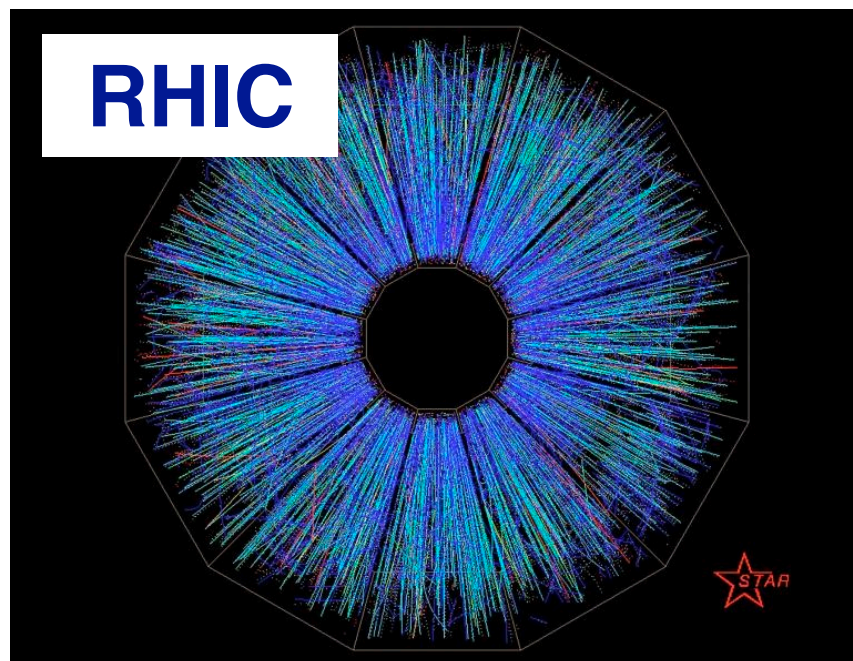


New **elementary particles**
beyond the **Standard Model?**

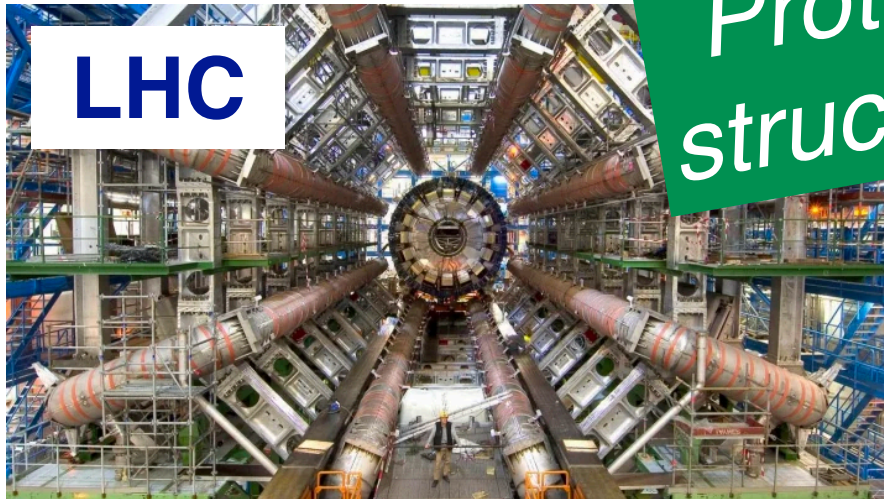
Origins and properties of
cosmic neutrinos?



Nature of **Quark-Gluon Plasma**
in **heavy-ion collisions?**



From colliders to the cosmos

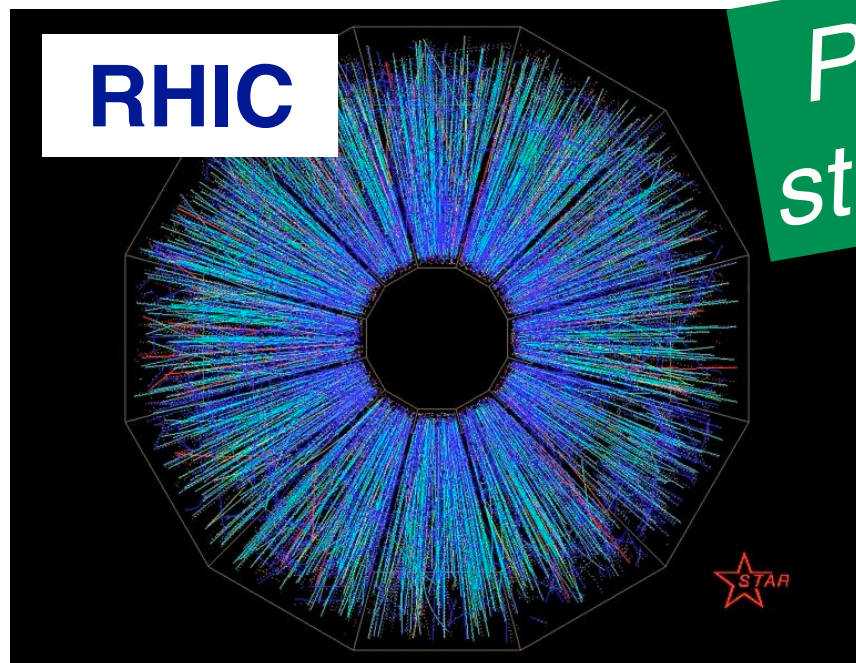


LHC

*Proton
structure*

New **elementary particles**
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Origins and properties of
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RHIC

*Proton
structure*

Nature of **Quark-Gluon Plasma**
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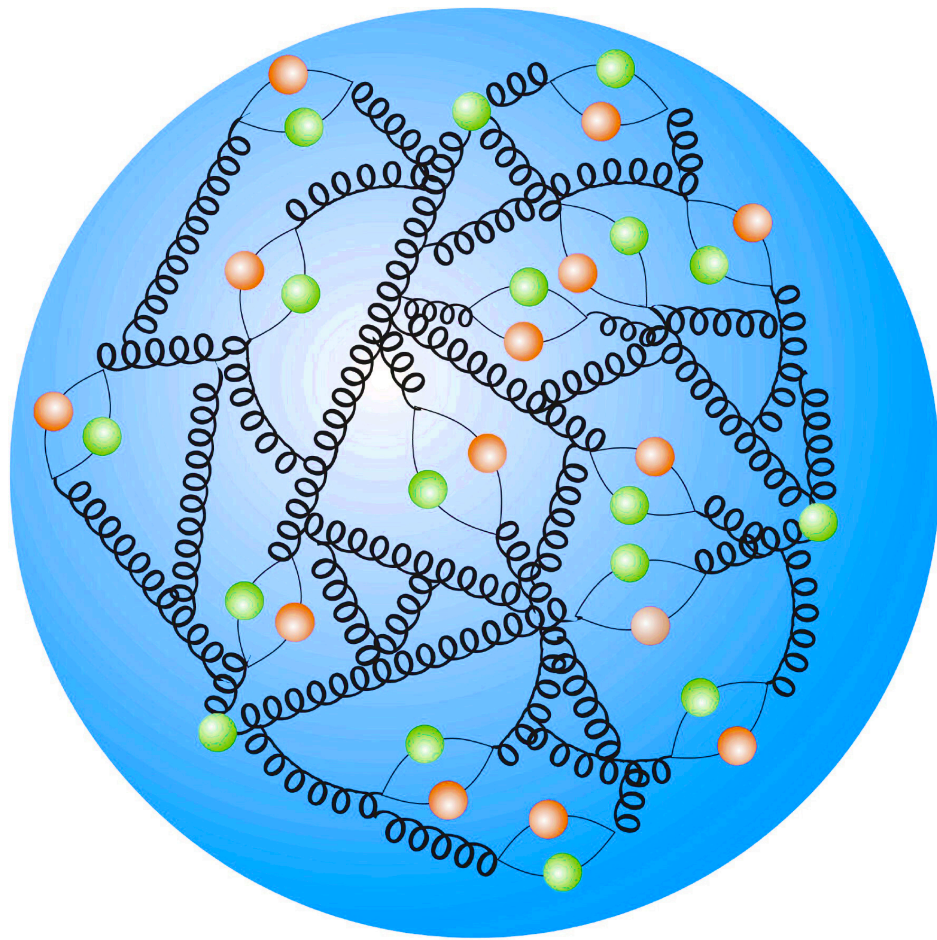
*Proton
structure*



IceCube

Parton Distributions

Proton energy divided among constituents: **quarks** and **gluons**



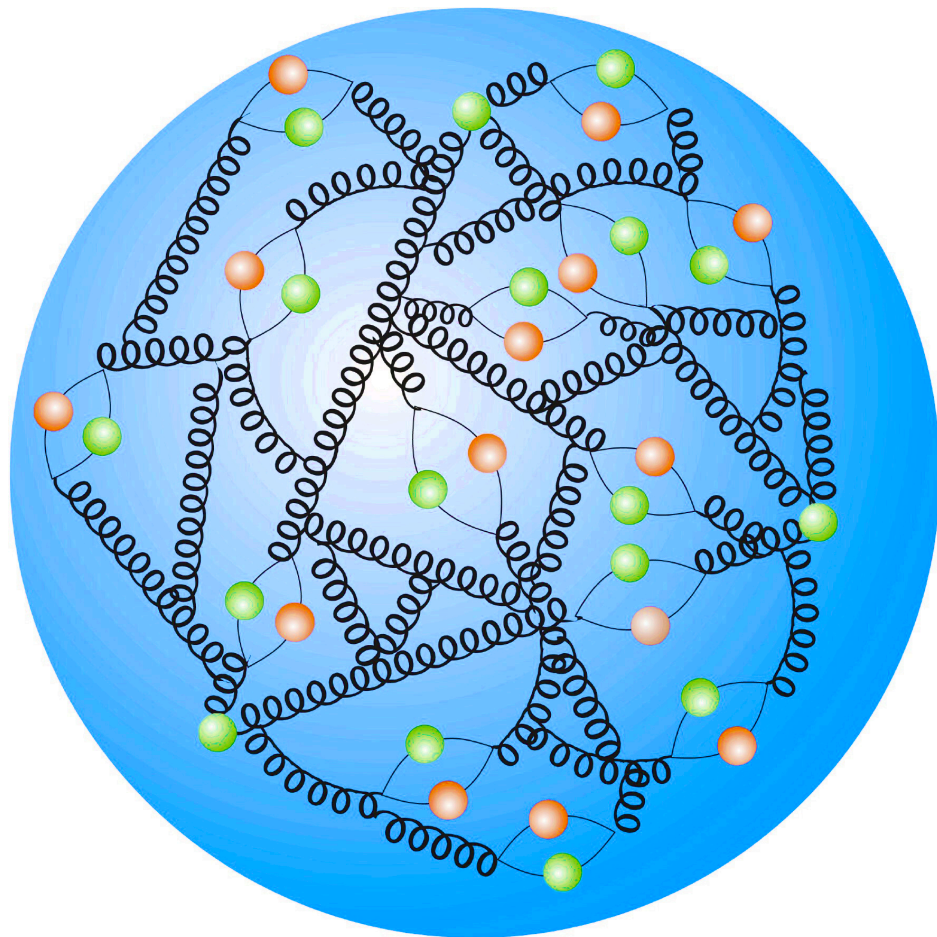
Parton Distribution Functions (PDFs)



Determine from **data**:
Global QCD analysis

Parton Distributions

Proton energy divided among constituents: **quarks** and **gluons**



Parton Distribution Functions (PDFs)



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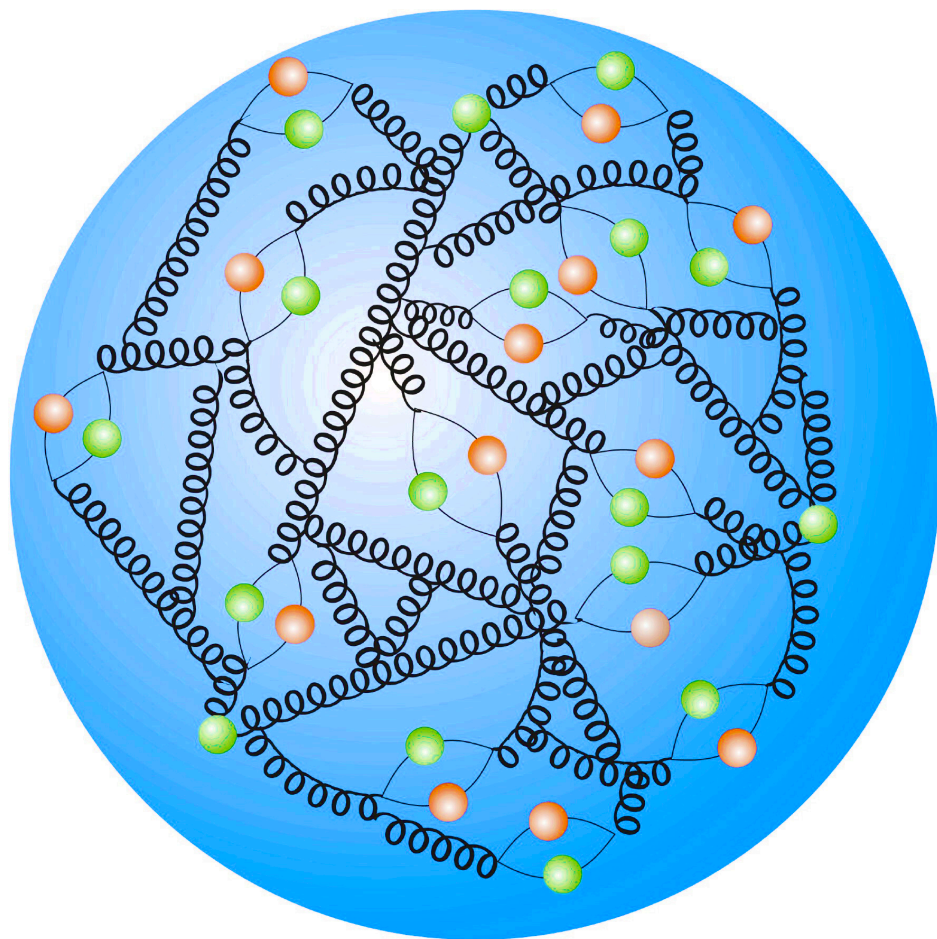
Lattice QCD starting to also make an impact

Parton Distributions

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Parton Distribution Functions (PDFs)

Determine from **data**:
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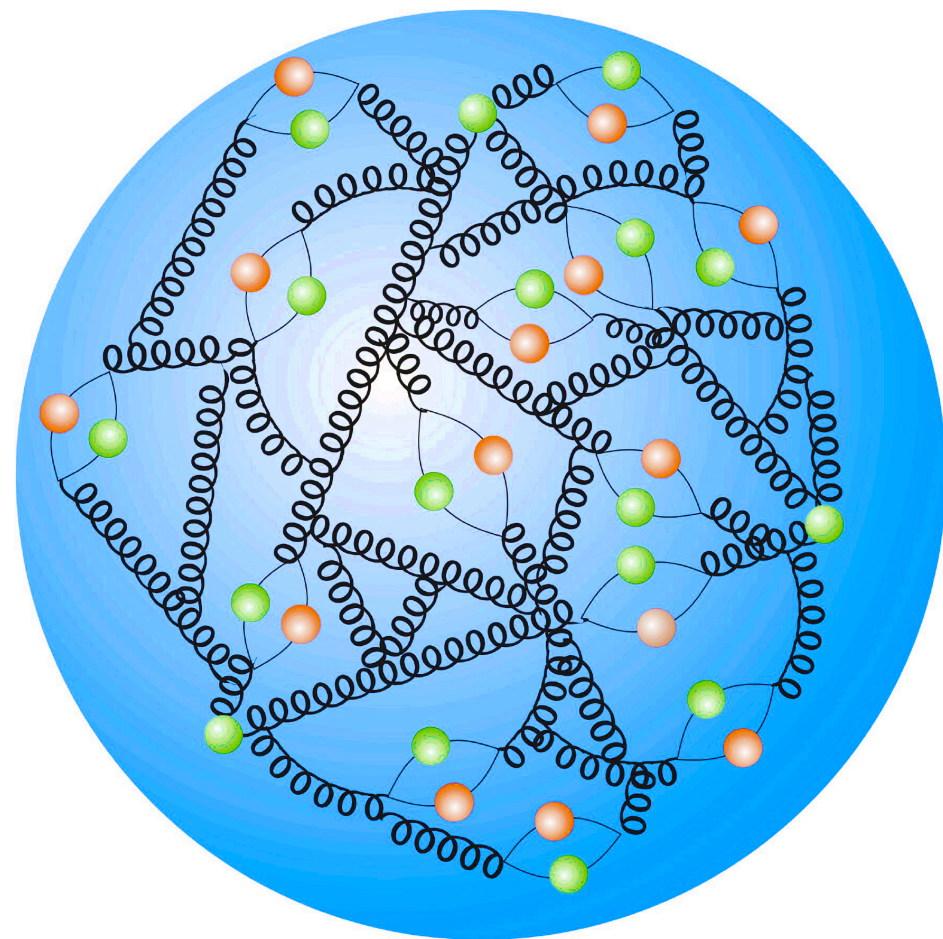
Mass? Spin?

Heavy quark content?

Novel QCD dynamics?

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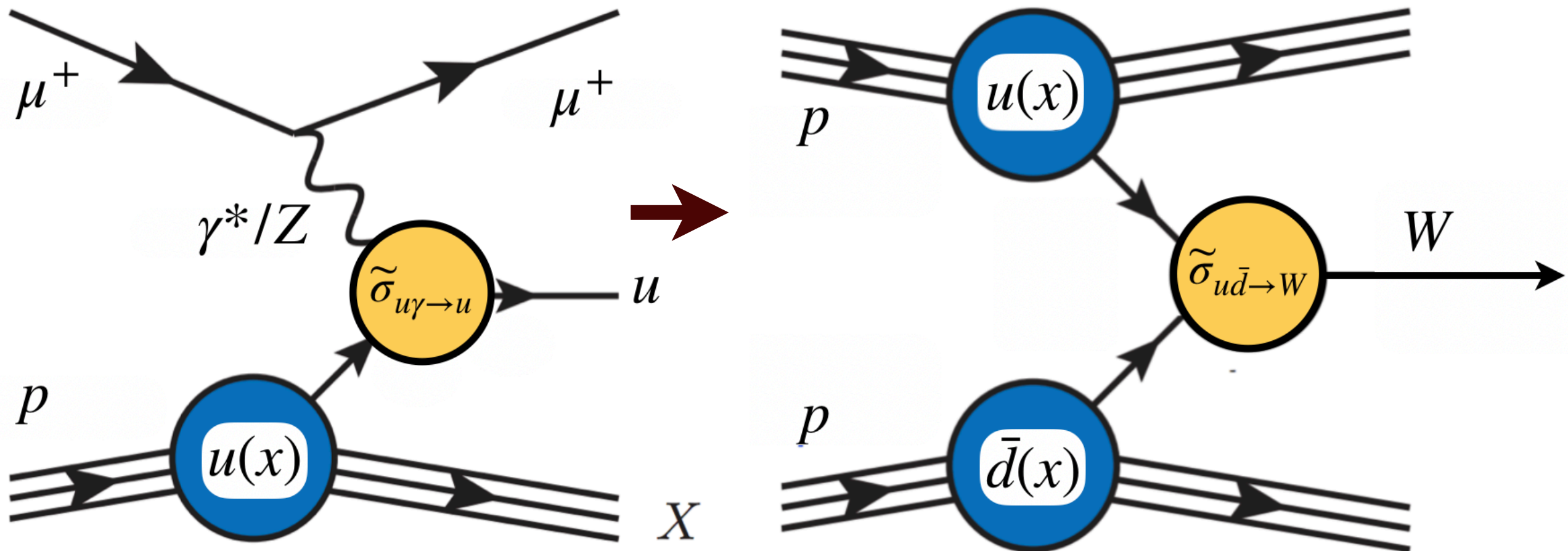
***Mass? Spin?
Heavy quark content?
Novel QCD dynamics?***

***Theoretical predictions
for LHC, RHIC, IceCube?***

The Global QCD analysis paradigm

QCD factorisation theorems: **PDF universality**

$$\sigma_{l p \rightarrow \mu X} = \tilde{\sigma}_{u\gamma \rightarrow u} \otimes u(x) \longrightarrow \sigma_{p p \rightarrow W} = \tilde{\sigma}_{u\bar{d} \rightarrow W} \otimes u(x) \otimes \bar{d}(x)$$



Determine PDFs from **deep-inelastic scattering...**

... and use them to compute predictions for **proton-proton collisions**

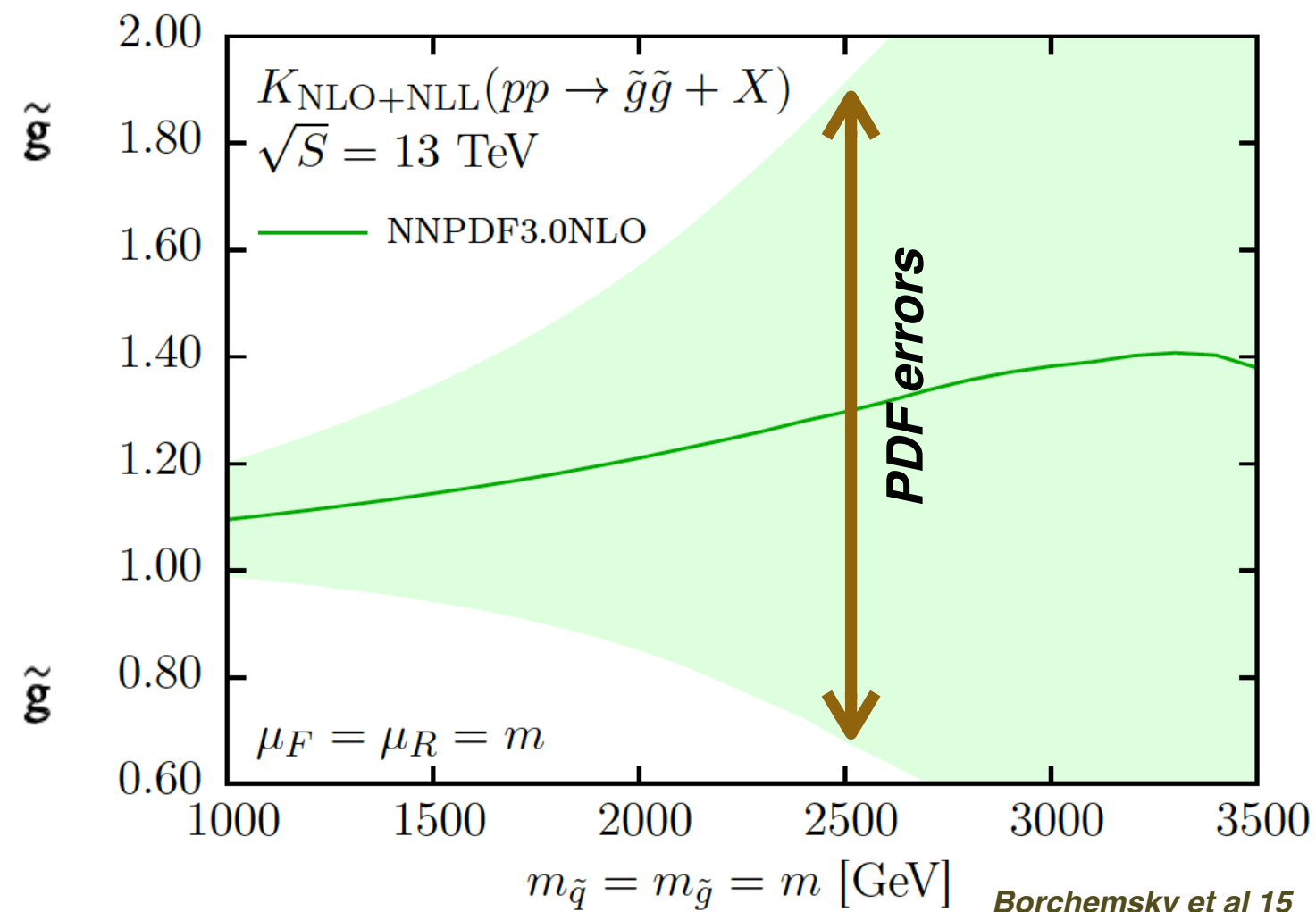
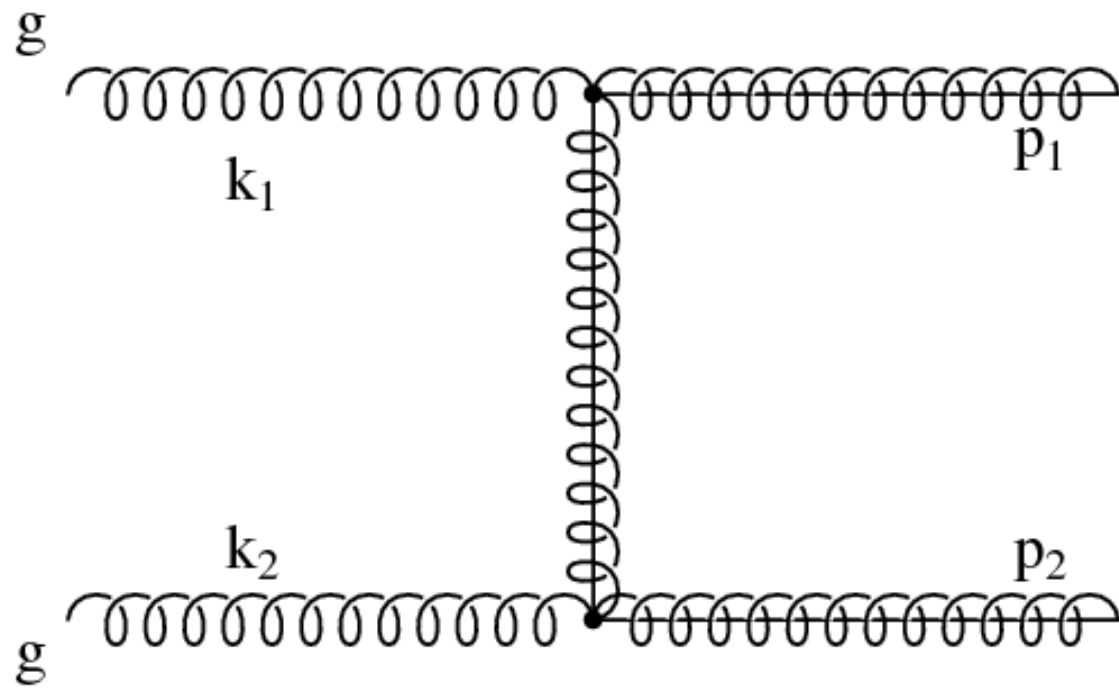
**Why do we need
better PDFs?**

Why do we need better PDFs?

PDF uncertainties in the production of **New Physics heavy resonances** up to **100%**

Due to limited coverage of the **large Bjorken-x** region

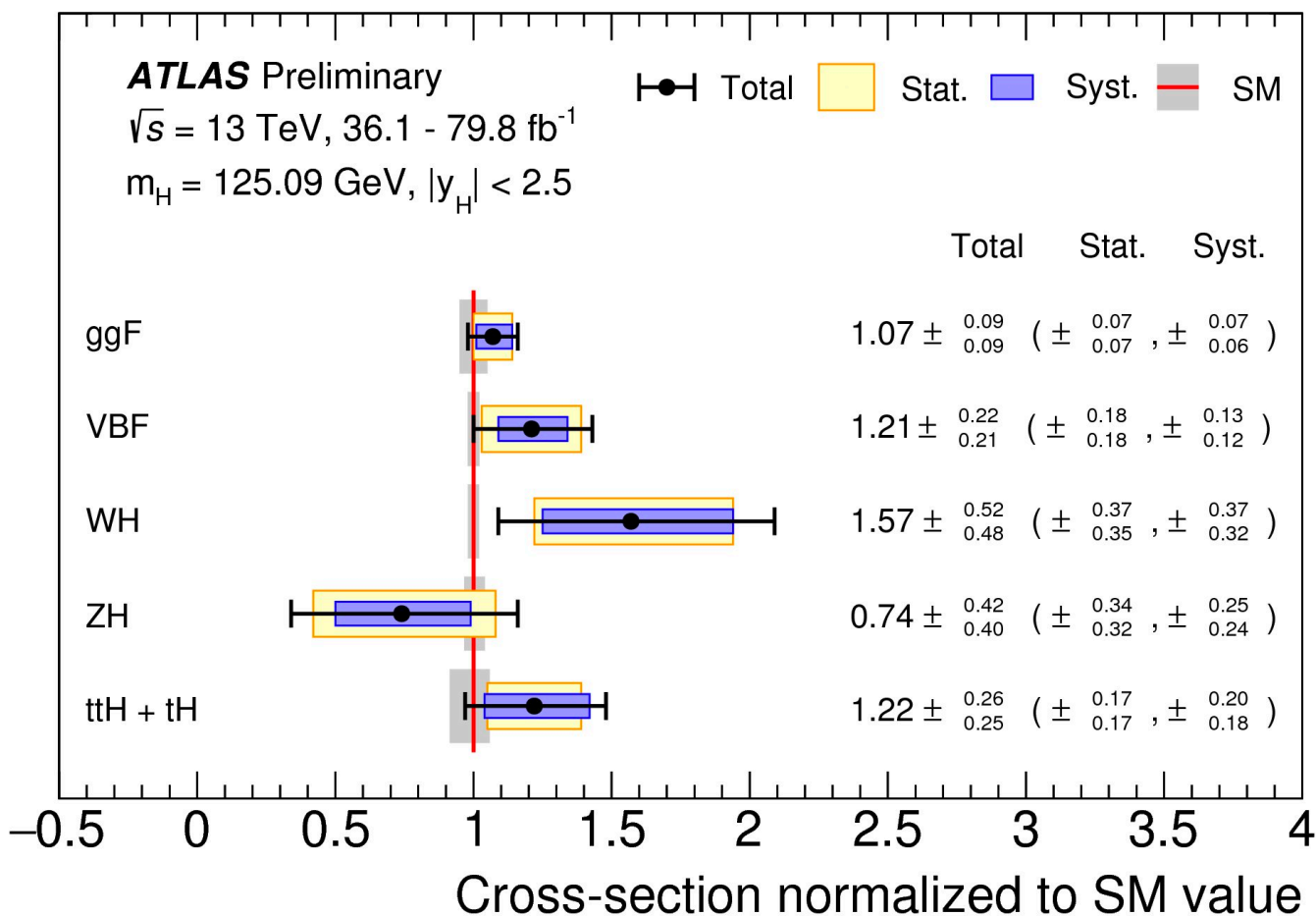
gluino-pair production in supersymmetry



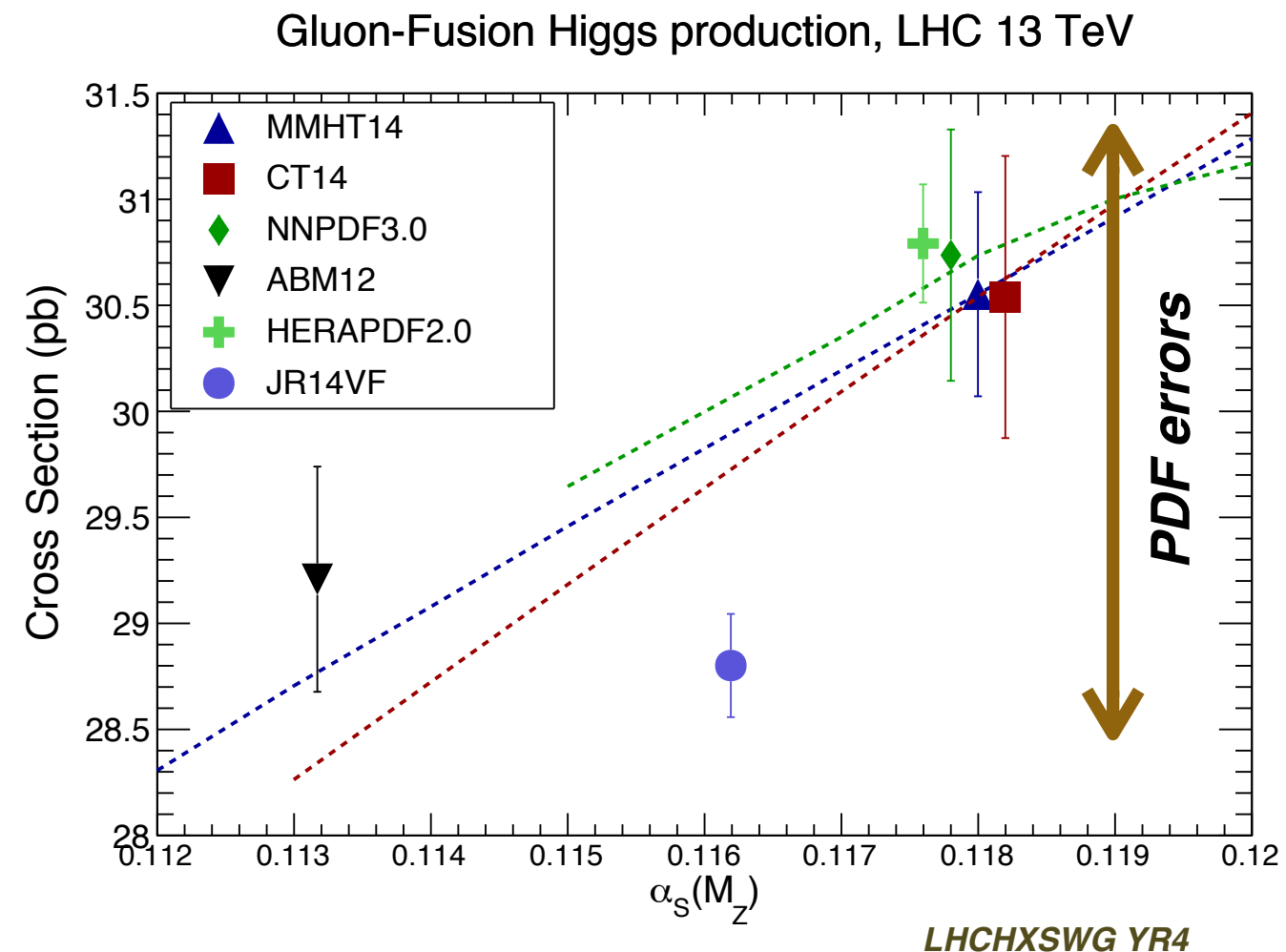
Why do we need better PDFs?

PDF uncertainties one of dominant theory errors in Higgs production cross-sections
any small **deviations of Higgs couplings** from SM predictions: **smoking gun for BSM**

Inclusive Higgs production rates



Juan Rojo

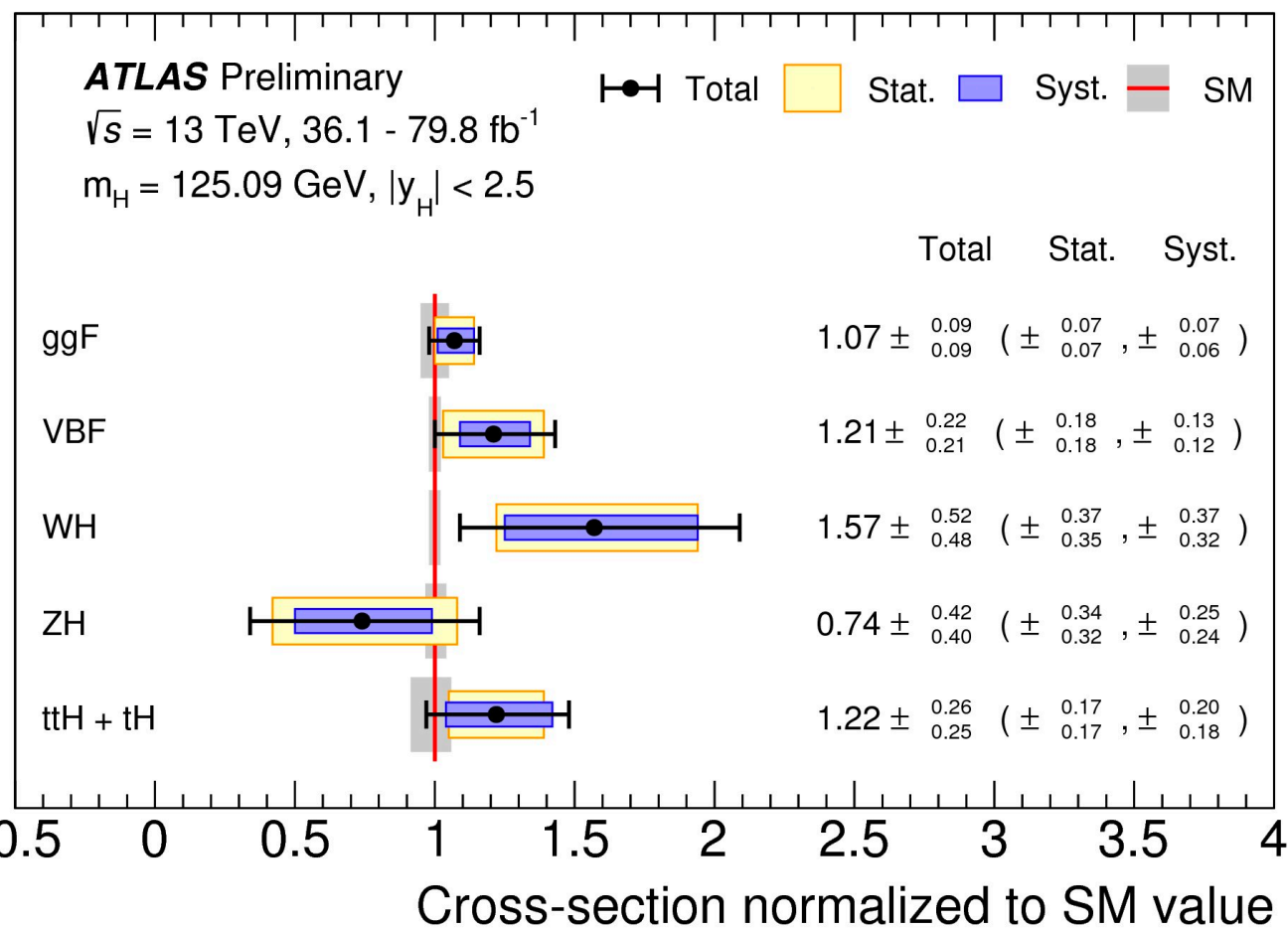


Proton Structure and PDFs, DIS2019

Why do we need better PDFs?

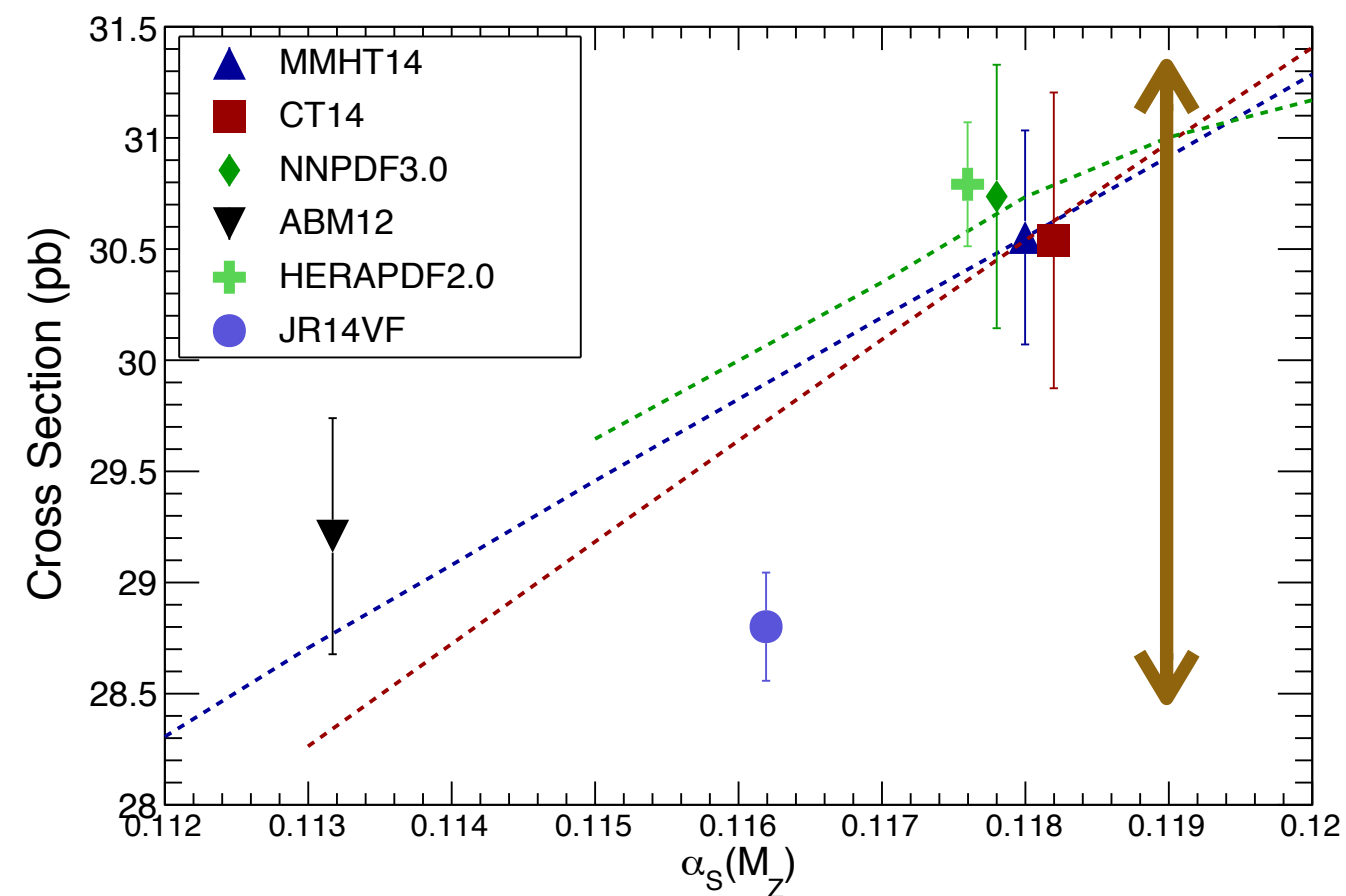
Snowmass 13

BSM model	Deviations in Higgs coupling to		
	W, Z weak bosons	bottom quarks	photons
New heavy Higgs boson	6%	6%	6%
Two-Higgs Doublet model	1%	10%	1%
Composite Higgs	-3%	-9%	-9%
New heavy top-like quark	-2%	-2%	+2%



Juan Rojo

Gluon-Fusion Higgs production, LHC 13 TeV



LHCHSWG YR4

Proton Structure and PDFs, DIS2019

Why do we need better PDFs?

- Heavy bSM physics beyond the direct reach of the LHC can be **parametrised in a model-independent** in terms of a **complete** basis of higher-dimensional operators: this is the **Standard Model Effective Field Theory**

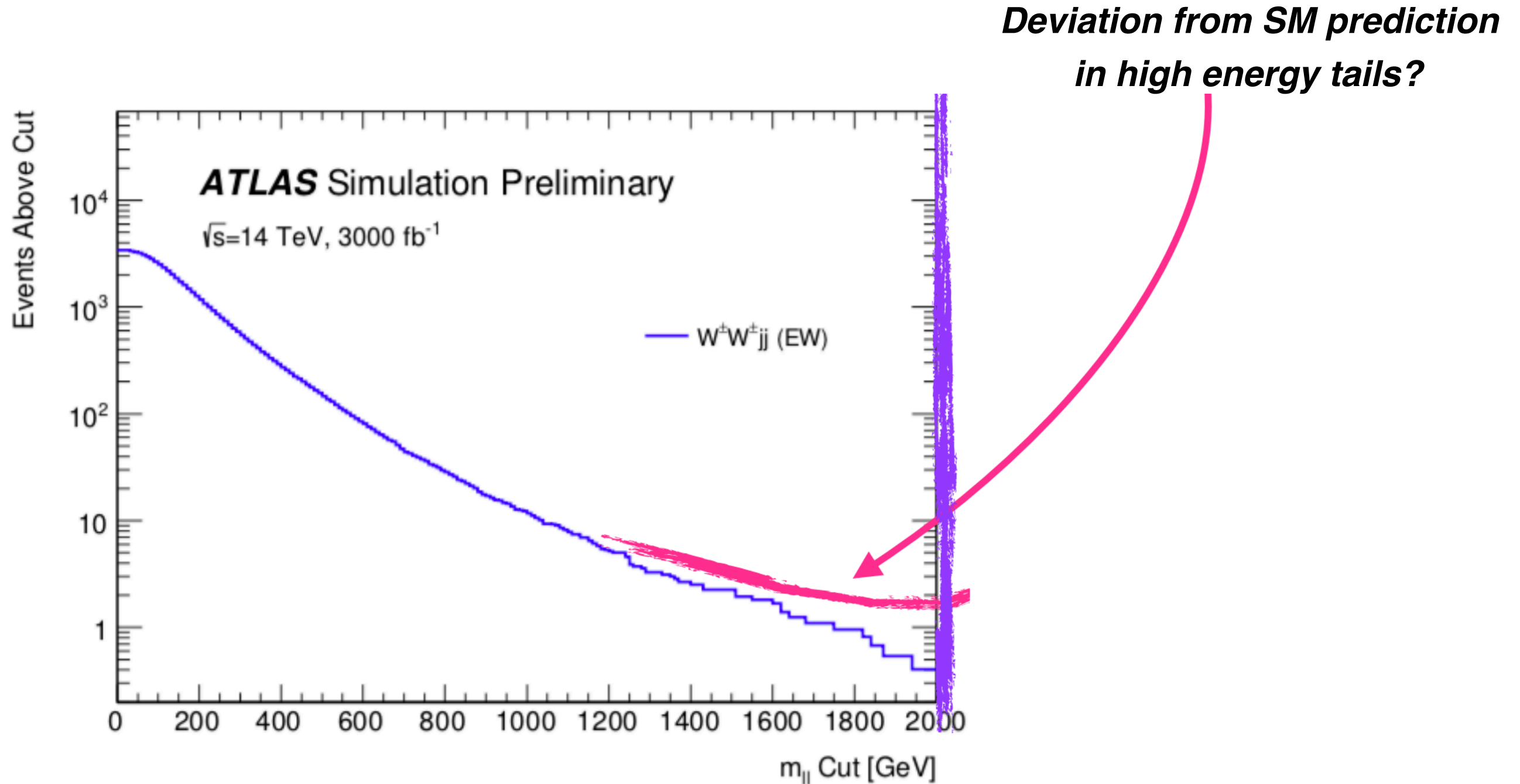
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i^{N_{d6}} \frac{c_i}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j^{N_{d8}} \frac{b_j}{\Lambda^4} \mathcal{O}_j^{(8)} + \dots,$$

- Some operators induce **growth with the partonic centre-of-mass energy**:
increased sensitivity in LHC cross-sections in the TeV region

$$\sigma(E) = \sigma_{\text{SM}}(E) \left(1 + \sum_i^{N_{d6}} \omega_i \frac{c_i m_{\text{SM}}^2}{\Lambda^2} + \sum_i^{N_{d6}} \tilde{\omega}_i \frac{c_i E^2}{\Lambda^2} + \mathcal{O}(\Lambda^{-4}) \right)$$

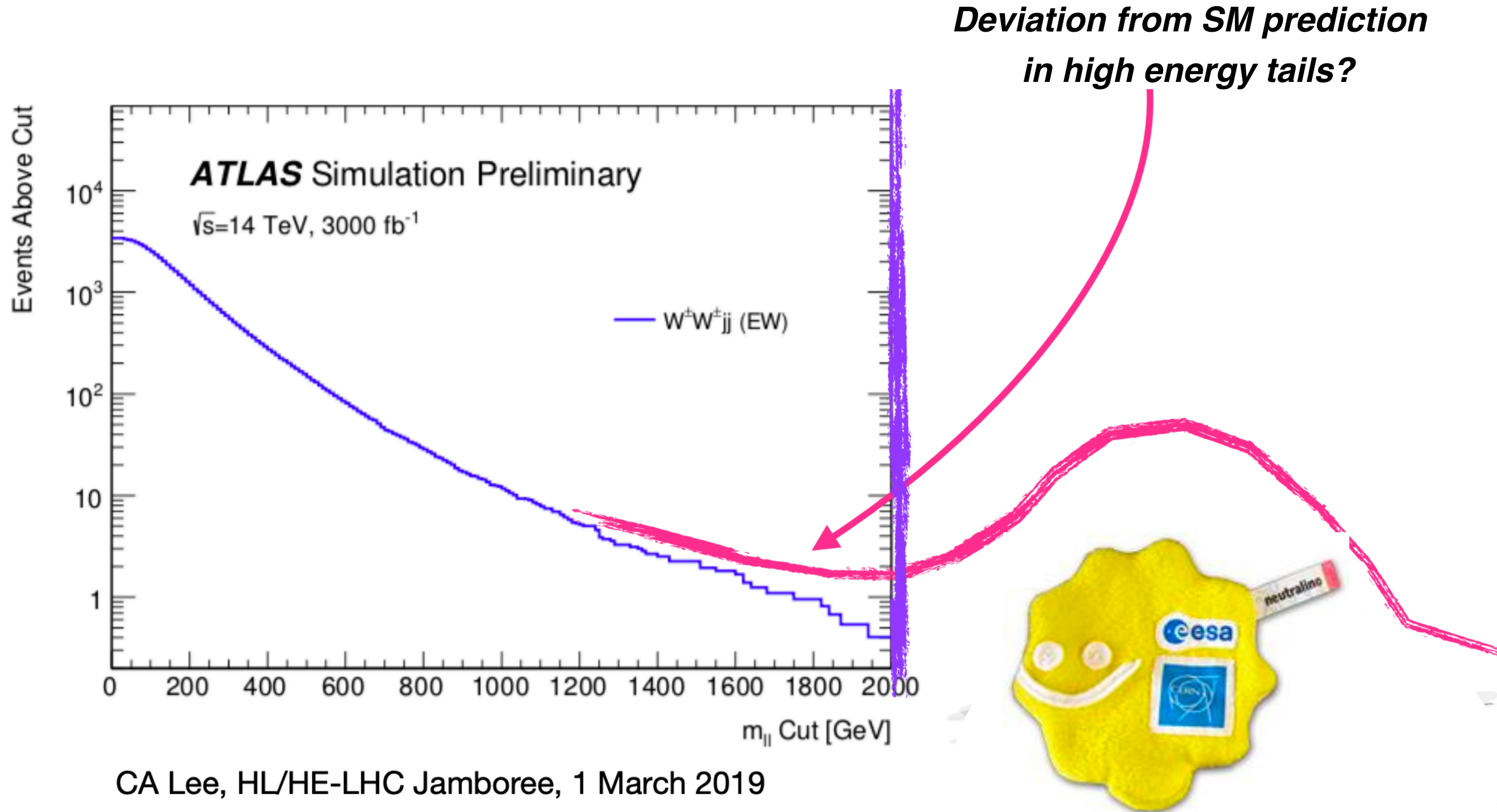

*enhanced sensitivity from **TeV-scale processes**:
unique feature of LHC*

Why do we need better PDFs?



CA Lee, HL/HE-LHC Jamboree, 1 March 2019

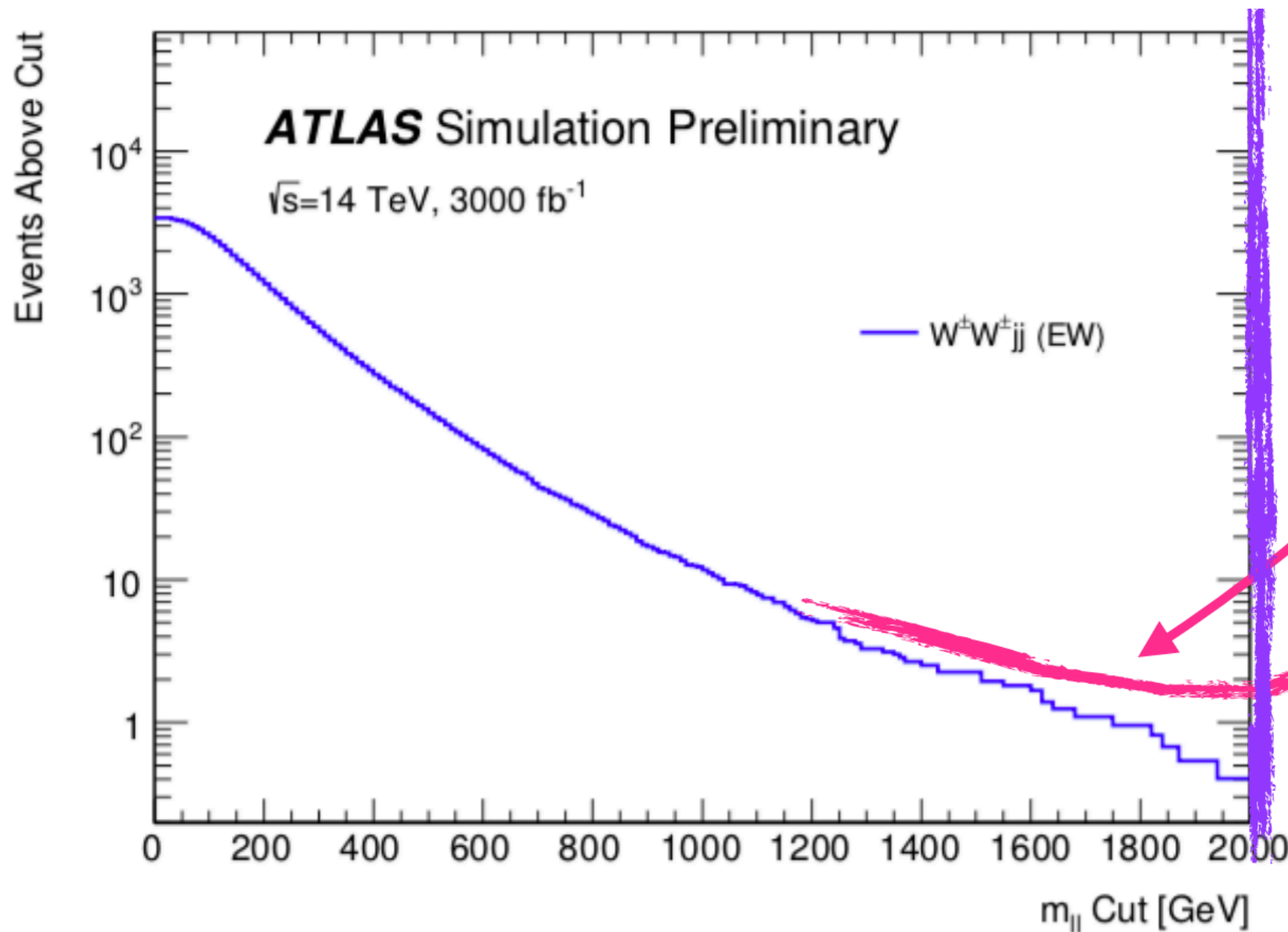
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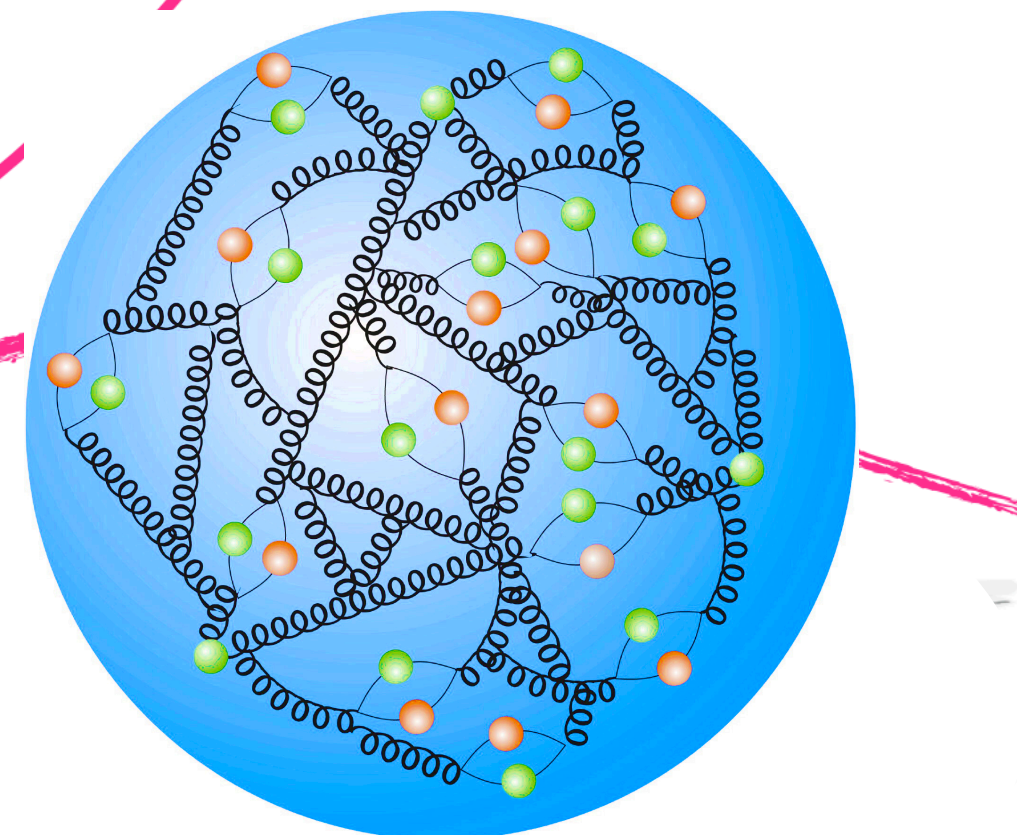
SMEFT interpretation: from a massive particle at high energies ...

Why do we need better PDFs?

*Deviation from SM prediction
in high energy tails?*



CA Lee, HL/HE-LHC Jamboree, 1 March 2019



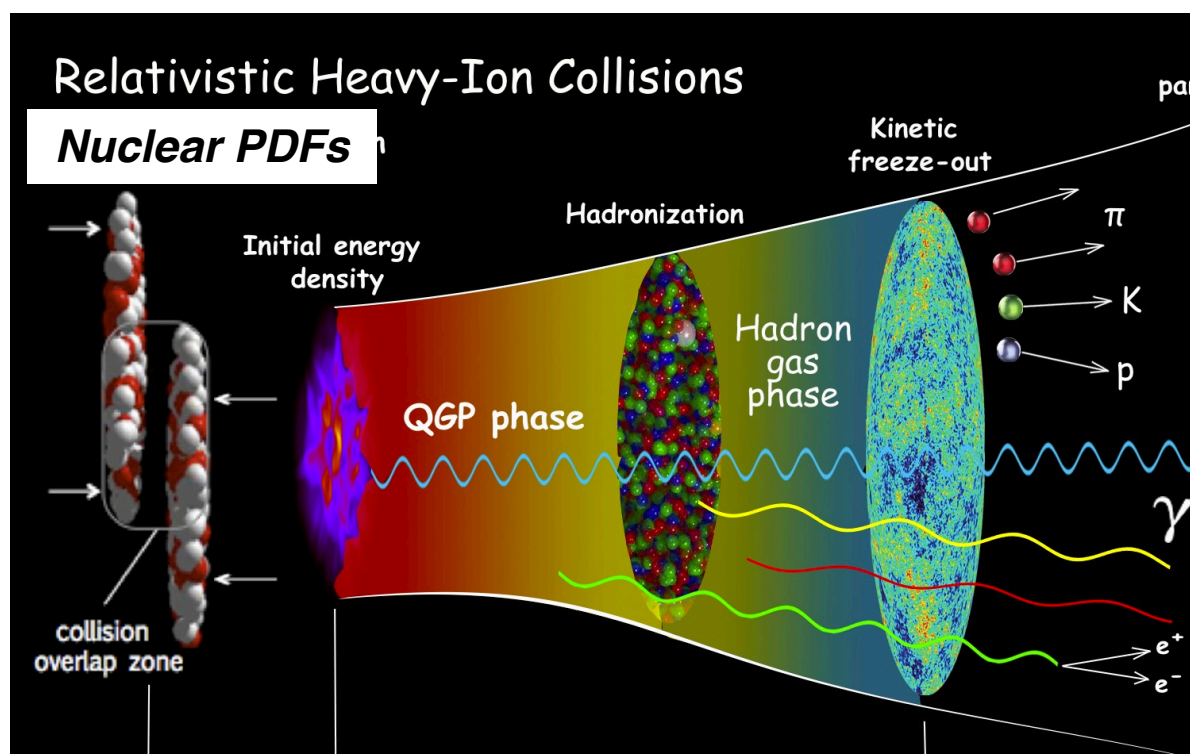
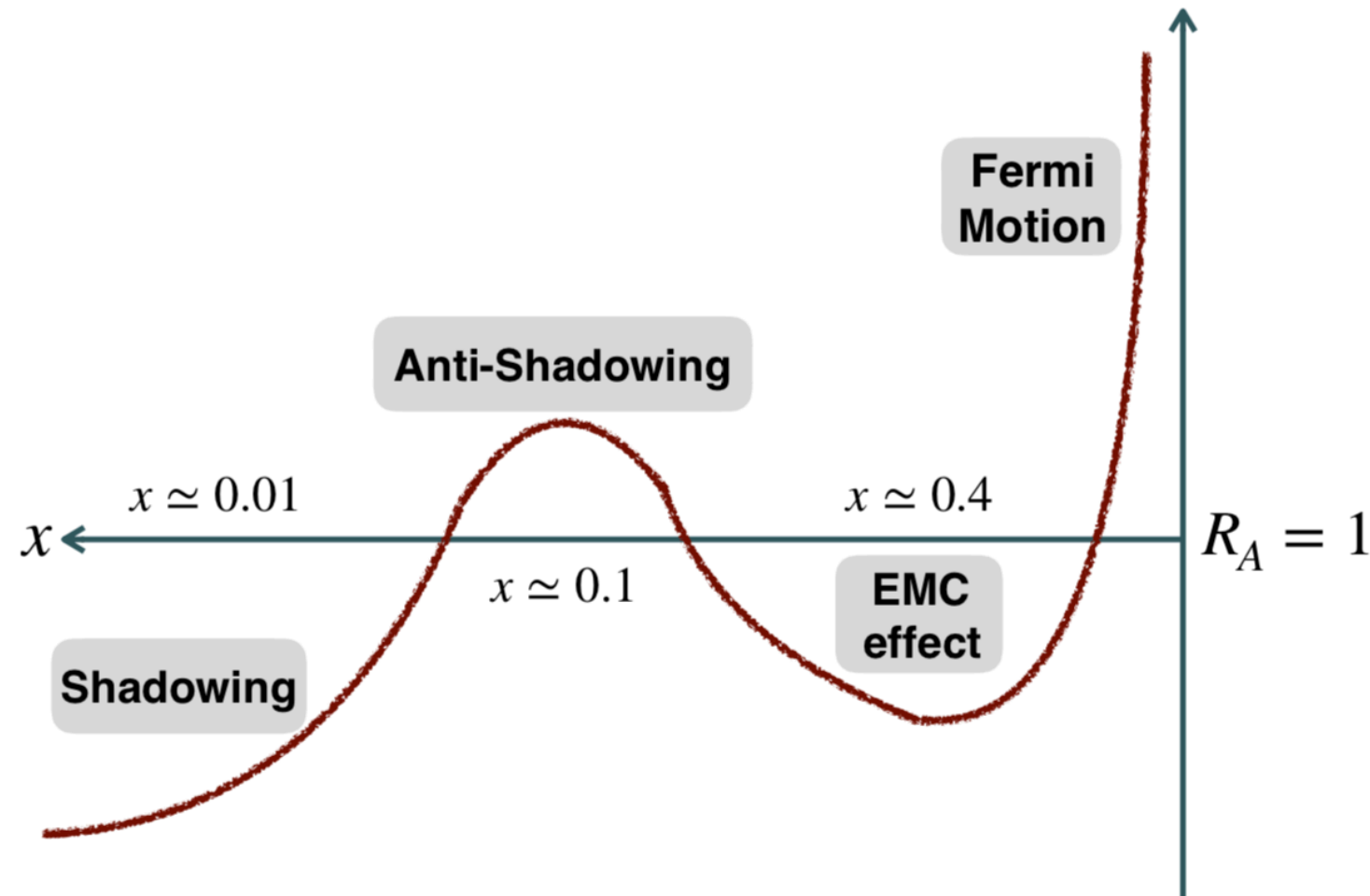
Iranipour WG3

...or reflecting our limited understating of proton structure?

Hot and Cold Nuclear Matter

- 📌 Cold nuclear matter effects modify the **PDFs of bound nucleons** as compared to the **free-proton case**
- 📌 Rich **QCD phenomenology**: EMC effect, shadowing, non-linear evolution,
- 📌 Onset of **new gluon-dominated state of matter**: the Color Glass Condensate

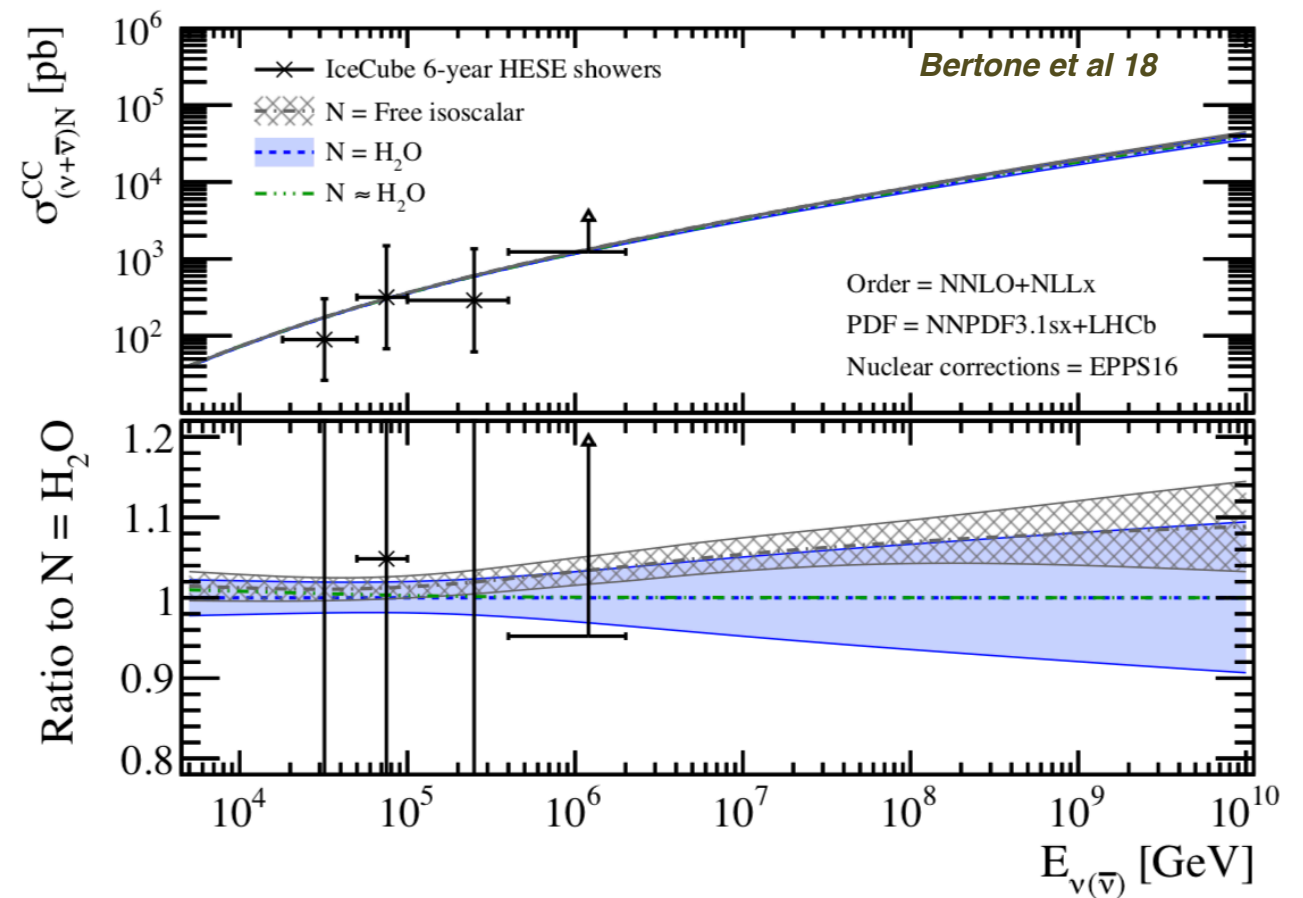
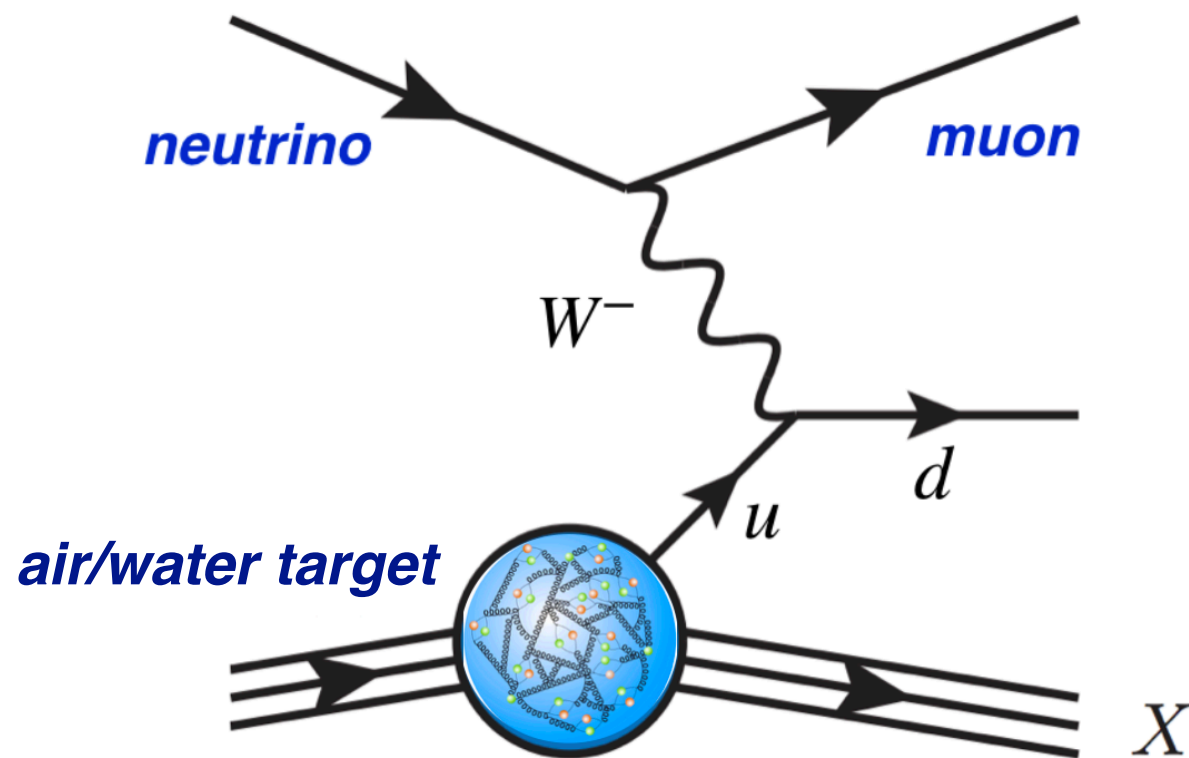
$$R_A = \frac{\text{Bound Nucleon PDF in Nucleus } A}{\text{Free nucleon PDF}}$$



- 📌 **Initial state of heavy-ion collisions:** Quark-Gluon Plasma characterisation
- 📌 nPDFs also required for **ultra-high-energy astrophysics** e.g. neutrino telescopes

Neutrino telescopes as QCD microscopes

*Ultra-high energy (cosmic) neutrino - nucleus scattering:
unique probe of **small- x PDFs and QCD***



JR WG1+WG7

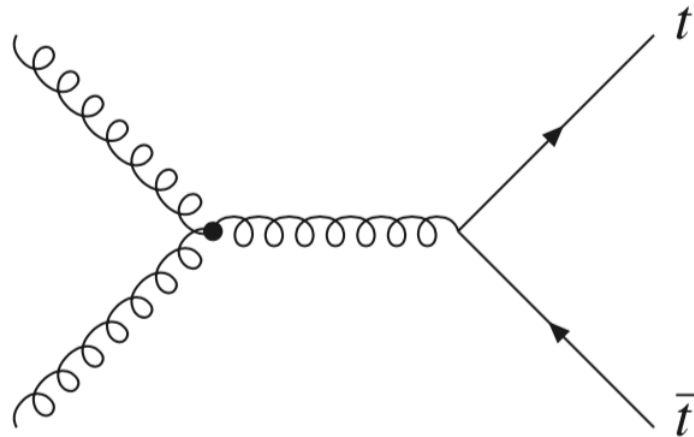
Sensitive to **small- x quarks** (and gluons via evolution)
down to $x \approx 10^{-8}$ at $Q \approx M_W$

Recent progress in the proton structure

PDF information from p+p collisions

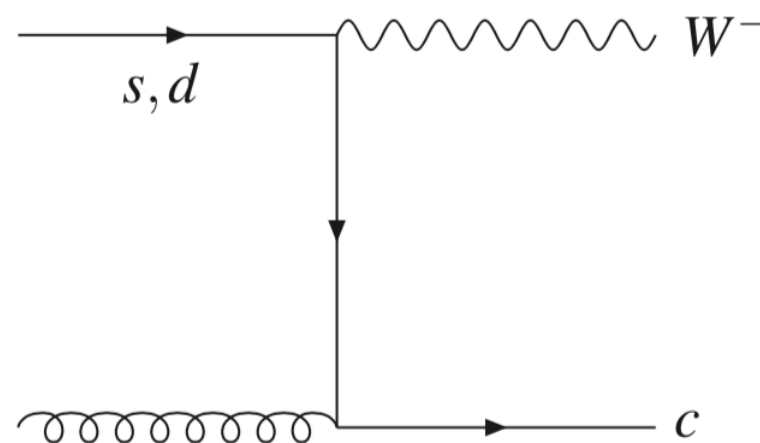
large-x gluon

Top quark pair production



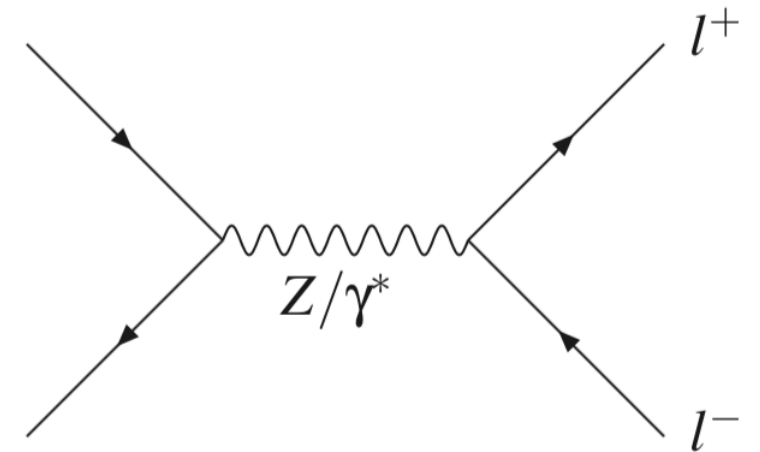
strangeness

$W + c$ production

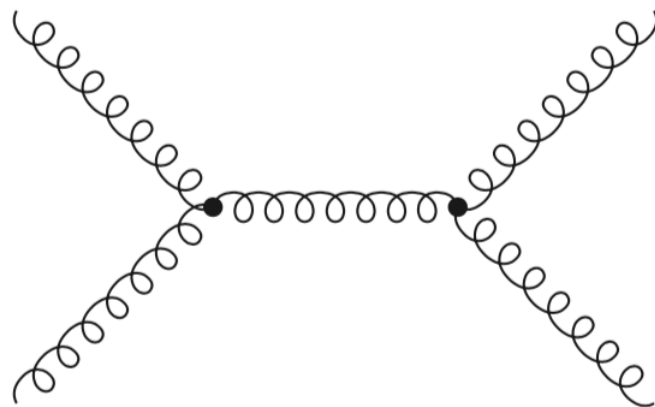


antiquarks

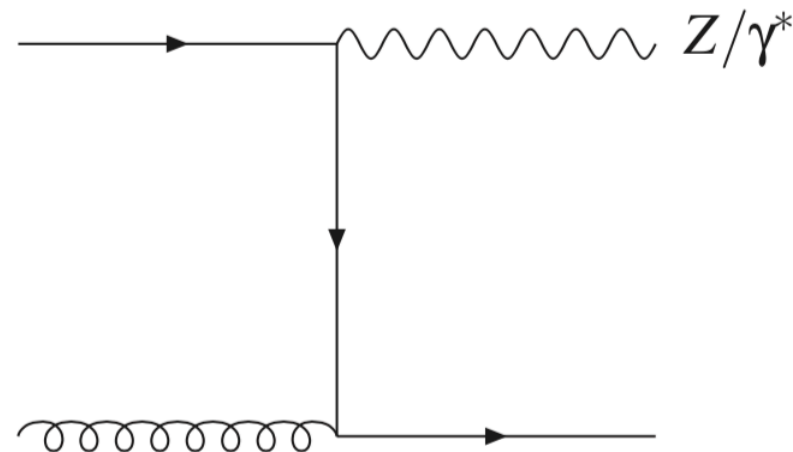
Drell–Yan production



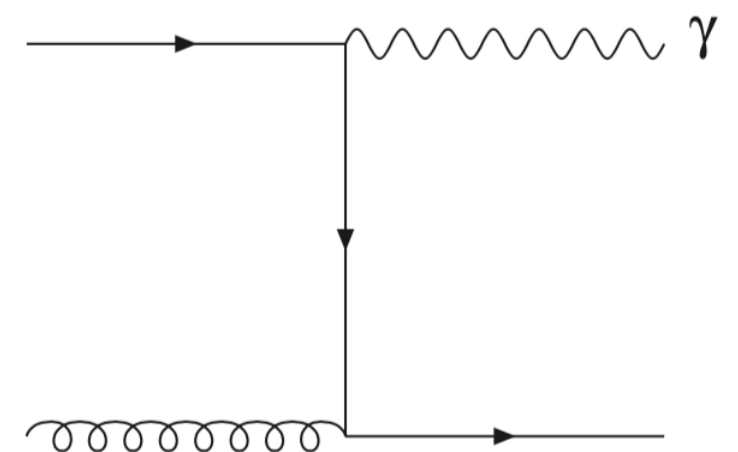
Jet production



$Z p_T$



Direct photon production



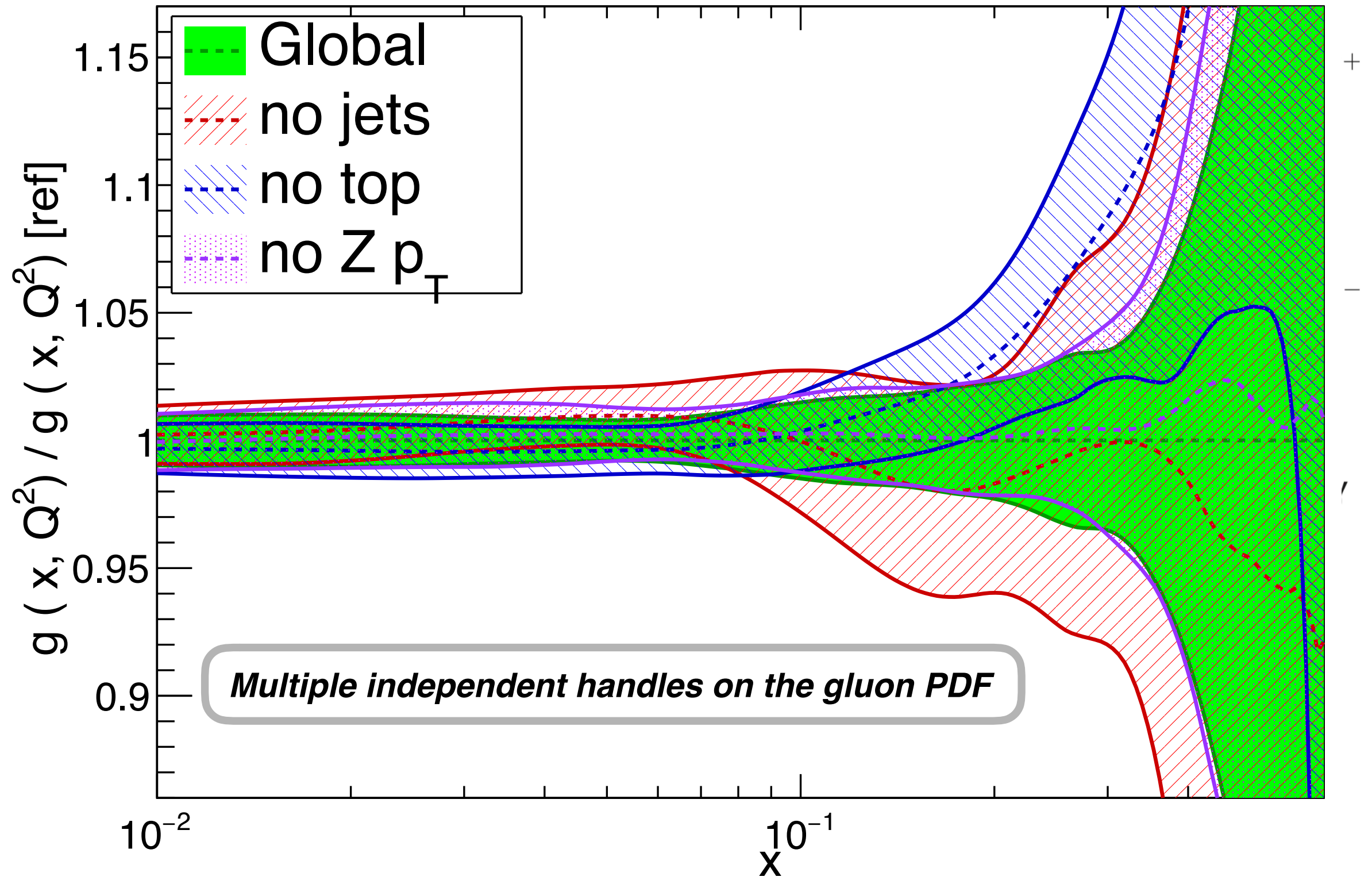
large-x gluon

medium-x gluon

medium-x gluon

One glue to bind them all

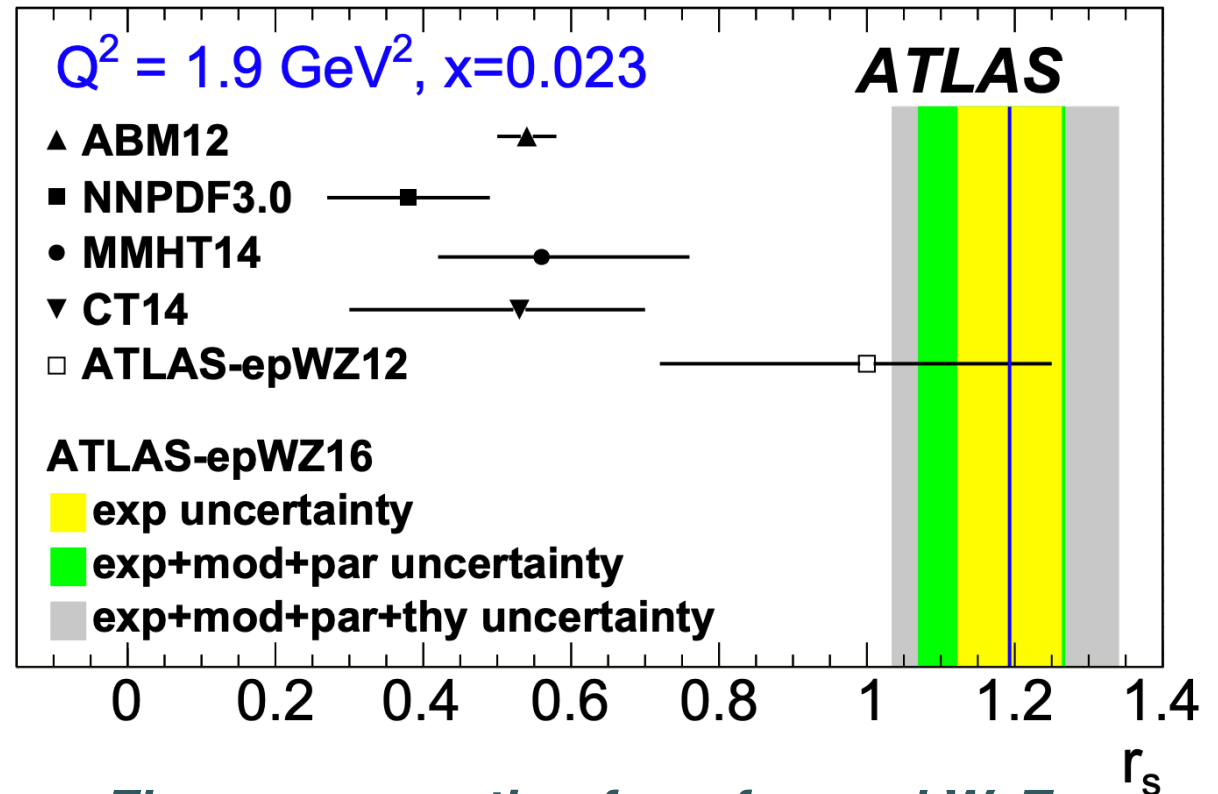
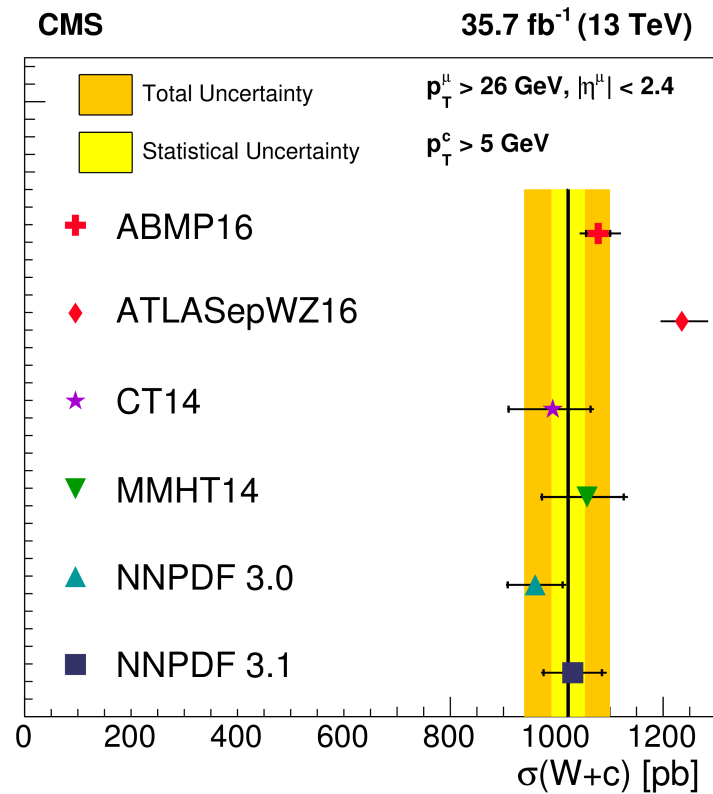
NNPDF3.1 NNLO, $Q = 100$ GeV



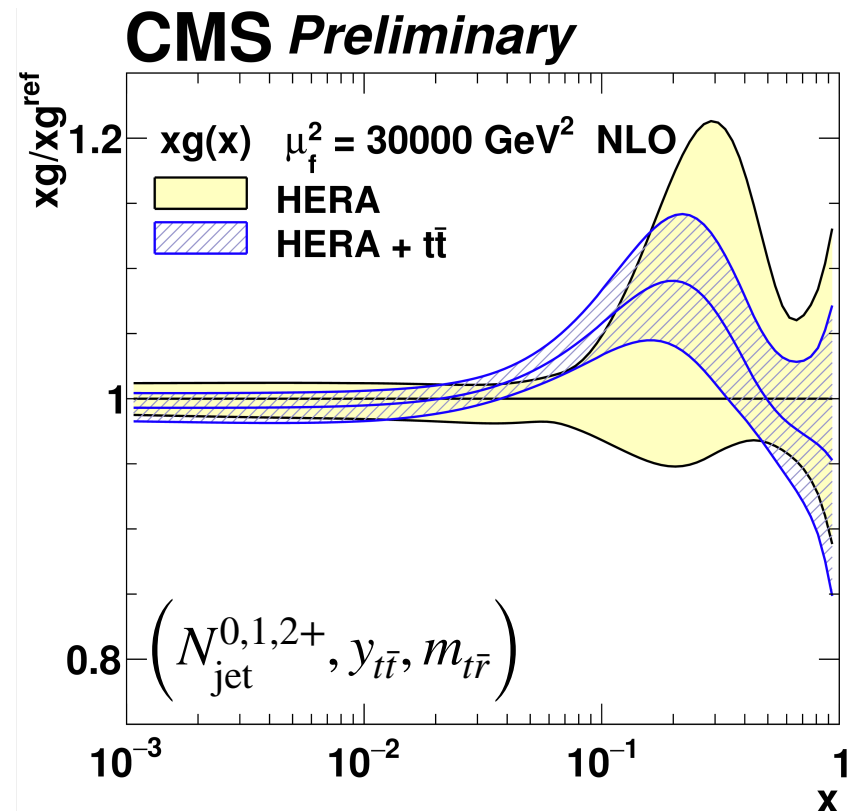
Constraints from LHC data

Combined interpretation
within global fit?

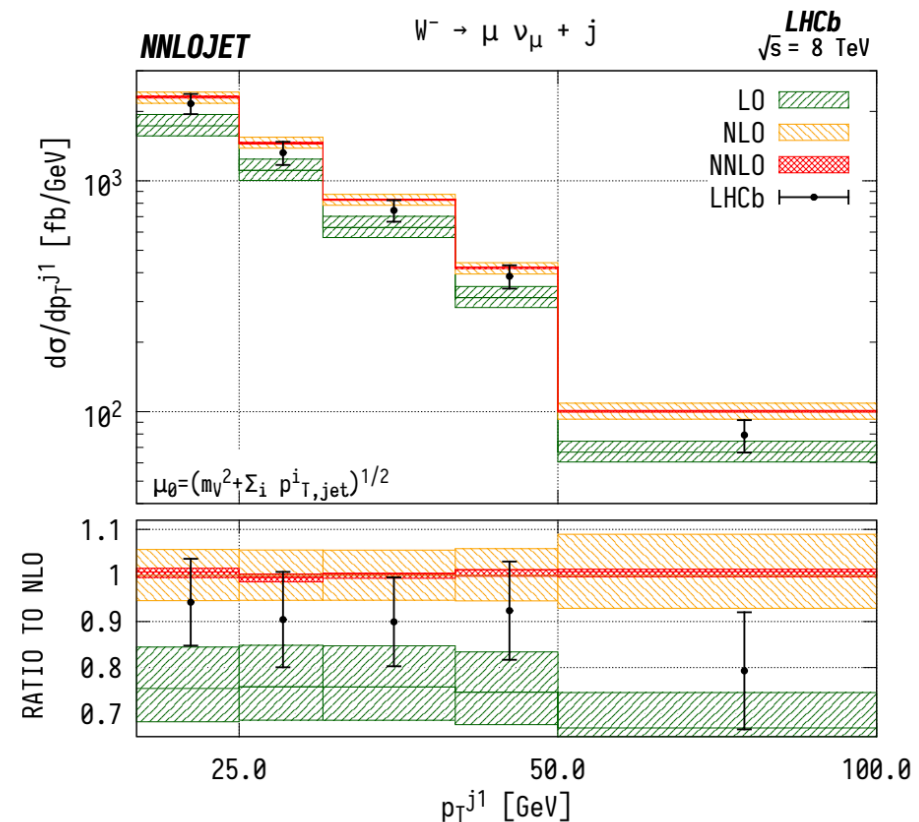
The strangeness conundrum: inclusive W,Z vs $W+c$



Multi-differential top quark pairs



Flavour separation from forward W, Z



Gehrmann-De Ridder
et al 19

QCD uncertainties in PDF fits

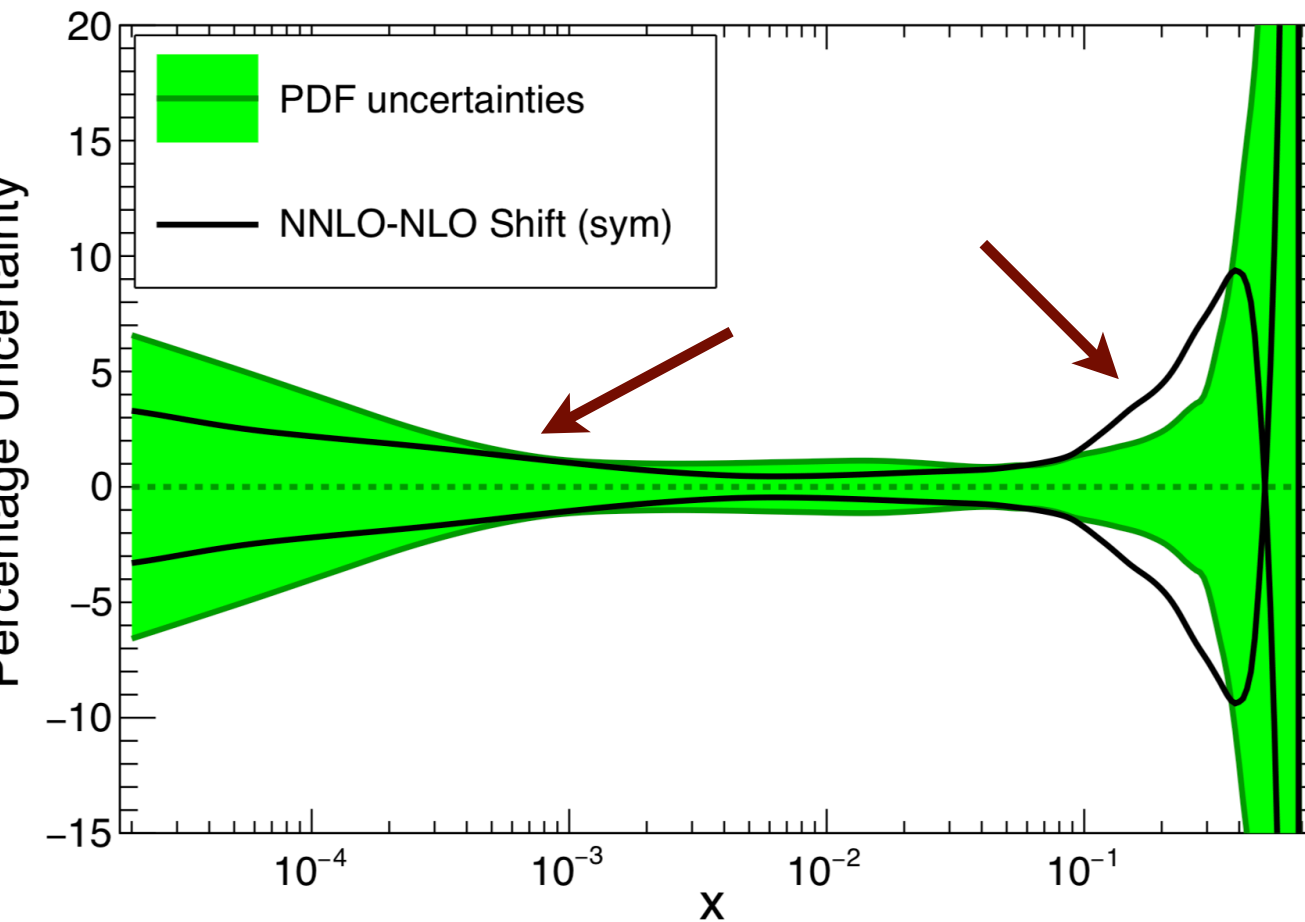
Standard global PDF fits are based on **fixed-order QCD calculations**

$$\sigma = \alpha_s^p \sigma_0 + \alpha_s^{p+1} \sigma_1 + \alpha_s^{p+2} \sigma_2 + \mathcal{O}(\alpha_s^{p+3})$$

The truncation of the perturbative series has associated a theoretical uncertainty: **Missing Higher Order (MHO)** uncertainty

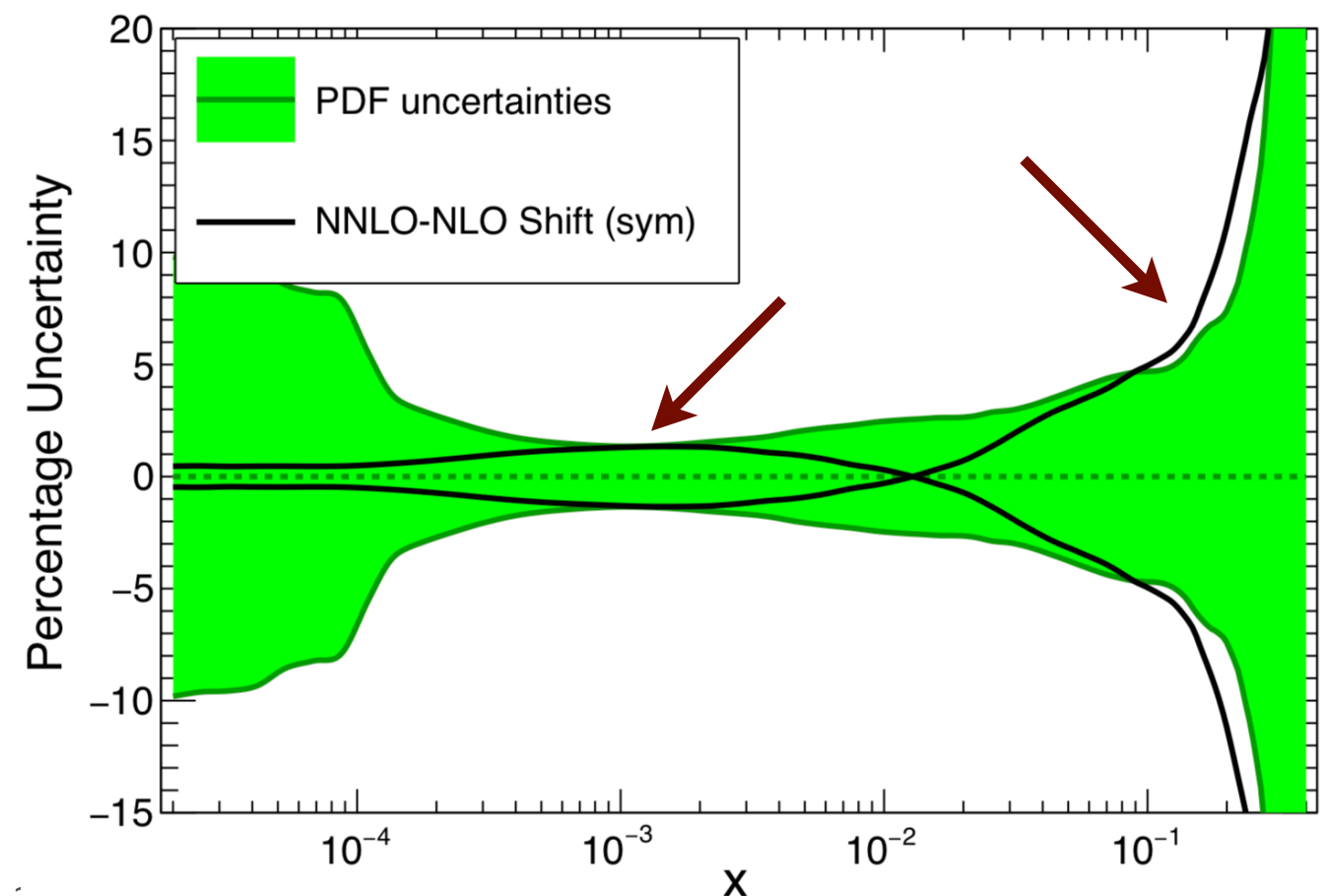
How severe is **ignoring MHOU**s in modern global PDFs fits?

$g(x, Q^2=100 \text{ GeV}^2)$, NNPDF3.1NLO



Juan Rojo

$\bar{u}(x, Q^2)$, NNPDF3.1NLO

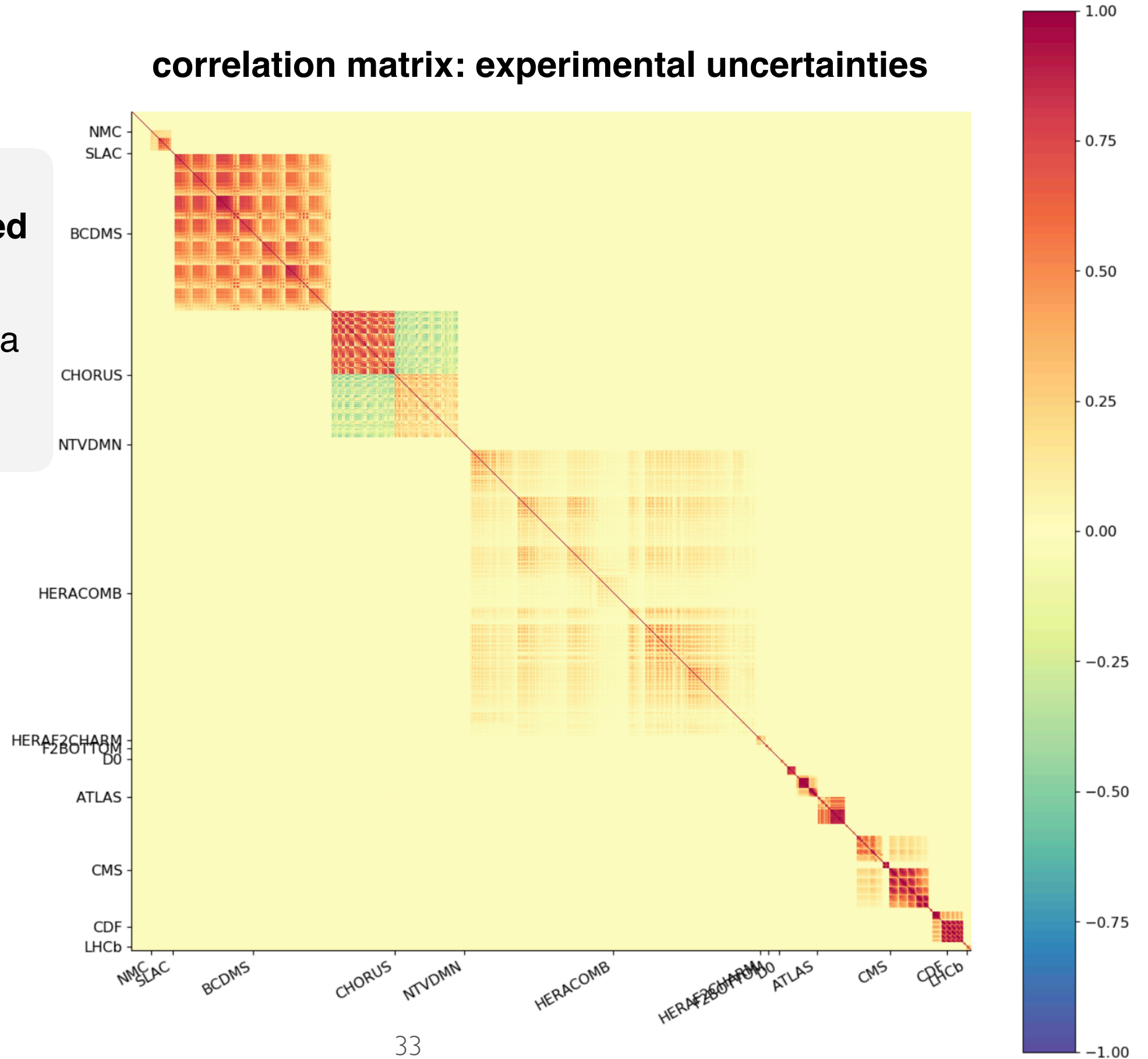


Proton Structure and PDFs, DIS2019

QCD uncertainties in PDF fits

correlation matrix: experimental uncertainties

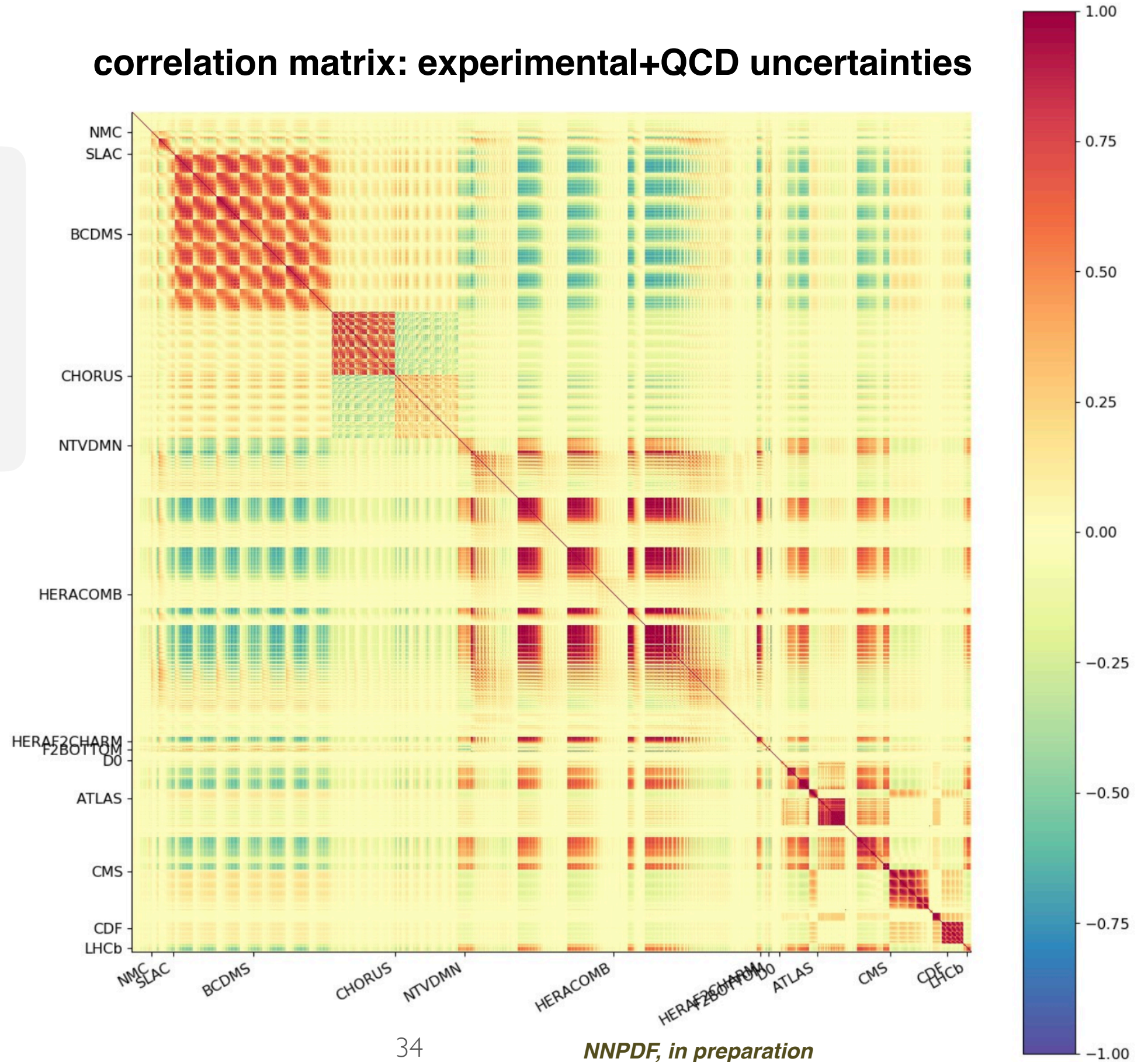
Experiment-induced correlations:
only present within a single dataset



QCD uncertainties in PDF fits

correlation matrix: experimental+QCD uncertainties

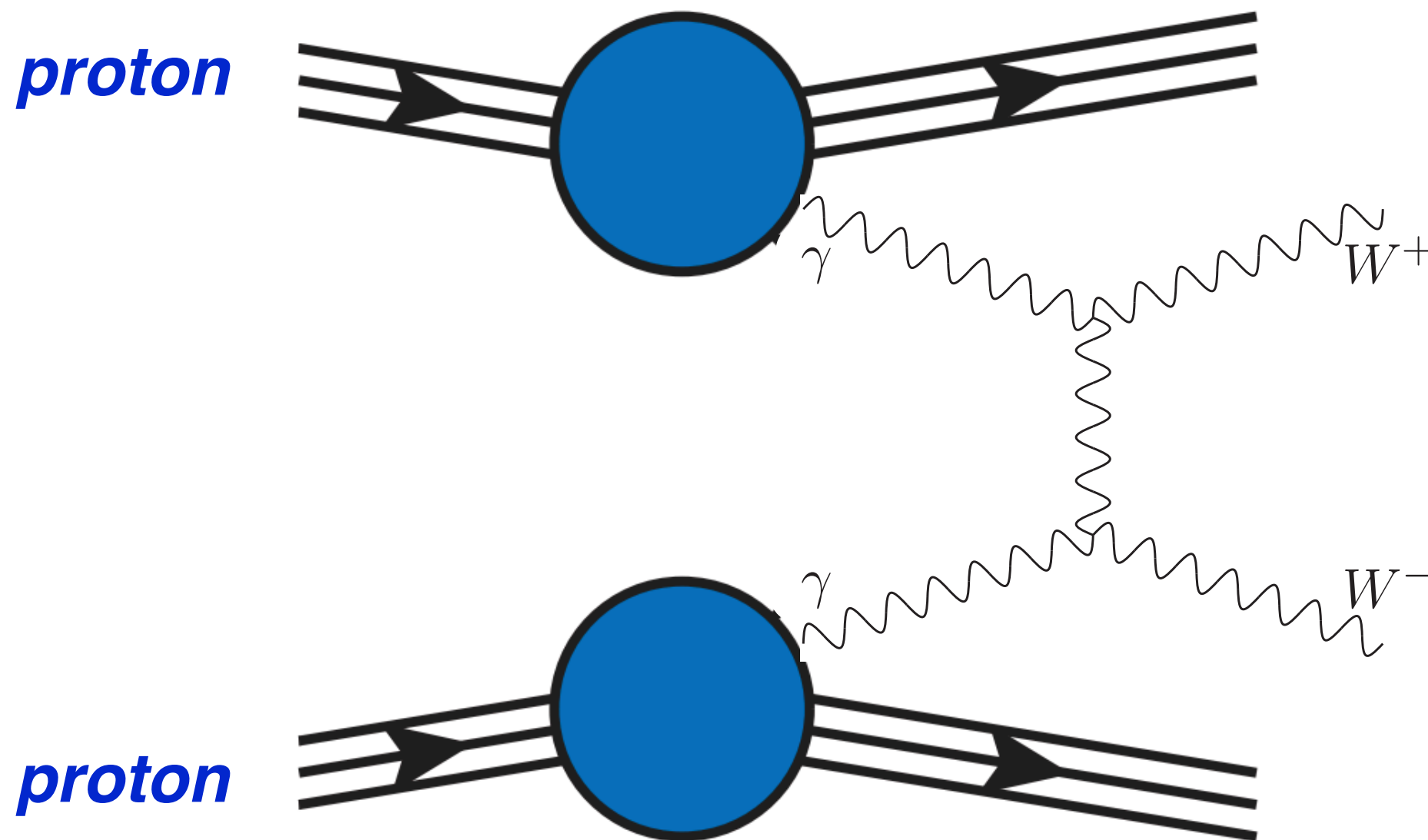
Theory-induced correlations
between different experiments
e.g. DIS and LHC



Harland-Lang, Voisey WG1

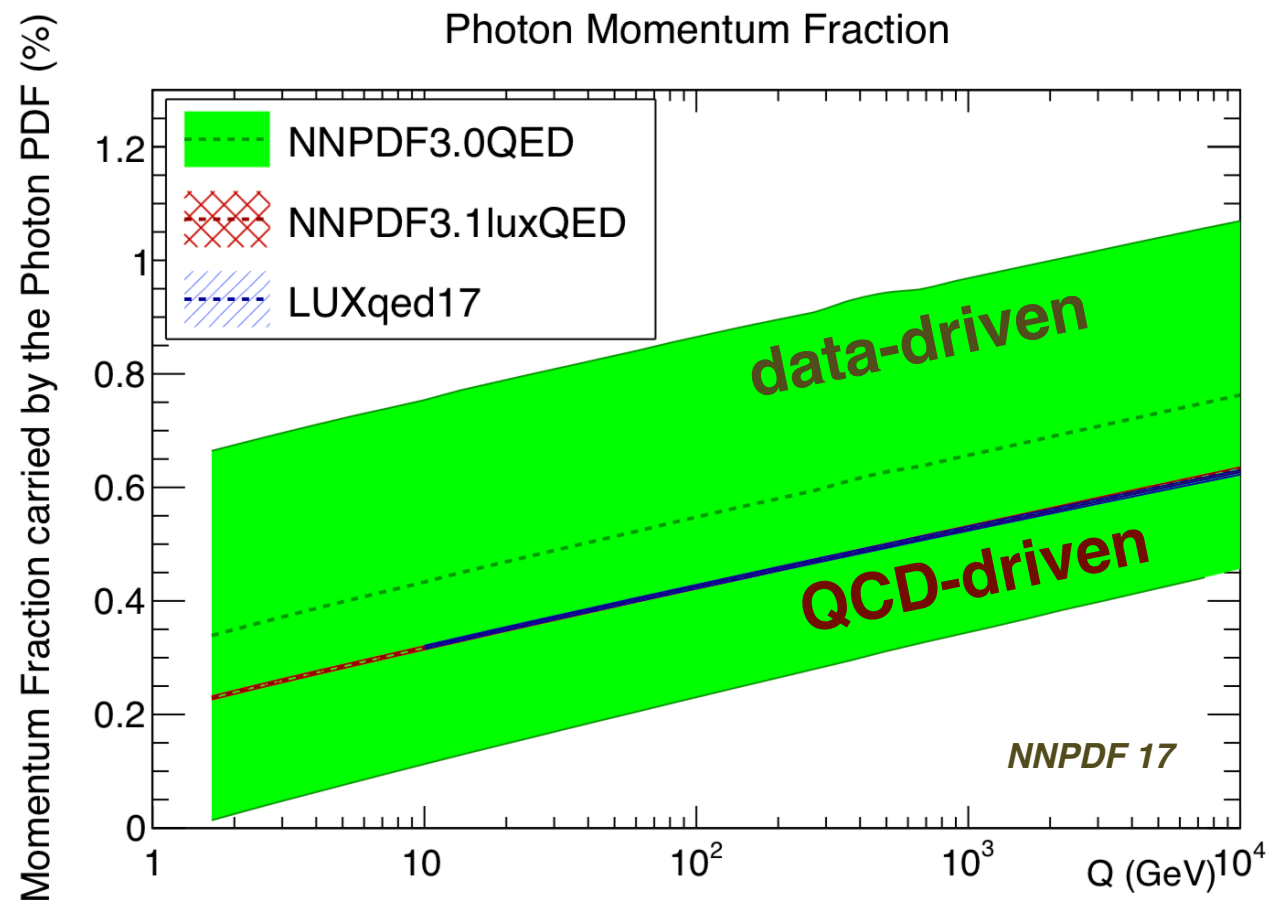
Let there be light: the photon PDF

- 📌 The proton contains not only quark and gluons as constituents: also **photons**!
- 📌 The photon PDF can be evaluated from deep-inelastic **structure functions F_2 and F_L**
LuxQED: Manohar et al 16,17
- 📌 Required for consistent implementation of **electroweak corrections** at the LHC



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*Up to 0.5% of proton's momentum
carried by photons*

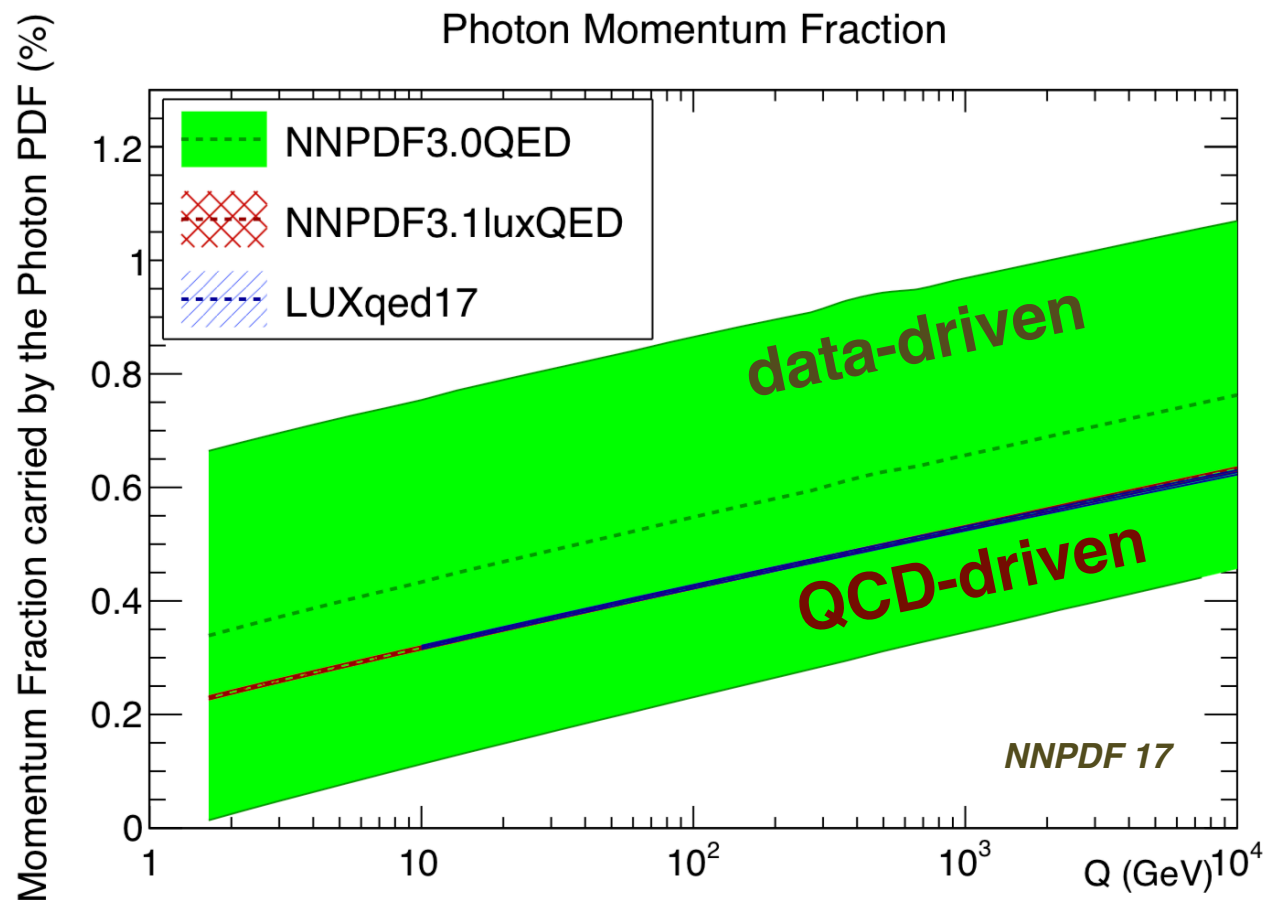
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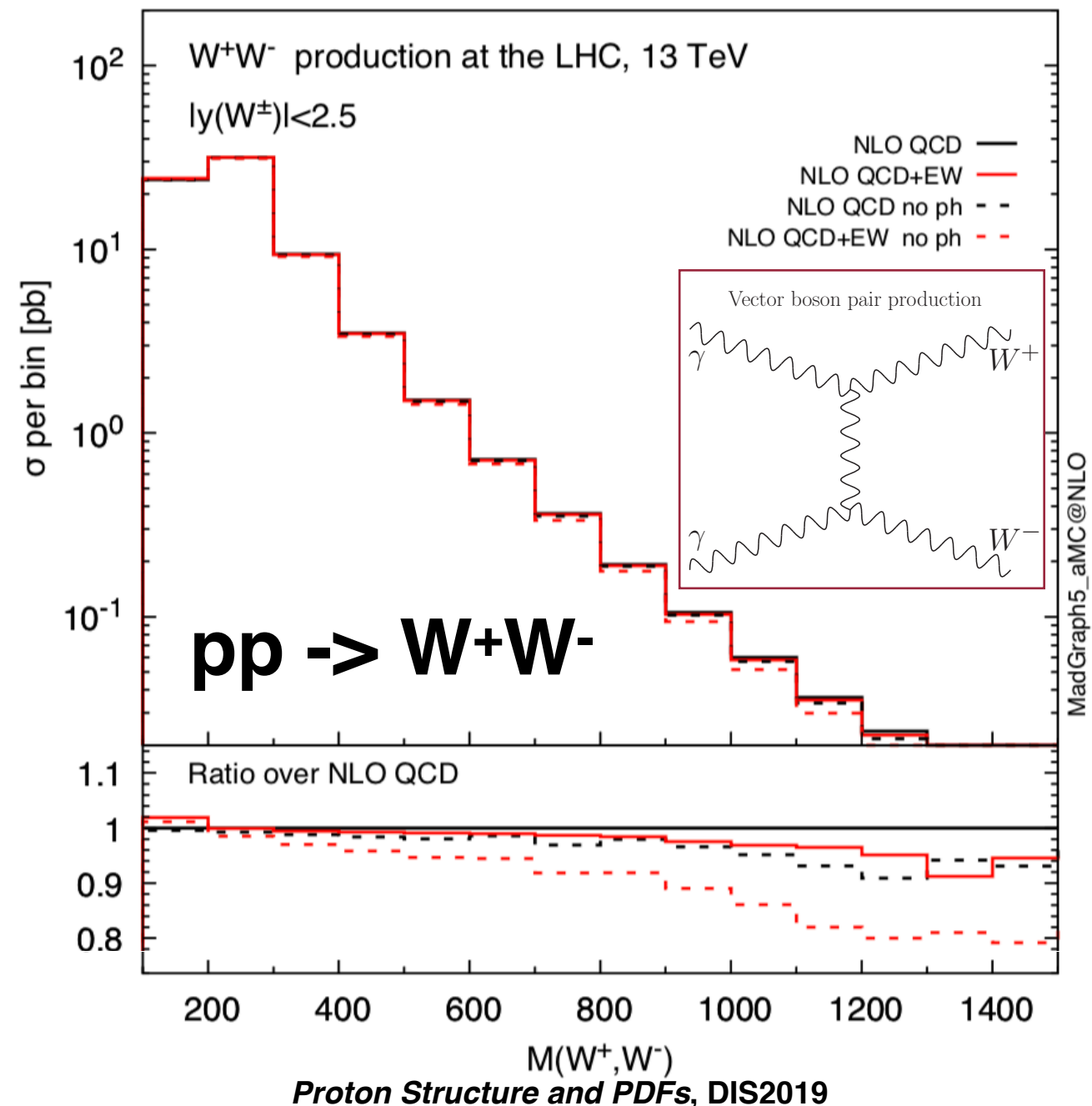
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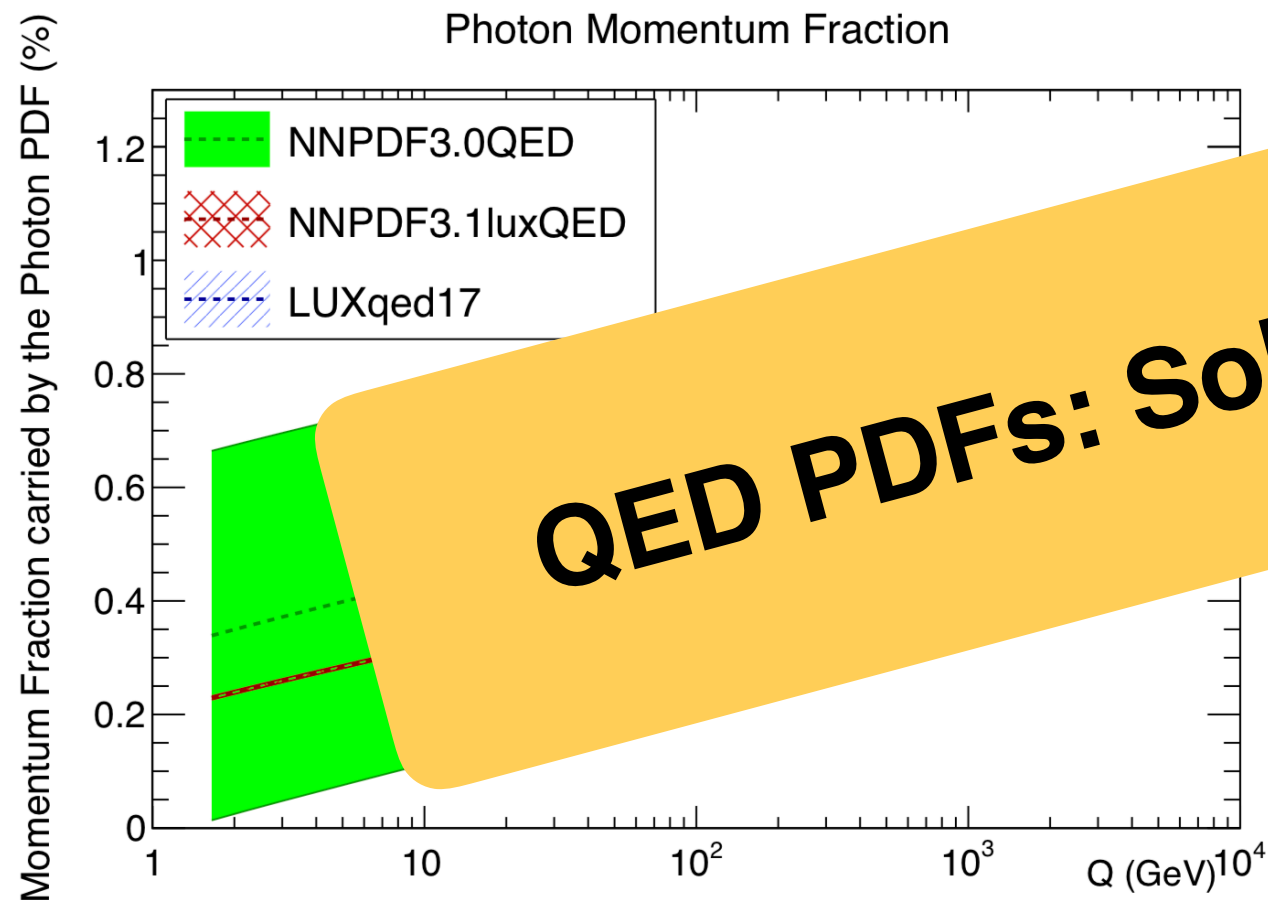


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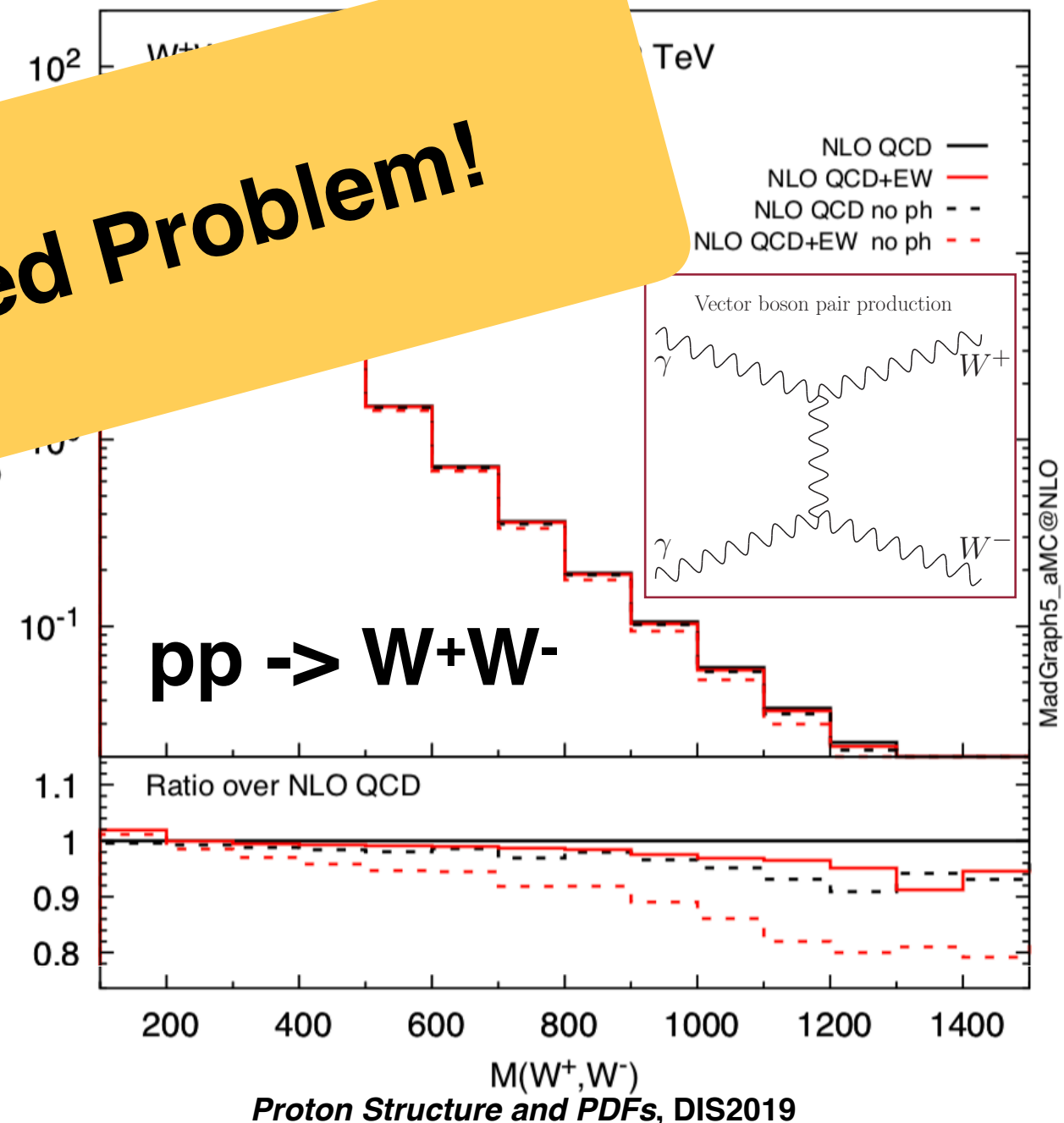
Let there be light: the photon PDF

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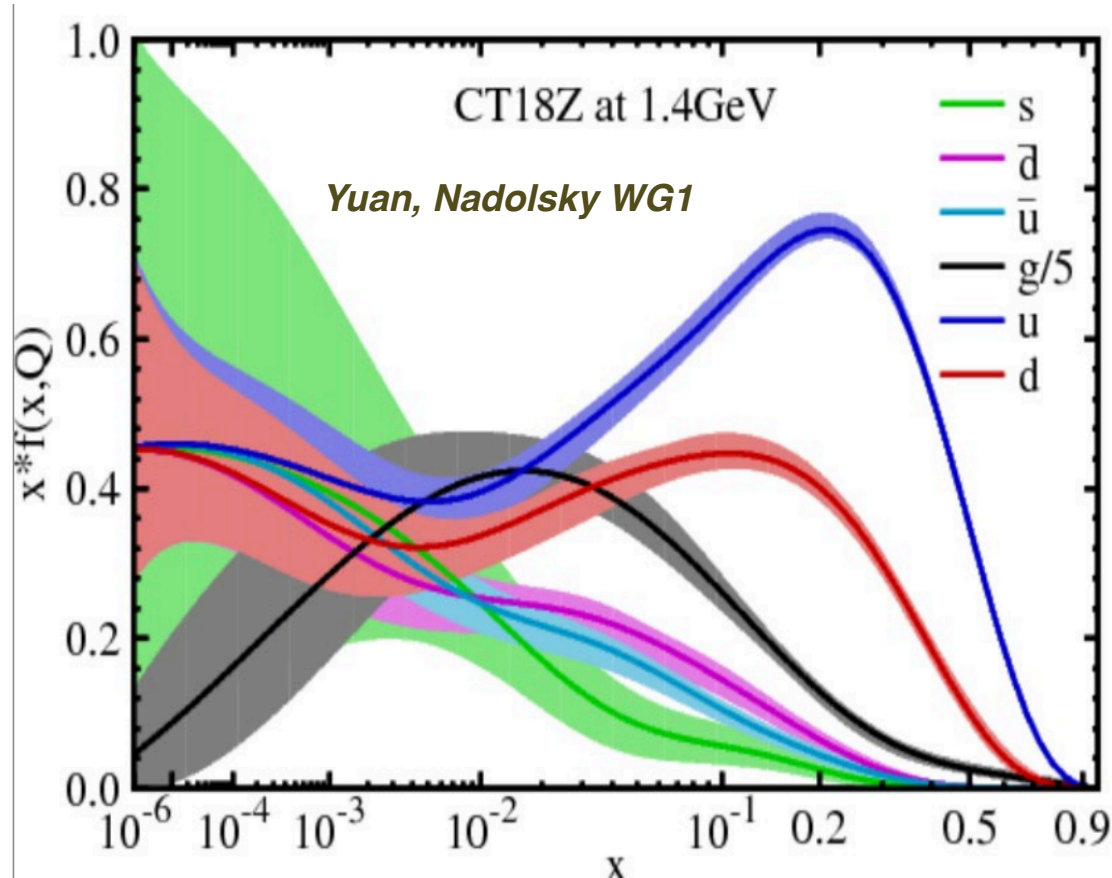


QED PDFs: Solved Problem!

*Up to 0.5% of proton's momentum
carried by photons*



Updating the global PDF analyses



- 📌 **CT18/CT18Z** is the follow-up of CT14 (without/with ATLAS W,Z 2011)

- 📌 Added new LHC measurements boosted by the **ePump** and **PDFsense** tools

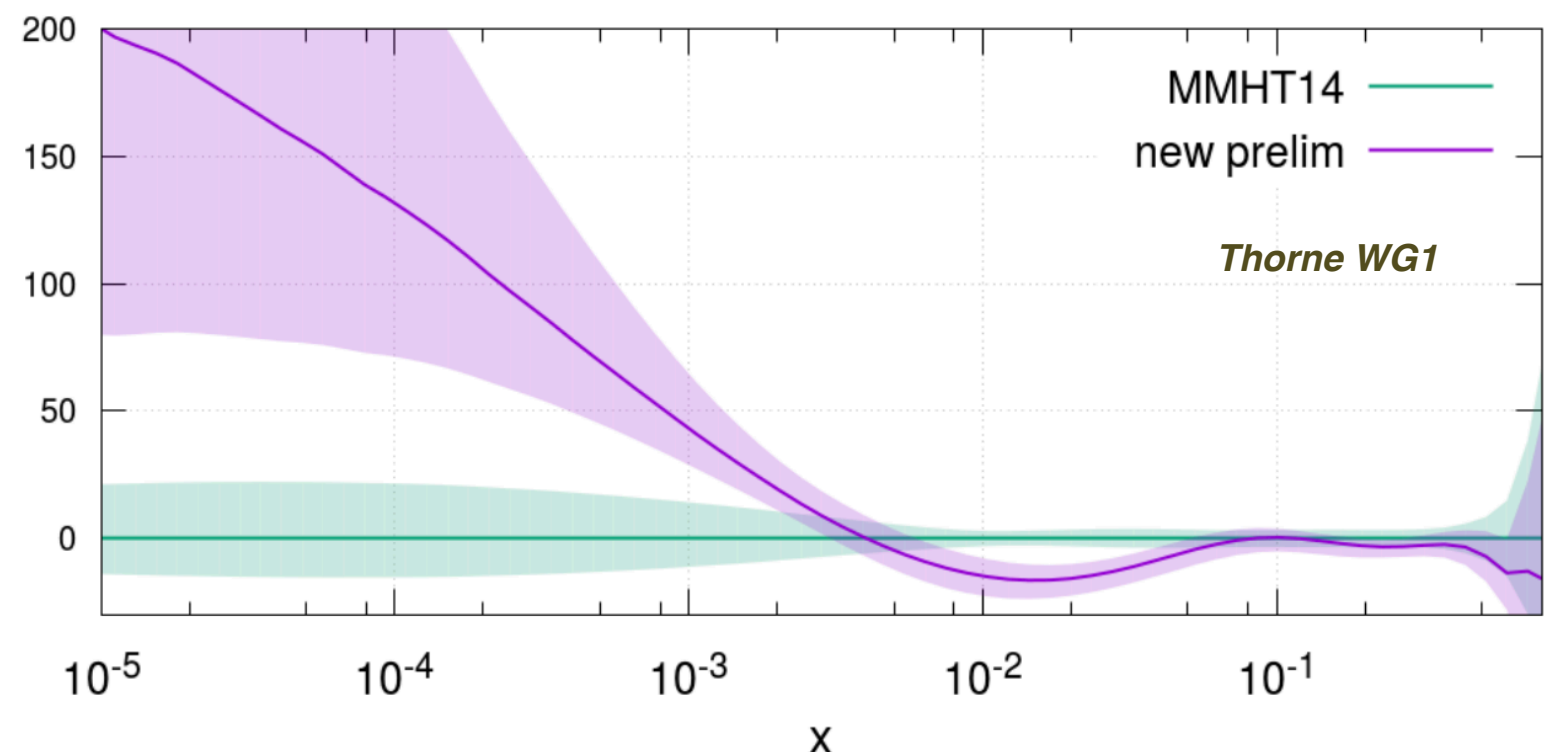
- 📌 Systematic studies of dataset compatibility

- 📌 **“MMHT” update** in progress

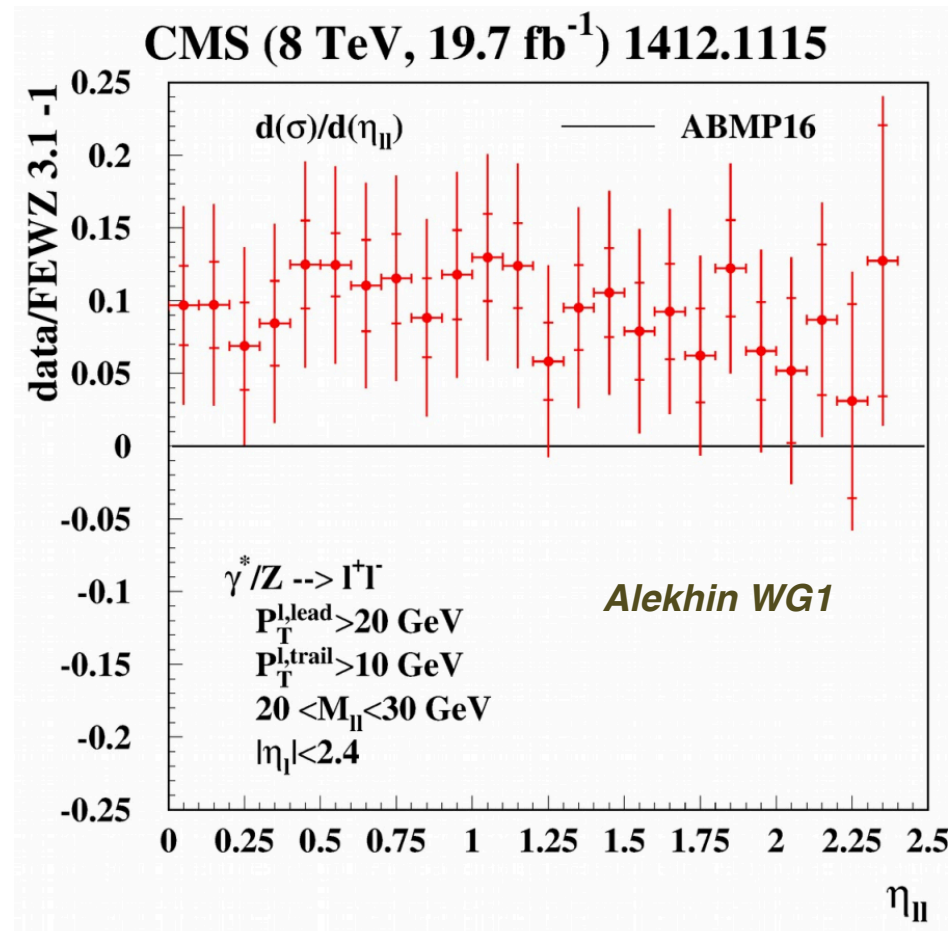
- 📌 Many new LHC data, extended parametrisation, NNLO calculations, ...

- 📌 QED sets ready for release

d_V (NNLO) percentage change from MMHT14 at $Q^2 = 100 \text{ GeV}^2$



Updating the global PDF analyses



📌 **NNPDF3.1** released in 2017, with QED, small- x (BFKL), and $\alpha_s(M_Z)$ fits follow-ups

📌 PDF sets **with MHOU**s near completion

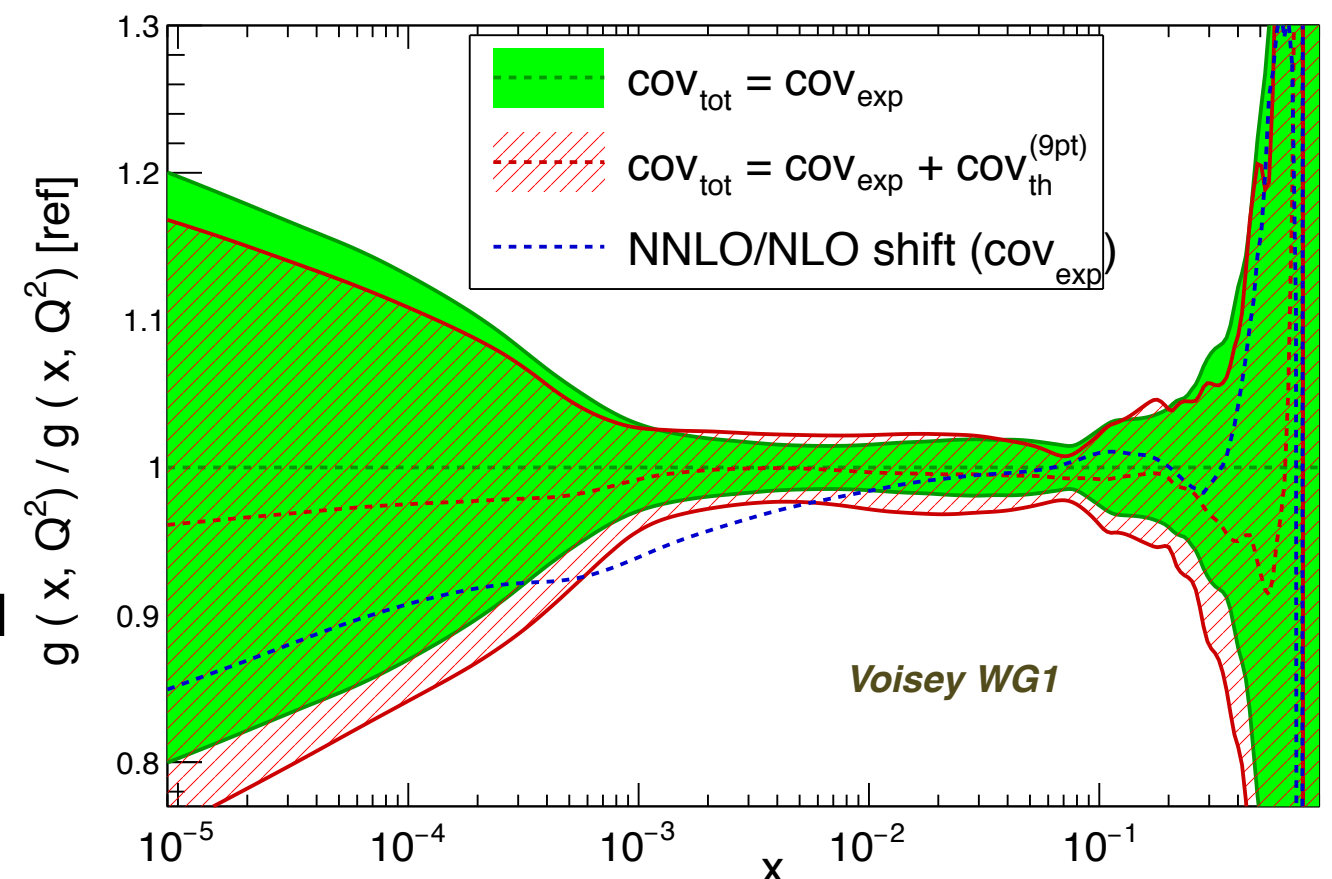
📌 Towards **NNPDF4.0**: new LHC data, improved methodology (TensorFlow for minimisation), MHOU's at NNLO, ...

📌 **ABM** updates focusing on adding LHC Drell-Yan and top quark production data

📌 Studies of the impact of **higher twists**

📌 Potential tensions between ATLAS and CMS?

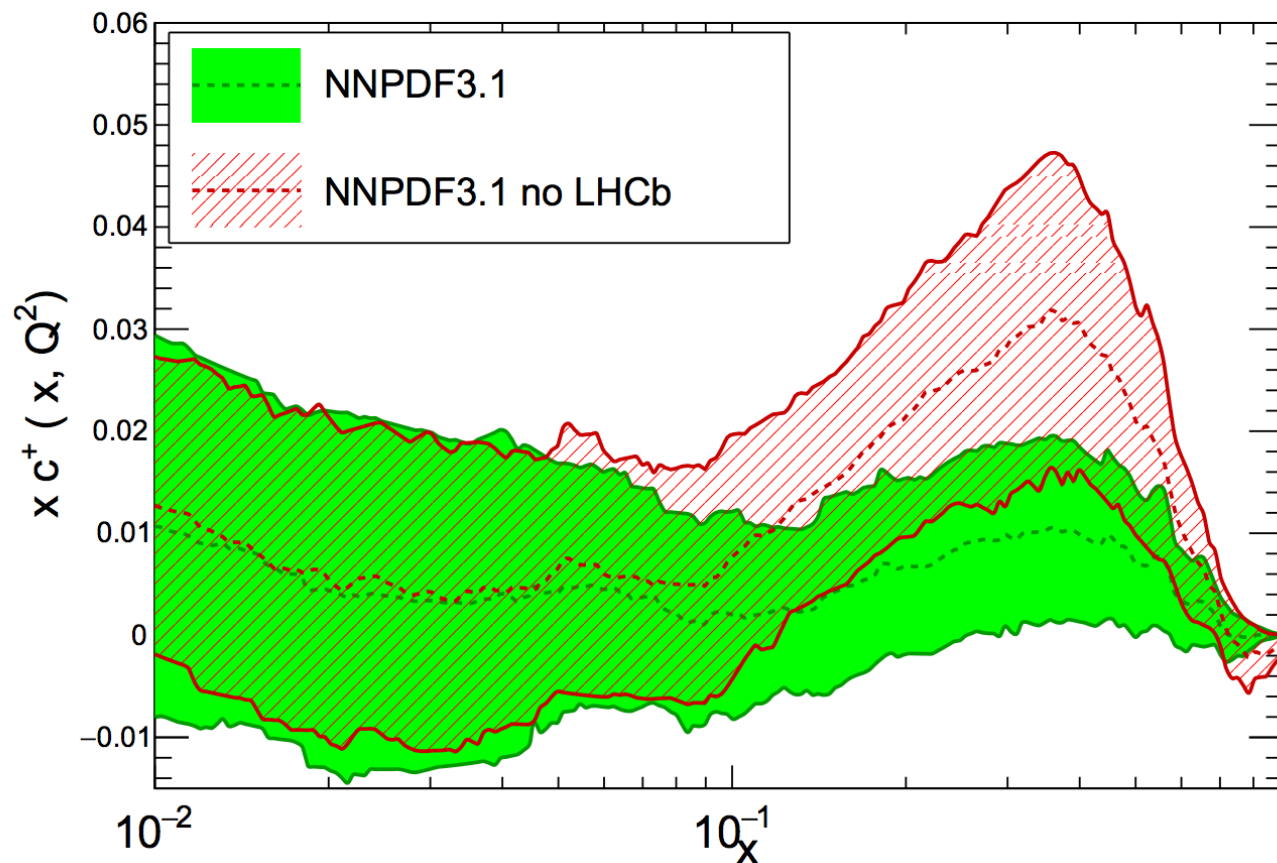
NNPDF3.1 Global NLO, $Q = 10 \text{ GeV}$



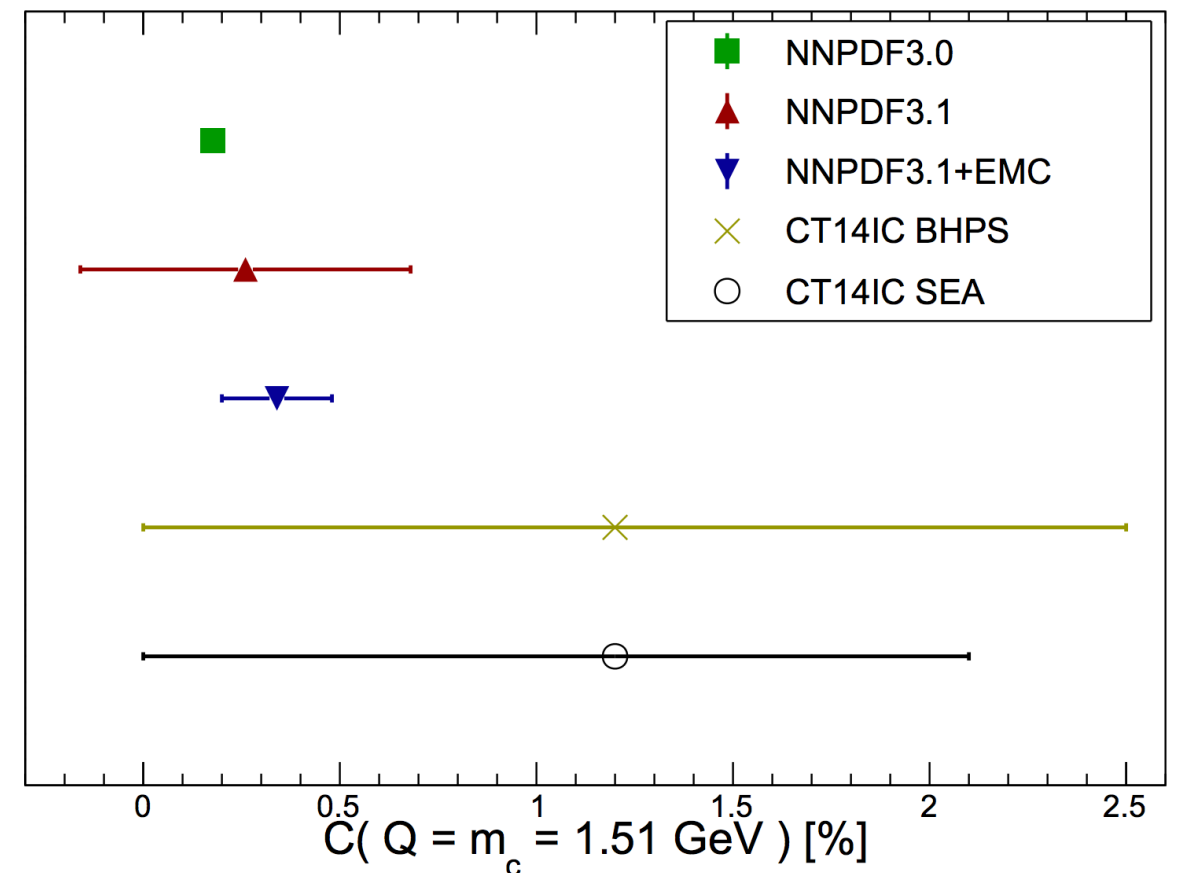
A charming proton?

Charm quark mass $\simeq$ proton mass: does the proton contain charm?

NNPDF3.1 NNLO, $Q = 1.7$ GeV



Momentum Fraction of Charm Quarks



LHC electroweak measurements provides information on the charm content of protons

Can be tested in $Z+D$, high p_T D , $photon+D$

$$C(Q^2) \equiv \int_0^1 dx x \left(c(c, Q^2) + \bar{c}(x, Q^2) \right)$$

Indications of a small but non-zero charm content of protons

Evidence for BFKL dynamics

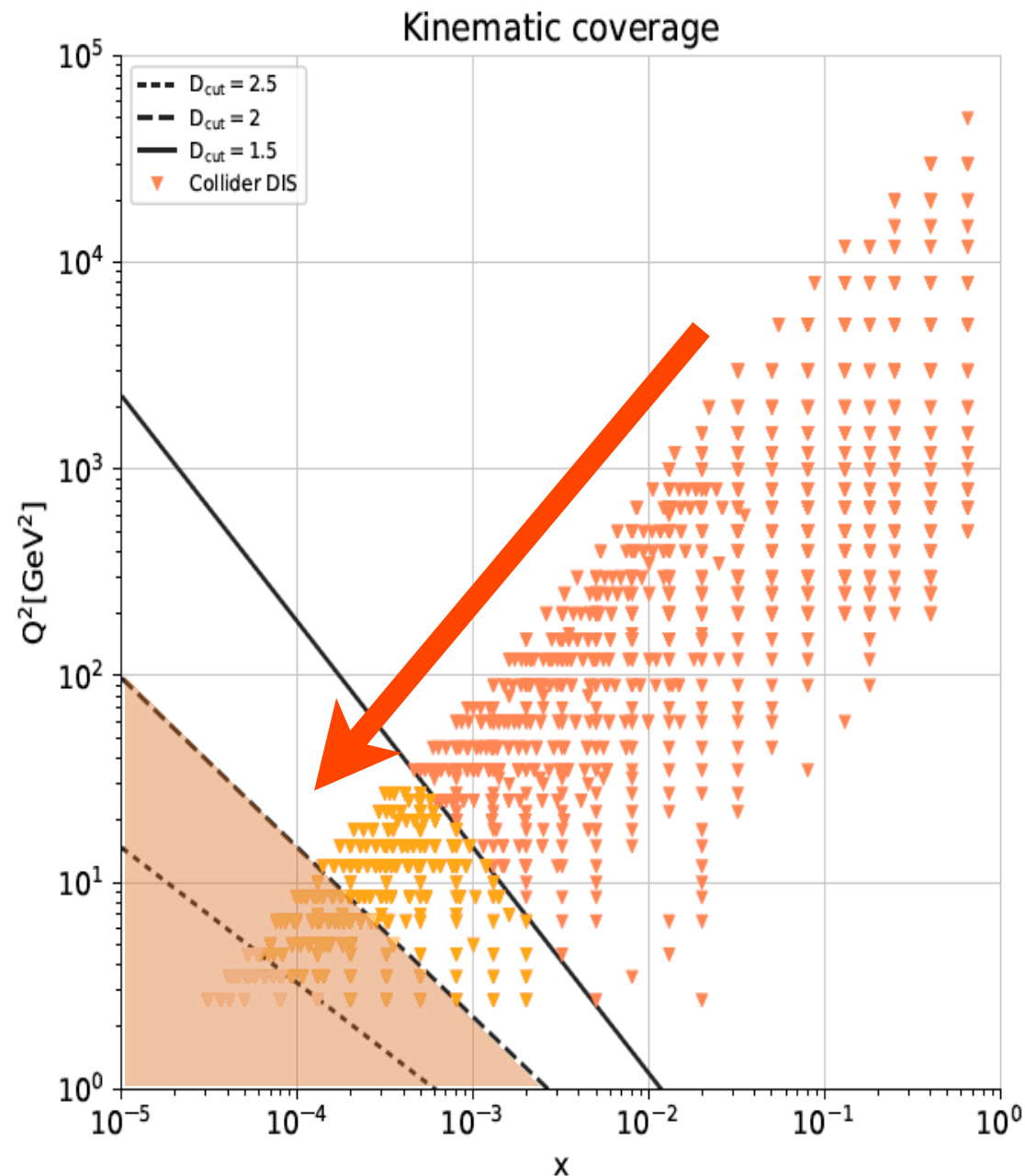
NNPDF3.1 fits based on **fixed order** (NNLO) and **small- x resummed** (NNLO+NLL x) theory

Altarelli, Ball, Forte 08, Bonvini et al 15,16

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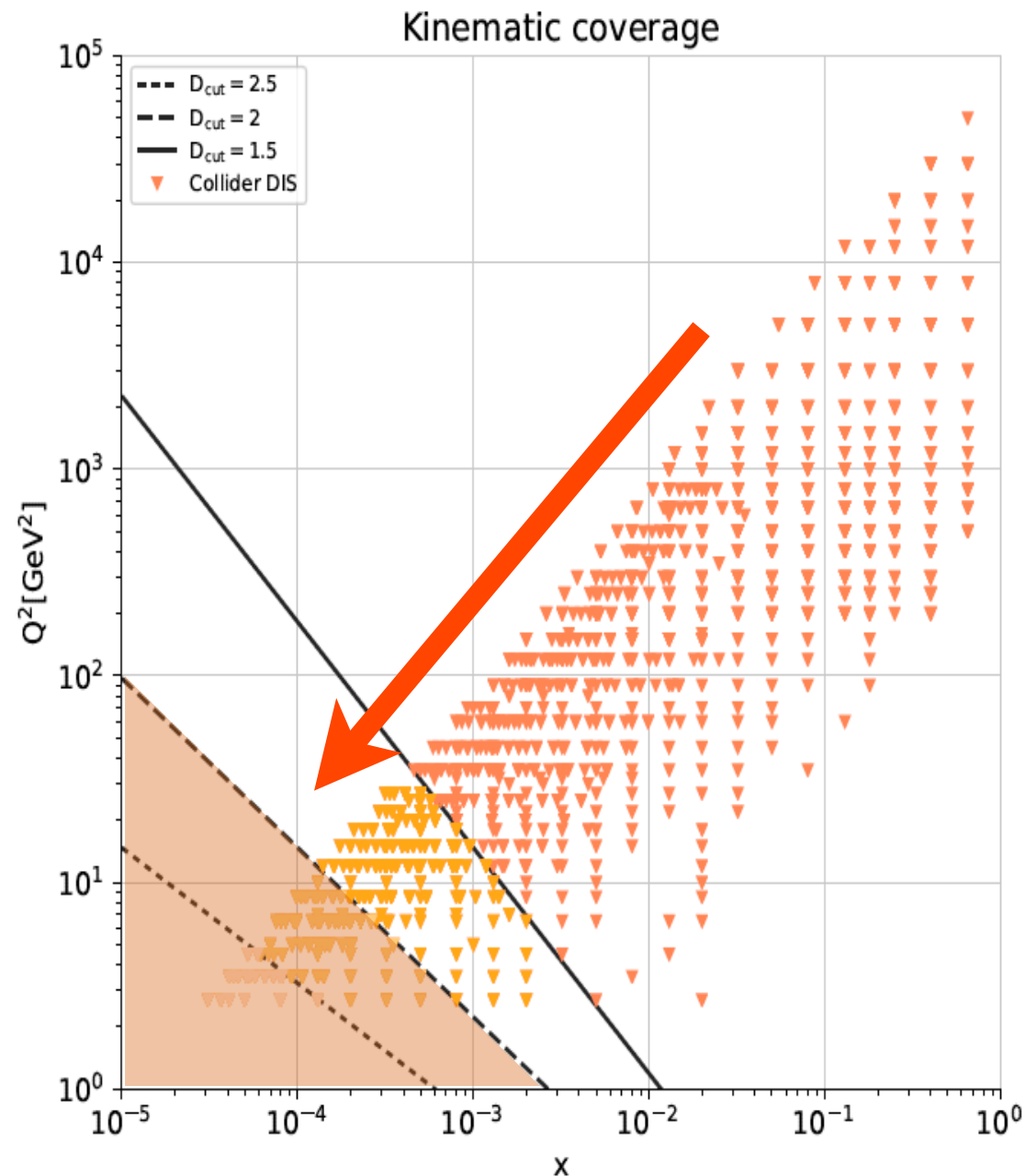
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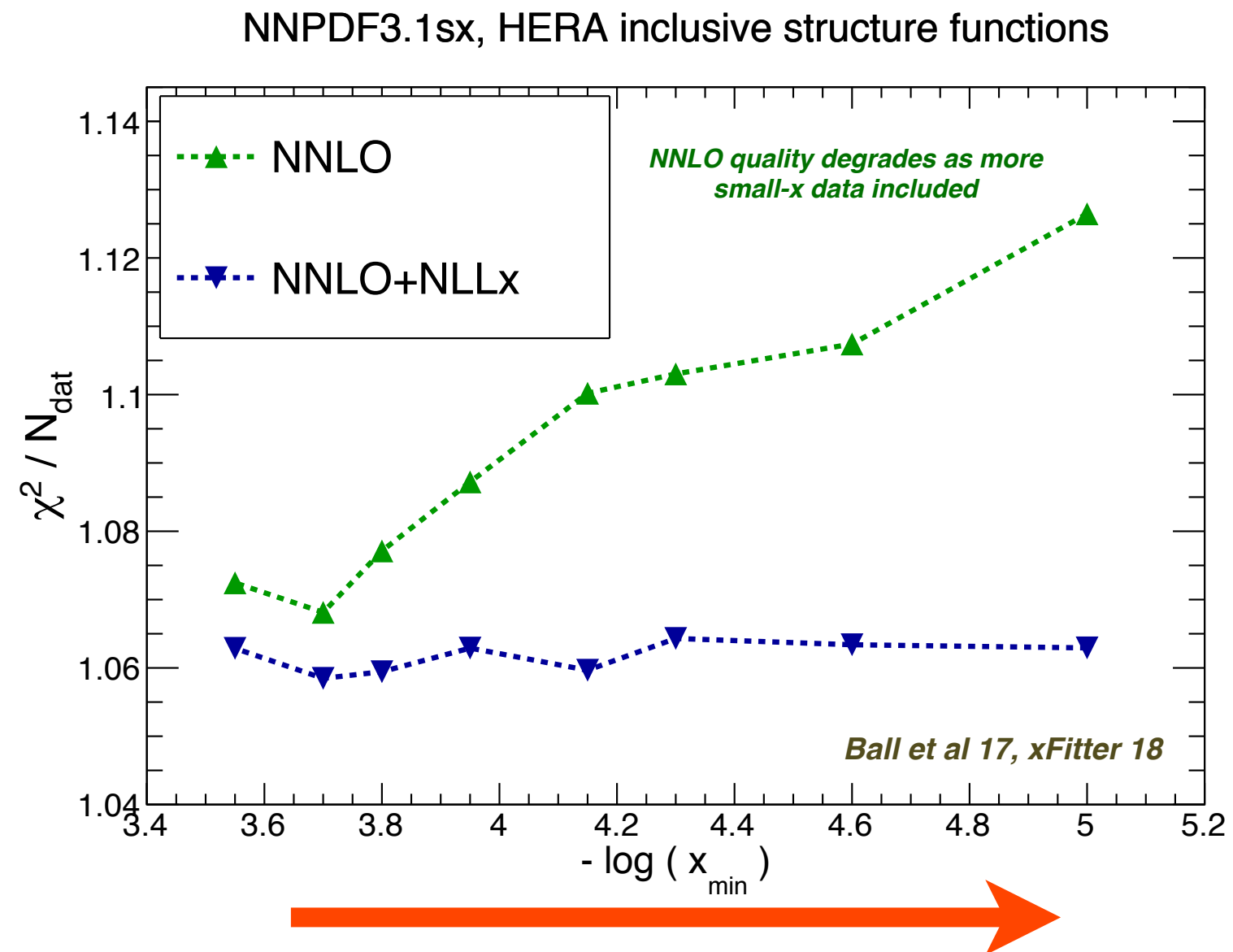
Monitor the **fit quality** as one includes more data from the **small- x region**

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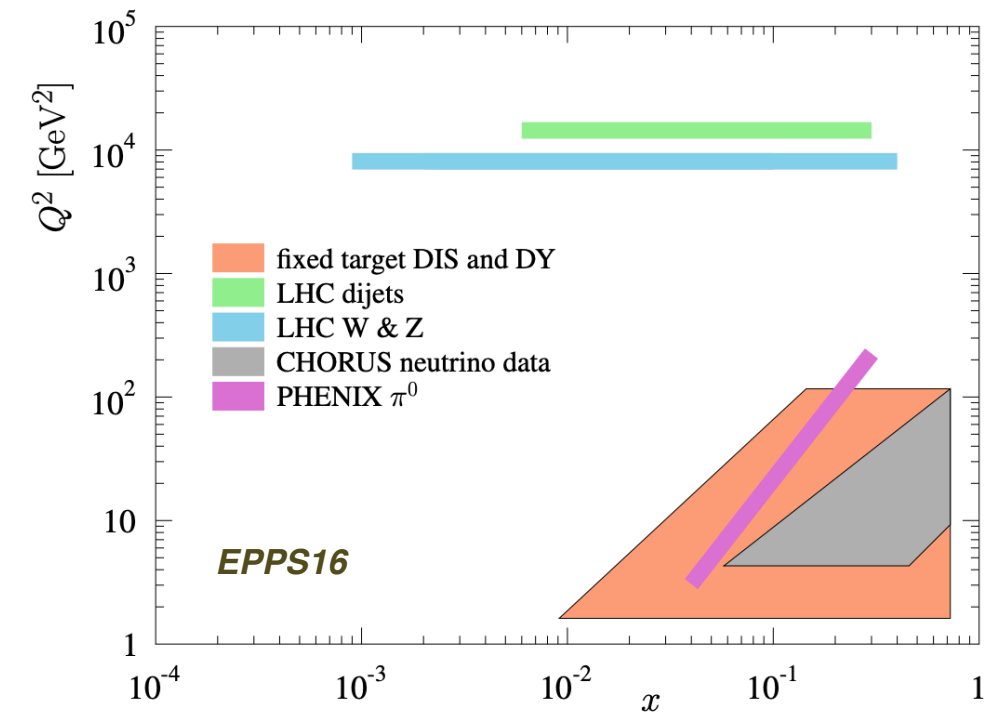


Best description of **small- x HERA data** only possible with **BFKL effects!**

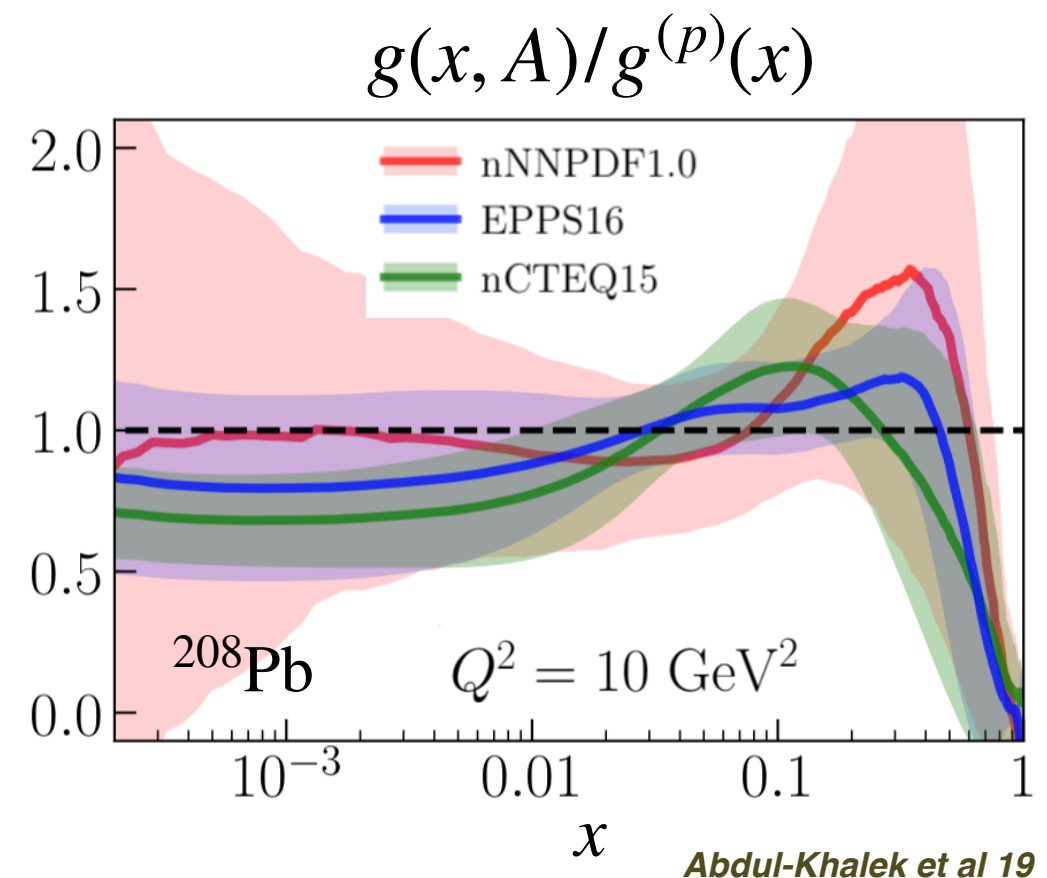
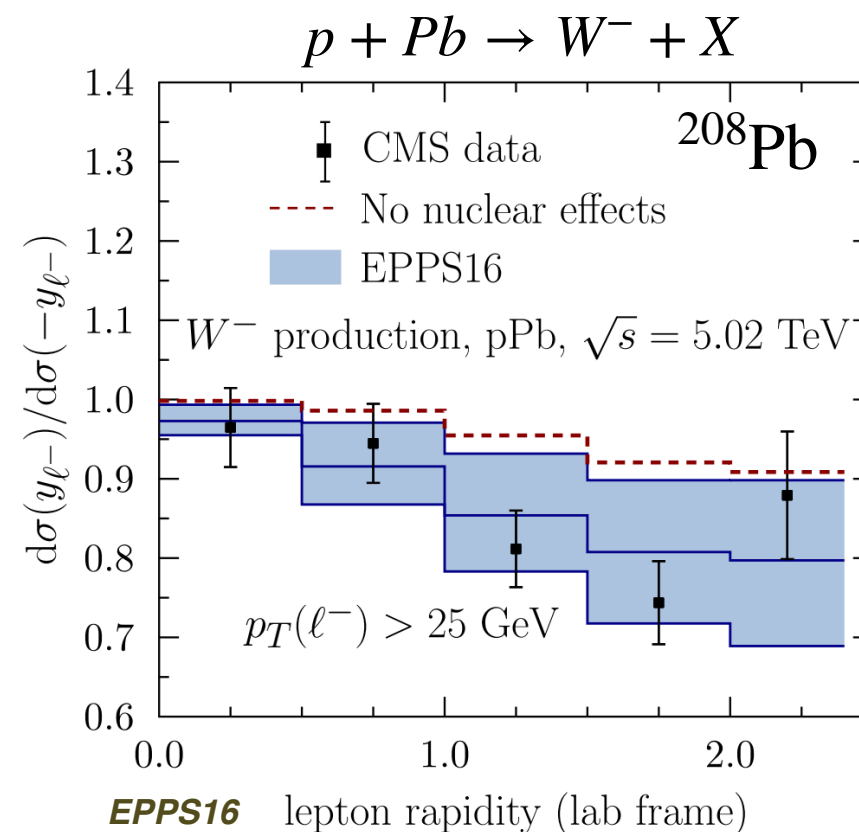
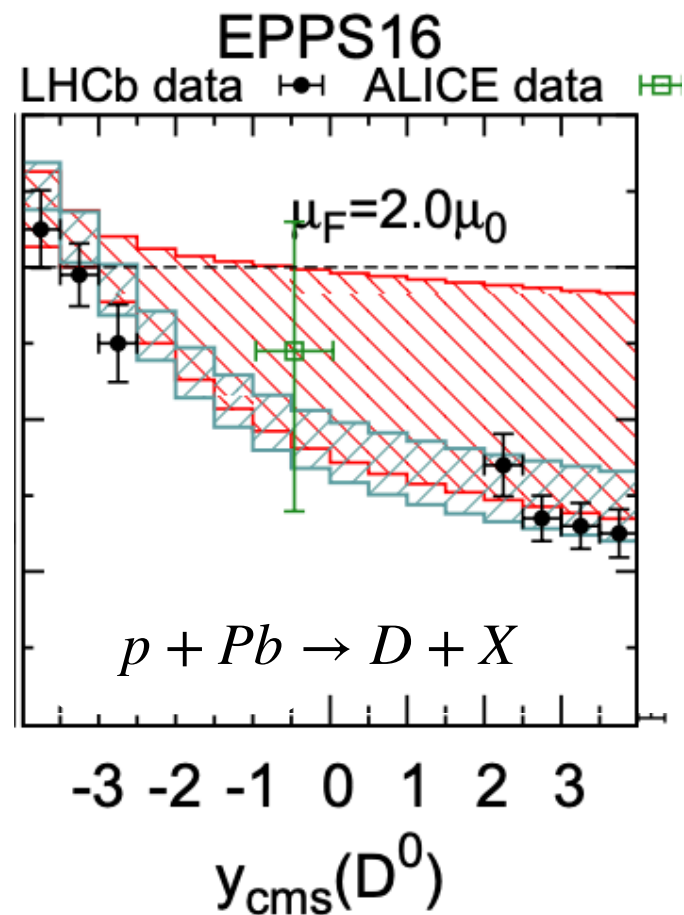
Progress in nuclear PDFs

📌 Constraints from **LHC proton+lead collisions**: W , Z , jets, charm, quarkonia, Drell-Yan, direct photons, ...

📌 **Theoretical and methodological** improvements: nPDF uncertainty estimate, NNLO QCD effects, ...



Kusina et al 17



Nuclear PDF analyses catching up with global proton PDF fits!

The lattice QCD frontier

PDFs are defined as nucleon matrix elements of quark fields separated in the **light-cone**

$$q(x) = \int_{-\infty}^{+\infty} d\xi^- e^{-ixP^+\xi^-} \langle N | \bar{\psi}(\xi^-) \Gamma W(\xi^-, 0) \psi(0) | N \rangle$$

Quasi-PDFs instead involve **euclidean separations** and can be computed on the lattice

$$\tilde{q}(x, P_3) = \int_{-z_{\max}}^{+z_{\max}} \frac{dz}{4\pi} e^{-ixP_3 z} \langle N | \bar{\psi}(0, z) \Gamma W(z, 0) \psi(0, 0) | N \rangle$$

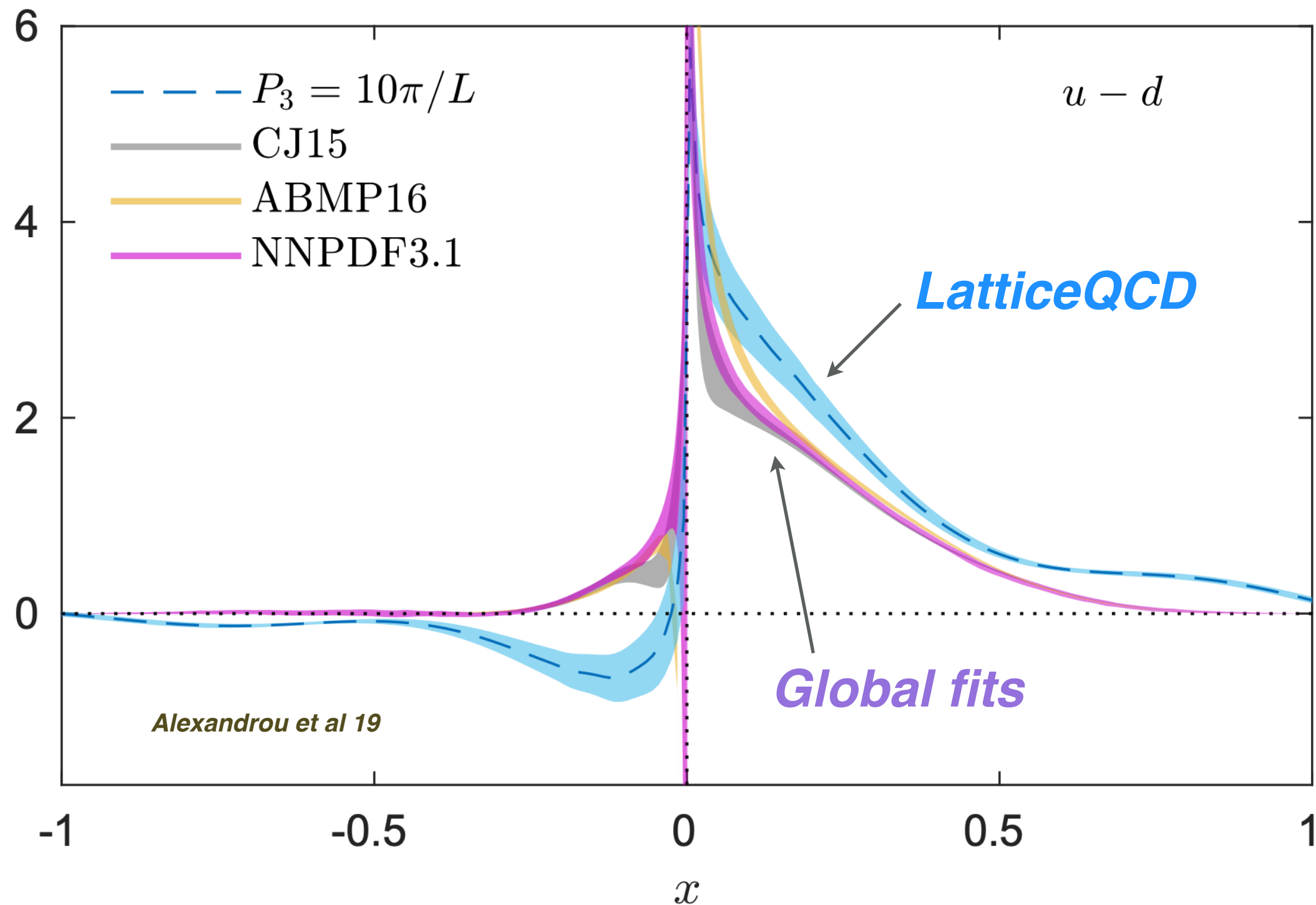
The two objects can be matched using the **Large Momentum EFT**:

$$q(x, \mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{P_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right) + O\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{QCD}^2}{P_3^2}\right)$$

Direct computation of **x-space PDFs** now feasible

*PDFLattice White Paper
Lin et al 17*

The lattice QCD frontier



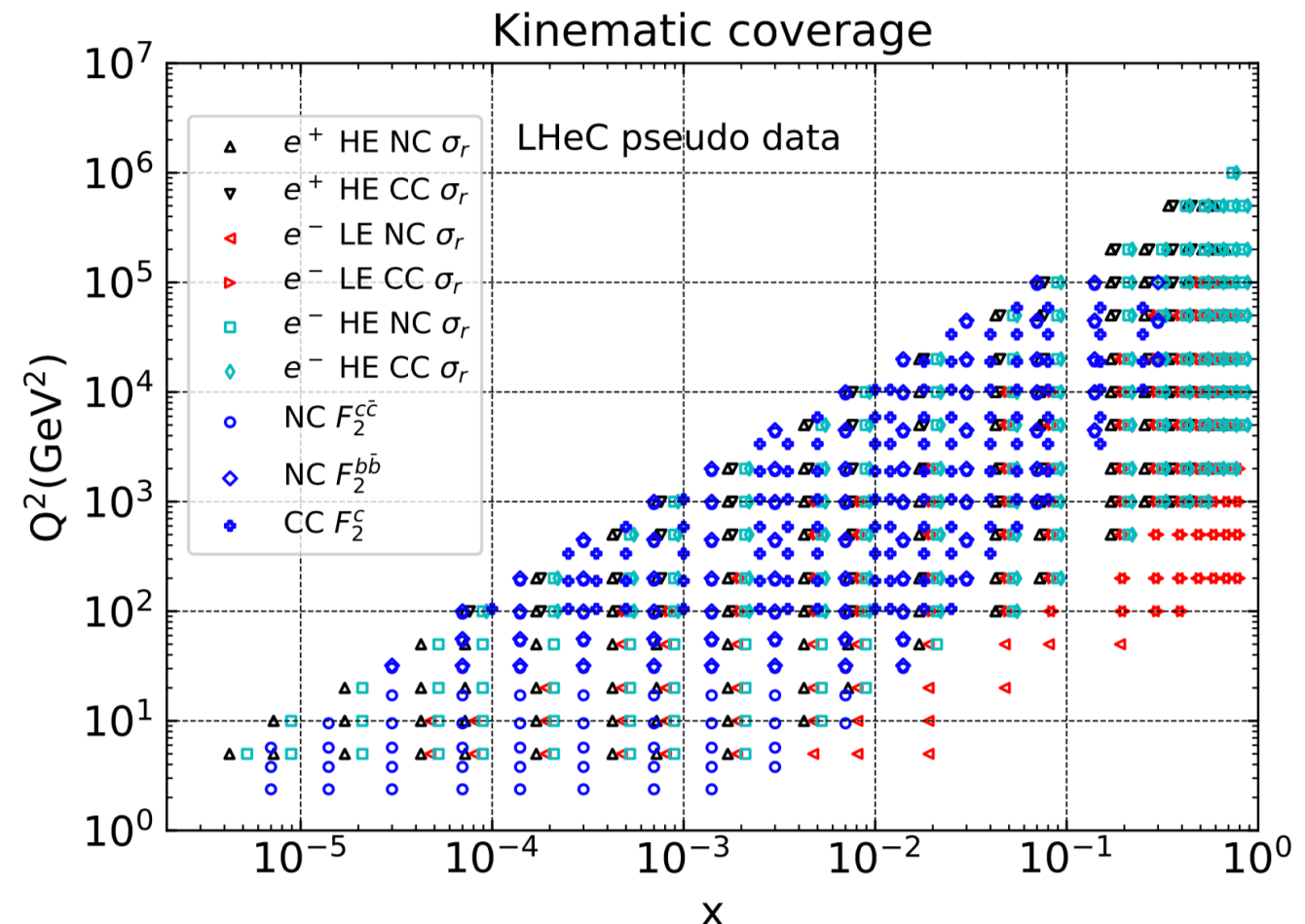
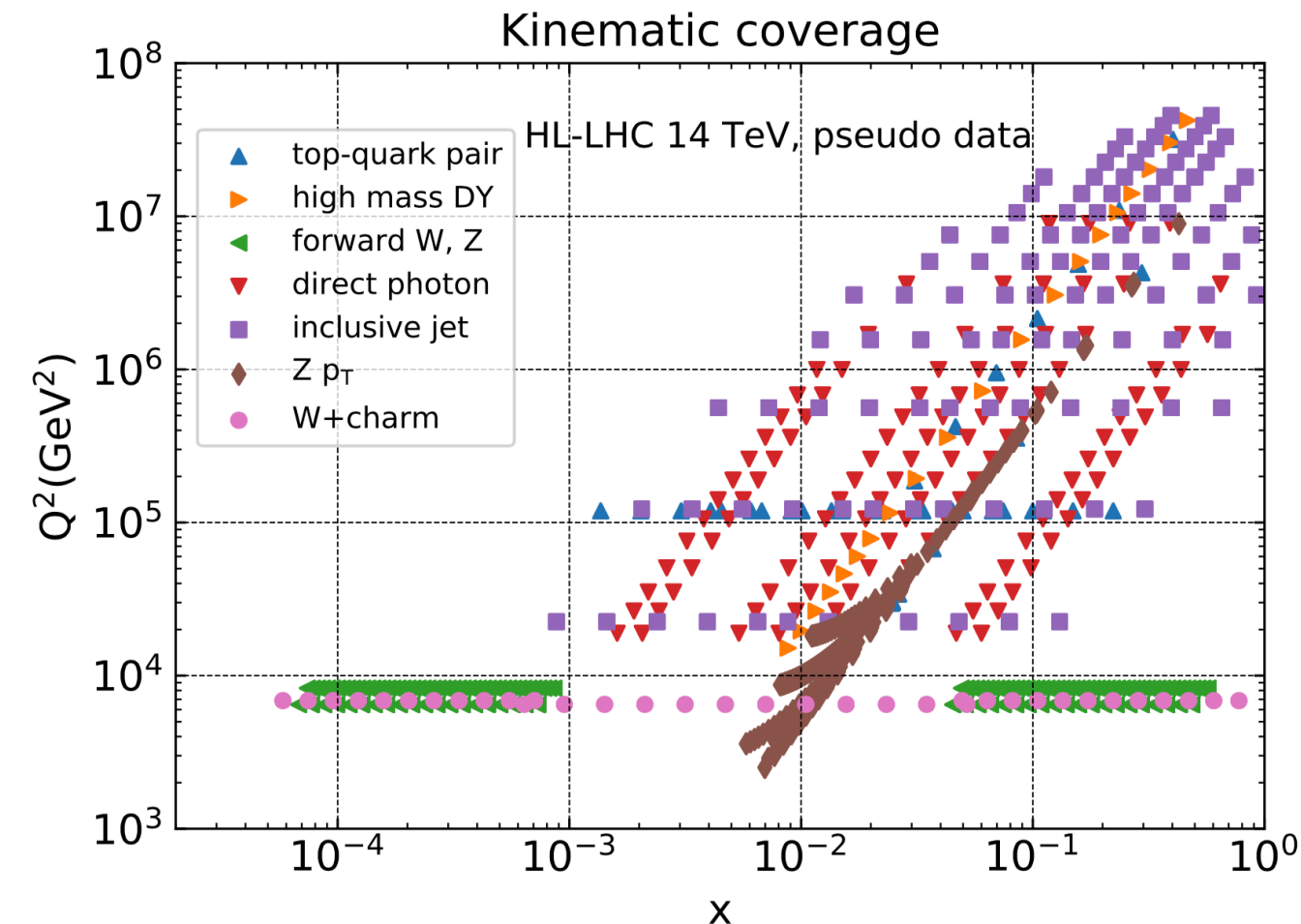
Direct computation of **x-space PDFs** now feasible

What next?

See also Rik's talk

Towards ultimate PDFs at the HL-LHC and LHeC

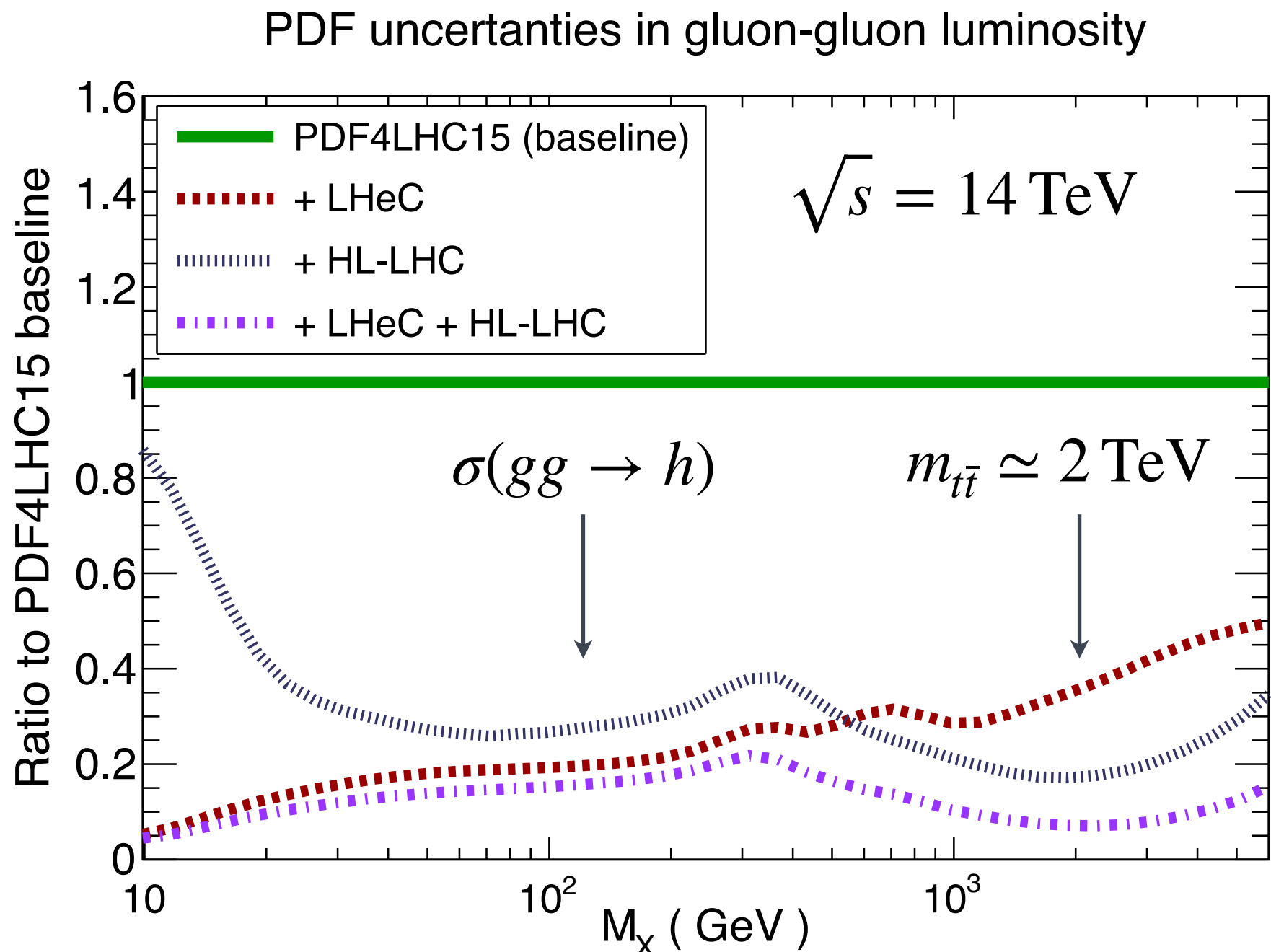
Exploit **novel facilities** for precision studies of the proton structure



Fully **complementary** in terms of PDF constraints, possible synchronous operation

Towards ultimate PDFs at the HL-LHC and LHeC

Exploit **novel facilities** for precision studies of the proton structure



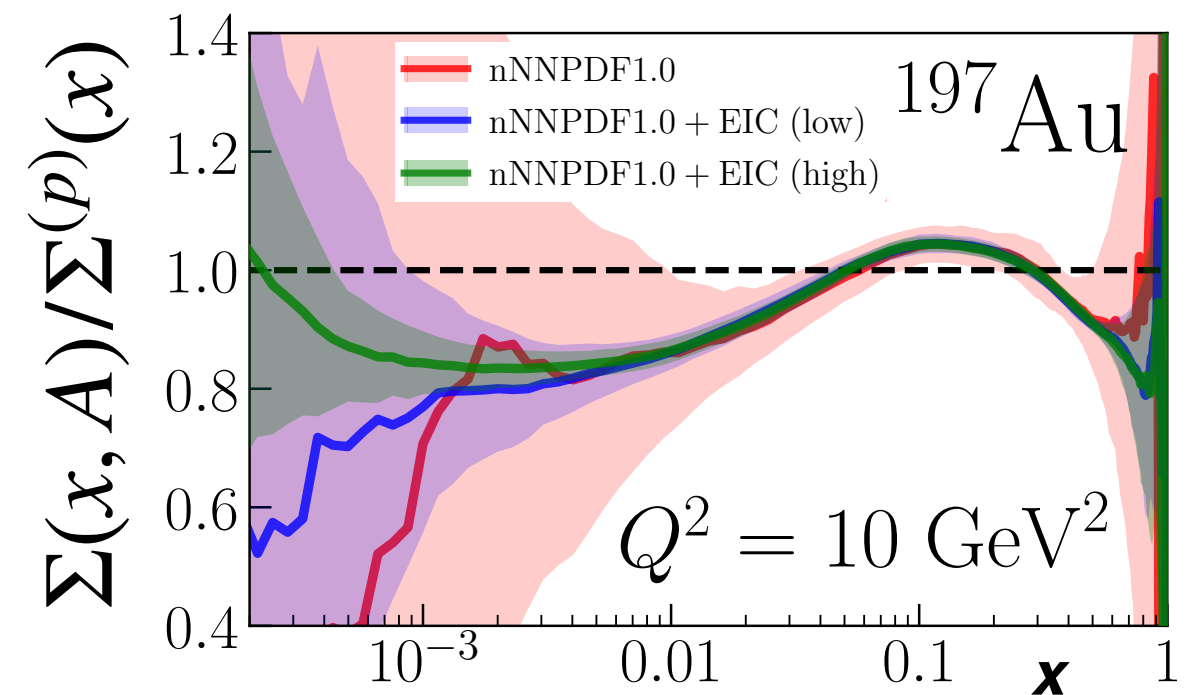
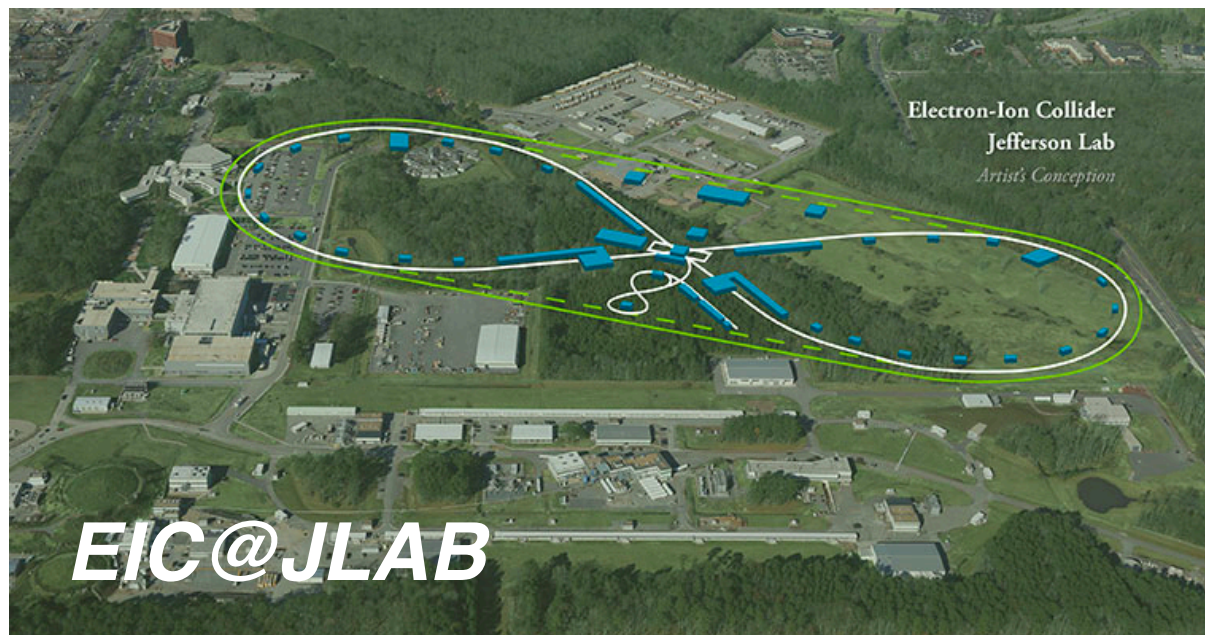
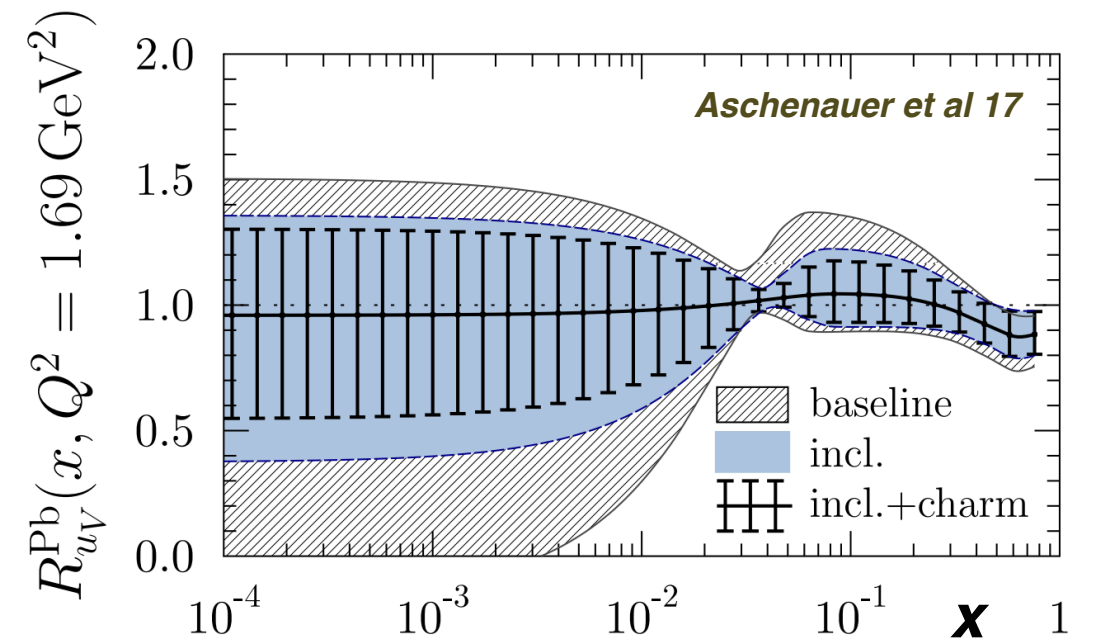
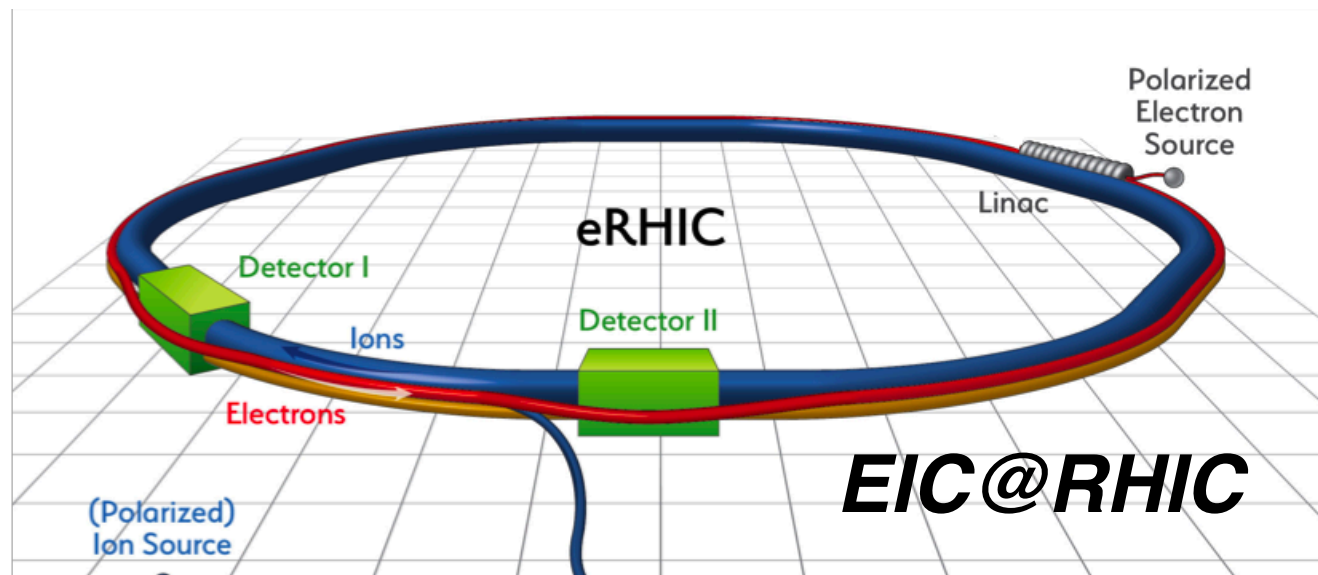
Abdul-Khalek et al. 18

*Harland-Lang, Gwenlan
WG1+WG7*

A reduction of PDF uncertainties by **up to a factor 10** could be within reach

Nuclear structure at the Electron-Ion Collider

Unique facility to study **QCD matter** and **nucleon/nuclear structure**

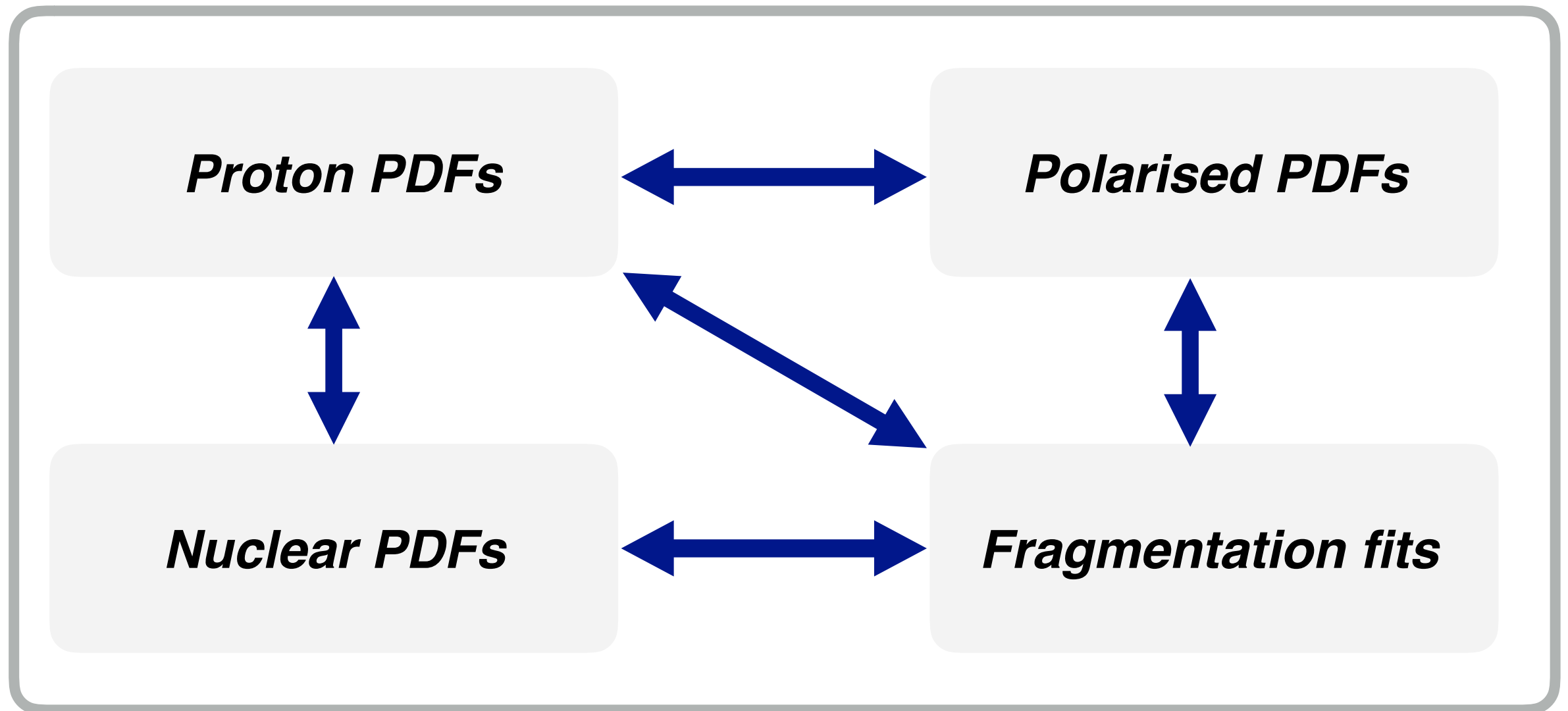


Pinning down nuclear PDFs at small- x : onset of gluon-dominated matter?

*Abdul-Khalek
et al 19*

Universal QCD fits

Pushing the **precision frontier** of **QCD fits** requires accounting for **cross-talk** between different **non-perturbative QCD** quantities

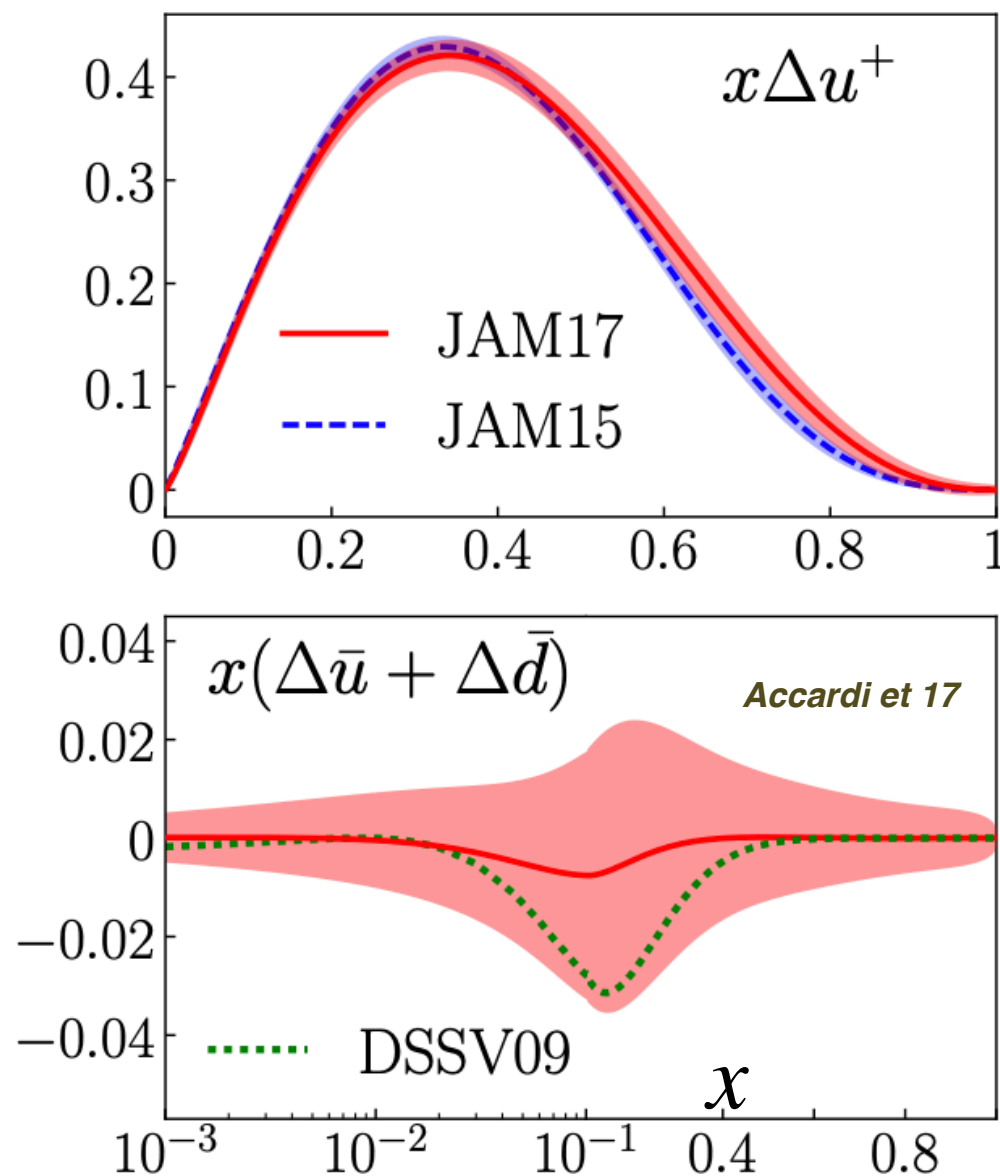


Towards universal/integrated global analyses of non-perturbative QCD

Universal QCD fits

Pushing the **precision frontier** of **QCD fits** requires accounting for **cross-talk** between different **non-perturbative QCD** quantities

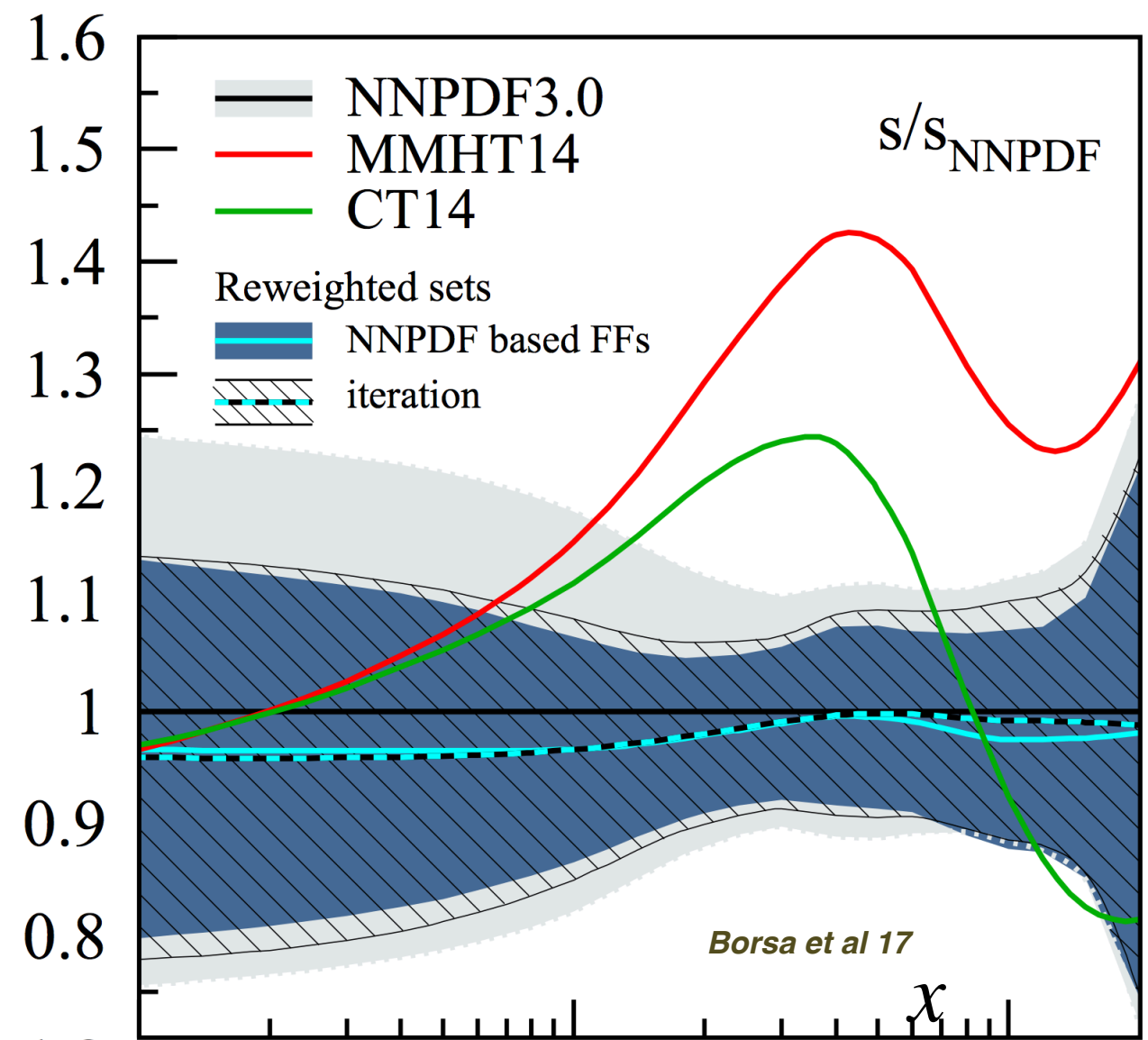
Polarised PDFs + FFs



Sato WG1

Juan Rojo

Unpolarised PDFs + FFs



Borsa et al 17

Summary and outlook

The accurate determination of the **quark and gluon structure of the proton** is an essential ingredient for **LHC phenomenology** and **beyond**

 **LHC measurements** start to dominate the global PDF fit results

Summary and outlook

The accurate determination of the **quark and gluon structure of the proton** is an essential ingredient for **LHC phenomenology** and **beyond**

- 📌 **LHC measurements** start to dominate the global PDF fit results
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The fascinating study of the proton structure never stops surprising us, stay tuned!

Summary and outlook

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Thanks for your attention!

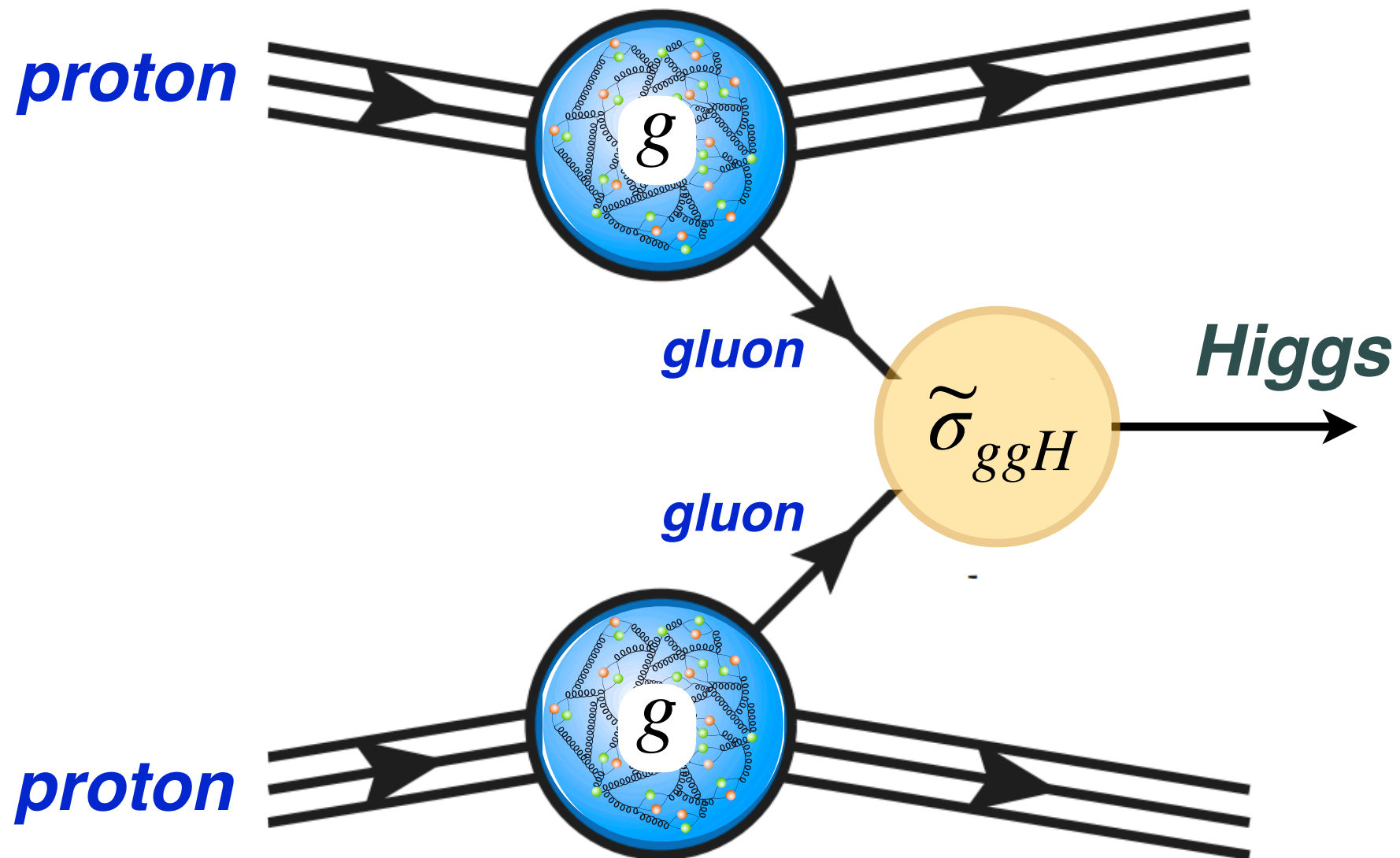
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Extra Material

Parton Distributions

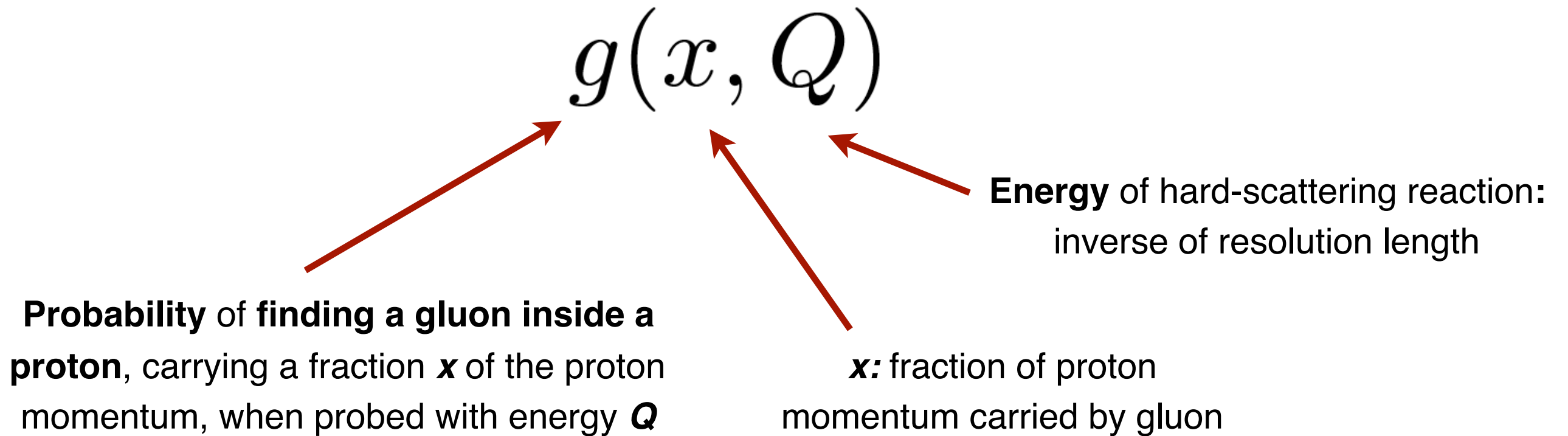
$$N_{\text{LHC}}(H) \sim g \otimes g \otimes \tilde{\sigma}_{ggH}$$

Parton Distributions



All-order structure: **QCD factorisation theorems**

Parton Distributions



Dependence on x fixed by **non-perturbative QCD dynamics**: extract from experimental data

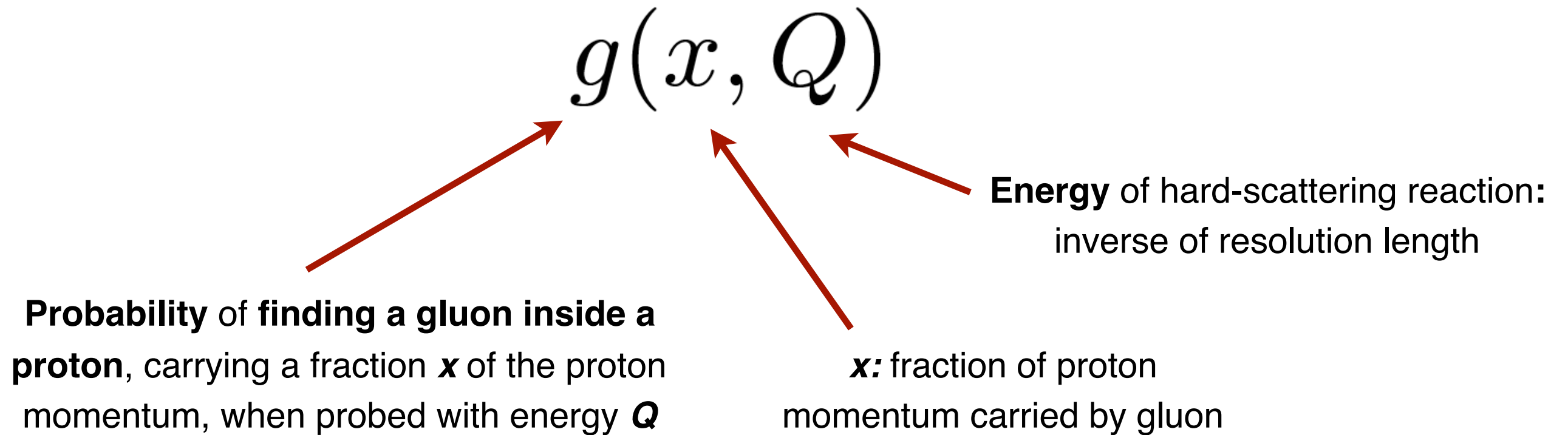
📌 **Energy conservation**: momentum sum rule

$$\int_0^1 dx \, x \left(\sum_{i=1}^{n_f} [q_i(x, Q^2) + \bar{q}_i(x, Q^2)] + g(x, Q^2) \right) = 1$$

📌 **Quark number conservation**: valence sum rules

$$\int_0^1 dx \, (u(x, Q^2) + \bar{u}(x, Q^2)) = 2$$

Parton Distributions

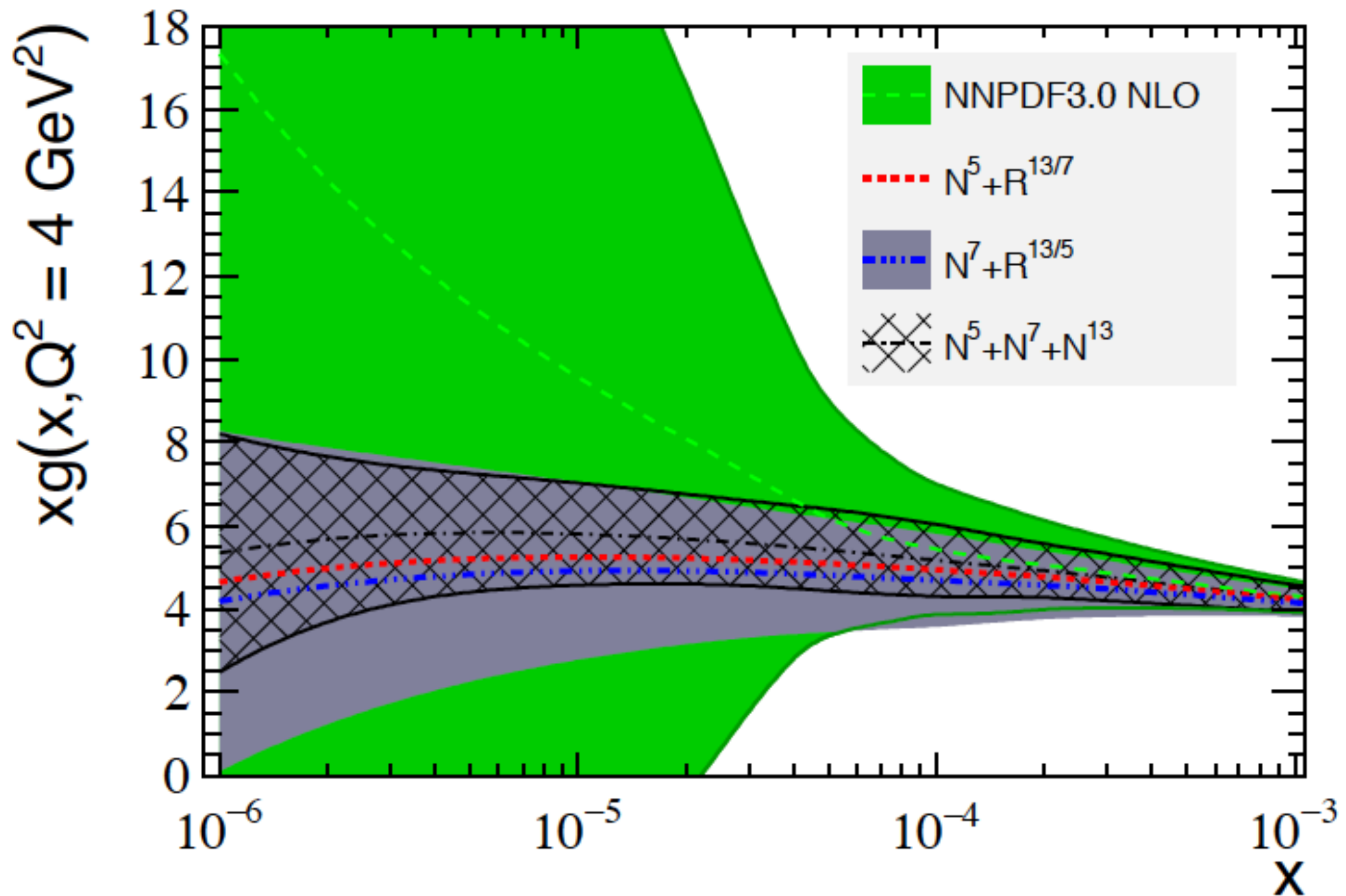


Dependence on Q fixed by **perturbative QCD dynamics**: computed up to $\mathcal{O}(\alpha_s^4)$

$$\frac{\partial}{\partial \ln Q^2} q_i(x, Q^2) = \int_x^1 \frac{dz}{z} P_{ij} \left(\frac{x}{z}, \alpha_s(Q^2) \right) q_j(z, Q^2)$$

DGLAP parton evolution equations

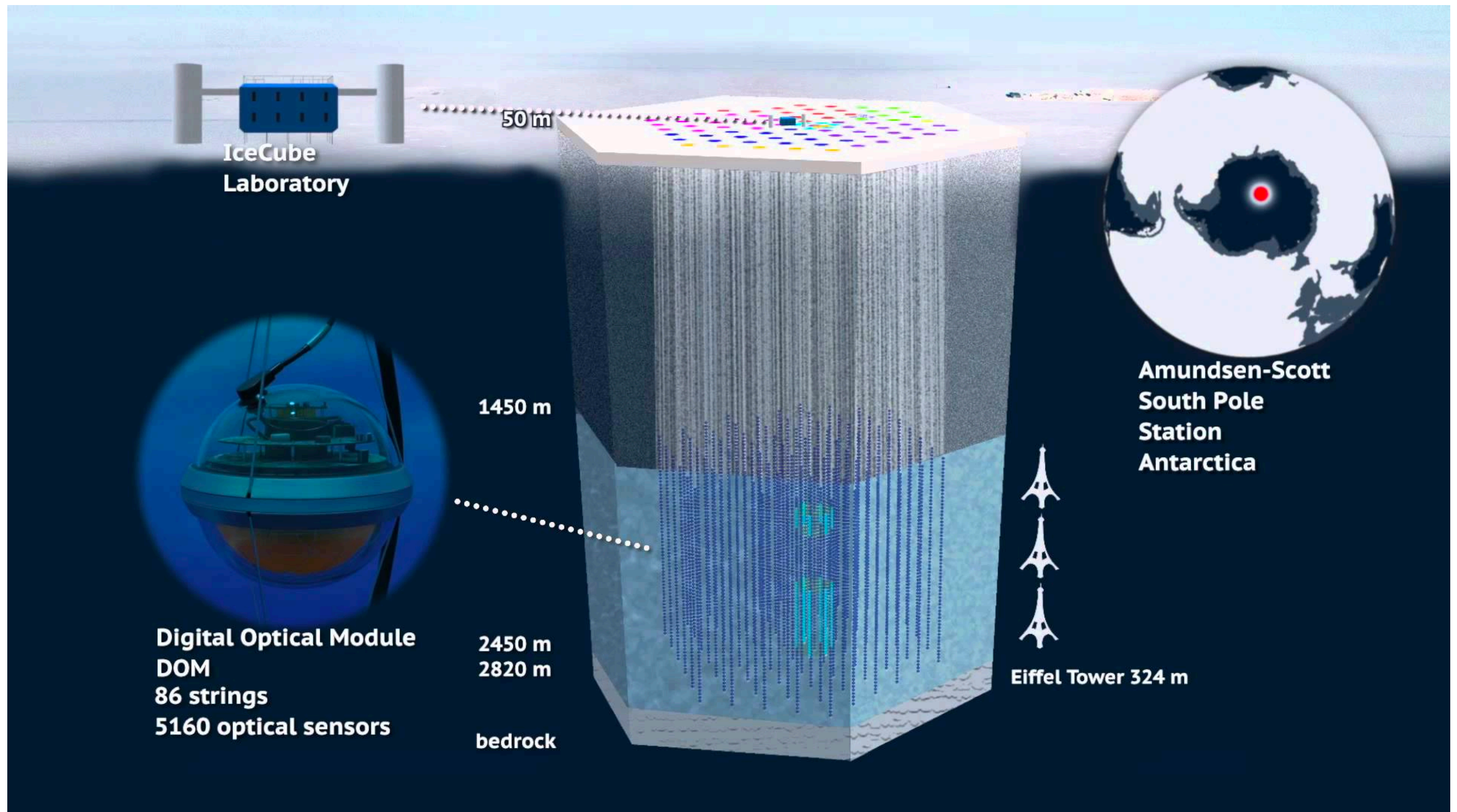
Forward charm production



gluon PDF uncertainties reduced by **factor 10** at $x \approx 10^{-6}$

Neutrino telescopes

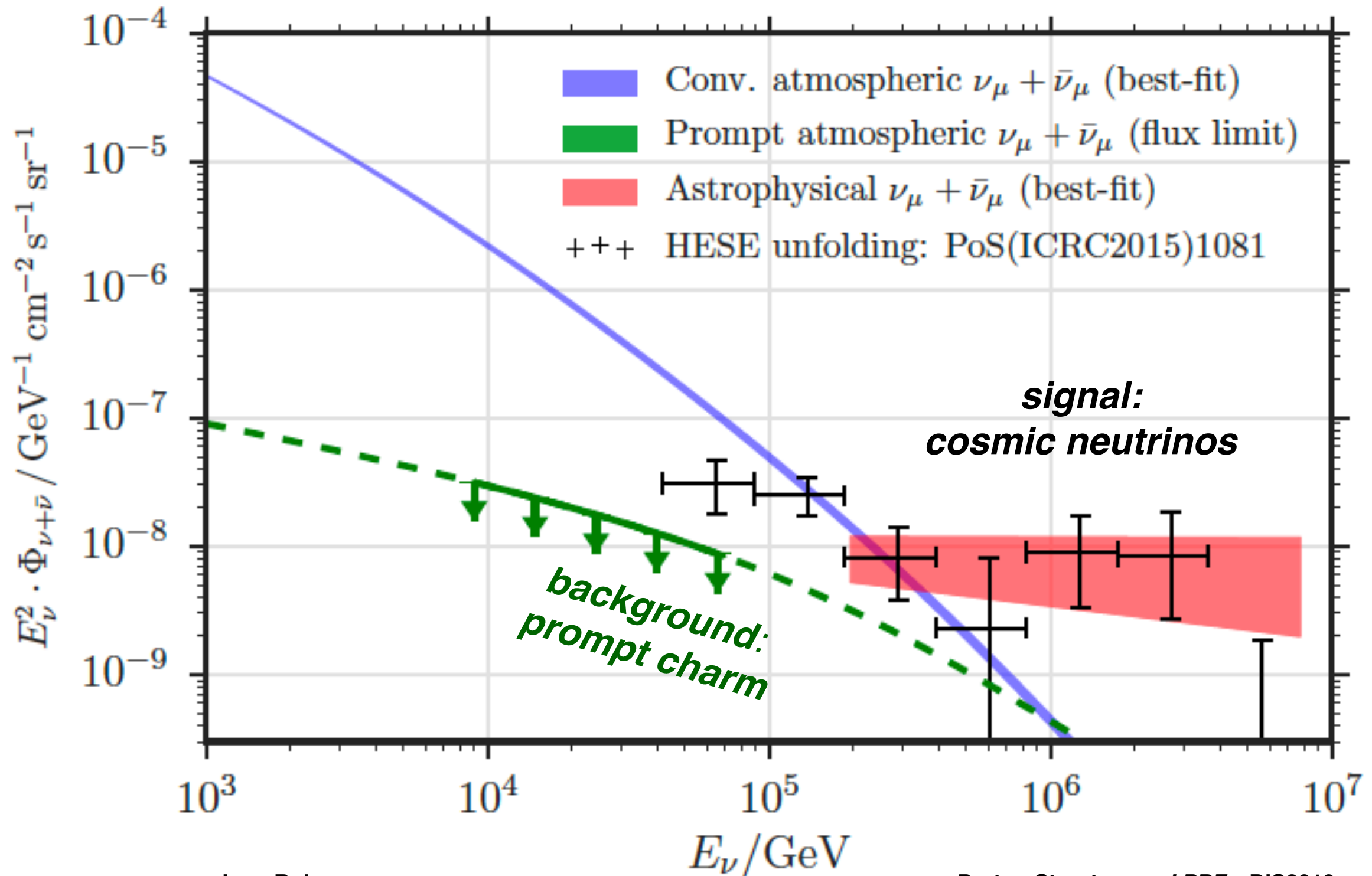
Ultra-high energy (UHE) neutrinos: novel window to the extreme Universe!



Neutrino telescopes as QCD microscopes

signal: cosmic neutrino - nucleus scattering

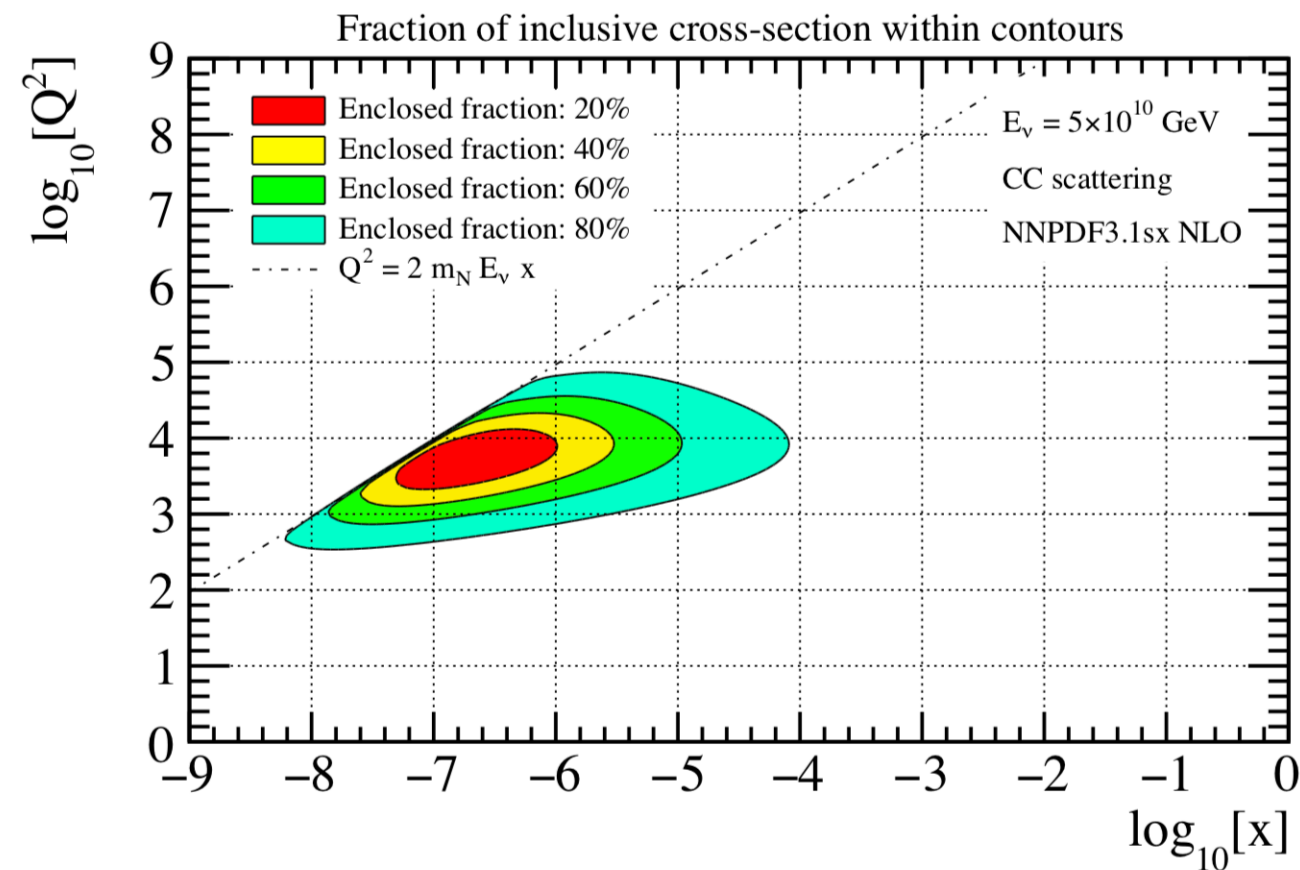
background: prompt charm production



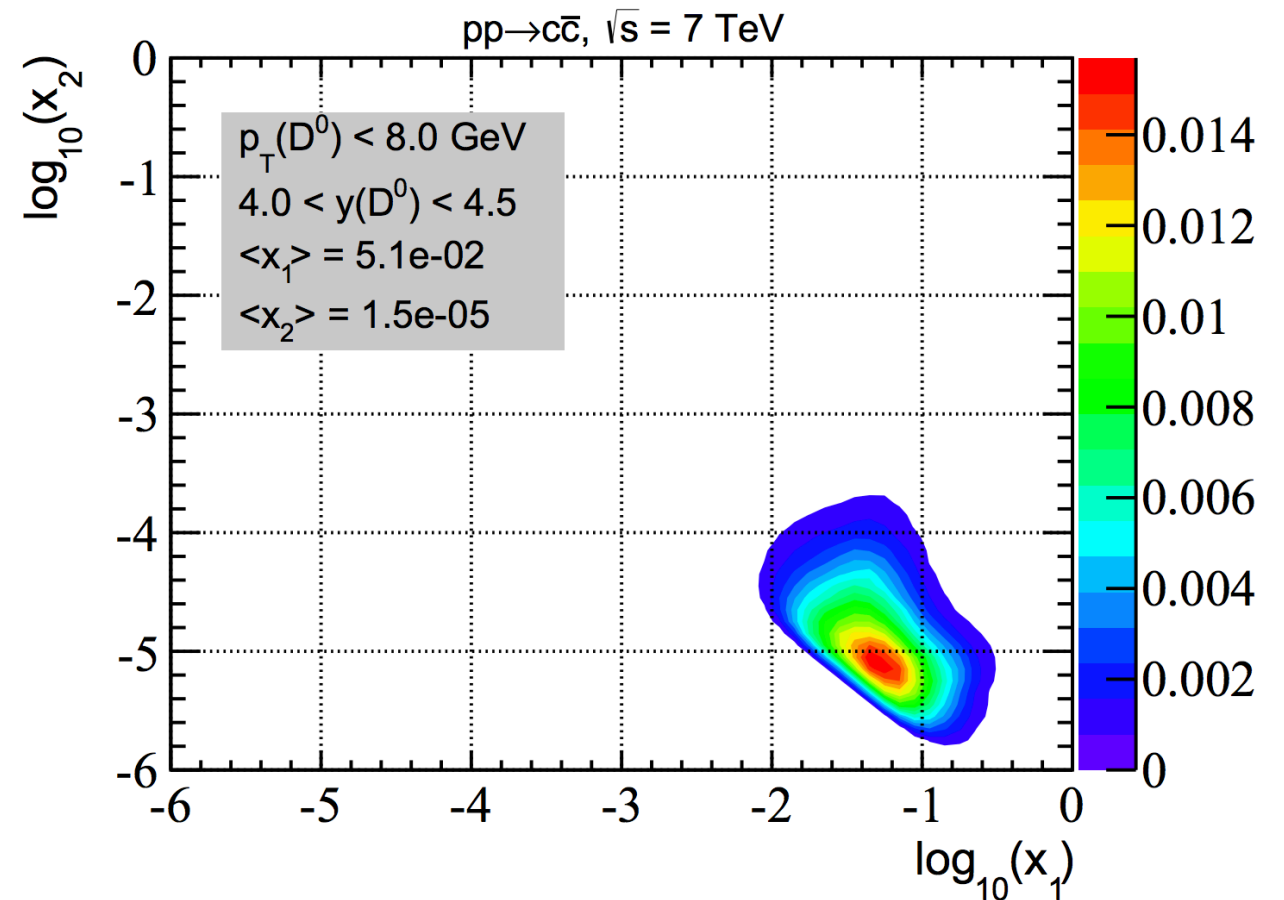
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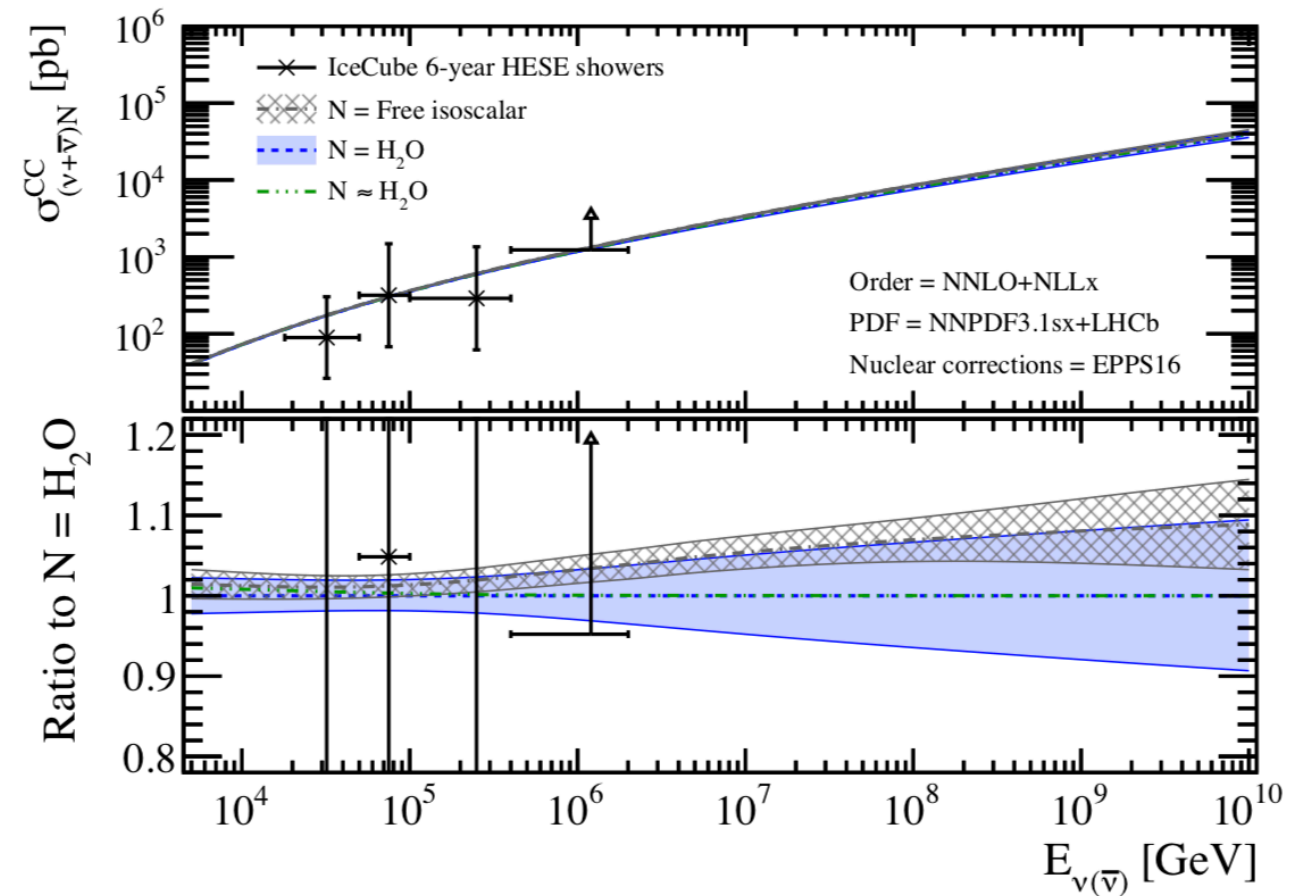
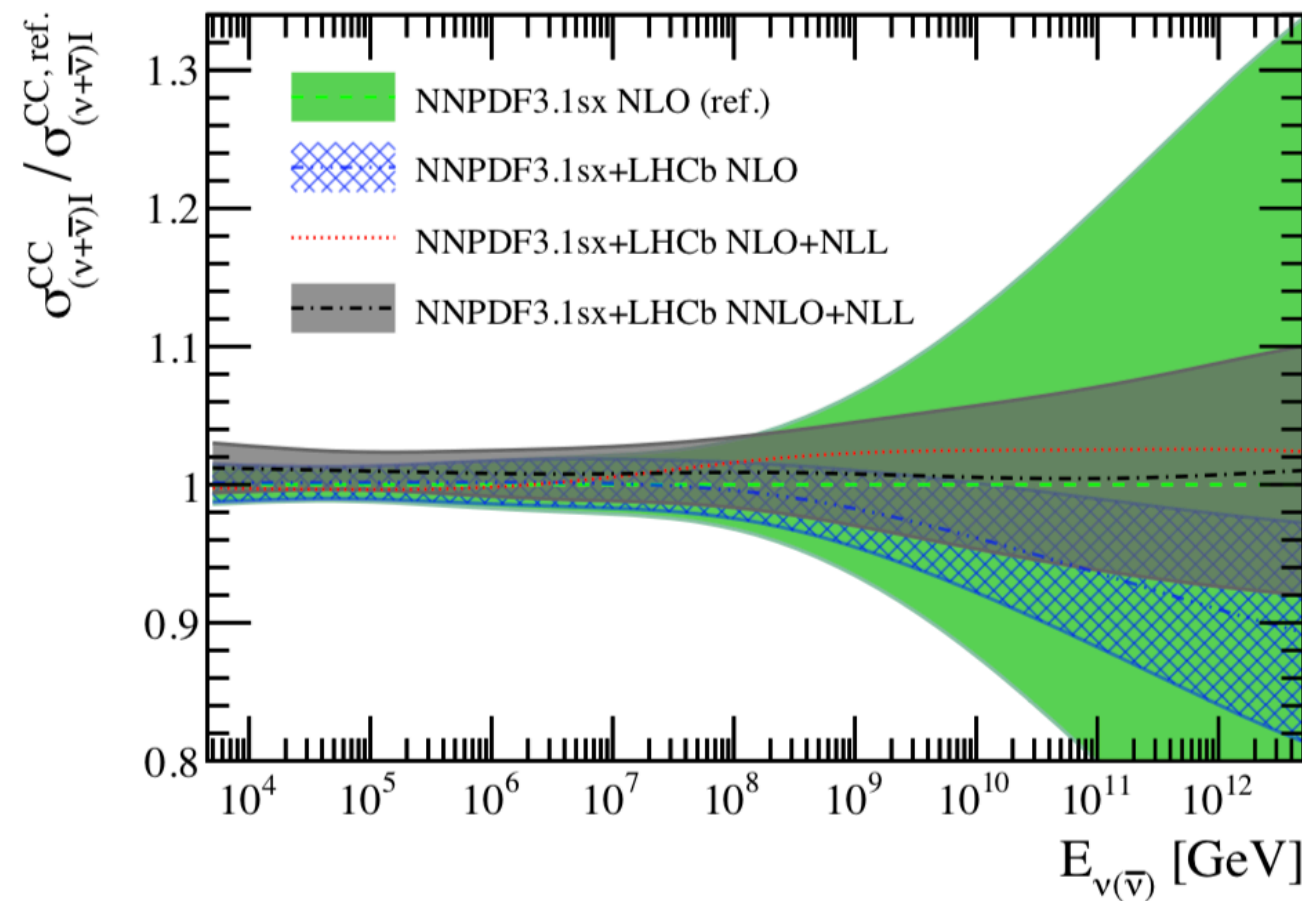


Sensitive to **small- x quarks** (and thus gluons via evolution) down to $x \approx 10^{-8}$ and $Q \approx M_W$



Sensitive to **small- x gluons** down to $x \approx 10^{-6}$ and $Q \approx M_{\text{charm}}$ in the **centre-of-mass frame**

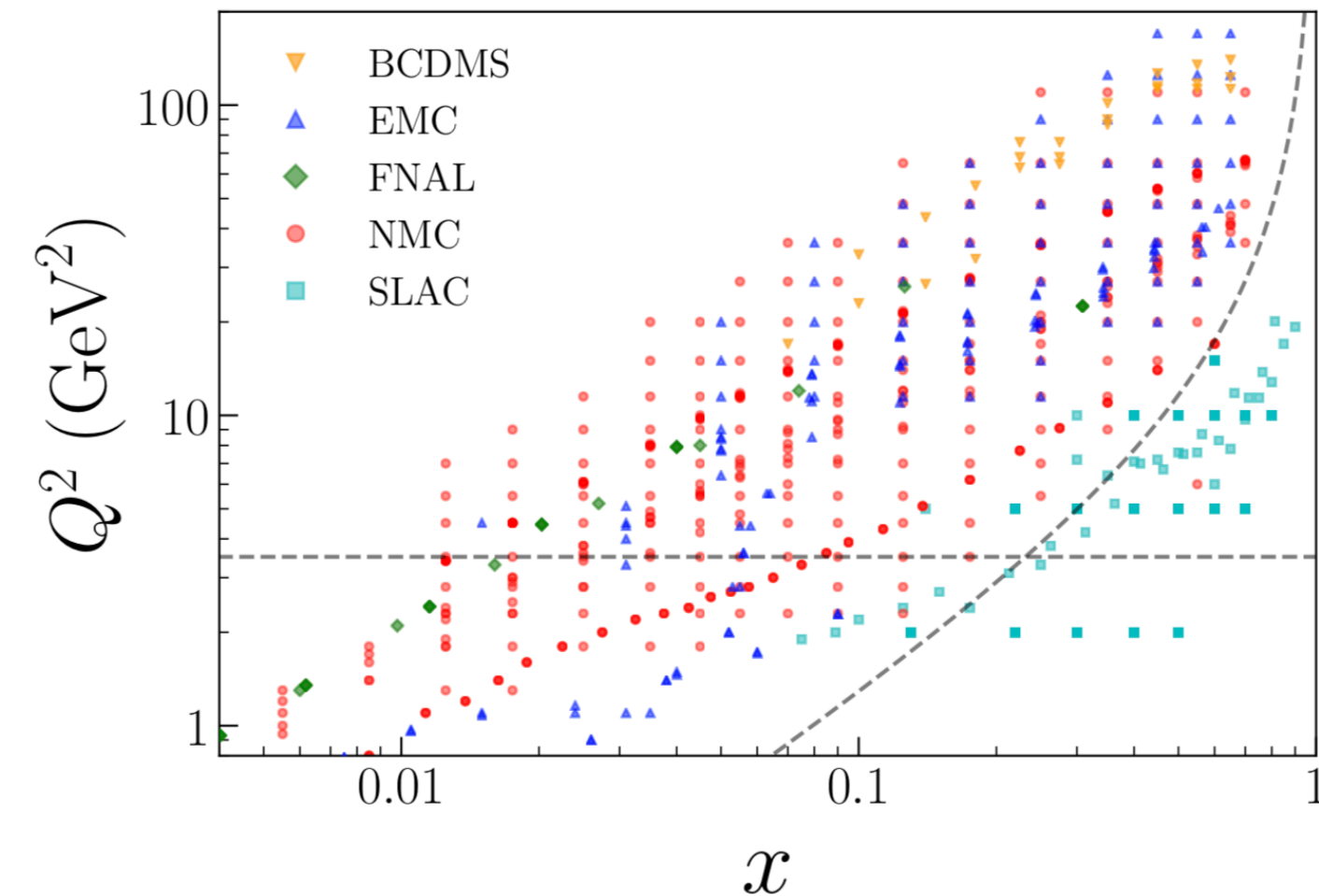
UHE neutrino-nucleus cross-section



State-of-the-art predictions for **ultra-high energy** neutrino interactions

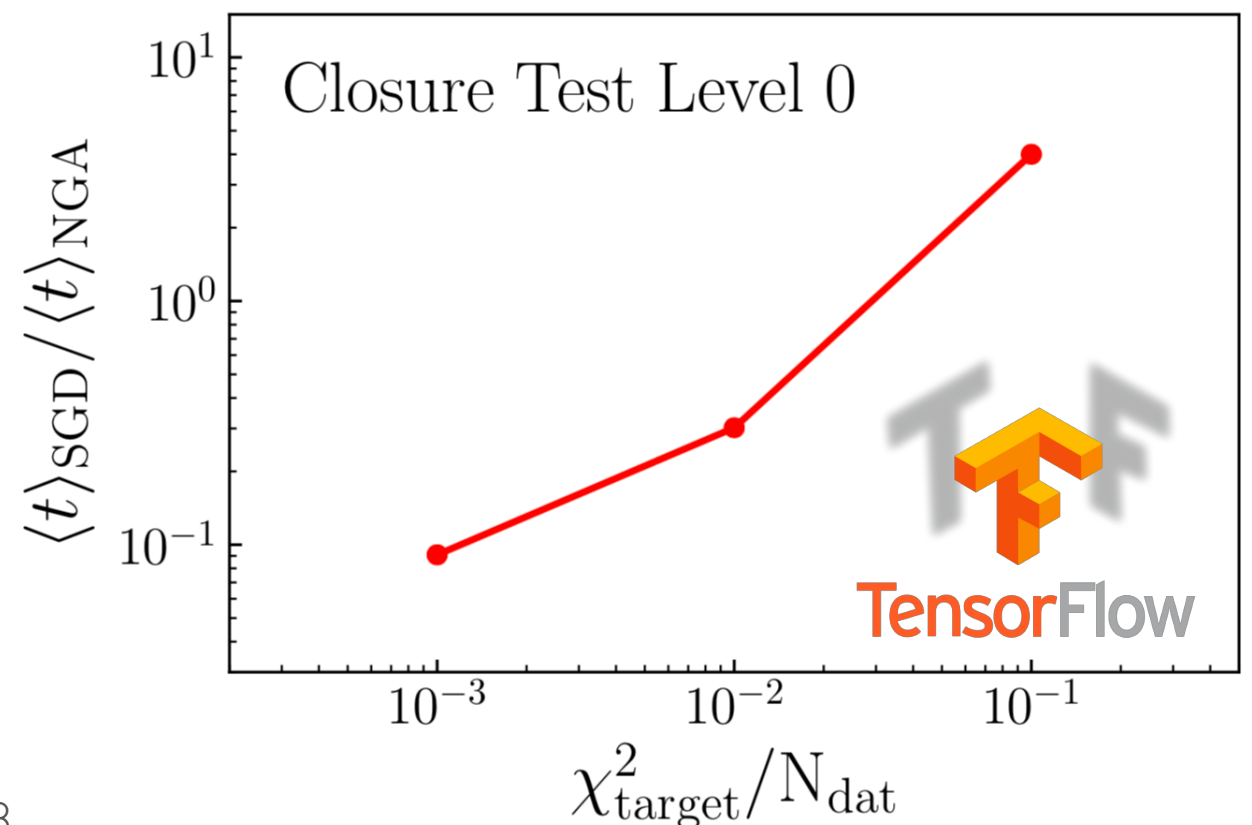
- 📌 **BFKL small- x resummation effects** in PDFs and structure functions
- 📌 Constraints on small- x PDFs from **LHCb charm production**
- 📌 IceCube and other neutrino telescopes are the **ultimate QCD microscopes!**

nNNPDF1.0



- NN training with Google's *TensorFlow*:
Stochastic Gradient Descent
- Factor 10 (or more) speed up as compared to evolutionary optimisers used in NNPDF

- First determination of **nuclear PDFs** based on the NNPDF framework
- Neutral-current DIS data for a range of A values
- Impose well-known free-nucleon PDFs as **boundary condition for $A=1$**



nNNPDF1.0

📌 Parametrize nPDFs with ANNs with $\mathbf{x}$, $\ln(\mathbf{x})$, $\mathbf{A}$ as input: **fully model-independent**

$$q_i(x, Q_0, A) = B_i x^{-\alpha_i} (1-x)^{\beta_i} \text{NN}(x, A), \quad i = g, \Sigma, T_8$$

📌 Gluon normalisation (A -dependent) fixed by the **momentum sum rule**

$$B_g(A) = \left(1 - \int_0^1 dx \, x \Sigma(x, Q_0, A) \right) / \int_0^1 dx \, x g(x, Q_0, A)$$

📌 **Proton boundary condition** implemented as a **penalty** in the figure of merit

$$\chi^2 = \sum_{j=1}^{n_{\text{dat}}} \frac{\left(F_j^{(\text{exp})} - F_j^{(\text{th})} \right)^2}{\sigma_j^{(\text{exp})2}} + \lambda \sum_{i=g, \Sigma, T_8} \sum_{k=1}^{n_x} \frac{\left(q_i(x_k, Q_0, A) - q_i^{(\text{ref})}(x_k, Q_0, A = 1) \right)^2}{\left(\delta q_i^{(\text{ref})}(x_k, Q_0, A = 1) \right)^2}$$

$q_i^{(\text{ref})}(x_k, Q_0, A = 1)$ Isoscalar **NNPDF3.1** NNLO global fit