

# Achievements and Open Issues in the Determination of Fragmentation Functions

XVI International Workshop on Hadron Structure and Spectroscopy

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# Foreword: the fragmentation realm

[Prog.Part.Nucl.Phys. 91 (2016) 136]

- Integrated (collinear) FFs (possibly polarised)

$$\Delta^{h/q}(z; P_h, S_h) = \sum_f \int \frac{d\xi^+}{2\pi} e^{ik^- \xi^+} \langle 0 | \mathcal{W}(\infty^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \\ \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle$$

- Transverse momentum dependent (TMD) FFs

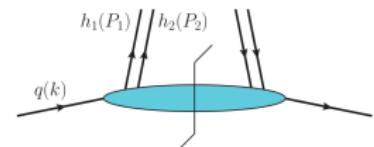
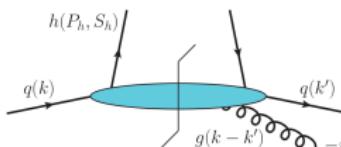
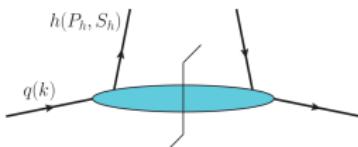
$$\Delta^{h/q}(z, \vec{k}_T; P_h, S_h) = \sum_f \sum \frac{d\xi^+ d^2 \vec{\xi}_T}{(2\pi)^3} e^{i(k^- \xi^+ - \vec{k}_T \cdot \vec{\xi}_T)} \langle 0 | \mathcal{W}_1(\infty, \xi) \psi_q(\xi^+, 0^-, \vec{\xi}_T) | P_h, S_h; X \rangle \\ \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}_2(0, \infty) | 0 \rangle$$

- Higher twist FFs

$$\Delta_F^{h/q;i}(z, z'; P_h, S_h) = \frac{1}{P_h^-} \sum_f \sum \frac{d\xi^+ d\xi'^+}{(2\pi)^2} e^{ik' - \xi^+} e^{i(k^- - k'^-) \zeta^+} \\ \times \langle 0 | \mathcal{W}(\infty^+, \zeta^+) i g F^{-i}(\zeta^+, 0^-, \vec{0}_T) \mathcal{W}(\zeta^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \\ \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle$$

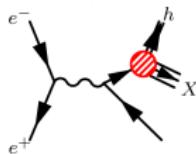
- Di-hadron FFs

$$\Delta^{h_1 h_2 / q}(z, \vec{k}_T; P_1, P_2) = \sum_f \int \frac{d\xi^+ d^2 \vec{\xi}_T}{(2\pi)^3} e^{i(k^- \xi^- - \vec{k}_T \cdot \vec{\xi}_T)} \langle 0 | \mathcal{W}_1(\infty, \xi) \psi_q(\xi^+, 0^-, \vec{\xi}_T) | P_1, P_2; X \rangle \\ \times \langle P_1, P_2; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}_2(0, \infty) | 0 \rangle$$

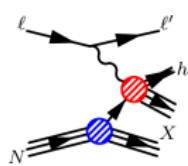


# Factorisation of physical observables

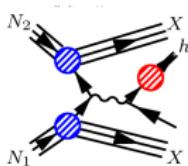
[Adv.Ser.Direct.HEP 5 (1988) 1]



$e^+ + e^- \rightarrow h + X$   
single-inclusive  
annihilation (SIA)



$\ell + N \rightarrow \ell' + h + X$   
semi-inclusive deep-  
inelastic scattering (SIDIS)



$N_1 + N_2 \rightarrow h + X$   
high- $p_T$  hadron production  
in  $pp$  collisions (PP)

$$\frac{d\sigma^h}{dz} = F_T^h(z, Q^2) + F_L^h(z, Q^2) = F_2^h(x, Q^2)$$

$$F_{k=T,L,2}^h = \frac{4\pi\alpha_{\text{em}}^2}{Q^2} \langle e^2 \rangle \left\{ D_\Sigma^h \otimes C_{k,q}^S + n_f D_g^h \otimes C_{k,g}^S + D_{\text{NS}}^h \otimes C_{k,q}^{\text{NS}} \right\}$$

up to NNLO [PLB 386 (1996) 422; NPB 487 (1997) 233; PLB 392 (1997) 207]

$$\frac{d\sigma^h}{dxdydz} = \frac{2\pi\alpha_{\text{em}}^2}{Q^2} \left[ \frac{1+(1-y)^2}{y} 2F_1^h + \frac{2(1-y)}{y} F_L^h \right]$$

$$2F_1^h = e_q^2 \left\{ q \otimes D_q^h + \frac{\alpha_s}{2\pi} \left[ q \otimes C_{qq}^1 \otimes D_q^h + q \otimes C_{gq}^1 \otimes D_g^h + g \otimes C_{qg}^1 \otimes D_q^h \right] \right\}$$

$$F_L^h = \frac{\alpha_s}{2\pi} \sum_{q,\bar{q}} e_q^2 \left[ q \otimes C_{qq}^L \otimes D_q^h + q \otimes C_{gq}^L \otimes D_g^h + g \otimes C_{qg}^L \otimes D_q^h \right]$$

up to NLO [NPB 160 (1979) 301; PRD 57 (1998) 5811]  
partial NNLO [PRD 95 (2017) 034027]

$$E_h \frac{d^3\sigma}{dp_{T,h}^3} = \sum_{a,b,c} f_a \otimes f_b \otimes \hat{\sigma}_{ab}^c \otimes D_c^h$$

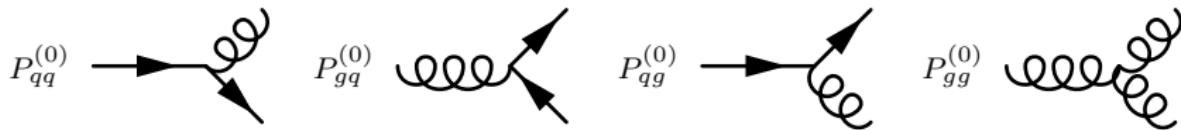
$$= \sum_{i,j,k} \int \frac{dx_a}{x_a} \int \frac{dx_b}{x_b} \int \frac{dz}{z^2} f^{i/p_a}(x_a) f^{j/p_b}(x_b) D^{h/k}(z) \hat{\sigma}^{ij \rightarrow k} \delta(\hat{s} + \hat{t} + \hat{u})$$

up to NLO [PRD 67 (2003) 054004; PRD 67 (2003) 054005]

# Evolution of FFs: DGLAP equations [NPB 126 (1977) 298]

A set of  $(2n_f + 1)$  integro-differential equations ( $n_f$ =number of active flavours)

$$\frac{\partial}{\partial \ln \mu^2} D_i(x, \mu^2) = \sum_j^{n_f} \int_x^1 \frac{dz}{z} P_{ji}(z, \alpha_s(\mu^2)) D_j\left(\frac{x}{z}, \mu^2\right)$$



LO [Sov. J. Nucl. Phys. 15 (1973) 438; NPB 126 (1977) 298; NPB 136 (1978) 445]

NLO [NPB 175 (1980) 27, PLB 97 (1980) 497, PRD 48 (1993) 116]

NNLO [PLB 638 (2006) 61, PLB 659 (2008) 290, NPB 854 (2012) 133]

Must be careful with fixed-order splitting functions as  $z \rightarrow 0$  ( $m = 1, \dots, 2k+1$ )

SPACE-LIKE CASE

$$P_{ji} \propto \frac{\alpha_s^{k+1}}{x} \log^{k+1-m} \frac{1}{x}$$

TIME-LIKE CASE

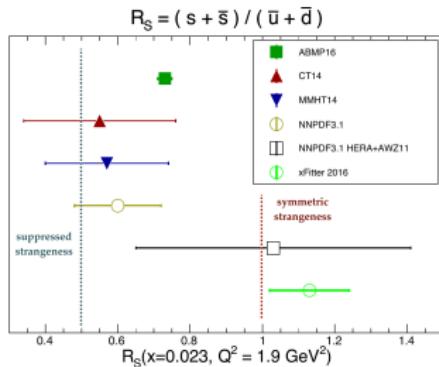
$$P_{ji} \propto \frac{\alpha_s^{k+1}}{z} \log^{2(k+1)-m-1} z$$

Soft gluon logarithms diverge more rapidly in the TL case than in the SL case: as  $z$  decreases, the unresummed SGLs spoil the convergence of the FO series for  $P(z, \alpha_s)$  if  $\log \frac{1}{z} \geq \mathcal{O}\left(\alpha_s^{-1/2}\right)$

# 1. Open issues

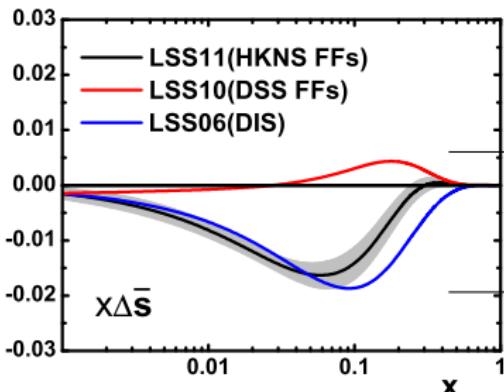
# Fragmentation functions: why should we bother?

Example 1: The strange (polarised) parton distribution and SIDIS

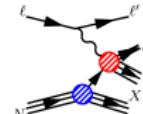


Can SIDIS data be used to determine  $s$ ?  
What is the bias induced by FFs onto PDFs?  
→ How well do we know kaon FFs?

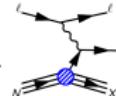
If SIDIS data is used to determine  $\Delta s$ ,  $K^\pm$  FFs  
for different sets lead to different results  
Such results may differ significantly among  
them and w.r.t. the results obtained from DIS  
→ How well do we know kaon FFs?



directly from SIDIS Kaon data



indirectly from DIS + SU(3)



# Fragmentation functions: why should we bother?

Example 2: Heavy quark fragmentation: the  $D^*$  case

Constrain the low- $x$  (gluon) PDFs through charm production in the forward region

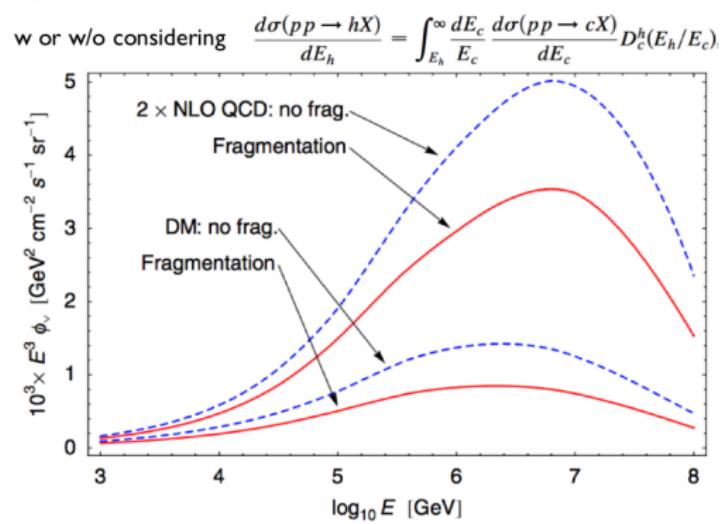
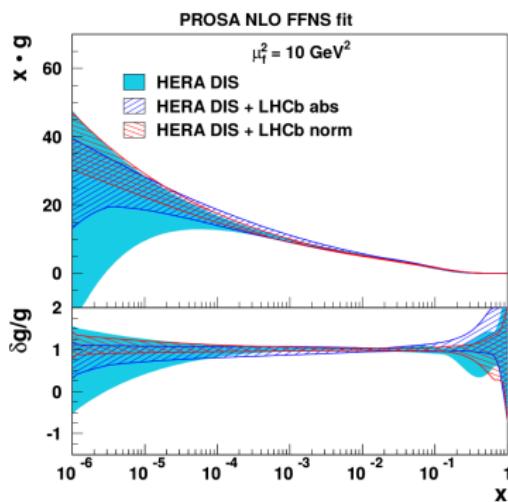
[EPJ C75 (2015) 396; JHEP 1602 (2016) 130]

Compute the prompt atmospheric neutrino flux

[PRD 78 (2008) 043005; JHEP 1506 (2015) 110]

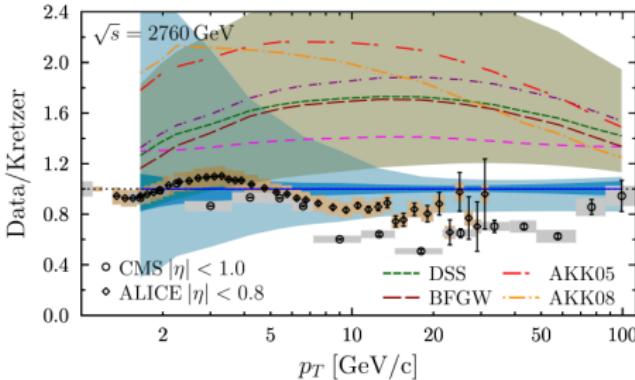
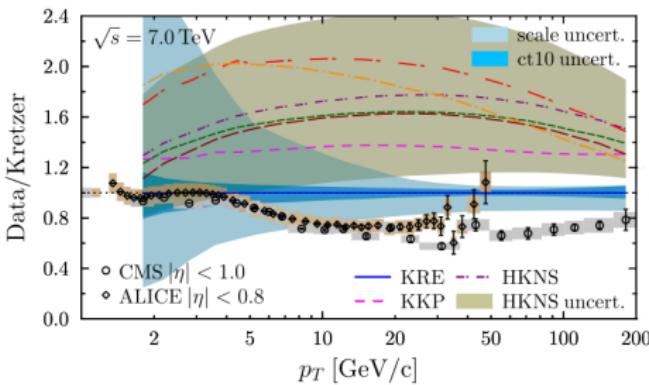
Extract information on the medium in heavy ion collisions

[JHEP 1703 (2017) 146]

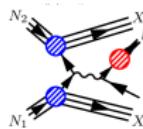
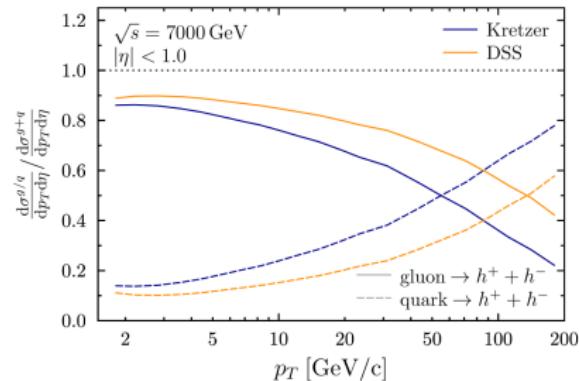


# Fragmentation functions: why should we bother?

**Example 3:** Ratio of the inclusive charged-hadron spectra measured by CMS and ALICE



Figures taken from [NPB 883 (2014) 615]



$$E \frac{d^3 \sigma}{dp_T^3} = \sum_{a,b,c} f_a \otimes f_b \otimes \hat{\sigma}_{ab}^c \otimes D_c^h$$

Predictions from all available FF sets are not compatible with CMS and ALICE data, not even within scale and PDF/FF uncertainties  
 → How well do we know the gluon FF?

## 2. Achievements

# Available fragmentation function sets (status 2019)

	DEHSS	HKNS	JAM	NNFF
DATA	SIA <input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
	SIDIS <input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/> <input checked="" type="checkbox"/>
	PP <input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/> ( $h^\pm$ )
METH.	statistical treatment	Iterative Hessian 68% - 90%	Hessian $\Delta\chi^2 = 15.94$	Monte Carlo
	parametrisation	standard	standard	Monte Carlo
THEORY	pert. order	(N)NLO	NLO	LO, NLO, NNLO
	HF scheme	ZM(GM)-VFN	ZM-VFN	ZM-VFN
hadron species		$\pi^\pm, K^\pm, p/\bar{p}, h^\pm$	$\pi^\pm, K^\pm, p/\bar{p}$	$\pi^\pm, K^\pm, p/\bar{p}, h^\pm$
latest update		PRD 91 (2015) 014035 PRD 95 (2017) 094019	PTEP 2016 (2016) 113B04	PRD 94 (2016) 114004 PRL 119 (2017) 132001
				EPJ C77 (2017) 516 EPJ C78 (2018) 651

+ many others (including analyses for specific hadrons)

Focus on  $\pi$  and  $K$  which constitute the largest fraction in measured yields

BKK96 [PRD 53 (1996) 3553]

DSV97 [PRD 57 (1998) 5811]

BFGW00 [EPJ C19 (2001) 89]

$K^0$

$\Lambda^0$

$h^\pm$

AESS11 [PRD 83 (2011) 034002]

AKSRV17 [PRD 96 (2017) 034028]

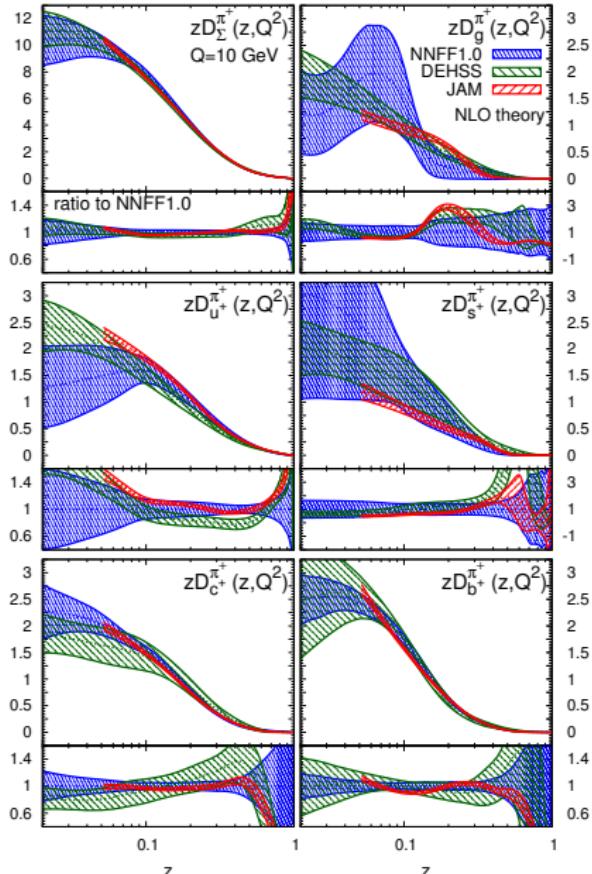
LSS15 [PRD 96 (2016) 074026]

$\eta$

$D^*$

SIDIS only

# Comparison at NLO ( $\pi^+$ ): NNFF1.0 - JAM - DEHSS



DEHSS [PRD 91 (2015) 014035]

(+SIDIS +PP)

JAM [PRD 94 (2016) 114004]

(almost same dataset as NNFF1.0)

different cuts at small  $z$

$D_{\Sigma}^{\pi^+}$ : excellent mutual agreement  
both c.v. and unc. (bulk of the dataset)

$D_g^{\pi^+}$ : slight disagreement  
different shapes, larger uncertainties  
DEHSS: data; JAM: parametrisation

$D_{u^+}^{\pi^+}, D_{s^+}^{\pi^+}$ : good overall agreement  
excellent with JAM, though larger uncertainties  
slightly different shape w.r.t. DEHSS (dataset)

$D_{c^+}^{\pi^+}, D_{b^+}^{\pi^+}$ : good overall agreement  
excellent with JAM, same uncertainties  
slightly different shape w.r.t. DEHSS (dataset)

# Comparison at NLO ( $K^+$ ): NNFF1.0 - JAM - DEHSS

DEHSS [PRD 95 (2017) 094019]  
 (+SIDIS +PP)

JAM [PRD 94 (2016) 114004]  
 (almost same dataset as NNFF1.0)

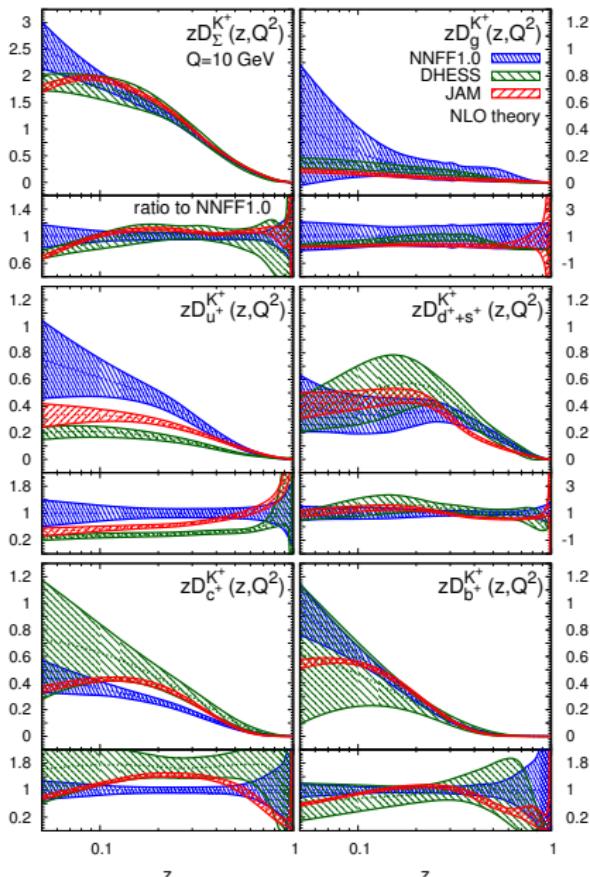
$D_{\Sigma}^{\pi^+}$ : excellent agreement (both c.v. and unc.)  
 bulk of the dataset

$D_g^{\pi^+}$ : good mutual agreement  
 similar shapes, larger uncertainties  
 DEHSS: data; JAM: parametrisation

$D_{u^+}^{\pi^+}$ : mutual sizable disagreement  
 differences in dataset and parametrisation  
 comparable uncertainties in the data region

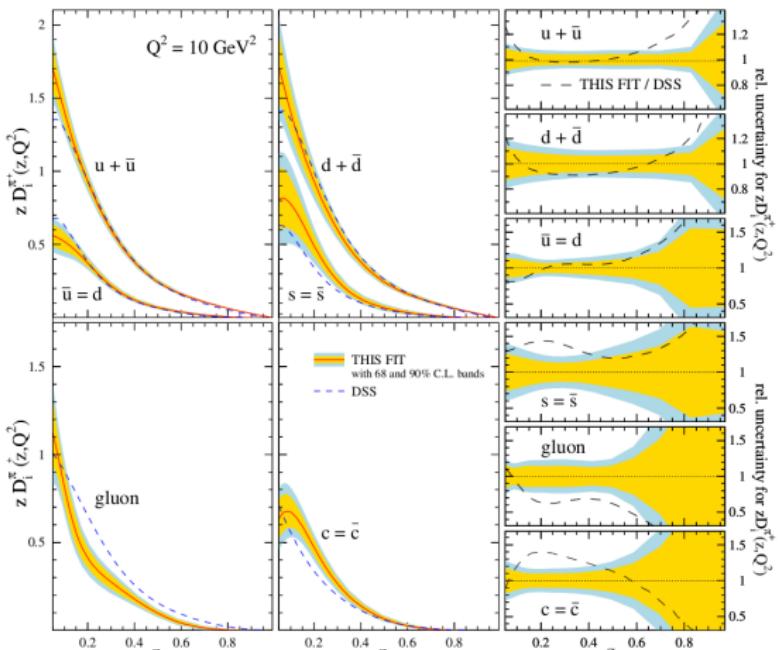
$D_{d^+}^{\pi^+} + D_{s^+}^{\pi^+}$ : fair mutual agreement  
 differences in dataset and parametrisation  
 comparable uncertainties in the data region

$D_{c^+}^{\pi^+}, D_{b^+}^{\pi^+}$ : excellent mutual agreement  
 uncertainties similar to JAM  
 DEHSS shows inflated uncertainties



# Global fit of pion fragmentation functions [PRD 91 (2015) 014035]

experiment	data type	norm. $N_i$	# data in fit	$\chi^2$
TPC [48]	incl.	1.043	17	17.3
	$uds$ tag	1.043	9	2.1
	$c$ tag	1.043	9	5.9
	$b$ tag	1.043	9	9.2
TASSO [49]	34 GeV	incl.	1.043	11
	44 GeV	incl.	1.043	7
SLD [19]	incl.	0.986	28	15.3
	$uds$ tag	0.986	17	18.5
	$c$ tag	0.986	17	16.1
	$b$ tag	0.986	17	5.8
ALEPH [16]	incl.	1.020	22	22.9
DELPHI [17]	incl.	1.000	17	28.3
OPAL [18, 20]	$uds$ tag	1.000	17	33.3
	$b$ tag	1.000	17	10.6
	incl.	1.000	21	14.0
	$u$ tag	0.786	5	31.6
BABAR [28]	$d$ tag	0.786	5	33.0
	$s$ tag	0.786	5	51.3
	$c$ tag	0.786	5	30.4
	$b$ tag	0.786	5	14.6
BELLE [29]	incl.	1.031	45	46.4
HERMES [30]	incl.	1.044	78	44.0
	$\pi^+$ (p)	0.980	32	27.8
	$\pi^-$ (p)	0.980	32	47.8
	$\pi^+$ (d)	0.981	32	40.3
COMPASS [31] prel.	$\pi^-$ (d)	0.981	32	59.1
	$\pi^+$ (d)	0.946	199	174.2
	$\pi^-$ (d)	0.946	199	229.0
	$\pi^0$ (p)	1.112	15	15.8
PHENIX [21]	$\pi^0$ (d)	1.161	7	5.7
	$0 \leq \eta \leq 1$	$\pi^0$	1.161	7
	$0.8 \leq \eta \leq 2.0$	$\pi^0$	0.954	7
	$ \eta  < 0.5$	$\pi^\pm$	1.071	8
STAR [33–36]	$ \eta  < 0.5$	$\pi^\pm$	1.071	8
	$0.5 <  \eta  < 0.9$	$\pi^+, \pi^-, \pi^0$	1.006	16
ALICE [32]	$0.9 <  \eta  < 1.2$	$\pi^0$	0.766	11
	7 TeV	$\pi^0$	0.766	11
<b>TOTAL:</b>		973		1154.6



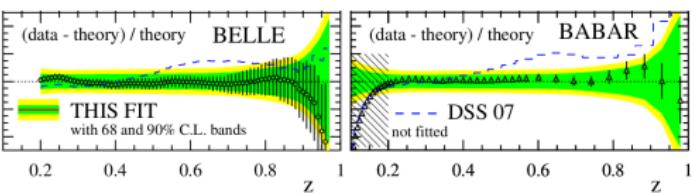
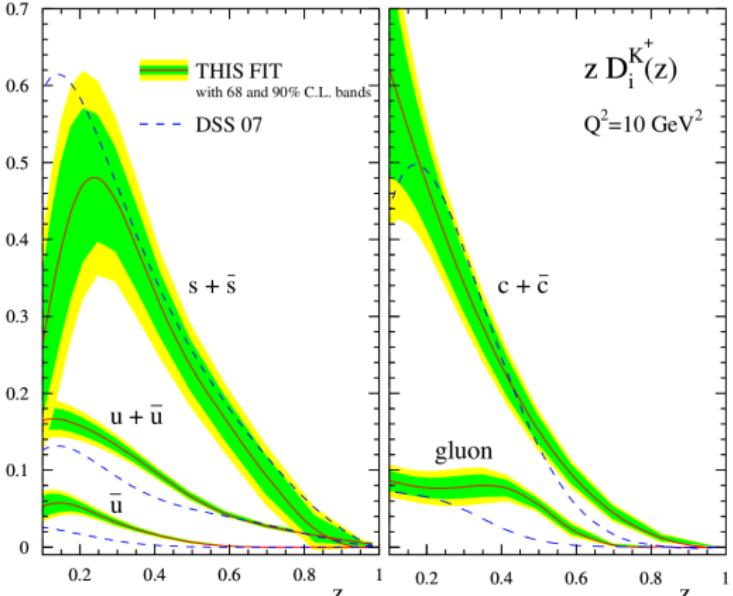
$D_{u+\bar{u}}$ : most precise ( $B$ -factory SIA data)

Very little or no charge symmetry breaking (SIDIS)

$D_g$ : significant shift of the central value ( $pp$  data)

# Global fit of kaon fragmentation functions [PRD 95 (2017) 094019]

experiment	data type	norm. $N_i$	# data in fit	$\chi^2$
TPC [37]	incl.	1.003	12	13.4
SLD [33]	incl.	1.014	18	17.2
	<i>uds</i> tag	1.014	10	31.5
	<i>c</i> tag	1.014	10	21.3
	<i>b</i> tag	1.014	10	11.9
ALEPH [30]	incl.	1.026	13	29.7
DELPHI [31]	incl.	1.000	12	6.9
	<i>uds</i> tag	1.000	12	13.1
	<i>b</i> tag	1.000	12	11.0
OPAL [34]	<i>u</i> tag	0.778	5	9.6
	<i>d</i> tag	0.778	5	7.7
	<i>s</i> tag	0.778	5	23.4
	<i>c</i> tag	0.778	5	42.5
	<i>b</i> tag	0.778	5	16.9
BABAR [17]	incl.	1.077	45	30.6
BELLE [18]	incl.	0.996	78	15.6
HERMES [19]	$K^+$ (p) $Q^2$	0.843	36	61.9
	$K^-$ (p) $Q^2$	0.843	36	29.6
	$K^+$ (p) $x$	1.135	36	75.8
	$K^-$ (p) $x$	1.135	36	42.1
	$K^+$ (d) $Q^2$	0.845	36	44.7
	$K^-$ (d) $Q^2$	0.845	36	41.9
	$K^+$ (d) $x$	1.095	36	48.9
	$K^-$ (d) $x$	1.095	36	44.4
COMPASS [22]	$K^+$ (d)	0.996	309	285.8
	$K^-$ (d)	0.996	309	265.1
STAR [24]	$K^+, K^- / K^+$	1.088	16	7.6
ALICE [23]	$K/\pi$	0.985	15	21.6
<b>TOTAL:</b>		1194		1271.7



Good flavour separation (SIDIS data)

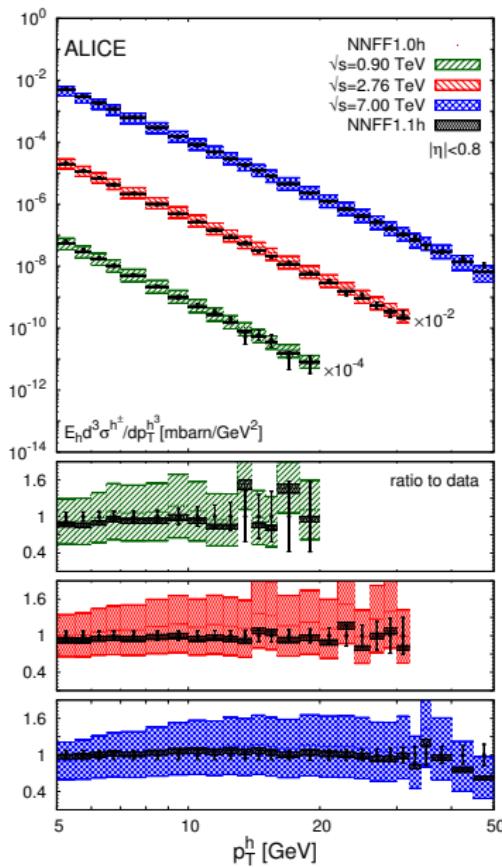
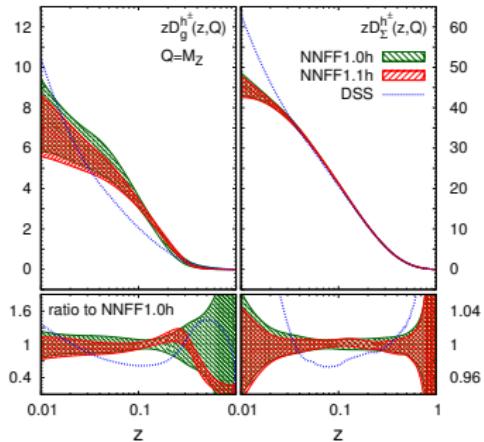
$D_g$ : significant shift ( $pp$  data)

Caution with mass corrections

$D_{u+\bar{u}}$ : most precise ( $B$ -factory SIA data)

# Global fit of unidentified charged hadron FFs [EPJ C78 (2018) 651]

Experiment	$\sqrt{s}$ [TeV]	$N_{\text{dat}}$	$\chi^2_b/N_{\text{dat}}$	$\chi^2_a/N_{\text{dat}}$
$e^+e^-$	various	471 (527)	0.83	0.83
ALICE	0.90	11 (54)	4.94	1.88
	2.76	27 (60)	13.3	0.82
	7.00	22 (65)	6.03	0.53
CMS	0.90	7 (20)	4.20	0.70
	2.76	9 (22)	10.6	1.24
	7.00	14 (27)	12.4	1.64
CDF	1.80	2 (49)	3.32	0.20
	1.96	50 (230)	2.93	1.23
		603 (1054)	6.54	1.11
				407



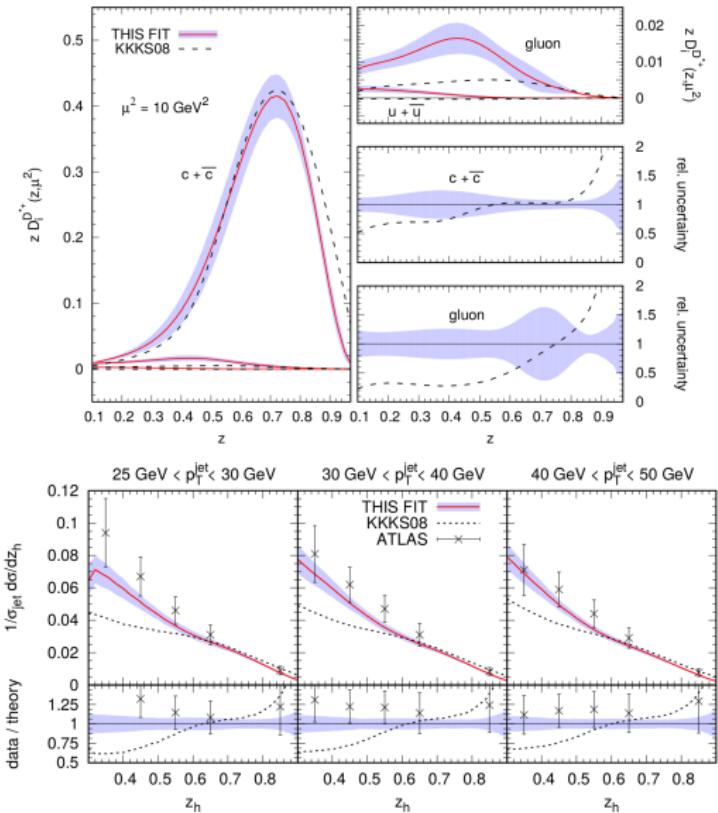
# First global fit of $D^*$ fragmentation functions [PRD 96 (2017) 034028]

Only  $g$ ,  $c$  and  $b$  FFs parametrised

Use of ZM-VFN scheme

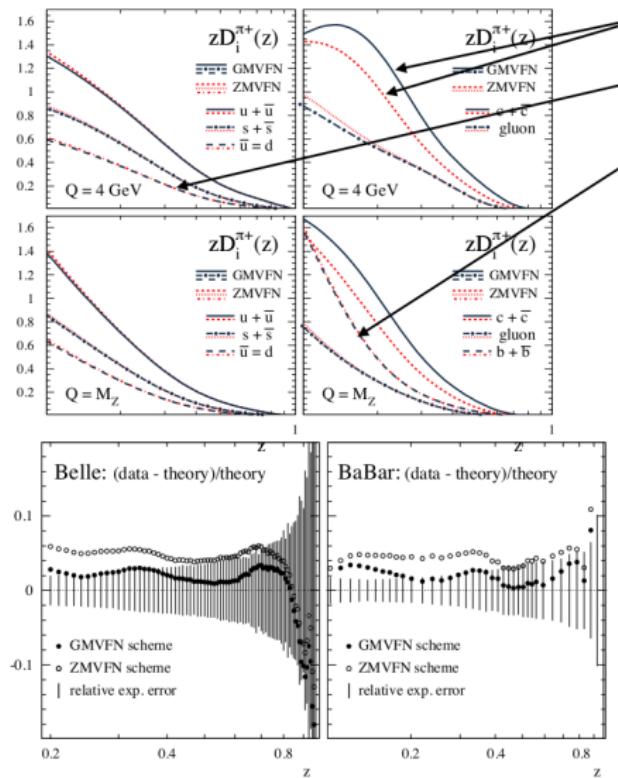
Kinematic cut  $p_T^h > 10 \text{ GeV}$

experiment		data type	#data in fit	$\chi^2$
ALEPH [80]	<a href="#">Eur. Phys. J. C 16, 597 (2000)</a>	incl.	17	33.738
OPAL [81]	<a href="#">Z. Phys. C 67, 27 (1995)</a>	incl.	9	6.999
		$c$ tag	9	8.388
		$b$ tag	9	5.342
ATLAS [94]	<a href="#">Nucl. Phys. B 907, 717 (2016)</a>	$D^{*\pm}$	5	3.598
ALICE [60, 61]	$\sqrt{S} = 7 \text{ TeV}$	$D^{*\pm}$	3	0.126
JHEP 1201, 128 (2012)	$\sqrt{S} = 2.76 \text{ TeV}$	$D^{*\pm}$	1	0.007
CDF [62]	<a href="#">Phys. Rev. Lett. 91, 241804 (2003)</a>	$D^{*\pm}$	2	1.289
LHCb [64]	$2 \leq \eta \leq 2.5$	$D^{*\pm}$	5	10.984
JHEP 1609, 013 (2016)	$2.5 \leq \eta \leq 3$	$D^{*\pm}$	5	2.607
	$3 \leq \eta \leq 3.5$	$D^{*\pm}$	5	8.229
	$3.5 \leq \eta \leq 4$	$D^{*\pm}$	2	10.411
ATLAS [68]	$25 \leq \frac{p_T^{\text{jet}}}{\text{GeV}} \leq 30 \text{ (jet } D^{*\pm})$	5	4.146	
<a href="#">Phys. Rev. D 85, 052005 (2012)</a>	$30 \leq \frac{p_T^{\text{jet}}}{\text{GeV}} \leq 40 \text{ (jet } D^{*\pm})$	5	1.977	
	$40 \leq \frac{p_T^{\text{jet}}}{\text{GeV}} \leq 50 \text{ (jet } D^{*\pm})$	5	0.659	
	$50 \leq \frac{p_T^{\text{jet}}}{\text{GeV}} \leq 60 \text{ (jet } D^{*\pm})$	5	0.791	
	$60 \leq \frac{p_T^{\text{jet}}}{\text{GeV}} \leq 70 \text{ (jet } D^{*\pm})$	5	1.333	
<b>TOTAL:</b>		97	100.980	



See also [JHEP 1605 (2016) 125]

# Pion fragmentation functions in the GM-VFNs [PR D94 (2016) 034037]



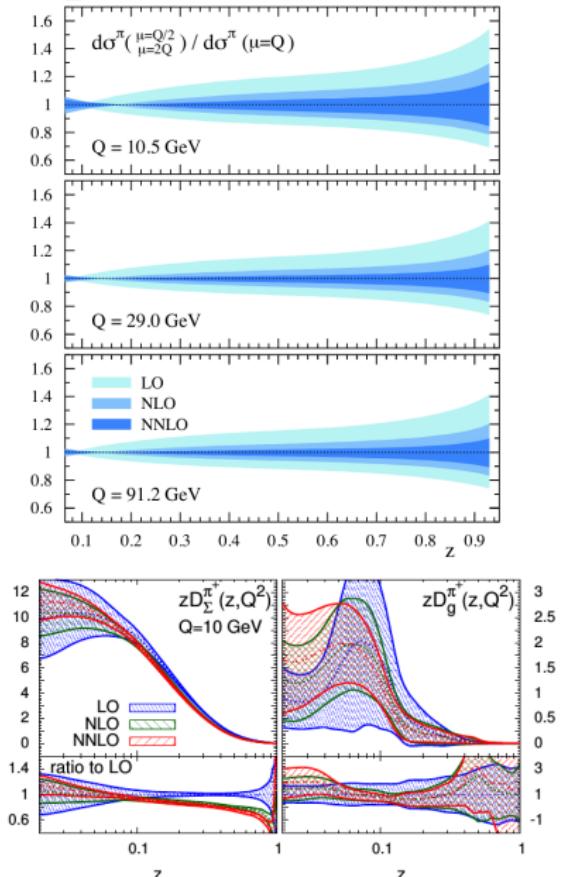
charm changes significantly  
light flavors constrained by sidis  
bottom constrained by high  $Q^2$

experiment	data type	# data in fit	ZMVFN	GMVFN		
			$N_i$	$\chi^2$	$N_i$	$\chi^2$
ALEPH [23]	incl.	22	0.968	21.6	0.994	23.3
BABAR [13]	incl.	39	1.019	76.7	1.002	58.2
BELLE [14]	incl.	78	1.044	19.5	1.019	11.0
DELPHI [24]	incl.	17	0.978	6.7	1.003	9.3
	<i>uds</i> tag	17	0.978	20.8	1.003	9.5
	<i>b</i> tag	17	0.978	10.5	1.003	7.8
OPAL [25]	incl.	21	0.946	27.9	0.970	15.9
SLD [26]	incl.	28	0.938	28.0	0.963	9.5
	<i>uds</i> tag	17	0.938	21.3	0.963	11.3
	<i>c</i> tag	17	0.938	34.0	0.963	19.8
	<i>b</i> tag	17	0.938	11.1	0.963	9.9
TPC [27]	incl.	17	0.997	31.7	1.006	27.9
	<i>uds</i> tag	9	0.997	2.0	1.006	2.0
	<i>c</i> tag	9	0.997	5.9	1.006	4.3
	<i>b</i> tag	9	0.997	9.6	1.006	10.9
COMPASS [28]	$\pi^\pm$ (d)	398	1.003	378.7	1.008	382.9
HERMES [29]	$\pi^\pm$ (p)	64	0.981	74.0	0.986	69.9
	$\pi^\pm$ (d)	64	0.980	107.3	0.985	103.7
PHENIX [30]	$\pi^0$	15	1.174	14.3	1.167	14.4
STAR [31]	$\pi^\pm, \pi^0$	38	1.205	31.2	1.202	33.8
ALICE [32]	$\pi^0$	11	0.696	33.3	0.700	31.2
<b>TOTAL:</b>		924		966.4		875.8

Slide: courtesy of R. Sassot

# Fragmentation functions at NNLO

[PRD92(2015)114017; EPJ C77(2017)516]



Exp.	$N_{\text{dat}}$	LO $\chi^2/N_{\text{dat}}$	NLO $\chi^2/N_{\text{dat}}$	NNLO $\chi^2/N_{\text{dat}}$
BELLE	70	0.60	0.11	0.09
BABAR	40	1.91	1.77	0.78
TASSO12	4	0.70	0.85	0.87
TASSO14	9	1.55	1.67	1.70
TASSO22	8	1.64	1.91	1.91
TPC	13	0.46	0.65	0.85
TPC-UDS	6	0.78	0.55	0.49
TPC-C	6	0.55	0.53	0.52
TPC-B	6	1.44	1.43	1.43
TASSO34	9	1.16	0.98	1.00
TASSO44	6	2.01	2.24	2.34
TOPAZ	5	1.04	0.82	0.80
ALEPH	23	1.68	0.90	0.78
DELPHI	21	1.44	1.79	1.86
DELPHI-UDS	21	1.30	1.48	1.54
DELPHI-B	21	1.21	0.99	0.95
OPAL	24	2.29	1.88	1.84
SLD	34	2.33	1.14	0.83
SLD-UDS	34	0.95	0.65	0.52
SLD-C	34	3.33	1.33	1.06
SLD-B	34	0.45	0.38	0.36
<b>TOTAL</b>	<b>428</b>	<b>1.44</b>	<b>1.02</b>	<b>0.87</b>

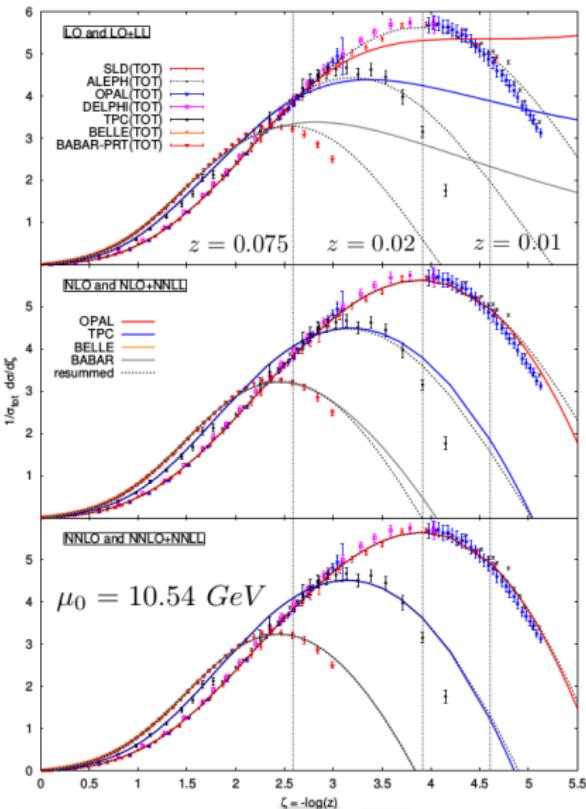
Excellent perturbative convergence  
FFs almost stable from NLO to NNLO  
LO FF uncertainties larger than HO  
Effects less evident for  $K^\pm$  and  $p/\bar{p}$

# Small- $z$ resummed fragmentation functions [PRD 95 (2017) 054003]

## — 436 Total data Points: —

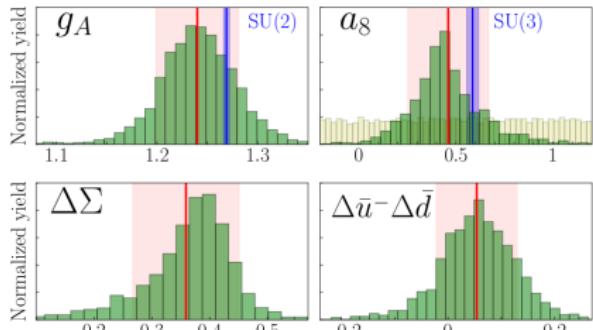
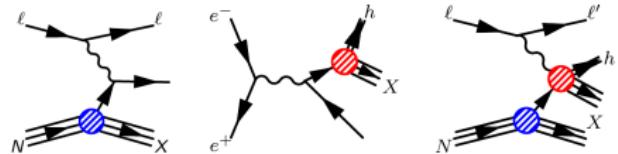
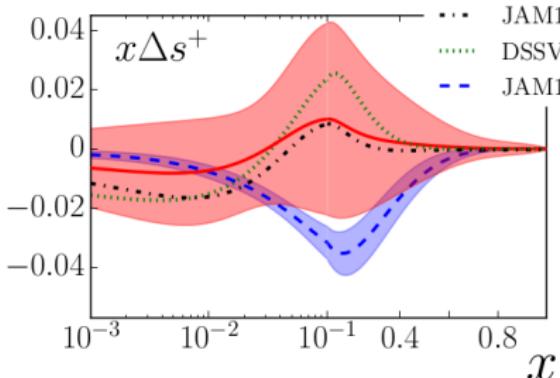
- LEP cut ( $z = 0.01$ ) due to inconsistency between OPAL and ALEPH
- TPC lower cut ( $z = 0.02$ ) based on difference of energy fraction  $z = 2E_h/Q$  and three momentum fraction  
 $x_p = z - 2m_h^2/(zQ^2) + \mathcal{O}(1/Q^4)$  in c.m.s being less than at least 15%

accuracy	$\chi^2$	$\chi^2/\text{dof}$
LO	1260.78	2.89
NLO	354.10	0.81
NNLO	330.08	0.76
LO+LL	405.54	0.93
NLO+NNLL	352.28	0.81
NNLO+NNLL	329.96	0.76



Slide: courtesy of D. P. Anderle

# Simultaneous fits of (pol.) PDFs and FFs [PRL 119 (2017) 132001]



$g_A = 1.24 \pm 0.04$        $a_8 = 0.46 \pm 0.21$   
 confirmation of SU(2) symmetry to  $\sim 2\%$

$\sim 20\% \text{ SU}(3) \text{ breaking} \pm 20\%$

$$\Delta s^+ = -0.03 \pm 0.09$$

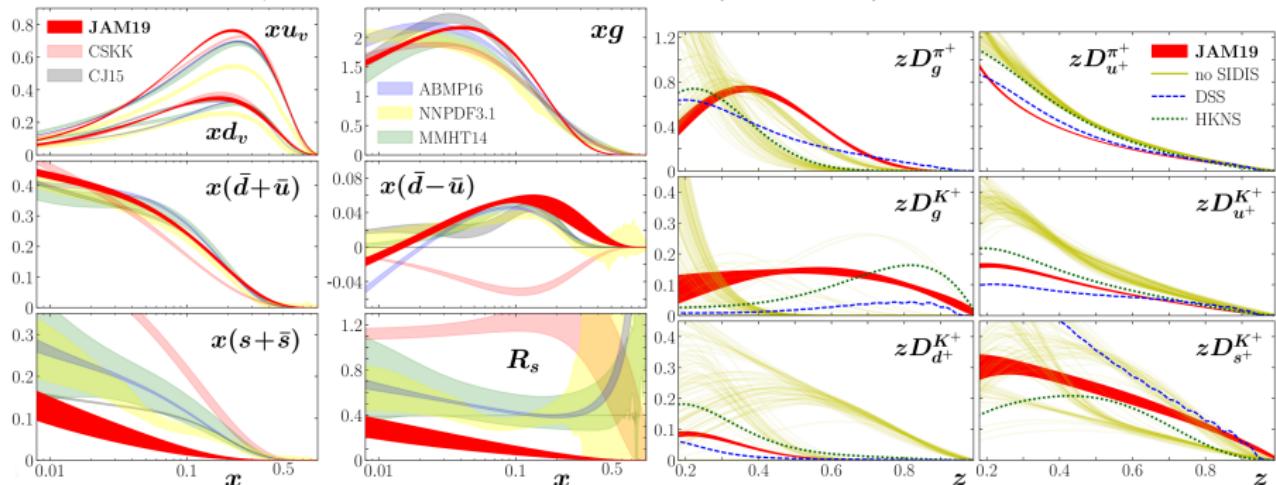
$$\Delta\Sigma = 0.36 \pm 0.09 \quad \Delta u - \Delta d = 0.05 \pm 0.08$$

# Simultaneous fits of (unp.) PDFs and FFs [arXiv:1905.03788]

Multi-step procedure

- sampling the posterior distributions from flat priors for fixed-target DIS data (BCDMS, SLAC, NMC)
- update these posteriors with collider DIS data (HERA I-II)
- update the resulting posteriors with DY data (E866)
- sampling the posterior distributions from flat priors for SIA data (DESY, SLAC, CERN, KEK)
- update the FF/PDF posteriors with SIDIS data (COMPASS)

Process	$N_{\text{dat}}$	$\chi^2/N_{\text{dat}}$
DIS	2680	1.28
SIDIS	992	1.25
DY	250	1.67
SIA	444	1.27
Total	4366	1.30



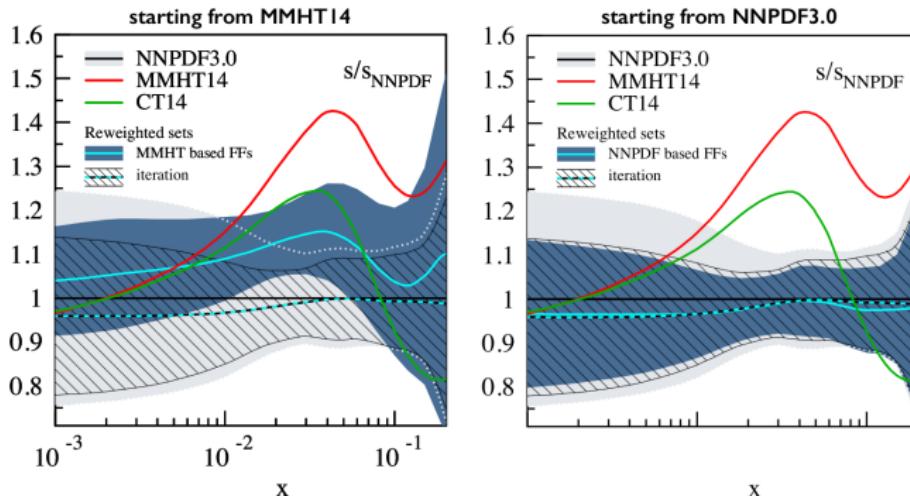
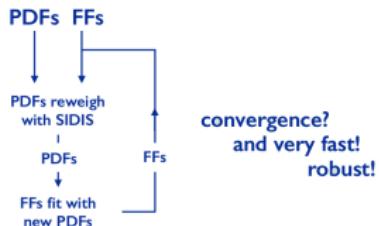
# Simultaneous fits of (unp.) PDFs and FFs [PRD 96 (2017) 094020]

IDEA:

iterative reweighting of PDFs and fit of FFs  
with kaon SIDIS data ( $N_{\text{dat}} = 906$ )

HERMES [PRD 87 (2013) 074029]

COMPASS [PLB 767 (2017) 133]



$$\chi^2_{FF} = 1271.7 \quad 1041.3 \quad 1002.3$$

$$1017.2 \quad 1005.3 \quad 1000.6$$

similar results with CT14 replicas

### 3. Conclusions

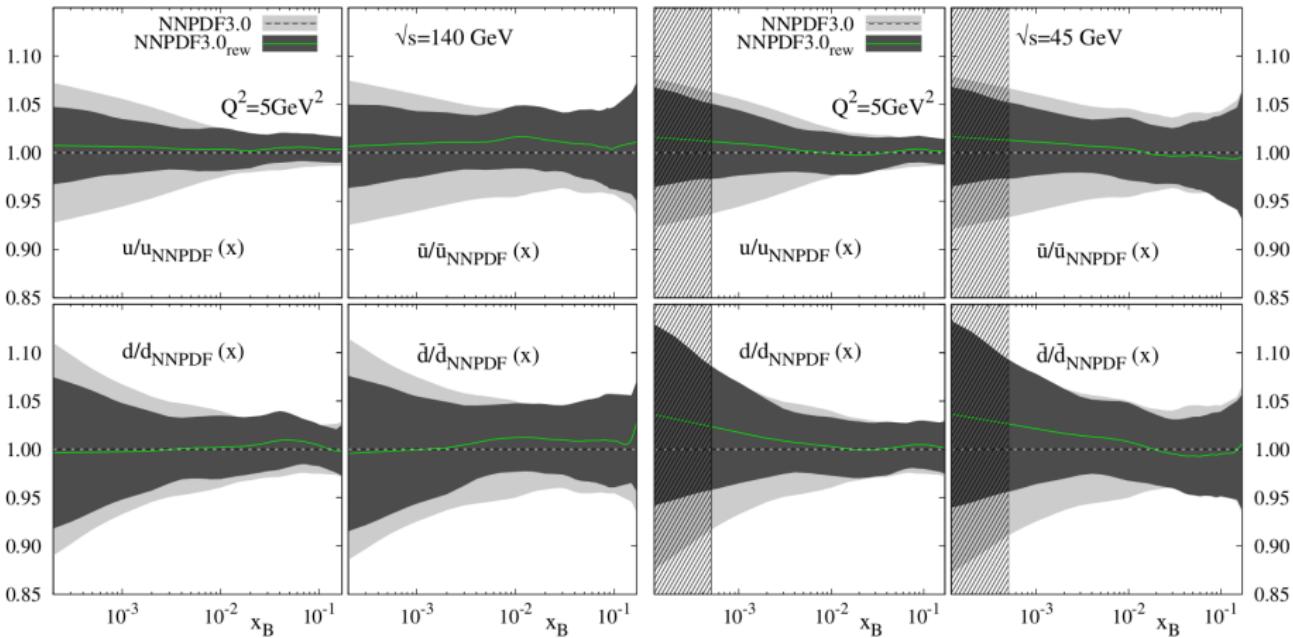
# Fragmentation Functions at a future EIC [PRD 99 (2019) 094004]

Assess the quantitative impact of simulated  $\pi^\pm$  and  $K^\pm$  DIS data at an EIC

Consider two kinematic scenarios for the EIC ( $\sqrt{s}=45$  GeV,  $\sqrt{s}=140$  GeV)

Use an iterative procedure based on reweighting

Appreciate the significant impact on sea quark PDFs and on all FFs



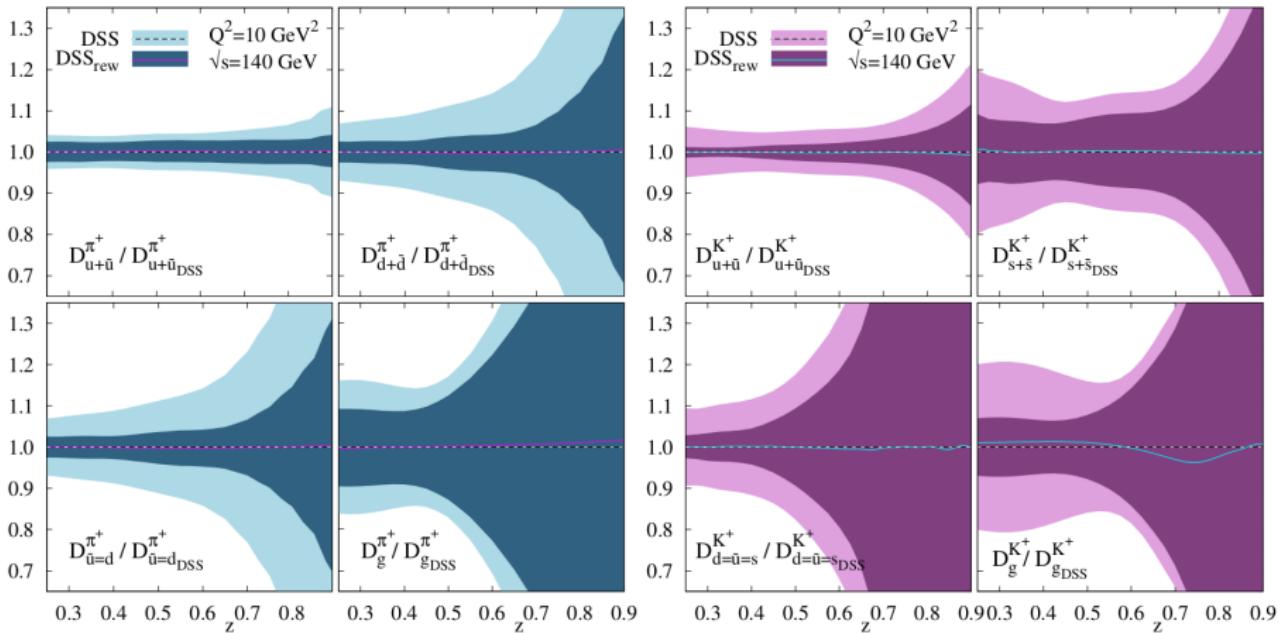
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# Summary

- ➊ A number of hard-scattering processes require an appropriate knowledge of FFs
  - ▶ probing nucleon momentum, spin and flavour
  - ▶ studying the prompt atmospheric neutrino flux
  - ▶ understanding spatial distributions and the dynamics of nuclear matter
- ➋ Significant role of new data, including LHC data
  - ▶ increased accuracy of fragmentation functions
  - ▶ increased precision of fragmentation functions
- ➌ Increasing sophistication of the QCD theory
  - ▶ needed to catch most of the features of the data
  - ▶ includes NNLO, heavy quark mass schemes, resummation
- ➍ Exploit the full potential of SIDIS to improve our knowledge of PDFs
  - ▶ simultaneous fits challenging, but feasible
  - ▶ combine simultaneous and global fits to make the most from the data

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**Thank you**

# Dependence on $\alpha_s$

