nNNPDF1.0 : Nuclear Parton Distributions from Neural Networks and the Impact of an Electron-Ion Collider

arXiv:1904.00018

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Talk at the University of MÜnster
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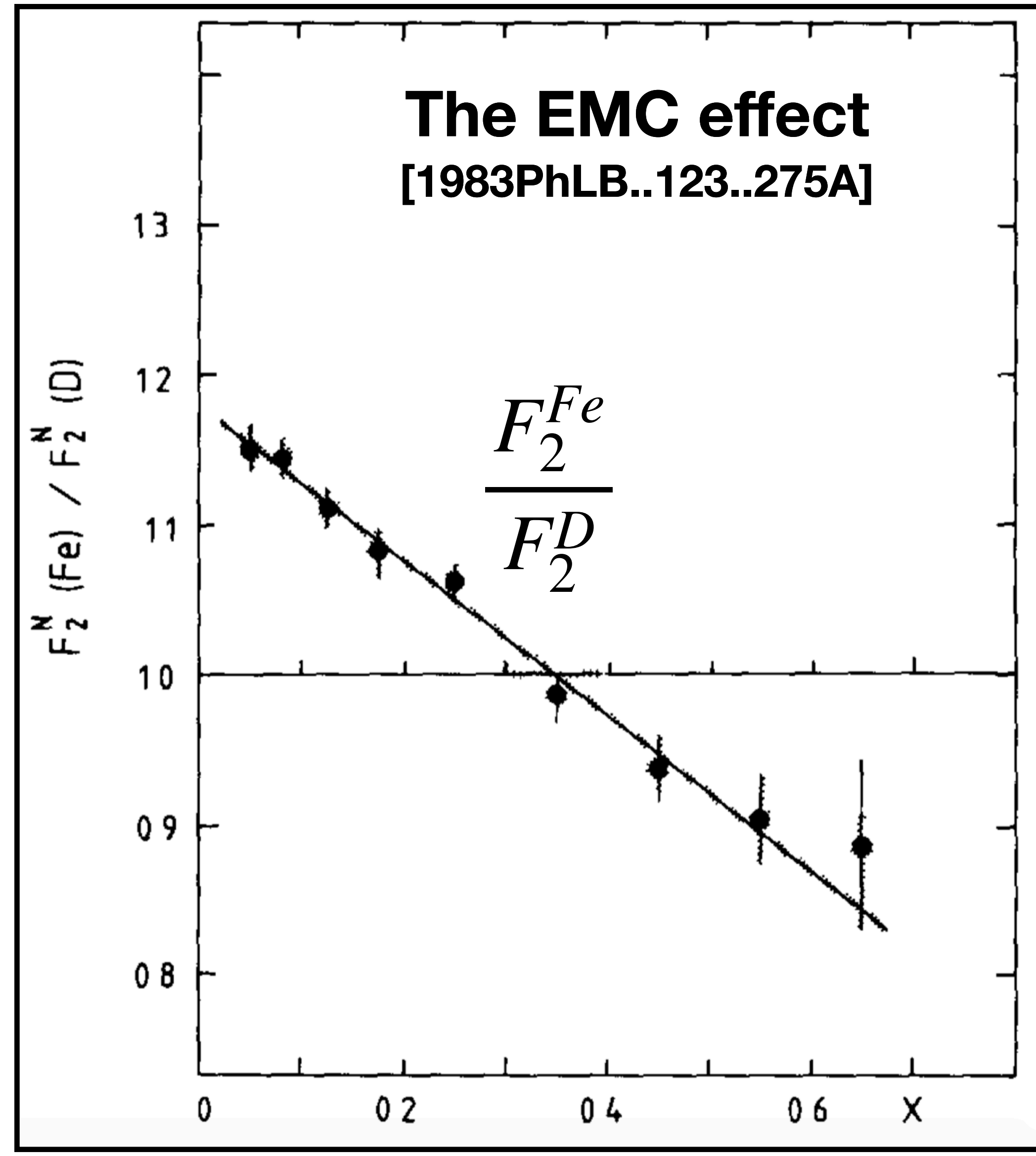
1 Introduction

- How it started
- Heavy-ion collisions
- Flavour separation
- Gluon saturation

Nuclear PDFs: How it started

Nuclear-binding effects (~MeV)
Momentum transfer (~GeV)

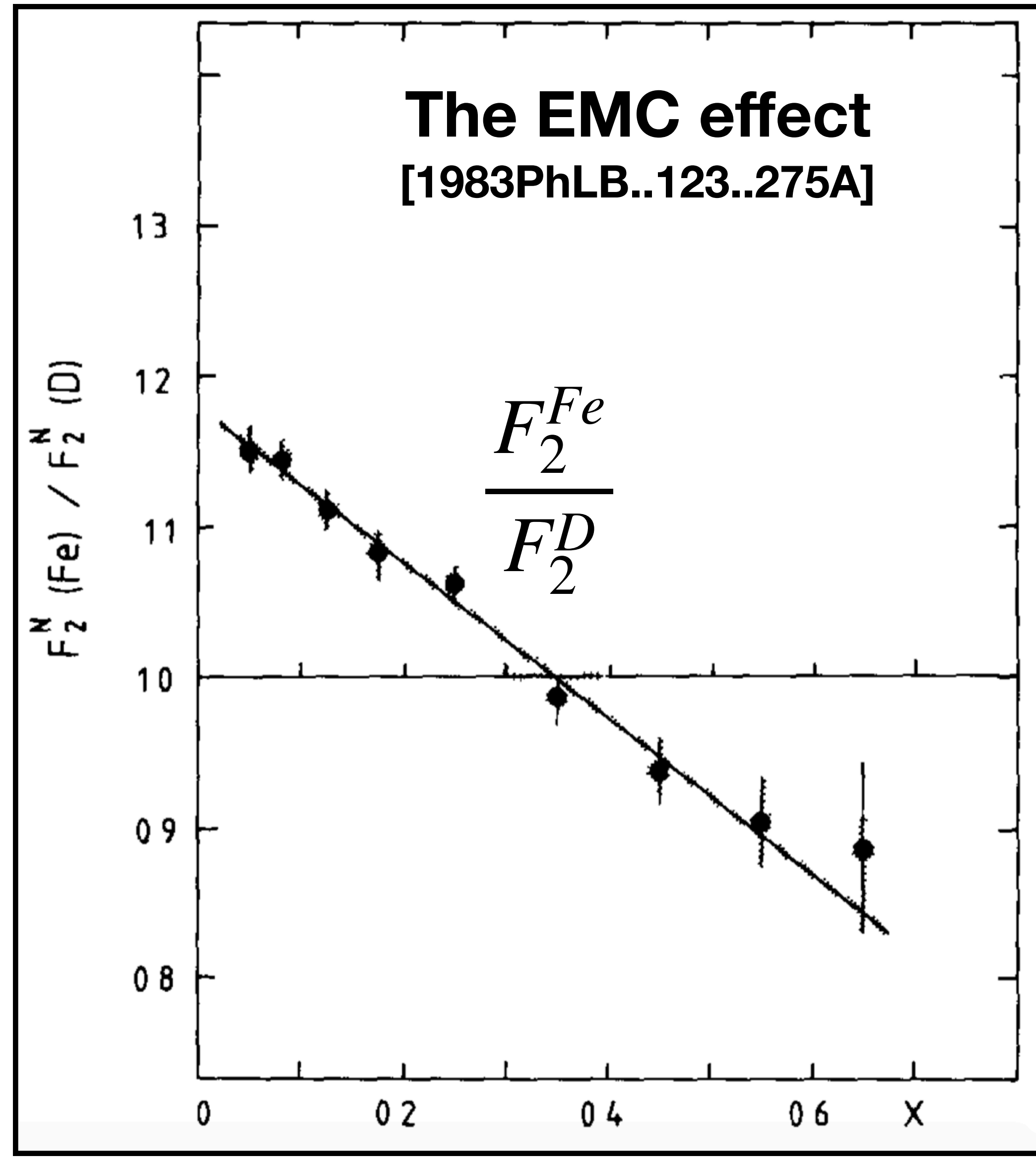


Nuclear PDFs: How it started

Nuclear-binding effects (~MeV)
Momentum transfer (~GeV)

Four equally possible scenarios:

a) the fundamental interactions are the same but PDFs are different.
b) The fundamental interactions are different in the medium but PDFs are the same.
c) Both a) and b).
d) The factorisation picture is no longer valid.



DIS Structure Function

$$F_2(x, Q^2, A) = \sum_i^{n_f} \sum_j^{n_f} \underbrace{C_i(x, Q^2)}_{\text{pQCD}} \otimes \underbrace{\Gamma_{ij}(x, Q^2)}_{\text{DGLAP}} \otimes \underbrace{q_j(x, Q_0^2, A)}_{\text{npQCD}}$$

Nuclear PDFs: How it started

Nuclear-binding effects (~MeV)
Momentum transfer (~GeV)

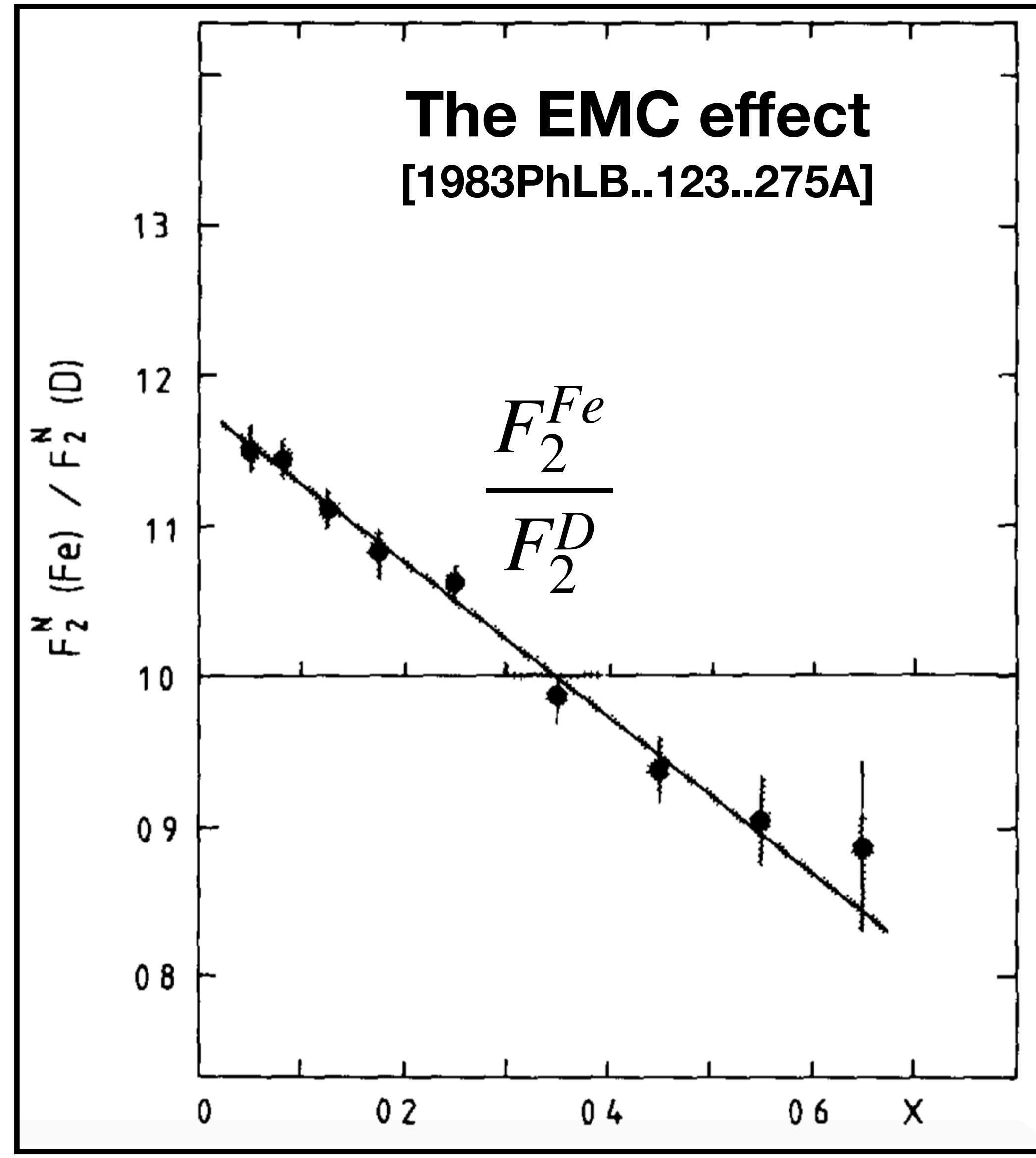
The simplest and most popular:

a) **the fundamental interactions are the same but PDFs are different.**

b) The fundamental interactions are different in the medium but PDFs are the same.

c) Both a) and b).

d) The factorisation picture is no longer valid.



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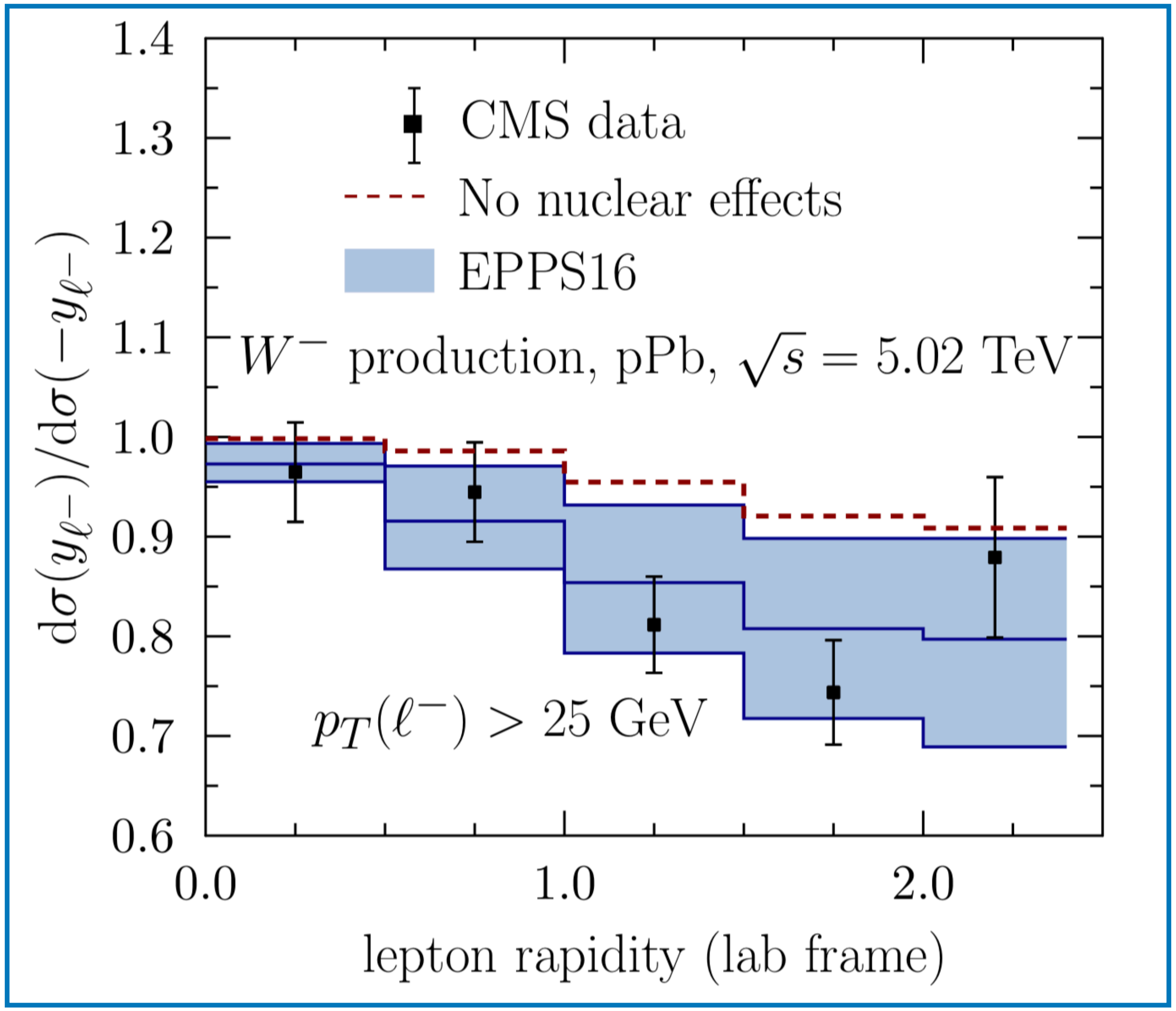
Heavy-Ion Collisions

W± Production prediction with/without nuclear effects

EPPS16, Fig.19, [1612.05741]



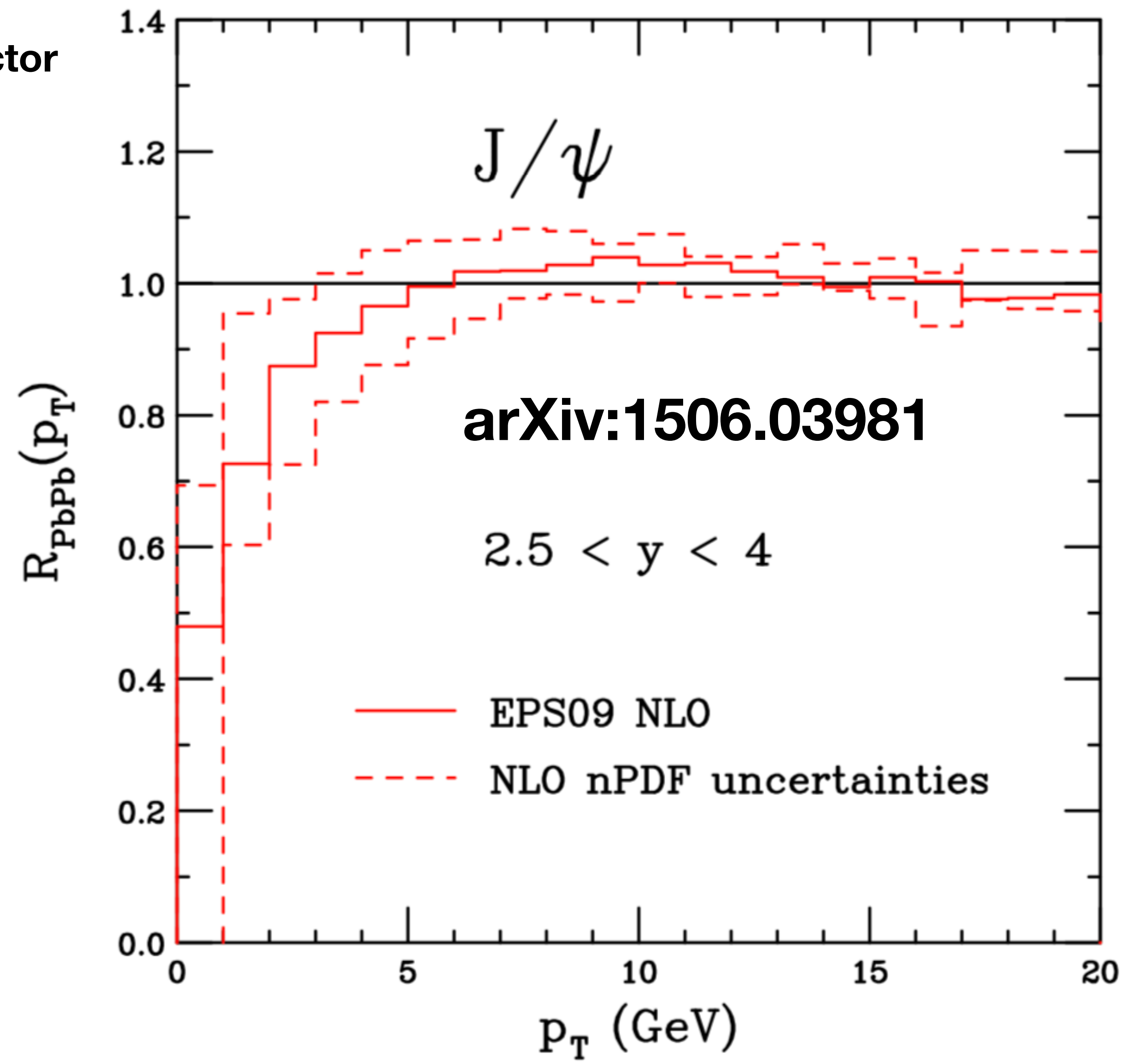
Better description of the data

Heavy-Ion Collisions

Heavy-ion collisions themselves cannot teach us much about the initial state, most of the details are wiped out during the evolution of the plasma.

The nuclear modification factor

$$R_{AA}(p_T) \propto \frac{dN_{AA}/dp_T}{d\sigma_{pp}/dp_T}$$



information on the initial state needs to be extracted from experiments on p+A and ultimately e+A with small and well understood final state effects.

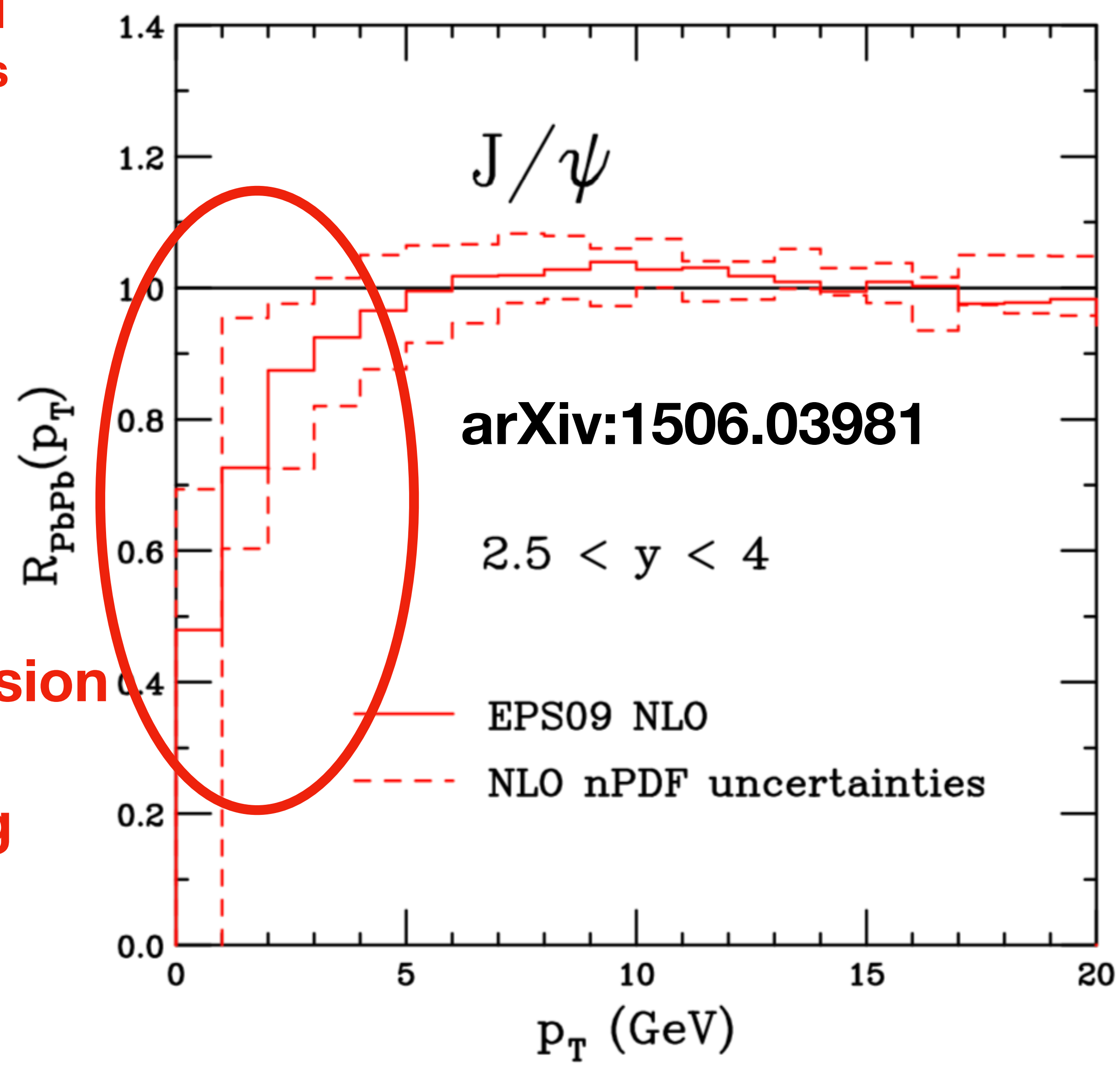
Heavy-Ion Collisions

Heavy-ion collisions themselves cannot teach us much about the initial state, most of the details are wiped out during the evolution of the plasma.

=1 in absence of initial and final states effects

$$R_{AA}(p_T) \propto \frac{dN_{AA}/dp_T}{d\sigma_{pp}/dp_T}$$

Quarkonium suppression in AA collisions Due to shadowing



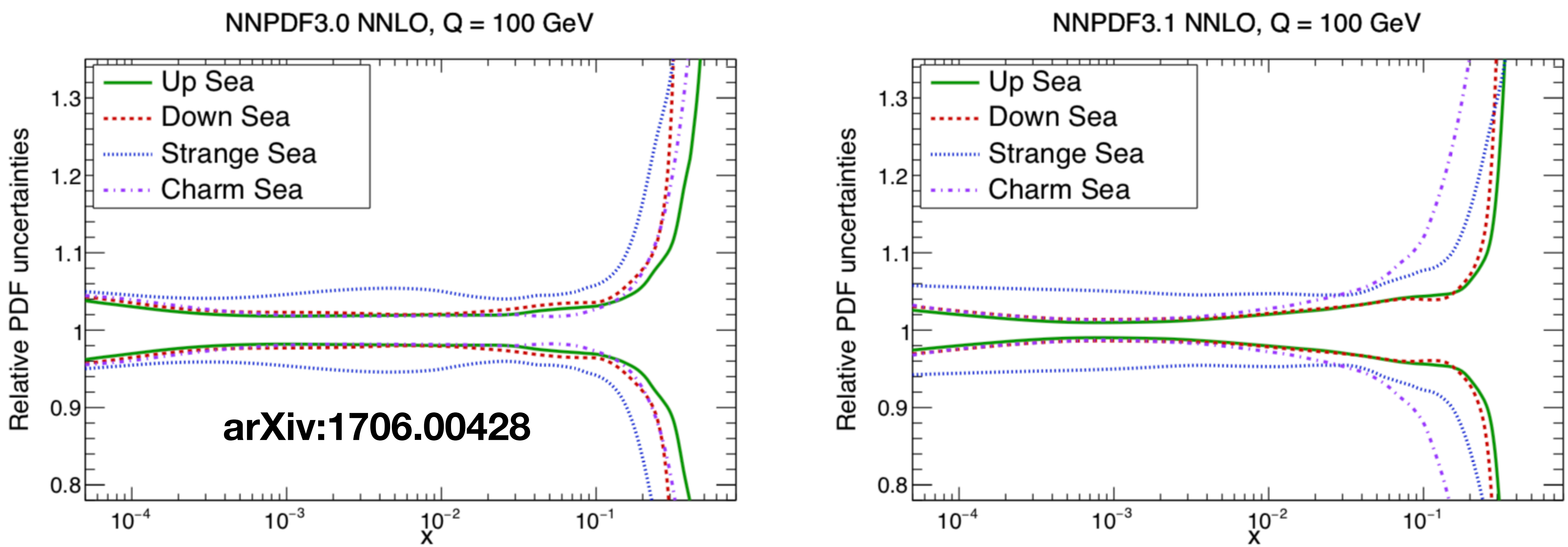
The final observables are sensitive to both, the initial state and the final state.

information on the initial state needs to be extracted from experiments on p+A and ultimately e+A with small and well understood final state effects.

Proton Flavour Separation

The W± mediated DIS access different combinations of quark and anti-quark flavours than photon-mediated DIS.

Additional constraint on the strange sea in the proton PDFs



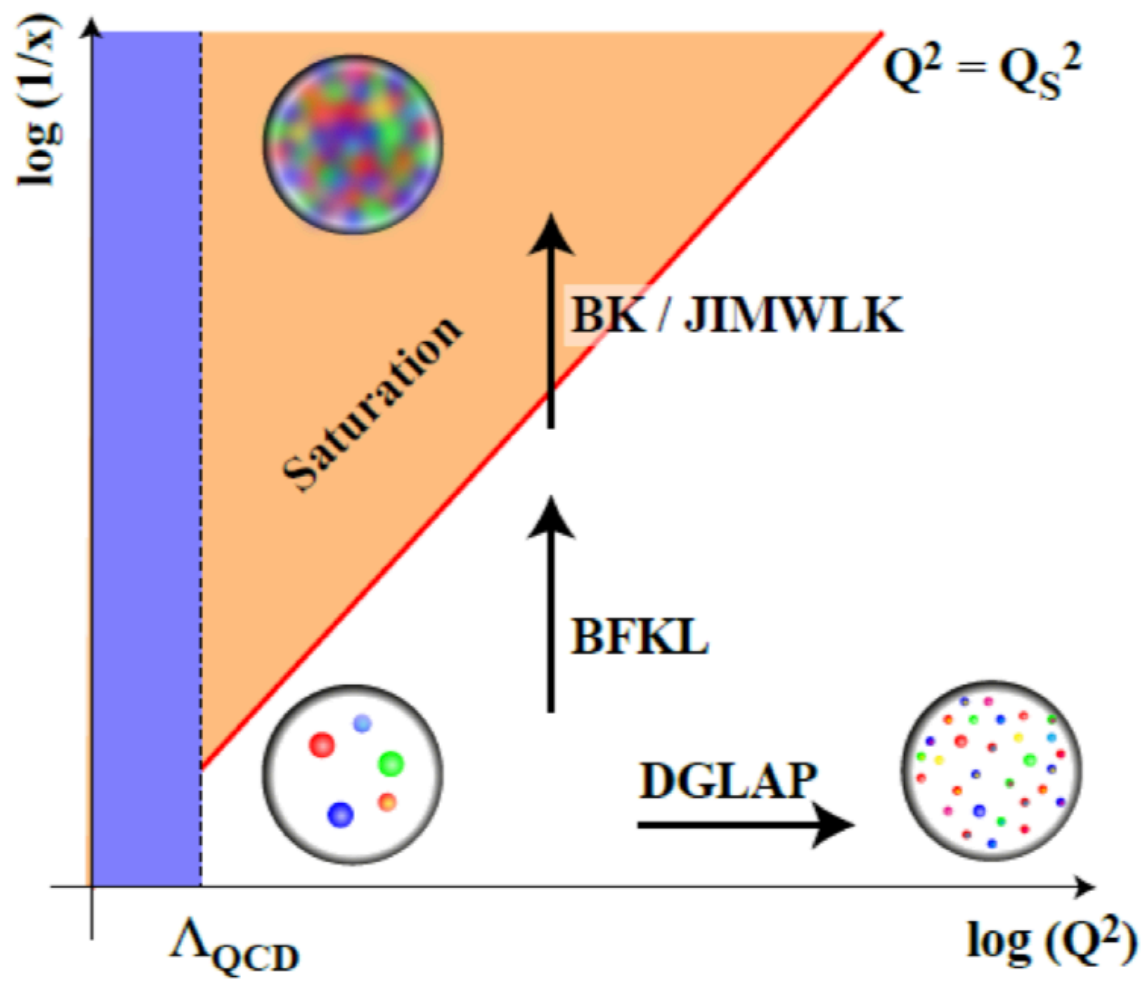
CC DIS provide access to flavour-dependent PDFs without the uncertainties from fragmentation functions (SIDIS)

Gluon Saturation in Nuclei

Saturation scale for gluon PDF can be estimated by:

$$Q_S^2 = \frac{\alpha_S}{\pi R^2} xG(x, Q^2) \propto \left(\frac{A}{x}\right)^{1/3}$$

For a given x,
<u>heavier</u> the nucleus,
<u>larger</u> is Q^2_S,
hence, more accessible.



saturation can be more easily probed
in collisions of protons on heavy nuclei targets

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2 nNNPDF1.0

- Data
- Kinematics
- Parametrisation
- Cost Function
- Minimisation
- Closure Tests
- Data vs. Theory
- NLO fit
- A dependence
- Boundary Condition
- Architecture

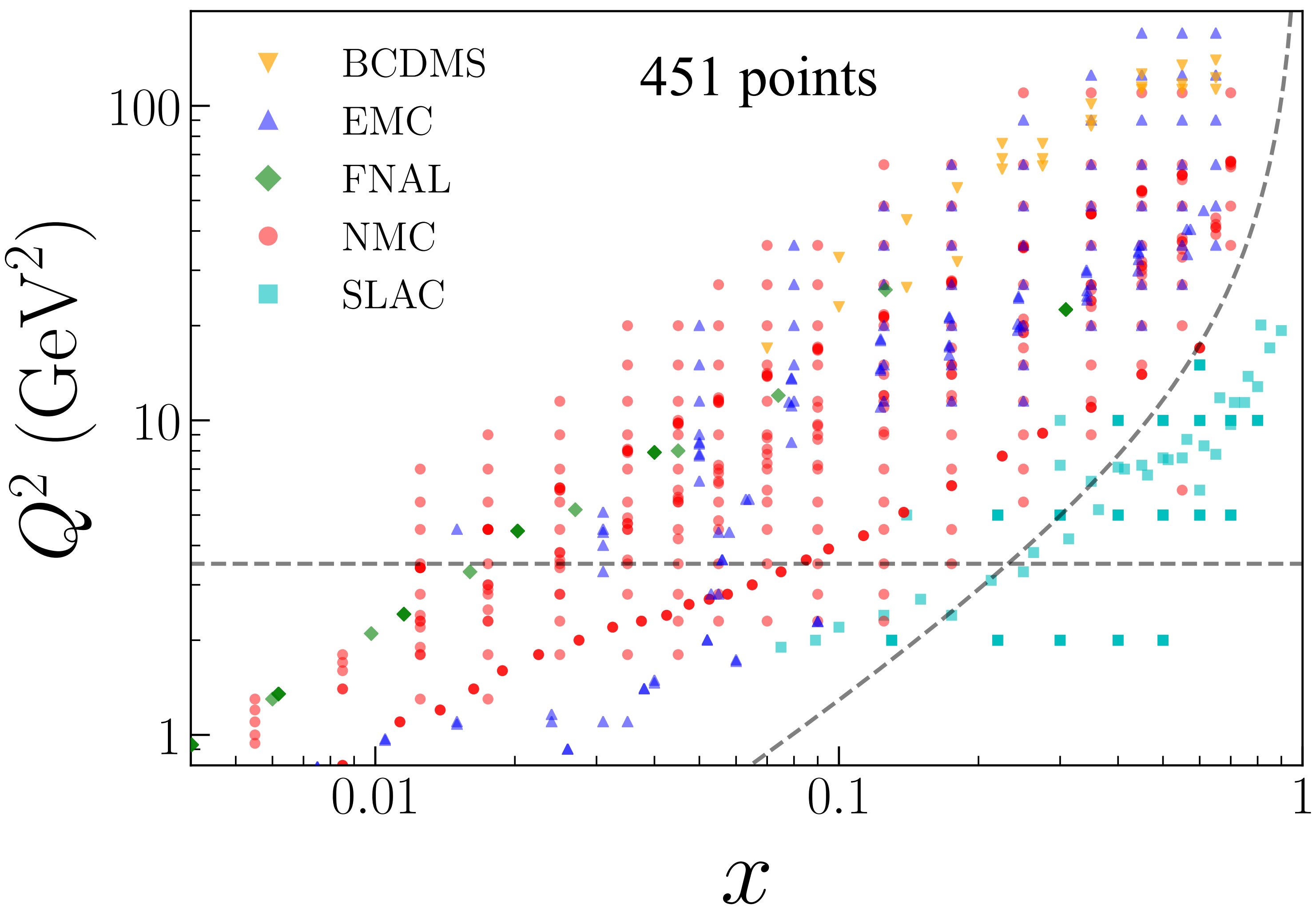
Data

	EPS09	DSSZ12	KA15	NCTEQ15	EPPS16	nNNPDF1.0
Order in α_s	LO & NLO	NLO	NNLO	NLO	NLO	NNLO
Neutral current DIS ℓ+A/ℓ+d	✓	✓	✓	✓	✓	✓
Drell-Yan dilepton p+A/p+d	✓	✓	✓	✓	✓	
RHIC pions d+Au/p+p	✓	✓		✓	✓	
Neutrino-nucleus DIS		✓			✓	
Drell-Yan dilepton π+A					✓	
LHC p+Pb jet data					✓	
LHC p+Pb W, Z data					✓	
Q cut in DIS	1.3 GeV	1 GeV	1 GeV	2 GeV	1.3 GeV	1.87 GeV
datapoints	929	1579	1479	708	1811	451
free parameters	15	25	16	17	20	183*
error analysis	Hessian	Hessian	Hessian	Hessian	Hessian	Monte Carlo rep
error tolerance $\Delta\chi^2$	50	30	not given	35	52	
Free proton baseline PDFs	CTEQ6.1	MSTW2008	JR09	CTEQ6M-like	CT14NLO	NNPDF3.1
Heavy-quark effects		✓		✓	✓	✓
Flavor separation				some	✓	
Reference	[JHEP 0904 065]	[PR D85 074028]	[PR D93, 014026]	[PR D93 085037]	[EPJ C77 163]	[arXiv:1904.00018]

Table 1, [1802.05927]

Kinematics

^{2}D, ^{4}He, ^{6}Li, ^{9}Be, ^{12}C, ^{14}N, ^{27}Al, ^{40}Ca, ^{56}Fe, ^{64}Cu, ^{108}Ag, ^{119}Sn, ^{131}Xe, ^{197}Au, ^{208}Pb



	nNNPDF1.0	nCTEQ15	EPPS16
$W^2_{\rm min}$	12.5 GeV^2	12.25 GeV^2	n/a
$Q^2_{\rm min}$	3.5 GeV^2	4 GeV^2	1.69 GeV^2

Kinematical cuts
Consistent with our
Proton PDF Boundary
NNPDF3.1

Parametrisation

Isoscalar nuclei F_2 as observable

$$xg(x, Q_0, A) = B_g(A) x^{-\alpha_g} (1-x)^{\beta_g} \xi_3^{(3)}(x, A)\,,$$

$$x\Sigma(x, Q_0, A) = x^{-\alpha_\Sigma} (1-x)^{\beta_\Sigma} \xi_1^{(3)}(x, A)\,,$$

$$xT_8(x, Q_0, A) = x^{-\alpha_{T_8}} (1-x)^{\beta_{T_8}} \xi_2^{(3)}(x, A)$$

Anti-correlated [next-slide]

Momentum Sum Rule

$$B_g(A) = \frac{1 - \int_0^1 dx\, x\Sigma(x, Q_0, A)}{\int_0^1 dx\, xg(x, Q_0, A)}\,.$$

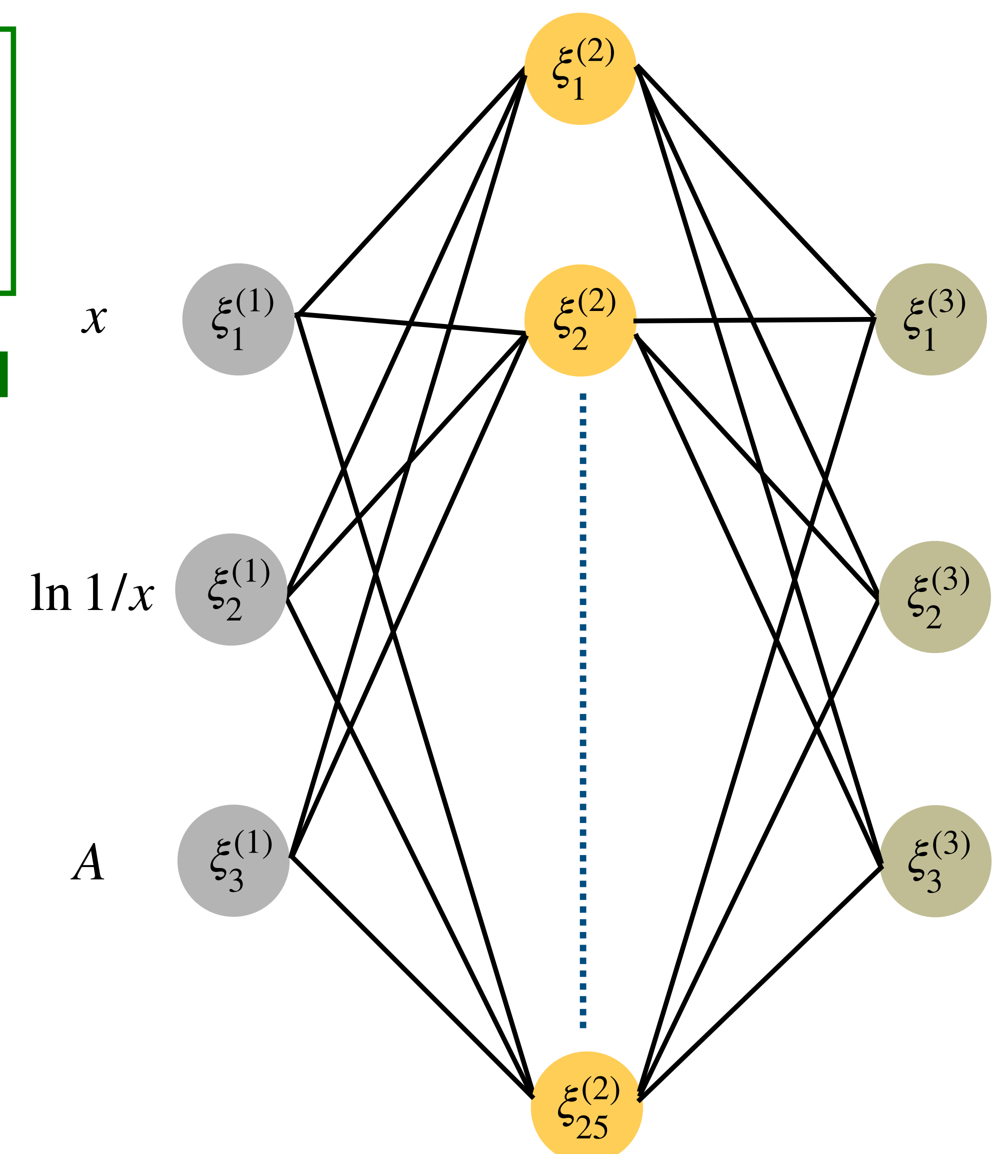
Evolution basis

$$\Sigma = \sum_{i=1}^{n_f} q_i^+$$ **where:** $q^\pm = q \pm \bar{q}$

$$\cancel{T_3 = u^+ - d^+}$$

No contribution from T_3 in isoscalar observables

$$T_8 = u^+ + d^+ - 2s^+$$



Parametrisation

Isoscalar nuclei F_2 as observable

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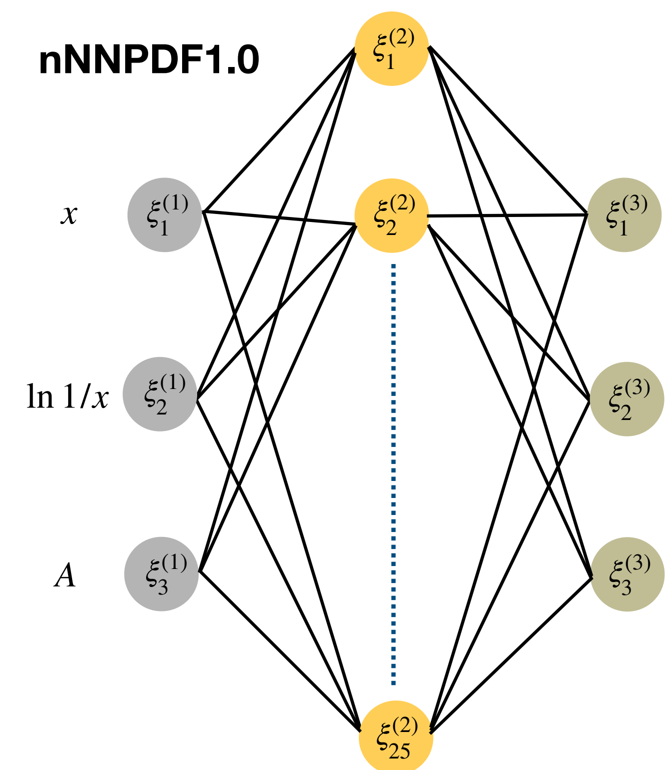
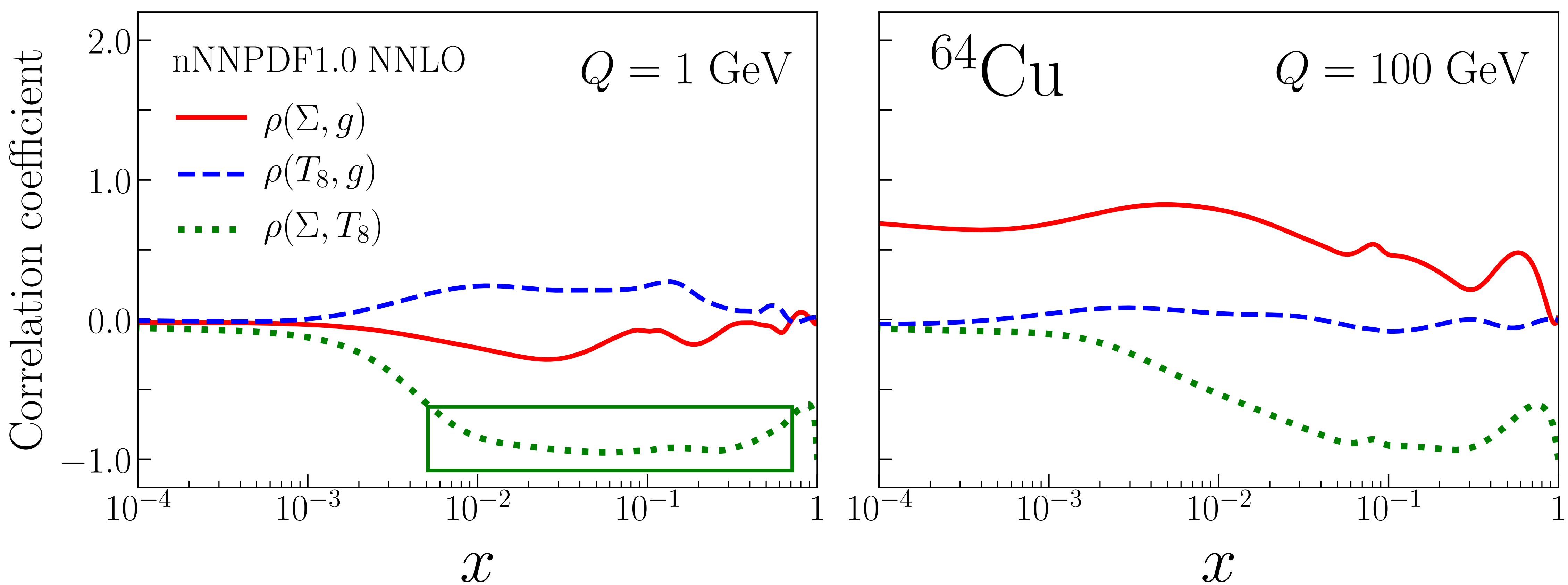


But in the data region…
Σ and T_8 are anti-correlated



Cost Function

$$\chi^2 \equiv \sum_{i,j=1}^{N_{\rm dat}} \left(R_i^{(\rm exp)} - R_i^{(\rm th)}(\{f_m\})\right) \left(\mathrm{cov}_{\rm t_0}\right)^{-1}_{ij} \left(R_j^{(\rm exp)} - R_j^{(\rm th)}(\{f_m\})\right)$$

$$+\lambda \sum_{m=g,\Sigma,T_8} \sum_{l=1}^{N_x} \left(f_m(x_l, Q_0, A) - f_m^{(p+n)/2}(x_l, Q_0)\right)^2 .$$

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<u>Per replica</u> boundary condition on the level of minimisation to reproduce NNPDF3.1 <u>central value and uncertainties</u> at A=1 with $\mathbf{x} \in [10^{-3}, 0.7]$

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Fitting pdf in ratio

$$\mathrm{R} = \frac{\mathrm{C} \otimes \mathrm{nNNPDF1.0(A)}}{\mathrm{C} \otimes \mathrm{nNNPDF1.0(A')}}$$

Minimisation

With TensorFlow

- Training and validation splitting
- Architecture: [3, 25, 3]
- Linear solver: Reverse Automatic Differentiation
- Minimisation: Backpropagation + gradient descent [ADAM]

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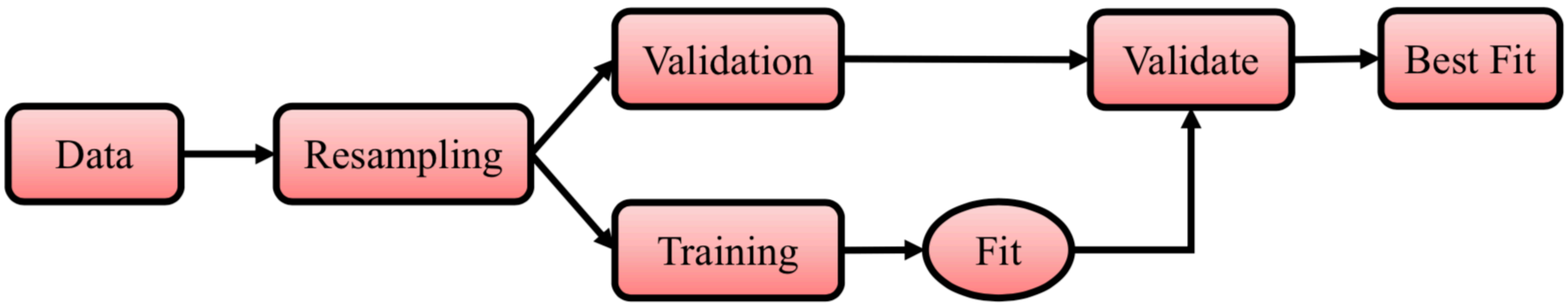

Minimisation

With TensorFlow

- Training and validation splitting
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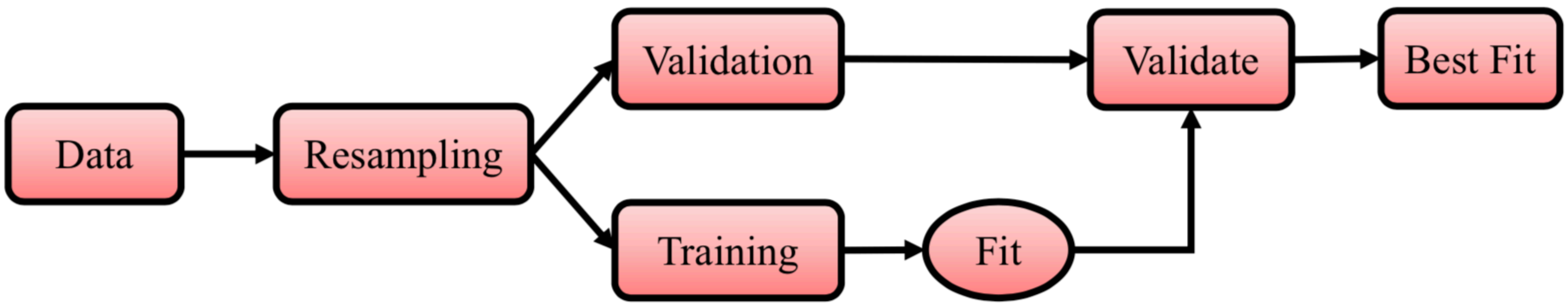
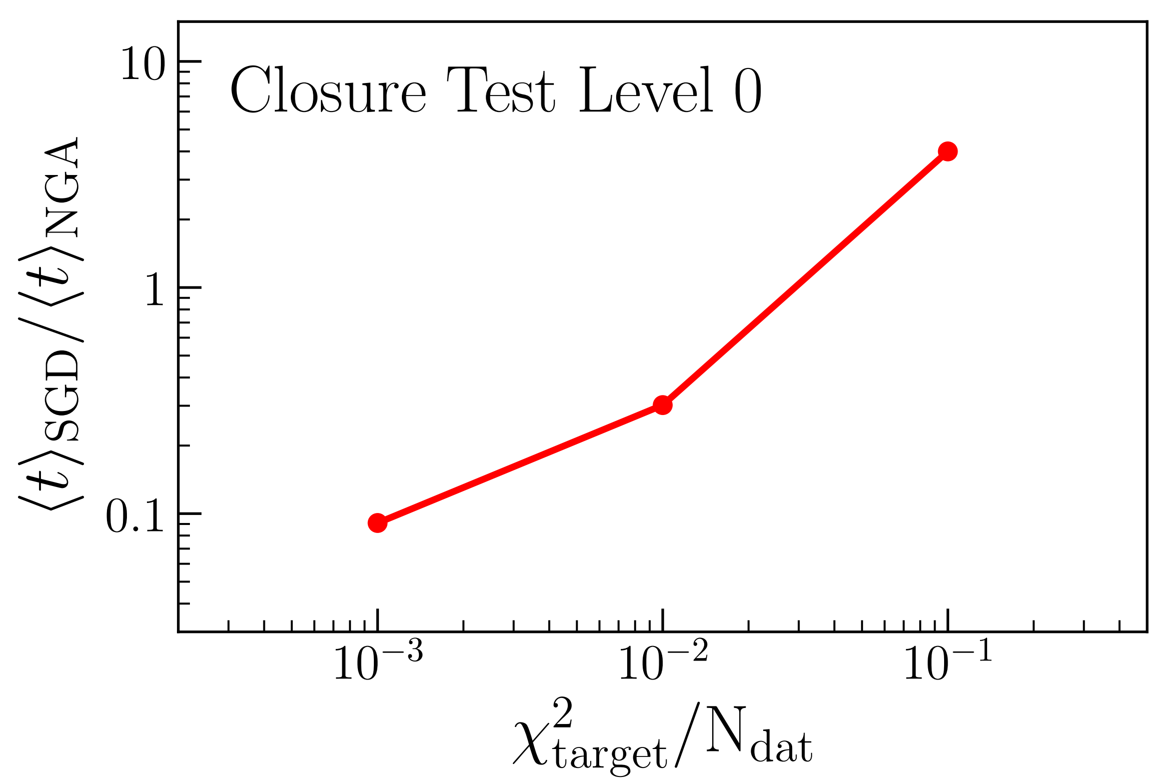




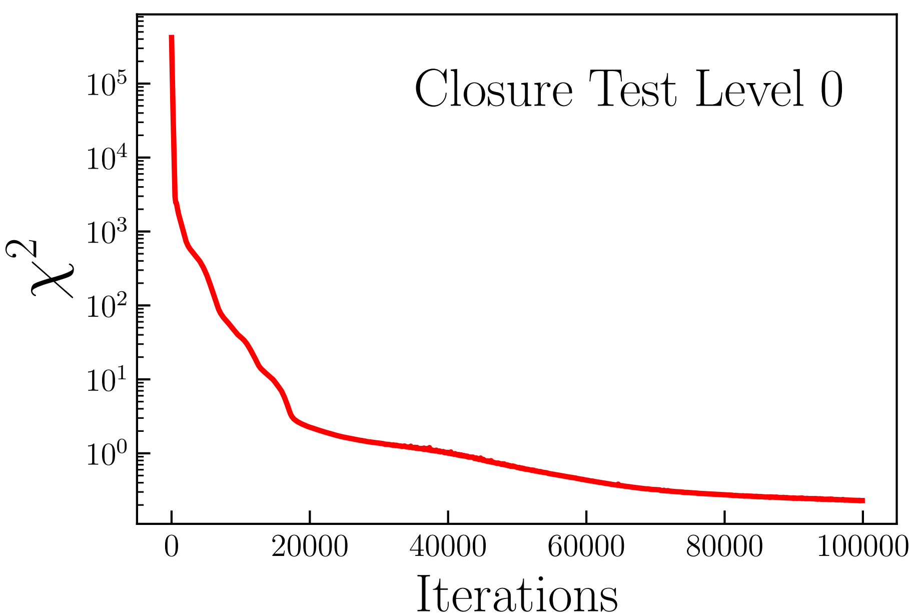
x11 faster!

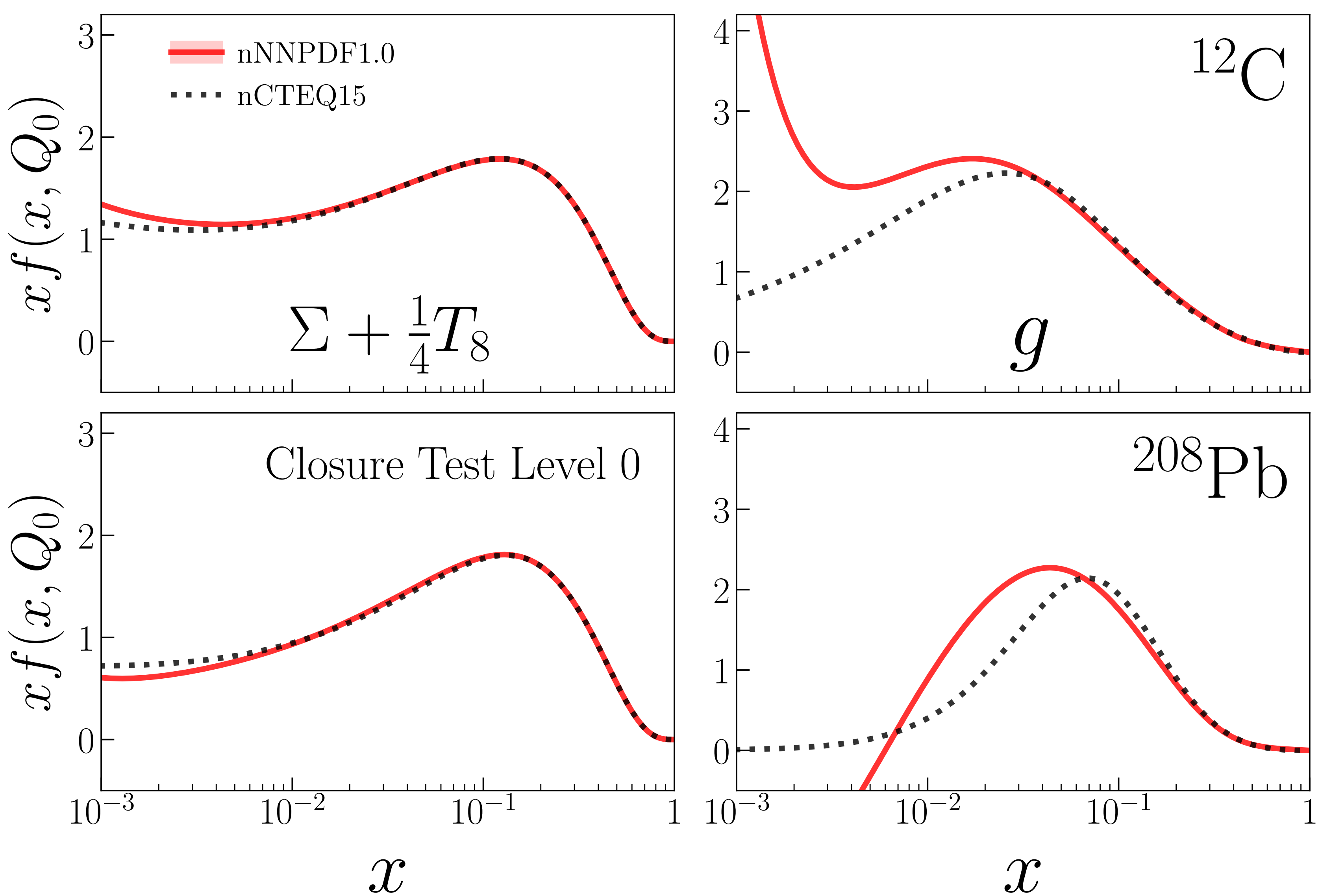
Methodology validation
Closure Tests

Closure Test Level 0

Fitting pseudo-data is generated from the nCTEQ distributions without any additional statistical fluctuations.

The uncertainties are taken to be the same as the experimental data.

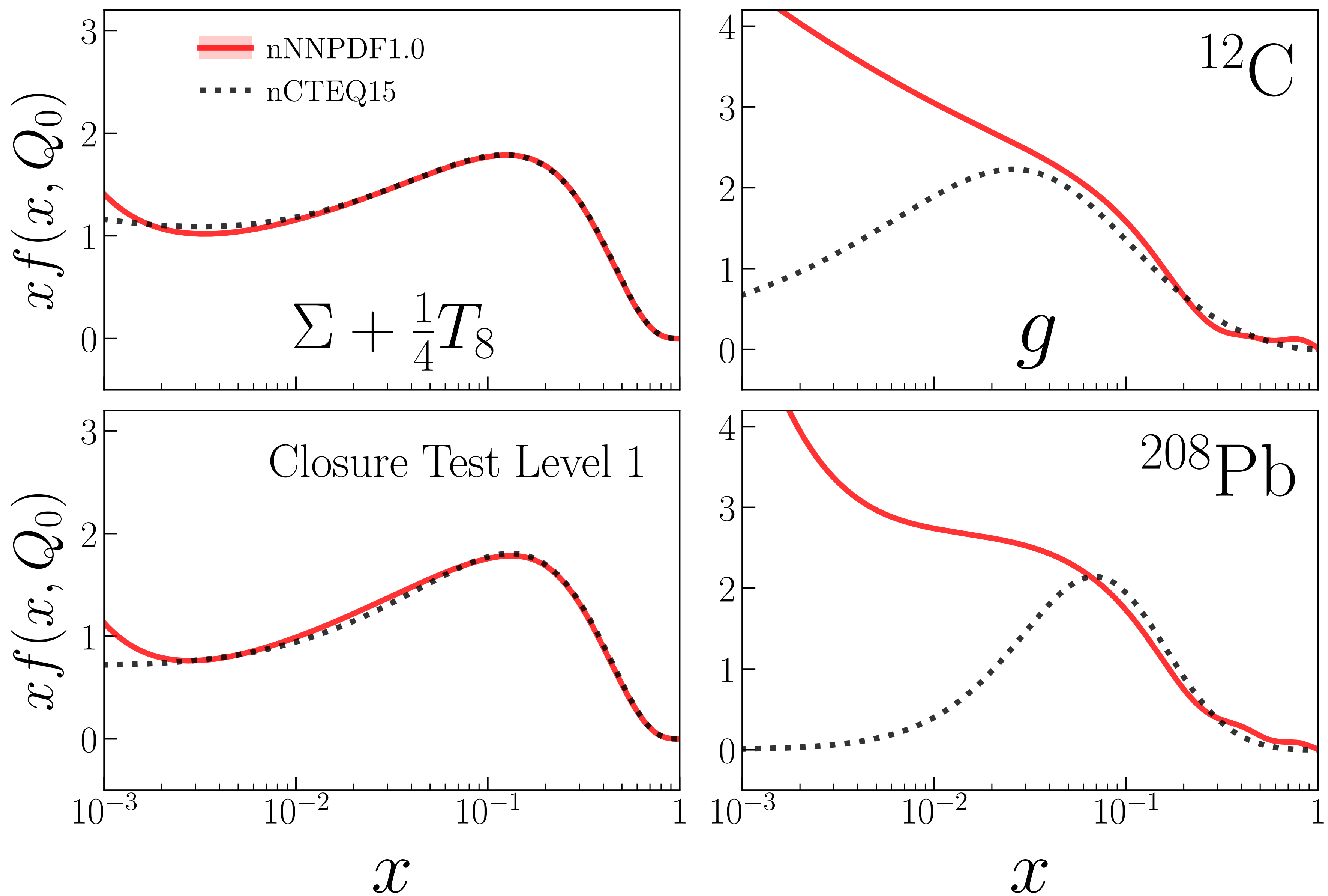




Closure Test Level 1

Pseudo-data is generated by adding statistical fluctuations to nCTEQ15 predictions.

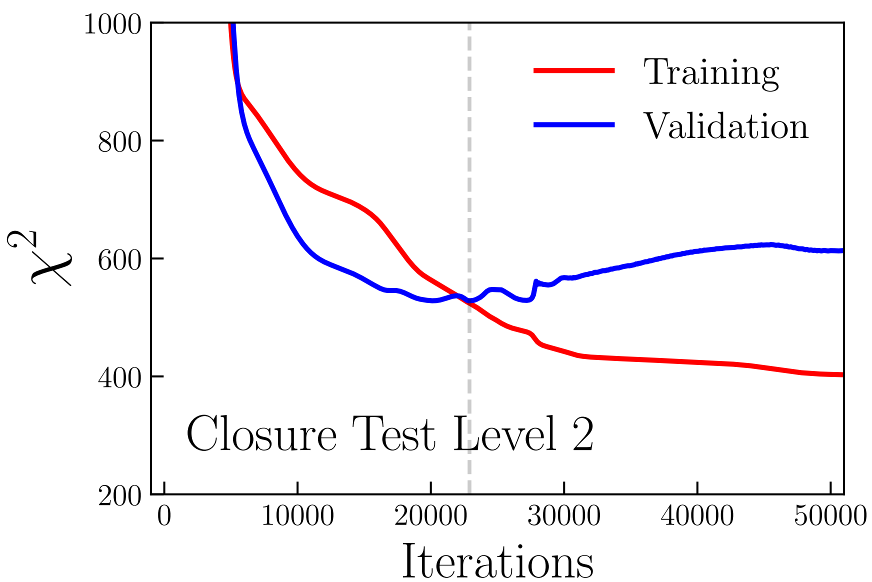
These fluctuations are dictated by the corresponding experimental statistical and systematic uncertainties, and are the same that enter in the t_0 covariance matrix.

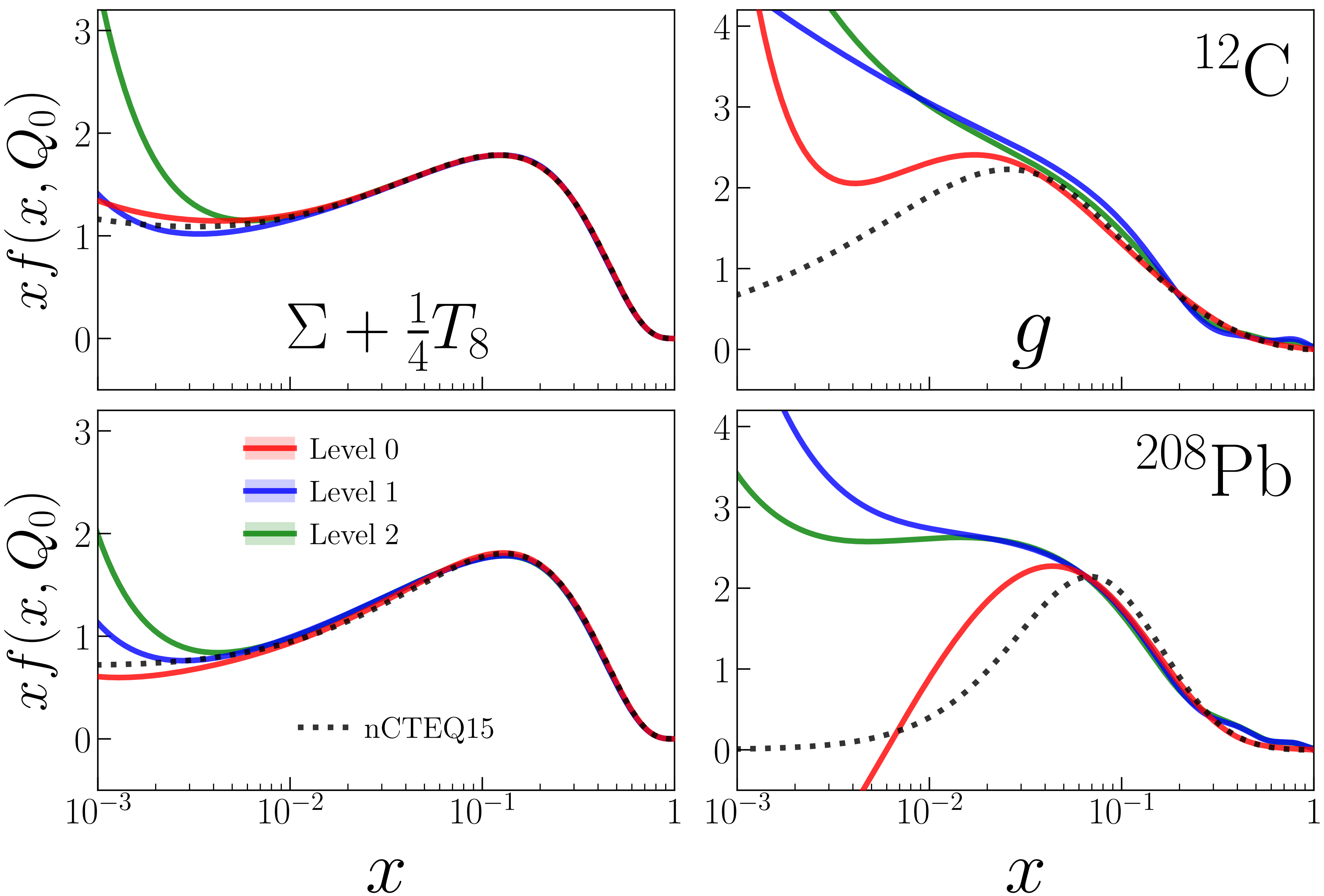


Closure Test Level 2

Pseudo-data generated in the L1 case is now used to produce a large N_{rep} number of Monte Carlo replicas.

A nuclear PDF fit is then performed for each replica, and look-back cross-validation is again activated to prevent over-fitting.

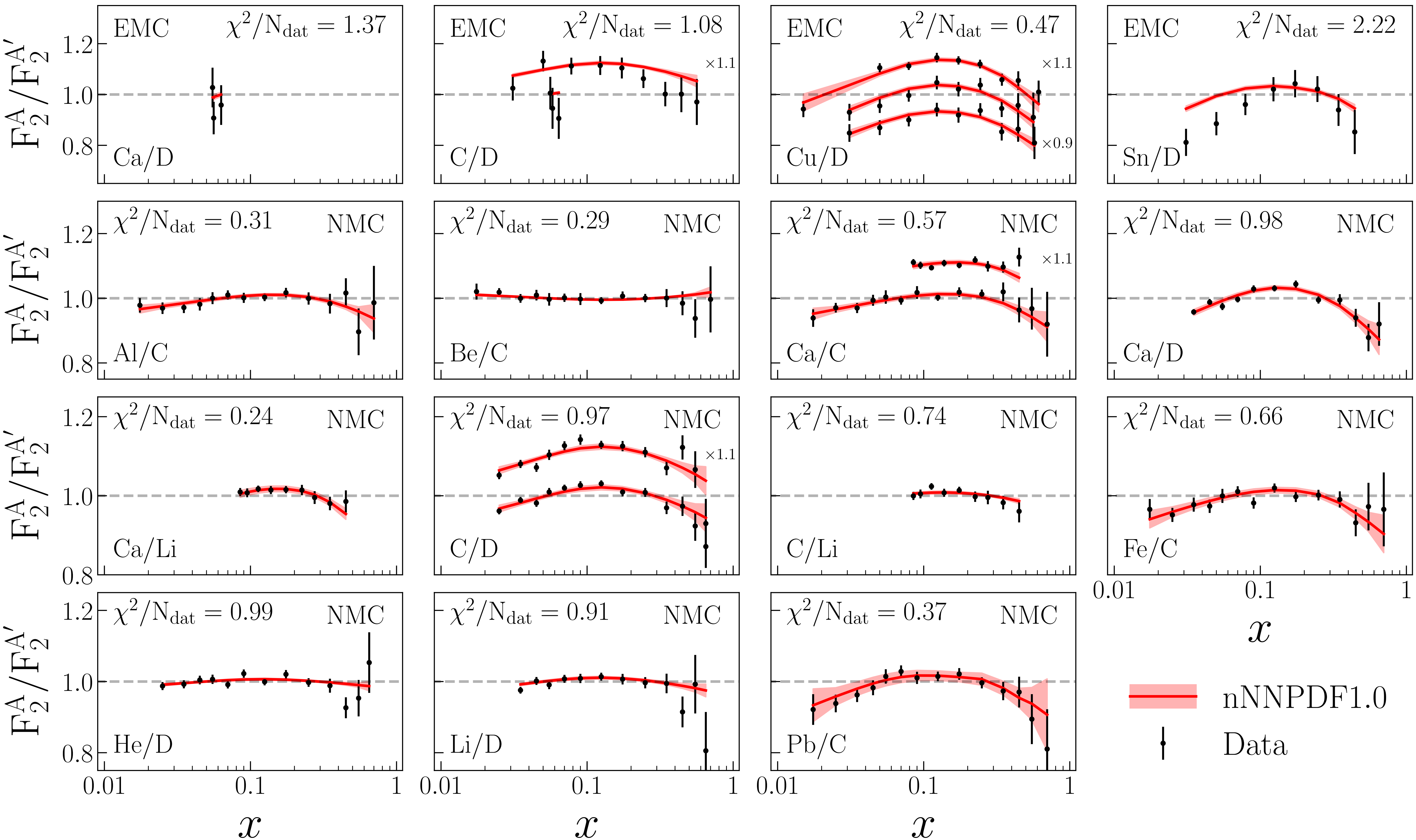




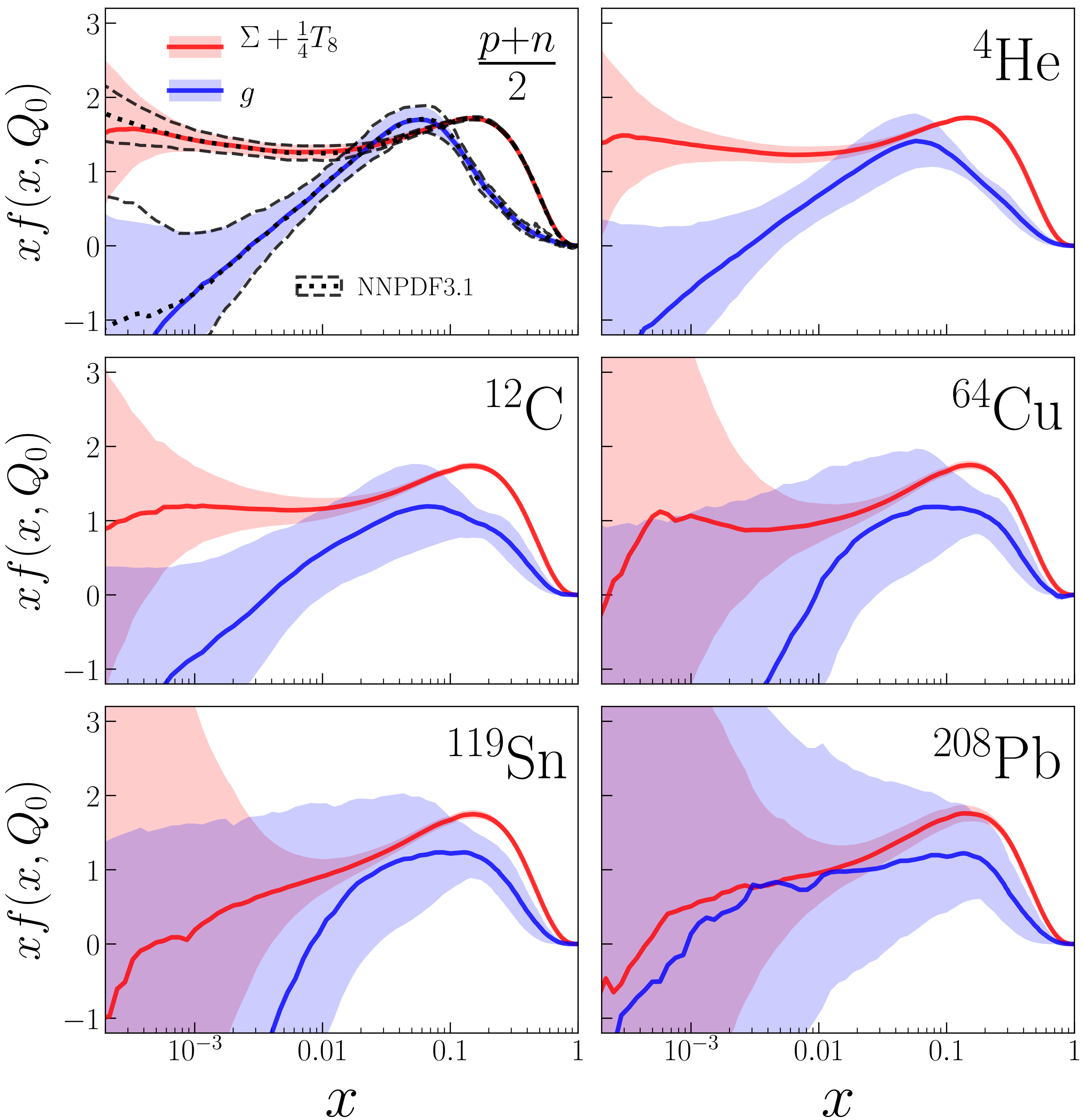
The Fit

Data vs. Theory

EMC - NMC



NLO fit



Σ +1/4 T_8
g

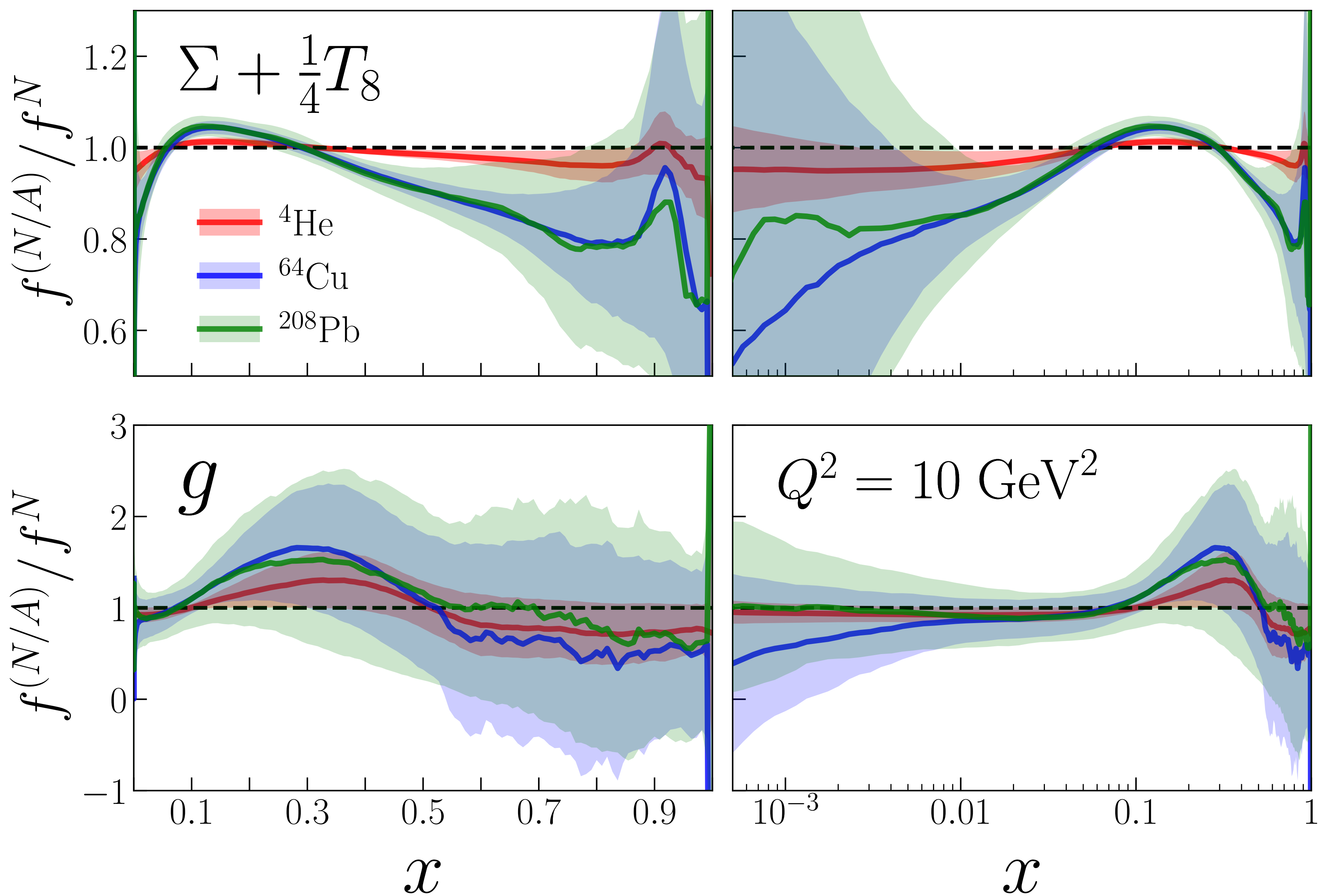
Errors computed as 90% CL (1k replicas)

NNPDF3.1 central value and uncertainties reproduced

$$f(x, Q, A = 1) = \frac{1}{2}\left[f_p(x, Q^2) + f_n(x, Q^2)\right]$$

The boundary condition constrain the nPDFs for low-A nuclei

A dependence

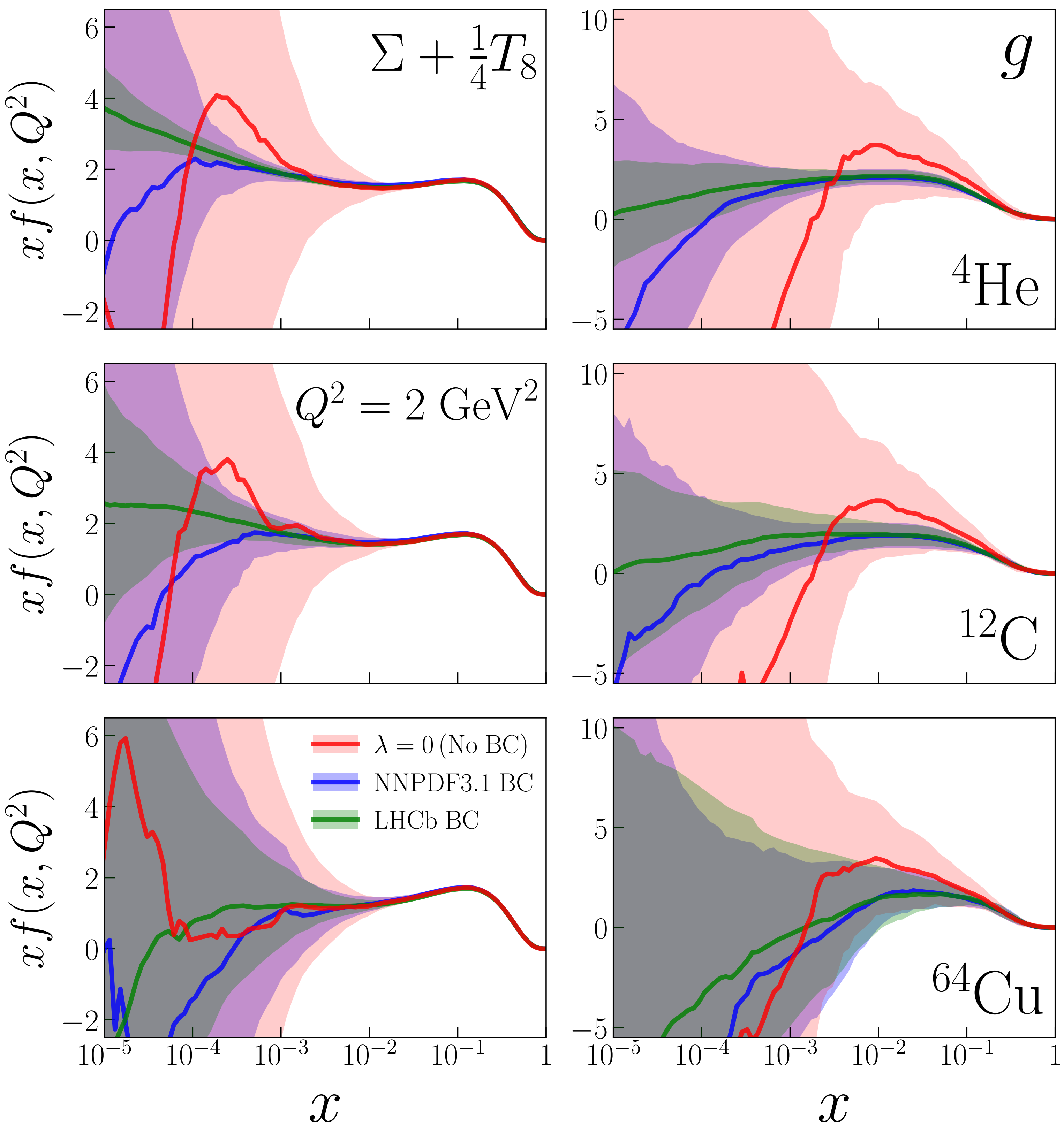


A=4 (He)
A=64 (Cu)
A=208 (Pb)

Nuclear effects more pronounced for larger A in the Σ + 1/4 T_8 combination

Gluon Uncertainties Within unity

Boundary Condition



No BC
NNPDF3.1
NNPDF3.1+LHCb

Imposed for
$\mathbf{x} \in [10^{-3}, 0.7]$

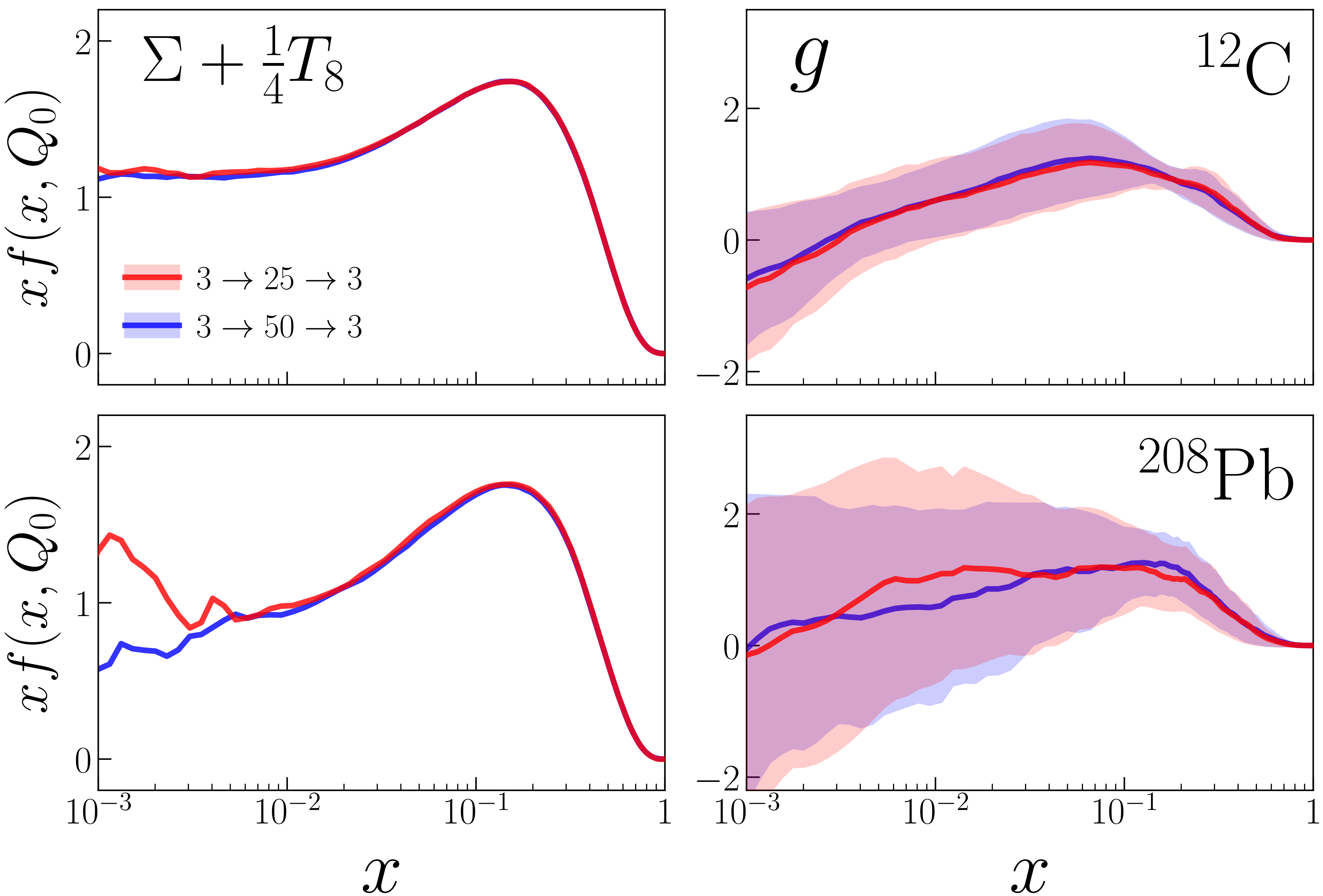
In the χ^2:

$$\lambda \sum_{m=g,\Sigma,T_8} \sum_{l=1}^{N_x} \left(f_m(x_l, Q_0, A) - f_m^{(p+n)/2}(x_l, Q_0) \right)^2$$

Important constraints on nPDF central values and uncertainties across A

Pronounced reduction of uncertainty due to the accurate determination of the proton's quark sea at small-x in NNPDF3.1 + LHCb

Architecture



Stable results for larger architecture (double the amount of parameters)

Results driven by the input data not by methodological choices such as degrees of freedom in the parametrisation

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- Architecture

3 | Comparisons

- A=1 limit
- nPDFs

A=1 limit

nNNPDF1.0

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- **Per replica boundary condition to reproduce NNPDF3.1 overall uncertainties (correlations included)**
- **Imposed on the level of minimisation**

A=1 limit

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EPPS16

Central value: $f_i^{p/A}(x, Q^2) = R_i^A(x, Q^2) f_i^{p}(x, Q^2),$ **CT14NLO**

Uncertainties:

- **Baseline parameters uncorrelated from the nPDF parameters**
- **On the level of observables:** $(\delta\mathcal{O}_{\rm total})^2 = (\delta\mathcal{O}_{\rm EPPS16})^2 + (\delta\mathcal{O}_{\rm baseline})^2$

A=1 limit

nNNPDF1.0

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nCTEQ15

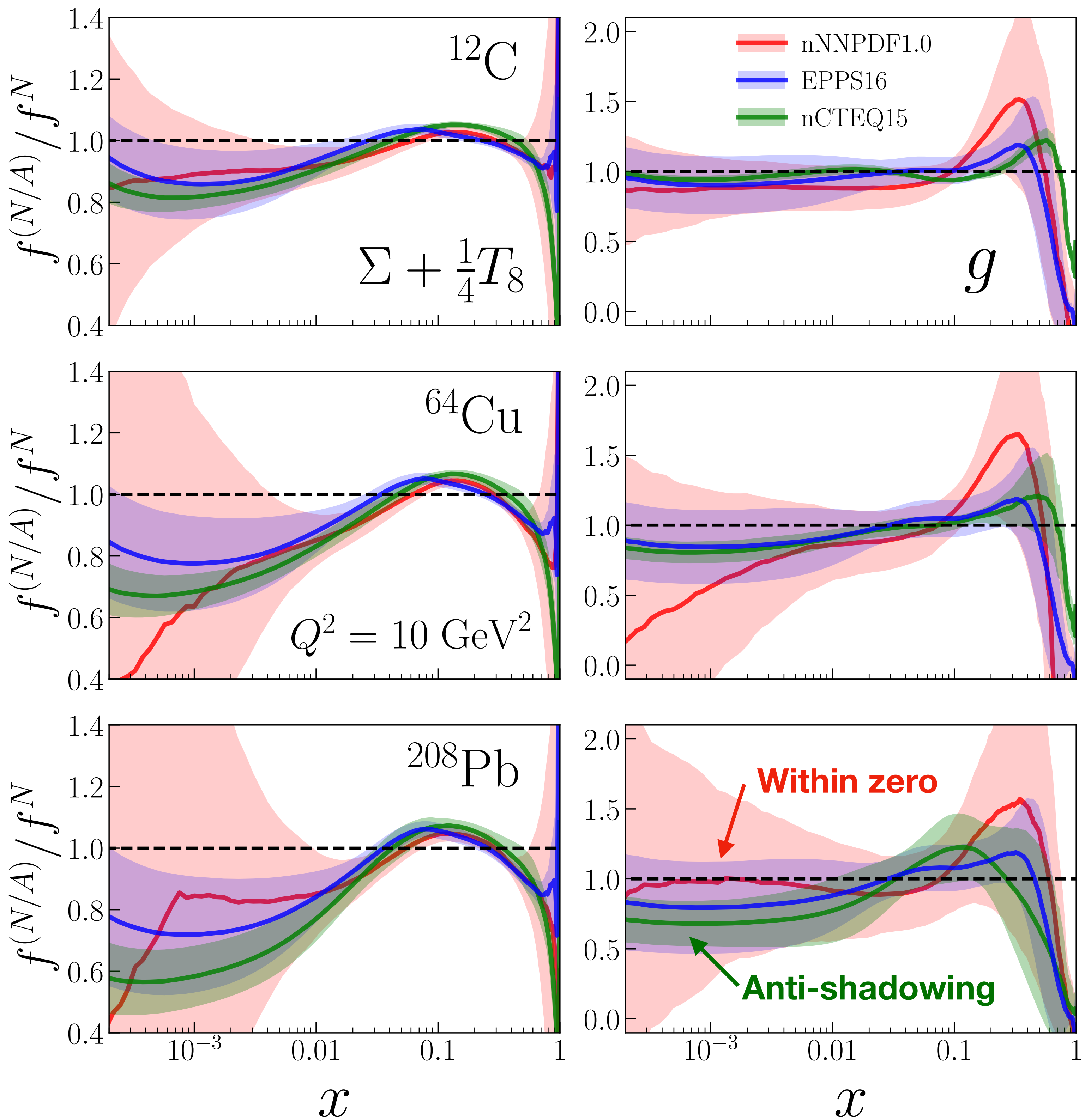
Central value:

$$x f_i^{p/A}(x, Q_0) = c_0\, x^{c_1}(1-x)^{c_2} e^{c_3 x}(1 + e^{c_4} x)^{c_5},$$
$$\text{for} \quad i = u_v, d_v, g, \bar{u} + \bar{d}, s + \bar{s}, s - \bar{s},$$

For A=1, PDF = Central Value of CTEQ6.1

$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right), \quad k = \{1, \ldots, 5\}.$$

nPDFs



nNNPDF1.0
EPPS16
nCTEQ15

All nPDFs are normalised to nNNPDF1.0(A=1) central value

90% CL for all nPDFs

- **Hessian for nCTEQ15 and EPPS16**
- **Monte Carlo for nNNPDF1.0**

Significant differences in uncertainties in non-data region

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3 Comparisons

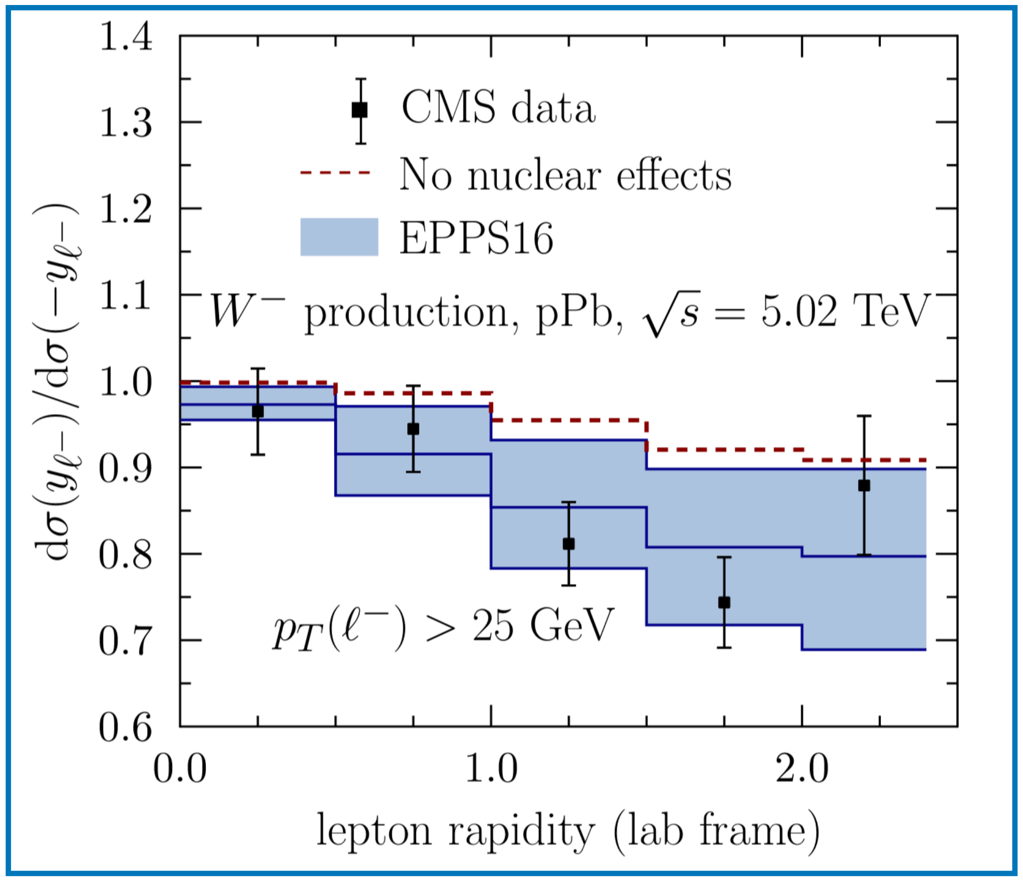
- A=1 limit
- nPDFs

4 EIC

- Motivation
- Kinematics
- Impact

The need for EIC

**Hera DIS data being the backbone of free-proton PDFs,
we need something similar for nPDFs**



EPPS16, Fig.19, [1612.05741]

EIC impact

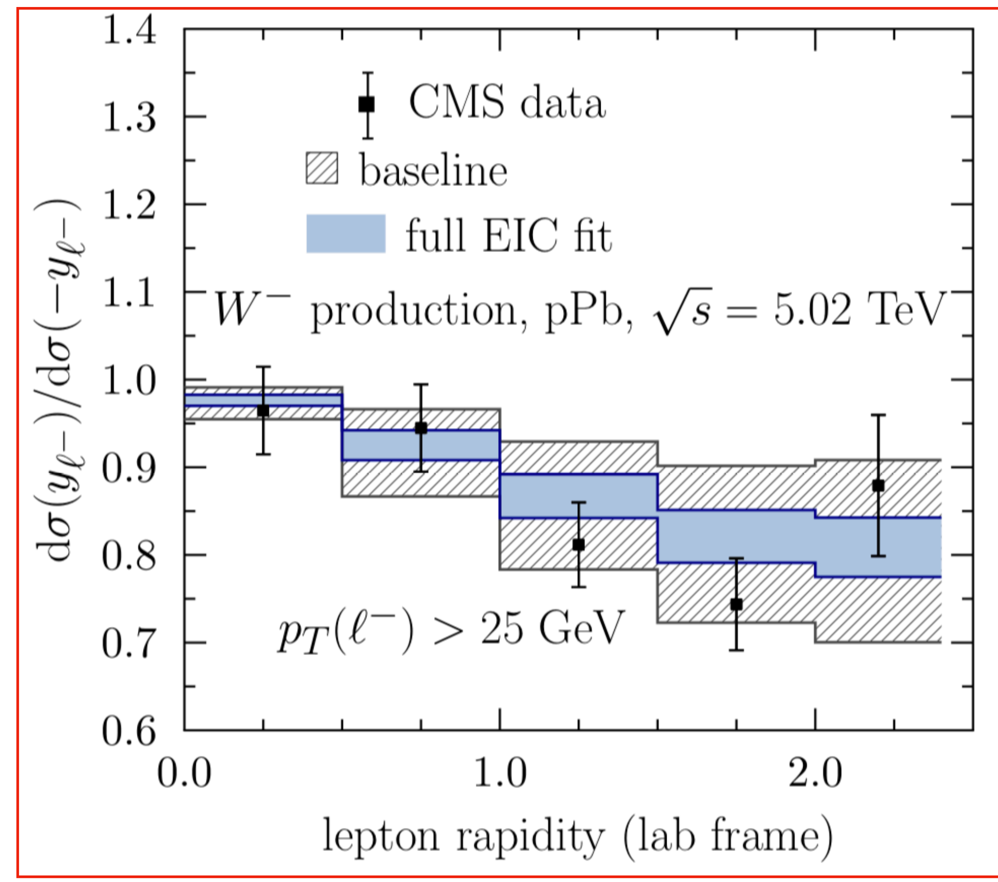


Fig.17, [1708.05654]

Reliable data to constrain the gluon at small-x and low-Q^2

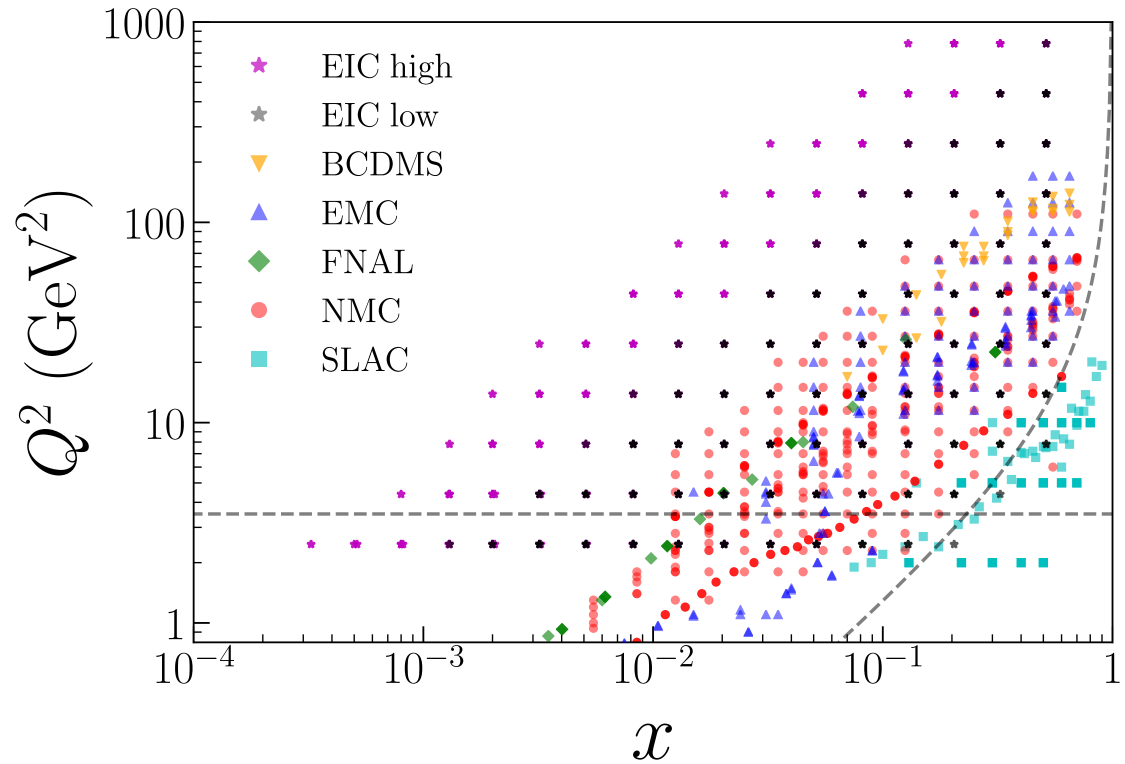
EIC kinematics

Scenario	A	E_e	E_A/A	$Q^2_{\max}$	$x_{\min}$	$N_{\rm dat}$
eRHIC_5x50C	12	5 GeV	50 GeV	440 GeV^2	0.003	50
eRHIC_5x75C	12	5 GeV	75 GeV	440 GeV^2	0.002	57
eRHIC_5x100C	12	5 GeV	100 GeV	780 GeV^2	0.001	64
eRHIC_5x50Au	197	5 GeV	50 GeV	440 GeV^2	0.003	50
eRHIC_5x75Au	197	5 GeV	75 GeV	440 GeV^2	0.002	57
eRHIC_5x100Au	197	5 GeV	100 GeV	780 GeV^2	0.001	64
eRHIC_20x50C	12	20 GeV	50 GeV	780 GeV^2	0.0008	75
eRHIC_20x75C	12	20 GeV	75 GeV	780 GeV^2	0.0005	79
eRHIC_20x100C	12	20 GeV	100 GeV	780 GeV^2	0.0003	82
eRHIC_20x50Au	197	20 GeV	50 GeV	780 GeV^2	0.0008	75
eRHIC_20x75Au	197	20 GeV	75 GeV	780 GeV^2	0.0005	79
eRHIC_20x100Au	197	20 GeV	100 GeV	780 GeV^2	0.0003	82

EIC projections, H. Paukkunen et al. [arXiv:1708.05654]

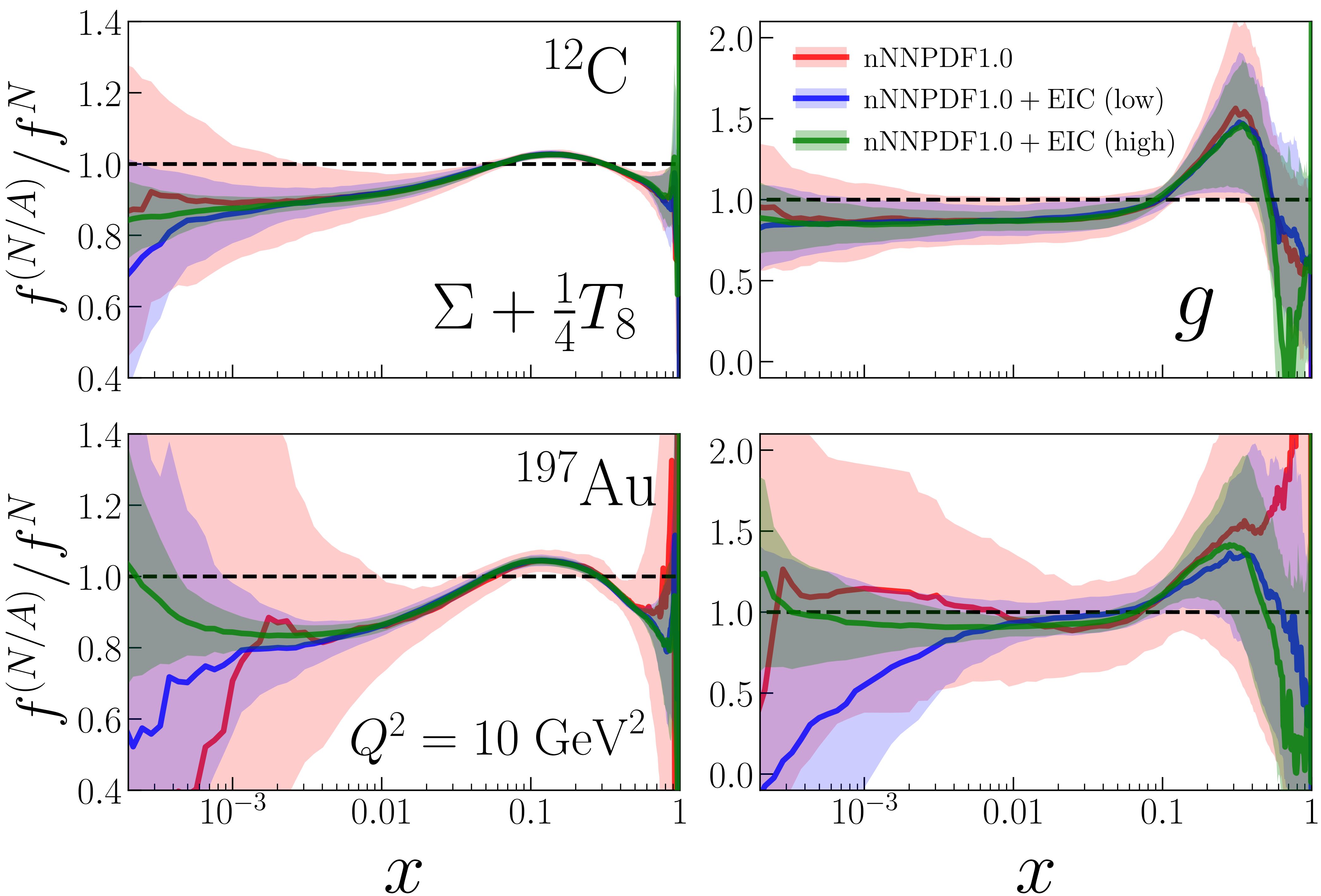
Two scenarios:

- **Low energy (5 GeV)**
- **High energy (20 GeV)**



Pseudo-data constructed With nNNPDF1.0 (Gold, Carbon)

EIC impact on nPDFs



Signification reduction of nPDF uncertainties at low-x for large A
Particularly for the higher energy scenario

Summary

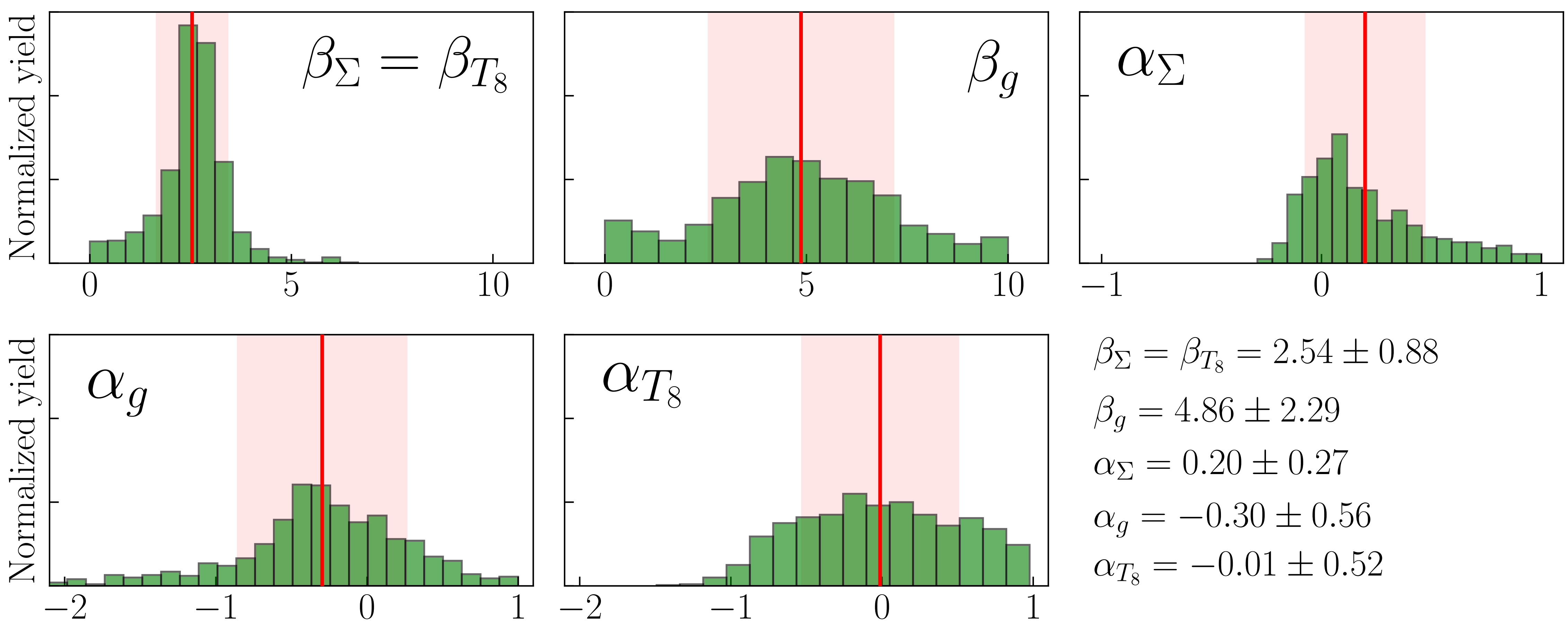
- First determination of nPDF using NNPDF methodology.
- Excellent agreement (NLO and NNLO) with all available NC DIS data (A=2 to A=208).
- Quark distributions are reasonably constrained for $x \geq 10^{-2}$.
- Significant methodology improvement lead by use of TensorFlow and stochastic gradient descent for the first time in NNPDF.
- Vitality of the boundary condition (A=1 limit), reproducing NNPDF3.1 central values and uncertainties of using consistently the same theory settings.
- Quantification of the future impact of e+A measurements from an Electron-Ion Collider constraining the quark and gluon nPDFs down to $x \approx 5 \times 10^{-4}$.
- LHAPDF sets available on NNPDF website.
- Future steps: including CC DIS data, LHC data in a global fit.

Backup Slides

Effective Exponents

In previous NNPDF analyses, the preprocessing exponents α_f and β_f were fixed to a randomly chosen value from a range that was determined iteratively.

Here instead we will fit their values for each Monte Carlo replica, so that they are treated simultaneously with the weights and thresholds of the neural network.



$\beta_\Sigma = \beta_{T_8} = 2.54 \pm 0.88$

$\beta_g = 4.86 \pm 2.29$

$\alpha_\Sigma = 0.20 \pm 0.27$

$\alpha_g = -0.30 \pm 0.56$

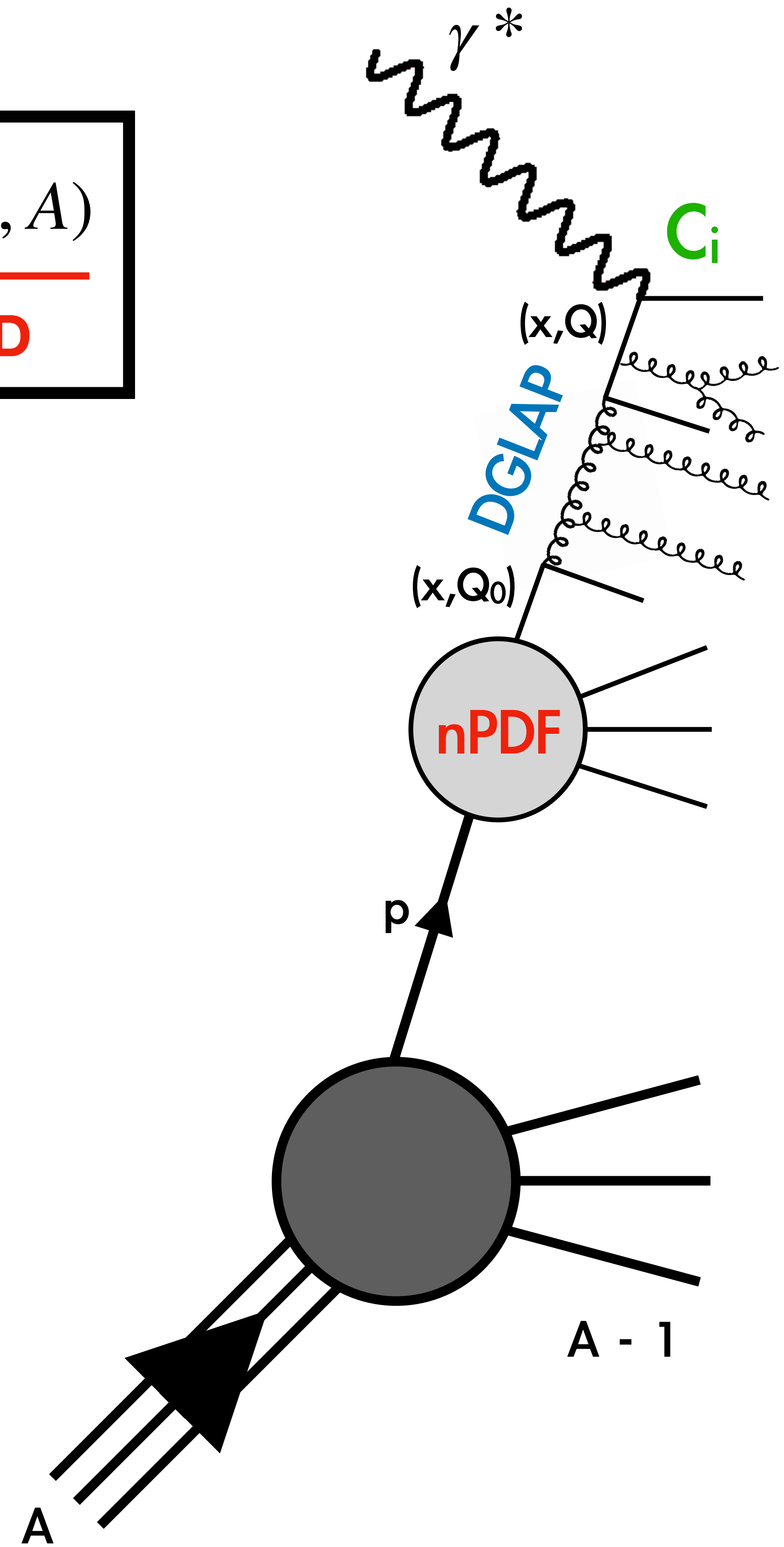
$\alpha_{T_8} = -0.01 \pm 0.52$

$$\alpha_f \in [-5, 1]\,, \qquad \beta_f \in [0, 10]\,, \qquad f = \Sigma, T_8, g\,.$$

Nuclear NC Inclusive DIS

EM F_2 ($Q < M_Z$)

$$F_2(x, Q^2, A) = \sum_i^{n_f} \sum_j^{n_f} \underbrace{C_i(x, Q^2)}_{\text{pQCD}} \otimes \underbrace{\Gamma_{ij}(x, Q^2)}_{\text{DGLAP}} \otimes \underbrace{q_j(x, Q_0^2, A)}_{\text{npQCD}}$$



Nuclear NC Inclusive DIS

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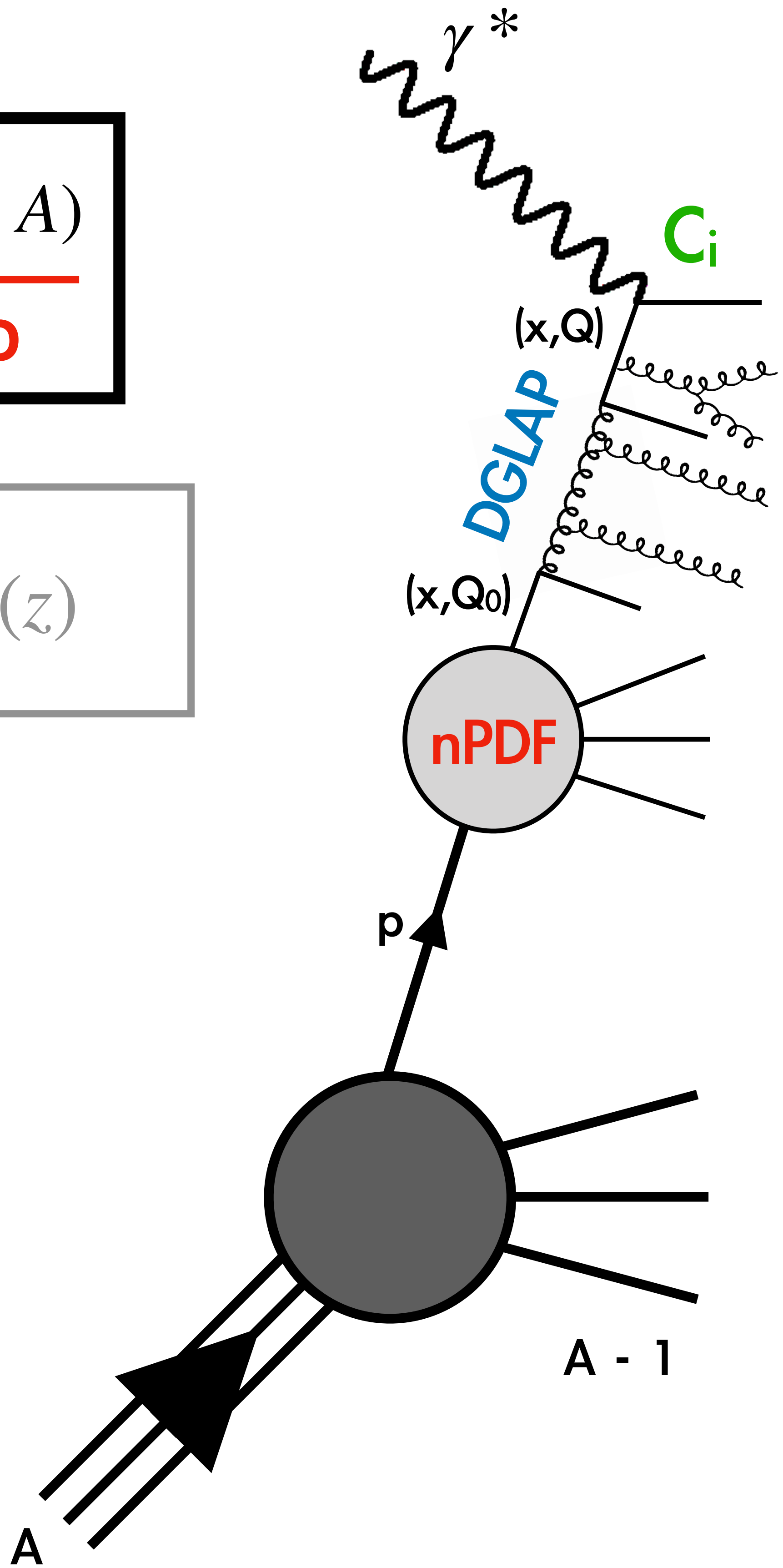
$$F_2(x, Q^2, A) = \sum_i^{n_f} \sum_j^{n_f} \underbrace{C_i(x, Q^2)}_{\text{pQCD}} \otimes \underbrace{\Gamma_{ij}(x, Q^2)}_{\text{DGLAP}} \otimes \underbrace{q_j(x, Q_0^2, A)}_{\text{npQCD}}$$

Convolution

$$f \otimes g = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) g(z)$$



Nuclear NC Inclusive DIS

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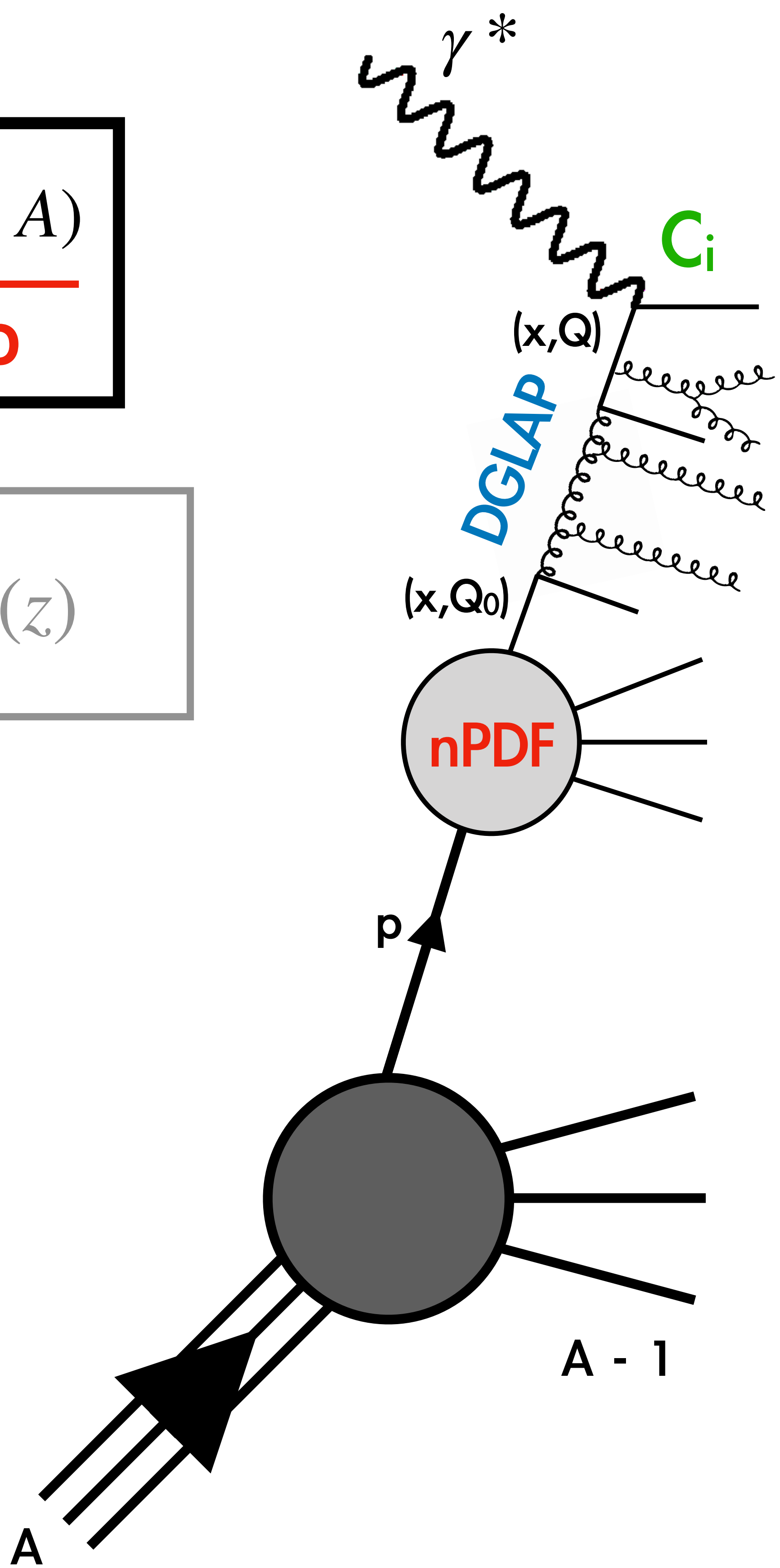
Coefficient Functions

$$C_i = C_i^{(0)} + \frac{\alpha_s}{4\pi} C_i^{(1)} + O(\alpha_s^2)$$

Convolution

$$f \otimes g = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) g(z)$$



Nuclear NC Inclusive DIS

EM F2 (Q < MZ)

$$F_2(x, Q^2, A) = \sum_i^{n_f} \sum_j^{n_f} \underbrace{C_i(x, Q^2)}_{\text{pQCD}} \otimes \underbrace{\Gamma_{ij}(x, Q^2)}_{\text{DGLAP}} \otimes \underbrace{q_j(x, Q_0^2, A)}_{\text{npQCD}}$$

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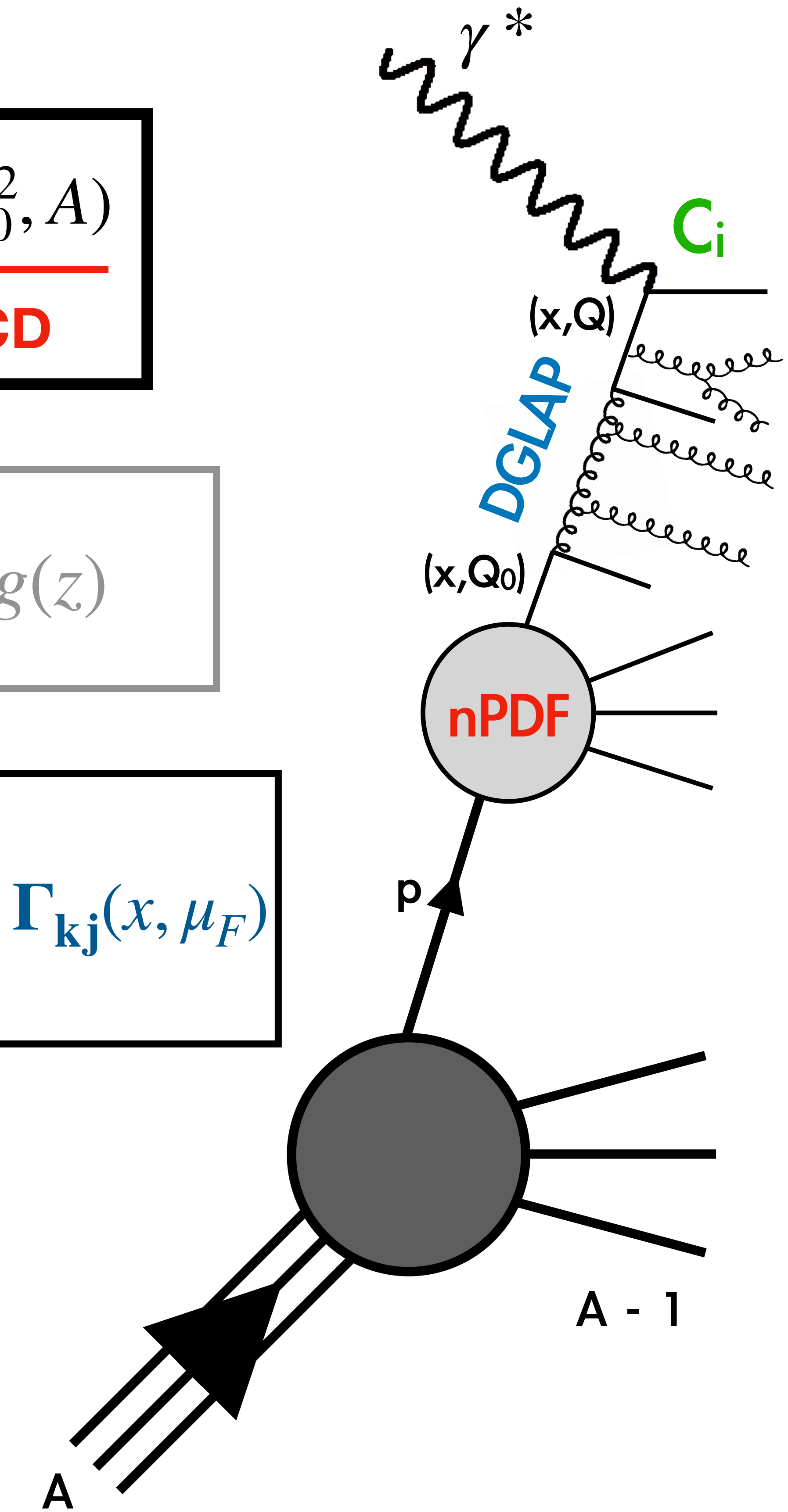
Convolution

$$f \otimes g = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) g(z)$$

DGLAP equation

$$\frac{\partial \Gamma_{\mathbf{ij}}}{\partial \ln \mu_F^2} = \sum_k \frac{\alpha_{\mathbf{s}}(\mu_{\mathbf{F}})}{4\pi} \left[P_{ik}^{(0)}(x) + \frac{\alpha_{\mathbf{s}}(\mu_{\mathbf{F}})}{4\pi} P_{ik}^{(1)}(x) + \ldots \right] \otimes \Gamma_{\mathbf{kj}}(x, \mu_F)$$



Nuclear NC Inclusive DIS

EM F2 (Q < MZ)

$$F_2(x,Q^2,A) = \sum_i^{n_f}\sum_j^{n_f} \underbrace{C_i(x,Q^2)}_{\text{pQCD}} \otimes \underbrace{\Gamma_{ij}(x,Q^2)}_{\text{DGLAP}} \otimes \underbrace{q_j(x,Q_0^2,A)}_{\text{npQCD}}$$

Coefficient Functions

$$C_i = C_i^{(0)} + \frac{\alpha_s}{4\pi}C_i^{(1)} + O(\alpha_s^2)$$

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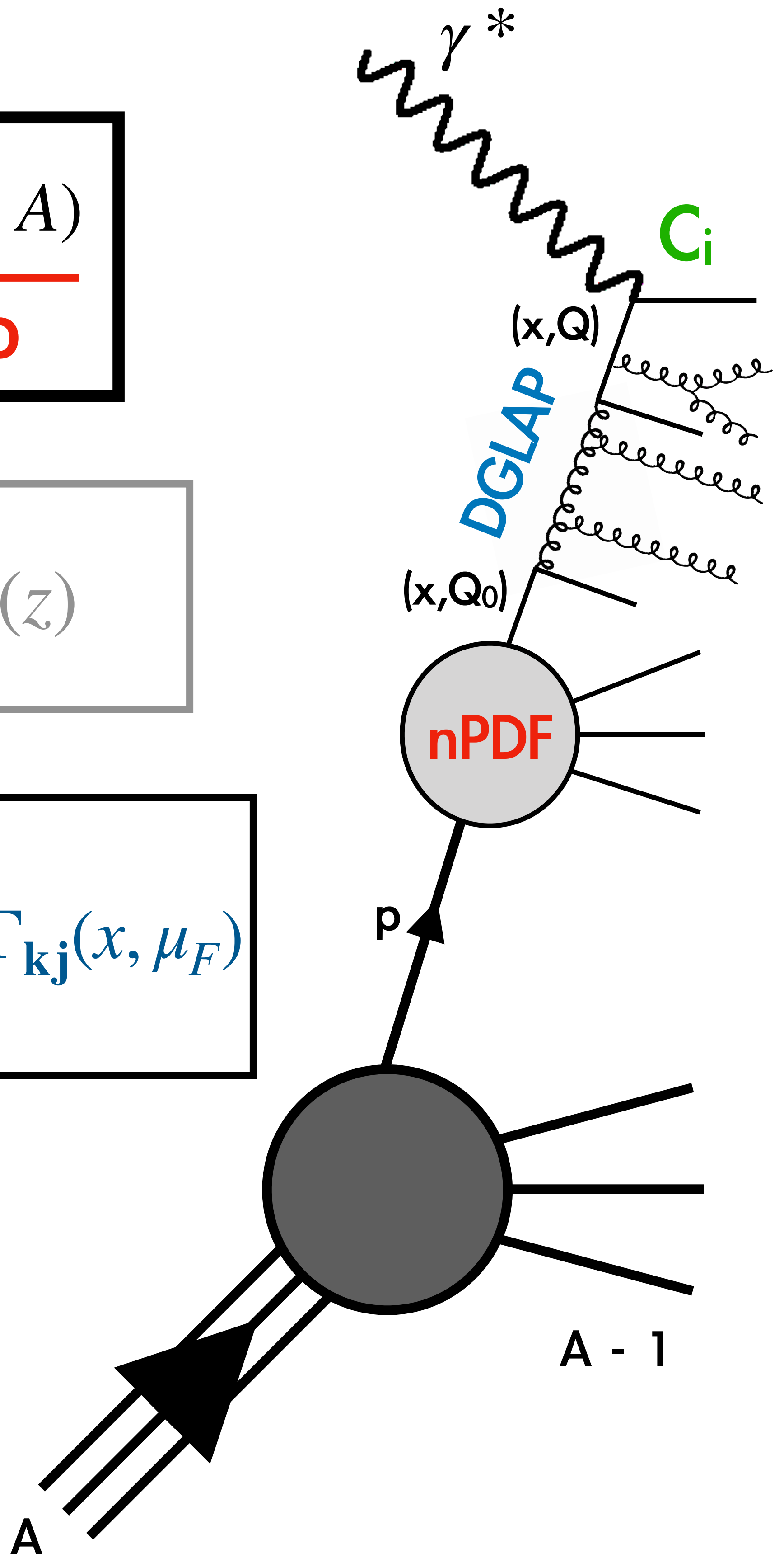
$$\frac{\partial \Gamma_{\mathbf{ij}}}{\partial \ln \mu_F^2} = \sum_k \frac{\alpha_{\mathbf{s}}(\mu_{\mathbf{F}})}{4\pi}\left[P_{ik}^{(0)}(x) + \frac{\alpha_{\mathbf{s}}(\mu_{\mathbf{F}})}{4\pi}P_{ik}^{(1)}(x) + \ldots\right] \otimes \Gamma_{\mathbf{kj}}(x,\mu_F)$$

RG equation

$$\frac{\partial \alpha_{\mathbf{s}}}{4\pi \cdot \partial \ln \mu_R^2} = -\left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^2 \left[\beta_0 + \frac{\alpha_s(\mu_R)}{4\pi}\beta_1 + \ldots\right]$$



Nuclear NC Inclusive DIS

EM F_2 ($Q < M_Z$)

$$F_2(x, Q^2, A) = \sum_i^{n_f} \sum_j^{n_f} \underbrace{C_i(x, Q^2)}_{\text{pQCD}} \otimes \underbrace{\Gamma_{ij}(x, Q^2)}_{\text{DGLAP}} \otimes \underbrace{q_j(x, Q_0^2, A)}_{\text{npQCD}}$$

$$= C^S_{2,q}(x, \alpha_s(Q)) \otimes \Sigma(x, Q^2) \longrightarrow \textbf{Singlet}$$

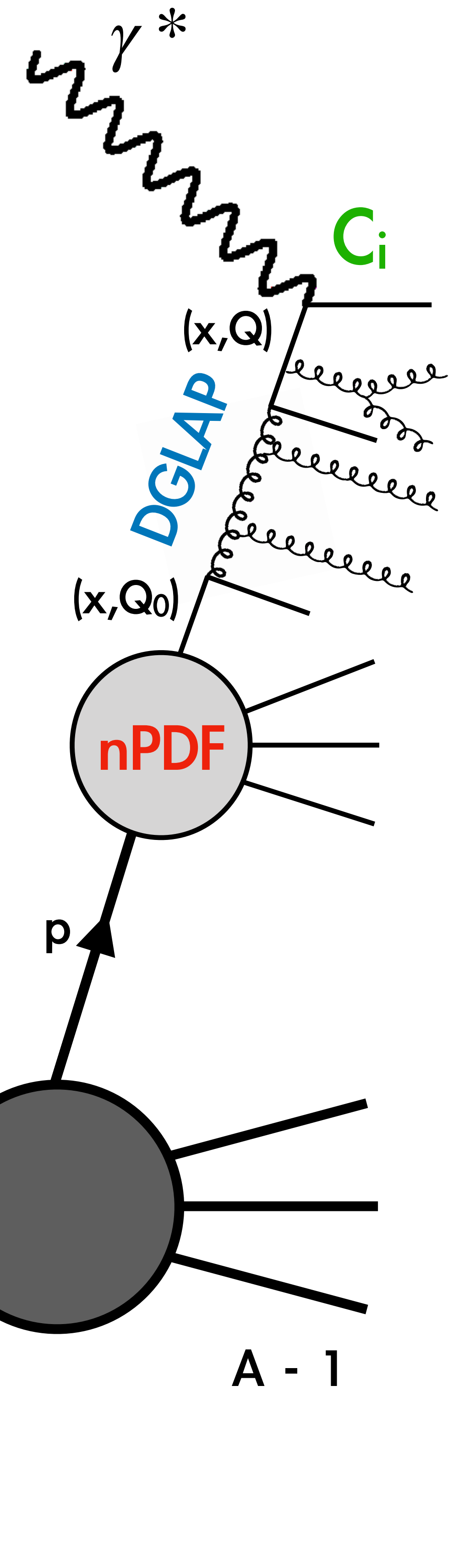
$$+ C^{NS}_{2,q}(x, \alpha_s(Q)) \otimes T(x, Q^2) \longrightarrow \textbf{Non-Singlet}$$

$$+ C^S_{2,g}(x, \alpha_s(Q)) \otimes g(x, Q^2) \longrightarrow \textbf{Gluon}$$

Theoretically, 3 Independent PDFs

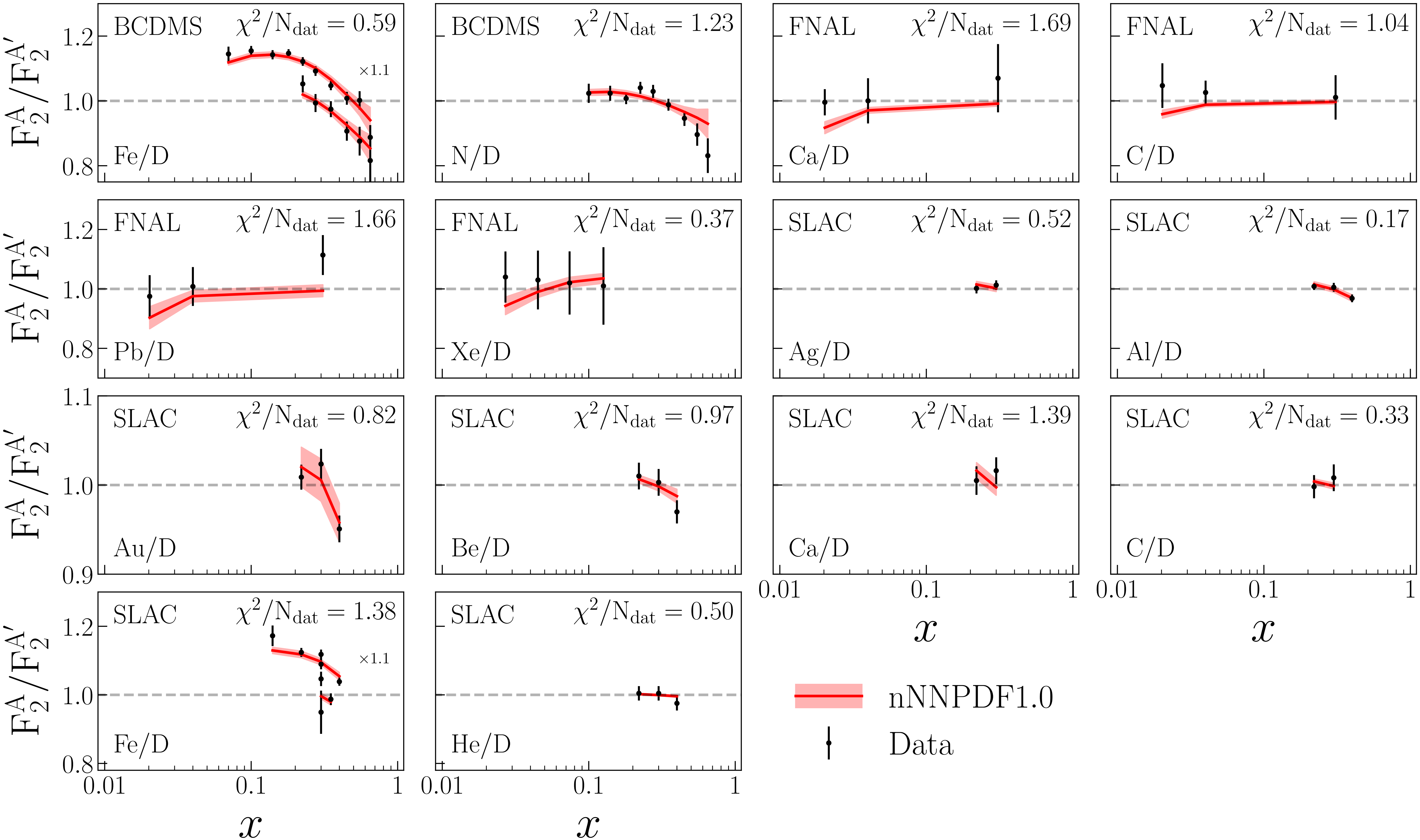
In practice, Data constrain only 2 PDFs,

gluon and a combination of Singlet and octet quarks

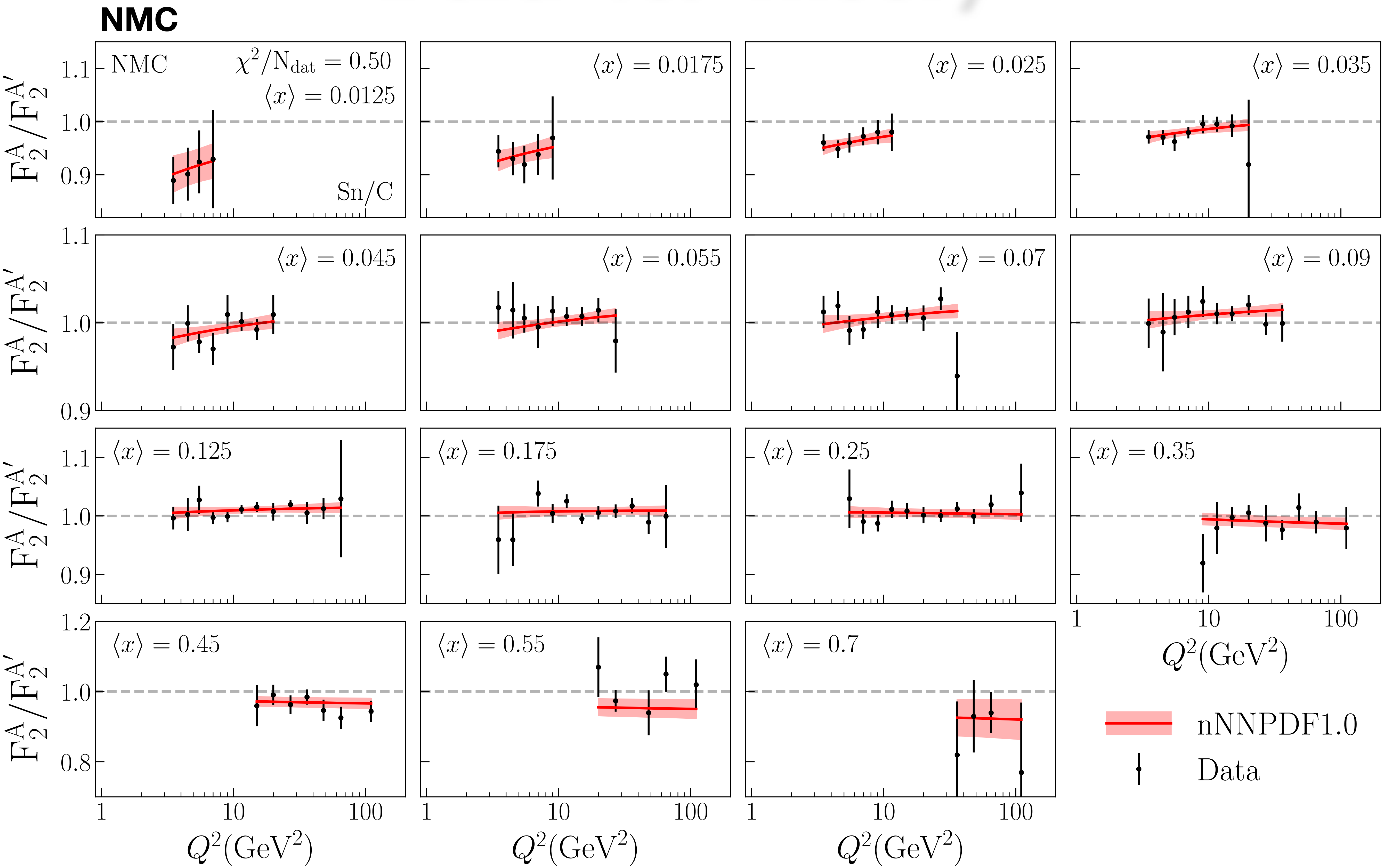


Data vs. Theory

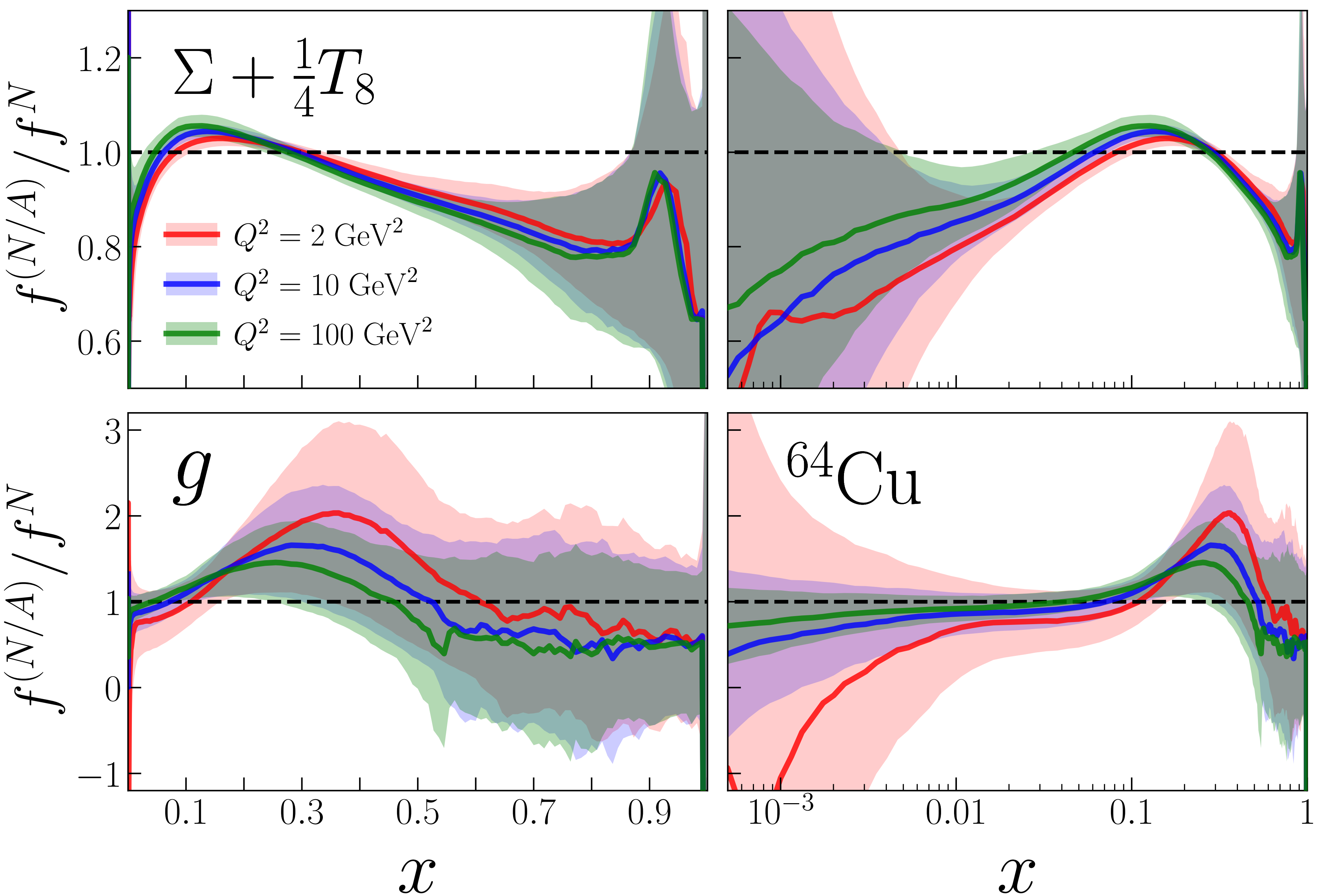
BCDMS - FNAL - SLAC



Data vs. Theory



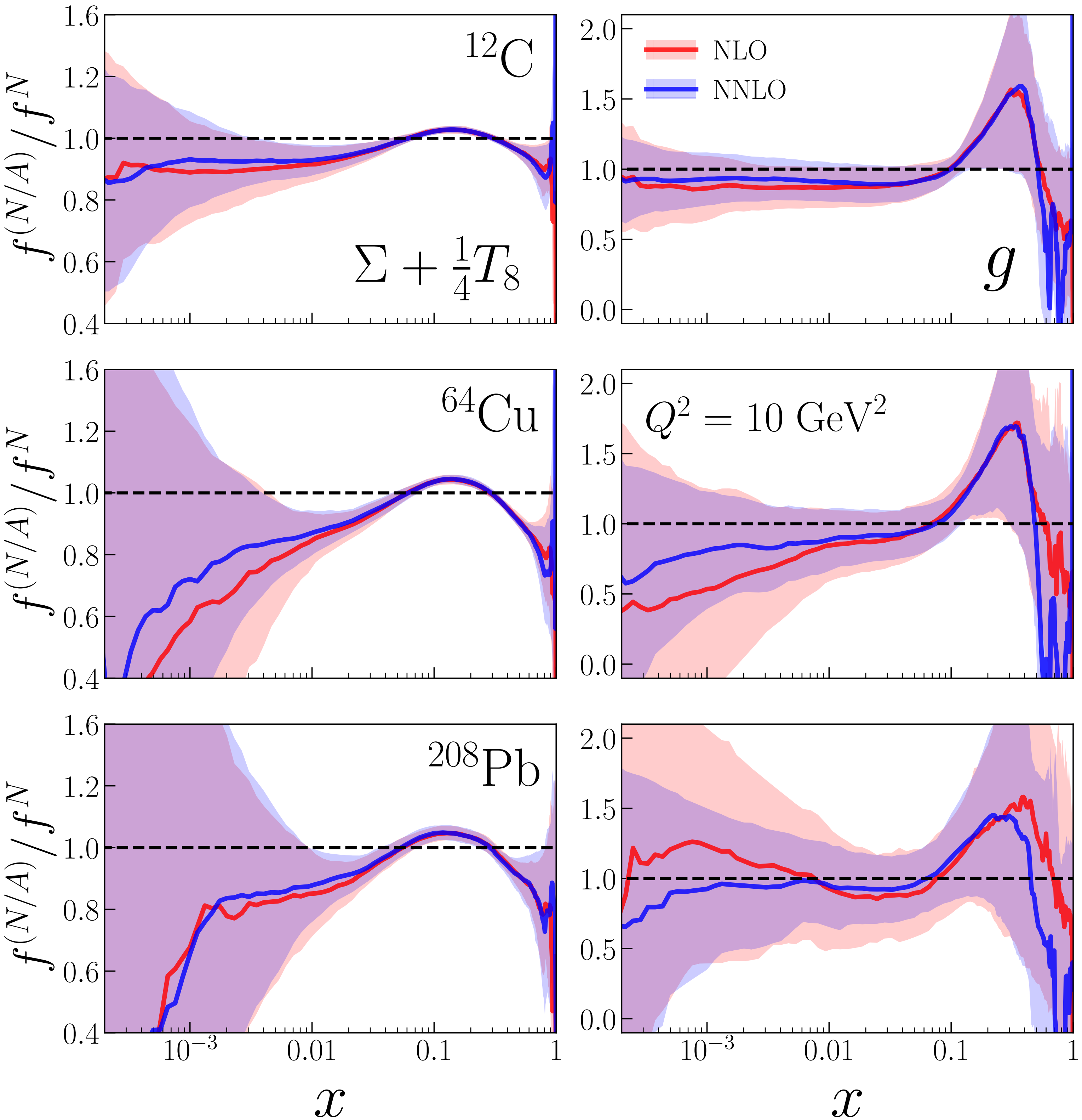
Q2 dependence



The sensitivity to nuclear modifications is reduced when going from low to high Q² in the small-x region

Large reduction in the gluon uncertainty at small-x due to DGLAP

NLO vs NNLO



Difference pronounced at large- and small- x

Reduction in uncertainty in NNLO

Parametrisation

nCTEQ15 [1509.00792]

$$x f_i^{p/A}(x, Q_0) = c_0\, x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4} x)^{c_5},$$
$$\text{for} \quad i = u_v, d_v, g, \bar{u} + \bar{d}, s + \bar{s}, s - \bar{s},$$

$$\frac{\bar{d}(x, Q_0)}{\bar{u}(x, Q_0)} = c_0\, x^{c_1} (1-x)^{c_2} + (1 + c_3 x)(1-x)^{c_4}.$$

$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right),$$
$$k = \{1, \ldots, 5\}.$$

EPPS16 [1612.05741]

$$f_i^{\mathrm{p}/A}(x, Q^2) = R_i^A(x, Q^2) f_i^{\mathrm{p}}(x, Q^2)$$

$$R_i^A(x, Q_0^2) = \begin{cases} a_0 + a_1(x - x_a)^2 & x \le x_a \\ b_0 + b_1 x^\alpha + b_2 x^{2\alpha} + b_3 x^{3\alpha} & x_a \le x \le x_e \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \le x \le 1, \end{cases}$$

nNNPDF1.0 [1904.00018]

$$xg(x, Q_0, A) = B_g x^{-\alpha_g}(1-x)^{\beta_g}\, \xi_3^{(3)}(x, A)\,,$$
$$x\Sigma(x, Q_0, A) = x^{-\alpha_\Sigma}(1-x)^{\beta_\sigma}\, \xi_1^{(3)}(x, A)\,,$$
$$xT_8(x, Q_0, A) = x^{-\alpha_{T_8}}(1-x)^{\beta_{T_8}}\, \xi_2^{(3)}(x, A)$$