DIS 2021

Can $\overline{\text{MS}}$ parton distributions be negative?

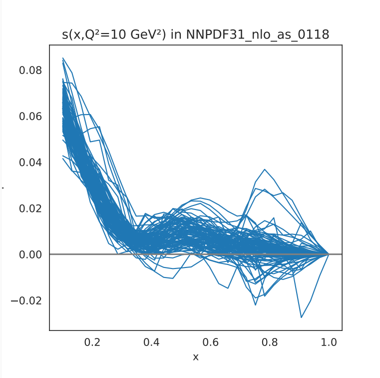
Alessandro Candido, Stefano Forte, and Felix Hekhorn in [JHEP 11 (2020) 129]

April, 2021

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No. 740006

Motivation

- PDFs at large x has been negative



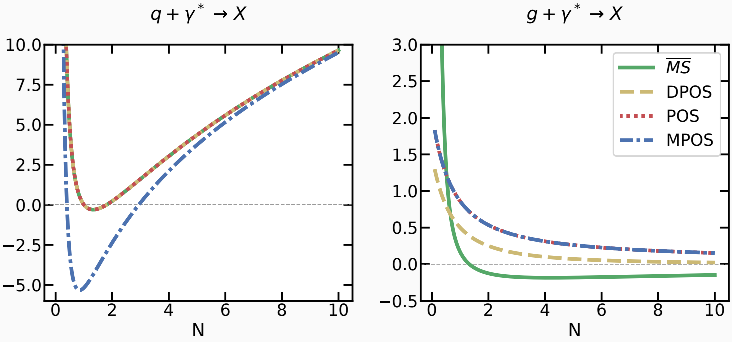
- we want physical xs: $\sigma \geq 0$
- we want safe BSM extrapolation

DIS

$$F(Q^2) = \sum_j c_j(Q^2) \otimes f_j(Q^2)$$

LO: Structure Function = PDF ✓

NLO:



DIS NLO Gluon Coefficient Function

- $C_g^{(1),\overline{\mathrm{MS}}}(z) = P_{qg}(z)\left(\ln\left(\frac{1-z}{z}\right) - 4\right) + 3\,T_R$

DIS NLO Gluon Coefficient Function

$$C_g^{(1),\text{bare}}(z, Q^2, \epsilon) = \frac{\Gamma(-\epsilon)\left(\frac{s}{4\pi\mu^2}\right)^{-\epsilon}\left[8P_{qg}(z) - 16T_R\epsilon(3-\epsilon(2-\epsilon))\right]}{16\pi(2-2\epsilon)\Gamma(3-2\epsilon)}$$

- $C_g^{(1),\overline{\text{MS}}}(z) = P_{qg}(z)\left(\ln\left(\frac{1-z}{z}\right) - 4\right) + 3T_R$

with $\mu_{\overline{\text{MS}}}^2 = Q^2$

$\frac{s}{4} = \frac{Q^2(1-z)}{4z}$

DIS NLO Gluon Coefficient Function

$$C_g^{(1),\text{bare}}(z, Q^2, \epsilon) = \frac{\Gamma(-\epsilon)\left(\frac{s}{4\pi\mu^2}\right)^{-\epsilon}\left[8P_{qg}(z) - 16T_R\epsilon(3-\epsilon(2-\epsilon))\right]}{16\pi(2-2\epsilon)\Gamma(3-2\epsilon)}$$

- $C_g^{(1),\overline{\mathrm{MS}}}(z) = P_{qg}(z)\left(\ln\left(\frac{1-z}{z}\right) - 4\right) + 3T_R$

  with $\mu_{\overline{\mathrm{MS}}}^2 = Q^2$
- $C_g^{(1),\mathrm{DPOS}}(z) = 3\left[T_R - P_{qg}(z)\right]$ ✓

  with $\mu_{\mathrm{DPOS}}^2 = (k_T^{max})^2 = \frac{s}{4} = \frac{Q^2(1-z)}{4z}$

DIS NLO Gluon Coefficient Function

$$C_g^{(1),\text{bare}}(z, Q^2, \epsilon) = \frac{\Gamma(-\epsilon)\left(\frac{s}{4\pi\mu^2}\right)^{-\epsilon}\left[8P_{qg}(z) - 16T_R\epsilon(3-\epsilon(2-\epsilon))\right]}{16\pi(2-2\epsilon)\Gamma(3-2\epsilon)}$$

- $C_g^{(1),\overline{\mathrm{MS}}}(z) = P_{qg}(z)\left(\ln\left(\frac{1-z}{z}\right) - 4\right) + 3T_R$

  with $\mu^2_{\overline{\mathrm{MS}}} = Q^2$
- $C_g^{(1),\mathrm{DPOS}}(z) = 3\left[T_R - P_{qg}(z)\right]$ ✓

  with $\mu^2_{\mathrm{DPOS}} = (k_T^{max})^2 = \frac{s}{4} = \frac{Q^2(1-z)}{4z}$
- $C_g^{(1),\mathrm{DIS}}(z) = 0$ ✓

$\Rightarrow$ Structure Function = PDF

DIS NLO Gluon Coefficient Function

$$C_g^{(1),\text{bare}}(z, Q^2, \epsilon) = \frac{\Gamma(-\epsilon)\left(\frac{s}{4\pi\mu^2}\right)^{-\epsilon}\left[8P_{qg}(z) - 16T_R\epsilon(3-\epsilon(2-\epsilon))\right]}{16\pi(2-2\epsilon)\Gamma(3-2\epsilon)}$$

- $C_g^{(1),\overline{\text{MS}}}(z) = P_{qg}(z)\left(\ln\left(\frac{1-z}{z}\right) - 4\right) + 3T_R$

  with $\mu_{\overline{\text{MS}}}^2 = Q^2$

- $C_g^{(1),\text{DPOS}}(z) = 3\left[T_R - P_{qg}(z)\right]$ ✓

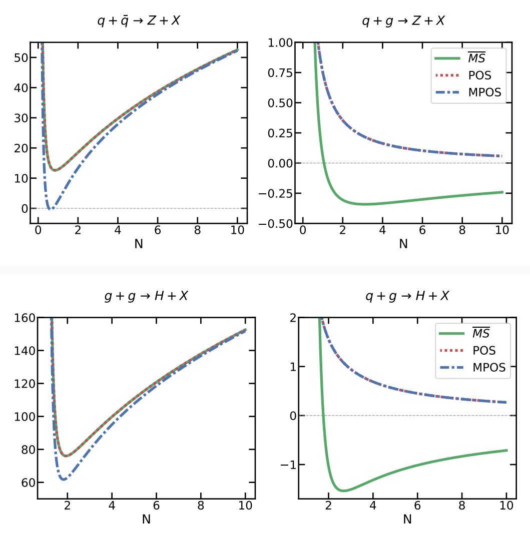
  with $\mu_{\text{DPOS}}^2 = (k_T^{max})^2 = \frac{s}{4} = \frac{Q^2(1-z)}{4z}$

- $C_g^{(1),\text{DIS}}(z) = 0$ ✓

$\Rightarrow$ Structure Function = PDF

$$K_{qg}^{\text{DPOS}}(z) = C_g^{(1),\overline{\text{MS}}}(z) - C_g^{(1),\text{DPOS}}(z)$$

Hadronic Processes



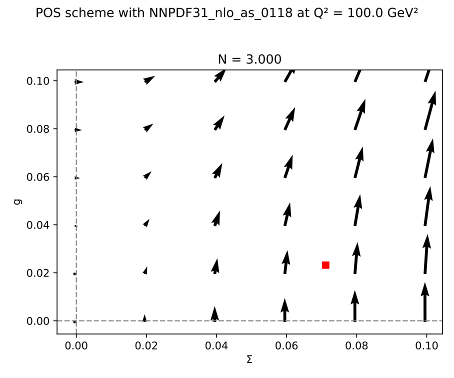
quark-initiated: Drell-Yan

gluon-initiated: Higgs

$$\mu^2_{\mathrm{POS}} = (k_T^{max})^2 = \frac{Q^2(1-z)^2}{4z} < \mu^2_{\mathrm{DPOS}} \quad \rightarrow K_{ij}^{\mathrm{POS}}$$

Scheme Change

$$f^{\overline{\mathrm{MS}}}(Q^2) = \left[\mathbb{I} + \frac{\alpha_s}{2\pi} K^{\mathrm{POS}} \otimes\right]^{-1} f^{\mathrm{POS}}(Q^2)$$



for $z < 1$ use perturbativity, for $z \to 1$ use *exact* transformation
$\Rightarrow \overline{\mathrm{MS}} > 0$

Usage

To obtain a physical xs is positivity **sufficient?** no!

or even **necessary?** no!

$\Rightarrow$ but we can prove they *are* positive

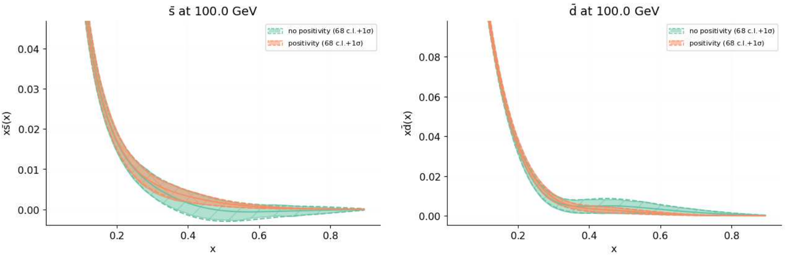
$\Rightarrow$ and so it adds a cut in PDF space!

Positivity - Implementation

Quarks, anti-quarks and gluon $\overline{MS}$ PDFs q_k have to be positive: we add a term in the χ^2 penalizing negative distributions

$$\chi^2_{tot} = \chi^2_{exp} + \sum_k \chi^2_{k,\text{pos}}\,,$$

$$\chi^2_{k,pos} = \Lambda_k \sum_i \Theta\left(-q_k\left(x_i, Q^2\right)\right)\,, \quad \text{with} \quad \Theta(t) = \begin{cases} t & \text{if } t > 0 \\ 0 & \text{if } t < 0 \end{cases}\,.$$



Thanks for your attention

Overall structure of the argument

1. $\overline{\text{MS}}$ partonic cross-sections are not positive, we found why - define a factorization scheme that prevents it
2. enforce momentum sum rules by slightly modifying the former, without affecting positivity
3. considering the scheme change we prove that positivity of PDFs in this last scheme proves positivity of PDFs in $\overline{\text{MS}}$

POS scheme

$$C^q{}_q^{(1)\,\mathrm{POS}}(x) = C^q{}_q^{(1)\,\overline{\mathrm{MS}}}(x)\,, \tag{1}$$

$$C^q{}_g^{(1)\,\mathrm{POS}}(x) = C^q{}_g^{(1)\,\overline{\mathrm{MS}}}(x) - K_{qg}^{\mathrm{POS}}(x)\,, \tag{2}$$

$$K_{qg}^{\mathrm{POS}}(x) = P_{qg}(x)\left[\log\left(\frac{(1-x)^2}{x}\right) - 1\right] \tag{3}$$

$$C^g{}_g^{(1)\,\mathrm{POS}}(x) = C^g{}_g^{(1)\,\overline{\mathrm{MS}}}(x)\,, \tag{4}$$

$$C^g{}_q^{(1)\,\mathrm{POS}}(x) = C^q{}_g^{(1)\,\overline{\mathrm{MS}}}(x) - K_{gq}^{\mathrm{POS}}(x)\,, \tag{5}$$

$$K_{gq}^{\mathrm{POS}}(x) = P_{gq}(x)\left[\log\left(\frac{(1-x)^2}{x}\right) - 1\right] \tag{6}$$

MPOS scheme

Impose momentum conservation

$$K_{qq} + K_{gq} = 2n_f K_{qg} + K_{gg}\Big|_{N=2} = 0 \tag{7}$$

using a “soft” enforcement function

$$f^{\mathrm{MOM}}(z) = 60z^2(1-z)^2\,, \quad \text{with } f^{\mathrm{MOM}}(N=2) = 1 \tag{8}$$

and so

$$K_{qq}^{\mathrm{MPOS}}(z) = -f^{\mathrm{MOM}}(z) K_{gq}^{\mathrm{POS}}\Big|_{N=2}\,, \tag{9}$$

$$K_{qg}^{\mathrm{MPOS}}(z) = K_{qg}^{\mathrm{POS}}(z)\,, \tag{10}$$

$$K_{gq}^{\mathrm{MPOS}}(z) = K_{gq}^{\mathrm{POS}}(z)\,, \tag{11}$$

$$K_{gg}^{\mathrm{MPOS}}(z) = -2n_f f^{\mathrm{MOM}}(z) K_{qg}^{\mathrm{POS}}\Big|_{N=2}\,. \tag{12}$$

but this does not affect the positivity

Inverting in the threshold limit

$$\left[q^{\mathrm{NS}}\right]^{\mathrm{DIS}}(\xi, Q^2) = \left[1 + \frac{\alpha_s}{2\pi}\Delta_q^{(1)\overline{\mathrm{MS}}} + \frac{\alpha_s}{2\pi} 2C_F \left[\frac{\ln(1-z)}{1-z}\right]_+ \otimes\right] \left[q^{\mathrm{NS}}\right]^{\overline{\mathrm{MS}}}(Q^2) + \mathrm{NLL}(1-\xi) \tag{13}$$

$$= \left(1 + \frac{\alpha_s}{2\pi}\Delta_q^{(1)\overline{\mathrm{MS}}}\right) \left[1 + c_{\mathrm{LL}} \left[\frac{\ln(1-z)}{1-z}\right]_+ \otimes\right] \left[q^{\mathrm{NS}}\right]^{\overline{\mathrm{MS}}}(Q^2) + \mathrm{NLL}(1-\xi) \tag{14}$$

$$c_{\mathrm{LL}} = \frac{\frac{\alpha_s}{2\pi} 2C_F}{1 + \frac{\alpha_s}{2\pi}\Delta_q^{(1)\overline{\mathrm{MS}}}} \tag{15}$$

$$\left[q^{\mathrm{NS}}\right]^{\overline{\mathrm{MS}}}(\xi, Q^2) = \frac{1}{1 + \frac{\alpha_s}{2\pi}\Delta_q^{(1)\overline{\mathrm{MS}}}} \times \left[1 - c_{\mathrm{LL}} \left[\frac{\ln(1-z)}{\left[1 + c_{\mathrm{LL}} \ln^2(1-z)/2\right]^2} \frac{1}{1-z}\right]_+ \otimes\right] \left[q^{\mathrm{NS}}\right]^{\mathrm{DIS}}(Q^2) + \mathrm{NLL}(1-\xi)\,. \tag{16}$$

Inverting the scheme change operator

Perturbative inverse:

$$\left[\mathbb{I} + \frac{\alpha_s}{2\pi} K^{\mathrm{POS}}\right]^{-1} = \left[\mathbb{I} - \frac{\alpha_s}{2\pi} K^{\mathrm{POS}}\right] \tag{17}$$

Exact inverse:

$$\left[\mathbb{I} + \frac{\alpha_s}{2\pi} K^{\mathrm{POS}}\right]^{-1} = \frac{1}{1 - \left(\frac{\alpha_s}{2\pi}\right)^2 K_{qg} K_{gq}} \left[\mathbb{I} - \frac{\alpha_s}{2\pi} K^{\mathrm{POS}}\right], \tag{18}$$

due to K^{POS} beeing purely off-diagonal