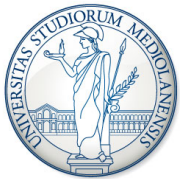


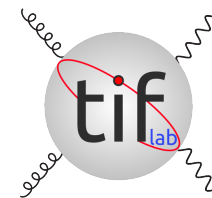


PDFs WITH 1% ACCURACY

STEFANO FORTE
UNIVERSITÀ DI MILANO & INFN



UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA



IRN TERASCALE MEETING

LPC-CLERMONT, NOV. 22, 2021

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 740006



PDFs WITH 1% ACCURACY

THE NNPDF COLLABORATION

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AMSTERDAM-CAMBRIDGE-EDINBURGH-INFN-MILAN-NIKHEF-SINGAPORE



IRN TERASCALE MEETING

LPC-CLERMONT, Nov. 22, 2021

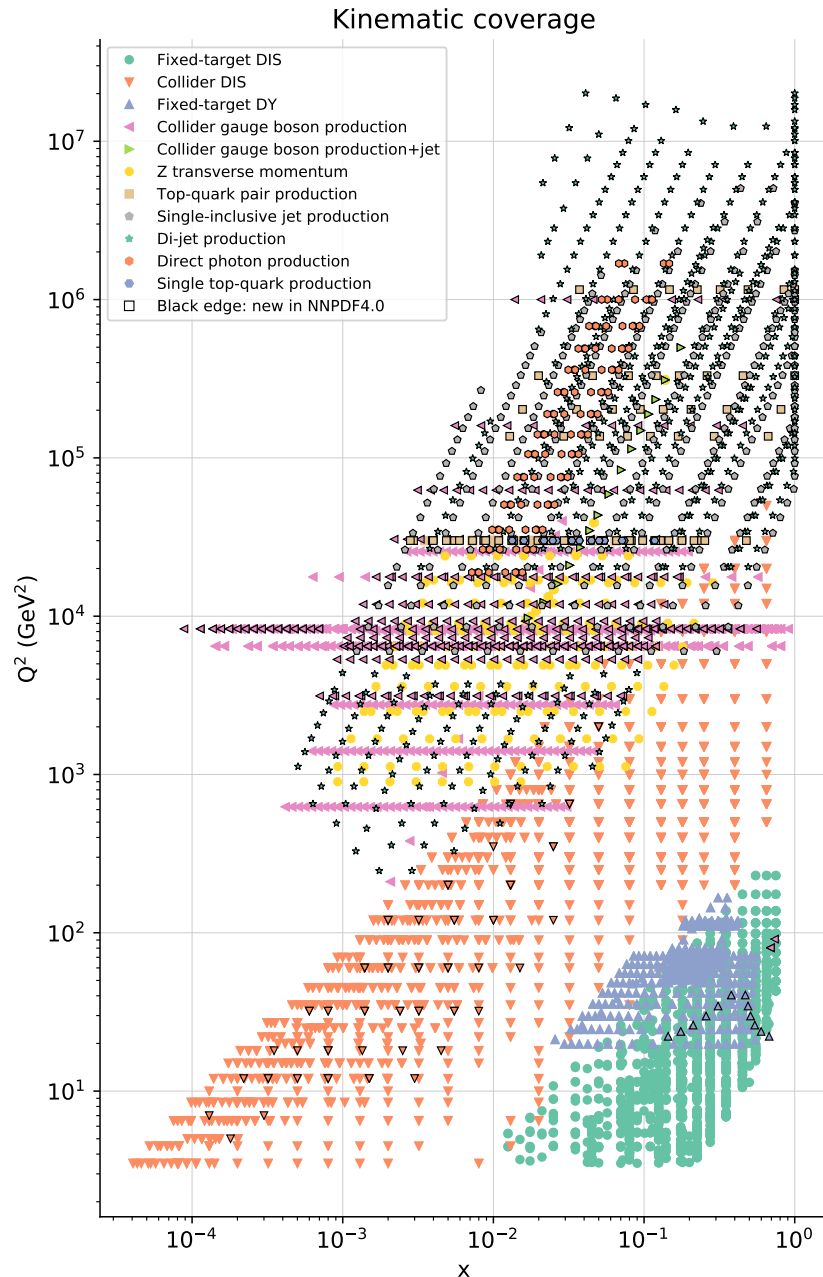
SUMMARY

NNPDF4.0

- DATA
- THEORY
- METHODOLOGY
- DATASET SELECTION
- UNCERTAINTIES
- VALIDATION
- STABILITY
- DELIVERY

DATA

DATA



- ABOUT **50 NEW DATASETS** & 400 EXTRA DATAPOINTS
- FULL DIS AND FT DY DATASET
 - AS IN NNPDF3.1: FINAL HERA, NMC, BCDMS, CHORUS, NuTeV
 - NOW ALSO **NOMAD NEUTRINO**
 - **SEAQUEST DY**
- FULL 7 TeV AND 8 TeV DATASET & EXTENSIVE USE OF **13 TeV** DATA:
 - W , Z PRODUCTION: RAPIDITY DISTRIBUTIONS, ASYMMETRIES, Z p_T DISTRIBUTIONS
 - TOP PAIR PRODUCTION: ALL AVAILABLE DISTRIBUTIONS
 - SINGLE-INCLUSIVE JETS
- SEVERAL **NEW PROCESSES**:
 - PROMPT PHOTON
 - SINGLE TOP
 - DIJETS
 - HERA JETS

TYPICAL PDF DETERMINATION: $\sim 10^{10}$ PREDICTIONS NEEDED

LHC DATA

LHCb

Data set	NNPDF4.0	NNPDF3.1	ABMP16	CT18	MSHT20
LHCb Z 940 pb	✓	✓	✗	✗	✓
LHCb $Z \rightarrow ee$ 2 fb	✓	✓	✓	✓	✓
LHCb $W, Z \rightarrow \mu$ 7 TeV	✓	✓	✓	✓	✓
LHCb $W, Z \rightarrow \mu$ 8 TeV	✓	✓	✓	✓	✓
LHCb $Z \rightarrow \mu\mu, ee$ 13 TeV	✓	✗	✗	✗	✗

ATLAS

Data set	NNPDF4.0	NNPDF3.1	ABMP16	CT18	MSHT20
ATLAS W, Z 7 TeV (2010)	✓	✓	✓	✓	✓
ATLAS W, Z 7 TeV (2011)	✓	✓	✗	✓	✓
ATLAS low-mass DY 7 TeV	✓	✓	✗	✗	✗
ATLAS high-mass DY 7 TeV	✓	✓	✗	✗	✓
ATLAS W 8 TeV	✓	✗	✗	✗	✓
ATLAS DY 2D 8 TeV	✓	✗	✗	✗	✓
ATLAS high-mass DY 2D 8 TeV	✓	✗	✗	✗	✓
ATLAS $\sigma_{W,Z}$ 13 TeV	✓	✗	✓	✗	✗
ATLAS W^+ +jet 8 TeV	✓	✗	✗	✗	✓
ATLAS $Z p_T$ 8 TeV	✓	✓	✗	✓	✓
ATLAS σ_{tt}^{tot} 7, 8 TeV	✓	✓	✓	✗	✗
ATLAS σ_{tt}^{tot} 13 TeV	✓	✓	✓	✗	✗
ATLAS $t\bar{t}$ lepton+jets 8 TeV	✓	✓	✗	✓	✓
ATLAS $t\bar{t}$ dilepton 8 TeV	✓	✗	✗	✗	✓
ATLAS single-inclusive jets 7 TeV, R=0.6	✗	✓	✗	✓	✓
ATLAS single-inclusive jets 8 TeV, R=0.6	✓	✗	✗	✗	✗
ATLAS dijets 7 TeV, R=0.6	✓	✗	✗	✗	✗
ATLAS direct photon production 13 TeV	✓	✗	✗	✗	✗
ATLAS single top R_t 7, 8, 13 TeV	✓	✗	✓	✗	✗
ATLAS single top diff. 7, 8 TeV	✓	✗	✗	✗	✗
ATLAS single top diff. 8 TeV	✓	✗	✗	✗	✗

- CUTOFF DATE AROUND 06/2020
- **DIJETS NOW INCLUDED** ALONG WITH JETS
CANNOT INCLUDE SIMULTANEOUSLY FROM SAME
UNDERLYING DATASET

CMS

Data set	NNPDF4.0	NNPDF3.1	ABMP16	CT18	MSHT20
CMS W electron asymmetry 7 TeV	✓	✓	✗	✓	✓
CMS W muon asymmetry 7 TeV	✓	✓	✓	✓	✗
CMS Drell-Yan 2D 7 TeV	✓	✓	✗	✗	✓
CMS W rapidity 8 TeV	✓	✓	✓	✓	✓
CMS $Z p_T$ 8 TeV	✓	✓	✗	✓	✗
CMS $W + c$ 7 TeV	✓	✓	✗	✗	✓
CMS $W + c$ 13 TeV	✓	✗	✗	✗	✗
CMS single-inclusive jets 2.76 TeV	✗	✓	✗	✗	✓
CMS single-inclusive jets 7 TeV	✗	✓	✗	✓	✓
CMS dijets 7 TeV	✓	✗	✗	✗	✗
CMS single-inclusive jets 8 TeV	✓	✗	✗	✓	✓
CMS 3D dijets 8 TeV	✗	✗	✗	✗	✗
CMS σ_{tt}^{tot} 5 TeV	✓	✗	✓	✗	✗
CMS σ_{tt}^{tot} 7, 8 TeV	✓	✓	✓	✗	✓
CMS σ_{tt}^{tot} 13 TeV	✓	✓	✓	✗	✗
CMS $t\bar{t}$ lepton+jets 8 TeV	✓	✓	✗	✗	✓
CMS $t\bar{t}$ 2D dilepton 8 TeV	✓	✗	✗	✓	✓
CMS $t\bar{t}$ lepton+jet 13 TeV	✓	✗	✗	✗	✗
CMS $t\bar{t}$ dilepton 13 TeV	✓	✗	✗	✗	✗
CMS single top $\sigma_t + \sigma_{\bar{t}}$ 7 TeV	✓	✗	✓	✗	✗
CMS single top R_t 8, 13 TeV	✓	✗	✓	✗	✗

THEORY

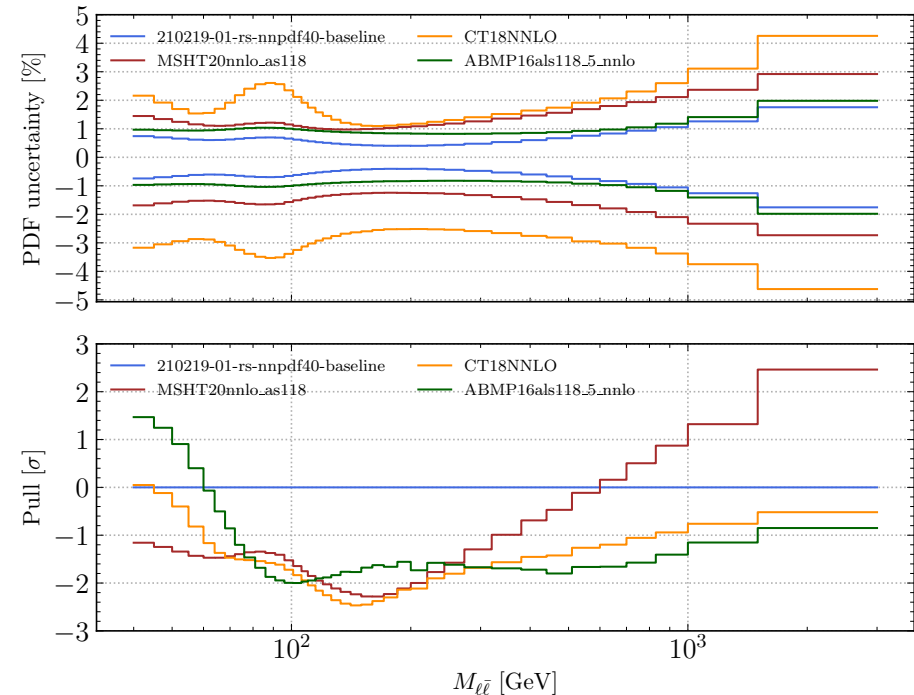
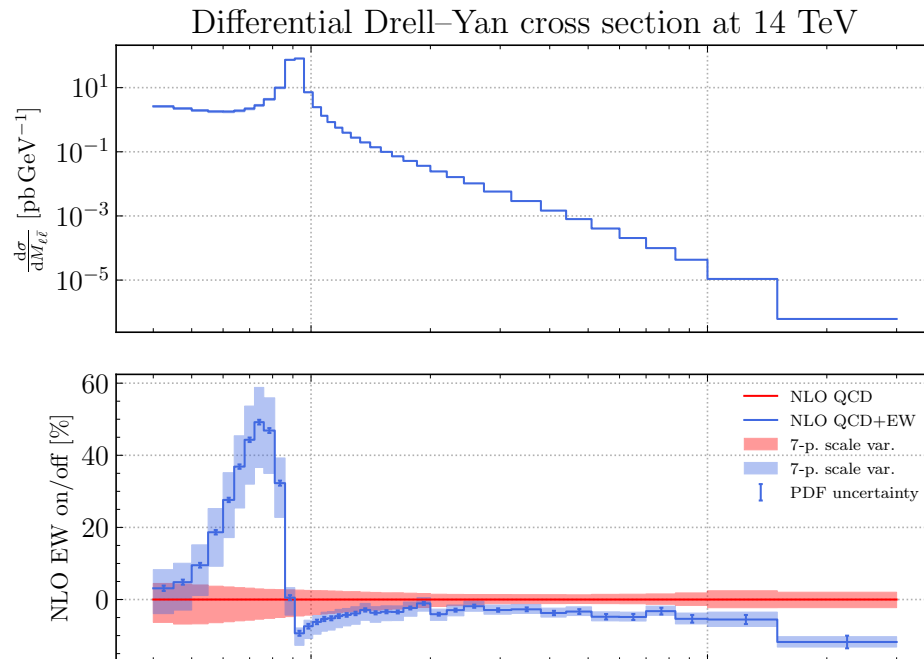
ELECTROWEAK CORRECTIONS

- PineAPPL **FAST INTERFACE** TO Madgraph5_aMC@NLO AVAILABLE
(Schwan, Carrazza, Nocera, Zaro 2020)
⇒ **FULL NLO EW+QCD** POSSIBLE
- **DATA W/O FSR & PHOTON-INITIATED** SUBTRACTION OFTEN **NOT AVAILABLE**
- **CURRENTLY USED FOR DATASET SELECTION:**
⇒ DISCARDED IF EW CORRNS EXCEED THRESHOLD

EXAMPLE: **DRELL-YAN AT 14 TeV**

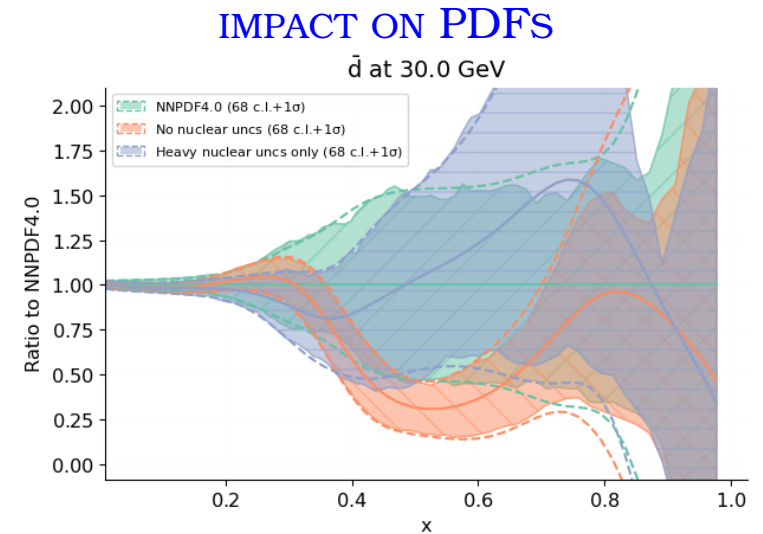
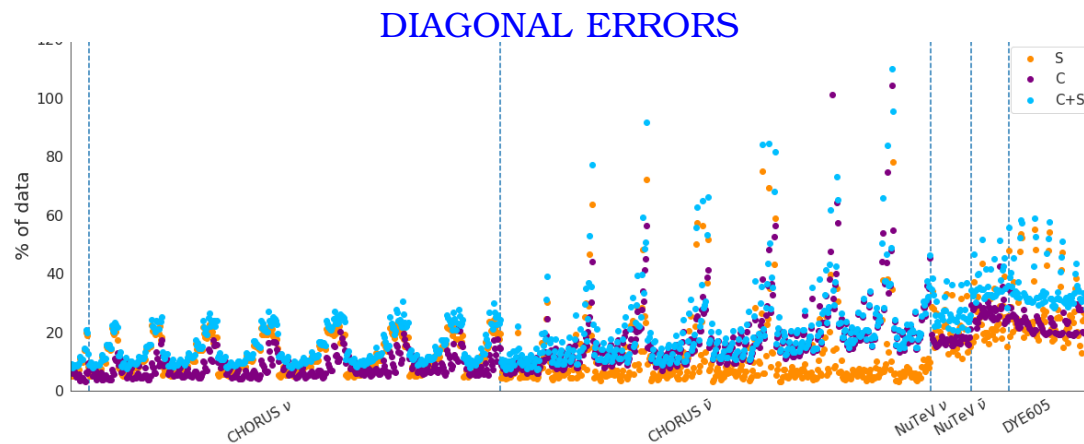
QCD+EW vs. QCD

PDF UNCERTAINTIES



NUCLEAR CORRECTIONS

- INCLUDED AS CONTRIBUTION TO COVARIANCE MATRIX (FULLY CORRELATED)
(Ball, Nocera, Pearson, 2019)
- COMPUTED AS SHIFT BETWEEN NUCLEAR & STANDARD PDF
- DEUTERIUM PDF DETERMINED FROM SELF-CONSISTENT NNPDF FIT
(Ball, Nocera, Pearson, 2019)
- NUCLEAR PDFs FROM nNNPDF2.0 (Abdul Khalek, Ethier, Rojo, van Weelden, 2020)

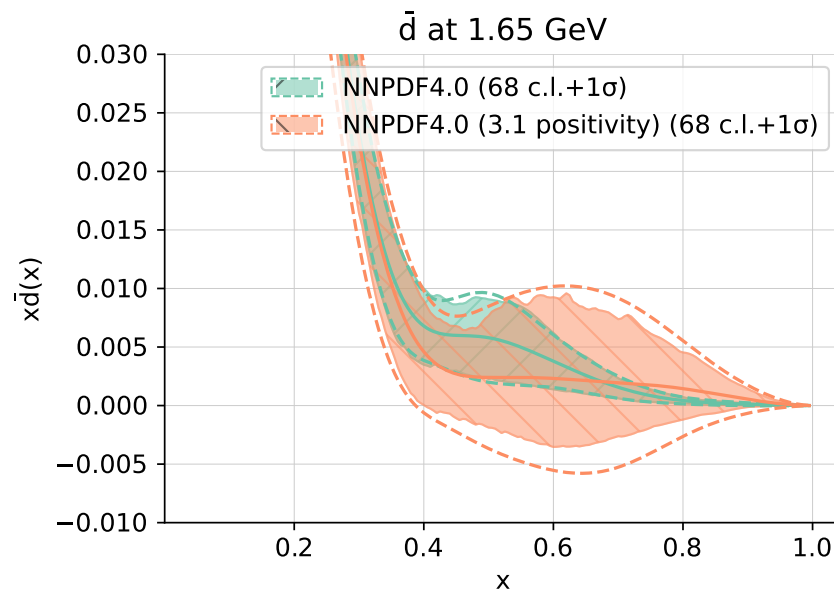


PDF POSITIVITY & INTEGRABILITY

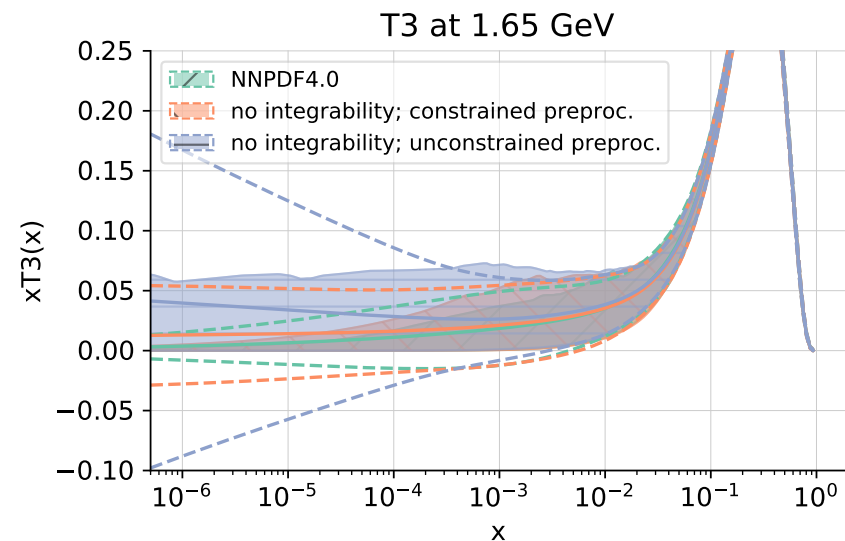
- $\overline{\text{MS}}$ PDFs ARE **NON-NEGATIVE!** (Candido, Hekhorn, Forte, 2020)
- PDF **POSITIVITY IMPOSED** (PREVIOUSLY: OBSERVABLE POSITIVITY)
 $\Rightarrow$ **SMALLER** LARGE x UNCERTAINTIES
- SEA **NONSINGLET** COMBINATIONS **INTEGRABLE:**
 GOTTFRIED $u + \bar{u} - (d + \bar{d})$
 STRANGENESS $u + \bar{u} + (d + \bar{d}) - 2(s + \bar{s})$
 $\Rightarrow$ **SMALLER** SMALL x UNCERTAINTIES

IMPACT ON PDFs

POS.: ANTIDOWN



INTEGR.: GOTTFRIED SR



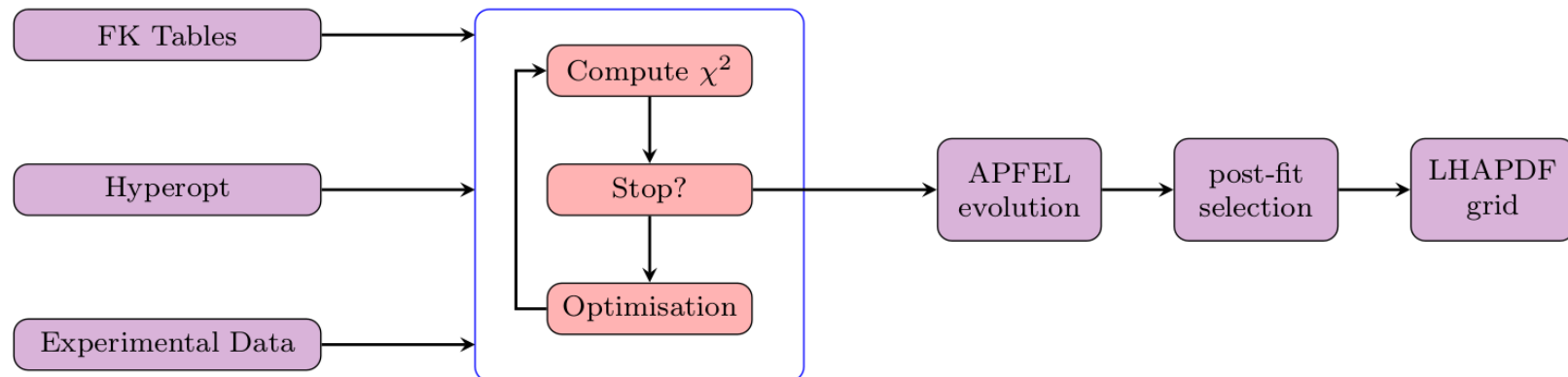
METHODOLOGY

THE NNPDF CODE STRUCTURE

- MODULAR PYTHON-BASED CODE
- HIGH DEGREE PARALLELIZATION & HARDWARE ACCELERATION

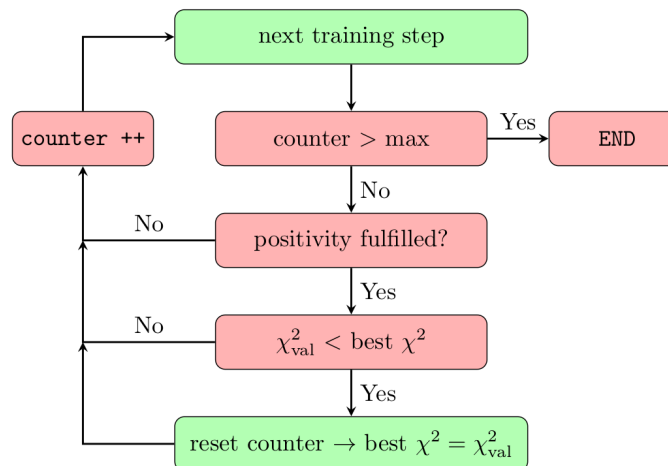
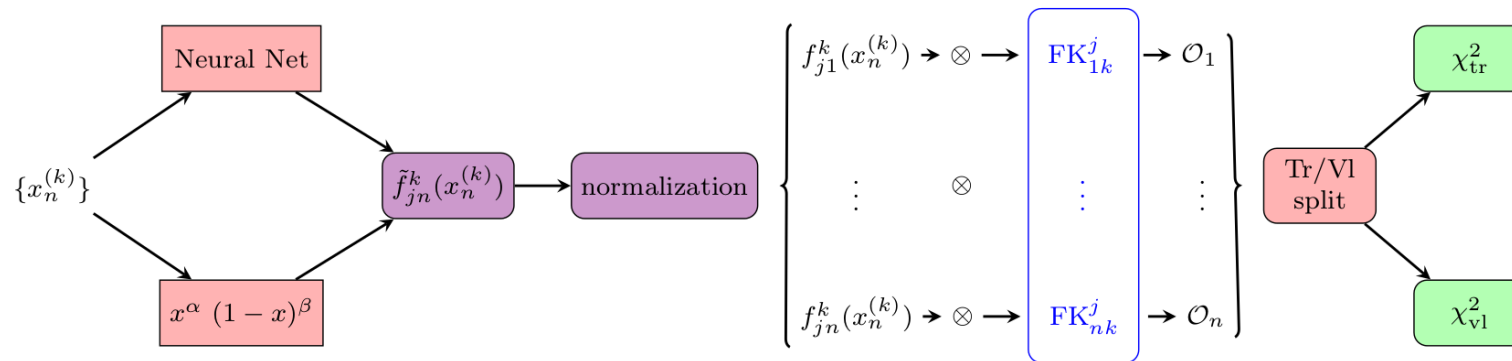
AVERAGE FITTING TIME PER REPLICA AND USE OF RESOURCES
SAME DATASET FOR OLD AND NEW METHODOLOGIES IN CPU AND GPU
CPU: INTEL(R) CORE(TM) I7-4770 AT 3.40GHz; GPU: NVIDIA TITAN V

	NNPDF31 CODEBASE	NNPDF40 CODEBASE IN CPU	NNPDF40 CODEBASE IN GPU
TIME	15.2 H.	38 ± 5 MIN.	6.6 MIN.
RAM USE	1.5 GB	6.1 GB	NA



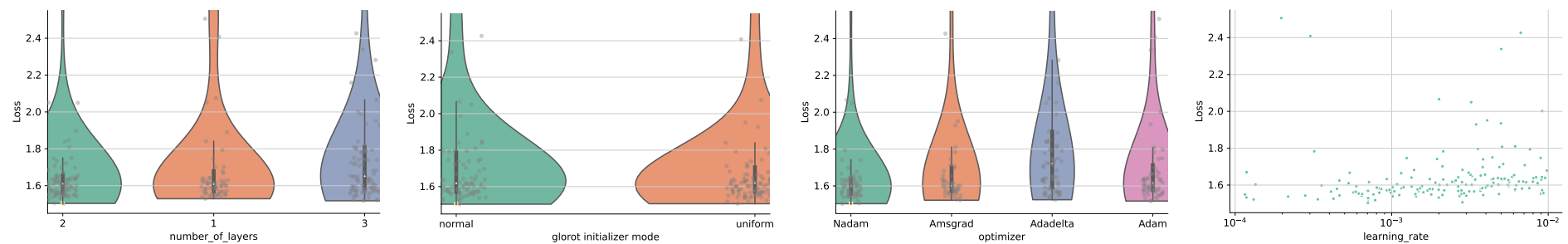
MINIMIZATION AND CROSS-VALIDATION

- DATA REPLICAS $\Rightarrow$ PDF REPLICAS
- EACH PDF REPLICA: PREPROCESSED NEURAL NET
- NEURAL NET $\Rightarrow$ OBSERVABLES
- RANDOM TRAINING-VALIDATION SPLIT, χ^2 TO TRAINING DATA REPLICAS MINIMIZED
- TRAINING STOPS IF VALIDATION χ^2 GROWS FOR A WHILE (PATIENCE)
- LOWEST VALIDATION $\chi^2 \Rightarrow$ OPTIMAL FIT

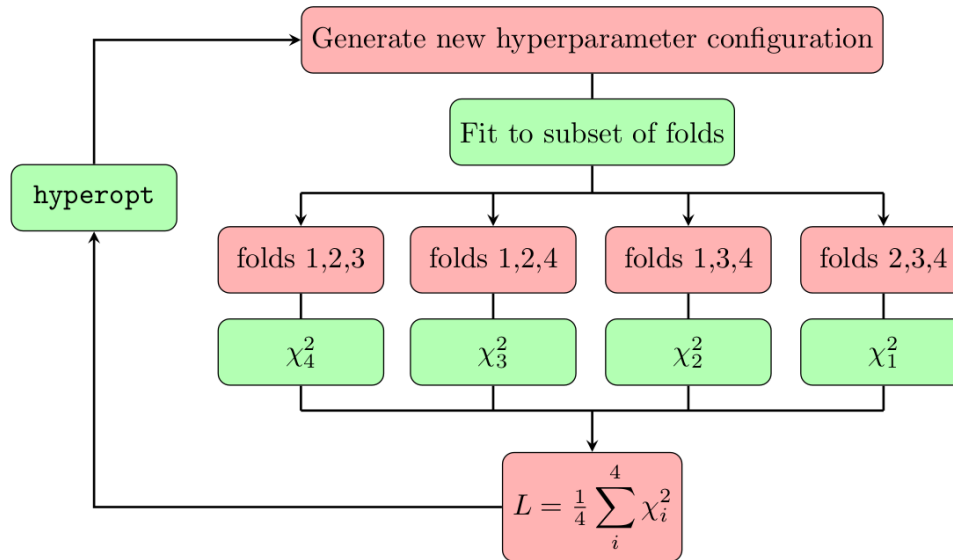


HYPEROPTIMIZATION

- PARAMETRIZATION AND MINIMIZATION PARAMETERS VARIED
- SCAN OF PARAMETER SPACE
- BAYESIAN UPDATING LEADS TO BEST METHODOLOGY



K-FOLDING



- HYPEROPTIMIZATION $\Rightarrow$ OVERFITTING (χ^2 TOO GOOD)
- CHECK GENERALIZATION POWER: K-FOLDING
 - DIVIDE DATA IN FOLDS
 - EXCLUDE ONE FOLD IN TURN FROM FIT
 - OPTIMIZE ON THE χ^2 OF THE EXCLUDED FOLDS
 - BEST AVERAGE OR BEST WORST

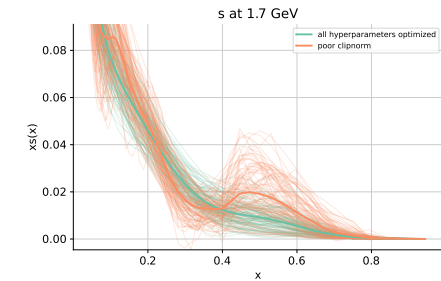
Fold 1		
CHORUS σ_{CC}^{ν}	HERA I+II inc NC e^+p 920 GeV	BCDMS p
LHCb Z 940 pb	ATLAS W, Z 7 TeV 2010	CMS Z p_T 8 TeV (p_T^H, y_H)
DY E605 σ_{DY}^{ν}	CMS Drell-Yan 2D 7 TeV 2011	CMS 3D dijets 8 TeV
ATLAS single- t y (normalised)	ATLAS single top R_t 7 TeV	CMS $t\bar{t}$ rapidity $y_{t\bar{t}}$
CMS single top R_t 8 TeV		

Fold 2		
HERA I+II inc CC e^-p	HERA I+II inc NC e^+p 460 GeV	HERA comb. σ_{bb}^{ed}
NMC p	NuTeV σ_e^{ν}	LHCb $Z \rightarrow ee$ 2 fb
CMS W asymmetry 840 pb	ATLAS Z p_T 8 TeV (p_T^H, M_H)	D0 $W \rightarrow \mu\nu$ asymmetry
DY E886 σ_{DY}^{ν}	ATLAS direct photon 13 TeV	ATLAS dijets 7 TeV, R=0.6
ATLAS single antitop y (normalised)	CMS $\sigma_{H^0}^{\text{tt}}$	CMS single top $\sigma_t + \sigma_{\bar{t}}$ 7 TeV

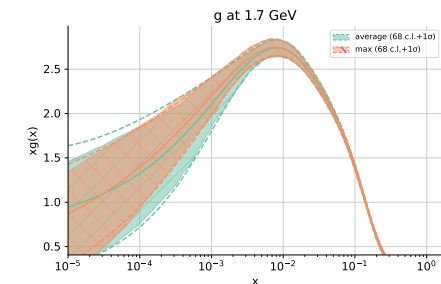
Fold 3		
HERA I+II inc CC e^+p	HERA I+II inc NC e^+p 575 GeV	NMC d/p
NuTeV σ_e^{ν}	LHCb $W, Z \rightarrow \mu$ 7 TeV	LHCb $Z \rightarrow ee$
ATLAS W, Z 7 TeV 2011 Central selection	ATLAS $W^+ + \text{jet}$ 8 TeV	ATLAS HM DY 7 TeV
CMS W asymmetry 4.7 fb	DYE 866 $\sigma_{DY}^{\nu}/\sigma_{DY}^p$	CDF Z rapidity (new)
ATLAS $\sigma_{H^0}^{\text{tt}}$	ATLAS single top y_t (normalised)	CMS $\sigma_{H^0}^{\text{tt}}$ 5 TeV
CMS $t\bar{t}$ double diff. $(m_{t\bar{t}}, y_t)$		

Fold 4		
CHORUS σ_{CC}^{ν}	HERA I+II inc NC e^+p 820 GeV	LHCb $W, Z \rightarrow \mu$ 8 TeV
LHCb $Z \rightarrow \mu\mu$	ATLAS W, Z 7 TeV 2011 Fwd	ATLAS $W^- + \text{jet}$ 8 TeV
ATLAS low-mass DY 2011	ATLAS Z p_T 8 TeV (p_T^H, y_H)	CMS W rapidity 8 TeV
D0 Z rapidity	CMS dijets 7 TeV	ATLAS single top y_t (normalised)
ATLAS single top R_t 13 TeV	CMS single top R_t 13 TeV	

NO K-FOLDING



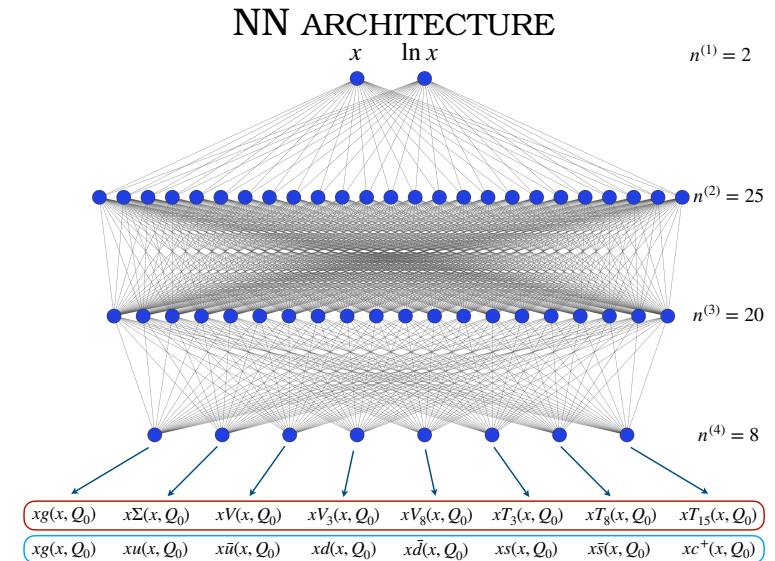
K-FOLDING VARIATION



THE ML METHODOLOGY

HYPEROPTIMIZED PARAMETERS

Parameter	NNPDF4.0	L as in Eq. (3.21)	Flavour basis Eq. (3.2)
Architecture	25-20-8	70-50-8	7-26-27-8
Activation function	hyperbolic tangent	hyperbolic tangent	sigmoid
Initializer	glorot_normal	glorot_uniform	glorot_normal
Optimizer	Nadam	Adadelta	Nadam
Clipnorm	6.0×10^{-6}	5.2×10^{-2}	2.3×10^{-5}
Learning rate	2.6×10^{-3}	2.5×10^{-1}	2.6×10^{-3}
Maximum # epochs	17×10^3	45×10^3	45×10^3
Stopping patience	10% of max epochs	12% of max epochs	16% of max epochs
Initial positivity $\Lambda^{(\text{pos})}$	185	106	2
Initial integrability $\Lambda^{(\text{int})}$	10	10	10



- HYPEROPT ADAPTS TO EXTERNAL CHOICES (E.G. PARAMETRIZATION BASIS)
- SIMILAR RESULTS CAN BE OBTAINED WITH RATHER DIFFERENT SETTINGS
- ~ 800 FREE PARAMETERS

DATASET SELECTION

DATA “TENSION” PROBLEMATIC DATA

$$\frac{\chi^2 - 1}{\sigma_{\chi^2}} \gg 1 \Leftrightarrow \text{POOR FIT QUALITY}$$

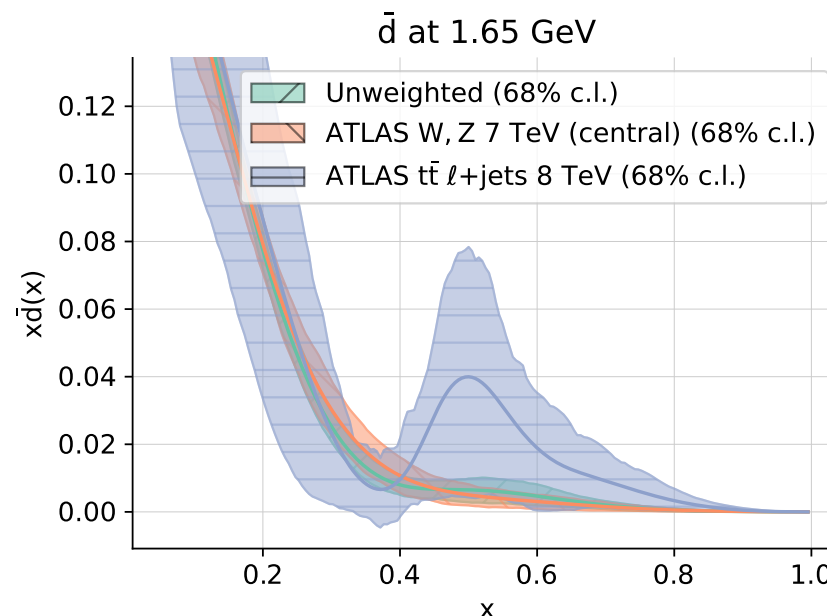
- MISSING HIGHER-ORDER CORRECTIONS
- NO RESUMMATION WHERE NEEDED
- ILL-CONDITIONED COVARIANCE MATRIX
- EXPERIMENTAL ISSUES

THE WEIGHTED FIT METHOD

- **FLAG** PROBLEMATIC DATASETS:
 - LARGE χ^2
 - LARGE FROBENIUS NUMBER OF COVMAT (EIGEN-VALUES TOO SMALL)
- **REPEAT GLOBAL** FIT WITH **LARGE WEIGHT** GIVEN TO EACH PROBLEMATIC DATASET IN TURN

- χ^2 OF DATASET
 - **UNCHANGED** $\Rightarrow$ INTERNAL INCONSISTENCY
 - **DECREASES** $\Rightarrow$ TENSION
- GLOBAL χ^2
 - **UNCHANGED** $\Rightarrow$ CONSISTENT, **KEEP**
 - **INCREASES** $\Rightarrow$ INCONSISTENT, **DISCARD**

INCONSISTENT!



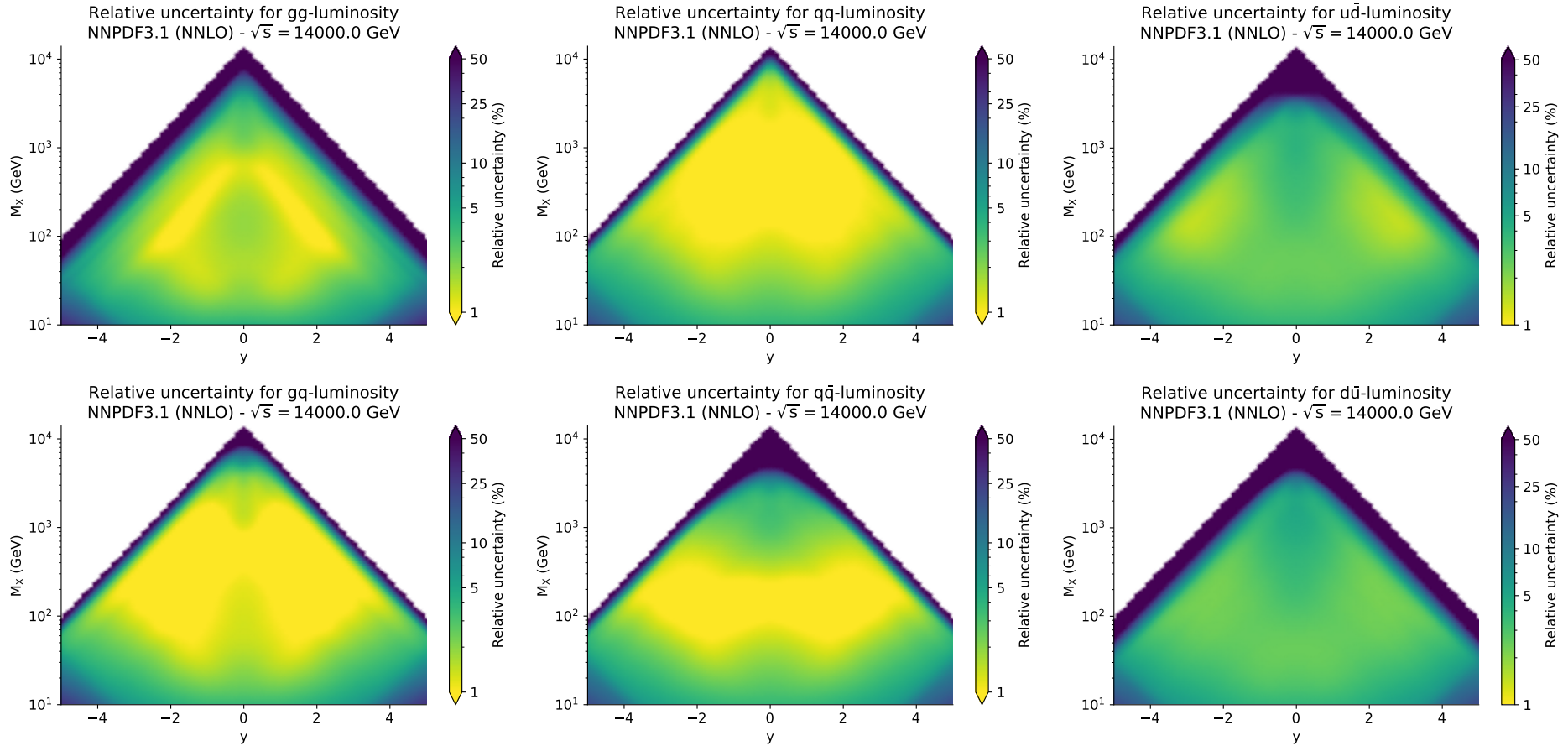
UNCERTAINTIES

UNCERTAINTIES: FROM NNPDF3.1...

GLUON

SINGLET

FLAVORS



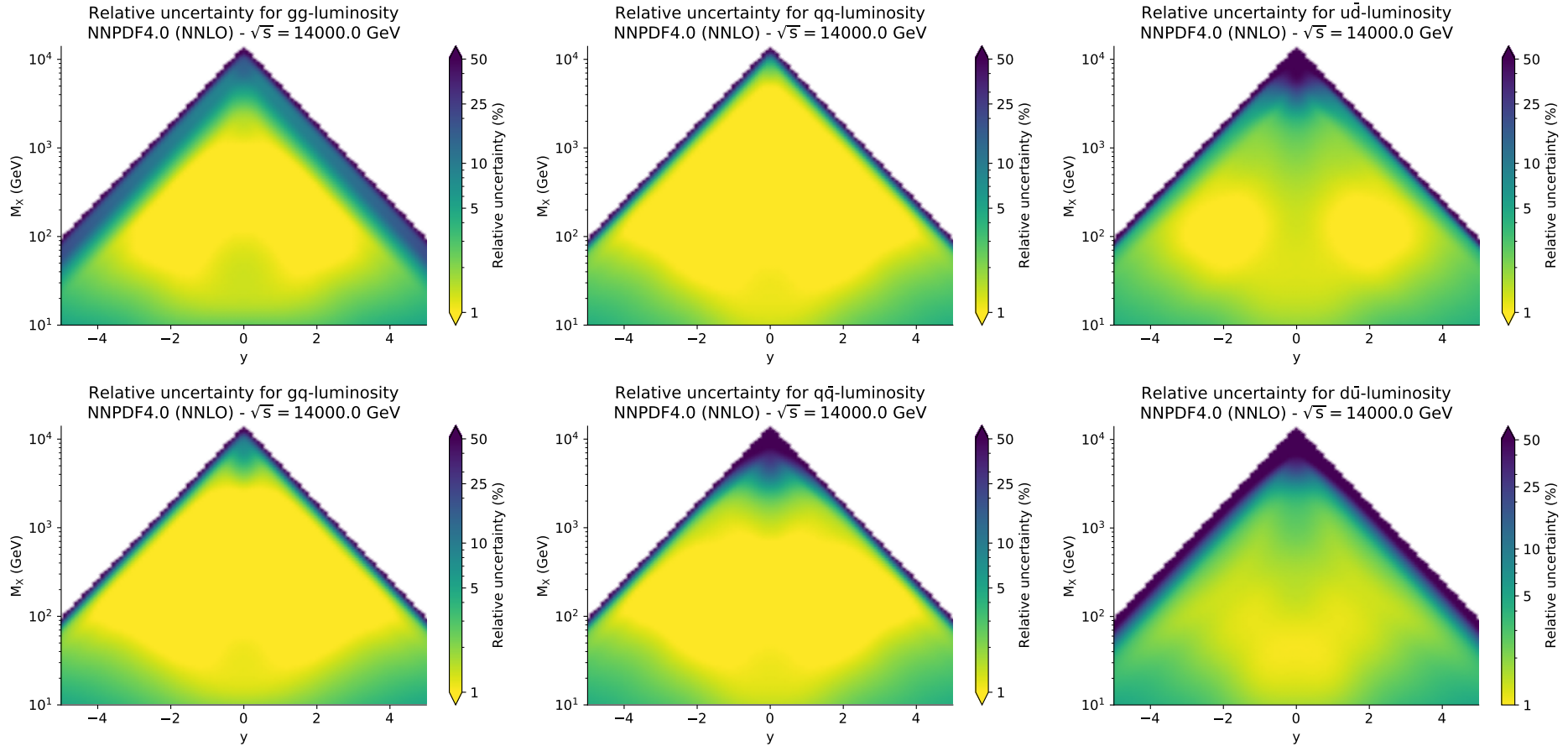
- TYPICAL UNCERTAINTIES IN DATA REGION: SINGLET $\sim 3\%$, NONSINGLET $\sim 5\%$
- DATA REGION: $10^2 \lesssim M_X \lesssim 10^3$ TeV, $-2 \lesssim y \lesssim 2$

UNCERTAINTIES: ...TO NNPDF4.0

GLUON

SINGLET

FLAVORS



- **TYPICAL** UNCERTAINTIES IN **DATA** REGION: SINGLET $\sim 1\%$, NONSINGLET $\sim 2 - 3\%$
- **DATA REGION:** $10 \lesssim M_X \lesssim 3 \cdot 10^3$ TeV, $-4 \lesssim y \lesssim 4$

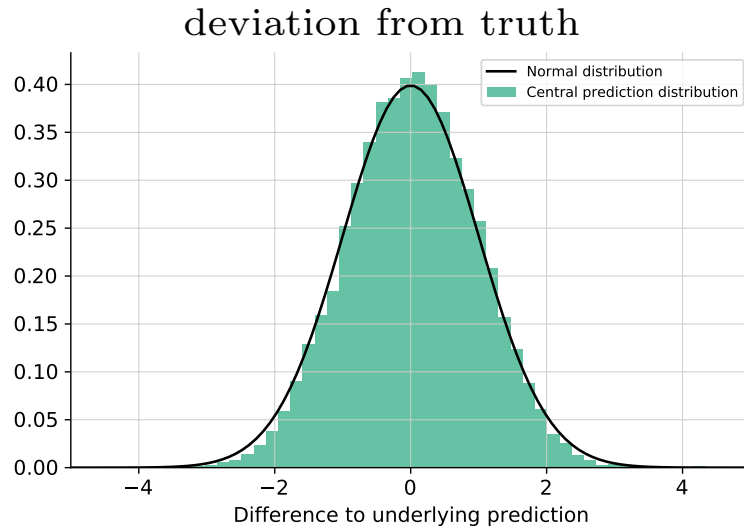
VALIDATION

CLOSURE TESTS

FAITHFUL UNCERTAINTIES IN DATA REGION?

- ASSUME “TRUE” UNDERLYING PDF $\Rightarrow$ E.G. SOME RANDOM PDF REPLICA
- GENERATE DATA DISTRIBUTED ACCORDING TO EXPERIMENTAL COVARIANCE MATRIX
- RUN WHOLE METHDOLOGY ON THESE DATA
- DO STATISTICS ON “RUNS OF THE UNIVERSE”, POSSIBLE THANKS TO EFFICIENT METHDOLOGY:
COMPARE TO TRUE PDFs, OR TO TRUE VALUES OF OBSERVABLES (NOT FITTED)
 - BIAS/VARIANCE: MEAN SQUARE DEVIATION WR TO TRUTH VS UNCERTAINTY
 - IS TRUTH WITHIN ONE SIGMA 68% OF TIMES?

RESULTS



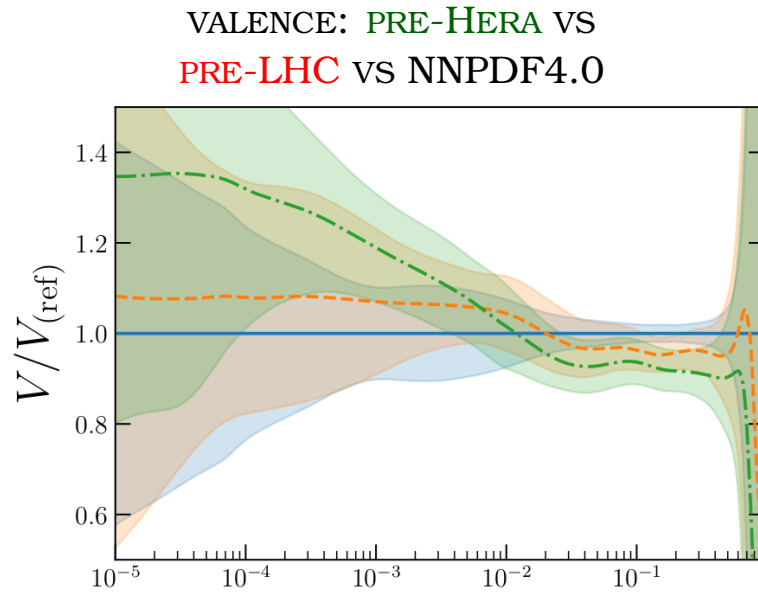
Dataset	$\sqrt{\text{bias/variance}}$	$\xi_{1\sigma}^{(\text{data})}$
DY	0.99 ± 0.08	0.69 ± 0.02
Top-pair	0.75 ± 0.06	0.75 ± 0.03
Jets	1.14 ± 0.05	0.63 ± 0.03
Dijets	0.99 ± 0.07	0.70 ± 0.03
Direct photon	0.71 ± 0.06	0.81 ± 0.03
Single top	0.87 ± 0.07	0.69 ± 0.04
Total	1.03 ± 0.05	0.68 ± 0.02

FUTURE TESTS

FAITHFUL UNCERTAINTIES IN EXTRAPOLATION?

- DETERMINE PDFs FROM A SUBSET OF CURRENT DATA: “PRE-HERA”, “PRE-LHC”,...
- COMPUTE χ^2 TO THE FULL CURRENT DATASET:
 - WITHOUT PDF UNCERTAINTIES $\Rightarrow$ IF $\gg 1$, MISSING INFORMATION
 - WITH PDF UNCERTAINTY $\Rightarrow$ IF ~ 1 , MISSING INFO REPRODUCED BY UNCERTAINTY

χ^2 WITHOUT/**WITH PDF UNC.**

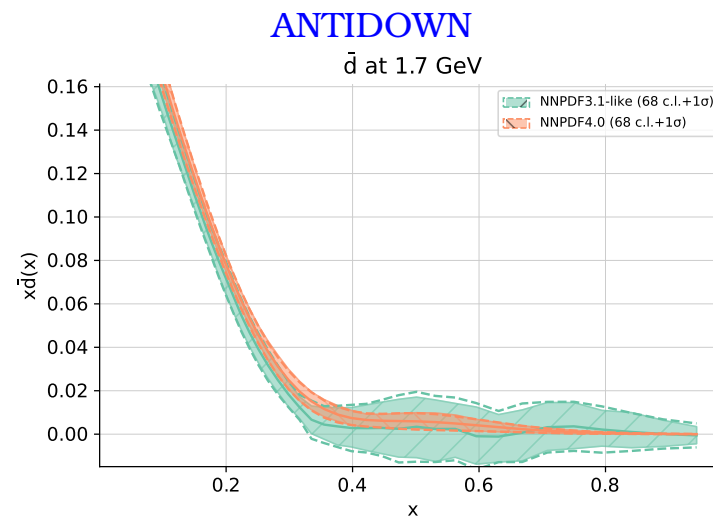
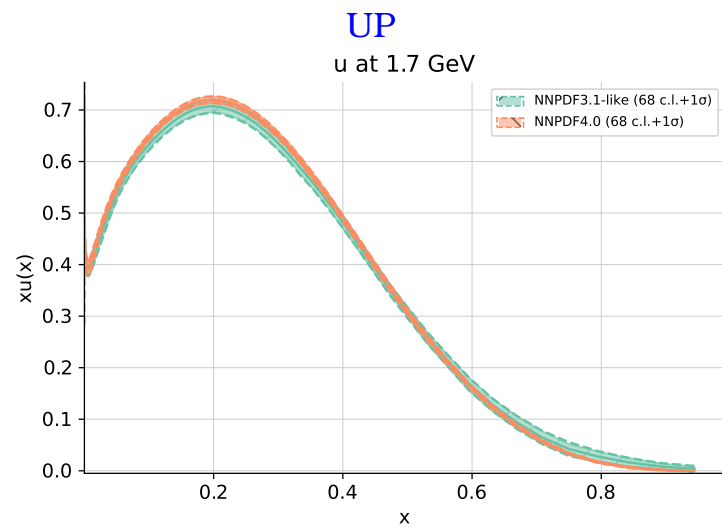
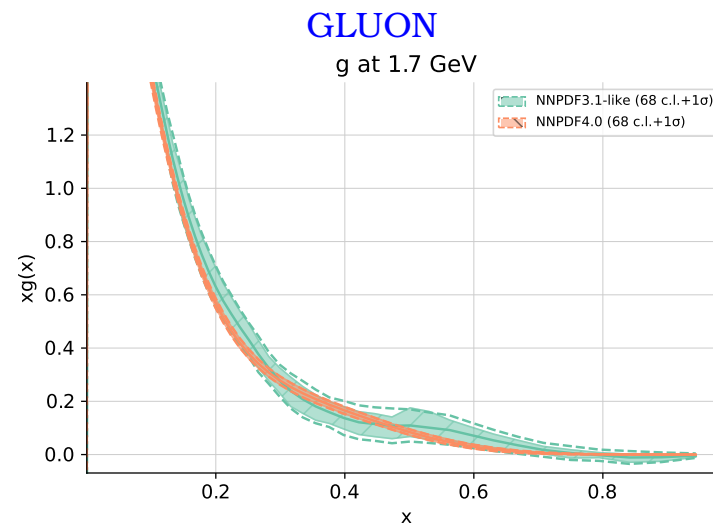
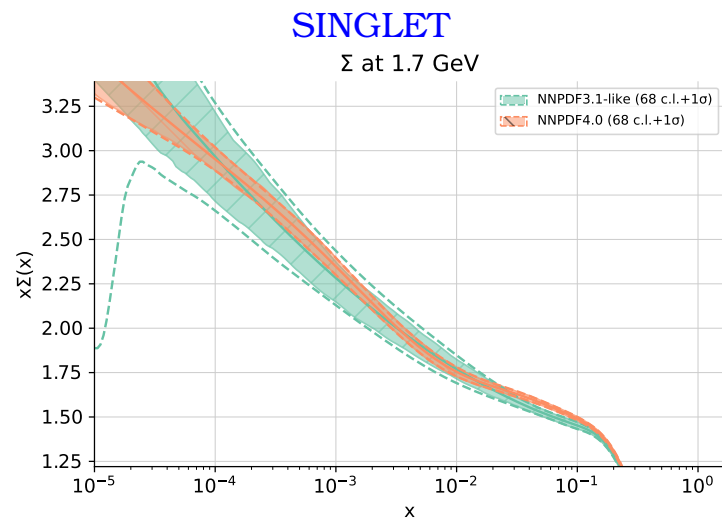


PROCESS	PRE-HERA	PRE-LHC	3.1-LIKE	4.0-GLOB
FT DIS (NC)	1.04	1.17	1.17	1.26
FT DIS (CC)	0.80	0.86	0.88	0.90
FT DY	0.93	1.27	1.43	1.59
HERA	24.01/ 1.12	1.22	1.21	1.21
COLL. DY (TeV.)	5.31/ 1.08	0.96	0.95	1.13
COLL. DY (LHC)	15.50/ 1.37	2.64/ 1.54	1.39	1.54
TOP QUARK	23.35/ 1.08	1.29/ 0.86	0.82	0.98
JETS	6.18/ 1.21	3.66/ 1.29	2.07/ 1.37	1.26
TOTAL	9.70	1.44	1.22	1.17

STABILITY

NNPDF4.0 vs. NNPDF3.1

- FULL BACKWARD COMPATIBILITY
- SUBSTANTIAL REDUCTION IN UNCERTAINTY

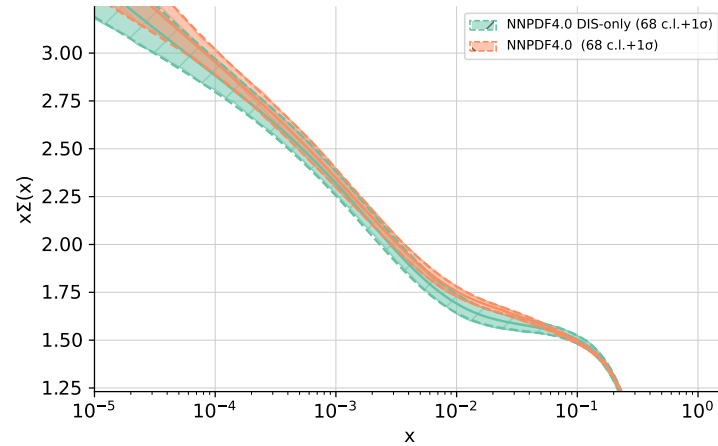


NNPDF4.0 vs DIS-ONLY

- DIS-ONLY FIT **NO LONGER COMPETITIVE**
- HADRONIC DATA **NEEDED FOR PRECISION**

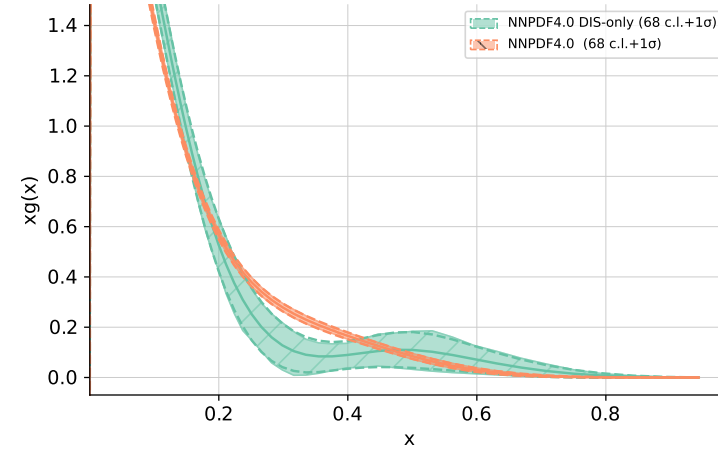
SINGLET

Σ at 1.7 GeV



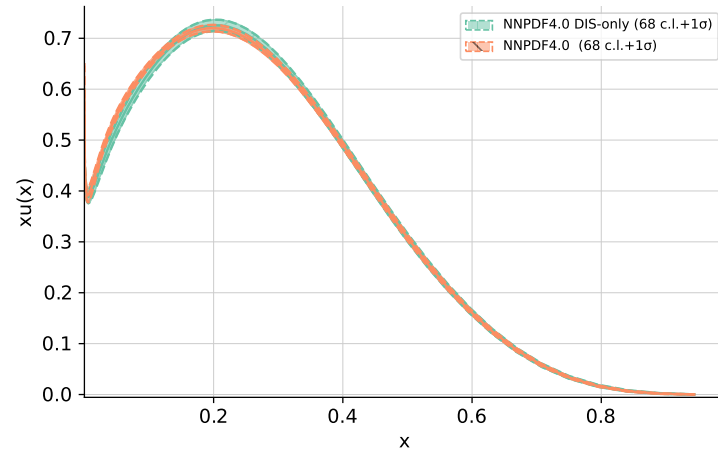
GLUON

g at 1.7 GeV



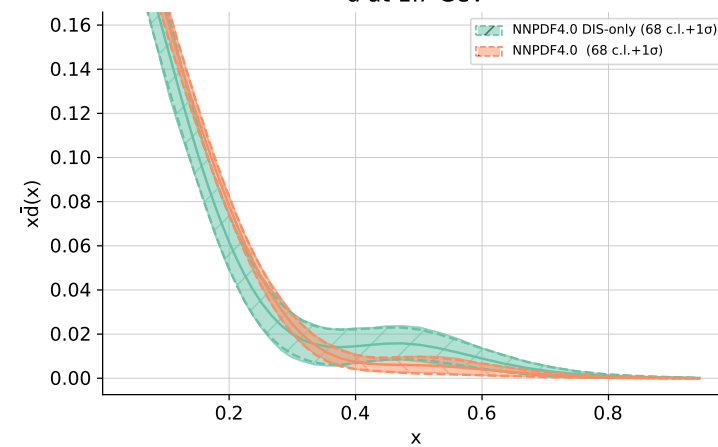
UP

u at 1.7 GeV



ANTIDOWN

$\bar{d}$ at 1.7 GeV

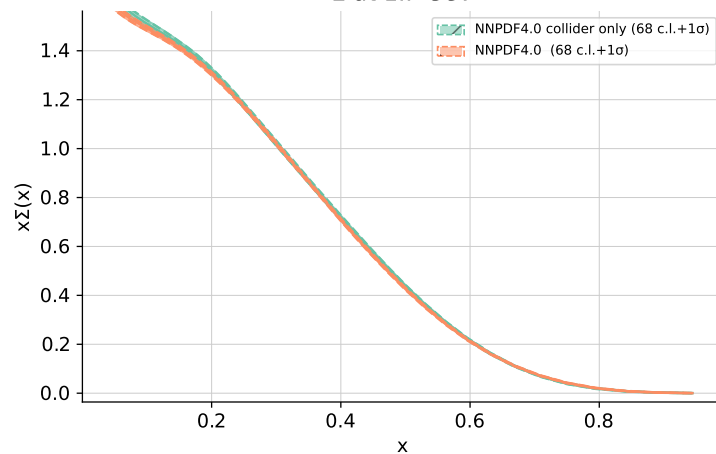


NNPDF4.0 VS COLLIDER ONLY

- COLLIDER ONLY **COMPETITIVE!**
- ONLY **DEUTERIUM FIXED-TARGET** DATA STILL RELEVANT

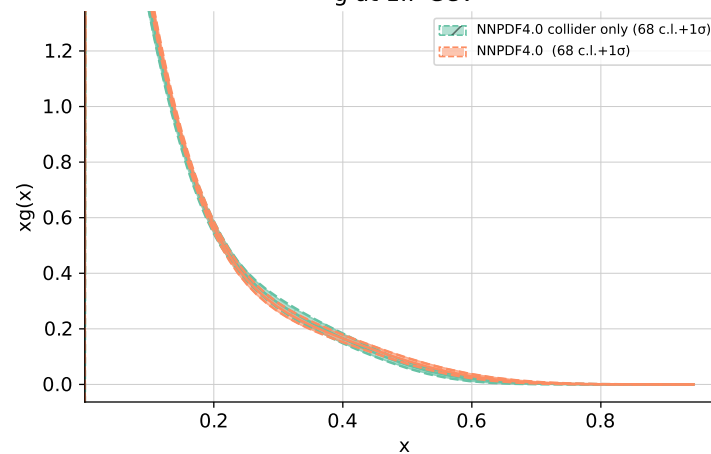
SINGLET

Σ at 1.7 GeV



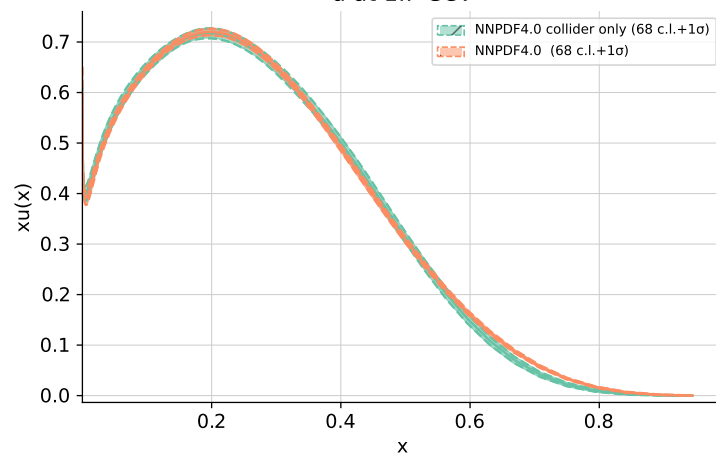
GLUON

g at 1.7 GeV



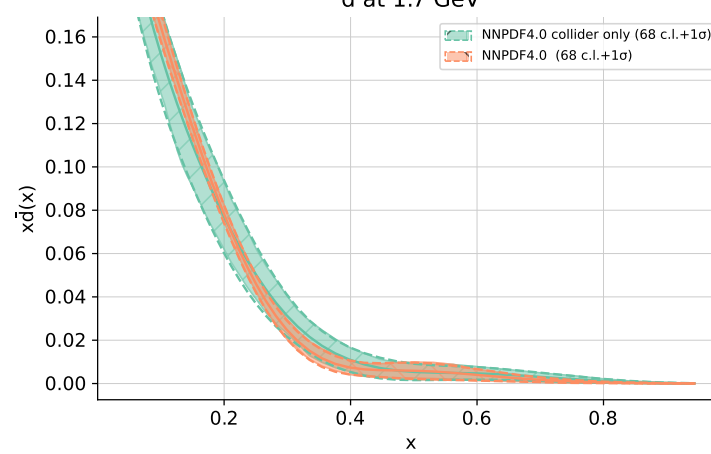
UP

u at 1.7 GeV



ANTIDOWN

$\bar{d}$ at 1.7 GeV



DELIVERY

AN OPEN SOURCE CODE!

- THE **FULL** NNPDF CODE HAS BEEN MADE **PUBLIC!**
- INCLUDING HYPEROPTIMIZATION, EVOLUTION, THEORY, FITTING, VISUALIZATION
- **FULLY DOCUMENTED** CODE

Eur. Phys. J. C (2021) 81:958
<https://doi.org/10.1140/epjc/s10052-021-09747-9>

THE EUROPEAN
PHYSICAL JOURNAL C



Special Article - Tools for Experiment and Theory

An open-source machine learning framework for global analyses of parton distributions

NNPDF Collaboration

<https://github.com/NNPDF/nnpdf>
<https://docs.nnpdf.science>

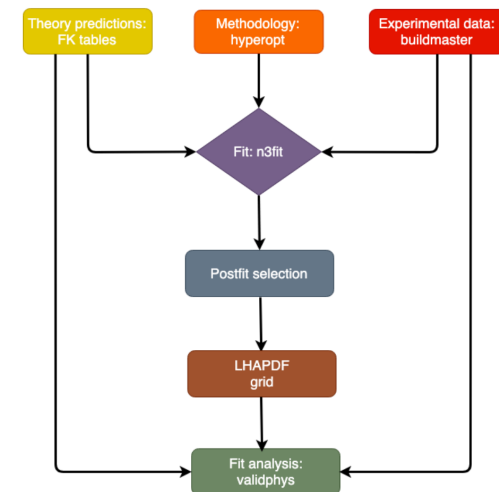


Fig. 2.1. Workflow for an NNPDF fit

OUTLOOK

THE ACCURACY CHALLENGE

- AT 1%, NOT ALL REDUCED DATASETS AGREE
- MUST INCLUDE MISSING HIGHER-ORDER CORRECTION UNCERTAINTIES
- INCLUDE EW CORRECTIONS
- GO BEYOND K -FACTORS
- WEIGHT SMALL DATASETS

THE ACCURACY CHALLENGE

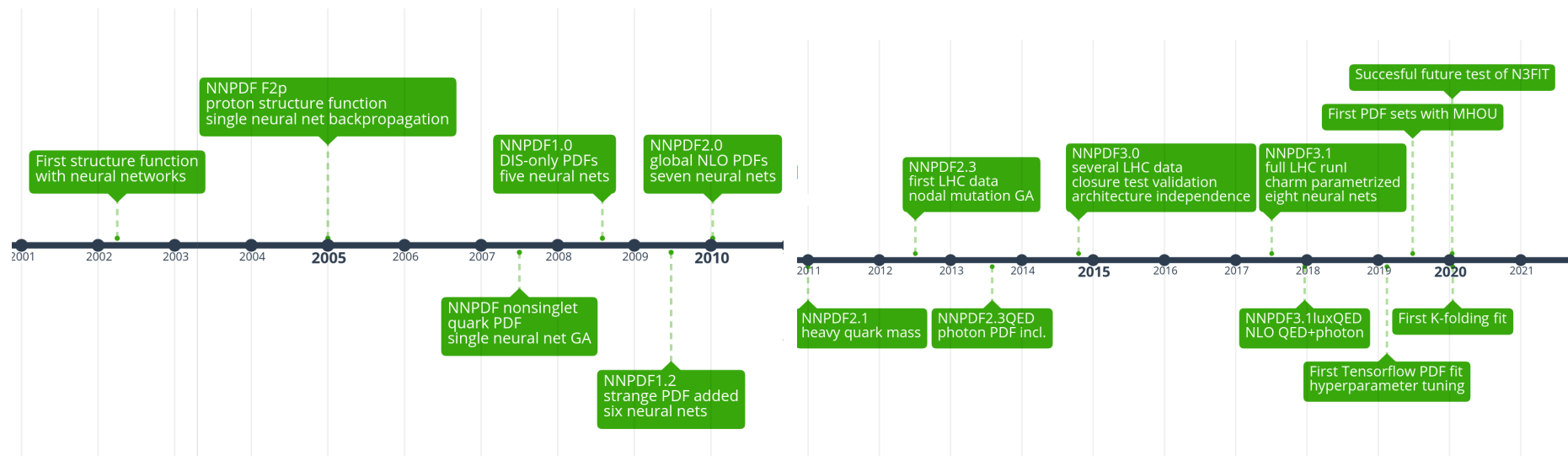
- AT 1%, NOT ALL REDUCED DATASETS AGREE
- MUST INCLUDE MISSING HIGHER-ORDER CORRECTION UNCERTAINTIES
- INCLUDE EW CORRECTIONS
- GO BEYOND K -FACTORS
- WEIGHT SMALL DATASETS

STAY TUNED FOR NNPDF4.1!!

EXTRAS

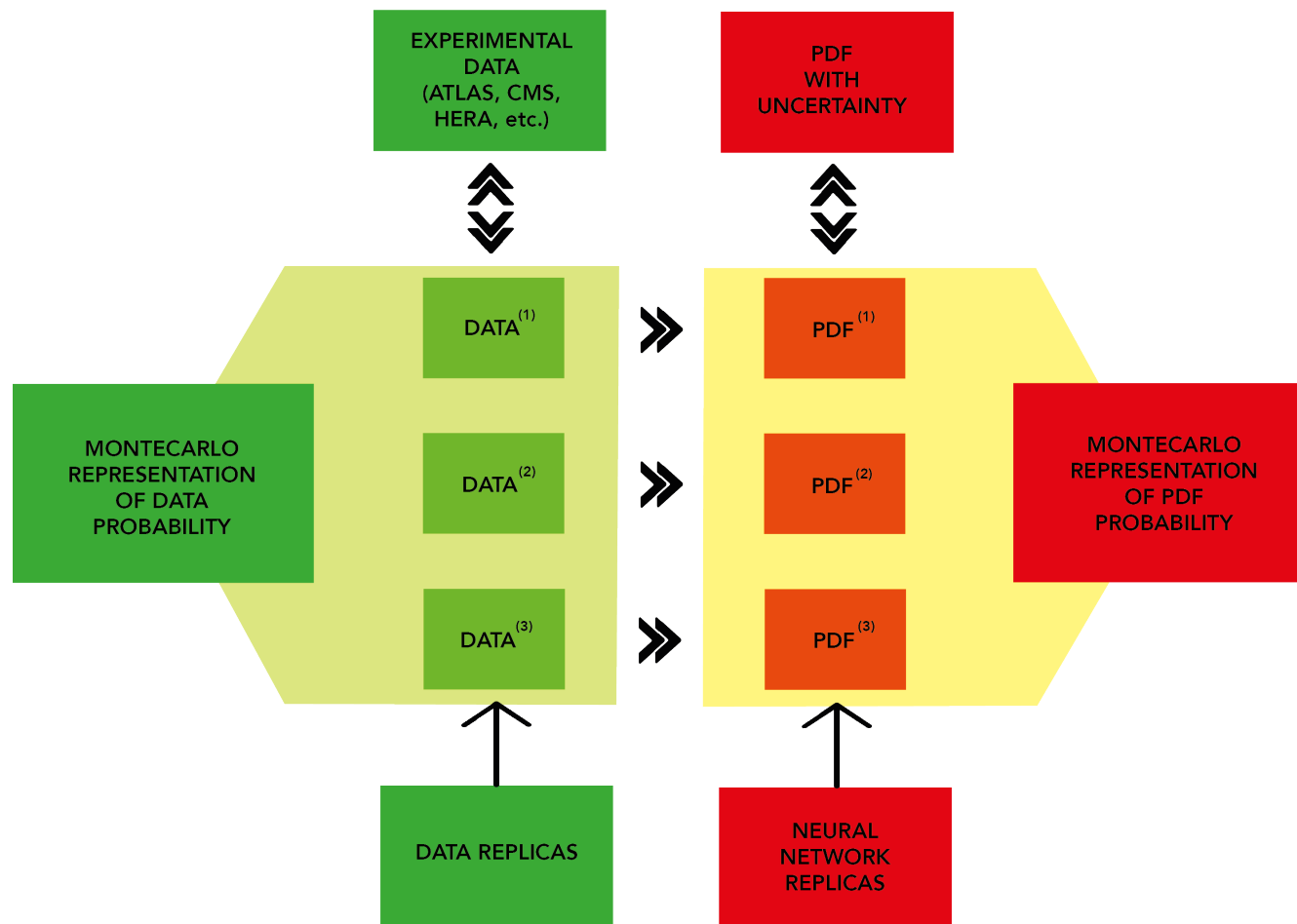
PROTON STRUCTURE AS AN AI PROBLEM:

NNPDF



AI FOR PDFS: THE NNPDF APPROACH THE FUNCTIONAL MONTE CARLO

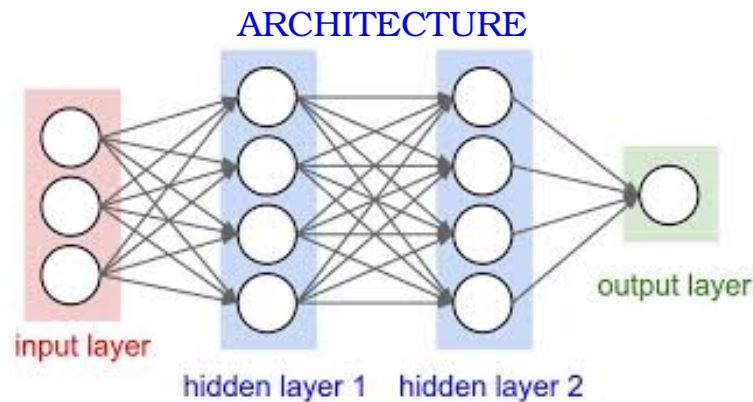
REPLICA SAMPLE OF FUNCTIONS $\Leftrightarrow$ PROBABILITY DENSITY IN FUNCTION SPACE
KNOWLEDGE OF LIKELIHOOD SHAPE (FUNCTIONAL FORM) NOT NECESSARY



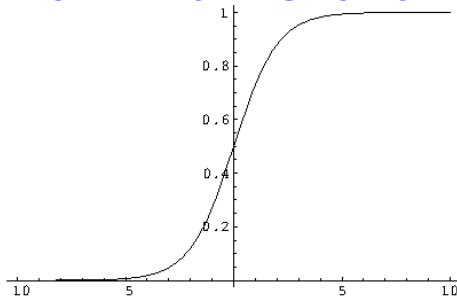
FINAL PDF SET: $f_i^{(a)}(x, \mu);$

$i = \text{up, antiup, down, antidown, strange, antistrange, charm, gluon}; j = 1, 2, \dots, N_{\text{rep}}$

ARTIFICIAL INTELLIGENCE NEURAL NETWORKS



ACTIVATION FUNCTION



PARAMETERS

- WEIGHTS ω_{ij}
- THRESHOLDS θ_i

$$F_{\text{out}}^{(i)}(\vec{x}_{\text{in}}) = F \left(\sum_j \omega_{ij} x_{\text{in}}^j - \theta_i \right)$$

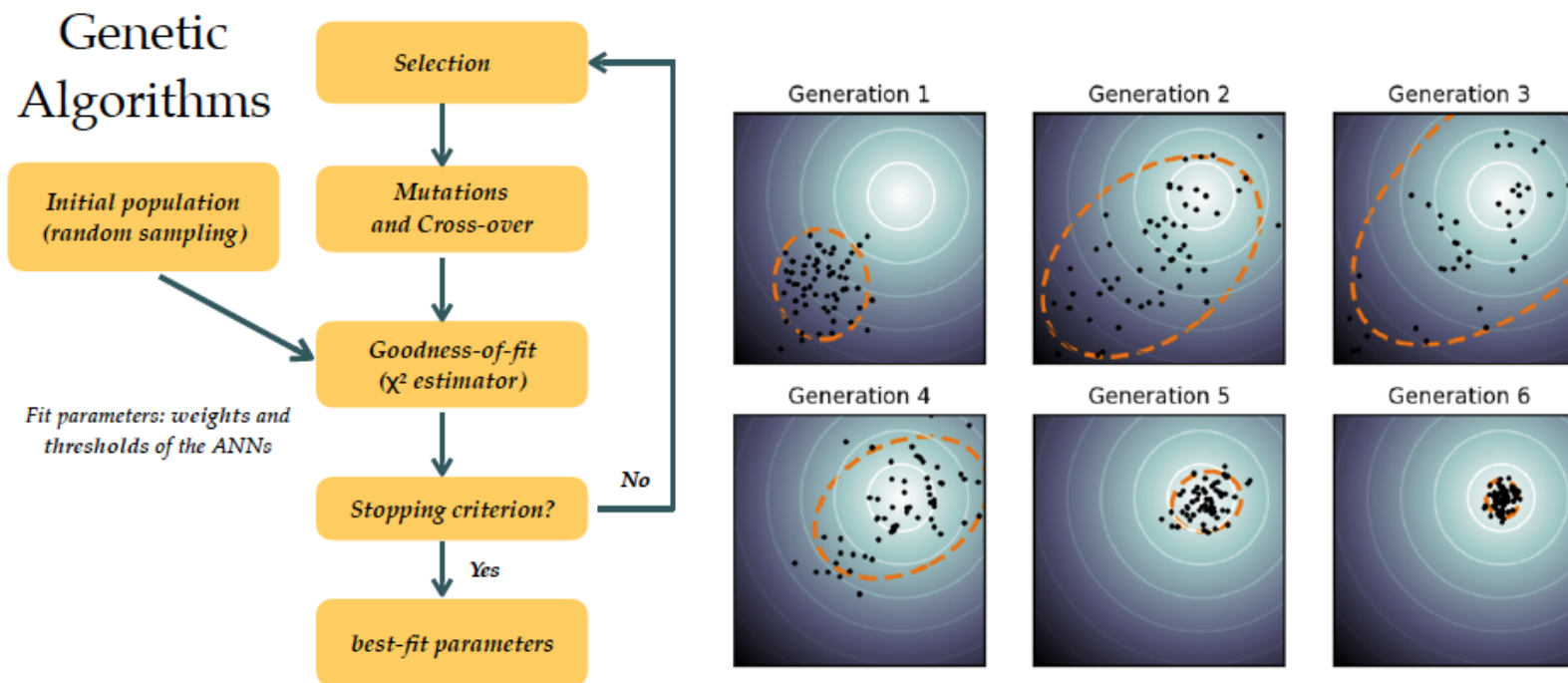
SIMPLEST EXAMPLE
1-2-1

$$f(x) = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}}}}$$

NNPDF: 2 – 5 – 3 – 1 NN FOR EACH PDF: $37 \times 8 = 296$ PARAMETERS

SUPERVISED LEARNING GENETIC ALGORITHMS

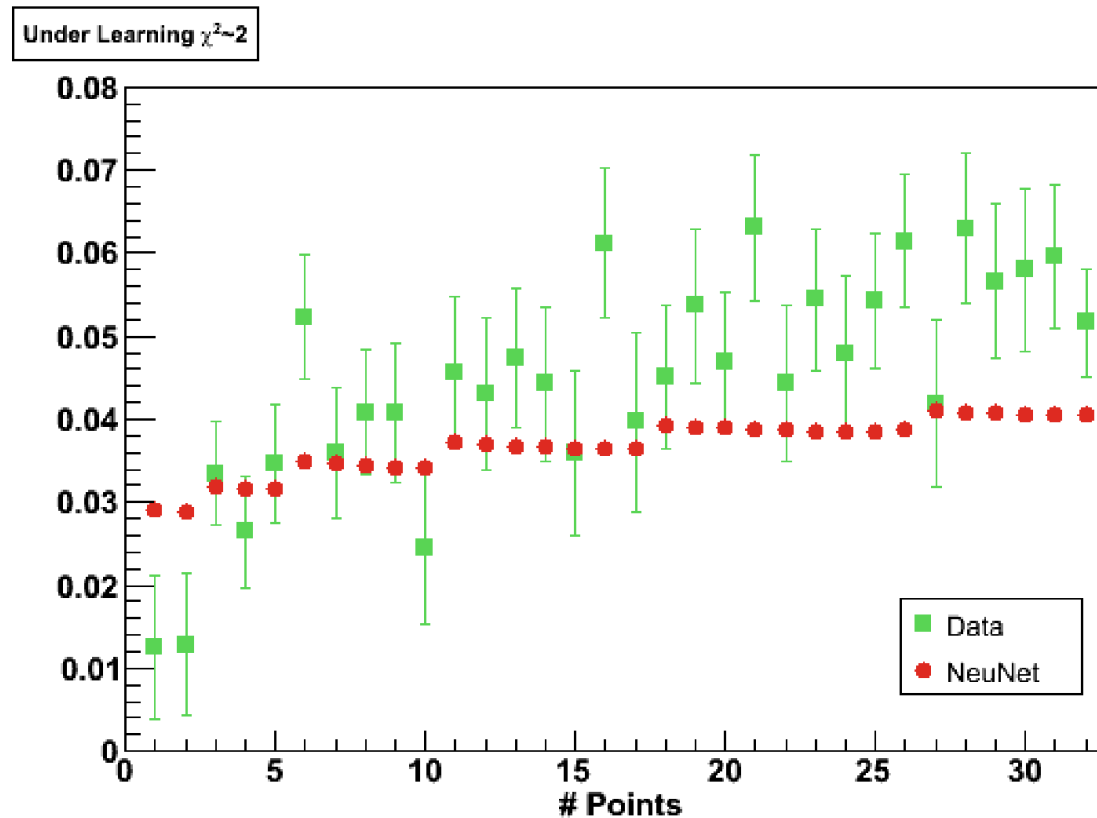
- BASIC IDEA: RANDOM MUTATION OF THE NN PARAMETER
- SELECTION OF THE FITTEST



NEURAL LEARNING

- COMPLEXITY INCREASES AS THE FITTING PROCEEDS
- UNTIL LEARNING NOISE
- WHEN SHOULD ONE STOP?

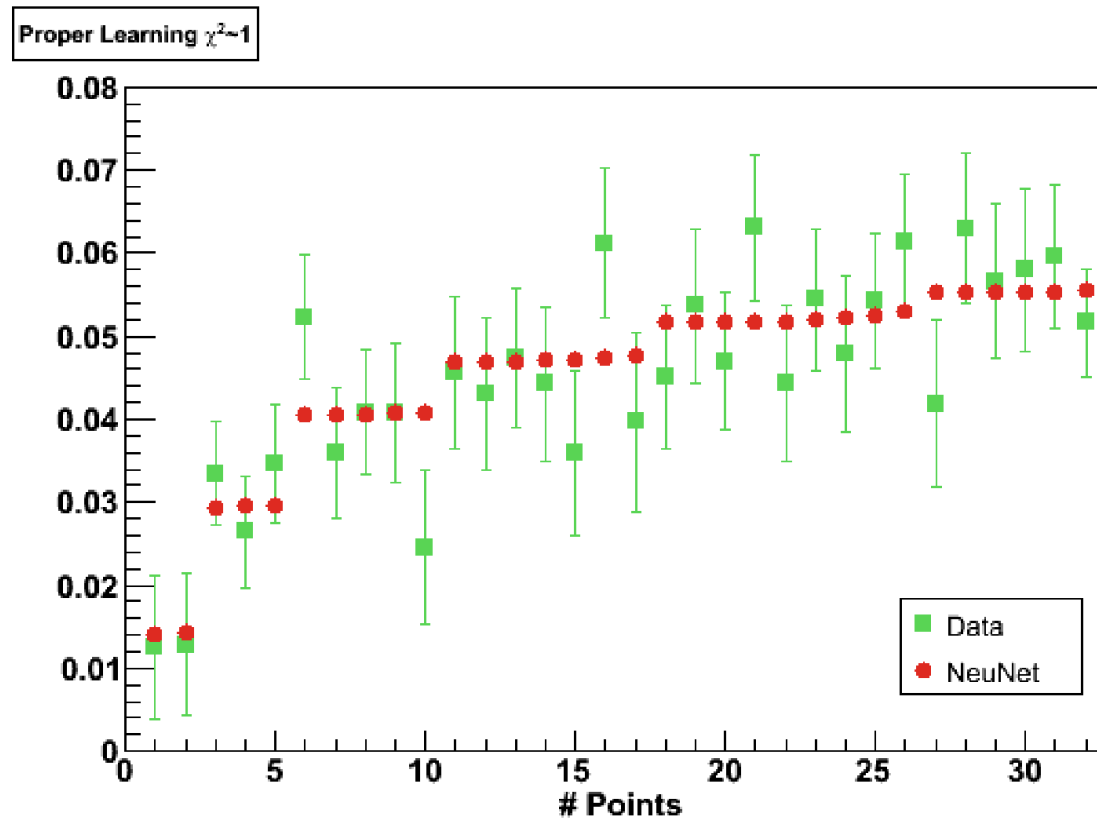
UNDERLEARNING



NEURAL LEARNING

- COMPLEXITY INCREASES AS THE FITTING PROCEEDS
- UNTIL LEARNING NOISE
- WHEN SHOULD ONE STOP?

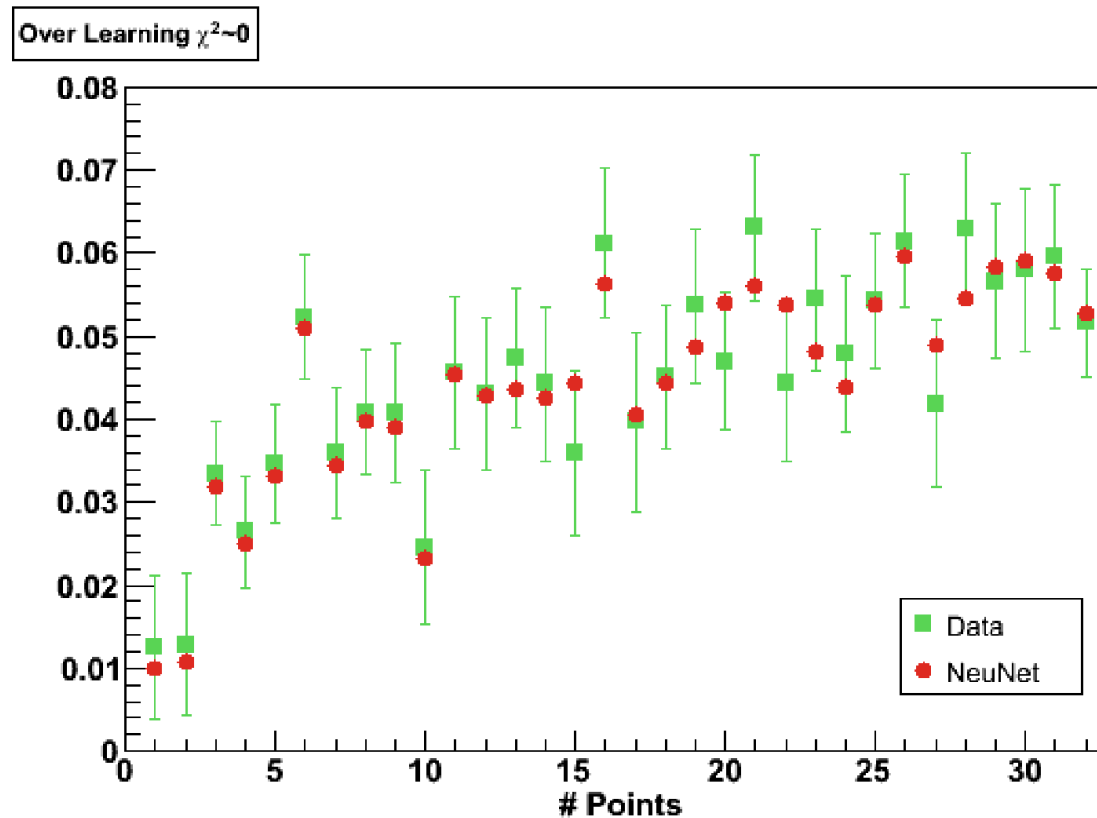
PROPER LEARNING



NEURAL LEARNING

- COMPLEXITY INCREASES AS THE FITTING PROCEEDS
- UNTIL LEARNING NOISE
- WHEN SHOULD ONE STOP?

OVERLEARNING

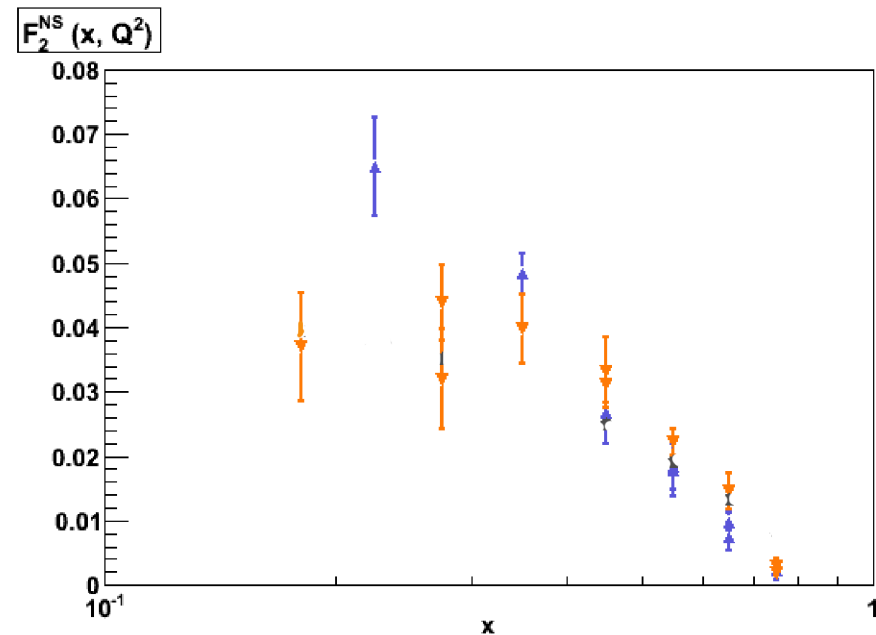


OPTIMAL FIT: CROSS-VALIDATION

GENETIC MINIMIZATION:

AT EACH GENERATION, χ^2 EITHER UNCHANGED OR DECREASING

- DIVIDE THE DATA IN TWO SETS: TRAINING AND VALIDATION
- MINIMIZE THE χ^2 OF THE DATA IN THE TRAINING SET
- AT EACH ITERATION, COMPUTE THE χ^2 FOR THE DATA IN THE VALIDATION SET (NOT USED FOR FITTING)
- WHEN THE VALIDATION χ^2 STOPS DECREASING, STOP THE FIT



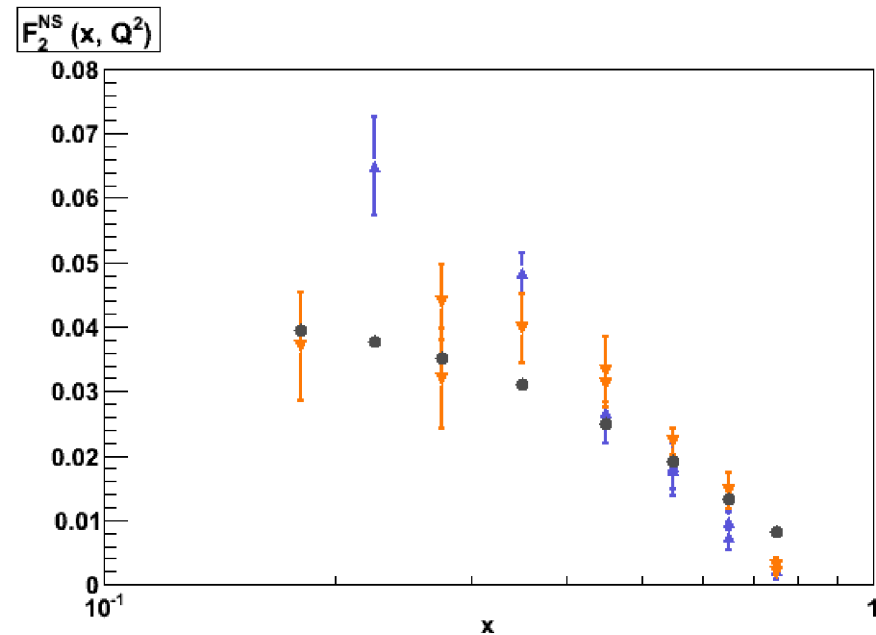
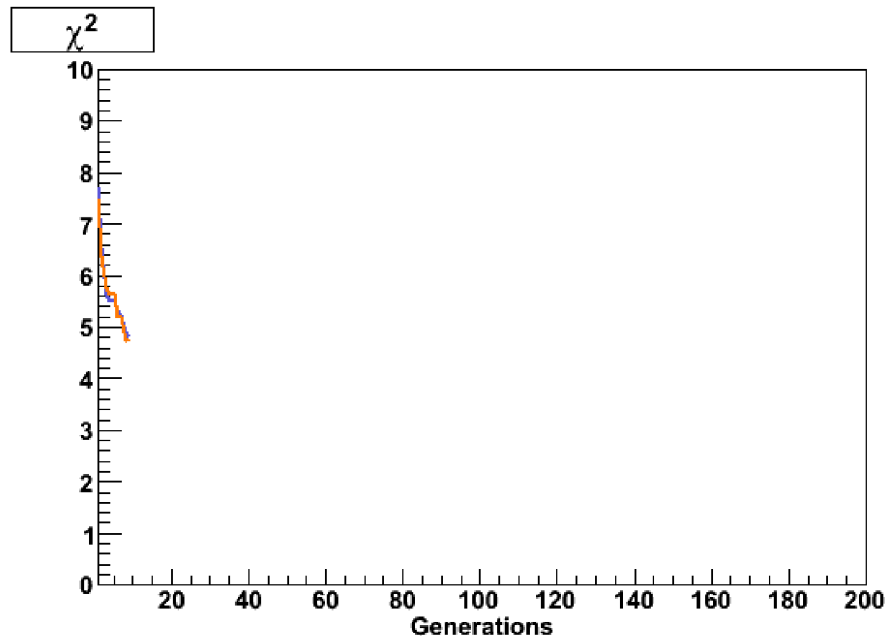
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- WHEN THE VALIDATION χ^2 STOPS DECREASING, STOP THE FIT

GO!



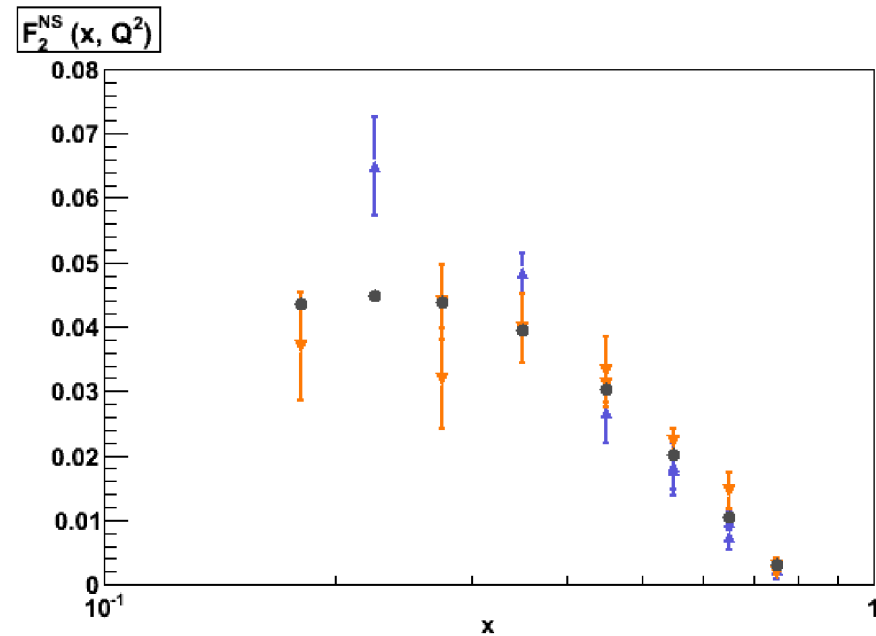
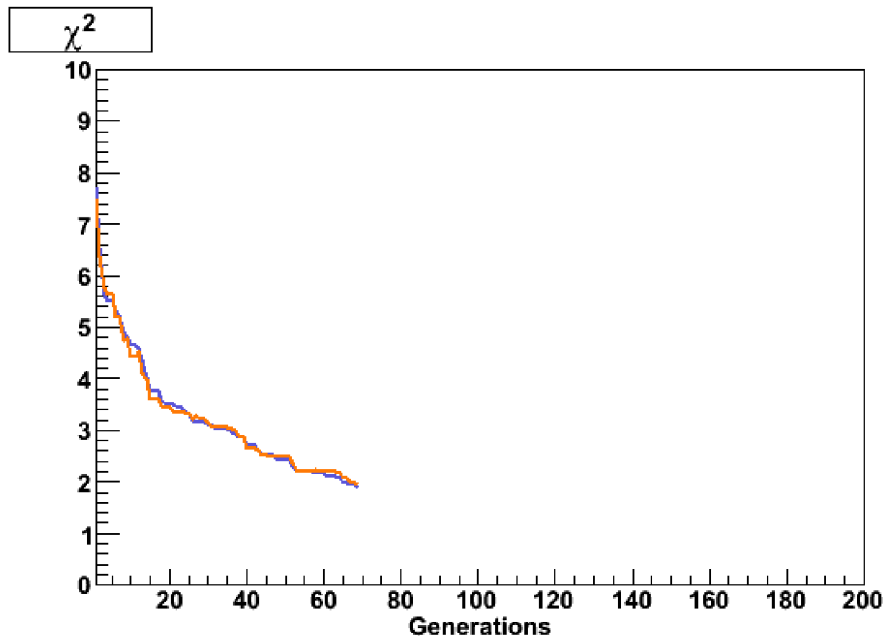
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- WHEN THE VALIDATION χ^2 STOPS DECREASING, STOP THE FIT

STOP!



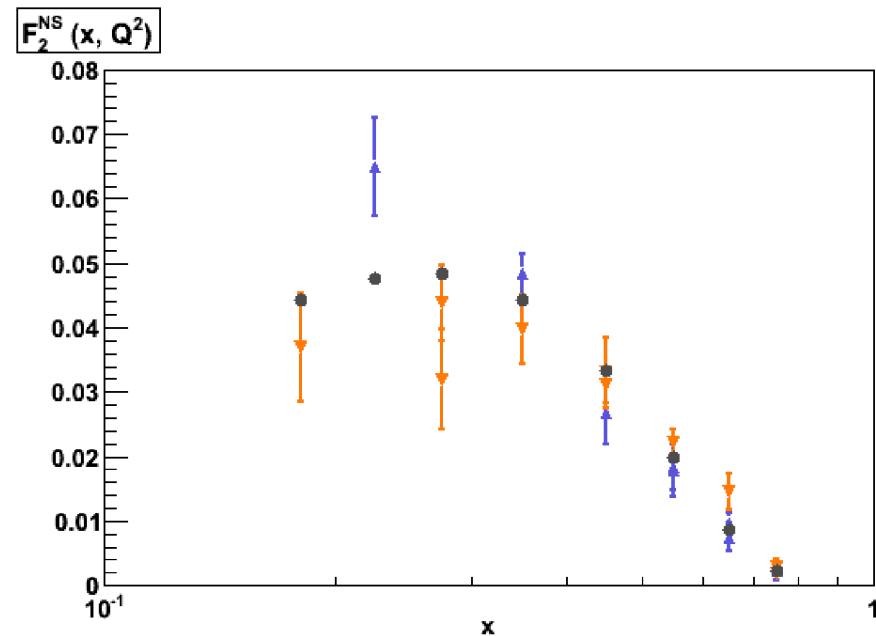
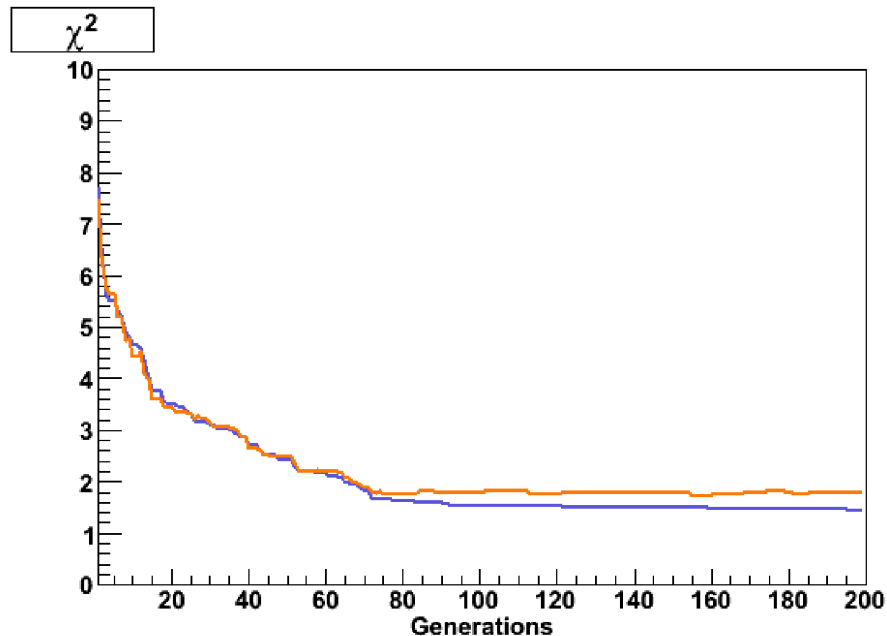
OPTIMAL FIT: CROSS-VALIDATION

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- WHEN THE VALIDATION χ^2 STOPS DECREASING, STOP THE FIT

TOO LATE!

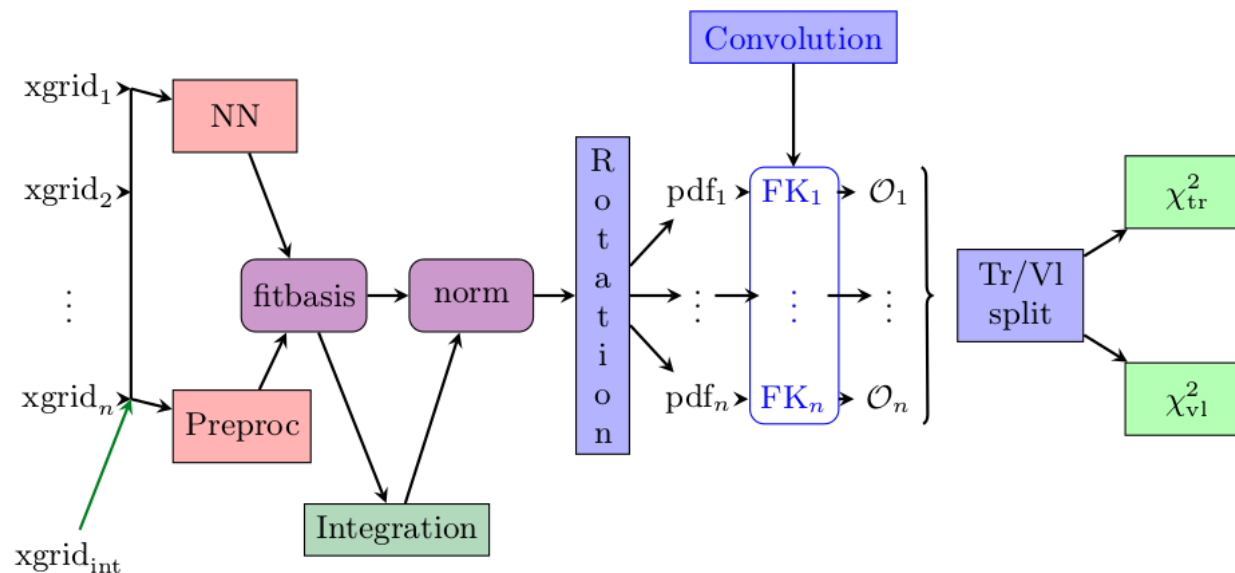


LEARNING THE METHODOLOGY

THE N3FIT PROJECT

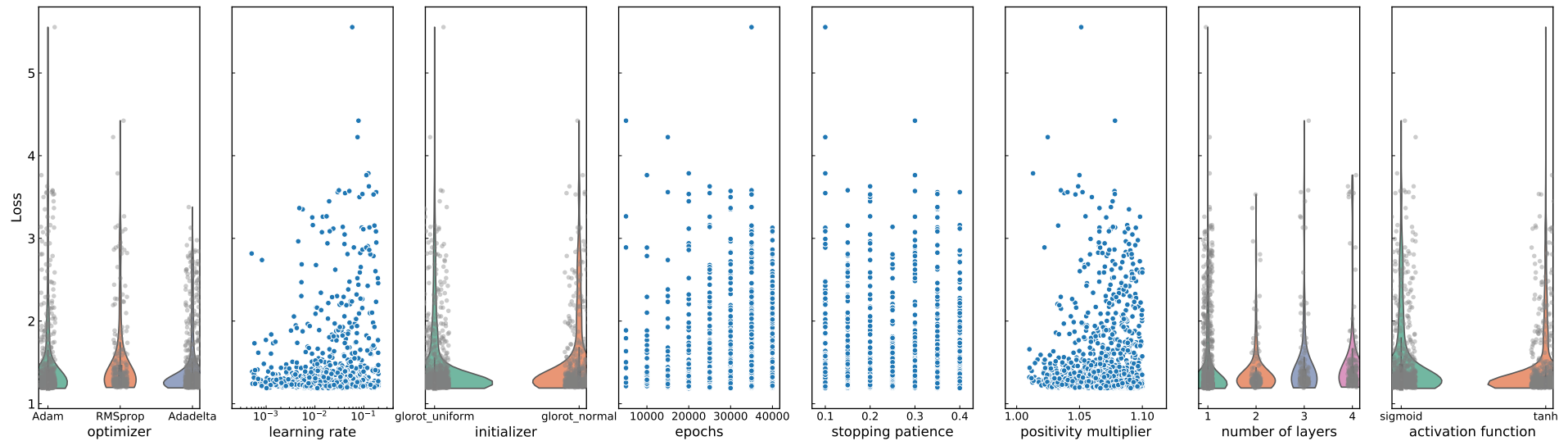
HOW DO WE KNOW THAT THE METHODOLOGY IS THE BEST?
“ACCUMULATED WISDOM” INEFFICIENT AND SLOW

CHANGE OF PHILOSOPHY $\Rightarrow$ DETERMINISTIC MINIMIZATION (GRADIENT DESCENT)
GO FOR THE ABSOLUTE MINIMUM, AND (HYPER)OPTIMIZE



- PYTHON-BASED KERAS + TENSORFLOW FRAMEWORK
- EACH BLOCK INDEPENDENT LAYER
- CAN VARY ALL ASPECTS OF METHODOLOGY

FITTING THE METHODOLOGY HYPEROPTIMIZATION SCANS



HYPEROPT PARAMETERS

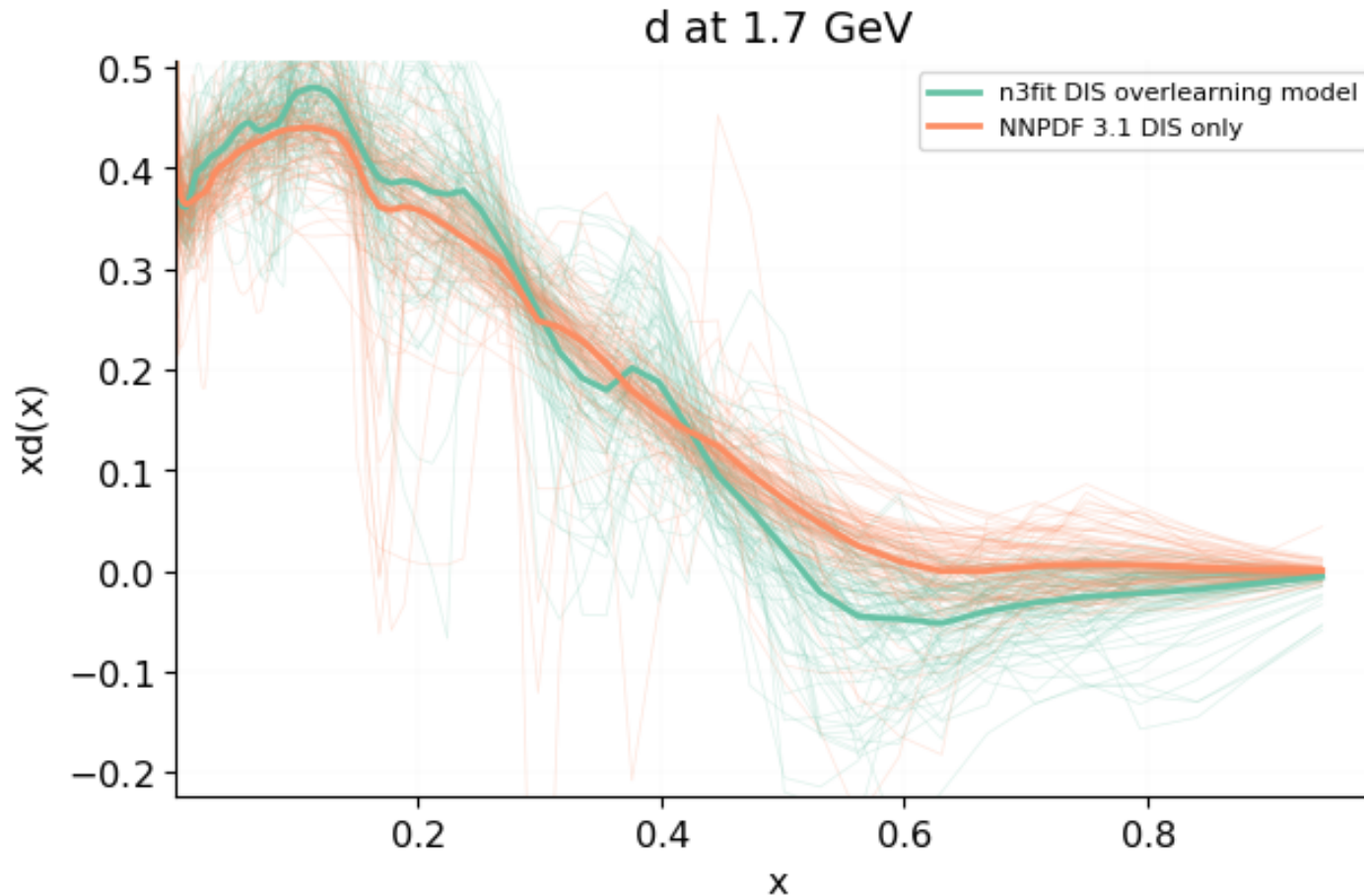
NEURAL NETWORK	FIT OPTIONS
NUMBER OF LAYERS (*)	OPTIMIZER (*)
SIZE OF EACH LAYER	INITIAL LEARNING RATE (*)
DROPOUT	MAXIMUM NUMBER OF EPOCHS (*)
ACTIVATION FUNCTIONS (*)	STOPPING PATIENCE (*)
INITIALIZATION FUNCTIONS (*)	POSITIVITY MULTIPLIER (*)

- **SCAN** PARAMETER SPACE
- **OPTIMIZE** FIGURE OF MERIT: **VALIDATION** χ^2
- **BAYESIAN** UPDATING

FITTING THE METHODOLOGY

THE OVERFITTING PROBLEM

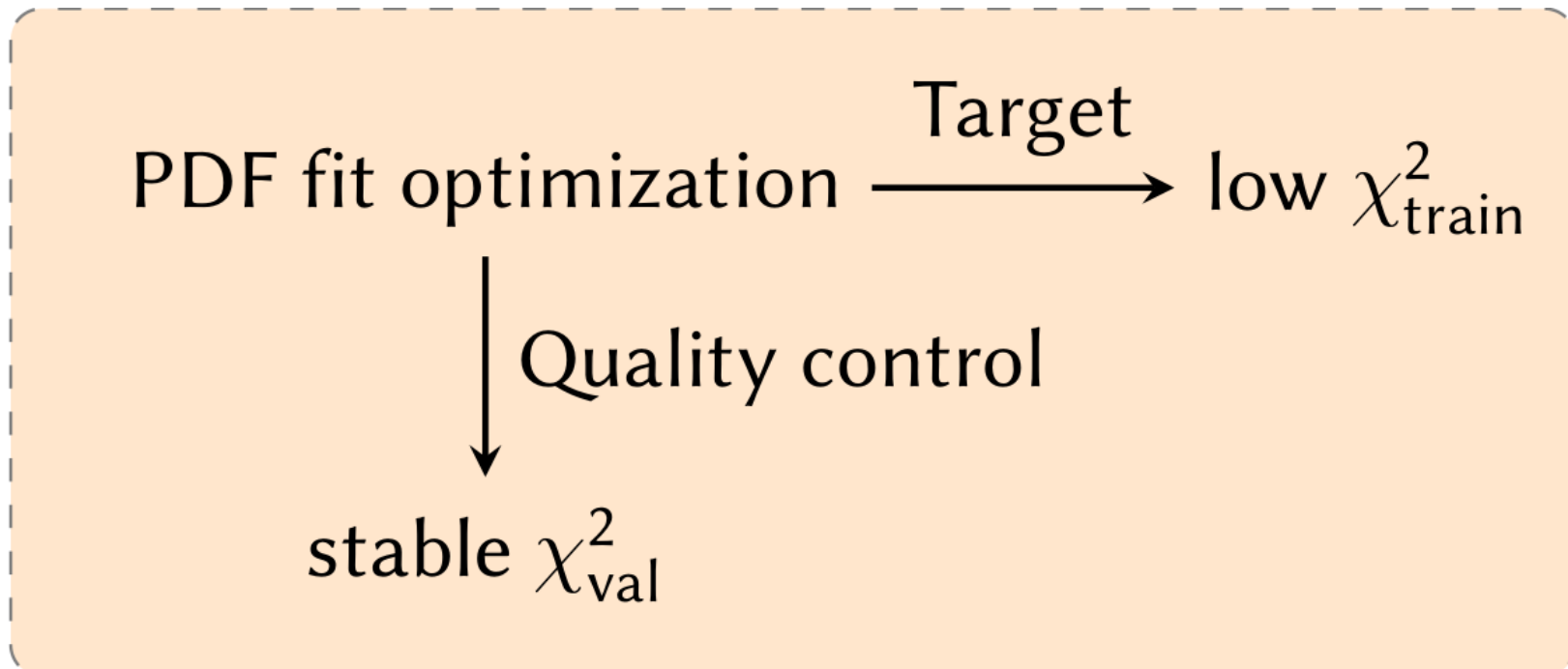
DOWN QUARK: HYPEROPTIMIZED VS. STANDARD



- **NNPDF3.1**: **WIGGLES**: **FINITE SIZE** $\Rightarrow$ WILL GO AWAY AS N_{rep} GROWS
- **N3FIT**: **WIGGLY** PDFS $\Leftrightarrow$ **OVERFITTING** $\Rightarrow$ WILL **NOT** GO AWAY ($\chi^2_{\text{train}} \ll \chi^2_{\text{valid}}$!!)

WHAT HAPPENED?

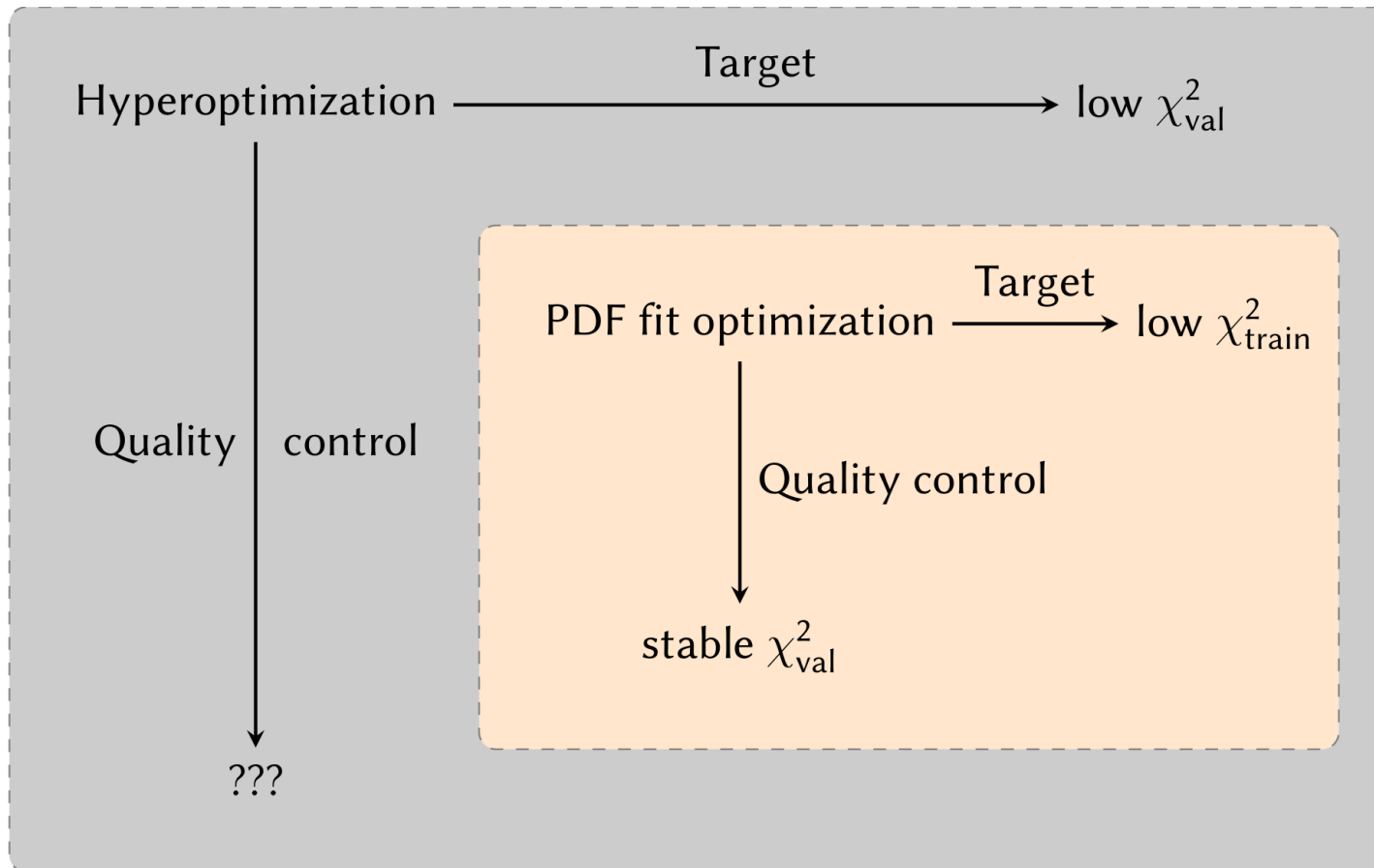
OPTIMIZATION



CROSS-VALIDATION SELECTS THE OPTIMAL MINIMUM

WHAT HAPPENED?

HYPEROPTIMIZATION

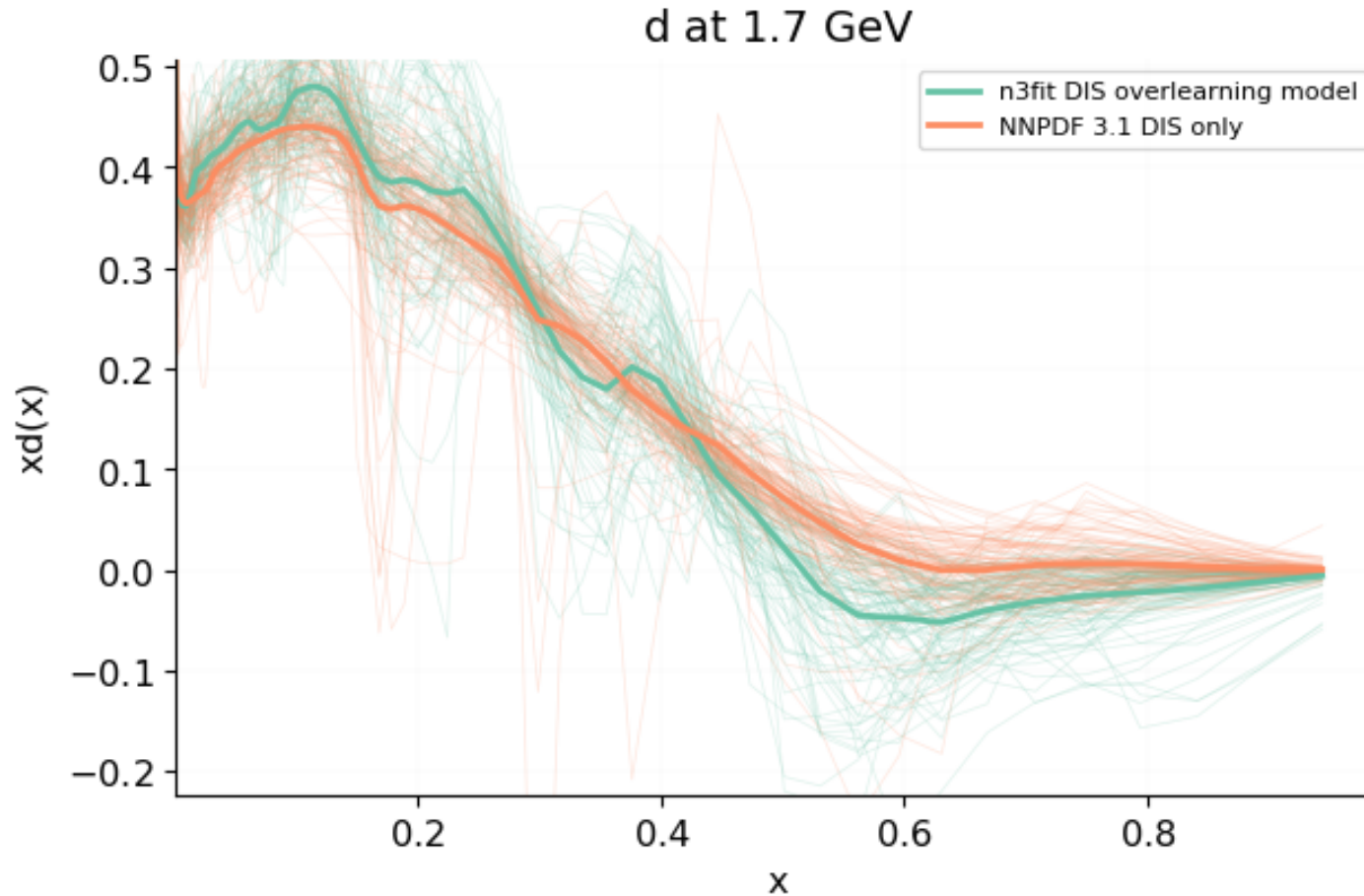


WE ARE MISSING A SELECTION CRITERION

FITTING THE METHODOLOGY

THE OVERFITTING PROBLEM

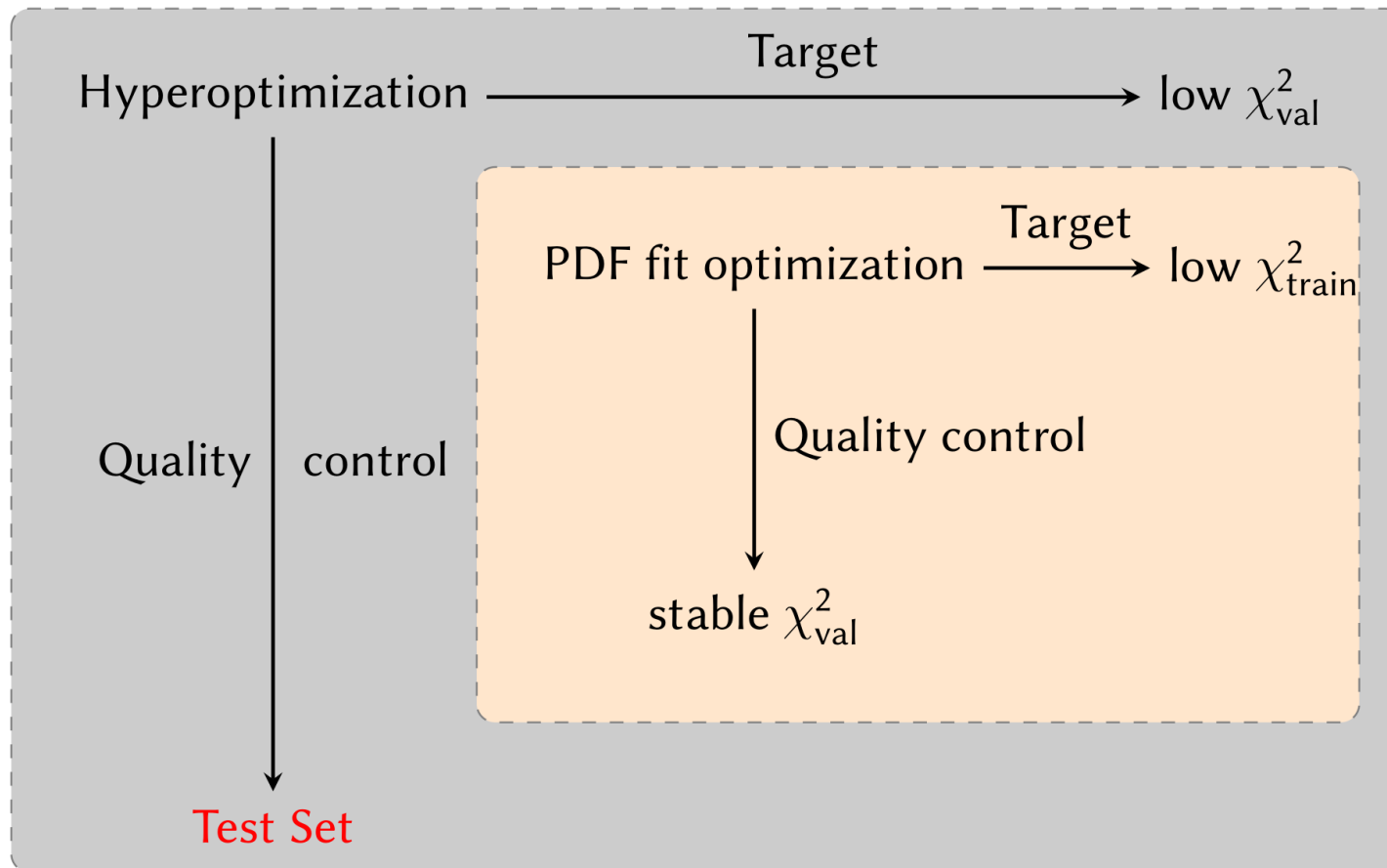
DOWN QUARK: HYPEROPTIMIZED VS. STANDARD



- **NNPDF3.1**: **WIGGLES**: **FINITE SIZE** $\Rightarrow$ WILL GO AWAY AS N_{rep} GROWS
- **N3FIT**: **WIGGLY** PDFS $\Leftrightarrow$ **OVERFITTING** $\Rightarrow$ WILL **NOT** GO AWAY ($\chi^2_{\text{train}} \ll \chi^2_{\text{valid}}$!!)
- **CORRELATIONS** BETWEEN TRAINING AND VALIDATION DATA

MACHINE LEARNING THE SOLUTION

TUNED HYPEROPTIMIZATION



COMPARE TO A **A TEST SET** (NEW SET OF DATA PREVIOUSLY NOT USED AT ALL)
TESTS **GENERALIZATION POWER**

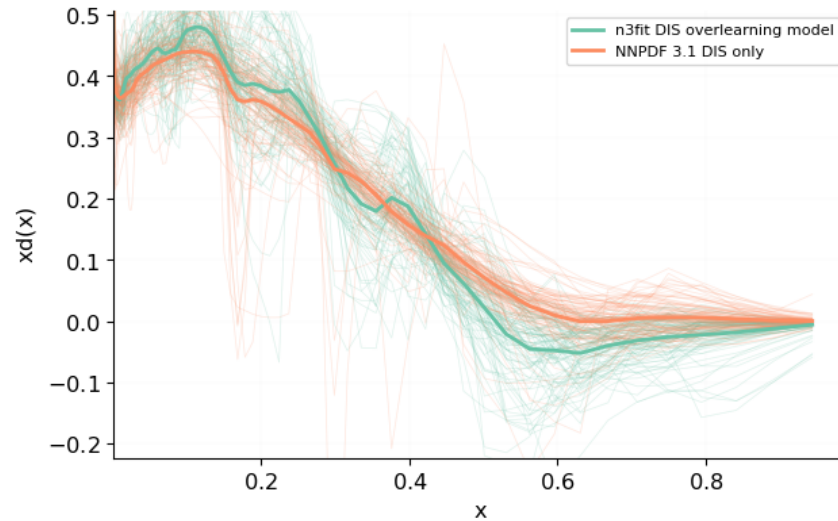
THE TEST SET METHOD

- COMPLETELY UNCORRELATED TEST SET
- OPTIMIZE ON WEIGHTED AVERAGE OF VALIDATION AND TEST
⇒ NO OVERLEARNING

HYPEROPTIMIZED PDFs DOWN QUARK

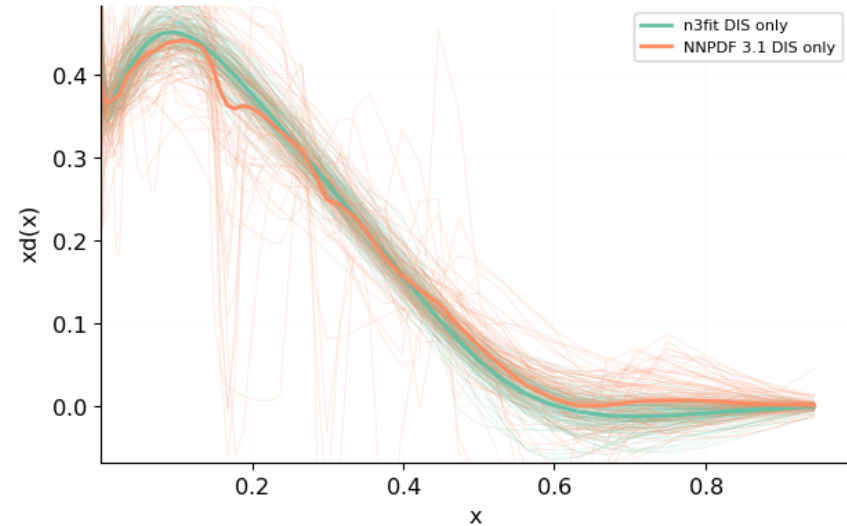
N3 OVERFIT VS NNPDF3.1

d at 1.7 GeV



N3FIT VS NNPDF3.1

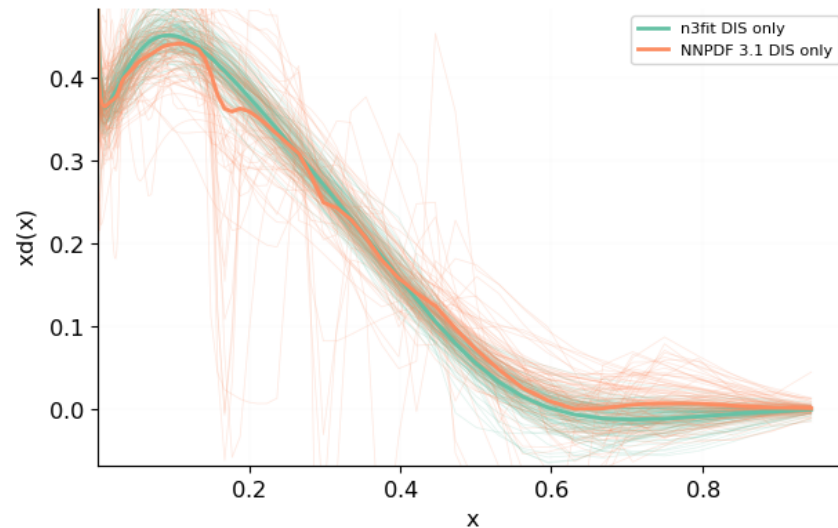
d at 1.7 GeV



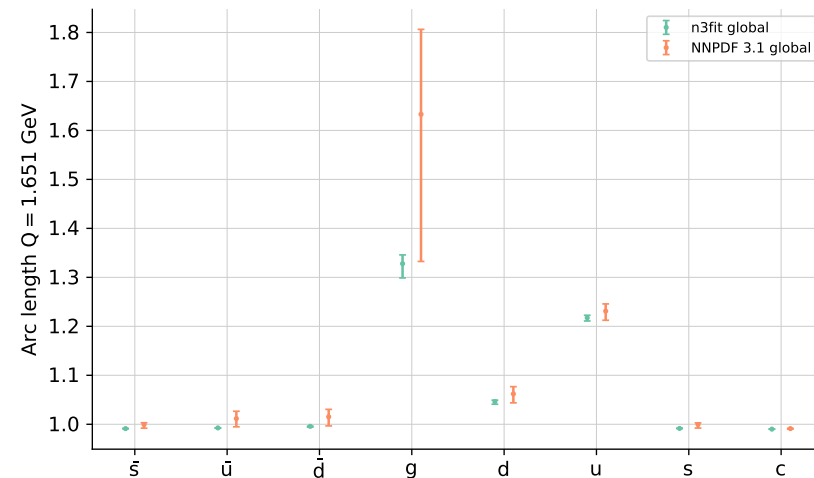
THE TEST SET METHOD

N3FIT vs NNPDF3.1

DOWN PDF
d at 1.7 GeV



ARCLENGTHS



- NO OVERFITTING
- COMPARED TO NNPDF3.1
 - MUCH GREATER STABILITY $\Rightarrow$ FEWER REPLICAS FOR EQUAL ACCURACY
 - UNCERTAINTIES SOMEWHAT REDUCED