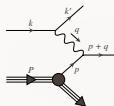


EKO  &
Evolution Kernel Operators



Yadism
Yet Another DIS Module

New Software Tools for DGLAP and DIS

Alessandro Candido, Felix Hekhorn, Giacomo Magni

DIS 2022, Santiago di Compostela, May 2022

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1. EKO [arXiv: 2202.02338]
2. Intrinsic Charm in the Proton [submitted]
3. yadism [in preparation]
4. Theory Prediction Pipeline

1. EKO [\[arXiv: 2202.02338\]](#)



DGLAP:

$$\mu_F^2 \frac{d\mathbf{f}}{d\mu_F^2}(\mu_F^2) = \mathbf{P}(a_s(\mu_R^2), \mu_F^2) \otimes \mathbf{f}(\mu_F^2)$$

as operator equation for the evolution kernel operator (EKO) $\mathbf{E}$:

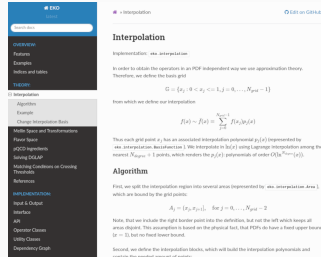
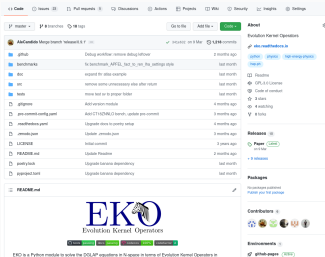
$$\mu_F^2 \frac{d}{d\mu_F^2} \mathbf{E}(\mu_F^2 \leftarrow \mu_{F,0}^2) = \mathbf{P}(a_s(\mu_R^2), \mu_F^2) \otimes \mathbf{E}(\mu_F^2 \leftarrow \mu_{F,0}^2)$$

with

$$\mathbf{f}(\mu_F^2) = \mathbf{E}(\mu_F^2 \leftarrow \mu_{F,0}^2) \otimes \mathbf{f}(\mu_{F,0}^2)$$

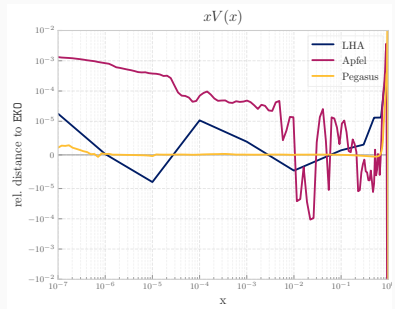
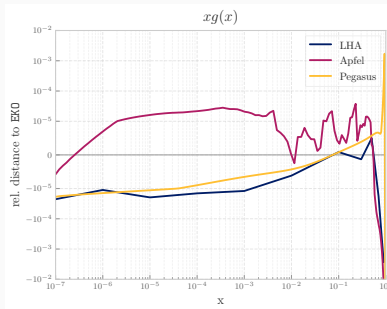
- independent of boundary condition $\rightarrow$ PDF fitting
- Mellin (N -) space solution, but momentum (x -) space delivery via piecewise Lagrange-interpolation
- Intrinsic heavy quark distributions $\rightarrow$ see part 2
- Backward VFNS evolution (i.e. across thresholds and with intrinsic) $\rightarrow$ see part 2

EKO Project Management



- Fully open source: <https://github.com/N3PDF/eko>
- Written in Python
- Fully documented: <https://eko.readthedocs.io/>
- A cornerstone in a new theory prediction suite → see part 4

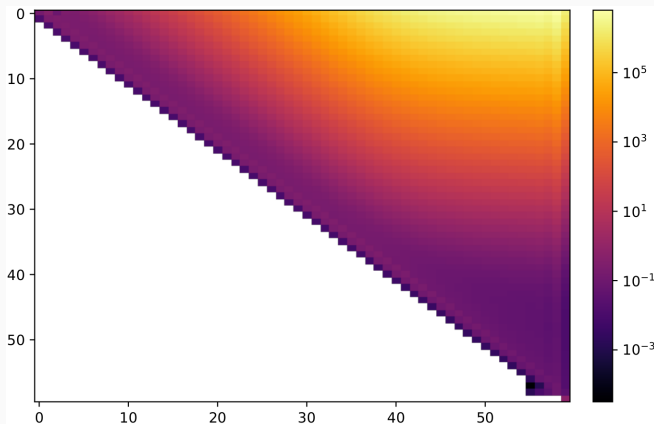
LHA benchmark [G+02][D+05]:



⇒ EKO is working!

EKO Snapshot: $\Sigma \leftarrow \Sigma$

FFNS LO LHA settings: $\Sigma(Q^2 = 10^4 \text{ GeV}^2) \leftarrow \Sigma(Q^2 = 2 \text{ GeV}^2)$



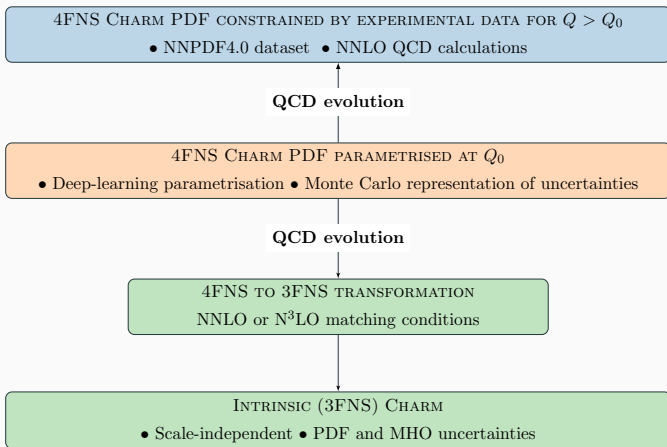
xgrid: $10^{-7}(0) \rightarrow \log \rightarrow \text{lin} \rightarrow 1(59)$,
axis: $x \sim \text{input } (2 \text{ GeV}^2)$, $y \sim \text{output } (10^4 \text{ GeV}^2)$

2. Intrinsic Charm in the Proton

[submitted]

Intrinsic Charm: Strategy

- see poster by G. Magni
- based on NNPDF4.0 [[arxiv:2109.02653](https://arxiv.org/abs/2109.02653)] - see talk by R. Stegeman



Matching Conditions and Backward Evolution

For (forward) evolution across a matching scale μ_h^2 :

$$\tilde{\mathbf{f}}^{(n_f+1)}(\mu_{F,1}^2) = \tilde{\mathbf{E}}^{(n_f+1)}(\mu_{F,1}^2 \leftarrow \mu_h^2) \mathbf{R}^{(n_f)} \tilde{\mathbf{A}}^{(n_f)}(\mu_h^2) \tilde{\mathbf{E}}^{(n_f)}(\mu_h^2 \leftarrow \mu_{F,0}^2) \\ \times \tilde{\mathbf{f}}^{(n_f)}(\mu_{F,0}^2)$$

with $\mathbf{R}^{(n_f)}$ a flavor rotation matrix and $\tilde{\mathbf{A}}^{(n_f)}(\mu_h^2)$ the operator matrix elements (partially known up to N³LO)

Matching Conditions and Backward Evolution

For (forward) evolution across a matching scale μ_h^2 :

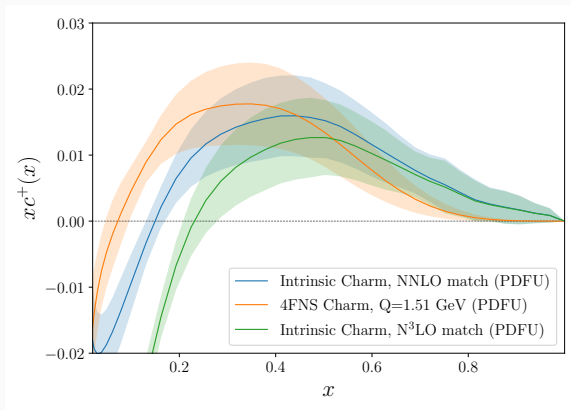
$$\tilde{\mathbf{f}}^{(n_f+1)}(\mu_{F,1}^2) = \tilde{\mathbf{E}}^{(n_f+1)}(\mu_{F,1}^2 \leftarrow \mu_h^2) \mathbf{R}^{(n_f)} \tilde{\mathbf{A}}^{(n_f)}(\mu_h^2) \tilde{\mathbf{E}}^{(n_f)}(\mu_h^2 \leftarrow \mu_{F,0}^2) \\ \times \tilde{\mathbf{f}}^{(n_f)}(\mu_{F,0}^2)$$

with $\mathbf{R}^{(n_f)}$ a flavor rotation matrix and $\tilde{\mathbf{A}}^{(n_f)}(\mu_h^2)$ the operator matrix elements (partially known up to N³LO)

for backward evolution:

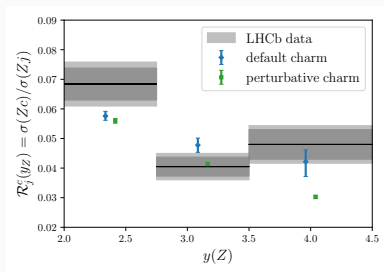
- invert $\tilde{\mathbf{E}}^{(n_f)}$: simple
- invert $\mathbf{R}^{(n_f)}$: simple
- invert $\tilde{\mathbf{A}}^{(n_f)}$: expanded or exact

Intrinsic Charm: PDF plot



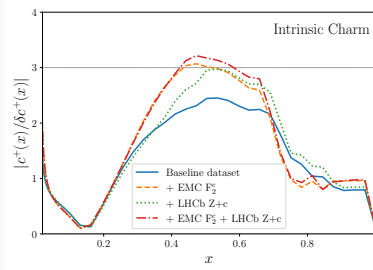
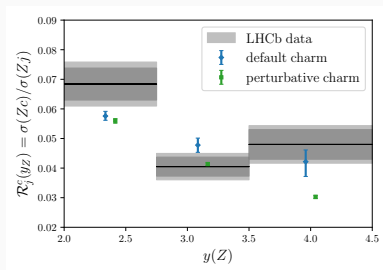
- in **3FNS** valence-like peak is still present
- for $x \leq 0.2$ the perturbative uncertainties are quite large
- the carried momentum fraction is within **1%**

Intrinsic Charm: LHCb and Significance



- match better recent **LHCb** $Z+c$ measurement [\[PRL128-082001\]](#)

Intrinsic Charm: LHCb and Significance



- match better recent **LHCb** $Z+c$ measurement [PRL128-082001]
- we find a **3σ** evidence of intrinsic charm
- result is **stable** with mass variation, dataset variation

3. yadism [in preparation]



- DIS coefficient function database
- independent of boundary condition $\rightarrow$ PDF fitting
- separate features: TMC, FNS, interpolation
- constant benchmark against APFEL (eventually discovering minor bugs there)

same improvement in terms of project management as EKO!

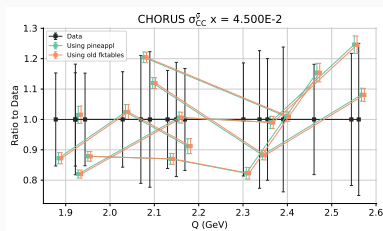
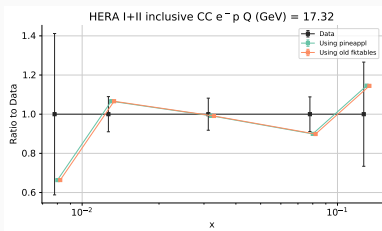
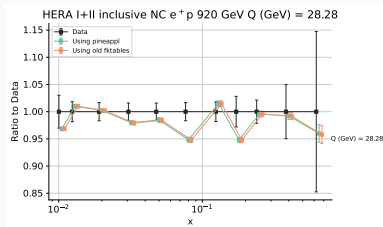
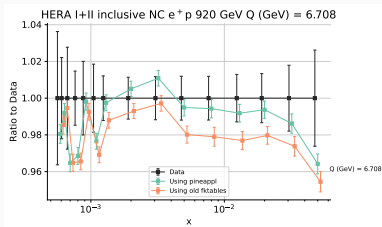
Coefficient Functions

- implemented coefficient functions:

	light	heavy	intrinsic
NC	$O(a_s^2)$ [VVM05,MVV05,MV00]	$O(a_s^2)$ [Hek19]	$O(a_s)$ [KS98]
CC	$O(a_s^2)$ [MRV08,MVV09]	$O(a_s)$ [GKR96]	$O(a_s)$ [in prep.]

- for CC intrinsic: see talk by K. Kudashkin
- implemented flavor number schemes: FFNS, ZM-VFNS, FONLL

Comparison yadism against APFEL



green, "pineappl" = yadism vs. orange, "old" = APFEL

4. Theory Prediction Pipeline

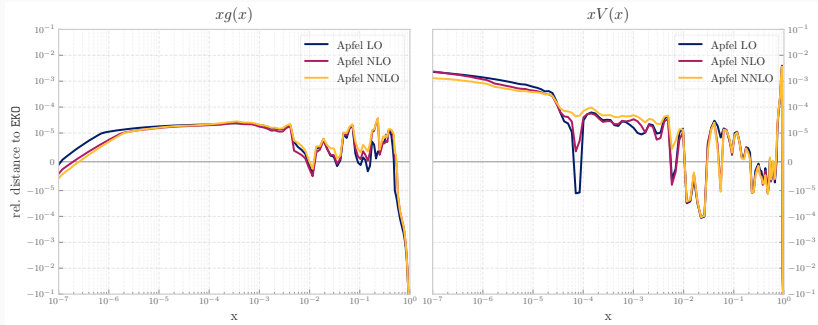
New Theory Prediction Pipeline

- We're about to develop a new pipeline for theory predictions around PineAPPL [\[arXiv:2008.12789\]](#)
- both, EKO and yadism, are interfaced with PineAPPL
- PineAPPL also has interfaces to mg5amc@nlo, APPLgrid, FastNLO
- aim: produce FastKernel tables used in PDF fitting

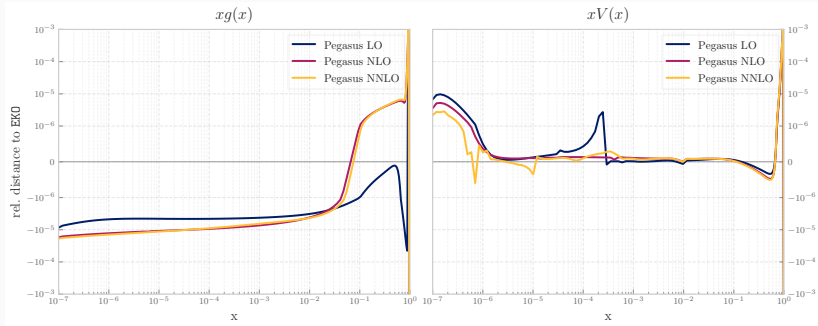
Thank you!

5. Backup slides

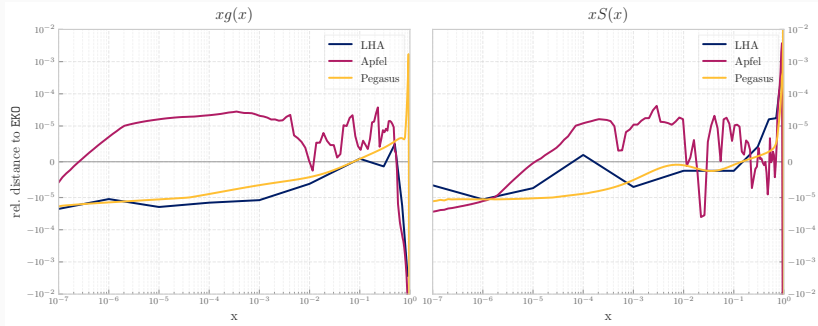
EKO APFEL benchmark



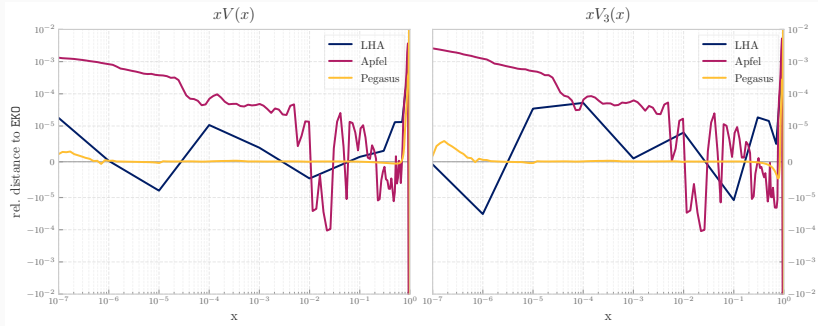
EKO PEGASUS benchmark



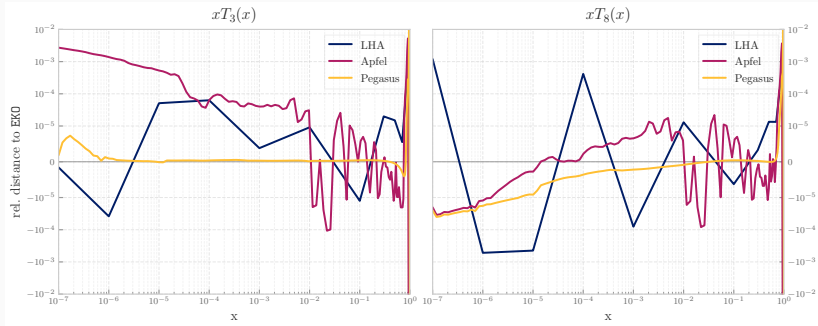
EKO LHA benchmark: g and Σ



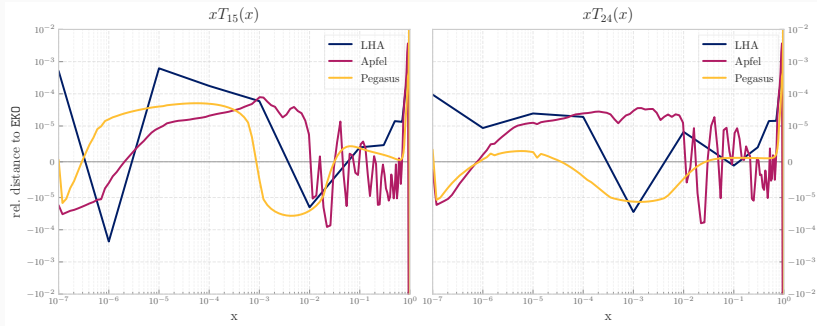
EKO LHA benchmark: V and V_3



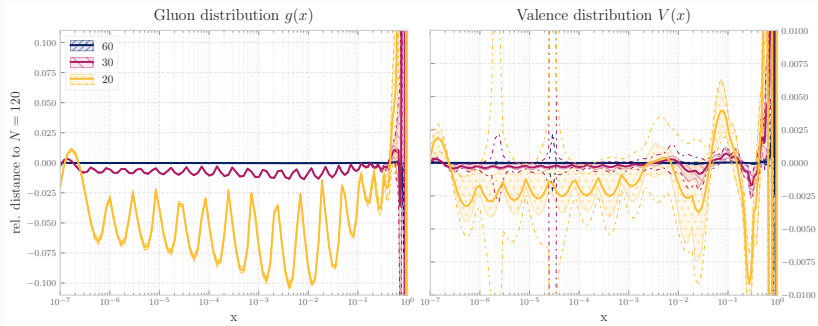
EKO LHA benchmark: T_3 and T_8



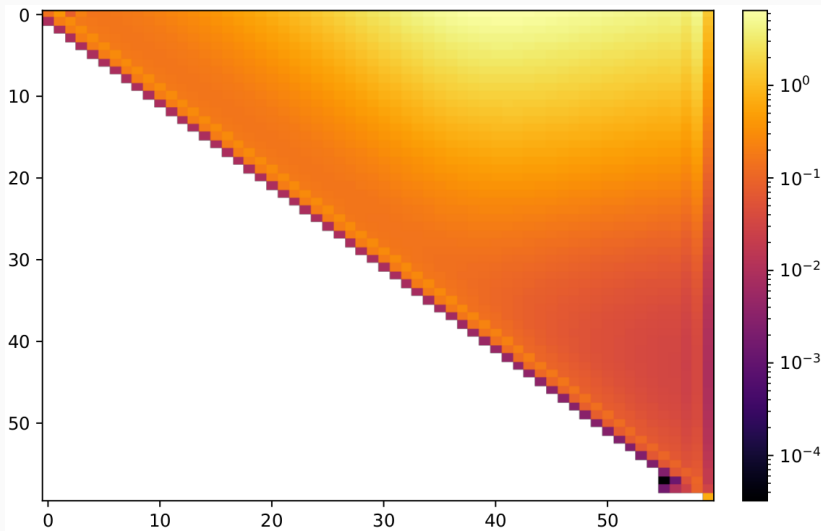
EKO LHA benchmark: T_{15} and T_{24}



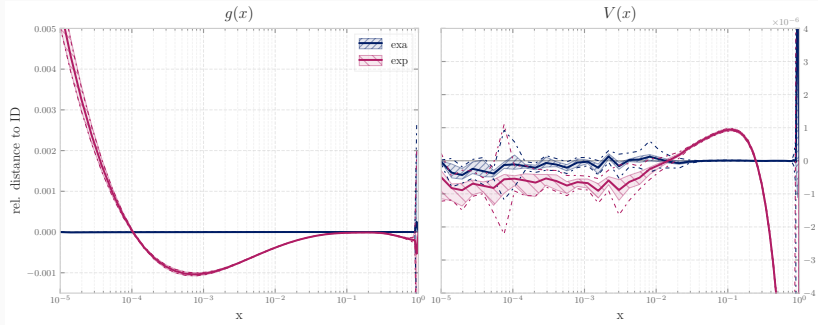
EKO Interpolation Error



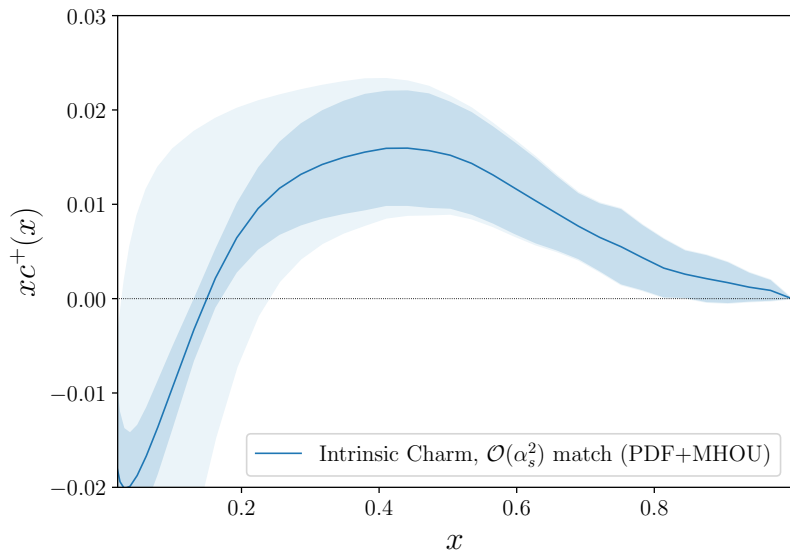
EKO Snapshot $V \leftarrow V$



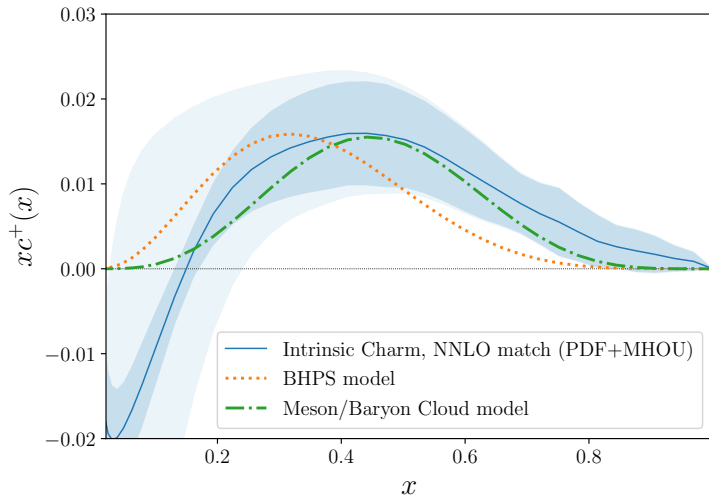
EKO Backward Evolution



IC - all uncertainties combined

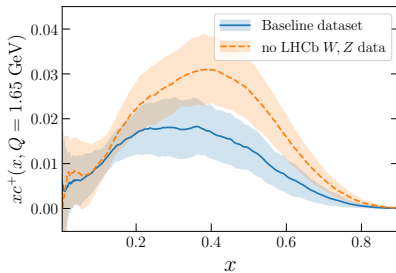
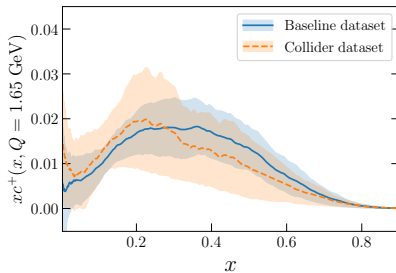
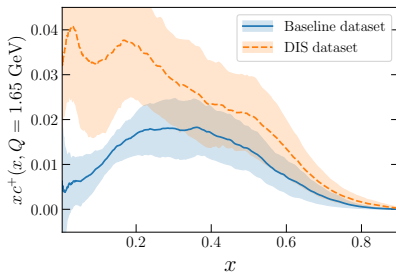
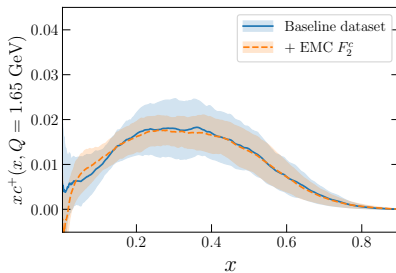


IC - model comparison

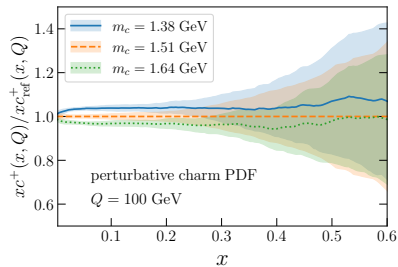
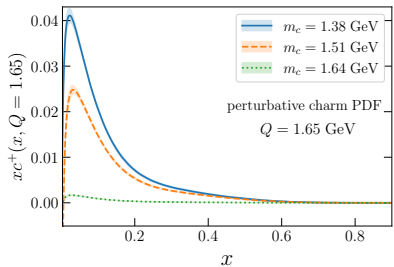
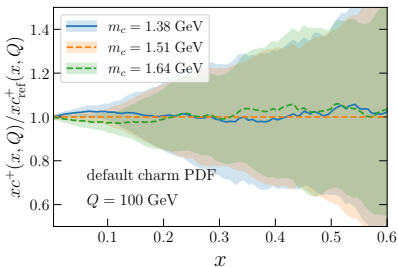
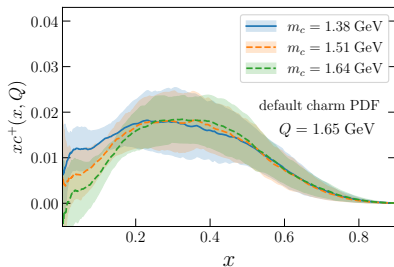


[BHPS] or [Meson/Baryon Cloud Model]

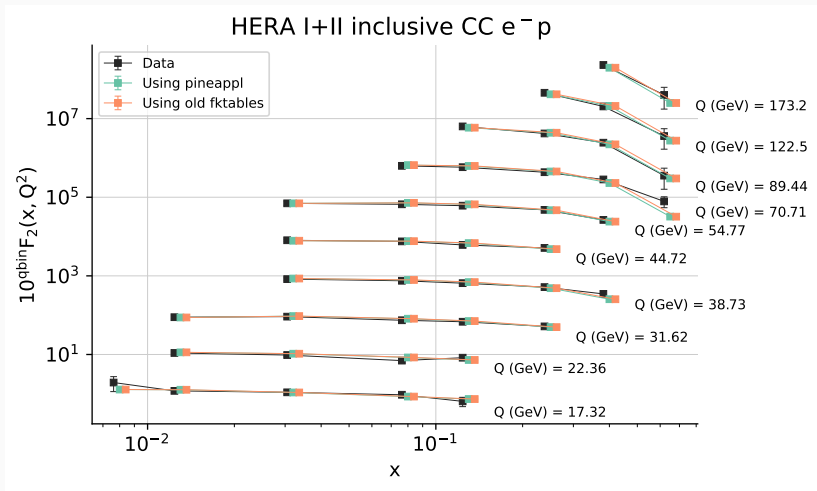
IC - dataset variation



IC - mass variation



Comparison yadism against APFEL



Comparison yadism against APFEL

