Precision QCD at Colliders

Juan Rojo, VU Amsterdam & Nikhef

QCD at hadron colliders

Master Formula for **LHC cross-sections:**

$$\sigma_{\rm LHC}(M,s) \propto \sum_{ij} \int_{M^2}^{s} d\hat{s}\ \mathscr{L}_{ij}(\hat{s},s)\ \widetilde{\sigma}_{ij}(\hat{s},\alpha_s(M))\,, \quad i,j = u,d,s,g,\ldots$$

partonic luminosity (non-perturbative QCD, phenomenological extraction from data)

hard-scattering matrix element (perturbative QCD, evaluate from Feynman diagrams)

$$\mathscr{L}_{ij}(Q,s) = \frac{1}{s}\int_{Q^2/s}^{1} \frac{dx}{x} f_i\left(\frac{Q^2}{sx},Q\right) f_j\,(x,Q)\,,$$

proton Parton Distribution Functions (PDFs)

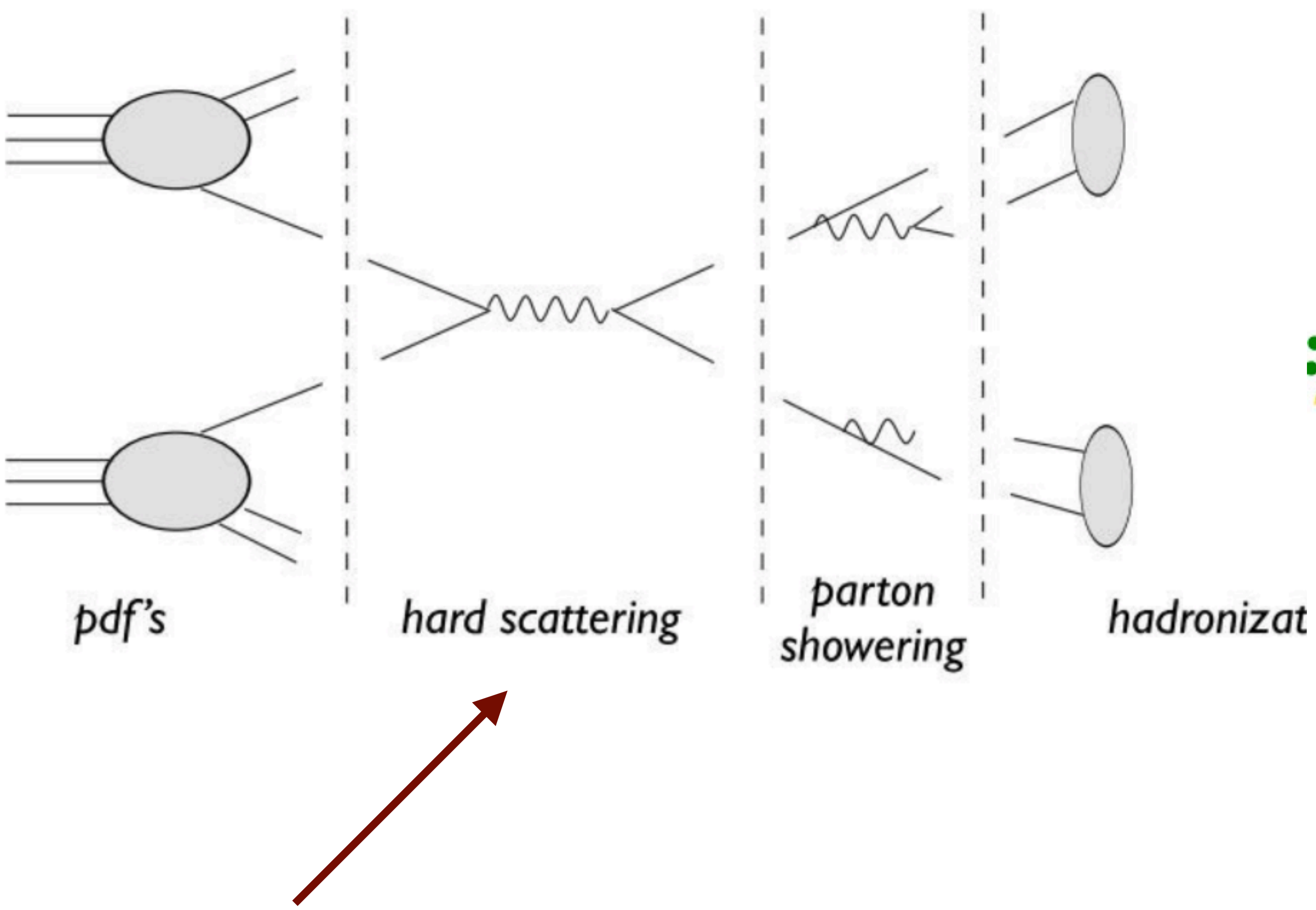
$$f_j\,(x,Q)$$

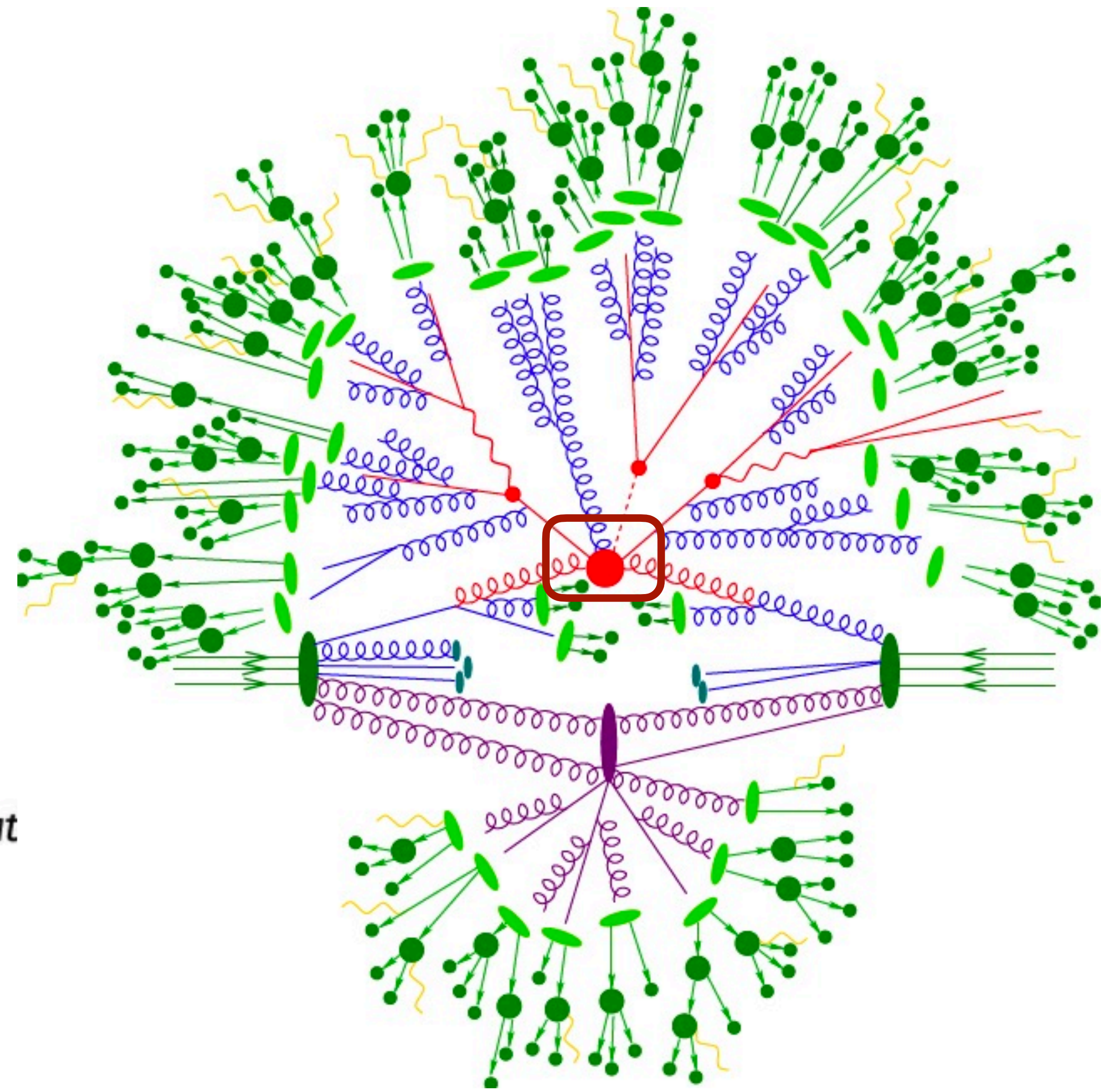
flavour index — *momentum fraction* — *energy scale of partonic scattering*

QCD at hadron colliders

Master Formula for **LHC cross-sections:**

$$\sigma_{\text{LHC}}(M,s) \propto \sum_{ij} \int_{M^2}^{s} d\hat{s}\ \mathscr{L}_{ij}(\hat{s},s)\, \tilde{\sigma}_{ij}(\hat{s},\alpha_s(M))\,, \quad i,j = u,d,s,g,\ldots$$





QCD at hadron colliders

focus on fixed-order QCD and EW **hard-scattering partonic cross-sections**

$$\sigma_{\rm LHC}(M,s) \propto \sum_{ij} \int_{M^2}^{s} d\hat{s}\ \mathscr{L}_{ij}(\hat{s},s)\, \tilde{\sigma}_{ij}(\hat{s},\alpha_s(M))\,, \quad i,j = u,d,s,g,\ldots$$

Other topics relevant for **precision QCD at colliders** covered in the talks by:

- PDFs and proton structure: ***Jun Gao***
- Parton showers and MC generators: ***Mrinal Dasgupta & Ilkka Helenius***
- Small-x and BFKL physics: ***Bowen Xiao***

My goal is to display **representative results** without being exhaustive.
Apologies in advance if your favourite calculation is missing!

Recent reviews on higher-order QCD & EW calculations:

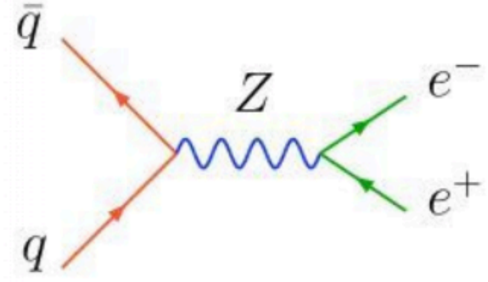
- *Collider Physics at the Precision Frontier*, G. Heinrich (2009.00516)
- *The path forward to N3LO,* F. Caola et al (2203.06730)
- *Rene Poncelet's review talk* at SM@LHC2022

The structure of higher orders

$$\widetilde{\sigma}(\alpha_s, \alpha) = \widetilde{\sigma}^{(0)}(\alpha_s, \alpha)$$

QCD coupling ***EW coupling*** ***Born (tree-level)***

LO

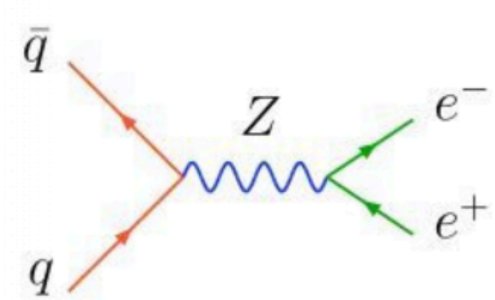


The structure of higher orders

$$\widetilde{\sigma}(\alpha_s, \alpha) = \widetilde{\sigma}^{(0)}(\alpha_s, \alpha)\left(1 + c_{1,0}\alpha_s + c_{0,1}\alpha + \ldots\right)$$

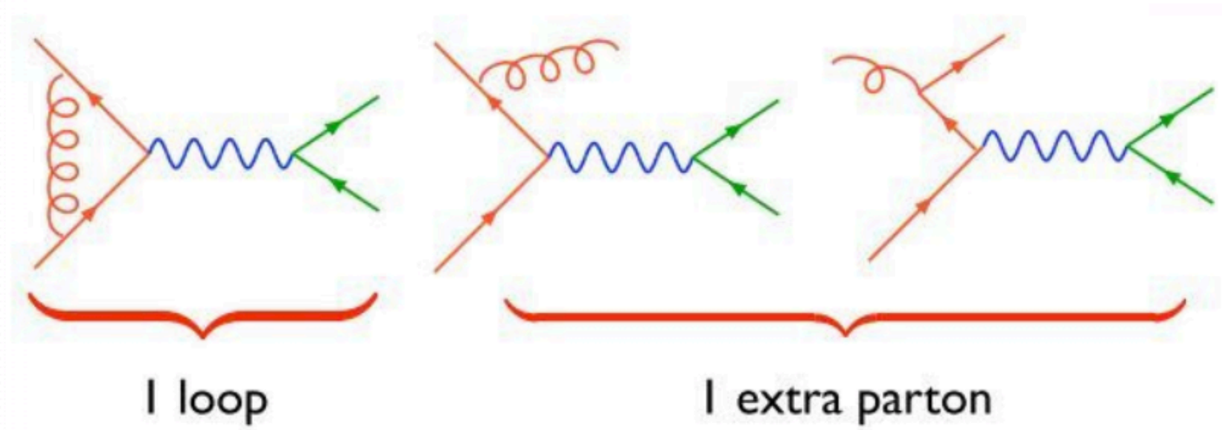
QCD coupling ***EW coupling*** ***Born (tree-level)*** ***NLO QCD correction*** ***NLO EW correction***

LO



virtual corrections *real emission* *new initial states*

NLO QCD

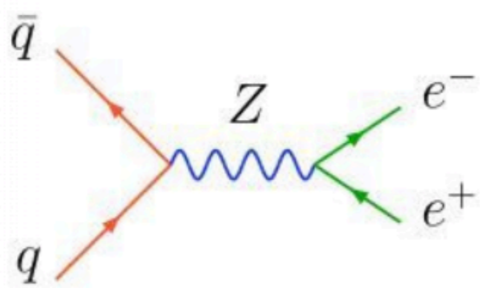


The structure of higher orders

$$\widetilde{\sigma}(\alpha_s, \alpha) = \widetilde{\sigma}^{(0)}(\alpha_s, \alpha)\Big(1 + c_{1,0}\alpha_s + c_{0,1}\alpha + c_{2,0}\alpha_s^2 + c_{3,0}\alpha_s^3 + \ldots\Big)$$

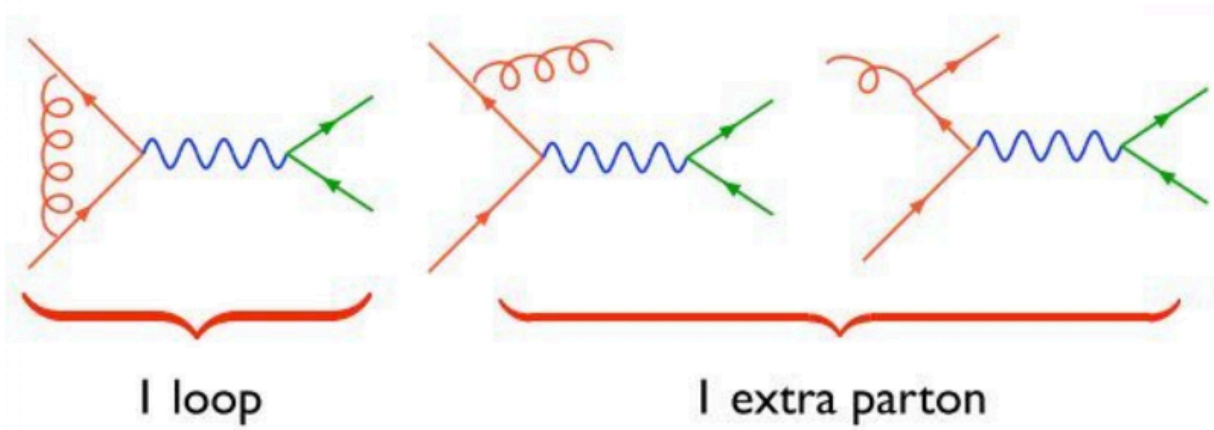
QCD coupling ***EW coupling*** ***Born (tree-level)*** ***NNLO QCD correction*** ***N3LO QCD correction***

LO

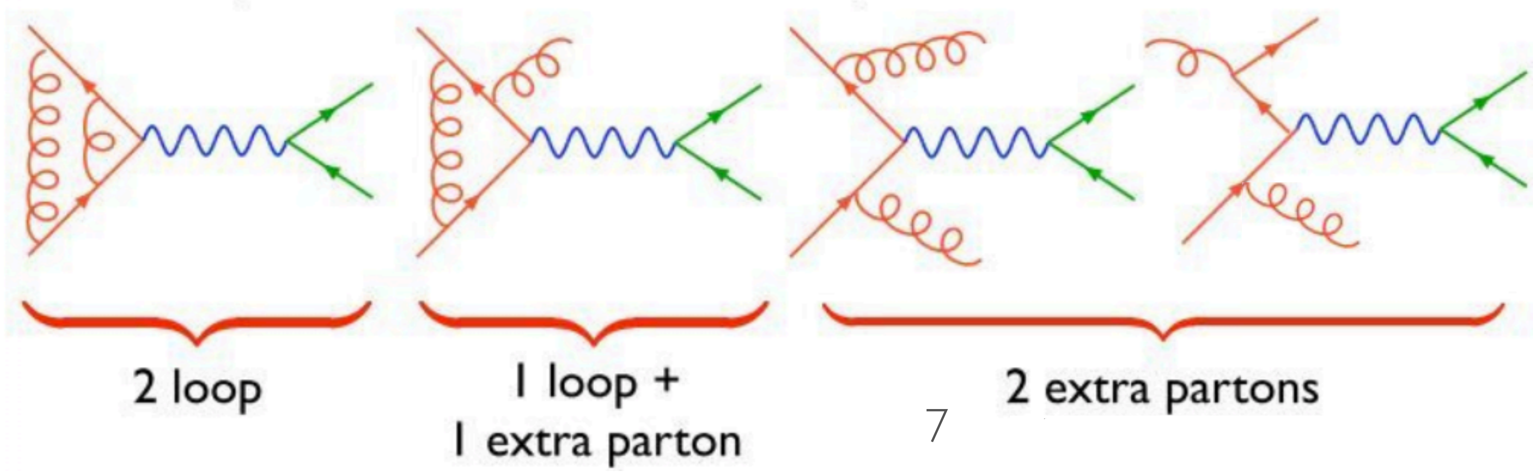


virtual corrections *real emission* *new initial states*

NLO QCD



NNLO QCD



double real, double virtual, real/virtual …

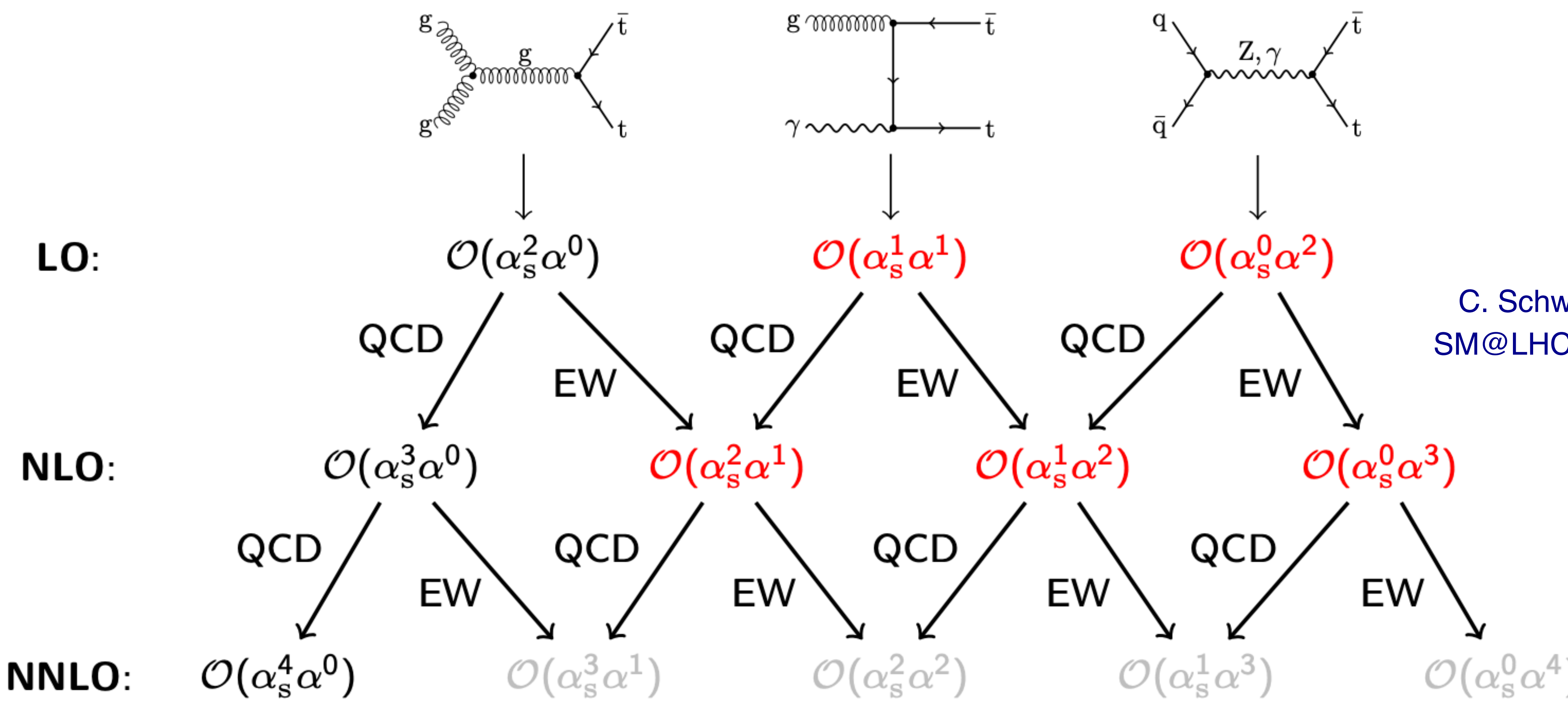
The structure of higher orders

$$\tilde{\sigma}(\alpha_s, \alpha) = \tilde{\sigma}^{(0)}(\alpha_s, \alpha)\Big(1 + c_{1,0}\alpha_s + c_{0,1}\alpha + c_{2,0}\alpha_s^2 + c_{3,0}\alpha_s^3 + c_{1,1}\alpha_s\alpha + c_{0,2}\alpha^2 + \ldots\Big)$$

NNLO mixed QCD/QED correction

NNLO EW correction

+ new ``LO" terms arise in the presence of EW corrections …



C. Schwan, SM@LHC2022

The structure of higher orders

$$\widetilde{\sigma}(\alpha_s, \alpha) = \widetilde{\sigma}^{(0)}(\alpha_s, \alpha)\Big(1 + c_{1,0}\alpha_s + c_{0,1}\alpha + c_{2,0}\alpha_s^2 + c_{3,0}\alpha_s^3 +$$
$$c_{1,1}\alpha_s\alpha + c_{0,2}\alpha^2 + \dots\Big)$$

+ new ``LO'' terms arise in the presence of EW corrections …

NNLO mixed QCD/QED correction

NNLO EW correction

power counting

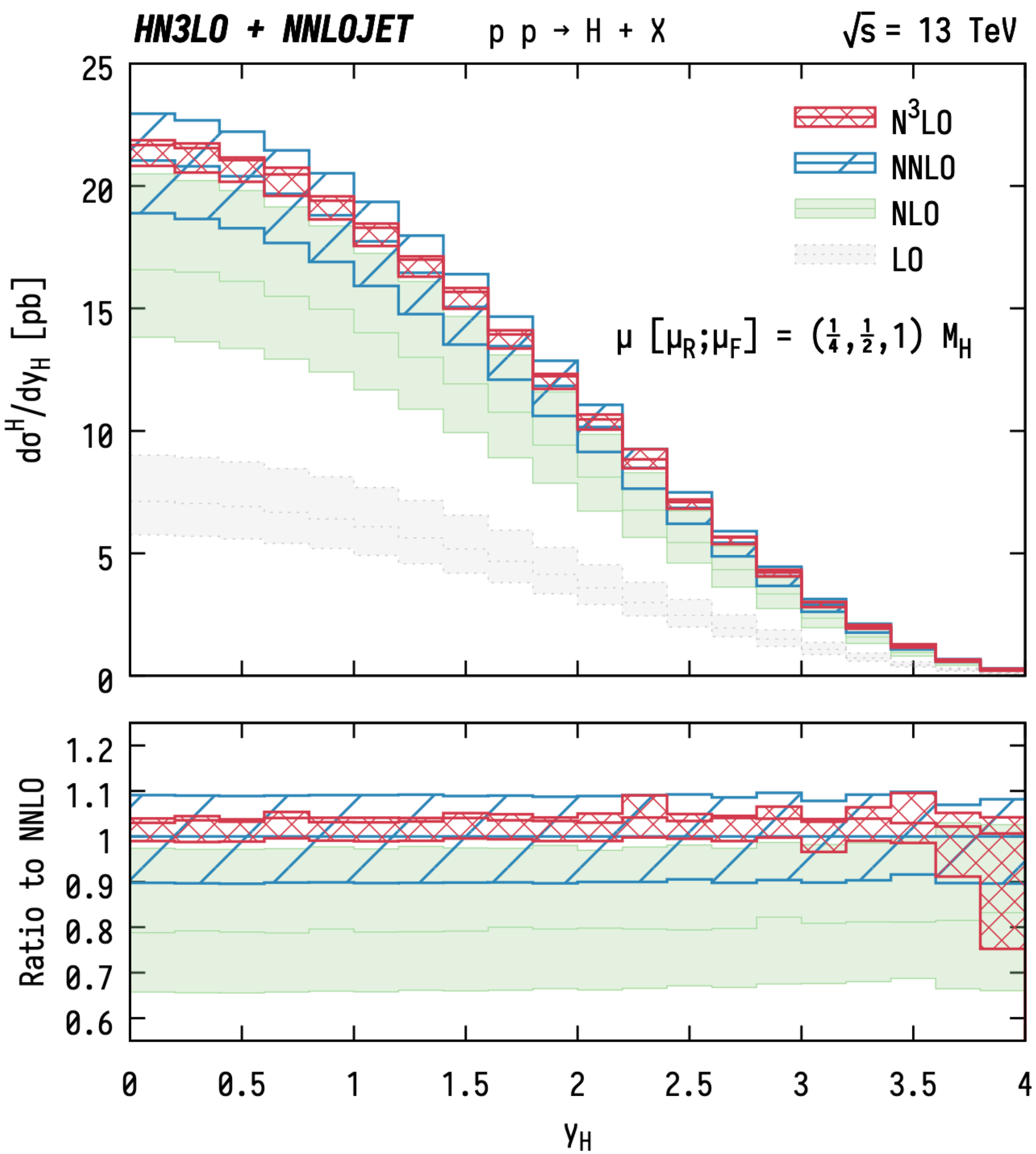
$$\alpha_s \sim 0.1, \quad \alpha \sim 0.01$$

Naive power counting often not reliable estimate relative impact of various terms, due to e.g.

- large electroweak (**Sudakov**) logarithms at high energy
- new **partonic subprocesses** becoming accessible at higher orders
- symmetry or accidental **cancelations** in the lower-order terms
- various types of **kinematic enhancements/suppressions**, observable-dependent

The structure of higher orders

Why **higher-order QCD calculations** are important?



Chen et al, 2102.07607

Fully differential N3LO **Higgs in gluon-fusion**

$$\tilde{\sigma}(\alpha_s, \alpha) = \tilde{\sigma}^{(0)} \left(1 + c_{1,0}\alpha_s + c_{2,0}\alpha_s^2 + c_{3,0}\alpha_s^3\right)$$

NLO ***NNLO*** ***N3LO***

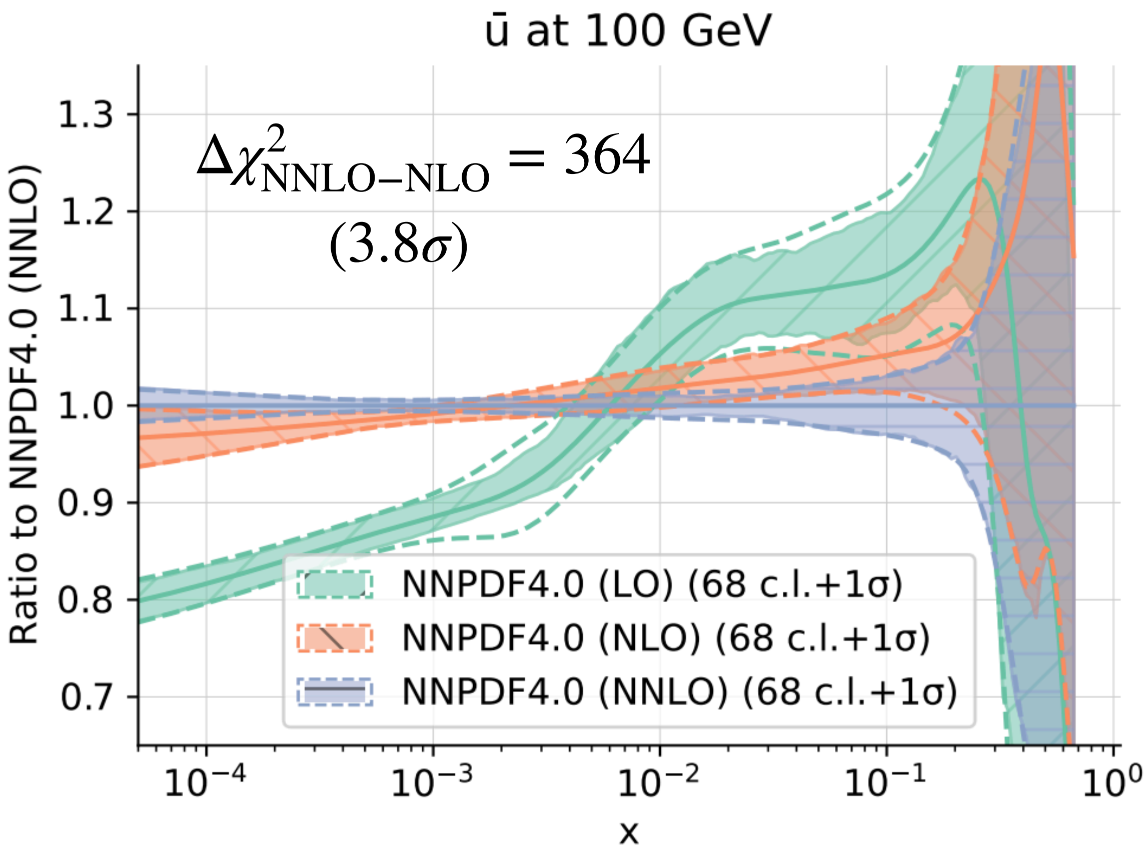
- Improved **precision & accuracy**: enhance physics reach of the same measurement
- Reliable estimate of **missing higher-order uncertainties** (MHOUs)
- Assess convergence of **perturbative expansion**

For Higgs rapidity distribution in gluon fusion:

- NLO: first sensible estimate of MHOUs
- NNLO: required for O(10%) precision
- N3LO: required for **few-percent** precision
- Good convergence of perturbative expansion

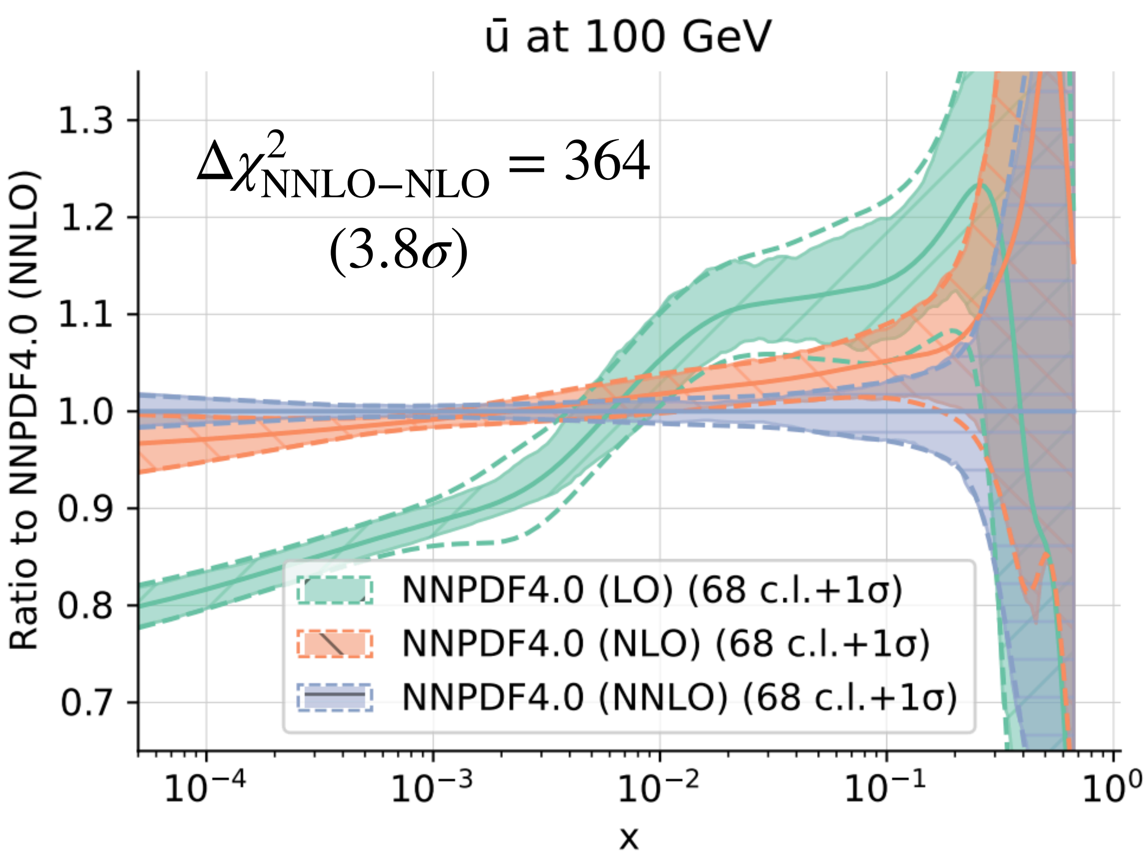
Why precision at colliders matters?

PDF determinations

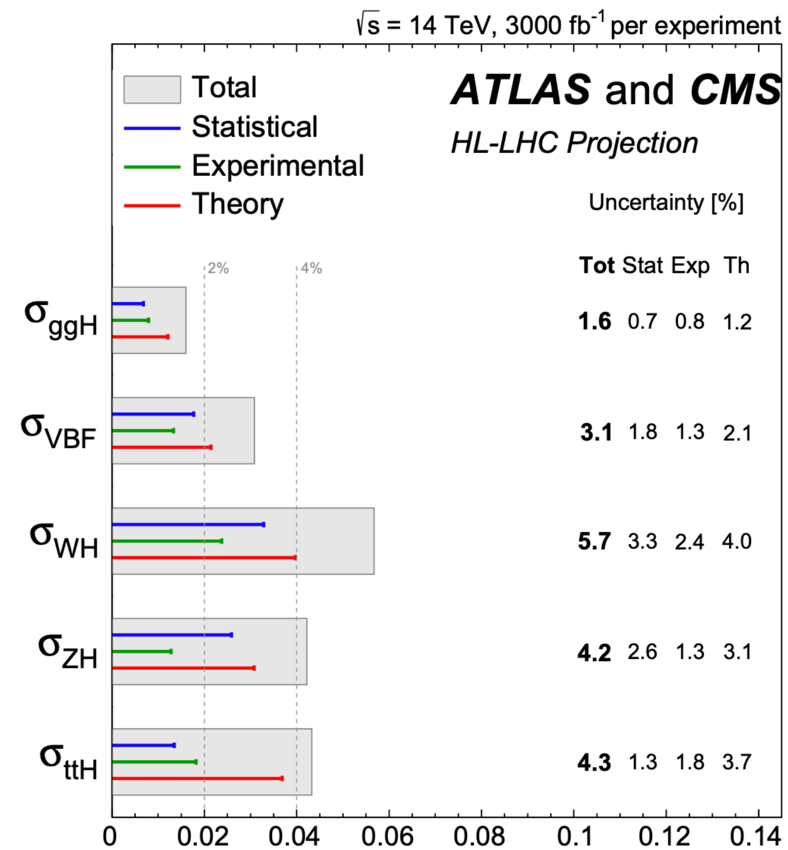


Why precision at colliders matters?

PDF determinations



Fingerprinting the Higgs sector



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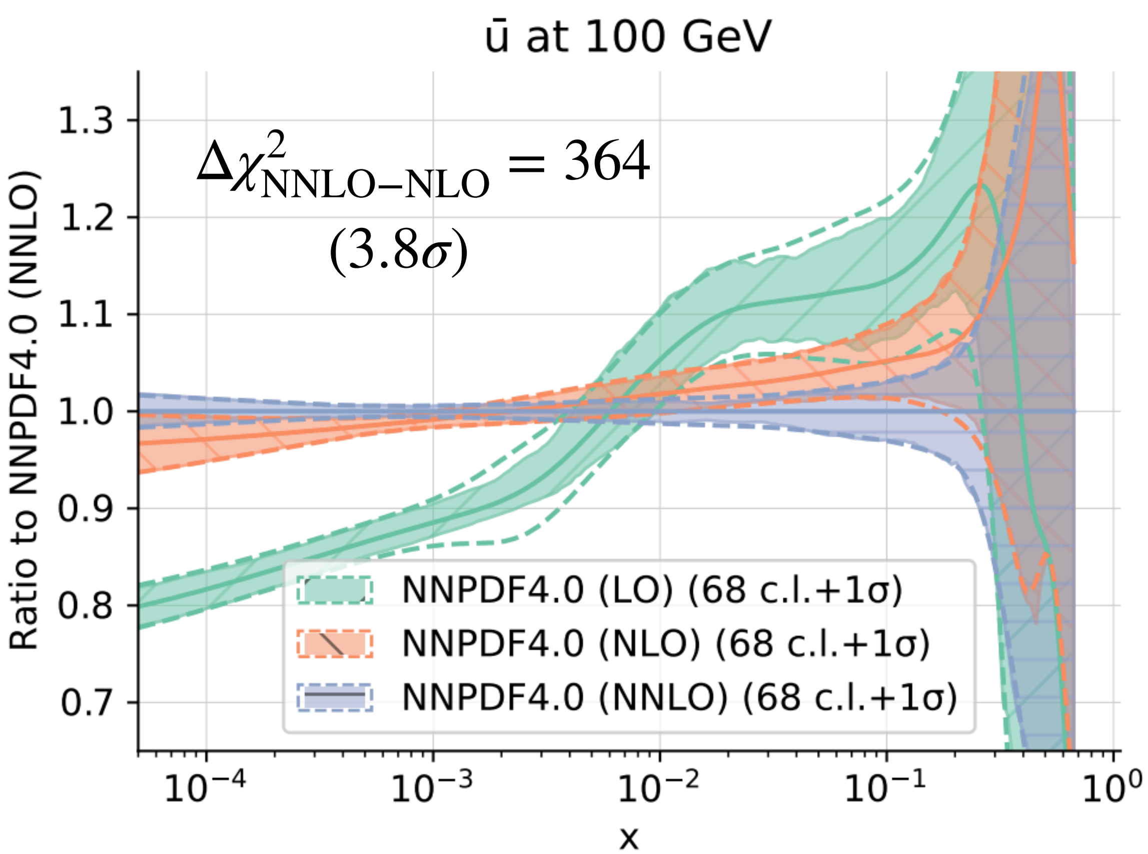
PDF determinations



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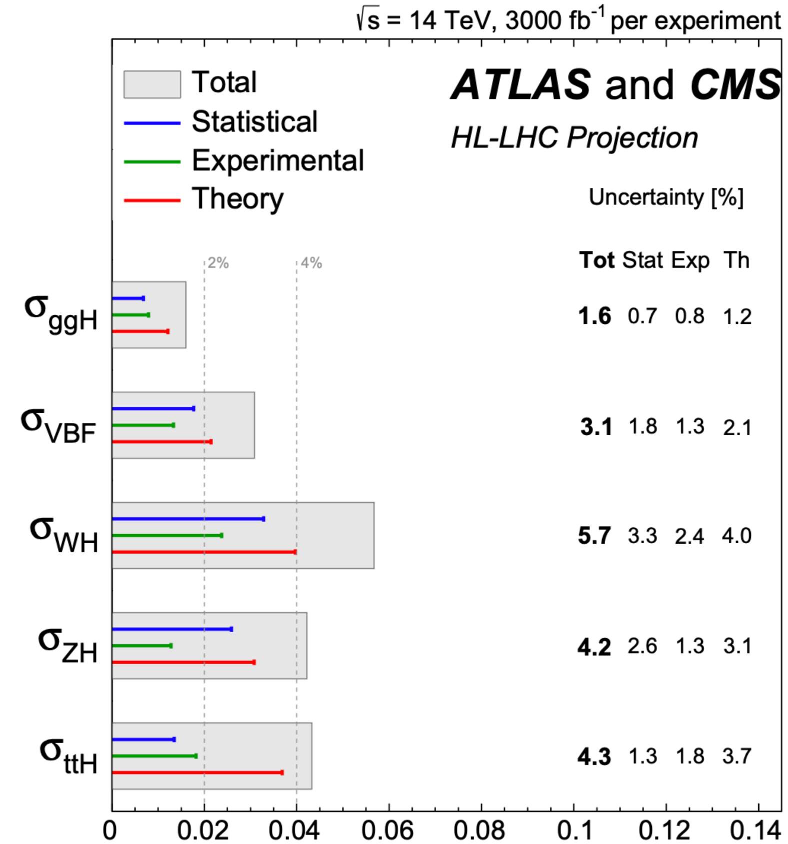
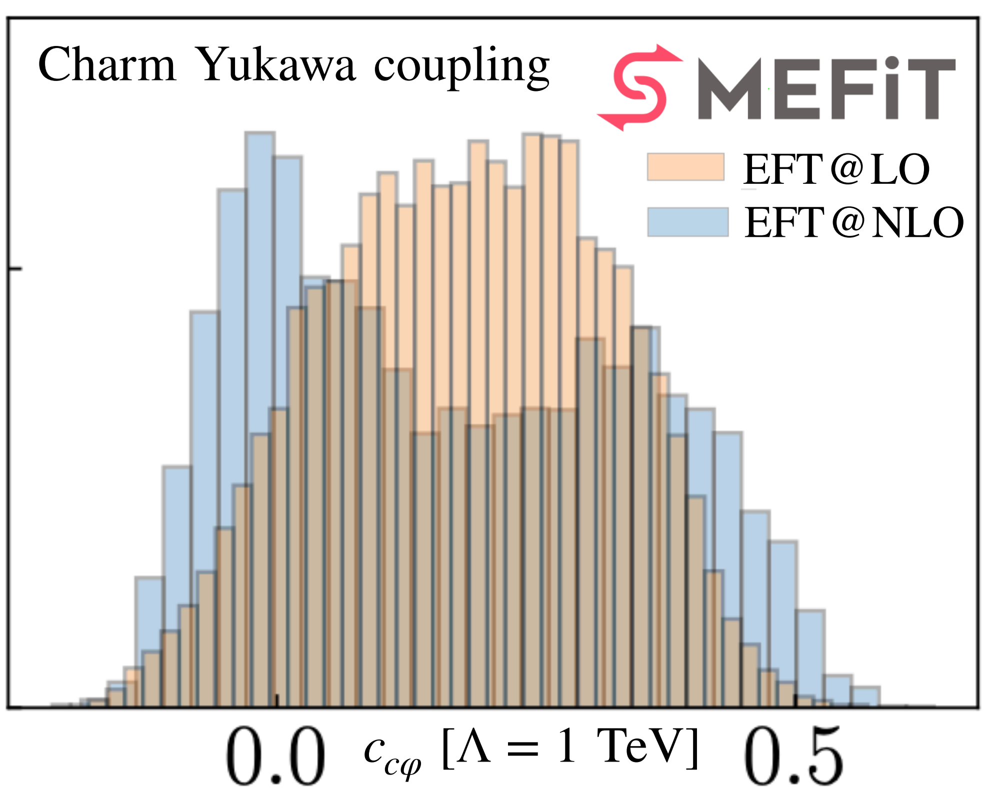
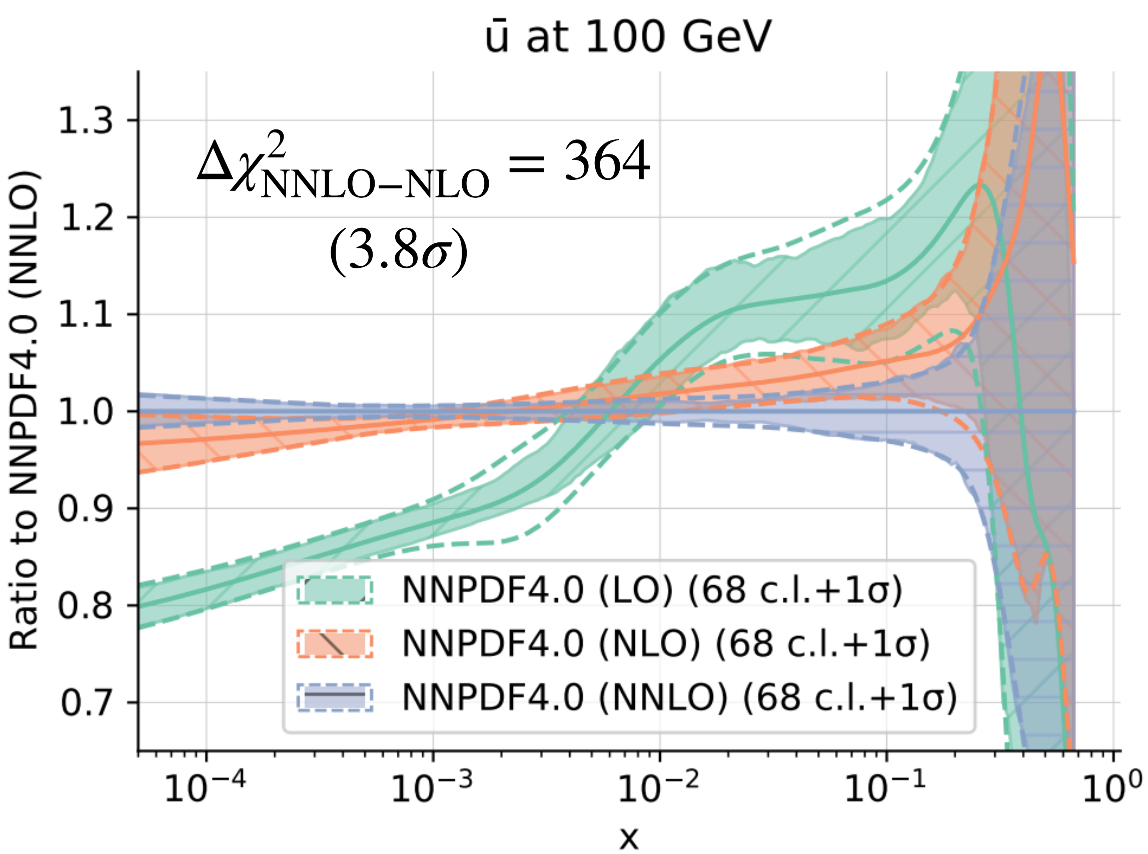


Model-independent (EFT) searches



Why precision at colliders matters?

PDF determinations



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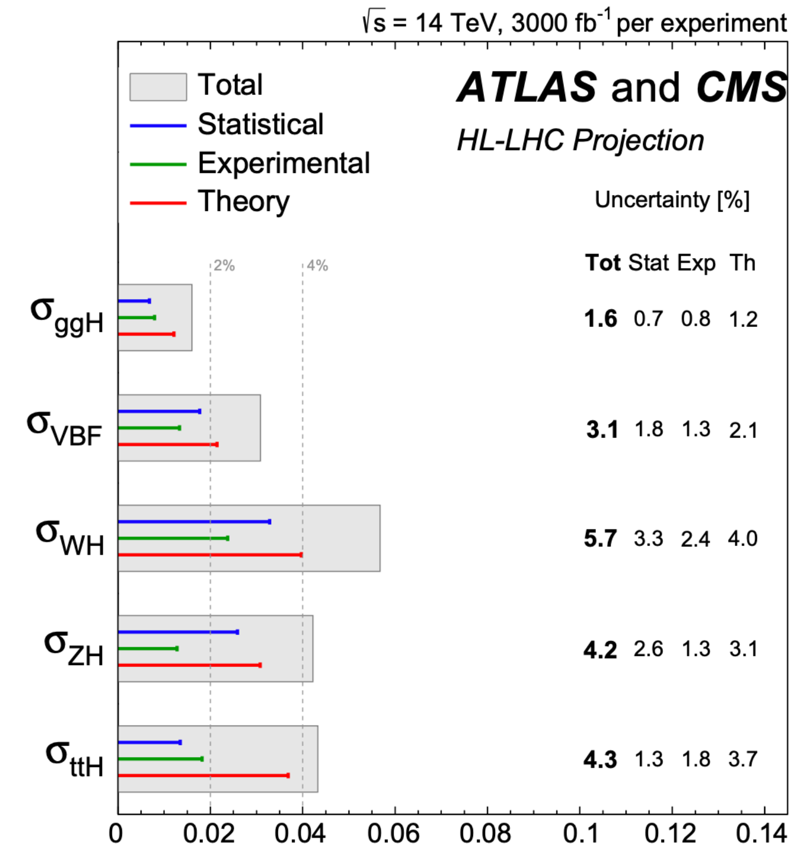

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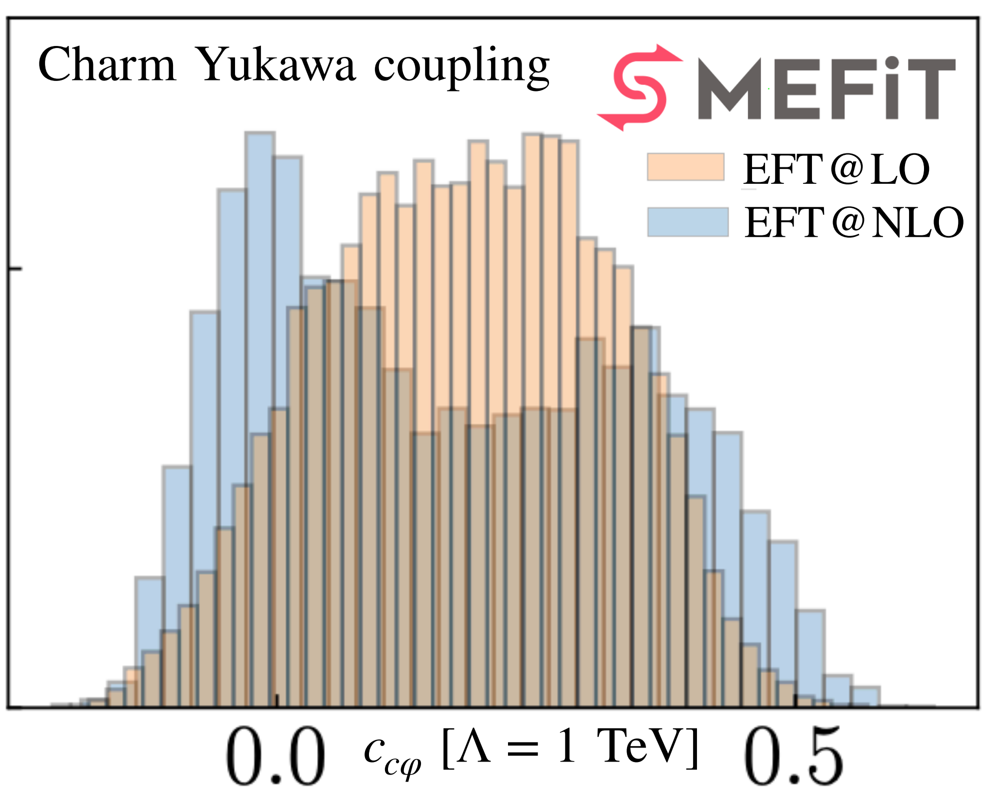
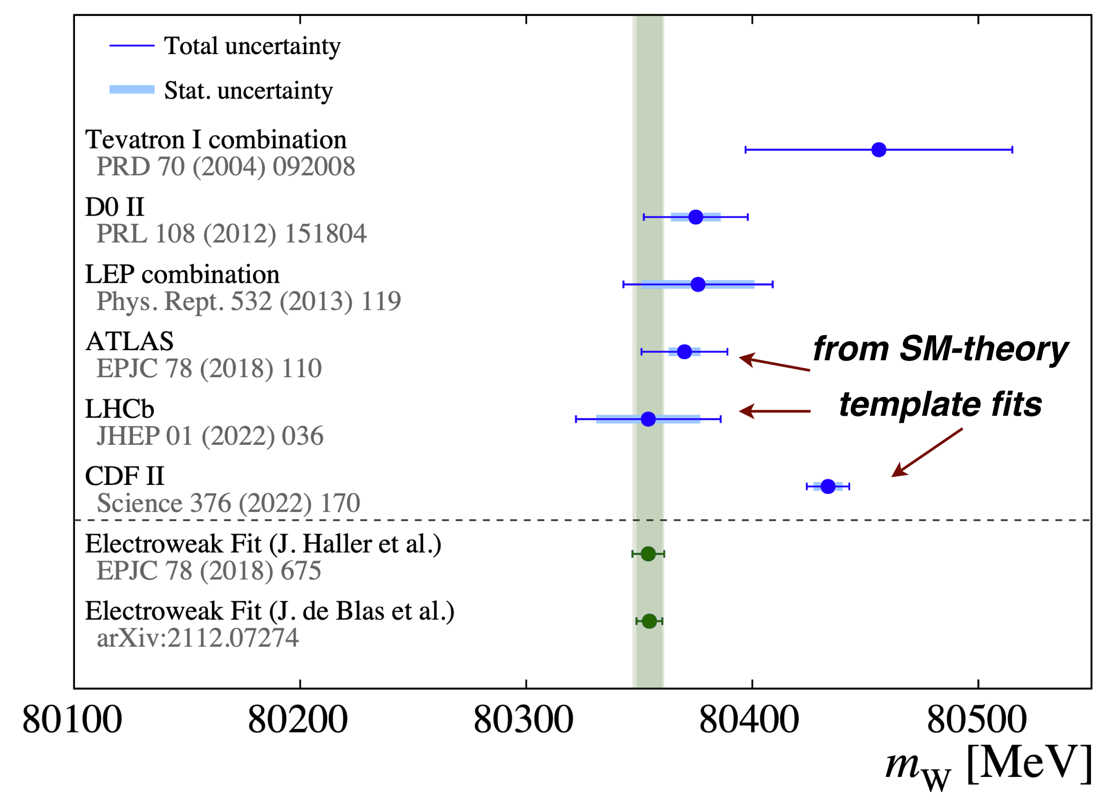
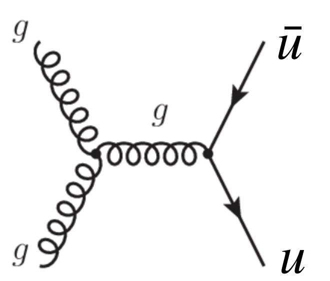
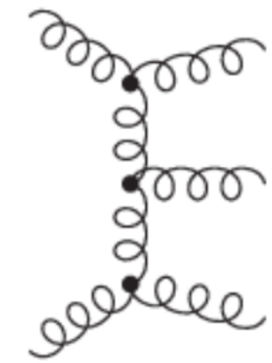


Extraction of fundamental SM parameters

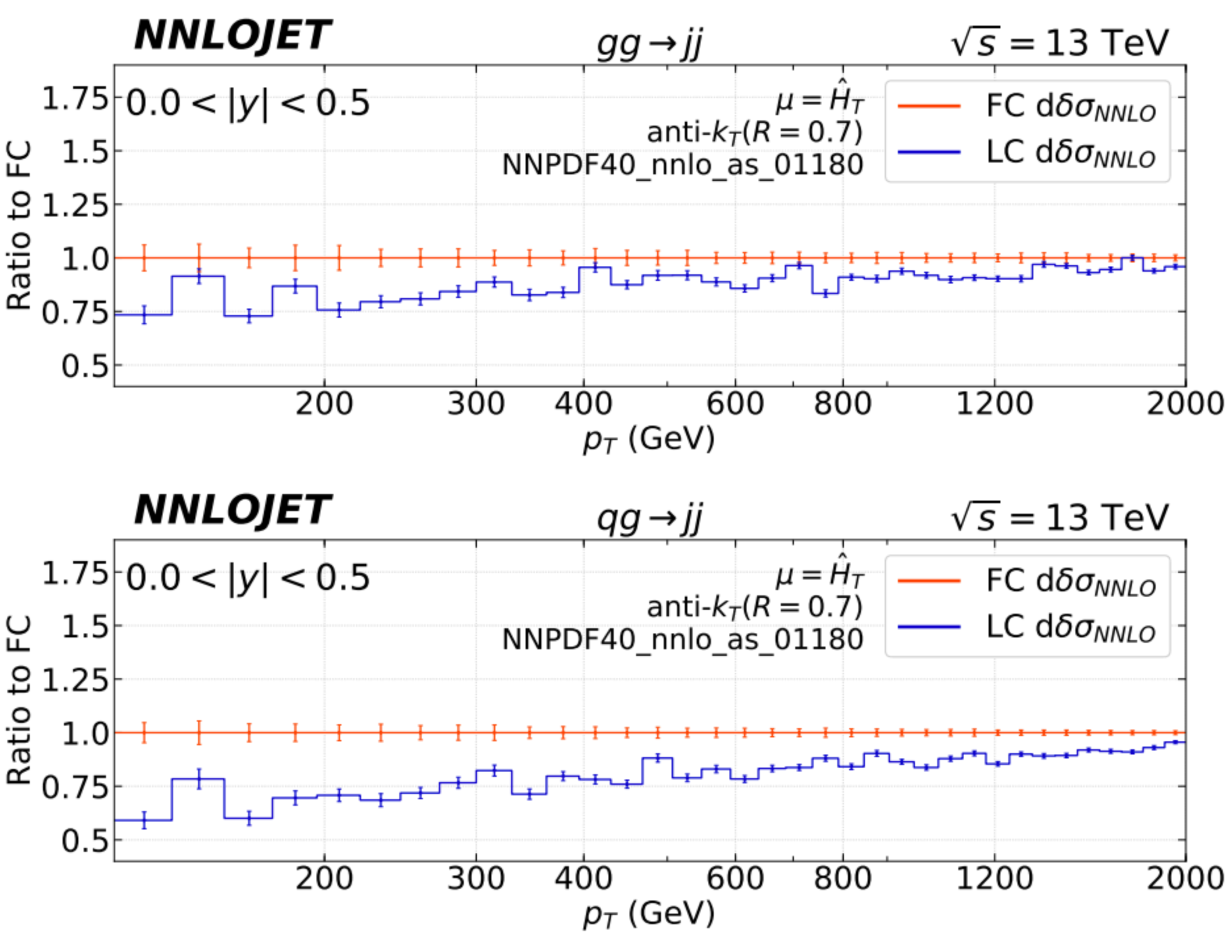


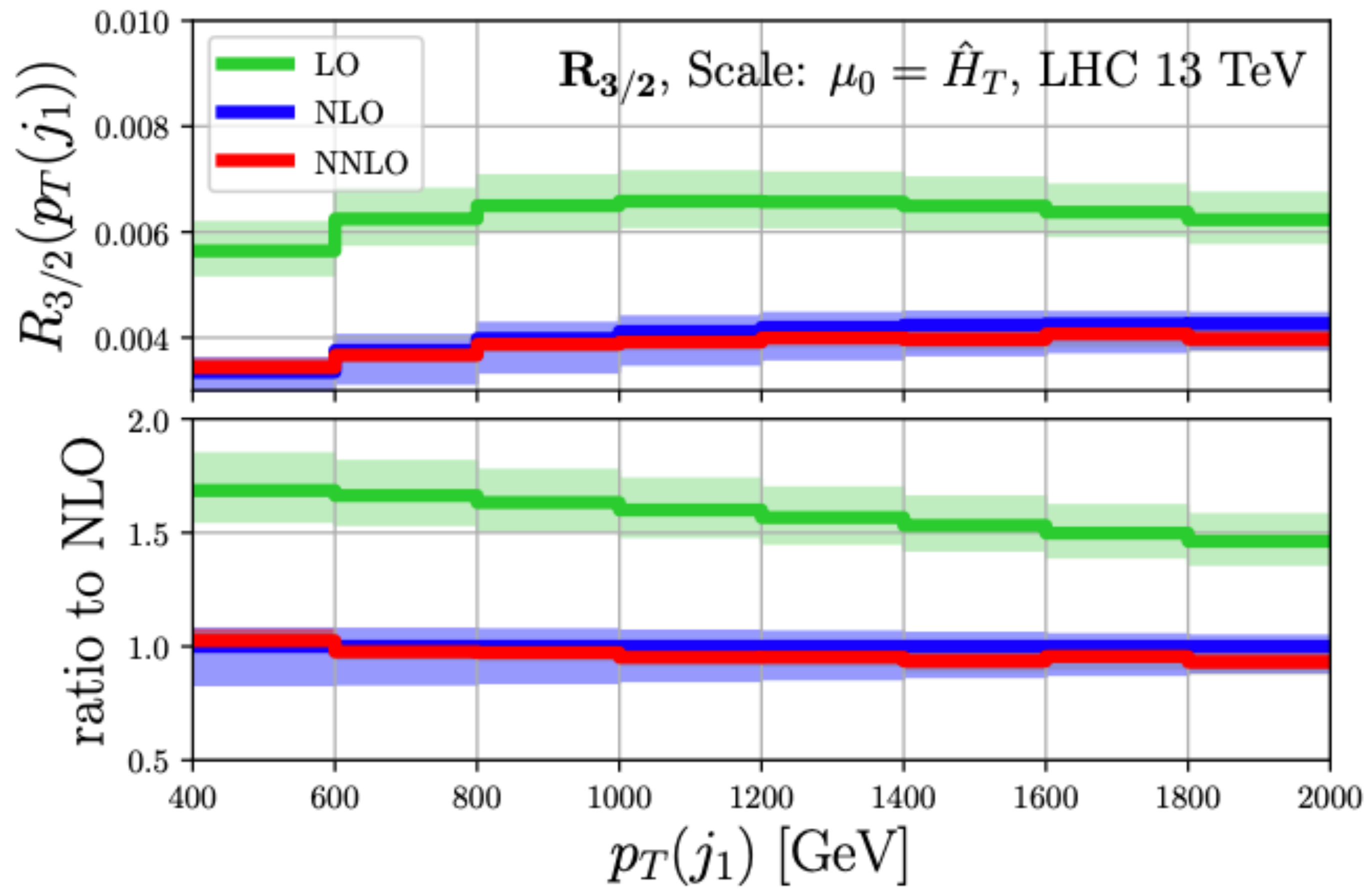


NNLO QCD for jet production



$$R_{3/2}(X, \mu_R, \mu_F) = \frac{d\sigma_3(\mu_R, \mu_F)/dX}{d\sigma_2(\mu_R, \mu_F)/dX}$$





NNLOJET, Chen et al, 2204.10173

Czakon et al, 2106.05331

NNLO jet & dijet production at the LHC with **full color**

*previously adopted leading color approximation undershoots the full result by up to 30%, important for triple differential dijets, relevant for **PDF fits** & **BSM searches***

NNLO **triple jet production** at the LHC

*Precise predictions for new physics backgrounds, extraction of strong coupling from **ratio $R_{3/2}$***

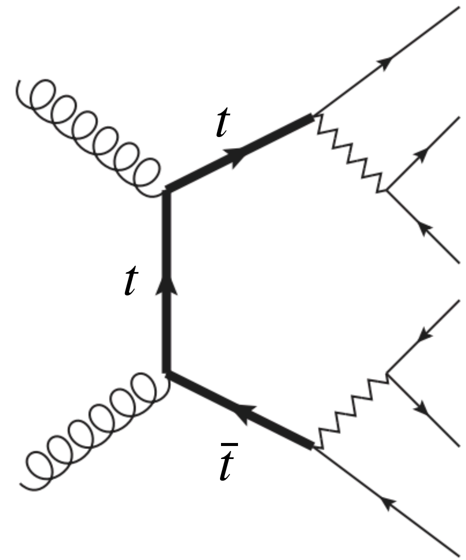
NNLO required for O(%) MHOUs

NNLO QCD for top production

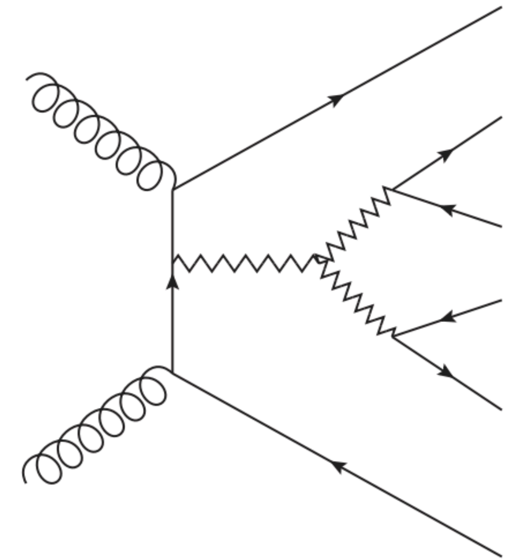
NNLO top quark pair production for **particle-level** fiducial cross-sections

$$pp \to b\bar{b}\ell^+\ell^-\nu\bar{\nu}$$

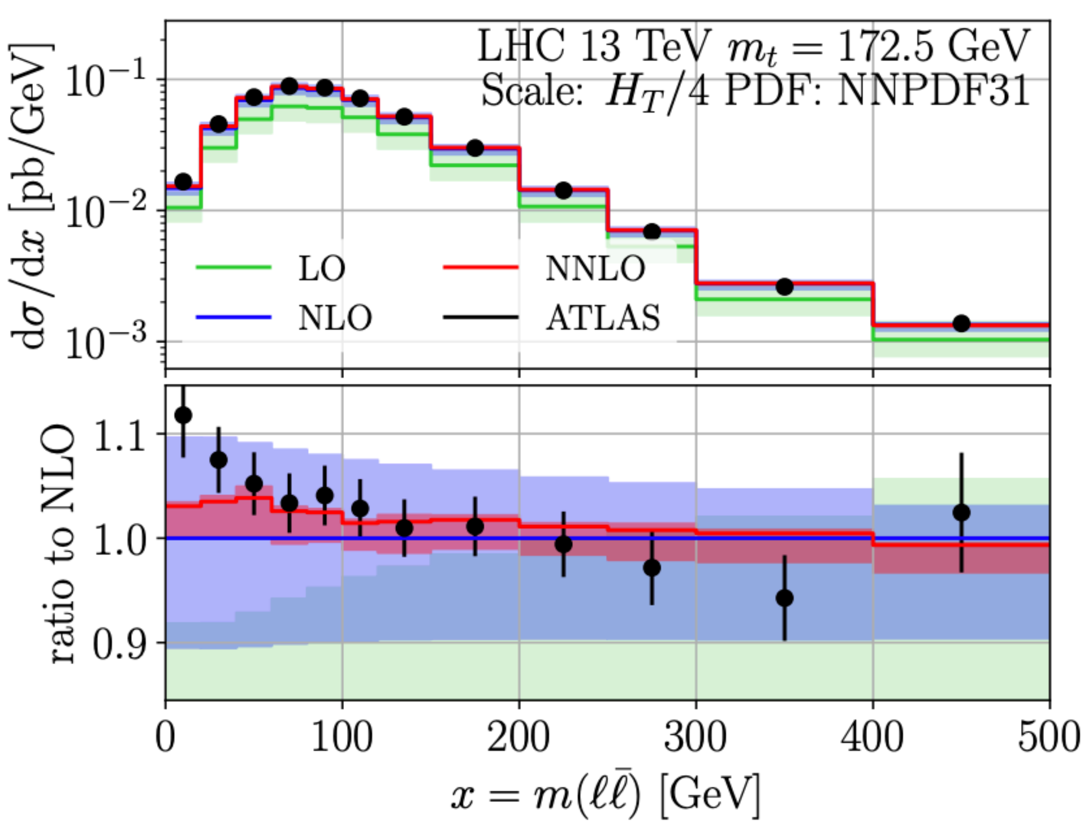
bypasses the need to unfold to **parton (top quark) level** observables

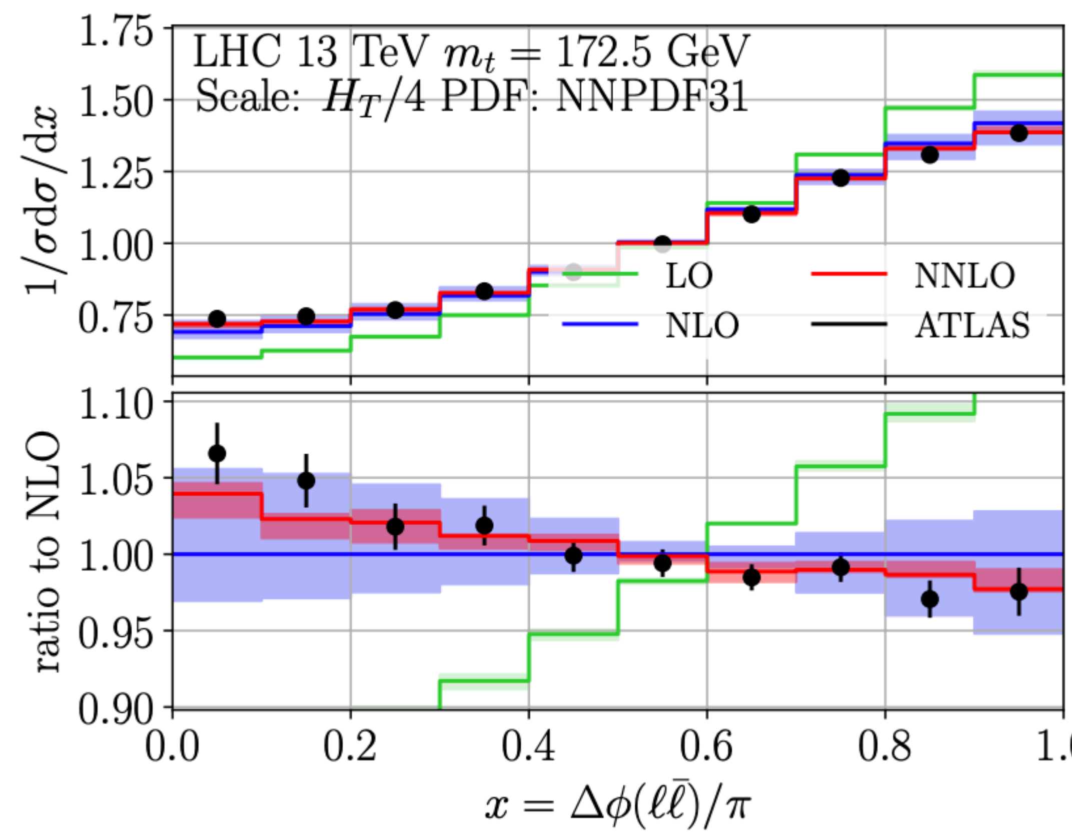


double-resonant



non resonant





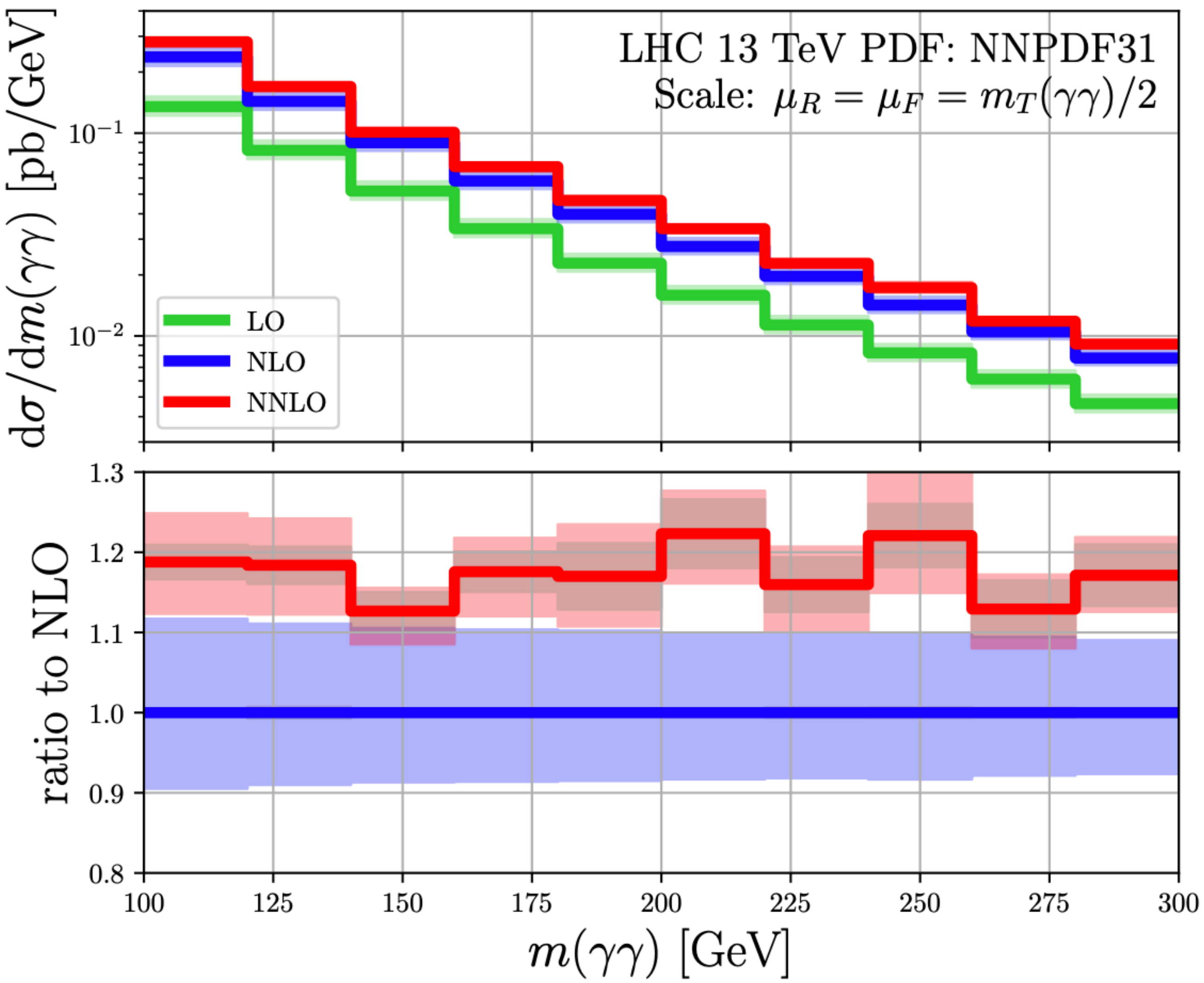
Czakon et al, 2008.11133

relevant for searches, EFT interpretations, PDF fits, SM parameter (m_t, α_s) extraction

related work: NNLO top quark pair production with running mass, Catani et al, 2005.00557

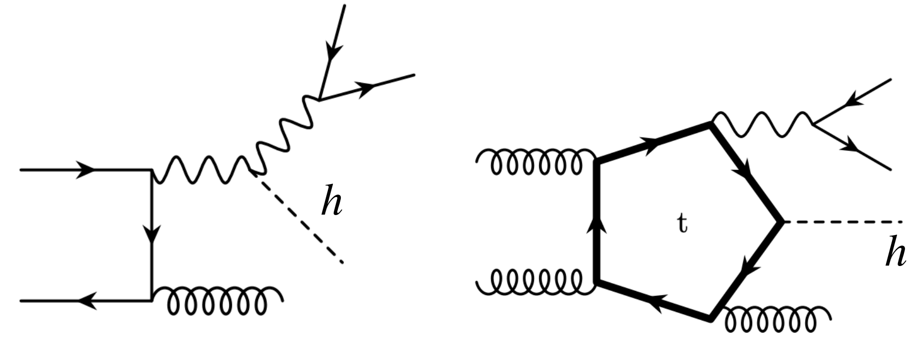
NNLO QCD for Higgs physics

Photon pair production with an extra jet: leading background for **Higgs to diphotons**

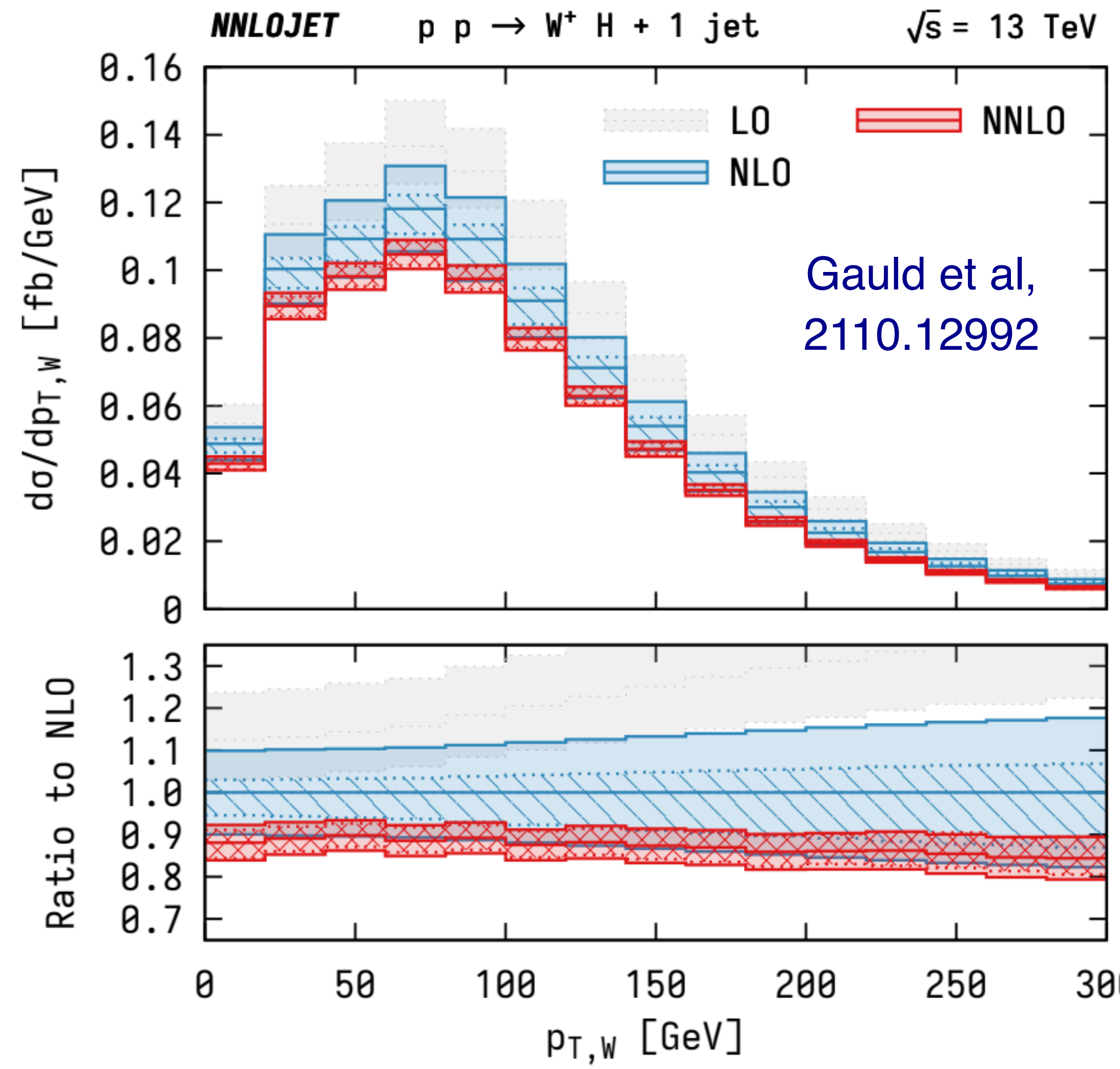


Chawdhry et al, 2105.06940

large NNLO K-factors, poor convergence of perturbative expansion



Higgs + *W/Z* with an extra jet

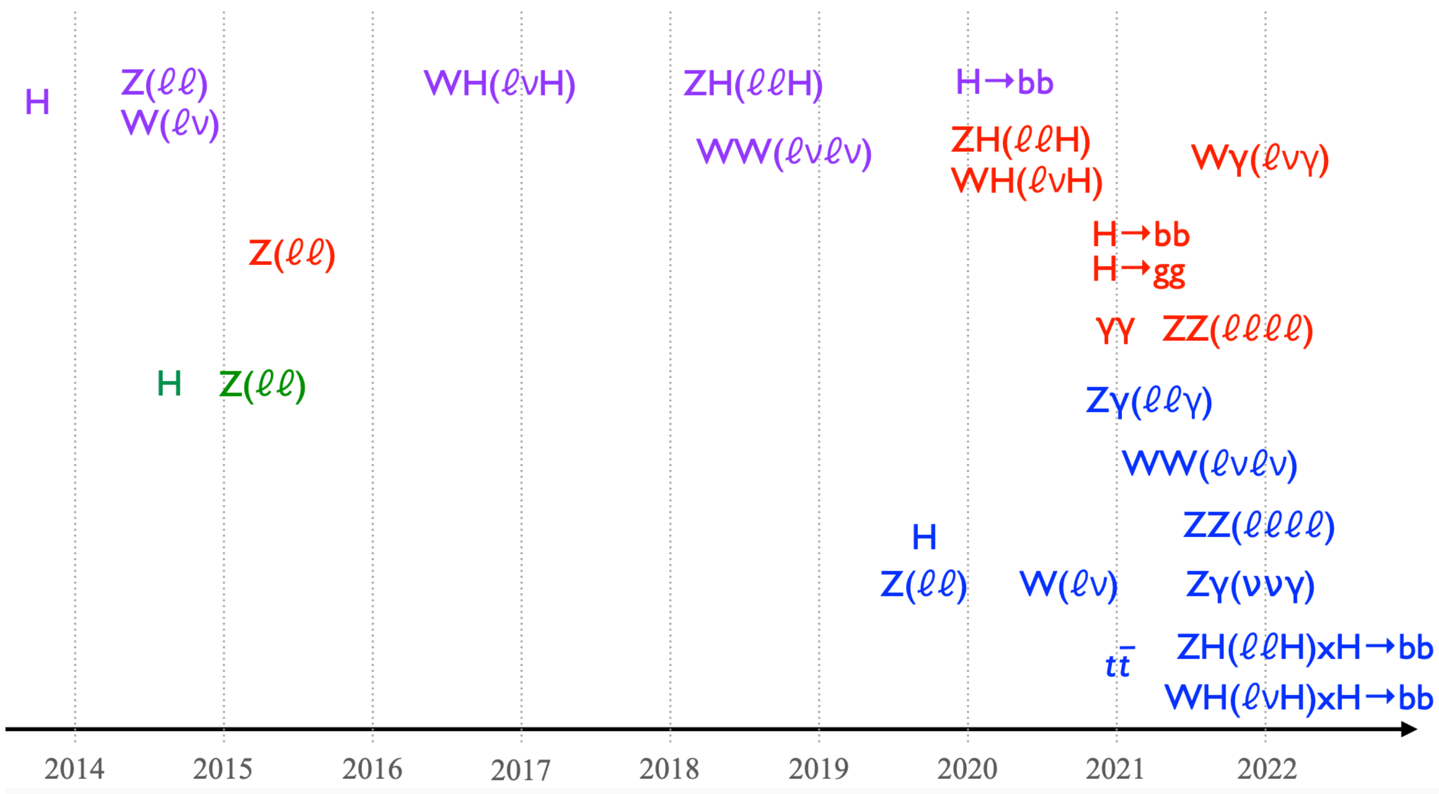


Gauld et al, 2110.12992

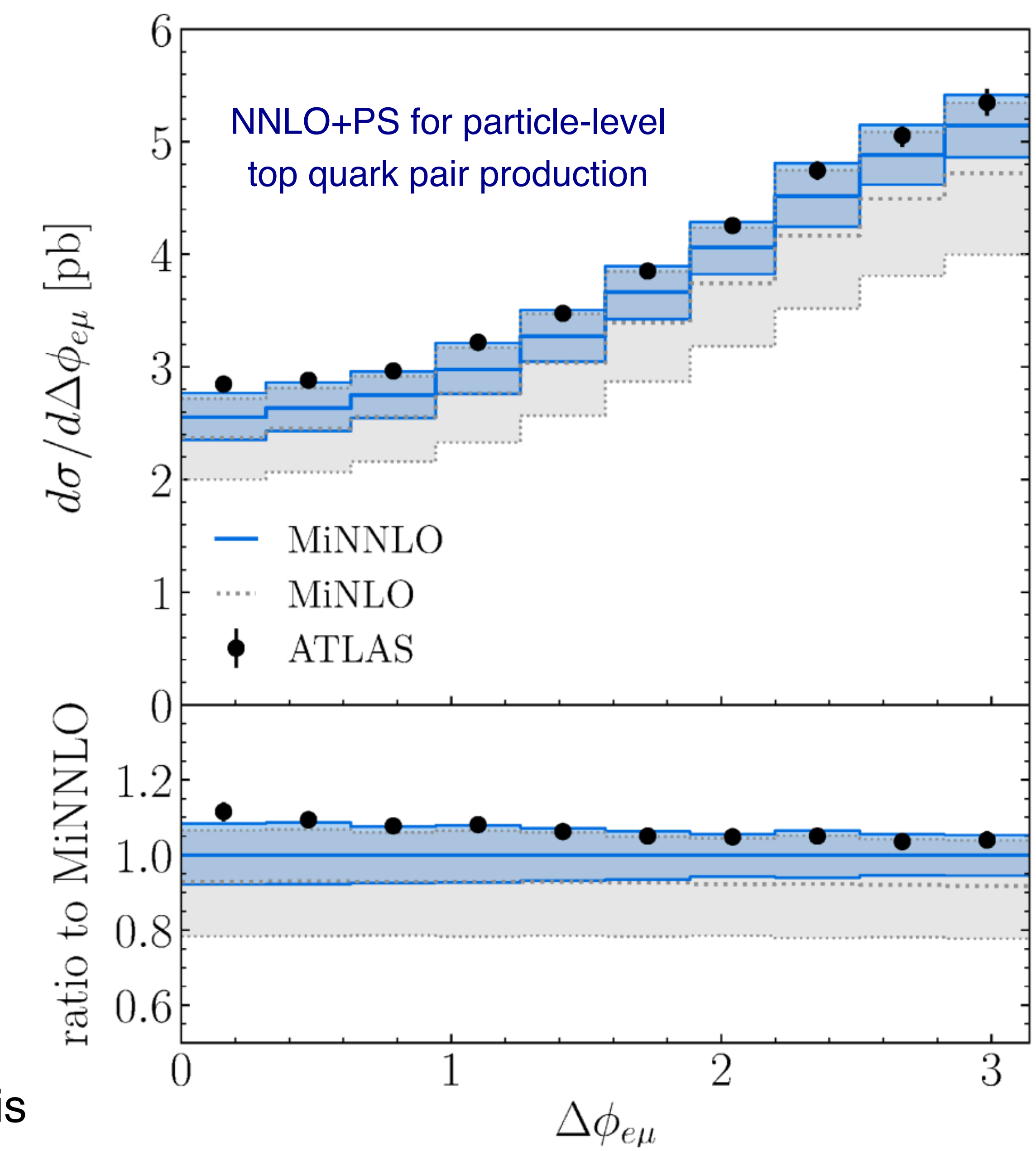
important for the precise modelling of ***Higgs kinematic distributions***

NNLO QCD matched to parton showers

Recent progress on matching **NNLO QCD** calculations to **parton showers** in **event generators**



Wiesemann, LHC EW WG Feb 22



Mazzitelli et al, 2112.12135

- Crucial to bridge the gap between **theory calculations** and **experimental measurements** (including detector simulation)

- **Logarithmic structure and accuracy of the parton shower** is maintained while keeping NNLO accuracy for the matrix element

- NNLO+PS simulations to become **precision standard at HL-LHC**

see the talks by Ilkka and Mrinal

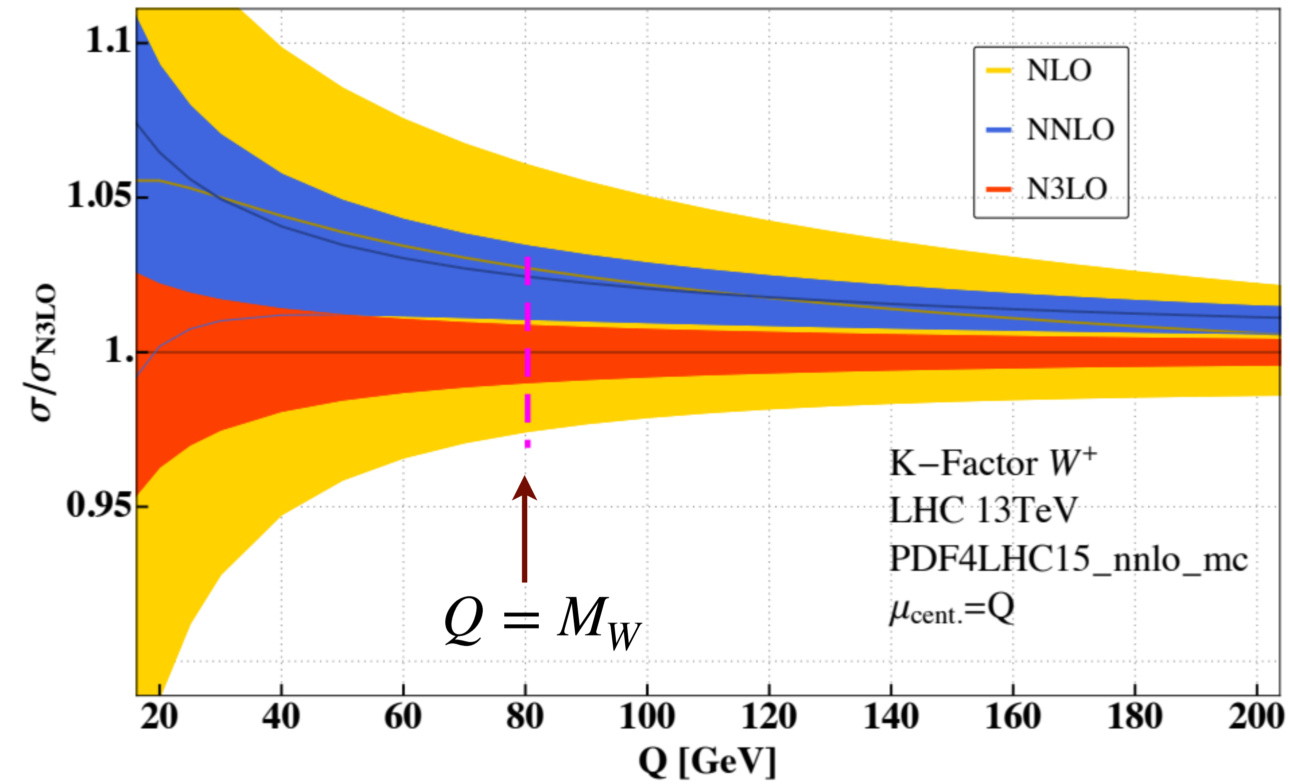
The N3LO frontier

Several key LHC processes are now available with **N^3LO QCD corrections** (inclusive and/or differential)

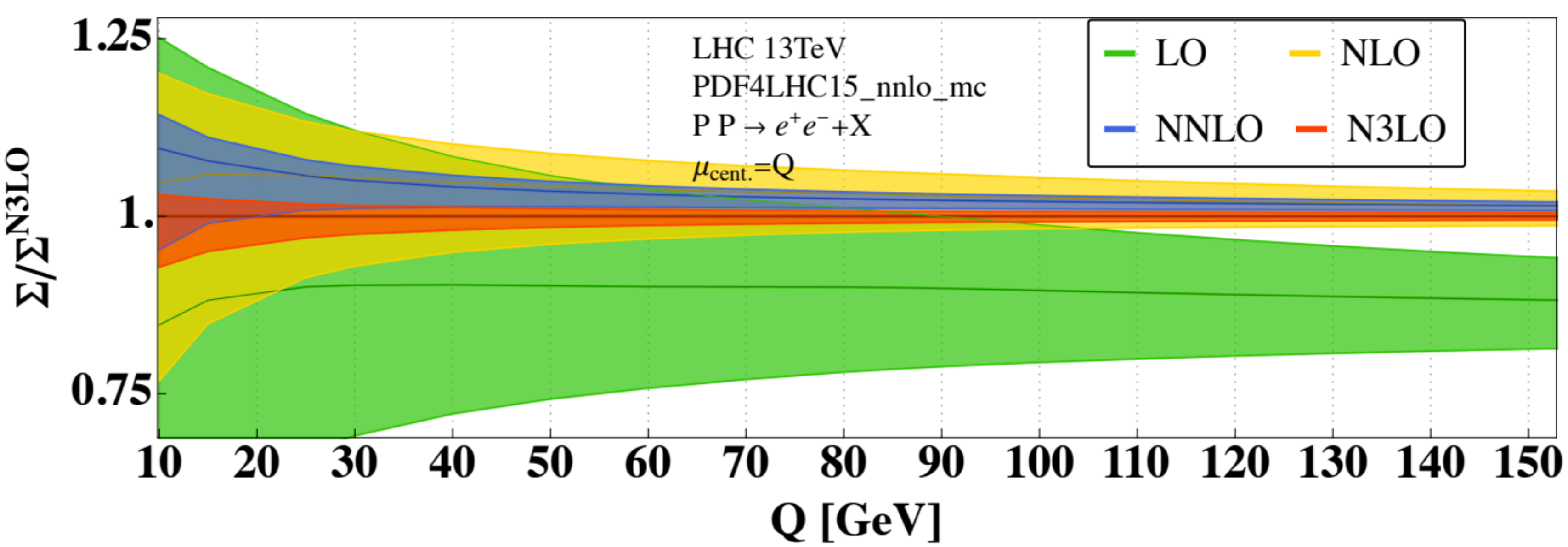
e.g. inclusive charged and neural-current **Drell-Yan**

Perturbative convergence not ideal: for both ***W*** and ***Z/γ**** production the NNLO and N^3LO bands do not overlap

nb all ``N3LO'' calculations rely on NNLO PDFs, hence we cannot claim **N3LO accuracy** yet



Duhr et al, 2007.13313

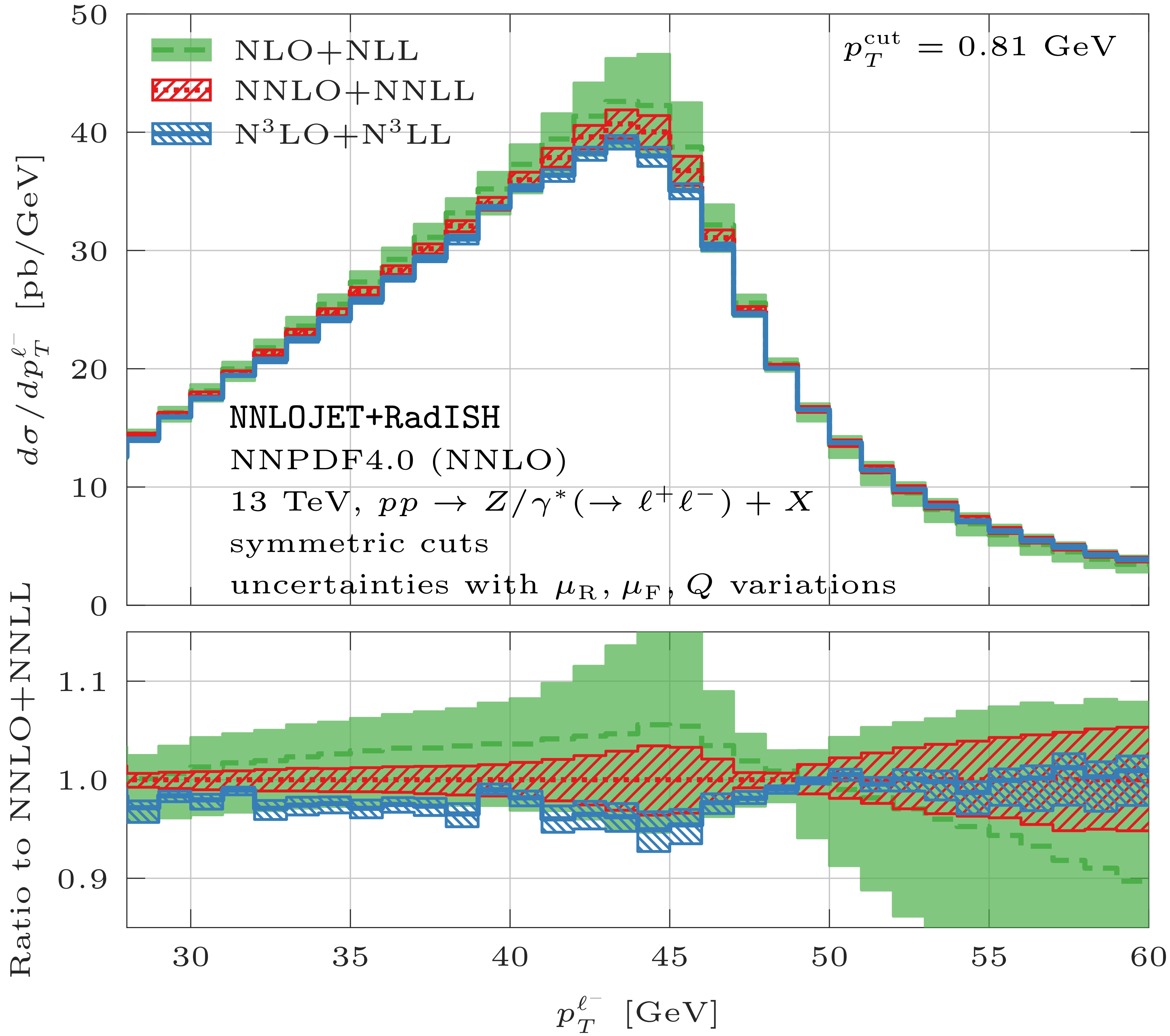


Duhr et al, 2111.10379

Several other N^3LO QCD calculations available: Higgs in gluon fusion (diff), VBF Higgs (diff), Higgs pair production, …

The N3LO frontier

Drell-Yan is also available at N3LO at the **fiducial level**, where **realistic kinematic cuts** can be applied



Chen at al, 2203.01565

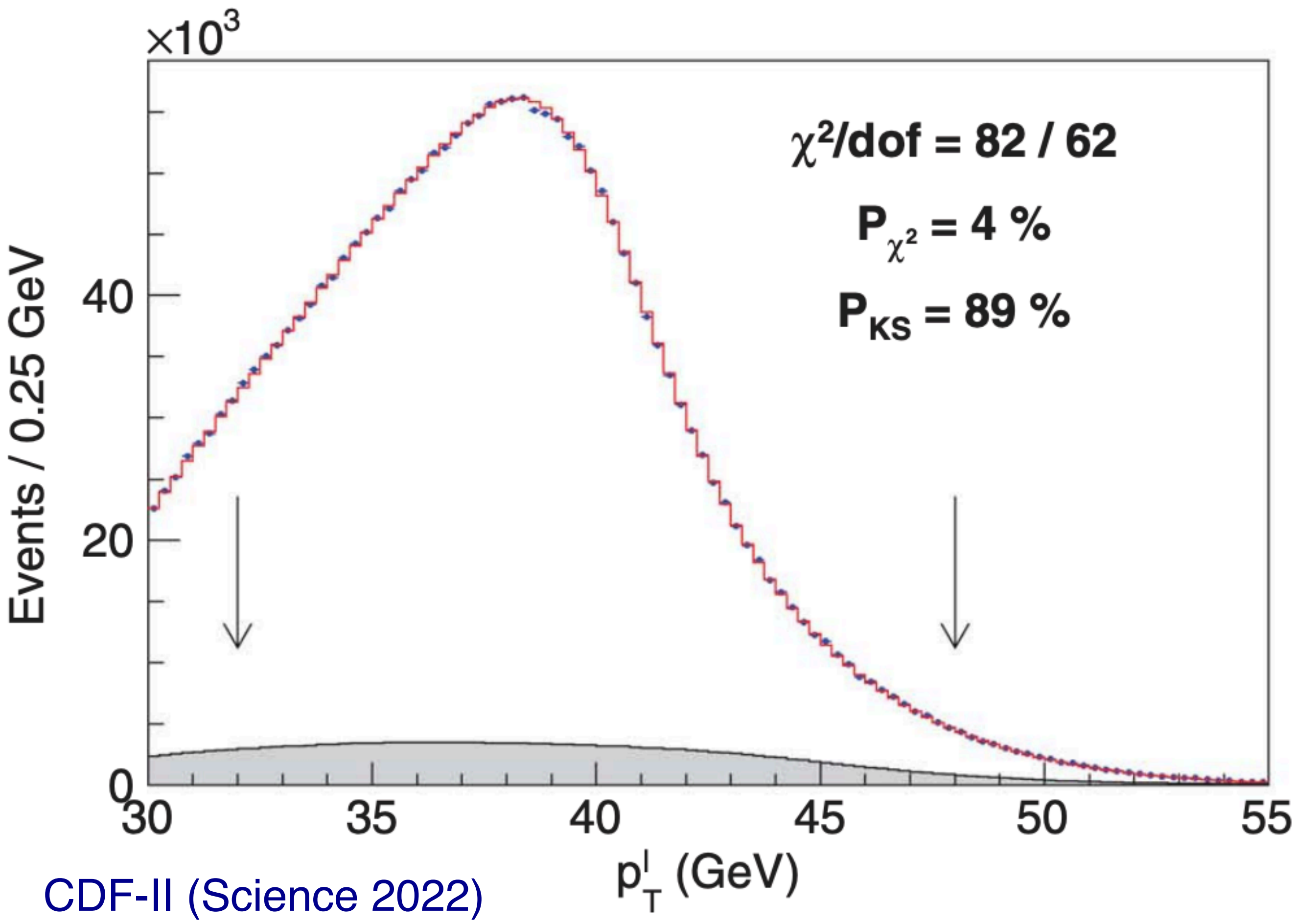
Potentially relevant for *W* mass determinations!

*perturbative stability can be optimised with tailored **kinematic cuts***

***precision** does not necessarily improve at N³LO ….*

mixed NNLO corrections to *W* & *Z* production

The recent measurement of **m_W at CDF-II** has brought the **precise modelling of W & Z kinematic distributions** under the spotlight

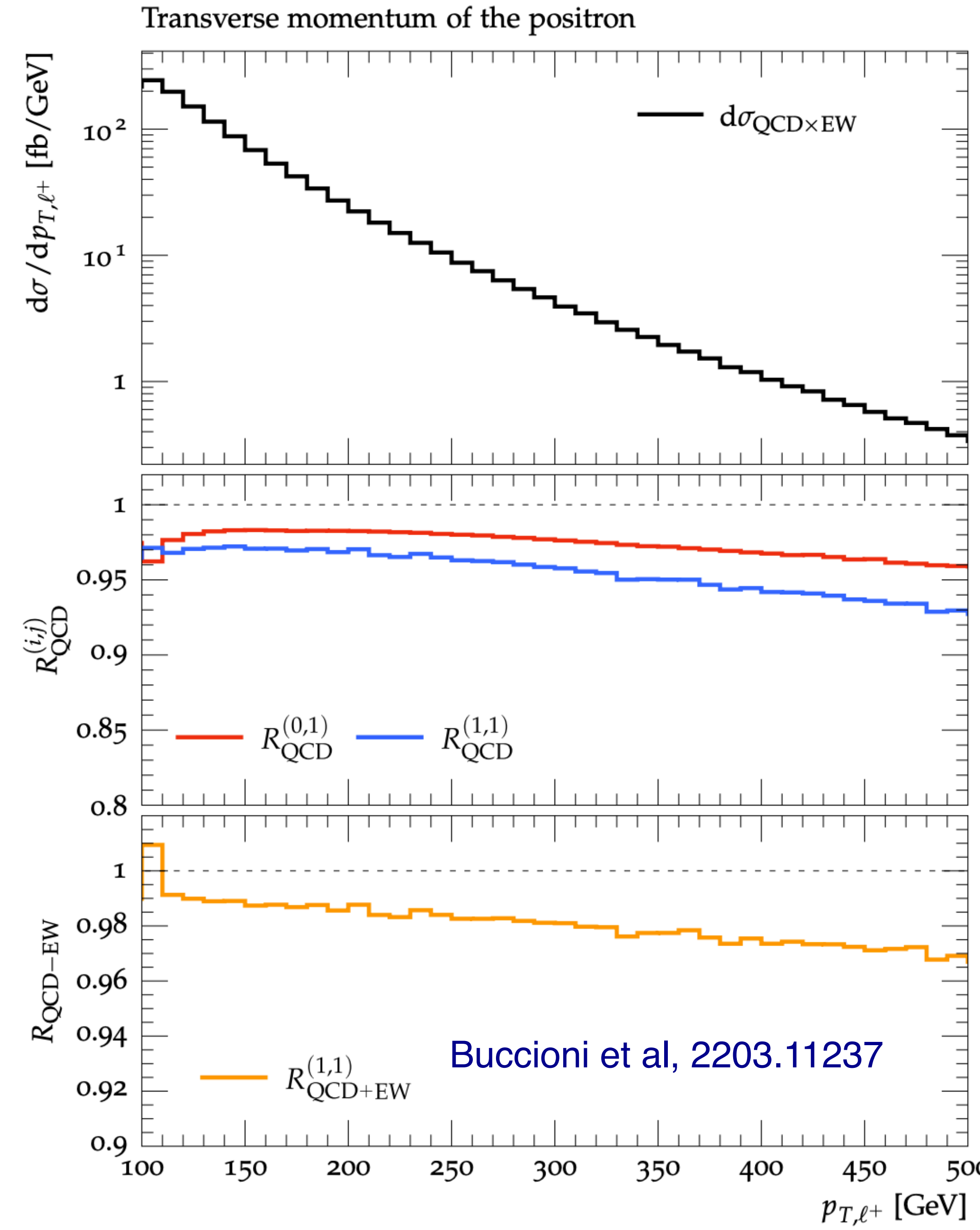


CDF-II (Science 2022)

mW fits very sensitive to distortions in the **p_T^l, M_T distributions**

	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: NLO-QCD+QCD$_{PS}$ Pseudodata accuracy	QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^l	 $W^+ \to e^+\nu$(dres) M_T	 p_T^l
1	NLO-QCD+(QCD+QED)$_{PS}$	PYTHIA	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED)$_{PS}$	PHOTOS	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED)$_{PS}$two-rad	PYTHIA	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED)$_{PS}$two-rad	PHOTOS	-88.6±0.6	-370±3	-39.2±0.6	-159±2

Chiesa et al, LHCP18



mixed NNLO QCD/EW corrections to fiducial *W* production

also for *Z* production: Bonciani et al, 2007.06518

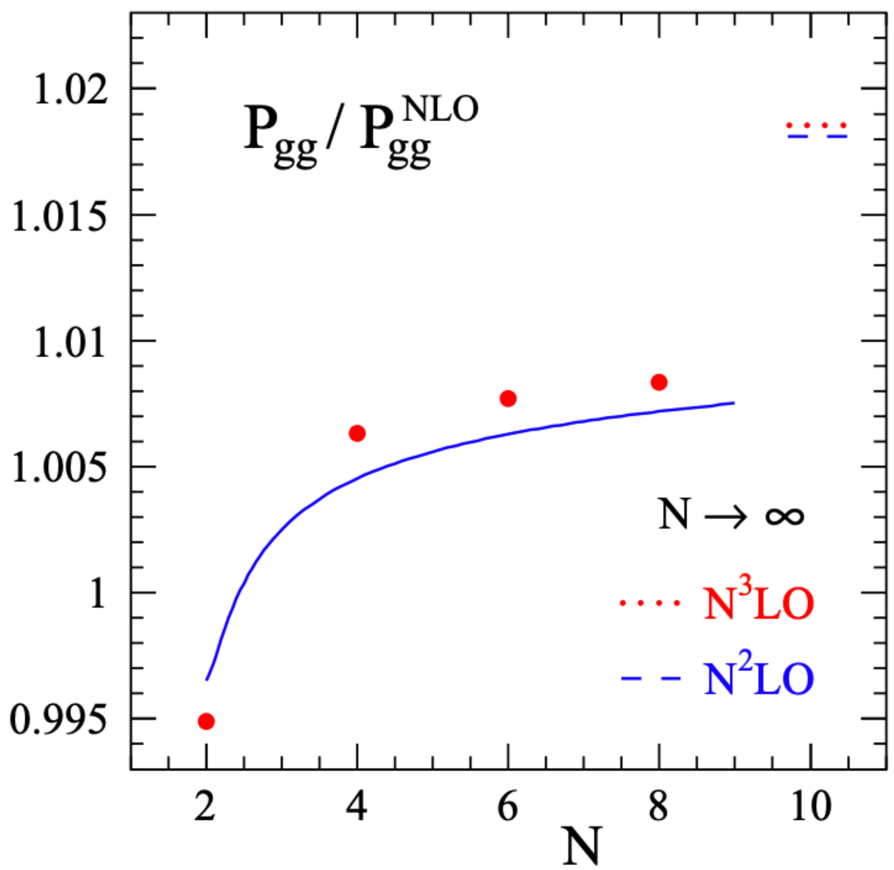
Towards N3LO proton structure

Fully exploiting recent progress in N³LO calculations requires **PDFs extracted at the same order**

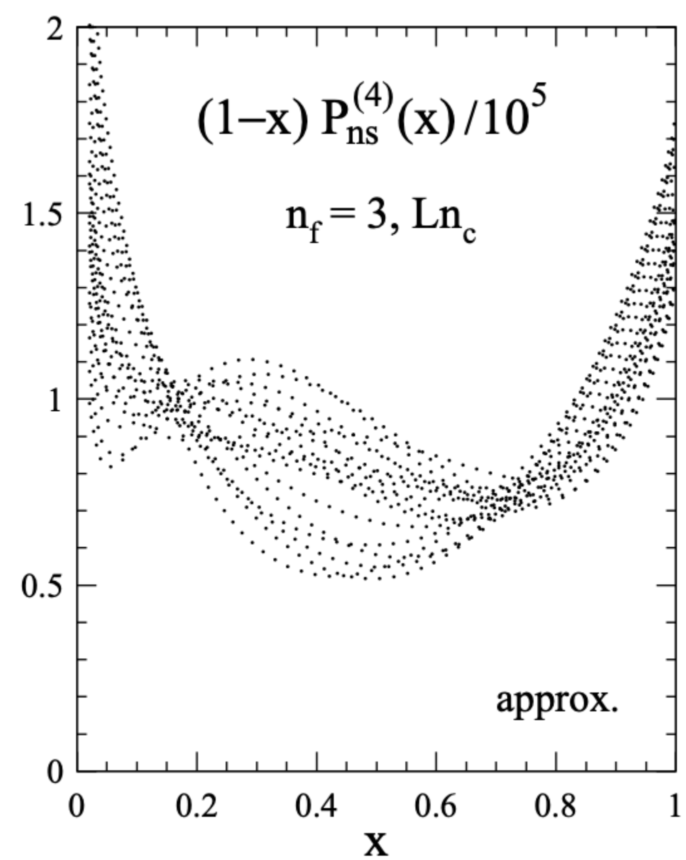
DGLAP splitting functions

$$P_{ij}(x,\alpha_s) = \alpha_s P_{ij}^{(0)}(x) + \alpha_s^2 P_{ij}^{(1)}(x) + \alpha_s^3 P_{ij}^{(2)}(x) + \alpha_s^4 P_{ij}^{(3)}(x) + \alpha_s^5 P_{ij}^{(4)}(x)$$

- ☑ LO (1973)
- ☑ NLO (1982)
- ☑ NNLO (2004)
- N³LO partial results
- N⁴LO partial results



Moch et al, 2111.15561



Moch et al, 1812.11818

What do we about about N³LO P_{ij} ?

- low moments of singlet & non-singlet
- large-x limits (threshold resummation)
- small-x limits (BFKL resummation)

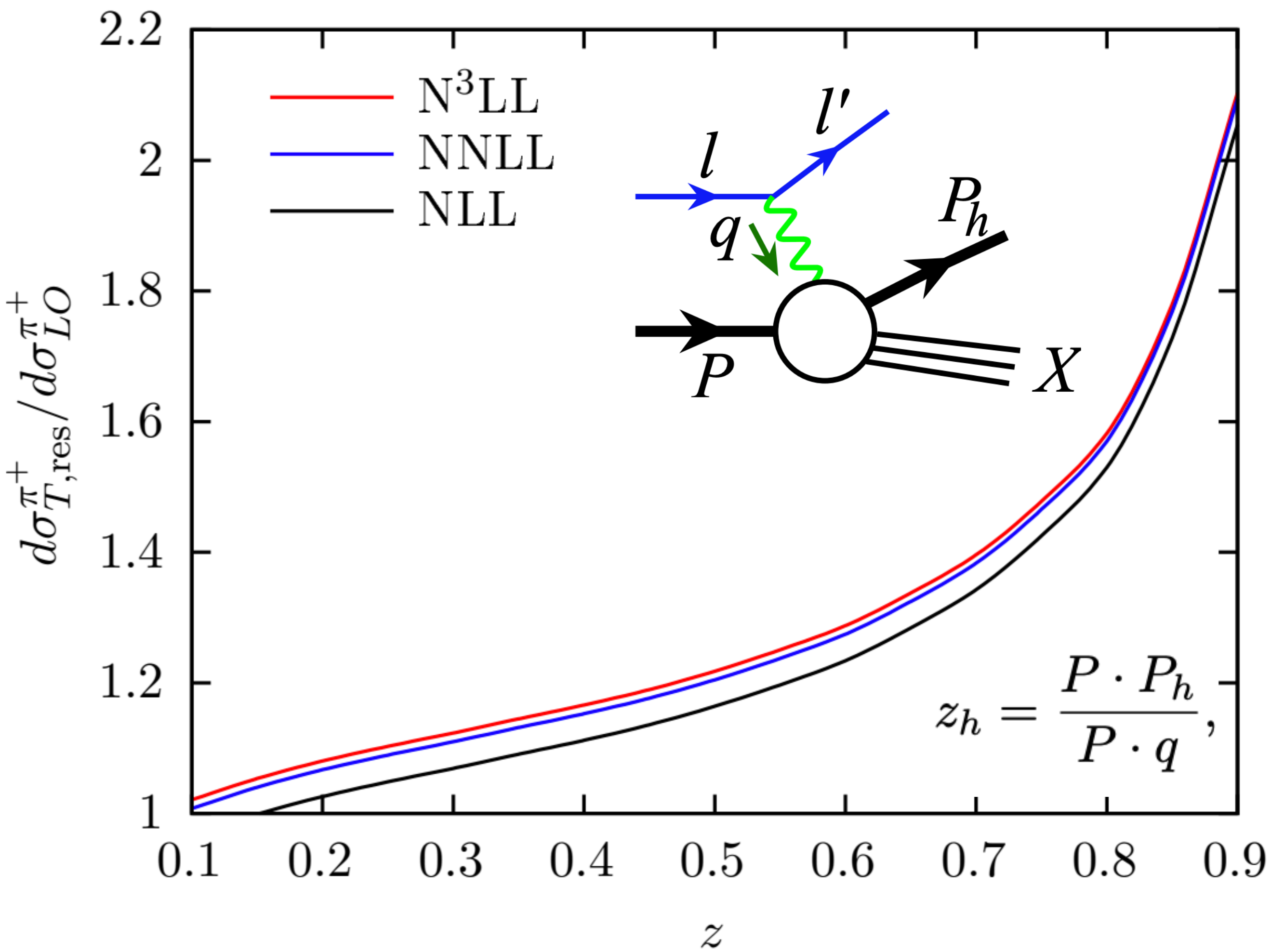
What else for a **N³LO PDF fit?**

- ☑ Massless and massive (partially) DIS **coefficient functions**
- ☑ Heavy quark **matching conditions**
- ☑ **N³LO K-factors** for LHC observables (subset, e.g. Drell-Yan)

Precision QCD for DIS

Progress in QCD calculations also instrumental for the interpretation of **lepton-nucleus collisions**

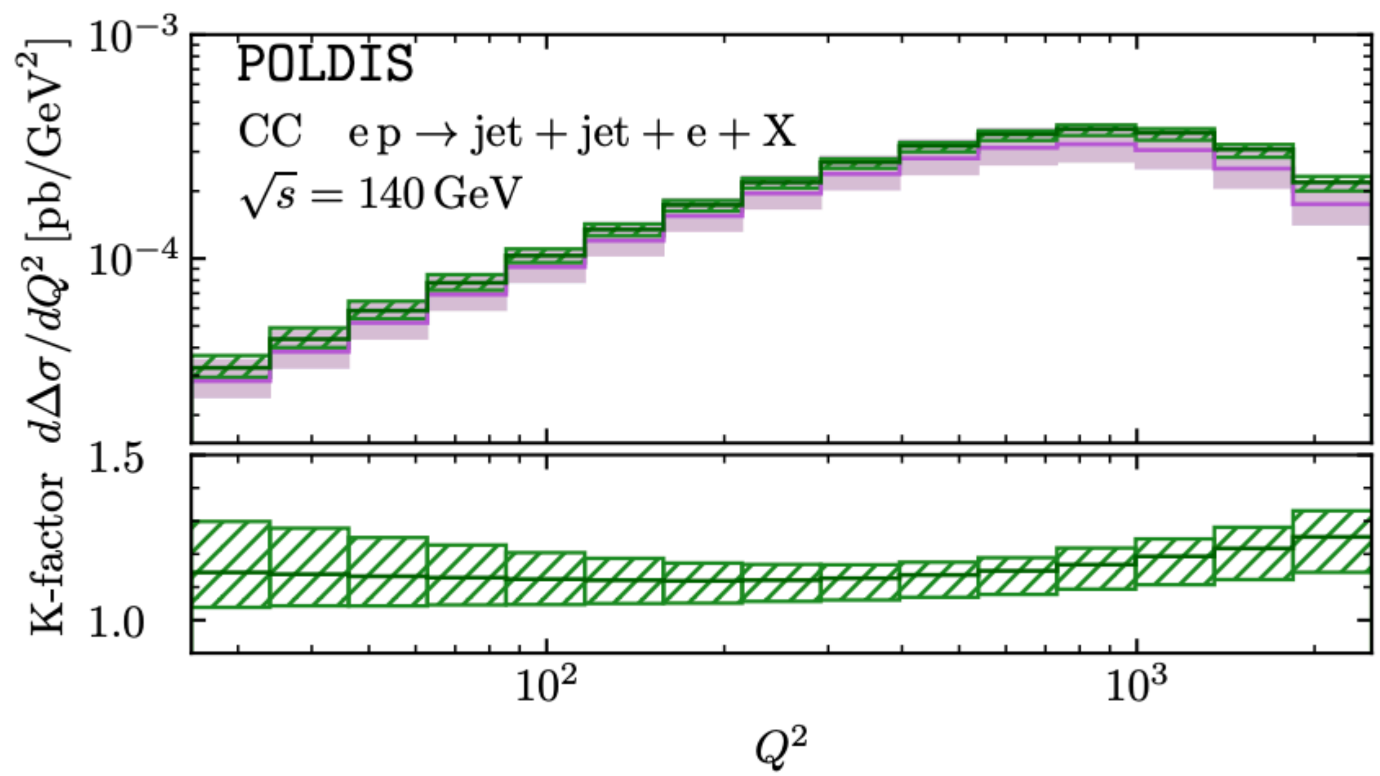
Full exploitation of the **Electron Ion Collider** scientific program requires **precision QCD for DIS!**



Abele et al, 2109.00847 & 2203.07928

N3LL & aN3LO for **semi-inclusive DIS**

Relevant for the determination of FFs and of unpolarised/polarised nucleon strangeness



Borsa et al, 2112.08223

Differential NLO for **dijet production** in NC & CC polarised DIS

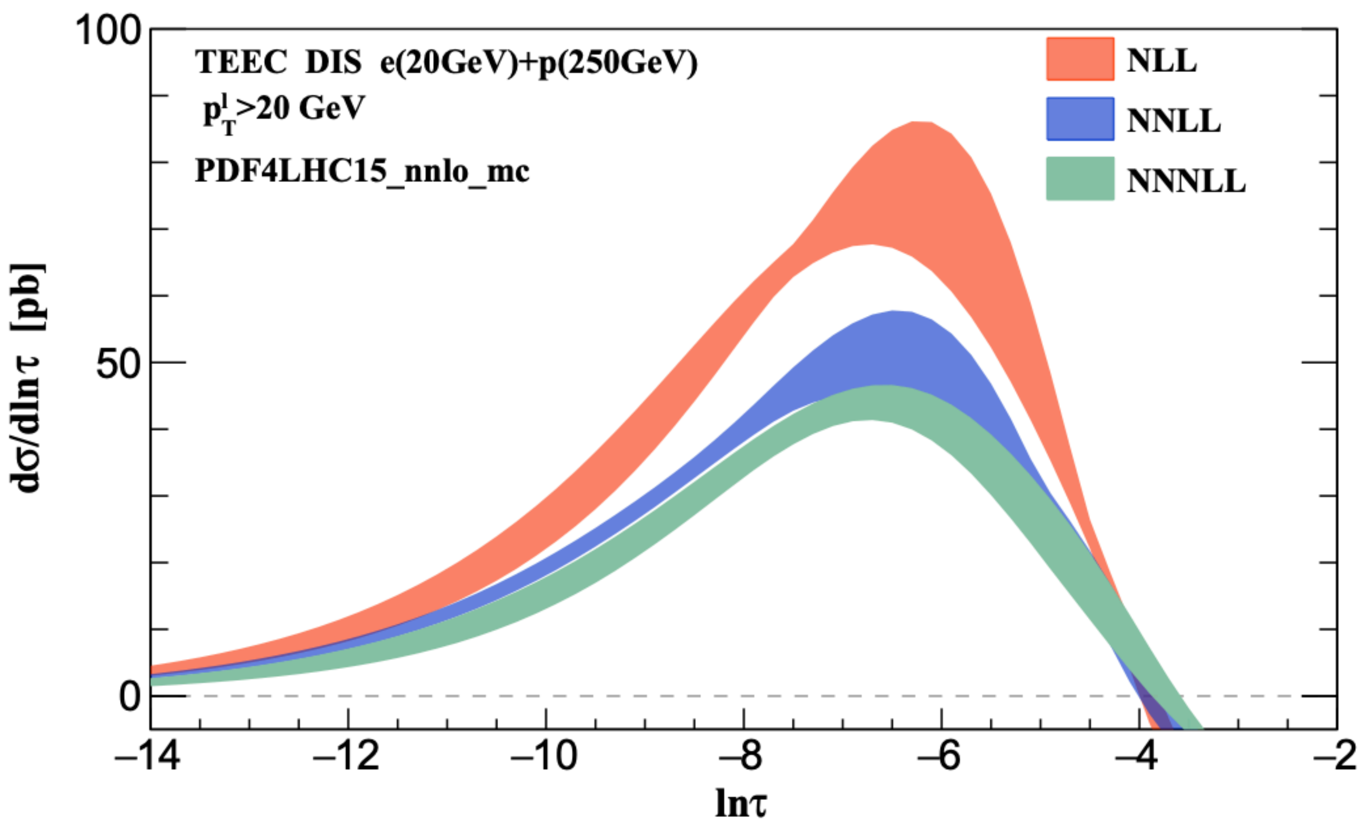
Constraints on gluon polarised PDFs and polarised quark flavour separation

Precision QCD for DIS

Progress in QCD calculations also instrumental for the interpretation of **lepton-nucleus collisions**

Full exploitation of the **Electron Ion Collider** scientific program requires **precision QCD for DIS!**

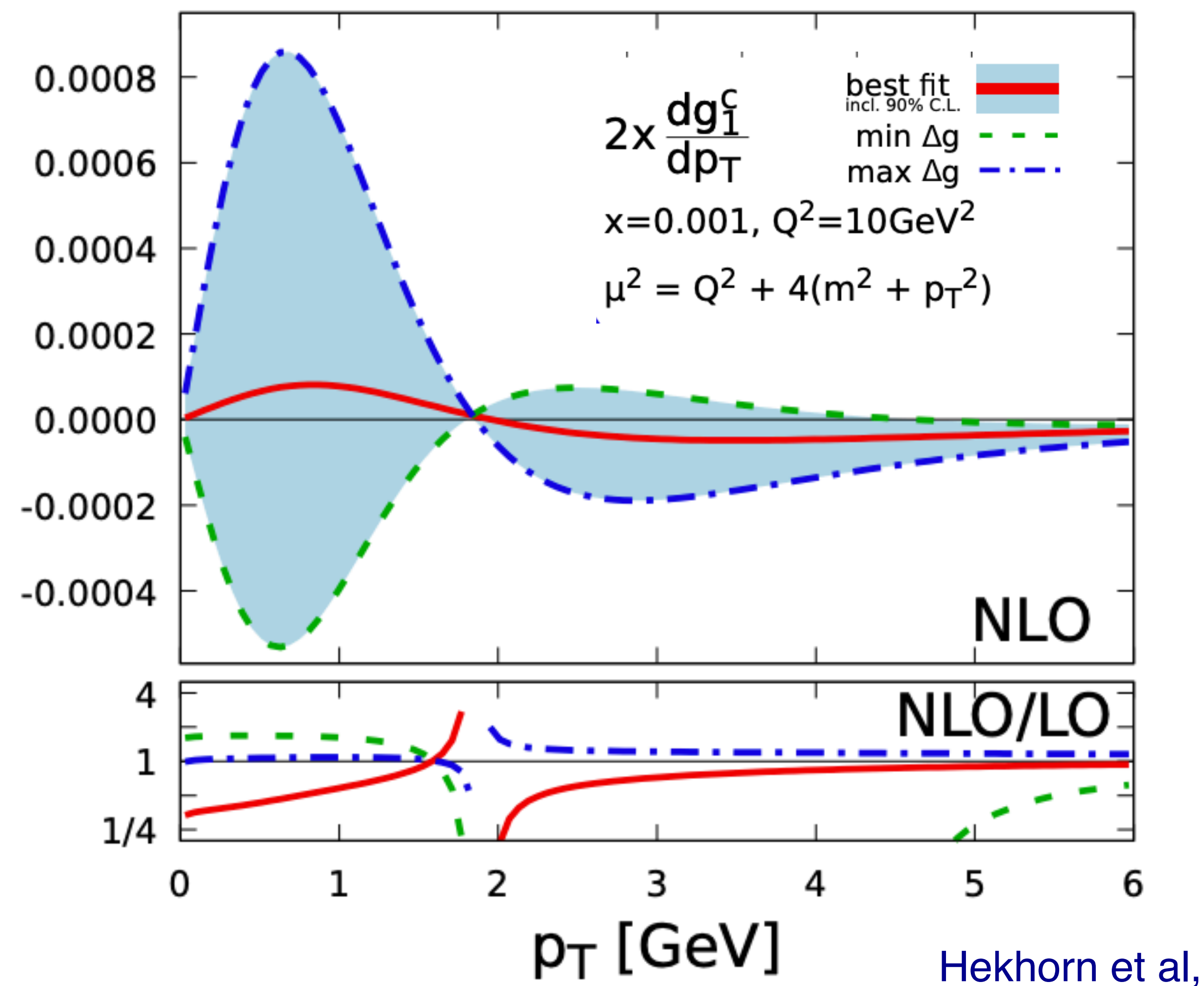
$$\mathrm{TEEC} = \sum_a \int d\sigma_{lp\to l+a+X} \frac{E_{T,l}E_{T,a}}{E_{T,l}\sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$$



Neill et al, 2203.07113

Event shapes in **semi-inclusive DIS**

transverse-energy-energy correlator (TEEC): direct probe of transverse hadron structure (TMDs)



Hekhorn et al, 2105.13944

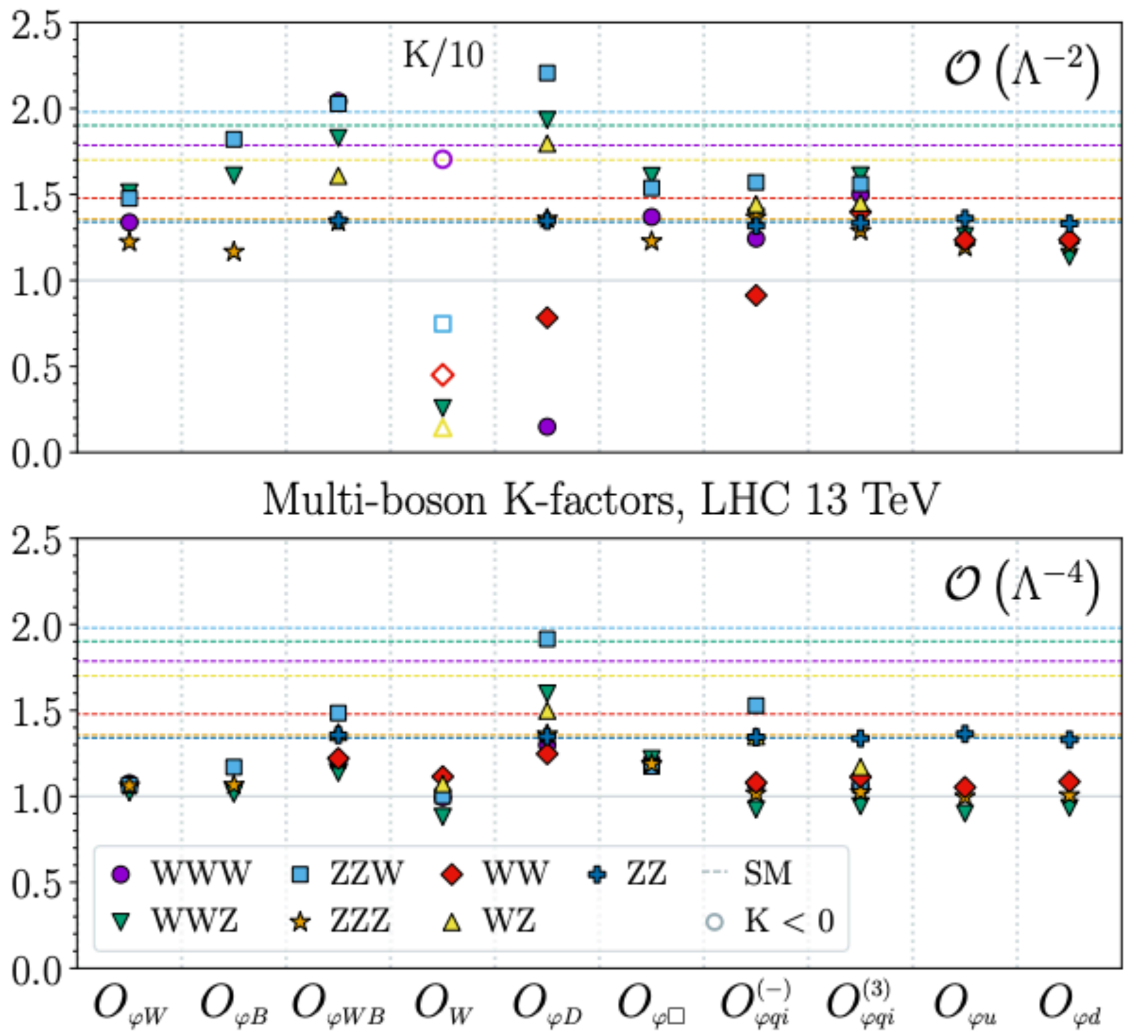
Exclusive **NLO charm structure functions** in polarised DIS

Clean probe of gluon polarisation at the EIC

Higher-order QCD meets BSM

Precise QCD & EW calculations are also crucial to accurately interpret collider data in **BSM scenarios**, such as in the SMEFT, and to derive bounds on the parameters of **UV-complete theories**

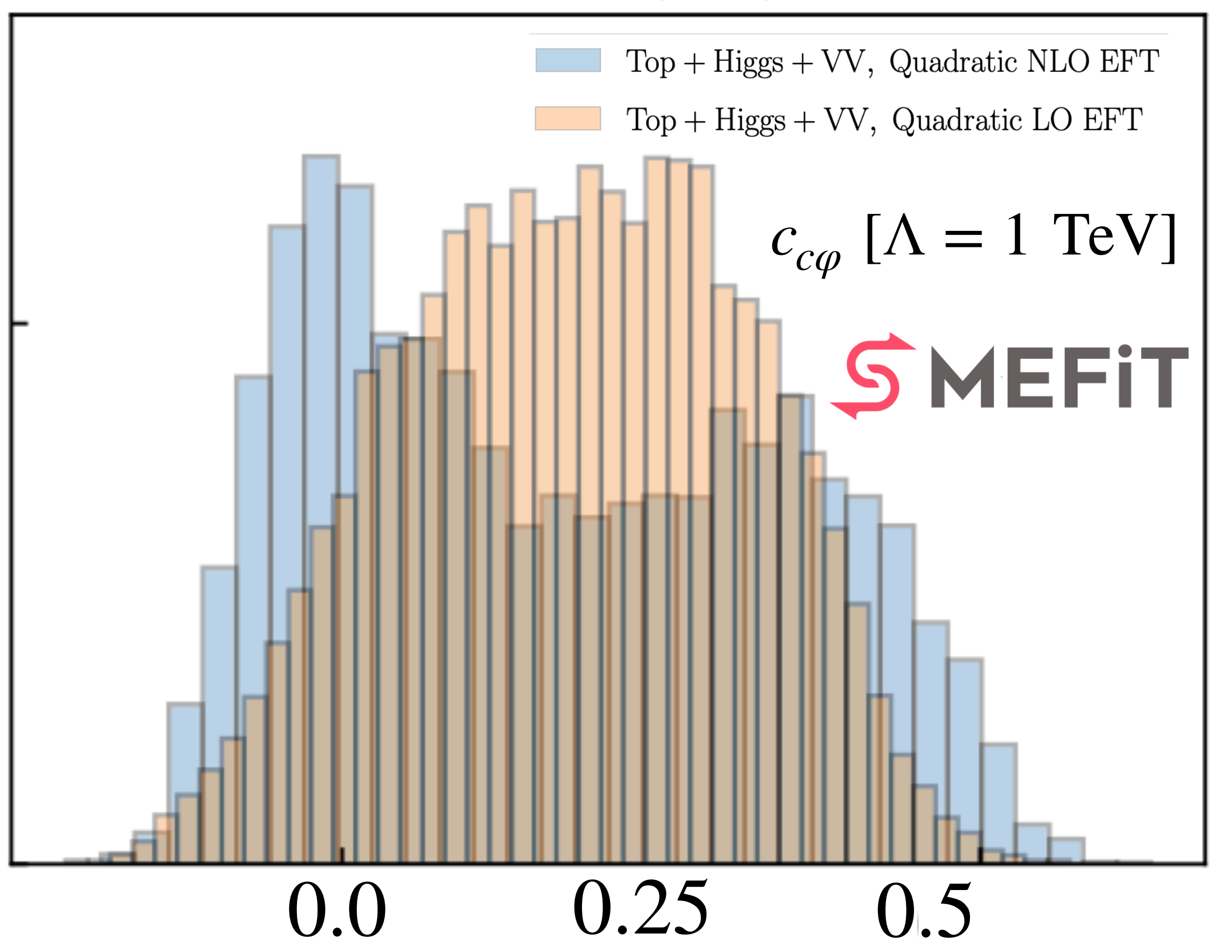
see also talk by Jorge de Blas



Degrande et al, 2008.11743

*large **NLO QCD K-factors for SMEFT** effects in multi-boson production at the LHC*

Charm Yukawa operator: $\left(\varphi^\dagger\varphi\right)\bar{q}_2\, c\, \tilde{\varphi} + \text{h.c.}$



SMEFiT collaboration, 2105:00006

***posterior distributions** in global SMEFT analysis distorted when NLO QCD effects ignored*

Fast interfaces

For many key applications (e.g. PDF fitting, SM parameter extraction, designing optimised observables) the availability of **fast interfaces** to **higher-order calculations** is essential

→ bottleneck is evaluation of the **partonic matrix elements**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}}(\mathcal{O},\xi_{\mathrm{R}},\xi_{\mathrm{F}}) = \sum_{a,b}\int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \int_{Q^2_{\mathrm{min}}}^{Q^2_{\mathrm{max}}} \mathrm{d}Q^2\, \underbrace{f_a(x_1,\xi_{\mathrm{F}}^2Q^2)f_b(x_2,\xi_{\mathrm{F}}^2Q^2)}_{\textit{fast (LHAPDF interpolation)}}\underbrace{\sigma_{ab}(x_1,x_2,Q^2,\xi_{\mathrm{R}},\xi_{\mathrm{F}})}_{\textit{slow...}}$$

The matrix element calculation can be classified by orders of **QCD & EW couplings**

$$\frac{\mathrm{d}\sigma_{ab}}{\mathrm{d}\mathcal{O}}(x_1,x_2,\mathcal{O},\xi_{\mathrm{R}},\xi_{\mathrm{F}}) = \sum_{k,l,m,n} \alpha_{\mathrm{s}}^k\left(\xi_{\mathrm{R}}^2Q^2\right)\alpha^l \log^m(\xi_{\mathrm{R}}^2)\log^n(\xi_{\mathrm{F}}^2)W_{ab}^{(k,l,m,n)}\left(x_1,x_2,Q^2,\mathcal{O}\right)$$

Carrazza et al, 2008.12789

QCD coupling ***EW coupling*** ***scale-variation logs*** ***matrix elements***

Two options to **re-evaluate hadronic cross-sections** upon varying e.g. the PDFs

store original ***n*-tuples**

$$\left\{x_1^i,x_2^i,Q_i^2,w_{ab}^{(k,l,m,n,o)}(x_1^i,x_2^i,Q_i^2,\mathcal{O}_i)\right\}_{i=1}^N$$

arbitrary observable/binning, ery heavy storage

interpolate the matrix elements

$$W_{ab}^{(k,l,m,n)}\left(x_1,x_2,Q^2,\mathcal{O}\right)$$

requires specifying observable (kin variables, binning)

Fast interfaces

Several **fast interfaces** to higher order calculations available, including

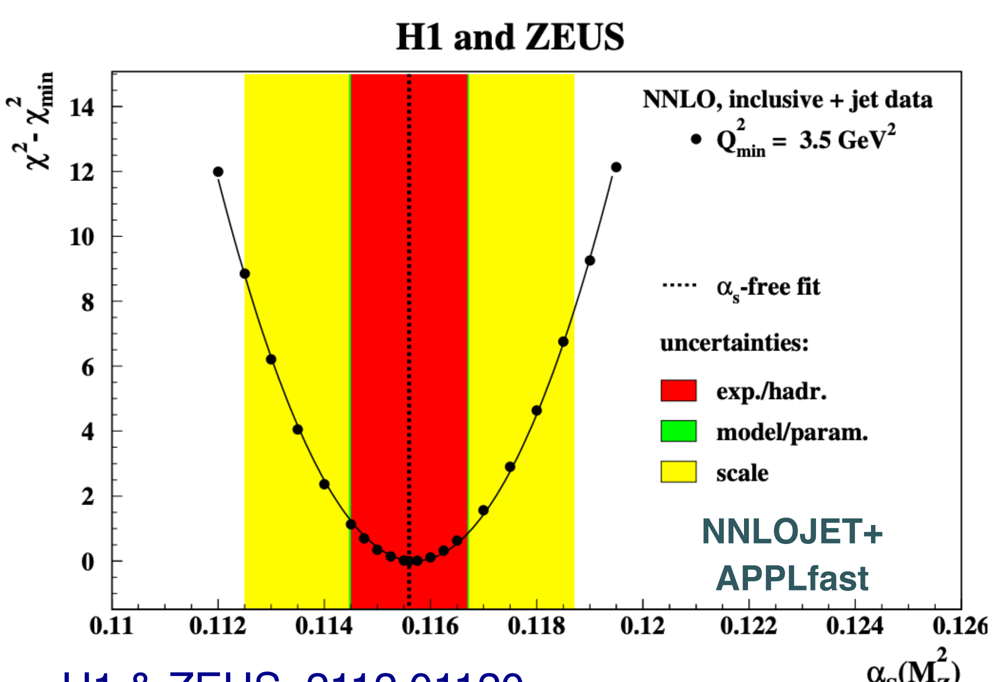
PineAPPL: interfaced to **mg5_aMC**, automates fast NLO QCD and electroweak calculations

FastNLO: interfaced to **NLOjet++**, fast NLO QCD for jet observables, and to **NNLO top differential**

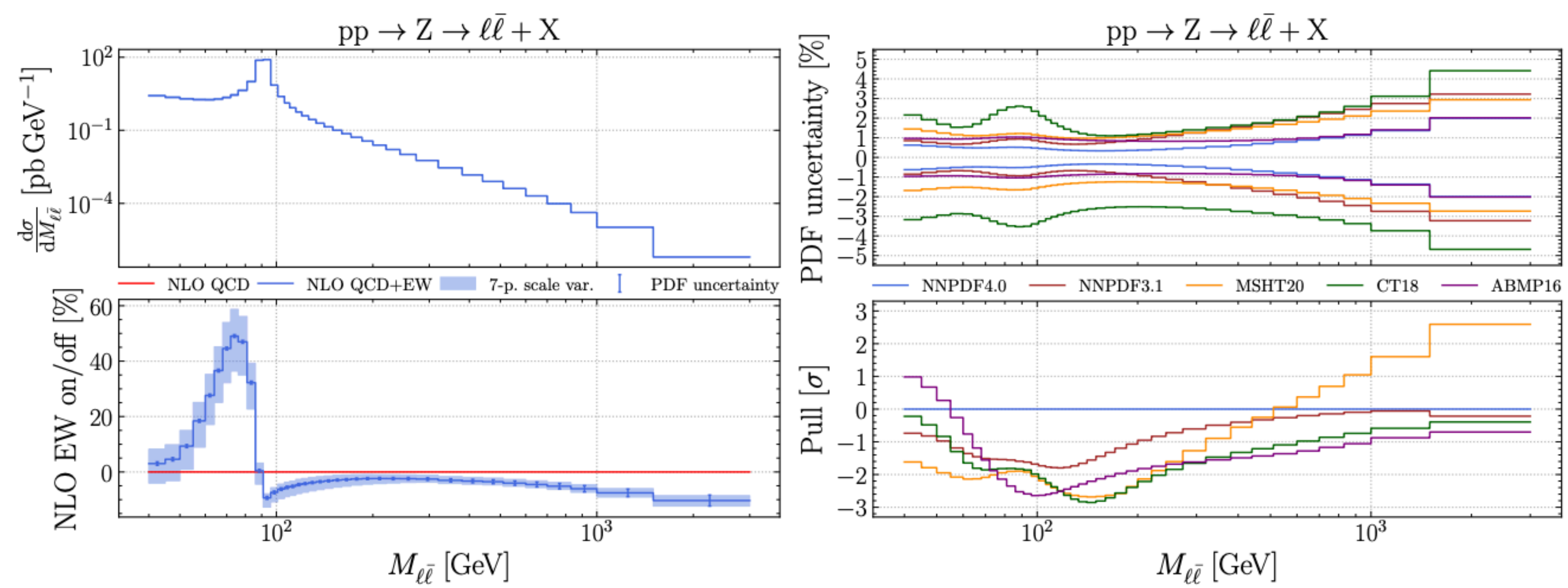
APPLgrid interfaced to **NLOjet++**, fast NLO QCD for jet observables, and to **NLO MCFM** (v6.8)

APPLfast: interface to NNLOJET for **NNLO** (pp & DIS) jets, $Z\ p_{T,}$ Higgs production

(private interface/grids)



H1 & ZEUS, 2112.01120



Carrazza et al, 2008.12789

*strong coupling from **NNLO DIS jets***

***PineAPPL**: fast variations of PDF sets in **NLO QCD+EW** calculations*

Estimating higher orders

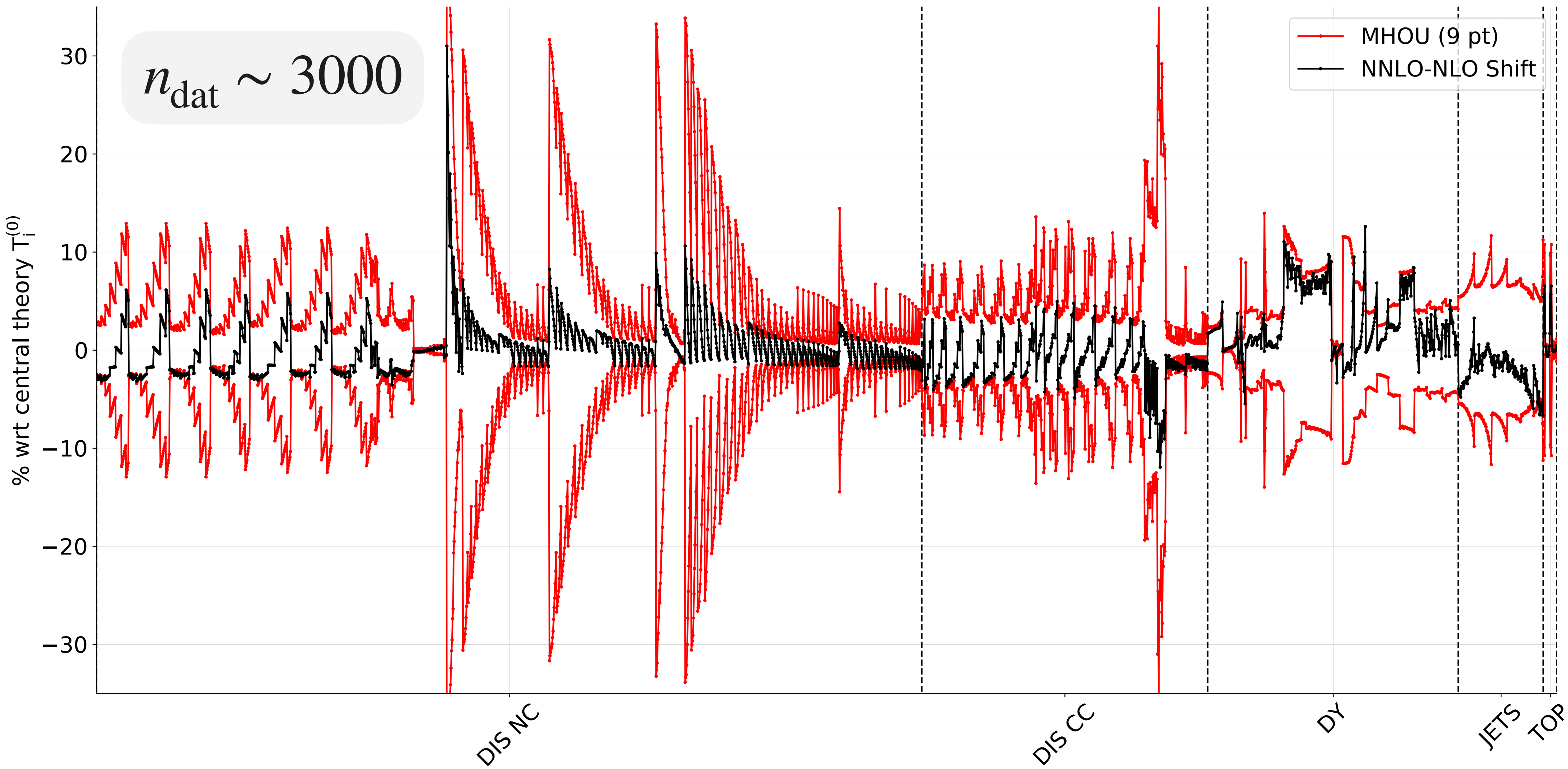
The more orders we evaluate in a expansion, the better (in principle) our **estimate of the residual terms**

$$\widetilde{\sigma}(\alpha_s) = \widetilde{\sigma}^{(0)} \left(\sum_{k=0}^{m} \alpha_s^k \right) + \mathcal{O}\left(\alpha_s^{m+1}\right)$$

for many processes, NLO is the first order where MHOUs are reliable

assuming that unknown orders resemble the known terms can fail e.g. when next channels open

standard practice is **scale variations** of the known $\mathrm{N}^k\mathrm{LO}$ terms to estimate the $\mathrm{N}^{k+1}\mathrm{LO}$ ones



MHOU estimate at NLO vs exact NNLO-NLO shift

NNPDF, 1906.10698

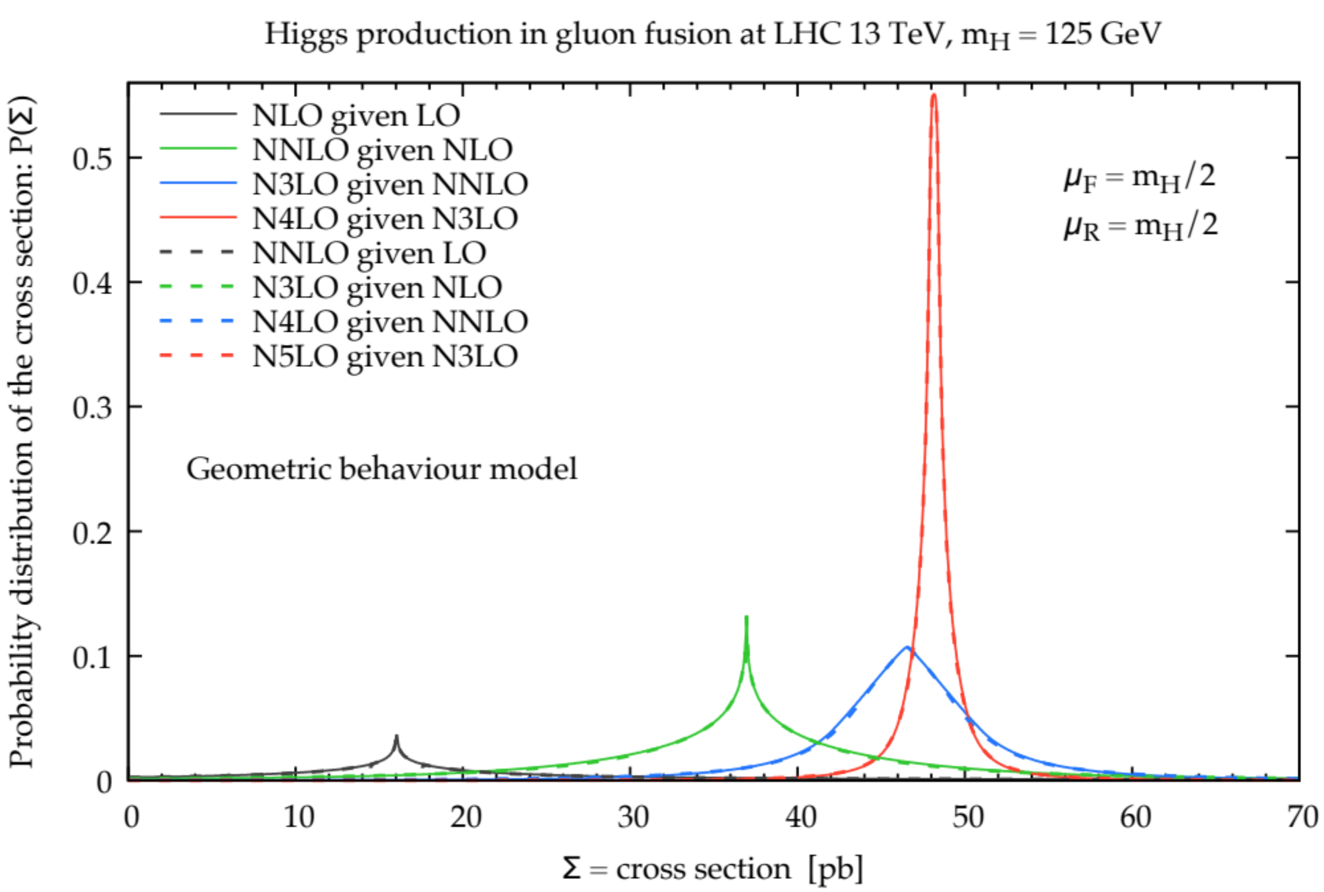
Estimating higher orders

Several approaches, mostly based on **Bayesian inference,** have been developed in order to construct a probabilistic interpretation of MHOUs

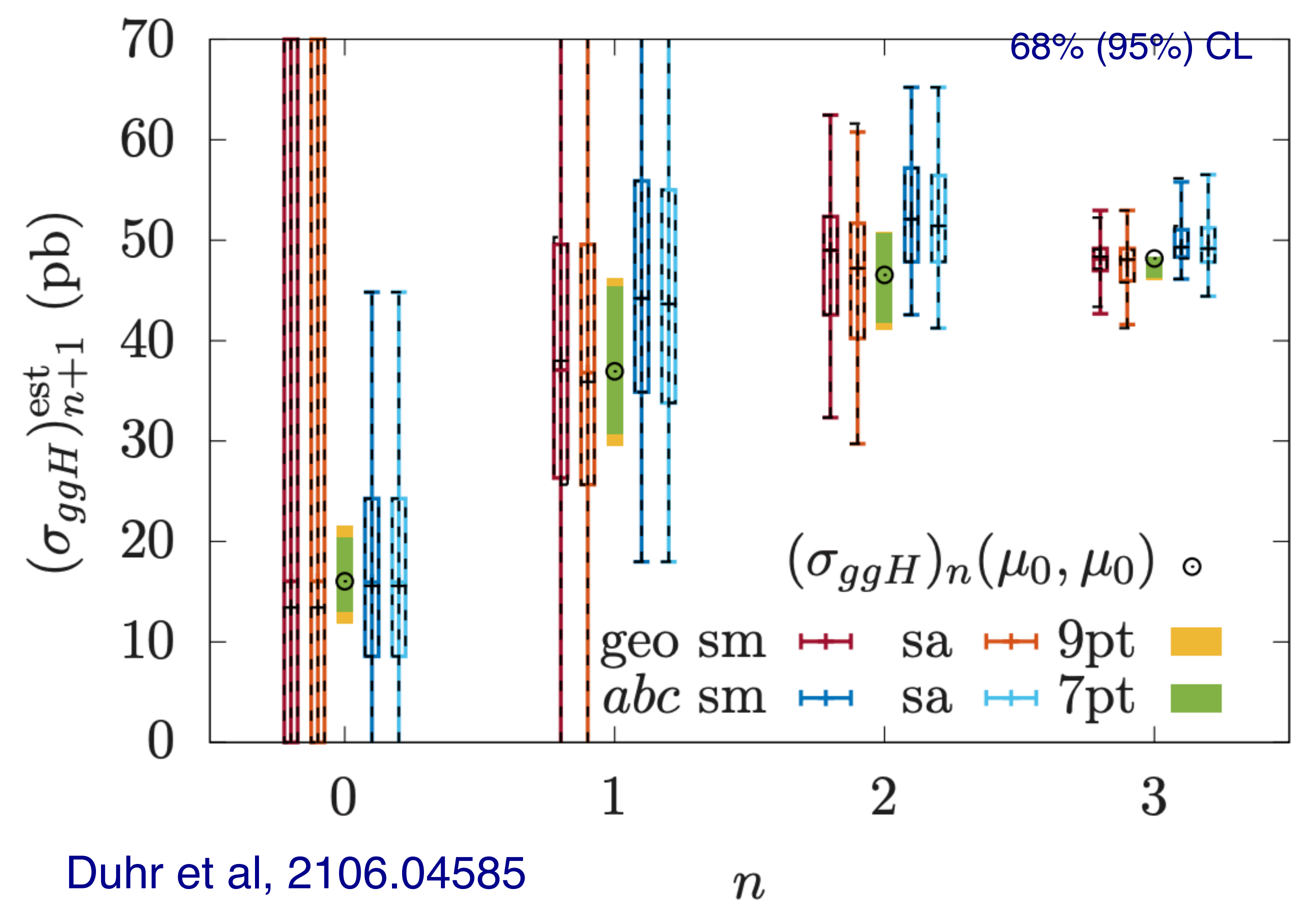
$$P(\Sigma|\underbrace{c_0(\mu), r_0(\mu), ..., c_n(\mu), r_n(\mu)}_{\text{known perturbative terms}}, H)$$

probability distribution for physical observable

assumptions (e.g. priors)



Bonvini, 2006.16293

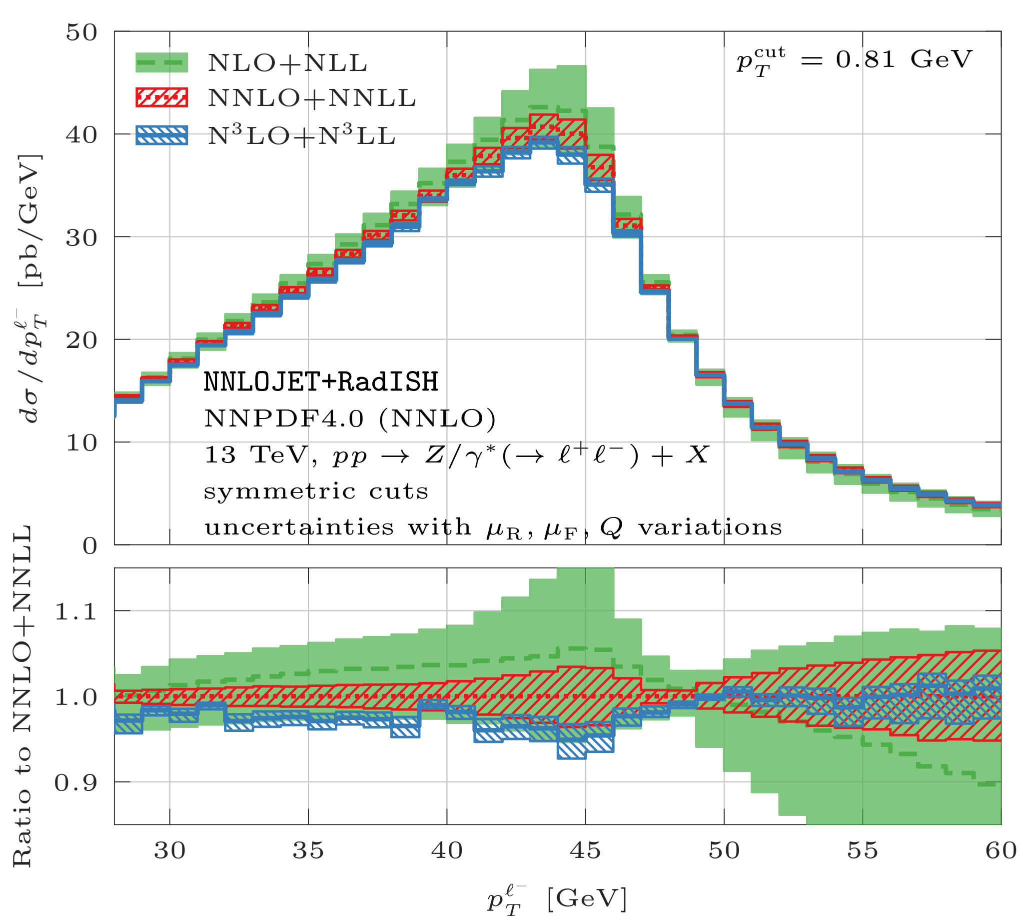


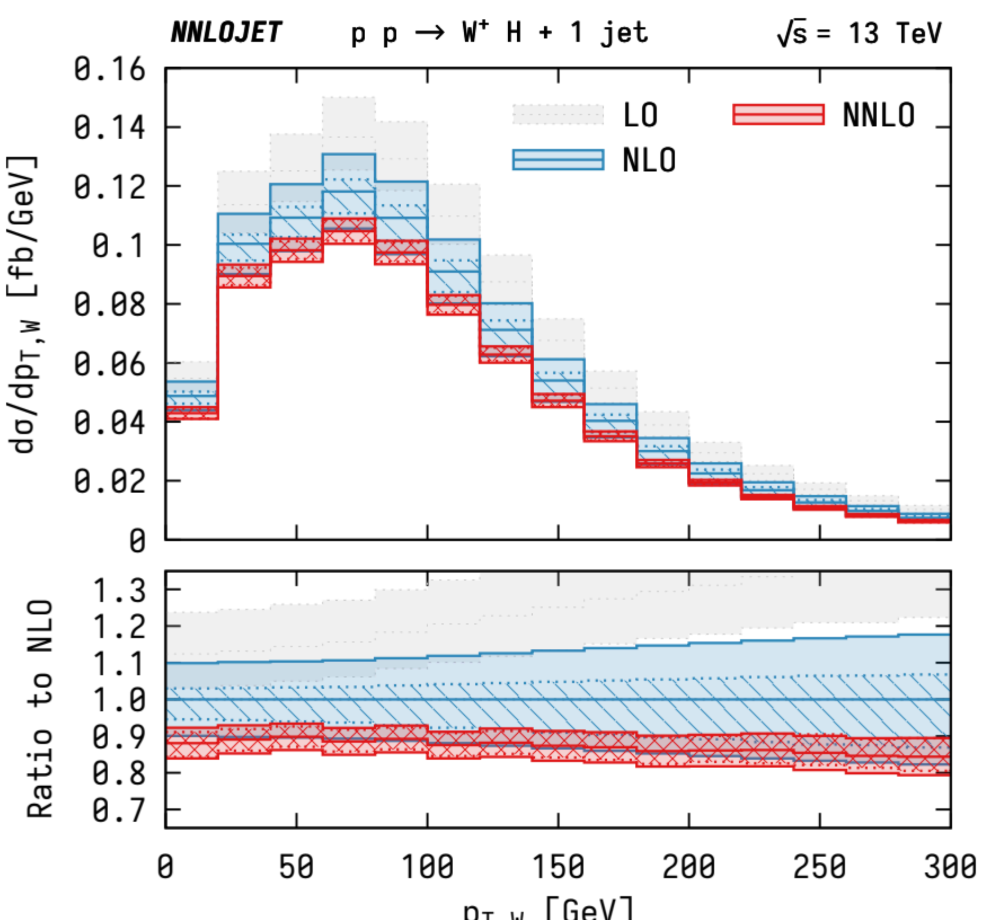
Duhr et al, 2106.04585

probabilistic approaches to MHOU estimates represent a **competitive alternative to scale variations**, but they still contain subjective assumptions related to model and priors

Summary and outlook

- Robust interpretation of collider data relies on **accurate and precise theory calculations**
- Most recent HEP anomalies rely on theory calculations: **discovery through precision?**
- Impressive progress in higher-order QCD&EW calculations, **differential N³LO QCD** current frontier: urgent need to produce a first **N³LO PDF determination**
- Fully exploiting this progress requires matching accuracy for **PDFs** and **parton showers/MCs**
- Beyond LHC physics, precision QCD relevant for the **Electron Ion Collider** program, astroparticle physics experiments (high-energy neutrinos) and the proposed **Forward Physics Facility** @ HL-LHC





Summary and outlook

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- Fully exploiting this progress requires matching accuracy f
- Beyond LHC physics, precision QC … icle physics experiment … LHC

Thanks for your attention!