## Recent Progress on PDF Determination: NNPDF4.0 Updates

Tanjona R. Rabemananjara
On behalf of the NNPDF Collaboration Low-x 2023 - September 3-8, 2023 Leros Island, Greece

VRIJE
UNIVERSITEIT AMSTERDAM

## OUTLINE OF THE TALK:

1. Introduction \& Motivations
2. Approximate $\mathrm{N}_{3} \mathrm{LO}\left(\mathrm{aN}_{3} \mathrm{LO}\right)$ PDFs
3. Theory Uncertainties in PDF Determination
4. PDF Determination with QED Corrections
5. Conclusions \& Outlook


# Introduction \& Motivations 

## Introduction: NNPDF Landscape

Significant improvements on all 3 FRONTS for NNPDF4.0:
■ Experiments: contains $\mathcal{O}(4500)$ datapoints, abundant LHC data from Run II, probe more processes and channels.

■ Methodology: SGD for NN minimisation, Automated Optimisation of Hyperparameters, Methodology validation using Closure Tests/ Future Tests/Parametrisation Basis independence.

- Theory: NNLO QCD with Electroweak corrections and Nuclear Unc.


## NRNPDF

(T) Tests passing DOI 10.5281/zenodo. 7554886

## NNPDF: An open-source machine learning framework for global analyses of parton distributions

The NNPDF collaboration determines the structure of the proton using Machine Learning methods. This is the main repository of the fitting and analysis frameworks. In particular it contains all the necessary tools to reproduce the NNPDF4.0 PDF determinations.

## Introduction: Challenges

- PDFs are becoming a bottleneck for LHC precision calculations with the largest uncertainties along with the incomplete knowledge of $\alpha_{s}$.
- Progress on N3LO calculations for Higgs (ggF, VBF, VH) \& NC/CC DY processes requires $\mathbf{N}_{3}$ LO PDFs.
- QED effects in PDFs are no longer negligible as experimental measurements become more precise and determination of parton densities more accurate.
- Theoretical uncertainties on PDFs are crucial to assess accuracy/uncertainties on MHOUs \& IHOUs.





## Approximate N3LO (aN3LO) PDFs

## What do we need for N3LO PDFs?

Several theory ingredients are required to achieve N3LO PDF fits:
O Splitting Functions/Anomalous Dimensions (AD) to evolve PDFs through the DGLAP equation

$$
\gamma\left(\alpha_{s}\right)=\alpha_{s} \gamma^{(0)}+\alpha_{s}^{2} \gamma^{(1)}+\alpha_{s}^{3} \gamma^{(2)}+\alpha_{s}^{4} \gamma^{(3)}+\cdots
$$

- Matching Conditions/Transition Matrix Elements to change number of PDF flavours at heavy-quark matching scales

$$
f_{\alpha}^{n_{f}+1}\left(x, Q^{2}\right)=\mathbf{A}_{\alpha \beta}^{\left(n_{f}\right)}\left(x, Q^{2} / m_{h}^{2}\right) \otimes f_{\beta}^{n_{f}}\left(x, Q^{2}\right)
$$

- DIS Coefficient Functions to compute structure functions

$$
F_{\alpha}\left(x, Q^{2}\right)=\sum_{\beta, \eta} \mathscr{C}_{\alpha, \beta, \eta}^{n_{\alpha}+1}\left(x, Q^{2}\right) \otimes \mathbf{A}_{\eta \xi}^{\left(n_{\xi}\right)}\left(x, Q^{2} / m_{h}^{2}\right) \otimes f_{\xi}^{n_{f}}\left(x, Q^{2}\right)
$$

O Hadronic Cross-Section k-factors

$$
\Sigma(x)=\Sigma_{0}(x)+\Sigma_{1}(x)+\Sigma_{2}(x)+\Sigma_{3}(x)+\cdots
$$

Several Pieces are still missing $\longrightarrow$ Need Reliable Approximations

## What do we know about 4-Loop Non-Singlet AD?

The complete N3LO Anomalous Dimensions are not known yet, but a lot of information is already available.

■ The case of NON-SINGLET sector:
○ $\mathcal{O}\left(n_{f}^{2}\right)$ and $\mathcal{O}\left(n_{f}^{3}\right)$ terms are known analytically Davies, Vogt, Ruijl, Ueda, Vermaseren [arXiv:1610.07477]

○ $\mathcal{O}\left(n_{f}^{0}\right)$ and $\mathcal{O}\left(n_{f}\right)$ terms are known in the Large $-N_{c}$ limit Moch, Ruijl, Ueda, Vermaseren, Vogt [arXiv:1707.08315]

O Small-N limit: coefficients of logarithms at pole $N=0$ are known numerically Davies, Kom, Moch, Vogt [arXiv:2202.10362]

$$
P_{\mathrm{ns}}(x) \supset \sum_{k=1}^{6} c_{k} \ln ^{k}(1 / x)
$$

O Large-N limit: some coefficients and constant terms are known Henn, Korchemsky, Mistlberger [arXiv:1911:10174]; Duhr, Mistlberger, Vita [arXiv:2205:04493]

$$
\gamma_{\mathrm{ns}}^{(3)}(N) \approx A_{4} S_{1}(N)-B_{4}^{+} C_{4} \frac{S_{1}(N)}{N}-D_{4} \frac{1}{N}
$$

O Results for Even/Odd Mellin Moments are known Moch, Ruijl, Ueda, Vermaseren, Vogt [arXiv:1707.08315]

The dependence of $\gamma_{i j}^{(3)}$ on the number of active flavours can be expressed as follows:

$$
\gamma_{i j}^{(3)}=\gamma_{i j}^{(3,0)}+n_{f} \gamma_{i j}^{(3,1)}+n_{f}^{2} \gamma_{i j}^{(3,2)}+n_{f}^{3} \gamma_{i j}^{(3,3)}
$$

|  | $n_{f}^{0}$ | $n_{f}^{1}$ | $n_{f}^{2}$ | $n_{f}^{3}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\gamma_{n s,-}^{(3)}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\gamma_{n s,+}^{(3)}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\gamma_{n s, s}^{(3)}$ |  | $\checkmark$ | $\checkmark$ |  |

## What do we know about 4-Loop Singlet AD?

The complete N3LO Anomalous Dimensions are not known yet, but a lot of information is already available.

- The case of SINGLET sector:

O Leading Large- $n_{f}$ contributions to $\mathcal{O}\left(n_{f}^{3}\right)$ terms are known analytically Davies, Vogt, Ruijl, Ueda, Vermaseren [arXiv:1610.07477]

- Small-N limit: BFKL limits of $\gamma_{q g}^{(3)}$ and $\gamma_{g g}^{(3)}$ are known up to LL and NLL, respectively. Coefficients of logarithms at pole $N=1$ are known numerically Bonvini and Marzani [arXiv:1805.06460]; Davies, Kom, Moch, Vogt [arXiv:2202.10362]

$$
P_{\mathrm{ns}}(x) \supset \sum_{k=0}^{3} c_{k} \frac{\ln ^{k}(1 / x)}{x}
$$

O Large-N limit: Diagonal $\left(\gamma_{g g}^{(3)}, \gamma_{q q, \mathrm{ps}}^{(3)}\right)$ and Off-diagonal $\left(\gamma_{q g}^{(3)}, \gamma_{g q}^{(3)}\right)$ need to be treated separately. Their coefficients in the expansion $1 / N$ are known numerically Duhr, Mistlberger, Vita [arXiv:2205.04493]; Henn, Korchemsky, Mistlberger [arXiv:1911.10174]; Soar, Moch, Vermaseren, Vogt [arXiv:0912.0369]

O Results for Even Mellin Moments are known Falcioni, Herzog, Moch, Vogt [arXiv:2302.07593]-[arXiv:2307.04158]; Moch, Ruijl, Ueda, Vermaseren, Vogt [arXiv:2111.15561]

The dependence of $\gamma_{i j}^{(3)}$ on the number of active flavours can be expressed as follows:

$$
\gamma_{i j}^{(3)}=\gamma_{i j}^{(3,0)}+n_{f} \gamma_{i j}^{(3,1)}+n_{f}^{2} \gamma_{i j}^{(3,2)}+n_{f}^{3} \gamma_{i j}^{(3,3)}
$$

|  | $n_{f}^{0}$ | $n_{f}^{1}$ | $n_{f}^{2}$ | $n_{f}^{3}$ |
| :--- | :---: | :---: | :---: | :---: |
| $\gamma_{g g}^{(3)}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\gamma_{g q}^{(3)}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\gamma_{q g}^{(3)}$ |  | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\gamma_{q q, p s}^{(3)}$ |  | $\checkmark$ | $\checkmark$ | $\checkmark$ |

## Approximating N3LO Anomalous Dimensions

The approximation to the full $\gamma_{i j}^{(3)}$ is done in Mellin space for each power of $n_{f}$ independently with the following steps:

- Select a basis function $G_{1}(N)$ for leading large- $N$ contributions
- Select a basis function $G_{2}(N)$ for leading small- $N$ contributions
- Select two basis functions $G_{1}(N), G_{2}(N)$ for subleading small- and large- $N$ contributions
n Varying subleading $G_{\ell}$ bases to produce Candidates $\longrightarrow$ IHOUs

Provided with the known ingredients, we can approximate $\gamma_{i j}^{(3)}$ (for a given $n_{f}^{\alpha}$ ) by parametrising the missing $\tilde{\gamma}_{i j}^{(3)}$ as follows:

$$
\gamma_{i j}^{(3)}=\gamma_{i j, n_{f}}^{(3)}+\gamma_{i j, N \rightarrow \infty}^{(3)}+\gamma_{i j, N \rightarrow 0}^{(3)}+\widetilde{\gamma}_{i j}^{(3)}
$$

With $\widetilde{\gamma}_{i j}^{(3)}$ expressed as a linear combination of interpolating functions

$$
\widetilde{\gamma}_{i j}^{(3)}(N)=\sum_{\ell=1}^{n_{\ell}} a_{\ell}^{i j} G_{\ell}(N)
$$

|  | $G_{1}(N)$ |
| :---: | :---: |
| $G_{2}(N)$ | $\mathcal{M}\left[(1-x) \ln ^{2}(1-x)\right]$ |
| $\gamma_{q q, \text { ps }}^{(3)}(N)$ | $-\frac{1}{(N-1)^{2}}+\frac{1}{N^{2}}$ |
|  | $G_{3}(N)$ |
|  | $\mathcal{M}\left[(1-x)^{2} \ln (1-x)^{2}\right], \frac{1}{N-1}-\frac{1}{N}, \mathcal{M}[(1-x) \ln (x)]$ |
| $G_{4}(N)$ | $\mathcal{M}[(1-x)(1+2 x)], \mathcal{M}\left[(1-x) x^{2}\right]$, |
|  | $\mathcal{M}[(1-x) x(1+x)], \mathcal{M}[(1-x)]$ |

## Approximating N3LO Anomalous Dimensions



## Comparisons with MSHTaN3LO



## N3LO DIS Coefficient Functions

Some of the perturbative ingredients to produce $\mathrm{N}_{3} \mathrm{LO}$ structure functions are not yet known.

- All Light Flavour Coefficient Functions both for NC and CC are known exactly Larin, Nogueira, Van Ritbergen, Vermaseren[arxiv:9605317]; Moch, Vermaseren, Vogt [arxiv:0411112], [arxiv:0504242]
- Some parts needed for the construction of Heavy Flavour Coefficient Functions are missing. Take the quark \& gluon coefficient functions:

$$
\mathscr{C}_{i}^{(3)}=\mathscr{C}_{i}^{(3,0)}+\mathscr{C}_{i}^{(3,1)} \ln \left(\frac{\mu^{2}}{m^{2}}\right)+\mathscr{C}_{i}^{(3,2)} \ln ^{2}\left(\frac{\mu^{2}}{m^{2}}\right)
$$

While $C_{i}^{(3,1)}$ and $C_{i}^{(3,2)}$ are known exactly $C_{i}^{(3,0)}$ can be constructed by combining known limits with some matching functions $f_{1}(x)$ and $f_{2}(z)$ that interpolate between the two limits:

$$
\mathscr{C}_{i, \text { approx }}^{(3,0)}(z)=\mathscr{C}_{i, z \rightarrow 0}^{(3,0)}(z) f_{1}(z)+\mathscr{C}_{i, z \rightarrow z_{\text {max }}}^{(3,0)}(z) f_{2}(z)
$$




## Matching Conditions \& GM-VFNS

Predictions for structure valid for all $Q^{2}$ require matching Mass effects in the Machine Scheme with Log resummation in the Massless scheme.

In NNPDF, DIS structure functions are computed using the FONLL method:

$$
\tilde{F}_{\alpha}\left(x, Q^{2}\right)=F_{\alpha}^{\left(n_{f}+1\right)}\left(x, Q^{2}\right)-F_{\alpha}^{\left(n_{f}, 0\right)}\left(x, Q^{2}\right)+F_{\alpha}^{\left(n_{f}\right)}\left(x, Q^{2}\right)
$$

PDFs defined in $\left(n_{f}+1\right)$ and $\left(n_{f}\right)$ are related via Matching Conditions:

$$
f_{\alpha}^{n_{f}+1}\left(x, Q^{2}\right)=\mathrm{A}_{\alpha i}^{\left(n_{f}\right)}\left(x, Q^{2} / m_{h}^{2}\right) \otimes f_{i}^{n_{f}}\left(x, Q^{2}\right)
$$

The full entries of the Matching Condition matrix elements are almost completely known except for $a_{H, g}^{(3)}$ Ablinger, Behring, Blümlein, De Freitas, Goedicke, von Manteuffel, Schonwald [arXiv:2211.0546]; Ablinger, Behring, Blümlein, De Freitas, Hasselhuhn, von Manteuffel, Round, Schneider, Wißbrock. [arXiv:1406.4654]; Bierenbaum, Blümlein, Klein [arXiv:0904.3563]


## Hadronic K-factors

https://github.com/jubaglio/n3loxs
■ Double Hadronic $\mathbb{K}$-factors are much less known than the other ingredients needed for (a) $\mathrm{N}_{3} \mathrm{LO}$

- Various calculations are available at N3LO but not useful for PDF fits: Higgs (ggF, VBF, VH) B. Mistlberger [arXiv:1802.00833]; A. Dreyer, A. Karlberg [arXiv:1606.00840], J. Baglio, C. Duhr, B. Mistlberger, R. Szafron [arXiv:2209.06138], Top N. Kidonakis, M. Guzzi, A. Toreno [arXiv: 2306.06166]

๕ In NNPDF, hadronic K-factors are computed using n3loxs for all NC/CC DY J. Baglio, C. Duhr, B. Mistlberger, R. Szafron [arXiv:2209.06138]:

Dataset: ATLASZHIGHMASS49FB



Estimating Theory Uncertainties

## IHOUs from N3LO Splitting Functions

How are higher-order uncertainties arising from the approximation of $\mathrm{N}_{3} \mathrm{LO}$ splitting functions propagated into the PDF Fit?

Variation of the Basis functions used to parametrise $\tilde{\gamma}_{i j}$ generate $N_{\gamma}$ variations. The spread of variation w.r.t. the Central Predictions is used to define a Covariance Matrix:

$$
\operatorname{Cov}_{i, j}^{\text {IHOU }}=\frac{1}{N_{\mathrm{var}}-1} \sum_{k=1}^{N_{\mathrm{var}}}\left(T_{i, k}-\bar{T}_{i}\right)\left(T_{j, k}-\bar{T}_{j}\right)
$$

Since theory uncertainties resulting from $\mathbf{I H O U s}$ are independent from experimental uncertainties, the two contributions can be added in Quadrature:

$$
\operatorname{Cov}_{i, j}=\operatorname{Cov}_{i, j}^{\exp }+\operatorname{Cov}_{i, j}^{\mathrm{IHOU}}
$$



## Missing Higher Order Uncertainties (MHOUs)

For a given observable $\mathcal{O}$, MHOUs are commonly estimated by varying the unphysical scales in the Parton evolutions and in the partonic cross-sections:

$$
\mathcal{O}\left(\alpha_{s}\left(\mu^{2}\right), \frac{Q^{2}}{\mu_{F}^{2}}, \frac{Q^{2}}{\mu_{R}^{2}}\right)=\mathscr{L}\left(\alpha_{s}\left(\mu_{F}^{2}\right), \frac{Q^{2}}{\mu_{F}^{2}}\right) \mathcal{O}\left(\alpha_{s}\left(\mu_{R}^{2}\right), \frac{Q^{2}}{\mu_{R}^{2}}\right)
$$



Variation of Factorisation Scale $\kappa_{F}=Q^{2} / \mu_{R}^{2}$ estimates MHOUs from Anomalous Dimensions in the evolution while variation of Renormalisation Scale $\kappa_{R}=Q^{2} / \mu_{R}^{2}$ estimates MHOUs from partonic cross-sections.

Similar to IHOUs, MHOUs can be added as a nuisance parameter to the Covariance Matrix NNPDF [arxiv:1906.10698], [arxiv:2105.05114]

$$
\operatorname{Cov}_{i, j}=\operatorname{Cov}_{i, j}^{\mathrm{exp}}+\operatorname{Cov}_{i, j}^{\mathrm{MHOU}}, \quad \operatorname{Cov}_{i, j}^{\mathrm{MHOU}}=\frac{1}{N_{\mathrm{var}}-1} \sum_{k=1}^{N_{\mathrm{var}}}\left(S_{i, k}-\bar{S}_{i}\right)\left(S_{j, k}-\bar{S}_{j}\right)
$$

7-point scale variation prescription is used. Points belonging to the same process are CORRELATED by $\kappa_{R}$-variation while $\kappa_{F}$ correlates all the points.


## Missing Higher Order Uncertainties (MHOUs)

We can check that the MHOU is Working by looking at the Diagonal Entries.


$$
\left(\frac{\sqrt{\operatorname{Cov}_{i i, \mathrm{NLO}}^{M H O U}}}{\mathcal{O}_{i}^{\mathrm{MHOU}}}\right) \times 100
$$

$$
\left(\frac{\widehat{O}_{i}^{\mathrm{NNLO}}-\widehat{O}_{i}^{\mathrm{NLO}}}{\widehat{O}_{i}^{\mathrm{NLO}}}\right) \times 100
$$

Good consistency between MHOUs \& NNLO-NLO Shifts

## NNLO MHOU Correlation Matrices

- First ever NNLO PDF determination with MHO Uncertainties
- MHOUs add CORRELATION between process not taken into account by experimental Covariance Matrix


Theory Correlation matrix (7 pt)


$$
\rho_{i j}=\frac{\operatorname{Cov}_{\mathrm{ij}}}{\sqrt{\operatorname{Cov}_{\mathrm{ii}}} \sqrt{\operatorname{Cov}_{\mathrm{jj}}}}
$$



## Impacts of MHOUs on PDFs



Compare Perturbative Convergence

## Phenomenological Impacts of MHOUs


n NLO-MHOU predictions are closer to NNLO than pure NLO.

- Very good agreement between NNLO and NNLO-MHOU.

MHOUs improve perturbative convergence from NLO to NNLO.
m NLO vs. NNLO exhibit the largest Uncertainty Pull.

## NNPDF4.0 @ aN3LO PDFs with MHOUs



PDF Determination with QED

## PDF Fits with QED Effects

M Photon PDF $\gamma\left(x, Q^{2}\right)$ can no longer be neglected as determination of parton densities become more accurate $\alpha \sim \mathcal{O}\left(\alpha_{s}^{2}\right) \sim \mathcal{O}(1 \%)$.
LuxQED: $\gamma$-PDF can be computed perturbatively using as inputs structure functions A.V. Manohar, P. Nason, G.P. Salam, G. Zanderighi [arXiv:1607.04266]-[arXiv:1708.01256]; NNPDF3.1 [arXiv:1712.07053]; MSHT2o [arXiv: 2111.05357]
Two main changes are required to account for QED effects in PDF fits:

M Modified QCD $\otimes$ QED DGLAP Evolution:

$$
\mu^{2} \frac{d f_{i}\left(N, \mu^{2}\right)}{d \mu^{2}}=\sum_{j} \gamma_{i j}\left(N, \alpha_{s}\left(\mu^{2}\right), \alpha\left(\mu^{2}\right)\right) f_{j}\left(N, \mu^{2}\right)
$$

■ Mixed QCD $\otimes$ QED Sum Rules:

$$
\int_{0}^{1} d x(x \Sigma+x g+x \gamma)=1
$$



$$
\mathbf{L O} \quad \alpha_{S}^{1} \alpha^{0} \quad \alpha_{S}^{0} \alpha^{1}
$$

$$
\text { NLO } \quad \alpha_{s}^{2} \alpha^{0} \quad \alpha_{s}^{1} \alpha^{1} \quad \alpha_{s}^{0} \alpha^{2}
$$

$$
\text { NNLO } \quad \alpha_{s}^{3} \alpha^{0} \quad \alpha_{s}^{2} \alpha^{1}
$$

## Determination of $\gamma$ PDF

Parton density functions and $\gamma$-PDF are determined such that they satisfy the following Sum Rule:

$$
\int_{0}^{1} d x\left(x \Sigma\left(x, Q^{2}\right)+x g\left(x, Q^{2}\right)+x \gamma\left(x, Q^{2}\right)\right)=1
$$

$$
a_{e m}=\alpha /(4 \pi)
$$

$$
F_{2, L}=\mathscr{C}_{2, L}^{i} \otimes f_{i}
$$

Where $\gamma\left(x, Q^{2}\right)$ is computed iteratively during the fit using structure function inputs:

$$
\begin{aligned}
& x \gamma\left(x, \mu^{2}\right)=\frac{2}{a_{e m}\left(\mu^{2}\right)} \int_{x}^{1} \frac{d z}{z}\left\{\int _ { \frac { m _ { D } ^ { 2 } x ^ { 2 } } { ( 1 - z ) } } ^ { \frac { \mu ^ { 2 } } { ( 1 - z ) } } \frac { d Q ^ { 2 } } { Q ^ { 2 } } a _ { e m } ^ { 2 } ( Q ^ { 2 } ) \left[-z^{2} F_{L}\left(x / z, Q^{2}\right)\right.\right. \\
&\left.\left.+\left(z P_{\gamma q}(z)+\frac{2 x^{2} m_{p}^{2}}{Q^{2}}\right) F_{2}\left(x / z, Q^{2}\right)\right]-a_{e m}^{2}\left(\mu^{2}\right) z^{2} F_{2}\left(x / z, \mu^{2}\right)\right\}
\end{aligned}
$$



## Mixed QCD $\otimes$ QED Evolution

Because photons couple differently to up-liike and down-like quarks
$\Longleftrightarrow$
QCD $\otimes$ QED Evolution is more difficult to Diagonalise

NNPDF4.oQED uses a so called Unified Evolution Basis $n_{f}$ active quarks are split into $n_{u}$ and $n_{f}$ flavors ( $n_{f}=n_{u}+n_{d}$ )

$$
\mu^{2} \frac{d}{d \mu^{2}}\left(\begin{array}{c}
g \\
\gamma \\
\Sigma \\
\Sigma_{\Delta}
\end{array}\right)=-\boldsymbol{\Gamma}_{s}\left(\begin{array}{c}
g \\
\gamma \\
\Sigma \\
\Sigma_{\Delta}
\end{array}\right), \quad \mu^{2} \frac{d}{d \mu^{2}}\binom{V}{V_{\Delta}}=\boldsymbol{\Gamma}_{V}\binom{V}{V_{\Delta}}, \quad \mu^{2} \frac{d}{d \mu^{2}} f_{n s, \pm}^{u / d}=\left(\gamma_{n s, \pm}+\tilde{\gamma}_{n s, \pm}^{u / d}\right) f_{n s, \pm}^{u / d}
$$

$$
f_{n s, \pm}^{u}=\left\{\begin{array}{l}
u^{ \pm}-c^{ \pm} \\
u^{ \pm}+c^{ \pm}-2 t^{ \pm}
\end{array}, \quad f_{n s, \pm}^{d}=\left\{\begin{array}{l}
d^{ \pm}-s^{ \pm} \\
d^{ \pm}+s^{ \pm}-2 b^{ \pm}
\end{array}, \quad \Sigma_{\Delta}=\frac{n_{d}}{n_{u}} \sum_{i=1}^{n_{u}} u_{i}^{+}-\sum_{i=1}^{n_{d}} d_{i}^{+} \quad V_{\Delta}=\frac{n_{d}}{n_{u}} \sum_{i=1}^{n_{u}} u_{i}^{-}-\sum_{i=1}^{n_{d}} d_{i}^{-}\right.\right.
$$

## NNLO QCD $\otimes$ QED PDFs







## Phenomenological Impacts of QCD $\otimes$ QED PDFs




$\mathrm{pp} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-} \rightarrow \ell \nu_{\ell} \ell^{\prime} \bar{\nu}_{\ell^{\prime}}+\mathrm{X}$


$p_{\mathrm{T}, \ell^{\prime} \bar{\ell}}[\mathrm{GeV}]$


Non-negligible corrections in highinvariant mass and high- $p_{T}$ regions


## Conclusions \& Outlook

## Conclusions \& Outlook

■ More precise and accurate PDF central values AND uncertainties are vital for precision and beyond the Standard Model Physics.
■ Inclusion of Electroweak corrections is becoming more relevant $\Leftarrow$ QED corrections \& photon $\gamma$-PDF

## Stay tuned for new NNPDF4.0 releases:

n NLO PDFs with Faithful estimation of MHO uncertainties $\Longrightarrow$ NNPDF4.0 MHOU

- PDF determination with QED $\Longrightarrow$ NNPDF4.0 QED
m Approximate $\mathrm{N}_{3} \mathrm{LO}$ PDF determination $\Longrightarrow$ NNPDF $_{4} .0$ aN3LO



## A new Toolchain for PDF predictions

```
Fitting Code can be anything:
    NNPDF, CTEQ, MSHT, etc.
```



## aN3LO PDFs: Perturbative Stability



## aN3LO PDFs: Luminosities








## MHOUs: closer look at CovMat



## QCD $\otimes$ QED Evolution



## Scale Dependence of $\gamma$-PDF



38

