HUGS2023

May 30 - June 16, 2023 • Newport News, VA

GLOBAL PDF FITS:
CONNECTING LOW TO HIGH ENERGY PHYSICS

LECTURE I & II

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Goal of the lectures

➡ Give an overview on our understanding on the structure of the proton: from Feynman parton model to modern QCD picture

➡ Introduce basic concepts and techniques behind PDF global fits

➡ Wealth of ingredients involved from low to high energy: non-perturbative effects, perturbative QCD, experimental measurements, statistical and mathematical problems, higher order predictions, phenomenology tools, machine learning.

➡ Discuss PDF-related phenomenology at the LHC (mostly), EIC and beyond

➡ Discuss current frontiers and challenges

Disclaimer: these lectures are far from providing a complete picture of the topic. You can find complementary information in excellent lectures on PDFs from W. Giele, G. Salam, A. Martin, P. Nadolsky, S. Forte, D. Stump, W. Melnitchouk, D. Stump, A. Guffanti, J. Rojo ... at recent graduate schools

References

- G. Ridolfi "Notes on deep-inelastic scattering and the Parton model"
- Ellis, Stirling and Webber "QCD and collider physics"
- Dissertori, Knowles, Schmelling "Quantum Chromo Dynamics"
- "Proton structure at the precision frontier" Snowmass 2021 Whitepaper arXiv:2203.13923
- Kovarik, Nadolsky, Soper, arXiv:1905.06957
- Gao, Harland-Lang, Rojo - *Phys.Rept. 742 (2018) 1-121*
- Forte, Watt - *Ann.Rev.Nucl.Part.Sci. 63 (2013)*
- Perez, Rizvi, *Rep.Prog.Phys. 76 (2013) 046201.*
- Accardi, et al., *Eur. Phys. J. C76 (8) (2016) 471*
- http://pdg.lbl.gov/2021/reviews/rpp2021-rev-structure-functions.pdf

List of references complemented by specific references during the lectures

Outline

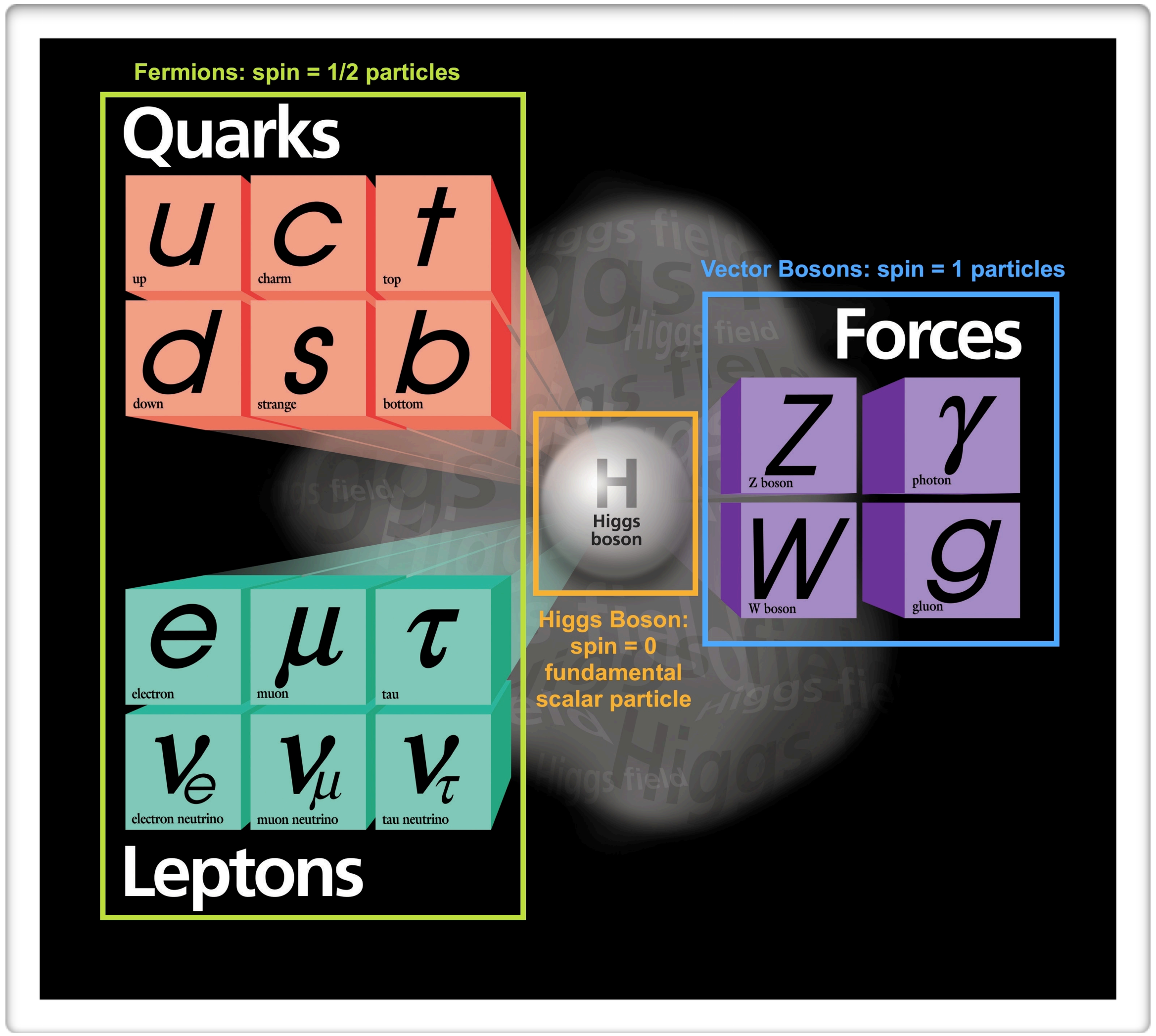
- **<u>First two lectures</u> (Today)**
- Third & fourth lectures (Wednesday)
    - Ingredients of a PDF global fits
    - Experimental input
    - Methodological aspects
    - Theoretical aspects
- Fifth lecture (Thursday)
    - New frontiers and challenges

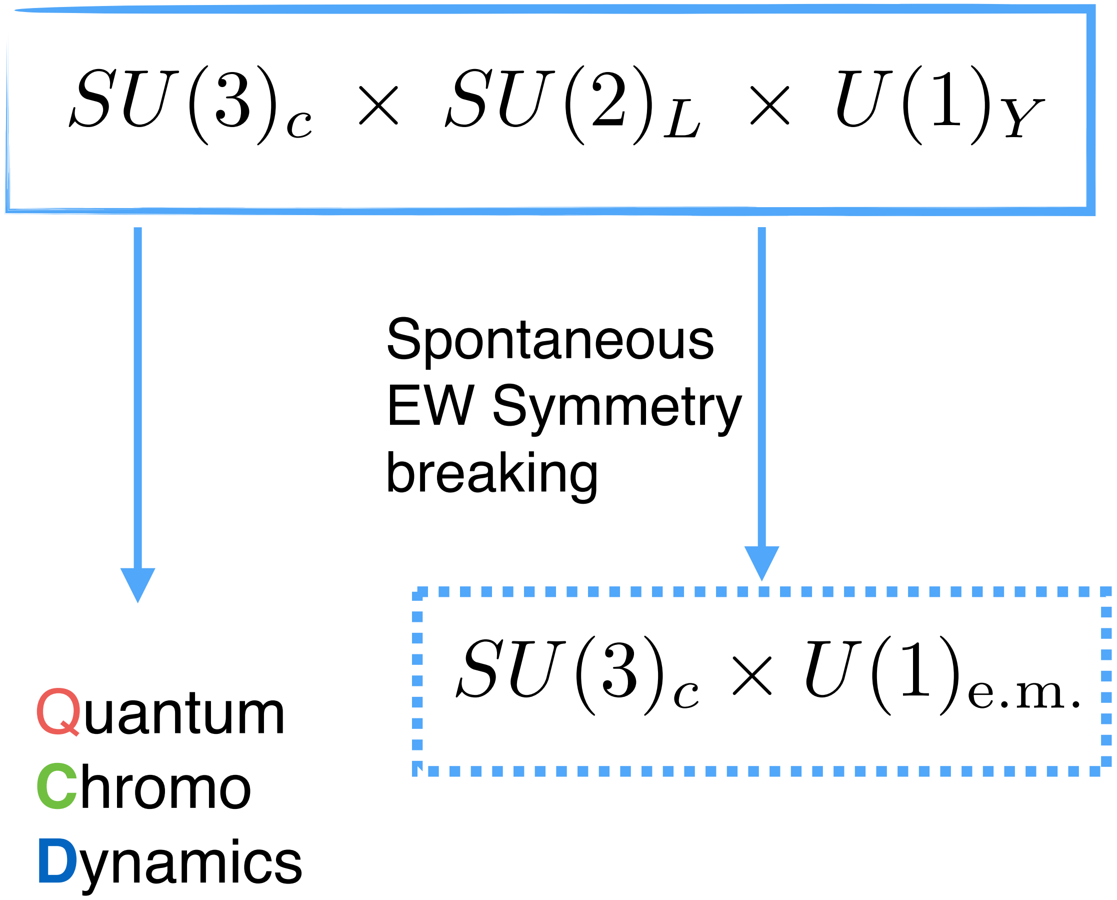
- Motivation: the high energy big picture
- Parton Model and QCD
- Collinear Factorisation

Motivation:
the high-energy big picture

Standard Model of particle physics

- Standard Model (SM) of particle physics one of the greatest triumph of Quantum Field Theories in the past century
- SM remarkably successful theory: no convincing deviations so far from its predictions, but necessarily incomplete





$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

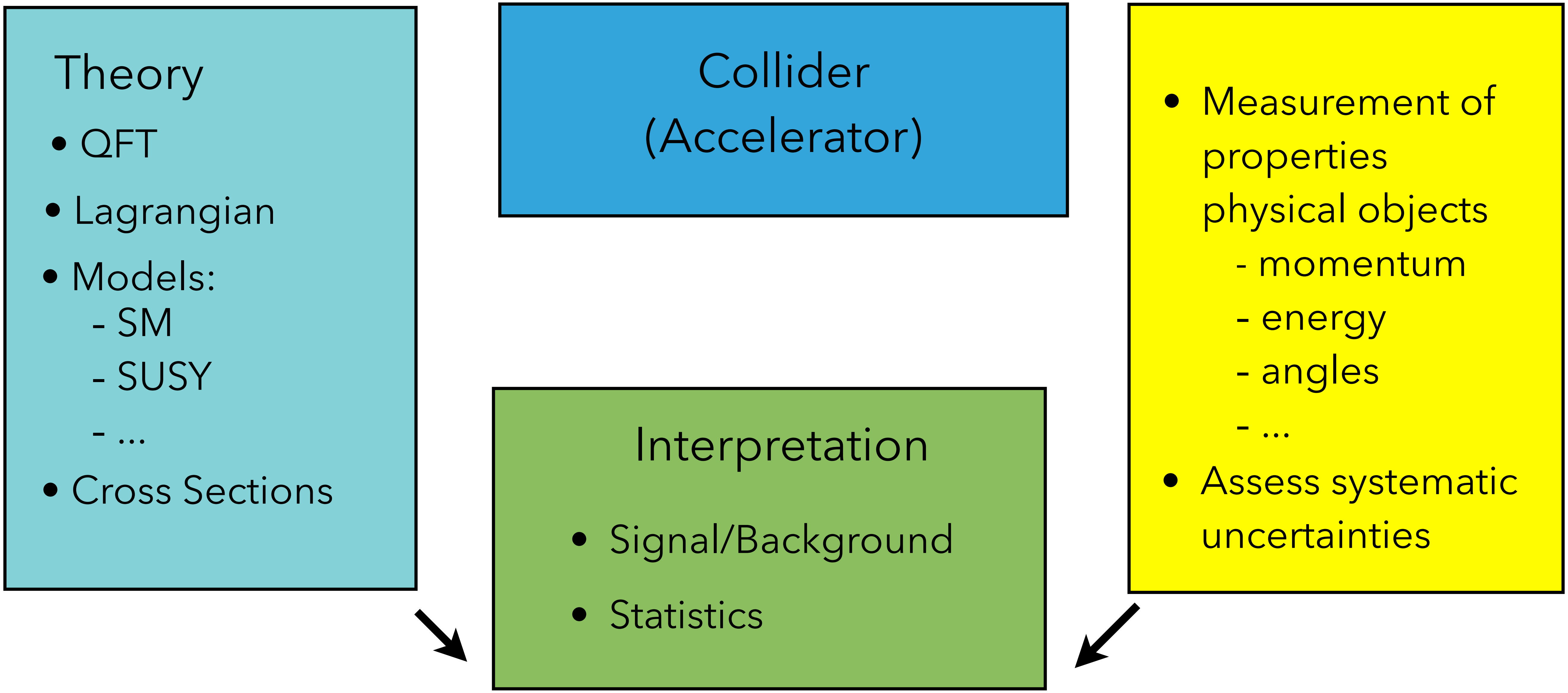
Spontaneous EW Symmetry breaking

Quantum **C**hromo **D**ynamics

$$SU(3)_c \times U(1)_{\mathrm{e.m.}}$$

The collider opportunity

- The collider era gives us the unique opportunity to test theoretical predictions experimentally in a controllable environment



Theory

- QFT
- Lagrangian
- Models:
  - SM
  - SUSY
  - …
- Cross Sections

Collider
(Accelerator)

- Measurement of properties physical objects
  - momentum
  - energy
  - angles
  - …
- Assess systematic uncertainties

Interpretation

- Signal/Background
- Statistics

The collider opportunity

$A + B \to M$ production in 2-particle collisions: $M^2 = (p_1 + p_2)^2$

fixed target:

$$p_1 \simeq (E, 0, 0, E)$$

$$p_2 = (m, 0, 0, 0)$$

$$M \simeq \sqrt{2mE}$$

- root E law: large energy loss in E_{kin}
- dense target: large collision rate / luminosity

circular collider:

$$p_1 = (E, 0, 0, E)$$

$$p_2 = (E, 0, 0, -E)$$

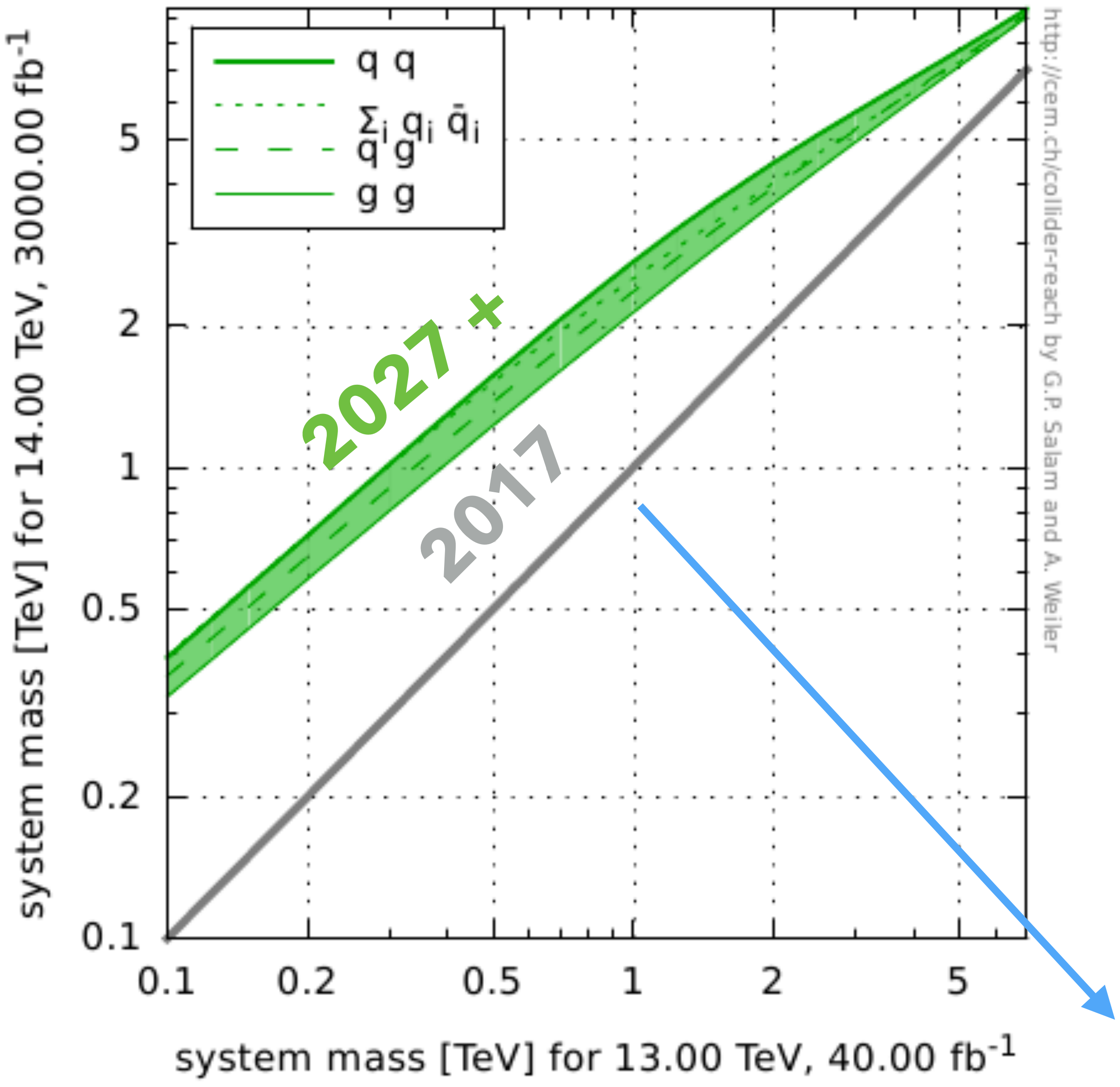
$$M \simeq 2E$$

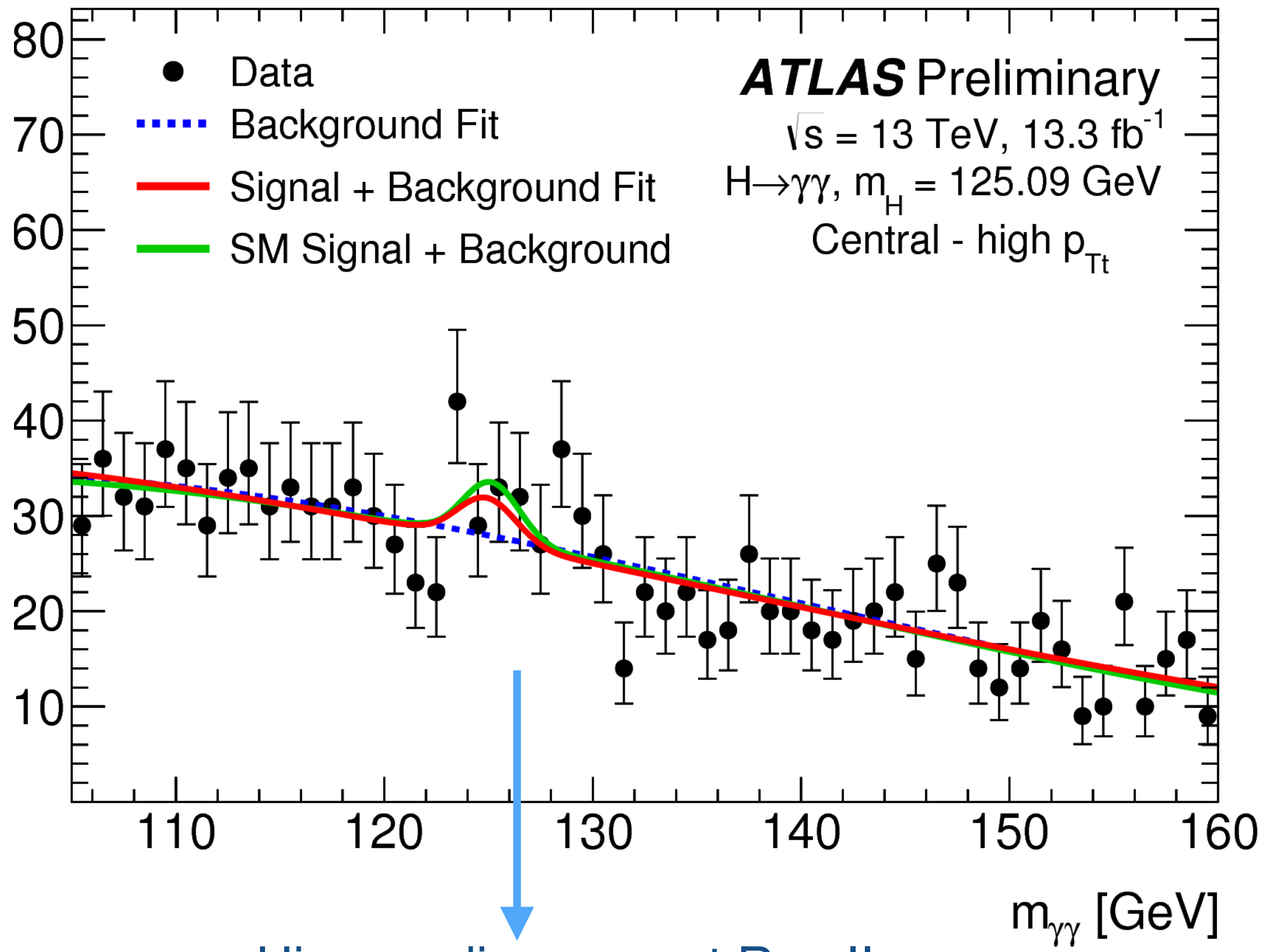
- linear E law: no energy loss
- less dense bunches: small collision rates
- synchrotron radiation: need powerful magnets

Collider	Site	Initial State	Energy	Discovery / Target
SPEAR (1972)	SLAC	e^+e^-	4 GeV	charm quark, tau lepton
PETRA (1978)	DESY	e^+e^-	38 GeV	gluon
SppS (1981)	CERN	$p\bar{p}$	600 GeV	W, Z bosons
LEP (1989)	CERN	e^+e^-	210 GeV	SM: elw and QCD
SLC (1989)	SLAC	e^+e^-	90 GeV	elw SM
HERA (1992)	DESY	$e\,p$	320 GeV	quark/gluon structure of proton
Tevatron (1987)	FNAL	$p\bar{p}$	2 TeV	top quark
BaBar / Belle (1999)	SLAC / KEK	e^+e^-	10 GeV	quark mix / CP violation
LHC (2010)	CERN	$p\,p$	7/8/14 TeV	Higgs, EW, QCD at high E, New Physics
EIC	BNL	e^-p/e^-A	20 - 140 GeV	structure of gluon-dominated matter
ILC	JAPAN?		> 200 GeV	hi. res of elw sb / Higgs couplings
CLIC	CERN		3 - 5 TeV	hi. res of elw sb / Higgs couplings
FCC	CERN		100 TeV	disc. multi-TeV physics

Let’s start from high energy…

- The Large Hadron Collider at CERN most powerful accelerator ever built
- Extremely successful Run I (7-8 TeV) and great performance at Run II (13-14 TeV)
- As luminosity increases, stronger probe on known processes (Higgs, Flavour anomalies…) & larger mass reach





Higgs rediscovery at Run II

With LHC upgrade in ten years
mass reach increase by ~2

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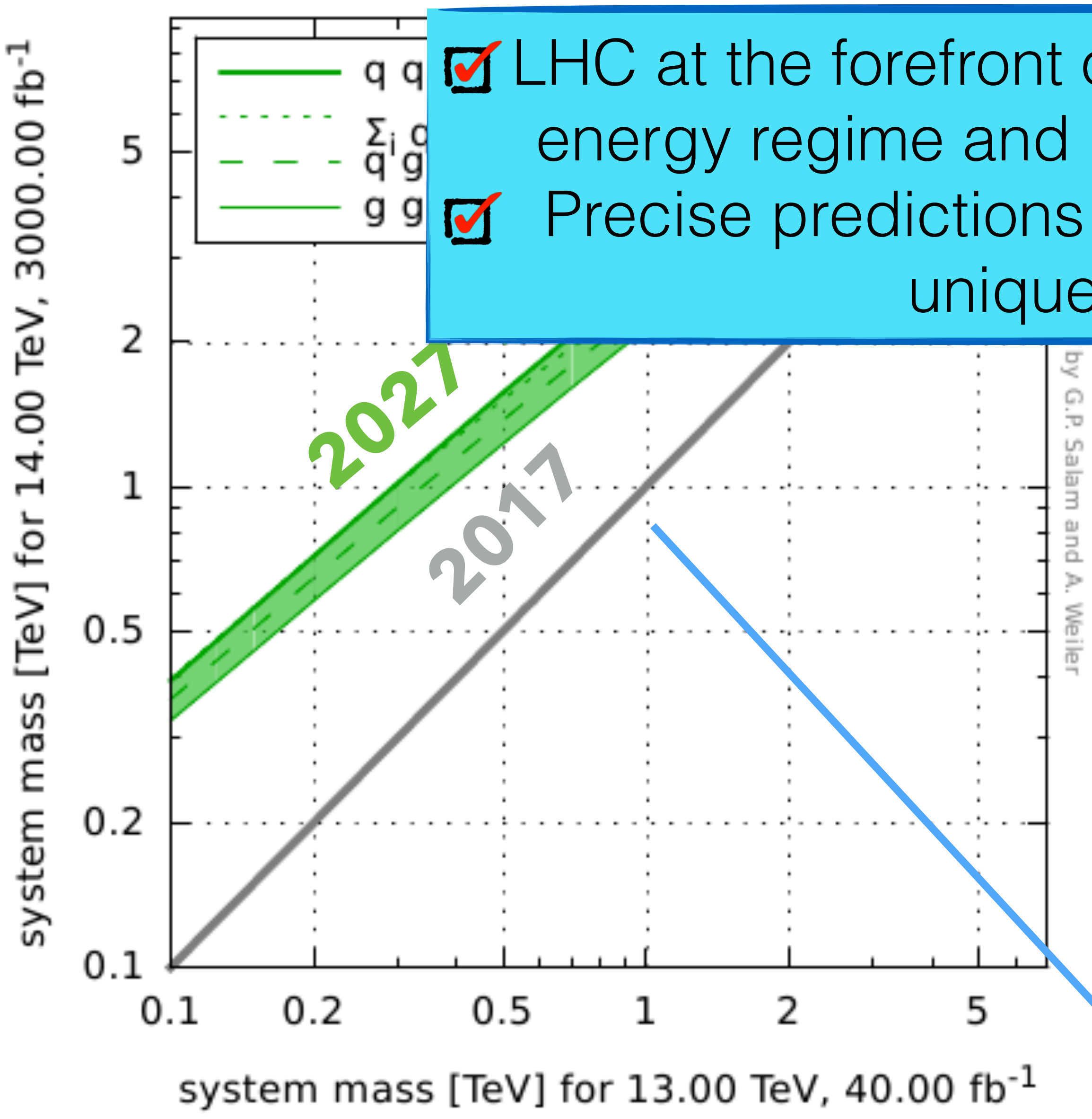
☑ LHC at the forefront of the exploration of the high energy regime and key to plan future colliders

☑ Precise predictions are the key not to miss this unique opportunity!



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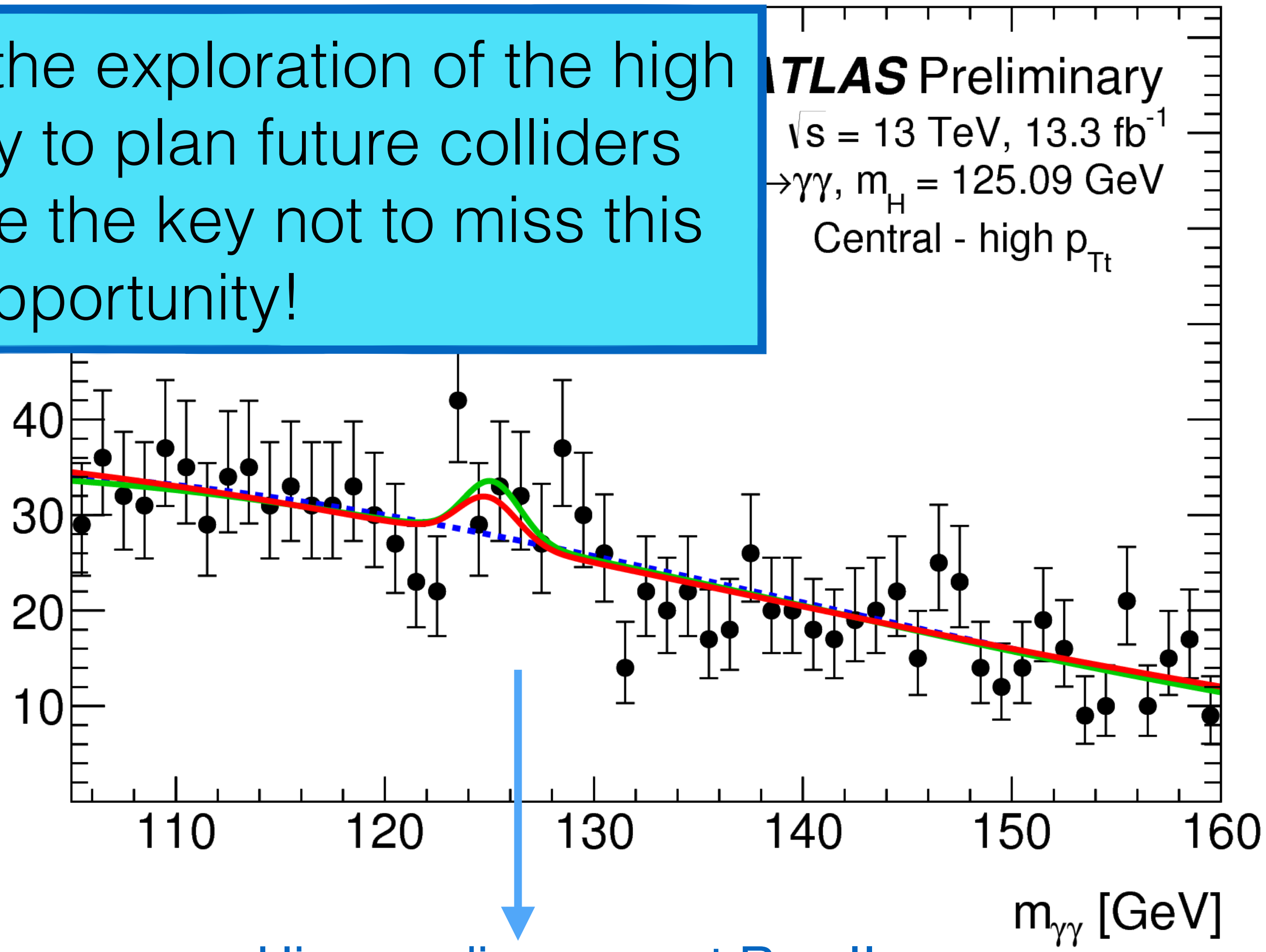
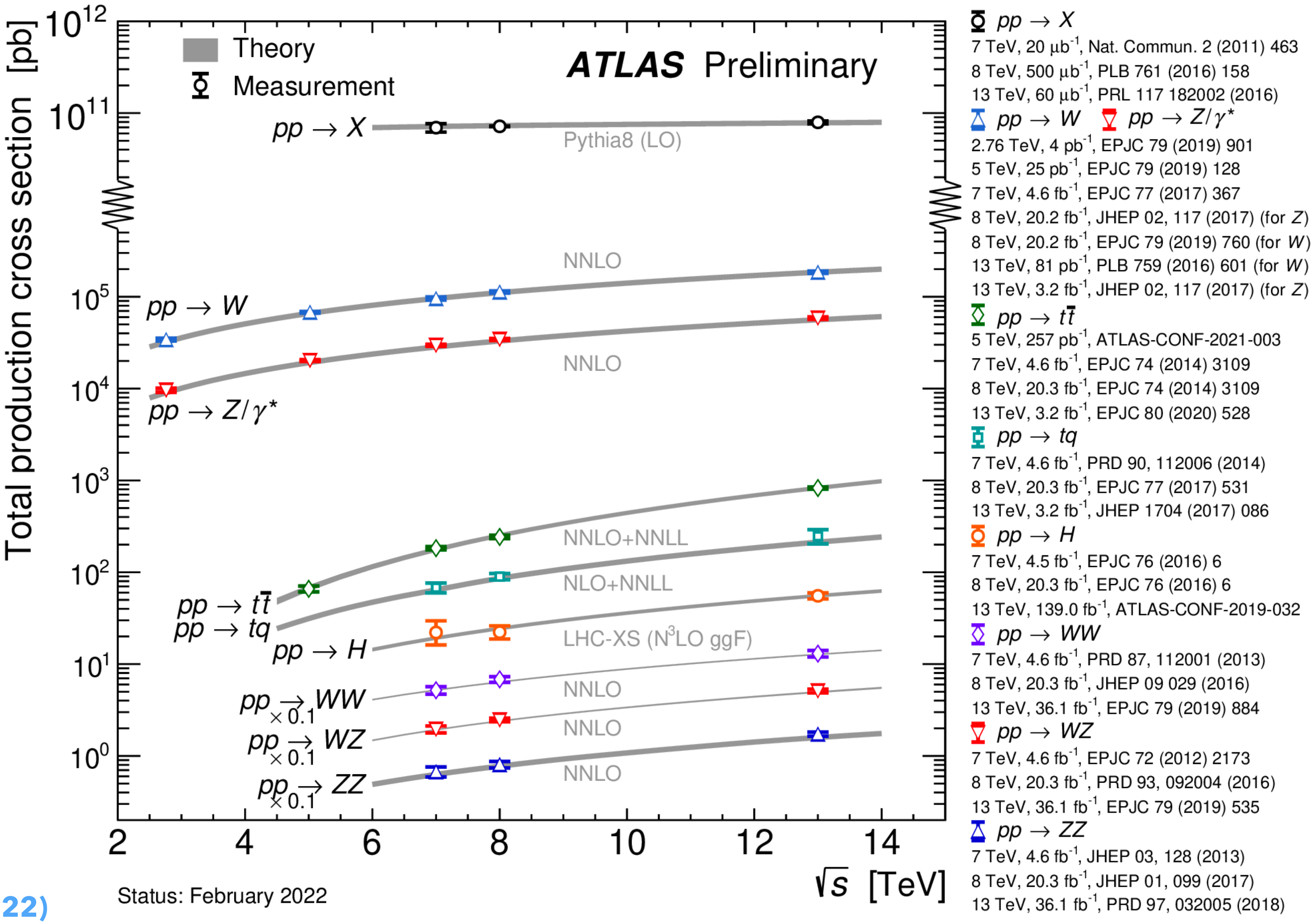


Higgs rediscovery at Run II

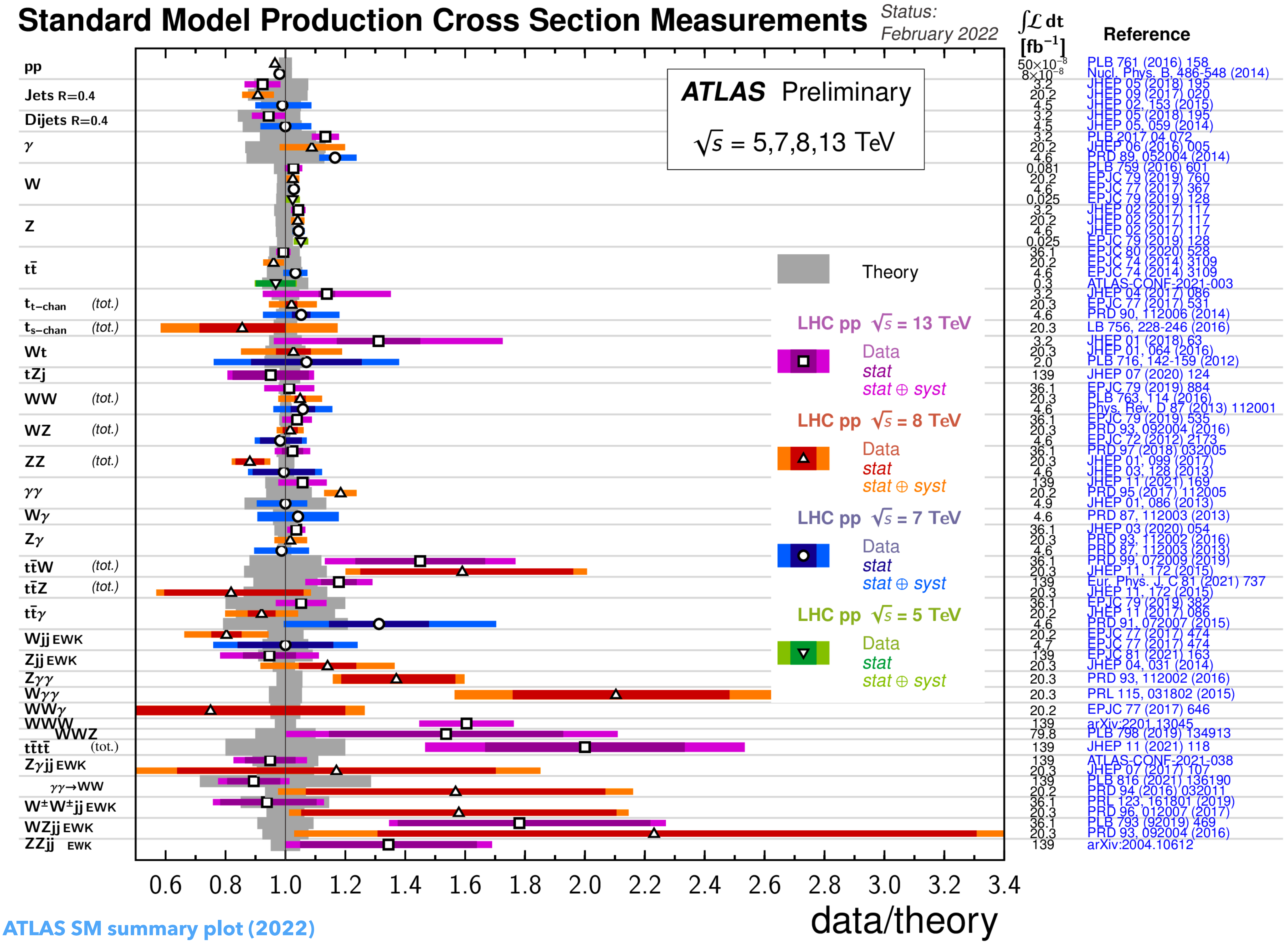
A precision era in particle physics

- Is precision physics really possible/necessary at hadron colliders?
  At the LHC a paradigm shift took place: discovery ➜ discovery through precision



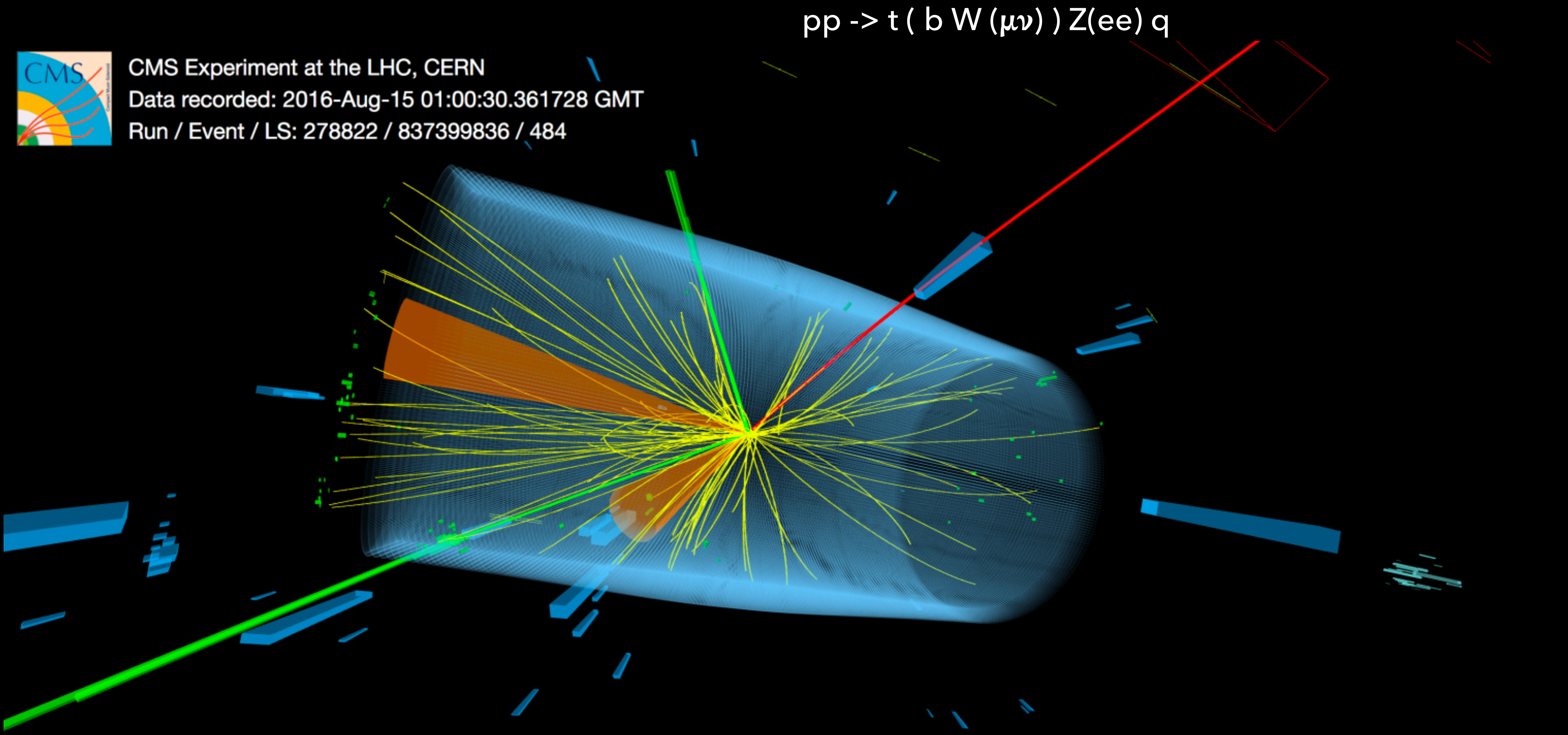
ATLAS SM summary plot (2022)

Standard Model Production Cross Section Measurements



ATLAS SM summary plot (2022)

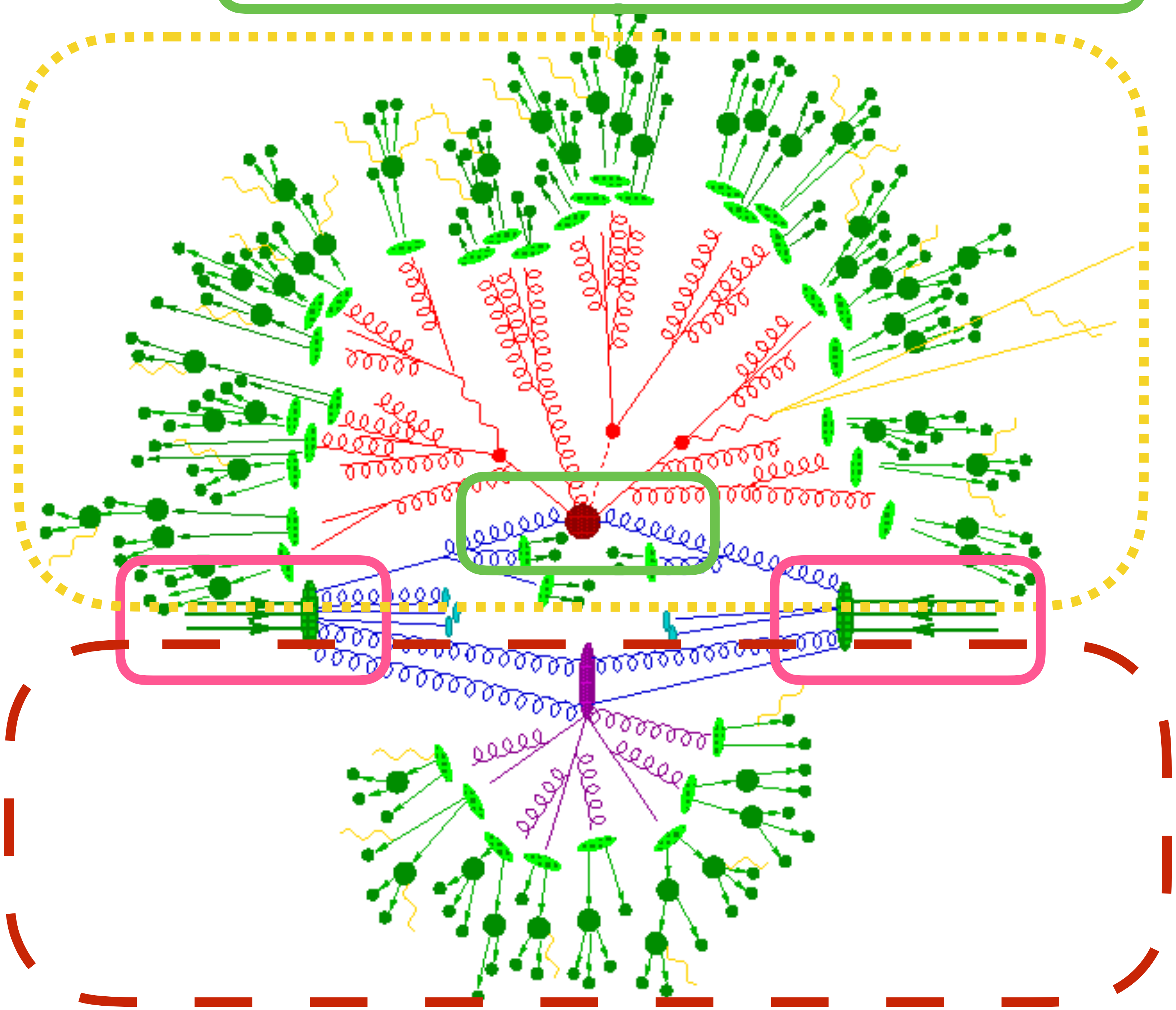
Collider events: an experimental view



A top + Z candidate collision recorded by CMS. The tZq state is characterised by three leptons (in this case two electrons and one muon), a jet produced from decay of a bottom quark, and a forward jet that is close to the LHC beam direction (Image: CMS/CERN)

Collider events: a theory view

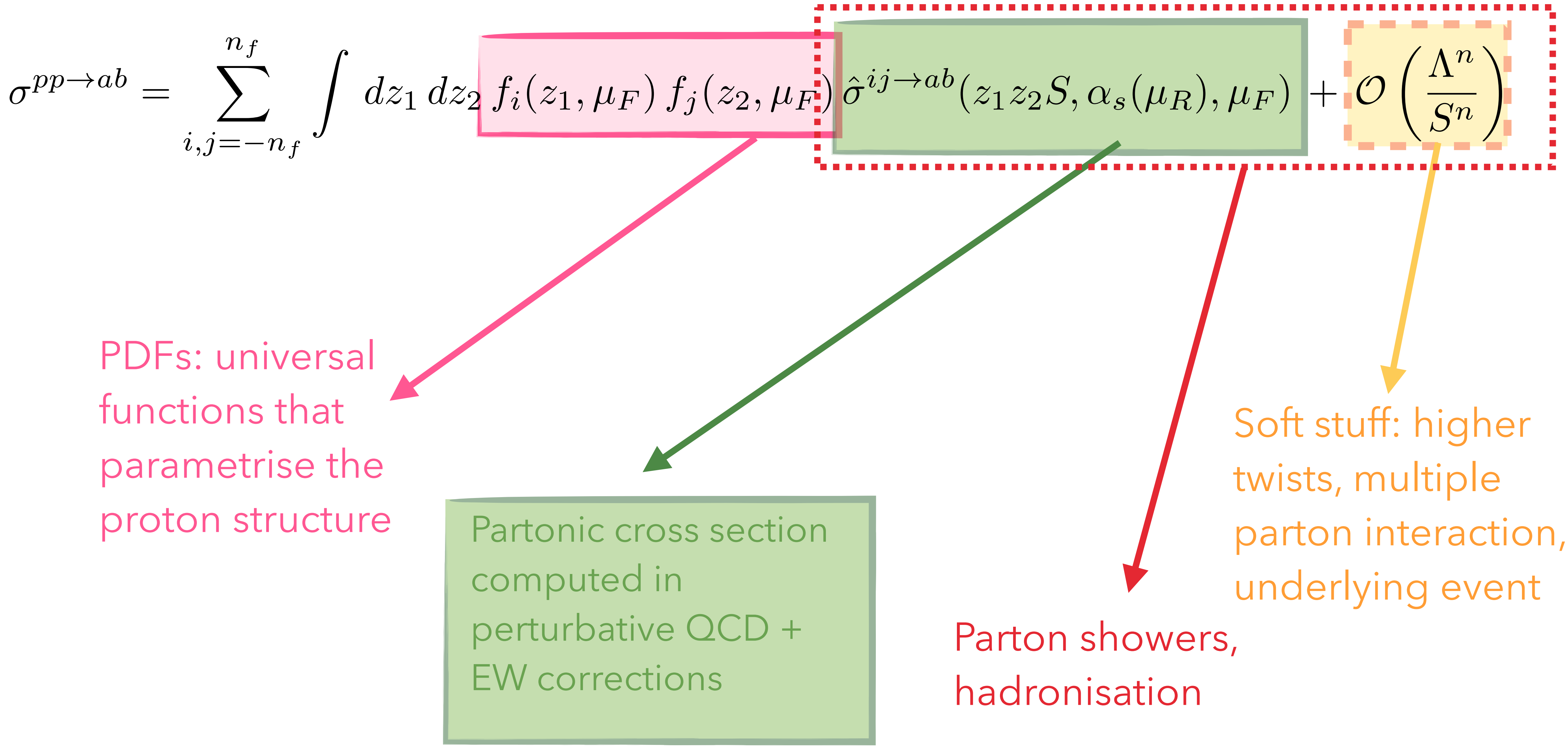
$$\sigma^{\rm th} = \boxed{\hat{\sigma}[\mathcal{O}(1)+\mathcal{O}(\alpha_s)+\mathcal{O}(\alpha_s^2)+\ldots]} \otimes \boxed{f_1 \otimes f_2} + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$



- Hard scattering of partons or Partonic cross section (Perturbative QCD+EW)
- Parton Distribution Functions
- Parton Showering and Hadronization
- Multiple Parton Interaction, Underlying Events

Collider events: a theory view

- Collinear factorisation: the LHC master formula
- Divide et impera!



$$\sigma^{pp\to ab} = \sum_{i,j=-n_f}^{n_f} \int dz_1\, dz_2\, f_i(z_1,\mu_F)\, f_j(z_2,\mu_F)\, \hat{\sigma}^{ij\to ab}(z_1 z_2 S, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

PDFs: universal functions that parametrise the proton structure

Partonic cross section computed in perturbative QCD + EW corrections

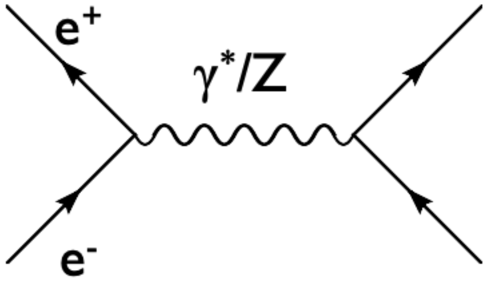
Parton showers, hadronisation

Soft stuff: higher twists, multiple parton interaction, underlying event

Perturbative expansion

- One of the first tests of QCD was the measurement of the R-ratio, defined as

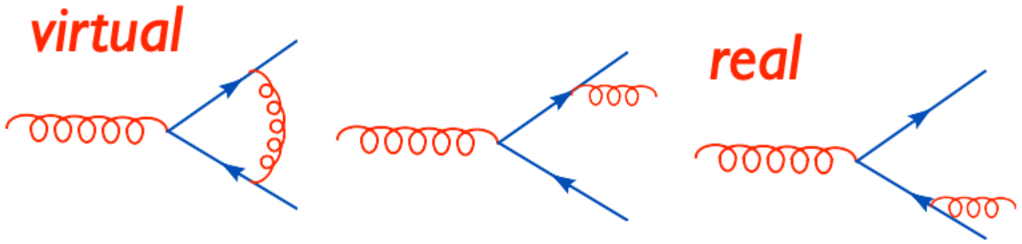
$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



- At leading order in QCD

$$R^{(0)} = N_c \sum_i e_i^2$$

- First order QCD corrections (Next-to-Leading Order)



$$R^{(1)} = R^{(0)} \left(1 + \frac{\alpha_S}{\pi}\right) \qquad \alpha_S \equiv \frac{g_S^2}{4\pi}$$

Perturbative expansion

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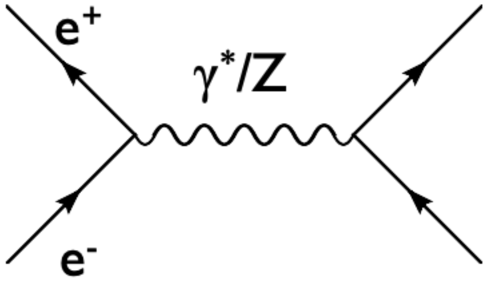
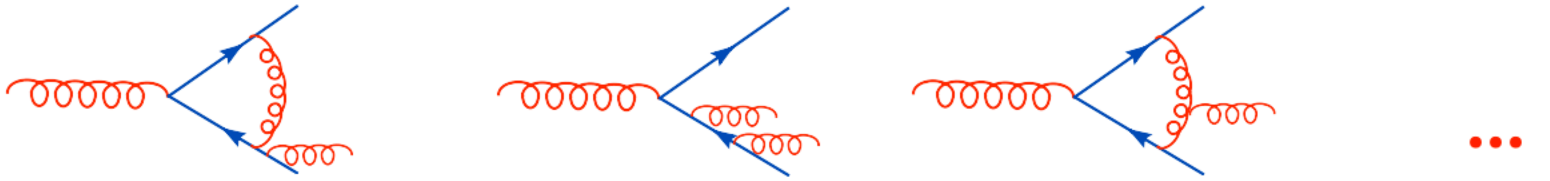


- Second order QCD correction (NNLO = next-to-next-to-leading order)



$$R^{(2)} = R^{(0)}\left(1 + \frac{\alpha_S}{\pi} + \left(\frac{\alpha_S}{\pi}\right)^2\left(c + \pi b_0 \log\left(\frac{M_{\rm UV}^2}{Q^2}\right)\right)\right) \qquad b_0 = \frac{11N_c - 4n_f T_R}{12\pi}$$

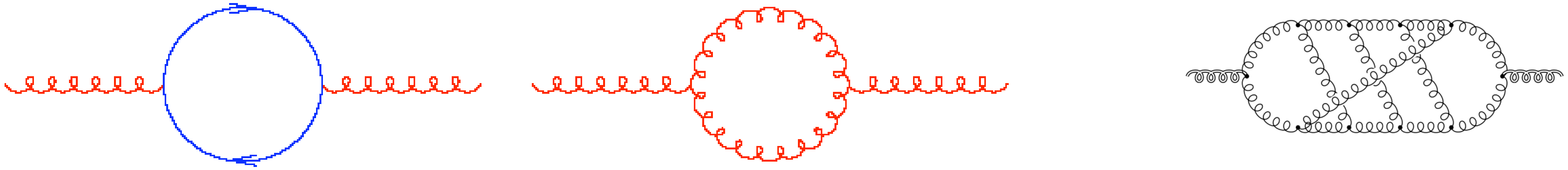
- UV divergences do not cancel => Renormalisation procedure: the UV divergence is dealt with renormalisation of bare coupling

$$\alpha_S(\mu) = \alpha_S^{\rm bare} + b_0 \log\left(\frac{M_{\rm UV}^2}{\mu^2}\right)(\alpha_S^{\rm bare})^2 \Rightarrow \mu^2\frac{d\alpha_S(\mu)}{\mu^2} = -b_0\alpha_S^2(\mu) + \ldots$$

The strong coupling constant

$$\mu^2 \frac{d\alpha}{d\mu^2} = \beta(\alpha) = -(b_0\alpha^2 + b_1\alpha^3 + b_2\alpha^4 + \cdots)$$

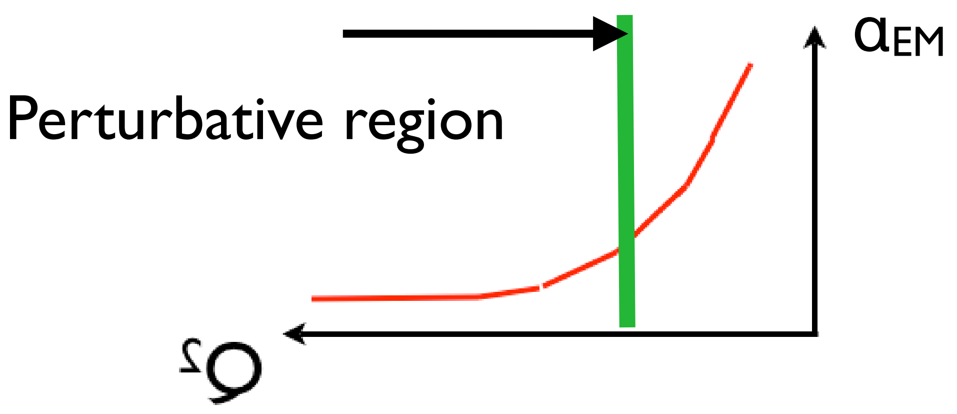
Known up to b_4 (5 loops):
arXiv: 1606.08659, … ,
arXiv: 1709.08541

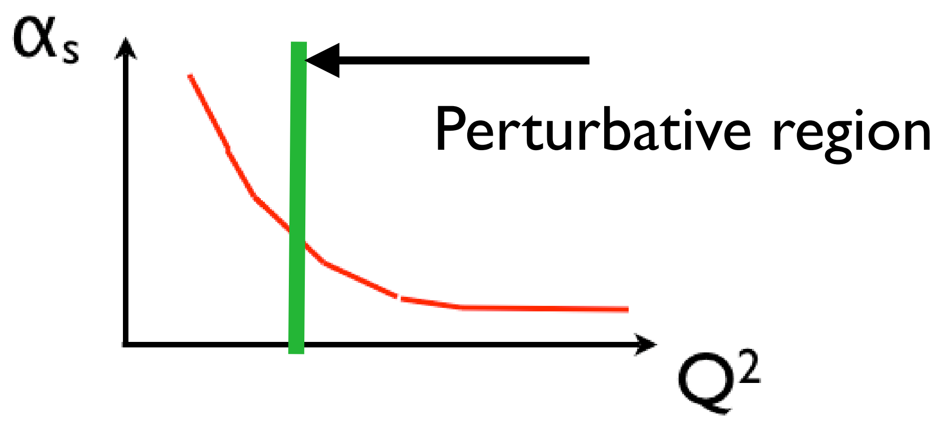


Roughly speaking, quark loop diagrams contribute with Nf negative terms in b_0, while the gluon loop, diagram gives a positive contribution proportional to N_c, which is dominant and make the overall beta function negative.

$$b_0 = \frac{11N_c - 2n_f}{12\pi} > 0 \qquad \Rightarrow \quad \beta(\alpha_S) < 0$$ in QCD

$$b_0 = -\frac{n_f}{3\pi} > 0 \qquad \Rightarrow \quad \beta(\alpha_{\rm EM}) > 0$$ in QED

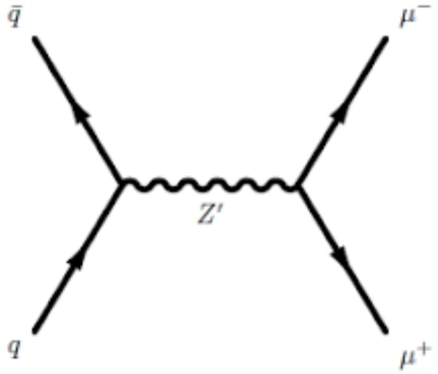




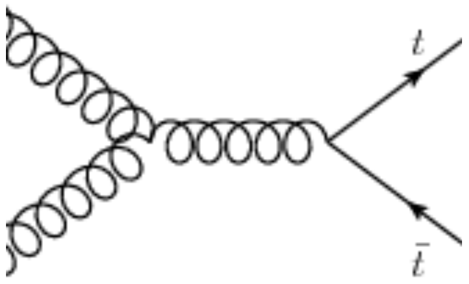
The strong coupling constant

- In pQCD all theoretical predictions are expressed in terms of the renormalised coupling $\alpha_S(\mu^2_R)$, a function of unphysical renormalization scale μ_R.

$$\hat{\sigma}^{ij\to ab} = (\alpha_s)^{k_0}\hat{\sigma}_0 + (\alpha_s)^{k_0+1}\hat{\sigma}_1 + (\alpha_s)^{k_0+2}\hat{\sigma}_2 + (\alpha_s)^{k_0+3}\hat{\sigma}_3 + \ldots$$



$K_0 = 0$



$K_0 = 2$

$$\boxed{\alpha_s \equiv \alpha_s(\mu_R^2)}$$

- When one takes μ_R close to the scale of the momentum transfer Q in a given process, then $\alpha_S(Q^2)$ is indicative of the effective strength of the strong interaction in that process

$$\mu_R \sim Q$$

The strong coupling constant

- Beside the quark masses, the only free parameter in the QCD Lagrangian is α_S

- The coupling constant not a physical observable, rather quantity defined in the context of perturbation theory, which enters predictions for measurable observables.

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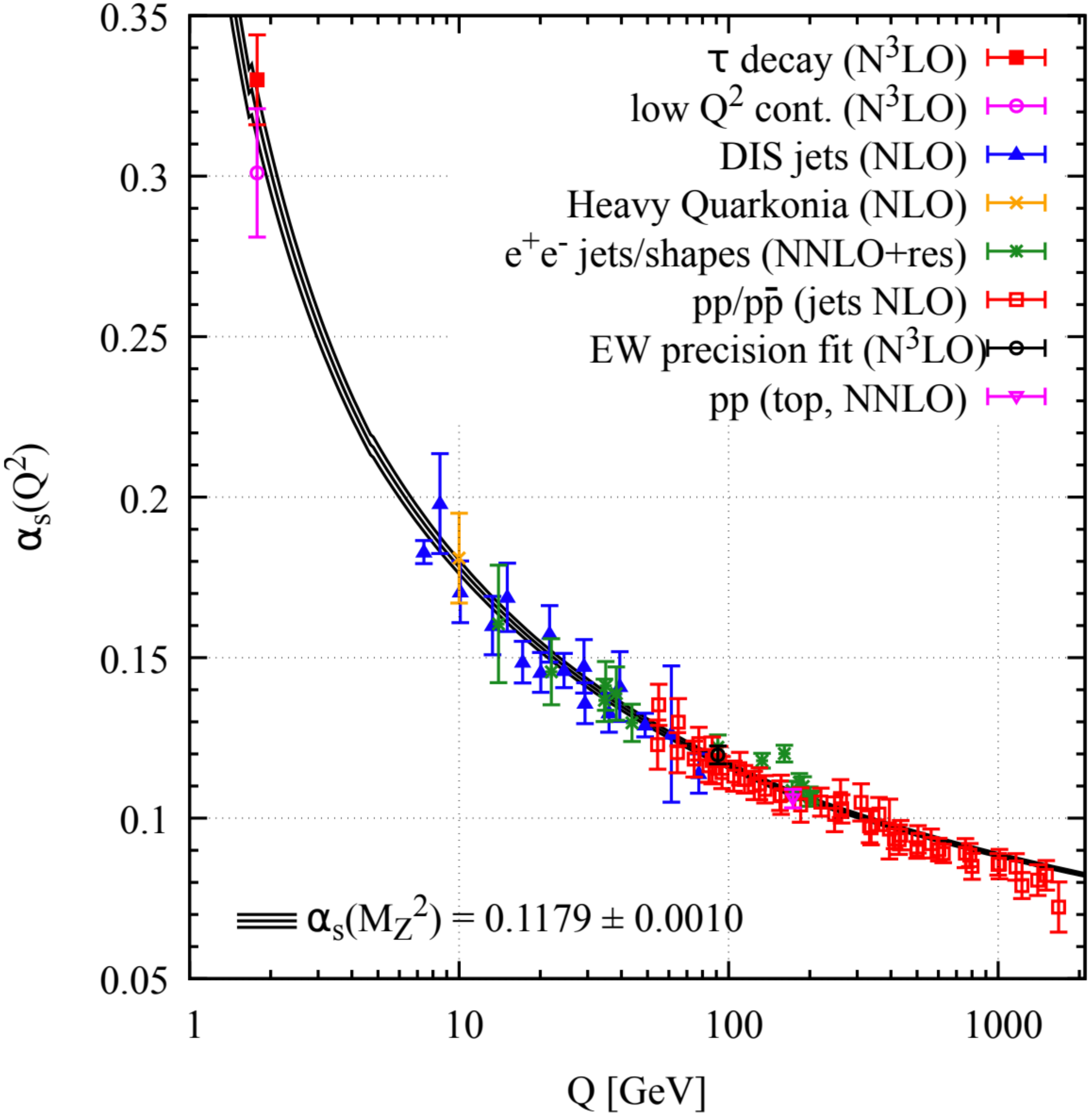
- PDG 2020 average from fit of specific experimental observables compared to predictions at a given order in pQCD



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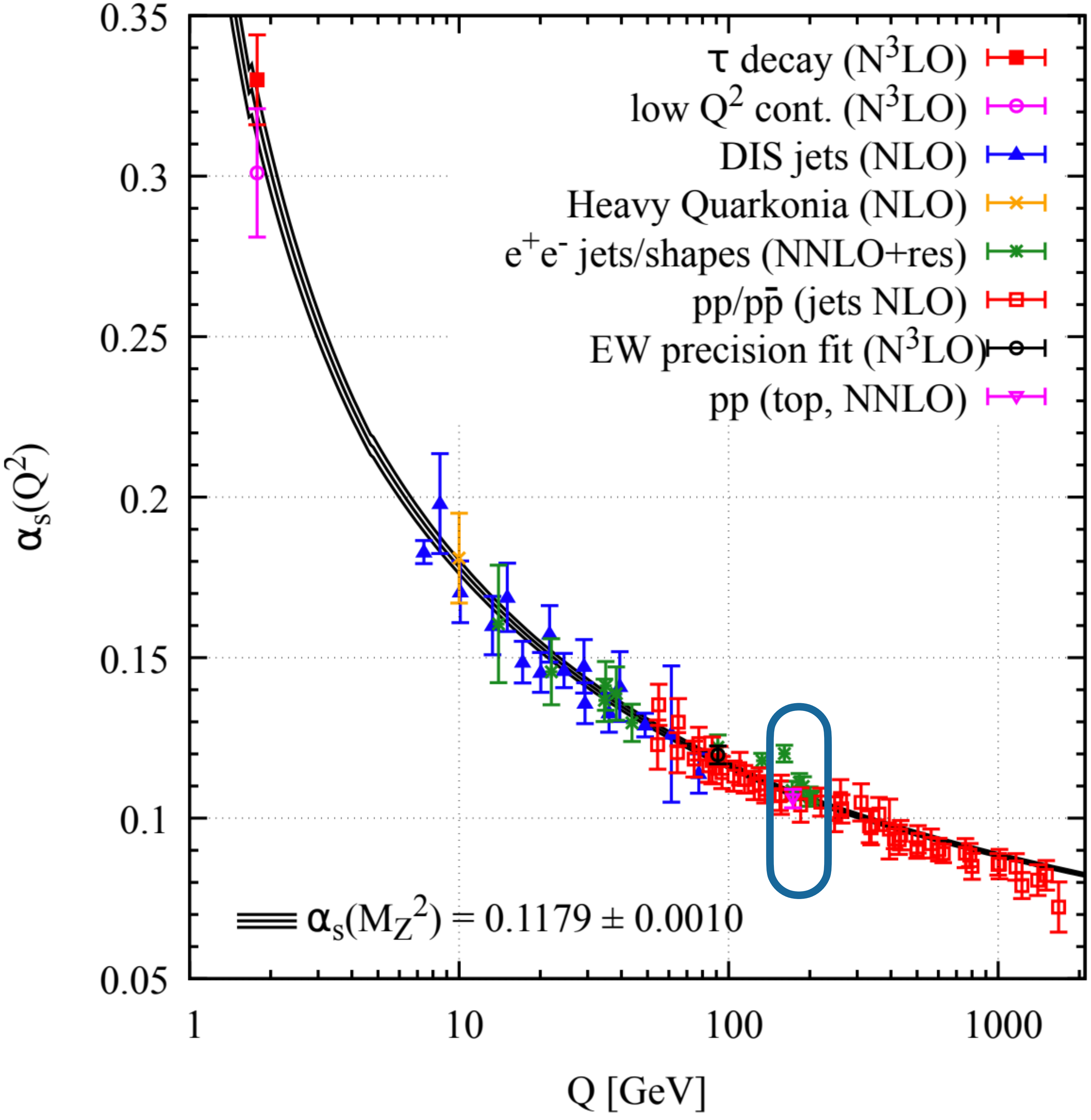
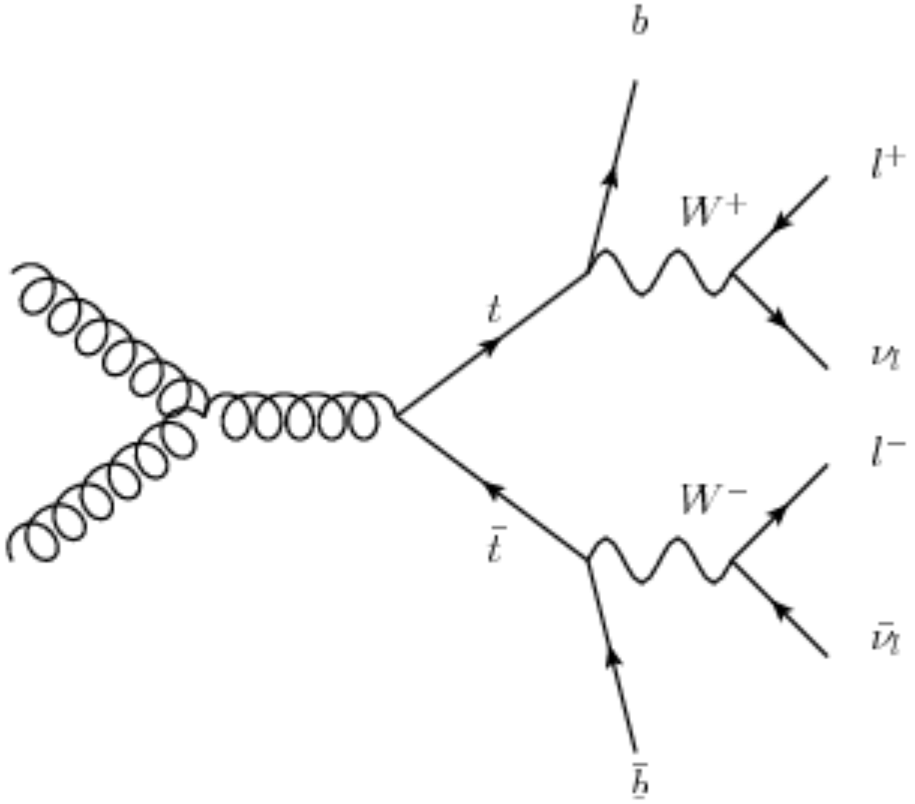
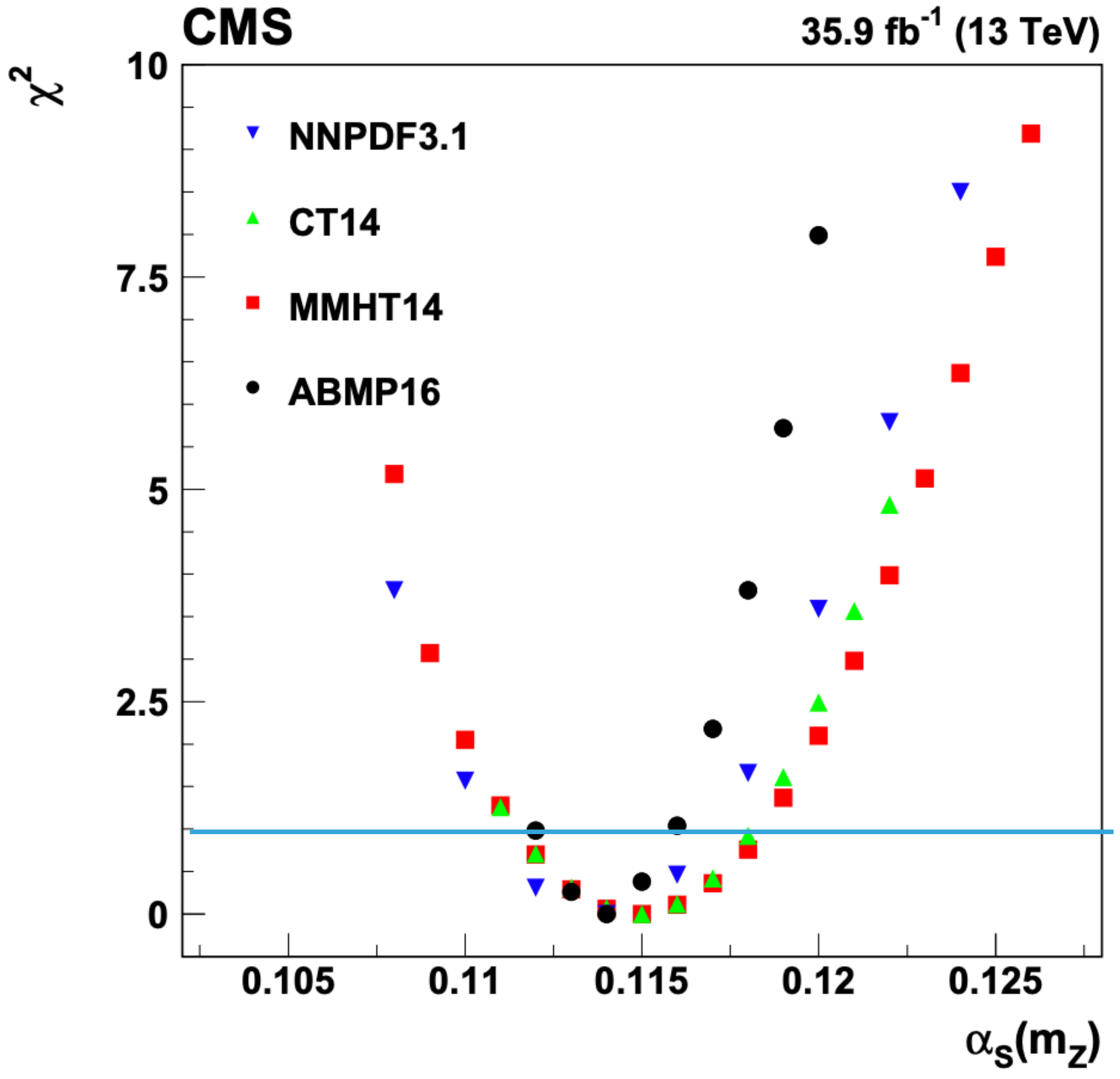


The strong coupling constant

- Example: CMS determination of m_t and $\alpha_S(M_Z)$ from tt~ production cross section in leptonic channel
- Results depend on experimental systematics (in D_i and cov_{ij}) and theoretical prediction T_i, which, for a given perturbative order depends on $\alpha_S(Q)$, which is the fit parameter

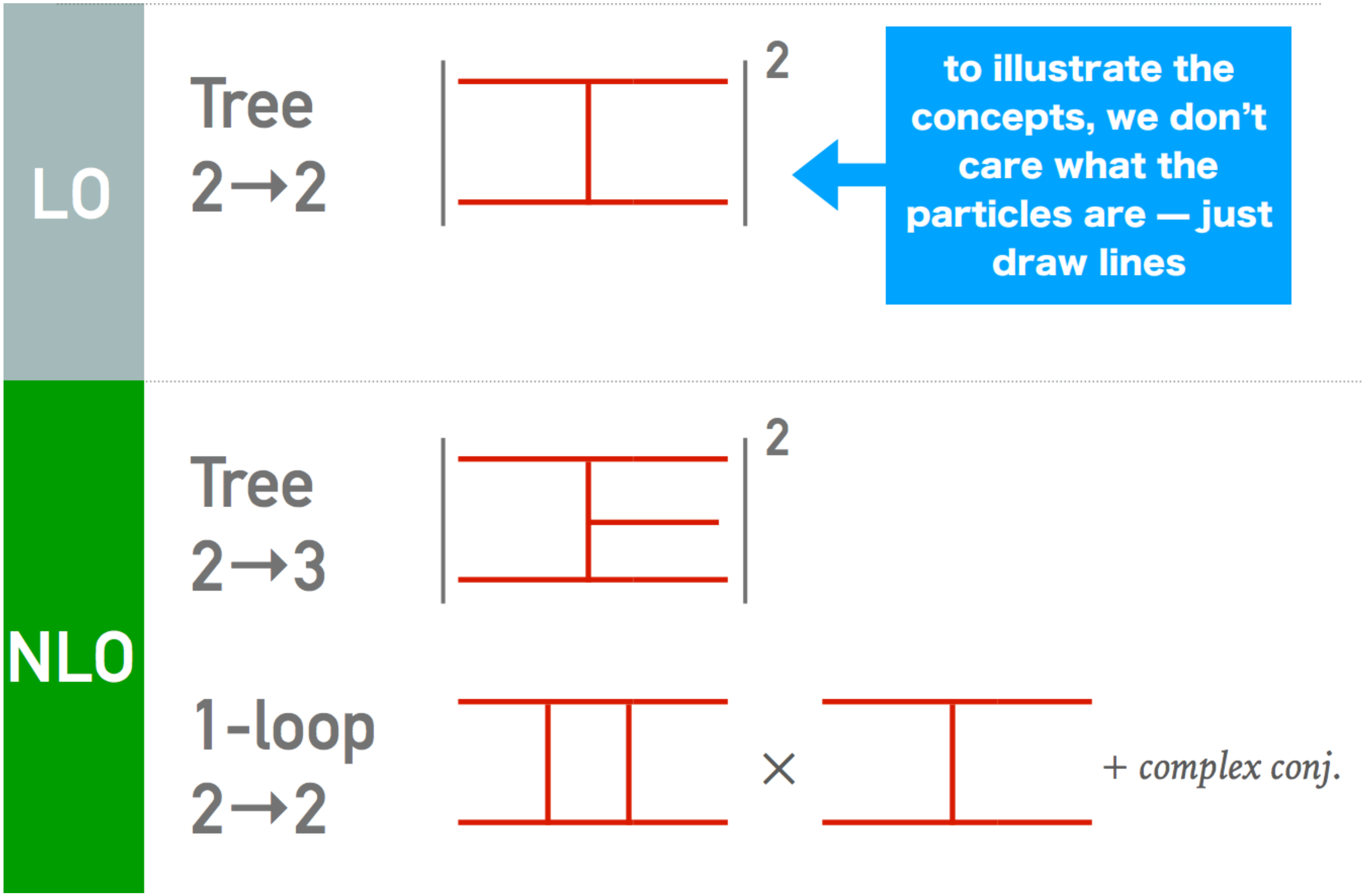


CMS collaboration
arXiv:1812.10505



$$\chi^2 = \sum_{i,j=1}^{N_{\mathrm{dat}}} (D_i - T_i)(\mathrm{cov})^{-1}_{ij}(D_j - T_j)$$

Partonic cross sections



LO

Tree 2→2 $\left| \ \ \right|^2$

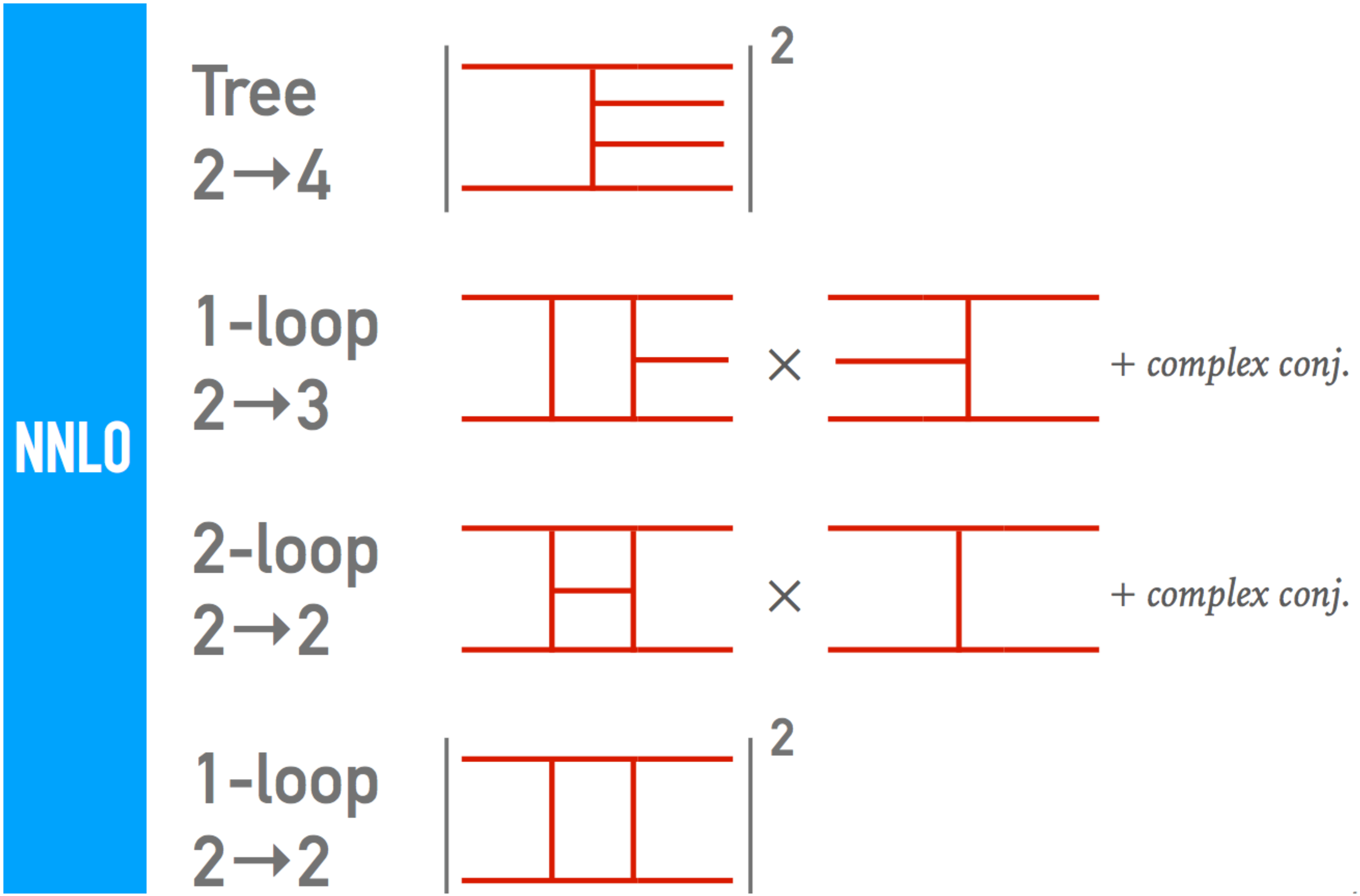
to illustrate the concepts, we don't care what the particles are — just draw lines

NLO

Tree 2→3 $\left| \ \ \right|^2$

1-loop 2→2 × + *complex conj.*

Partonic cross sections



Partonic cross sections

$$\sigma(pp \to H) = (961\,\mathrm{pb})\times(\alpha_s^2 + 10.4\alpha_s^3 + 38\alpha_s^4 + 48\alpha_s^5 + \cdots)$$

$$\alpha_s \equiv \alpha_s(M_H/2)$$

$$\sqrt{s_{pp}} = 13\,\mathrm{TeV}$$

Anastasiou et al., 1602.00695 (ggF, hEFT)

pp → H (via gluon fusion) is one of the
5 hadron-collider processes known at N3LO.

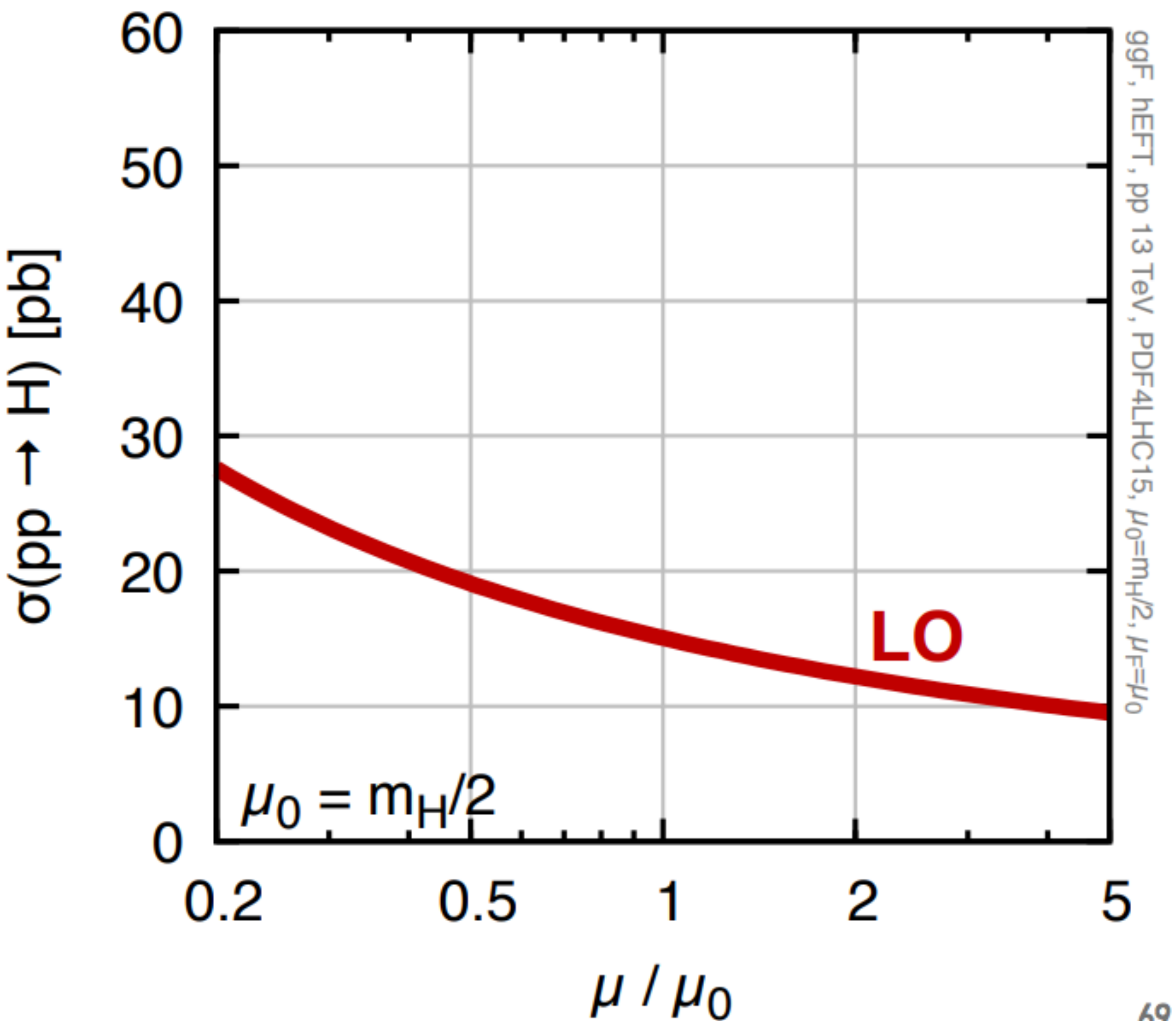
The perturbative series does not converge well!

Partonic cross sections

- On previous page, we wrote the series in terms of powers of $\alpha_S(M_H/2)$
- But we are free to rewrite it in terms of $\alpha_S(\boldsymbol{\mu})$ for any choice of renormalisation scale **μ**

LO

$$\sigma(pp \to H) = \sigma_0 \times \alpha_s^2(\mu)$$



Slide from Gavin Salam lectures
Quy Nhon Vietnam 2018

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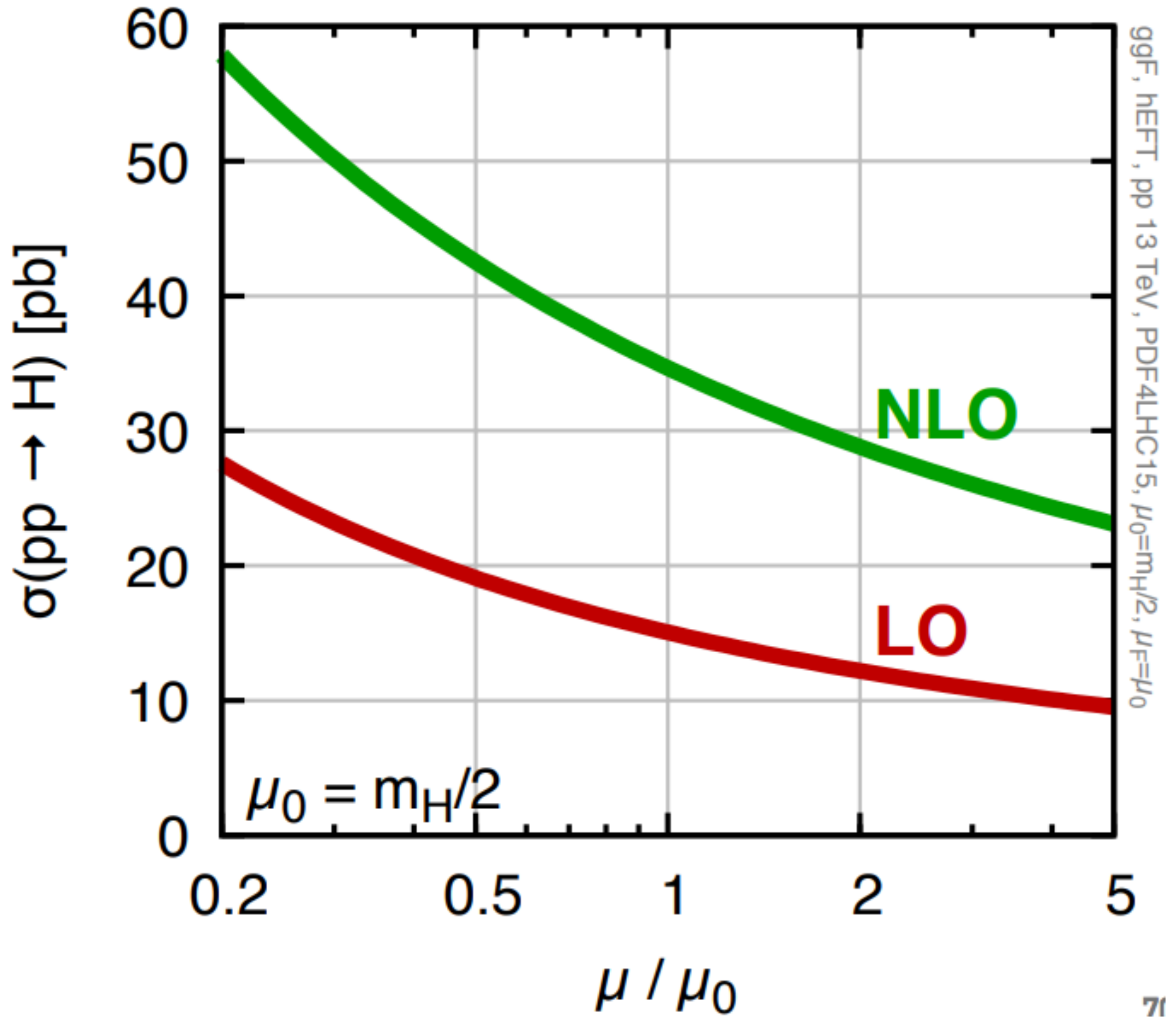
NLO

$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu) + (10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu)\Big)$$



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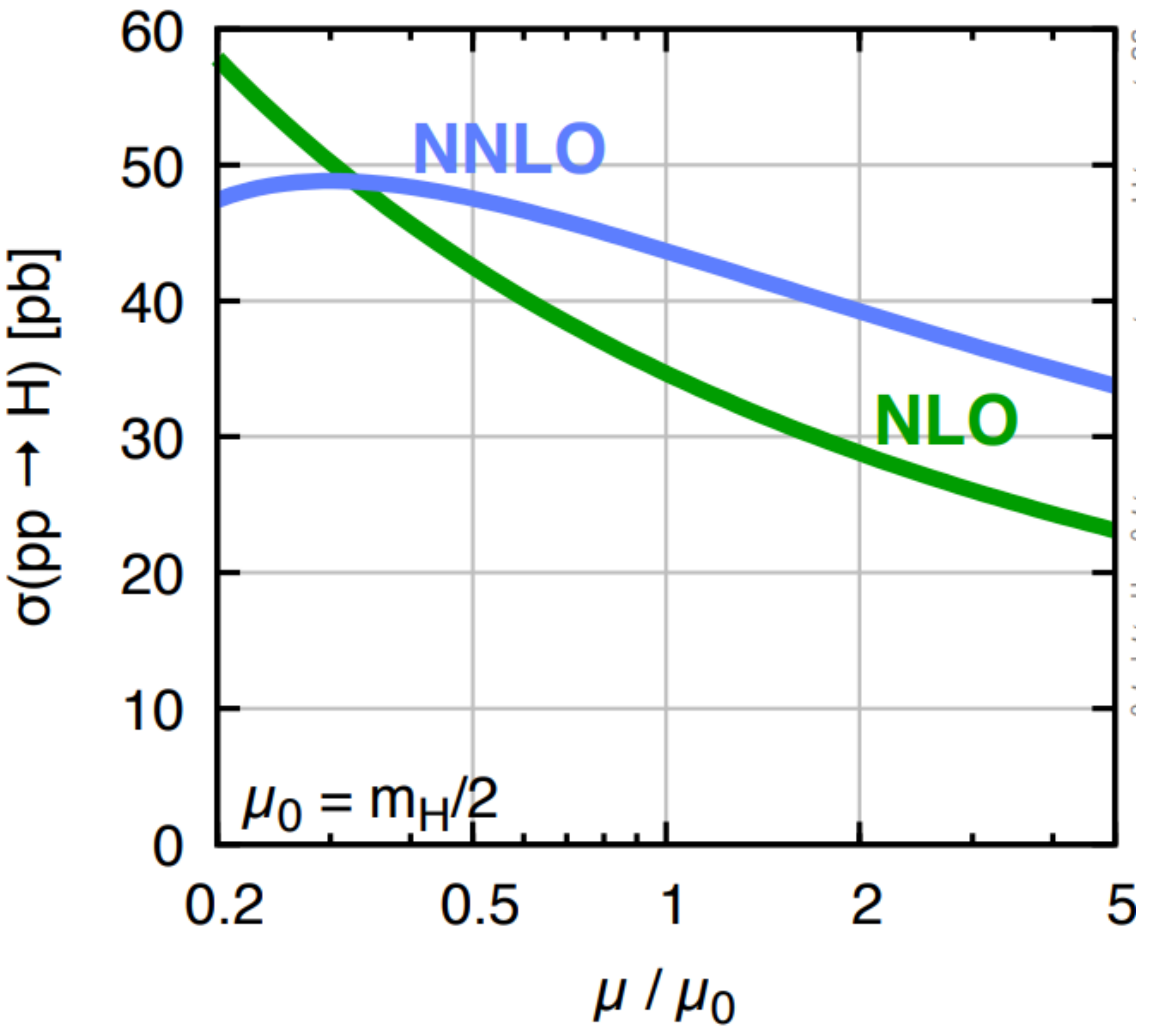
NNLO

$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu) + (10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu) + c_4(\mu)\alpha_s^4(\mu)\Big)$$



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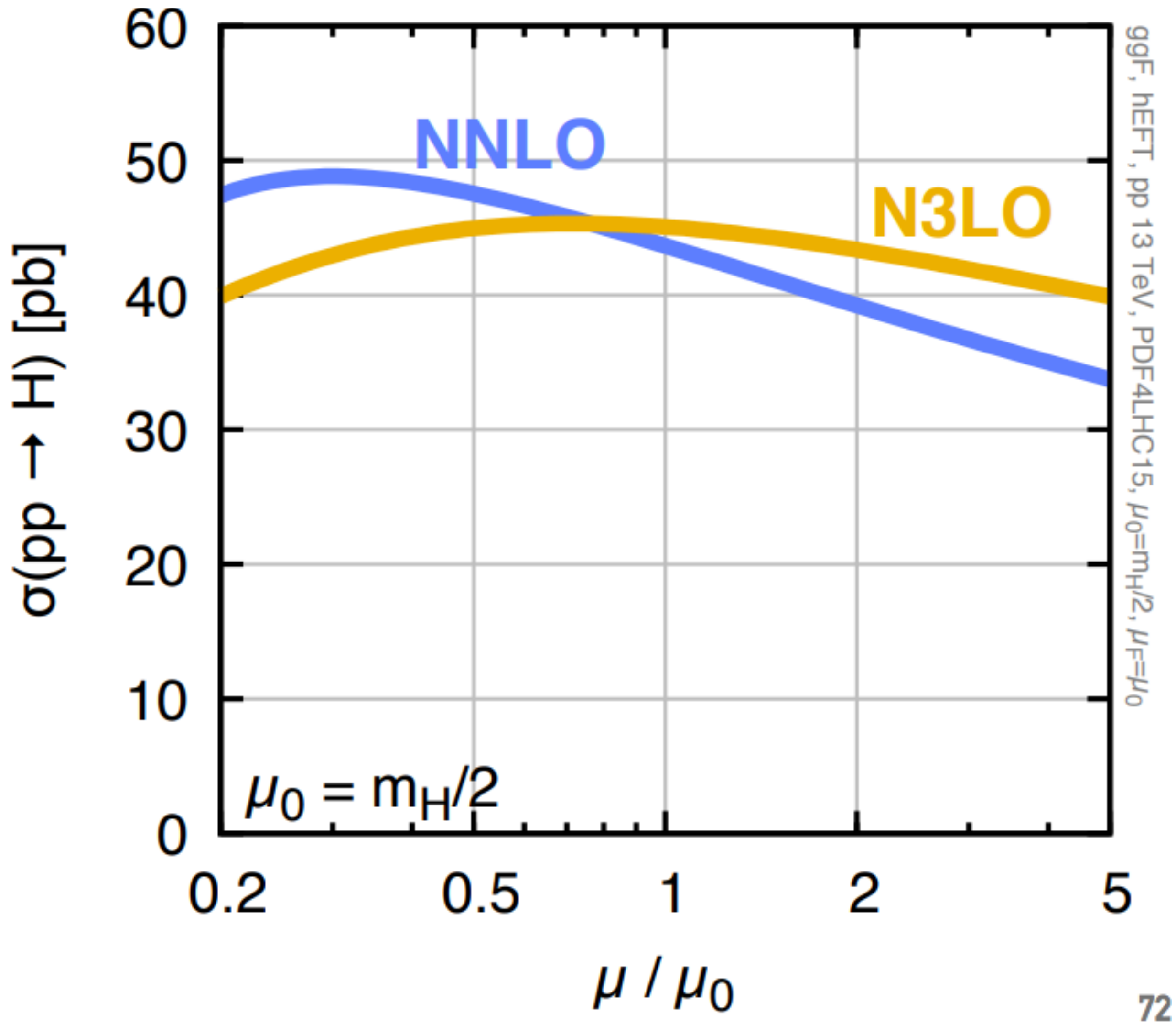
N3LO

$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu)$$
$$+ (10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu)$$
$$+ c_4(\mu)\alpha_s^4(\mu) + c_5(\mu)\alpha_s^5(\mu)\Big)$$



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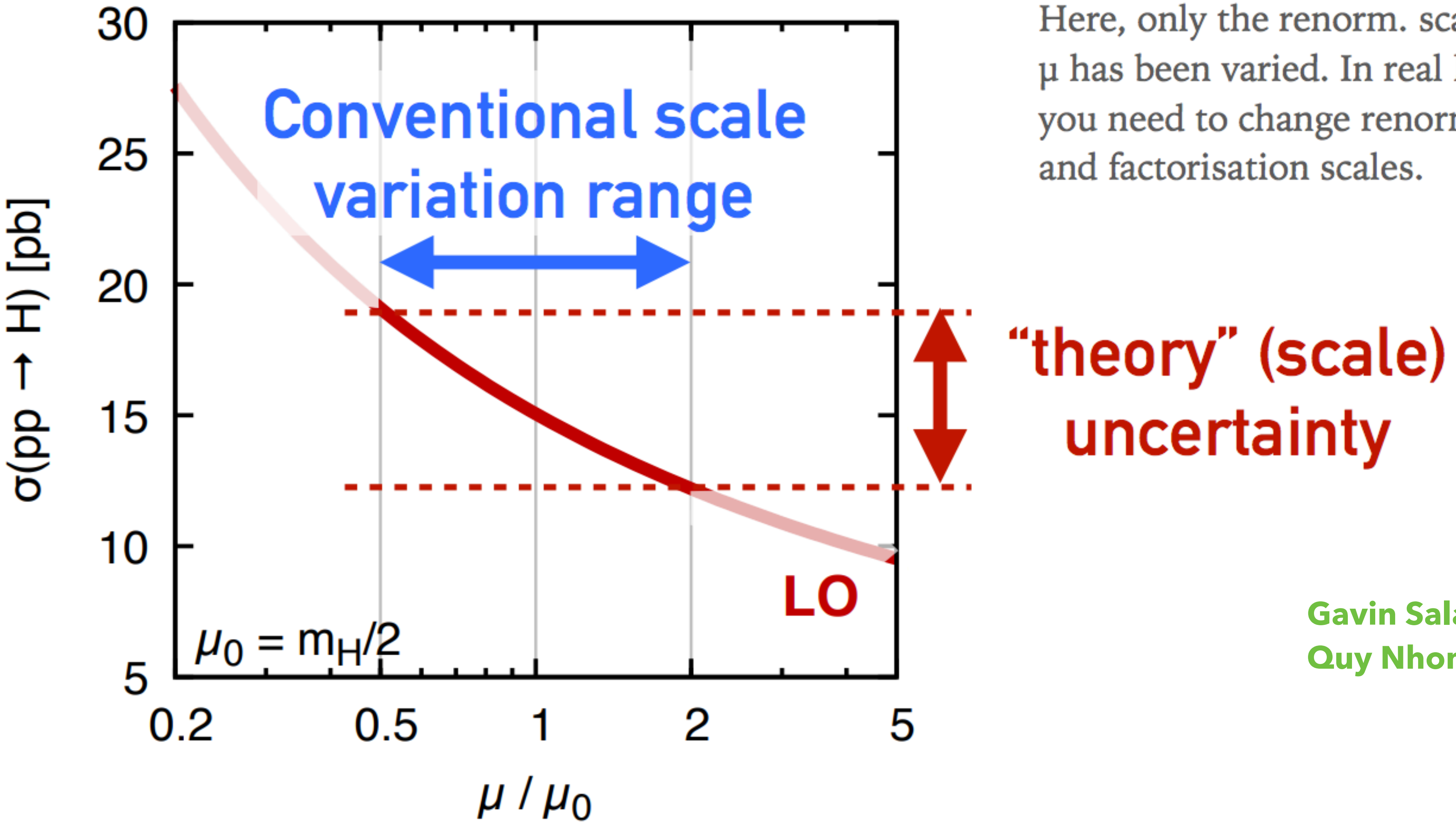
Partonic cross sections

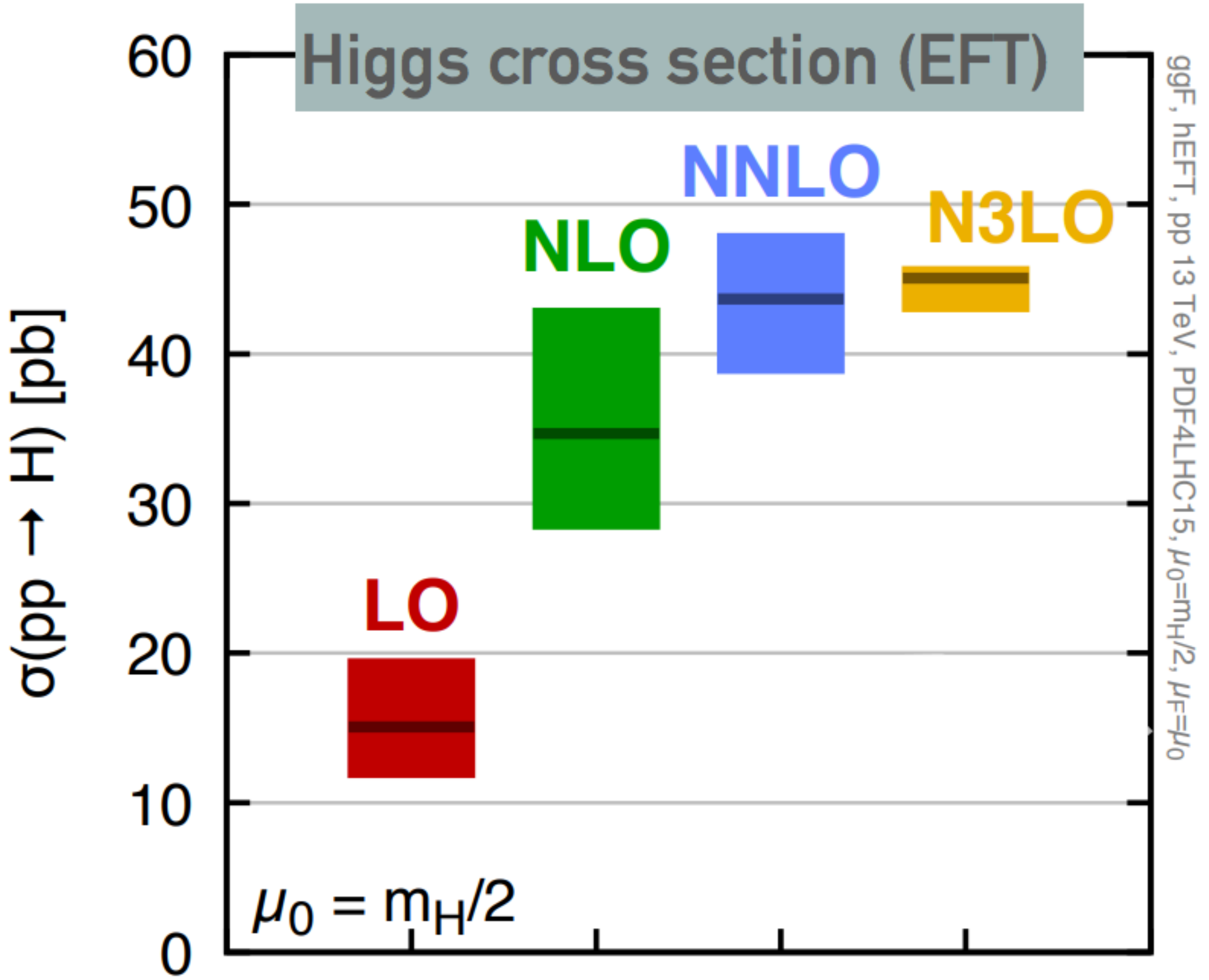


Here, only the renorm. scale μ has been varied. In real life you need to change renorm. and factorisation scales.

Gavin Salam lectures
Quy Nhon Vietnam 2018

MHOU is estimated by change of cross section values when varying factorization and renormalization scales in range 1/2 - 2 around central scales

Partonic cross sections

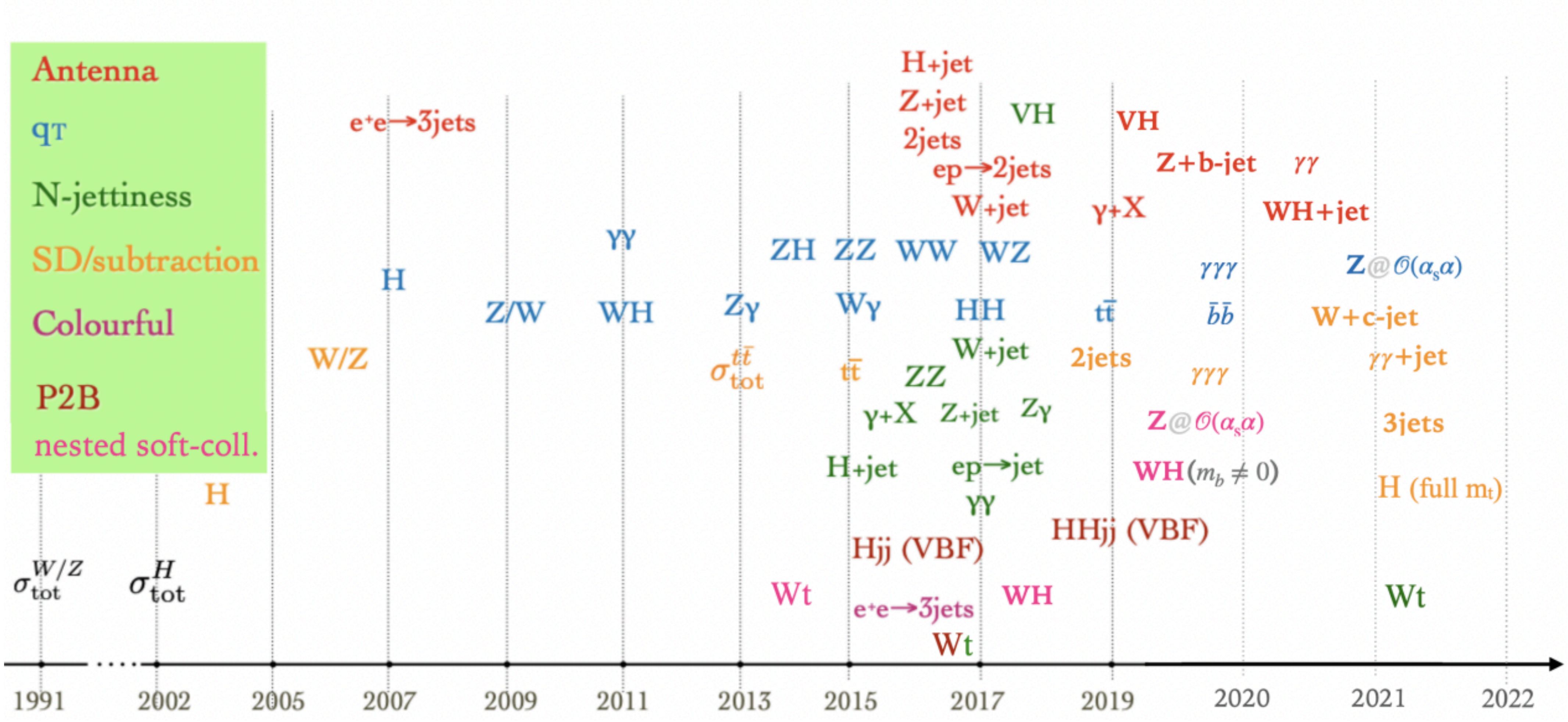


Here, only the renorm. scale $\mu(\equiv\mu_R)$ has been varied. In real life you need to change renorm. and factorisation (μ_F) scales.

Scale dependence is one of the theory uncertainties that characterise theory predictions

MHOU is estimated by change of cross section values when varying factorization and renormalization scales in range 1/2 - 2 around central scales

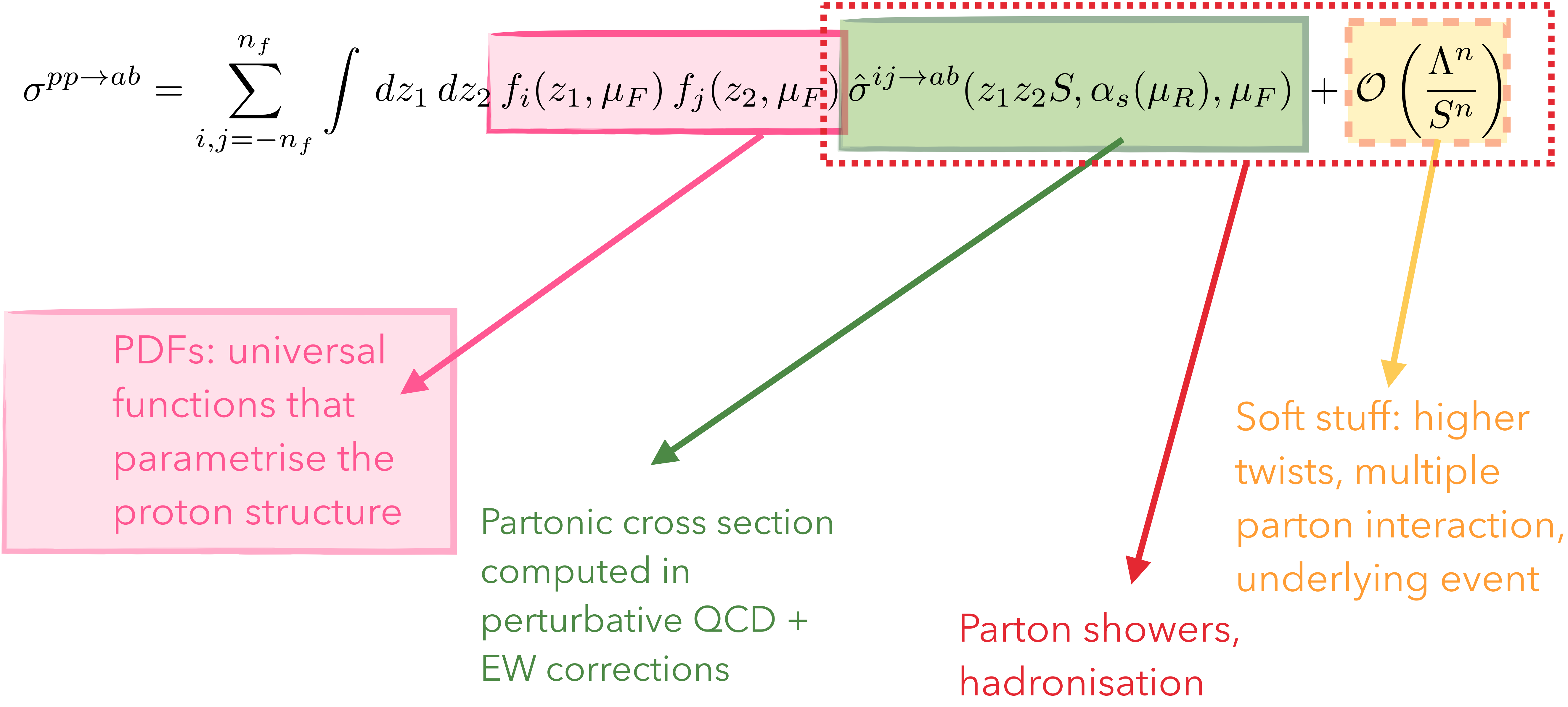
Partonic cross sections


NNLO status (Slide from Giulia Zanderighi, LHCP 2023)

- LO: almost all processes
- NLO: most processes (automated calculations)
- NNLO: all 2 → 1, most 2→2 (explosion of calculations in the past few years)
- N3LO: Higgs gluon fusion, Higgs via vector boson fusion, bb → H, W and Z

Collider events: a theory view

- Collinear factorisation: the LHC master formula
- Divide et impera!



$$\sigma^{pp\to ab} = \sum_{i,j=-n_f}^{n_f} \int dz_1\, dz_2\, f_i(z_1,\mu_F)\, f_j(z_2,\mu_F)\, \hat{\sigma}^{ij\to ab}(z_1 z_2 S, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

PDFs: universal functions that parametrise the proton structure

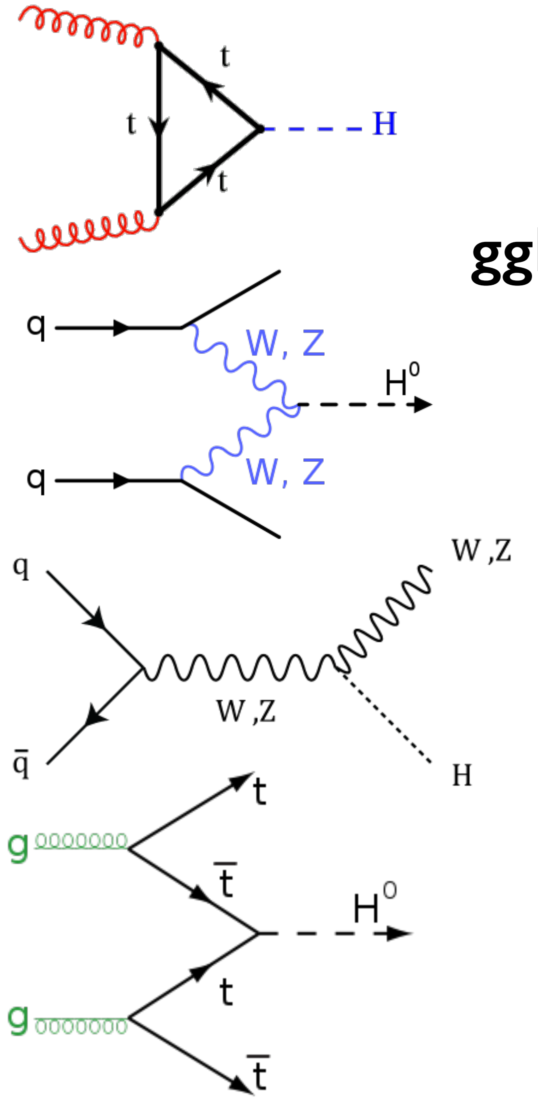
Partonic cross section computed in perturbative QCD + EW corrections

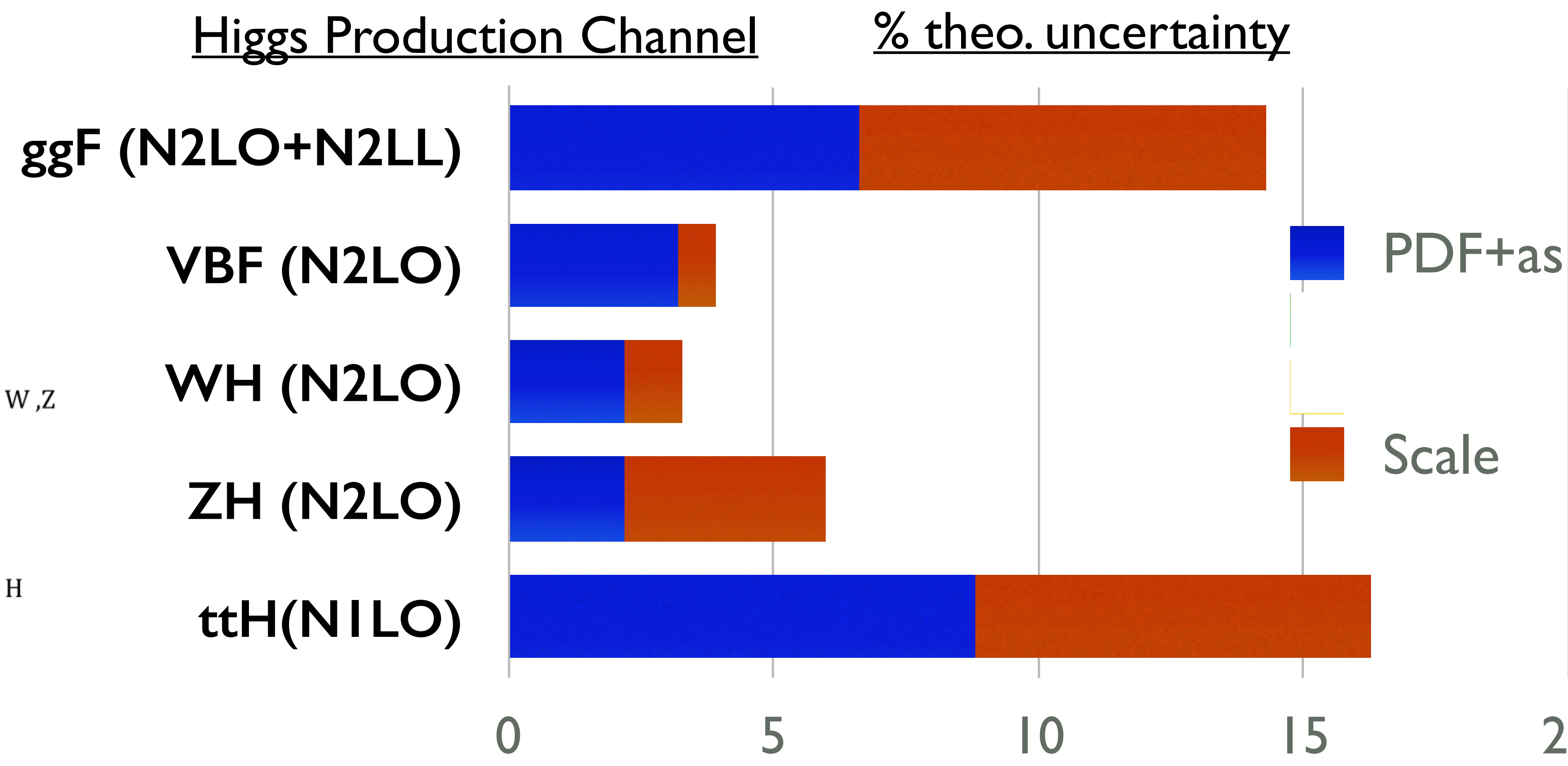
Parton showers, hadronisation

Soft stuff: higher twists, multiple parton interaction, underlying event

PDF uncertainties

Yellow Report 3 (2013)





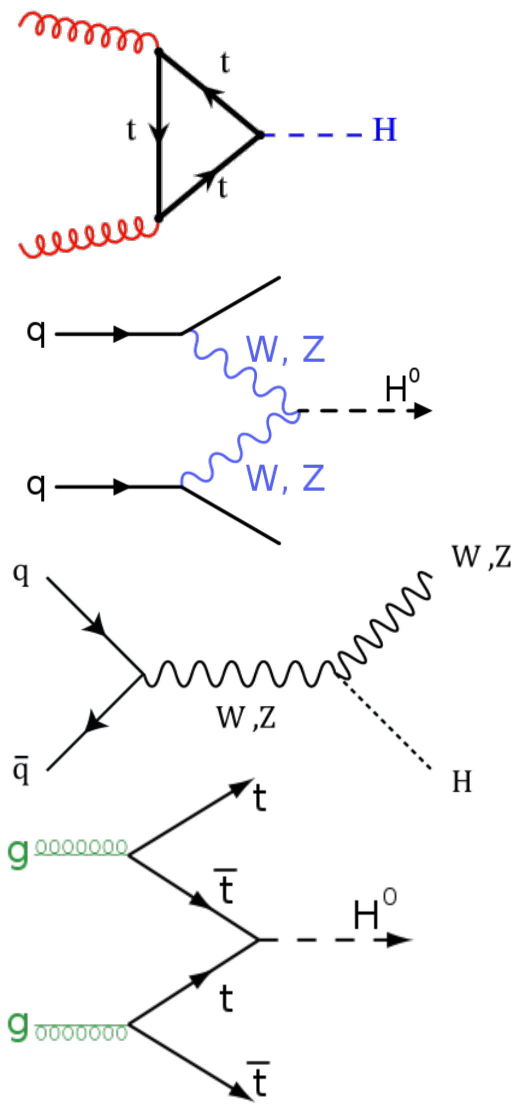
Higgs Production Channel	σ@13 TeV
ggF (N2LO+N2LL)	**43.9 pb**
VBF (N2LO)	**3.75 pb**
WH (N2LO)	**1.38 pb**
ZH (N2LO)	**0.87 pb**
ttH(N1LO)	**0.51 pb**

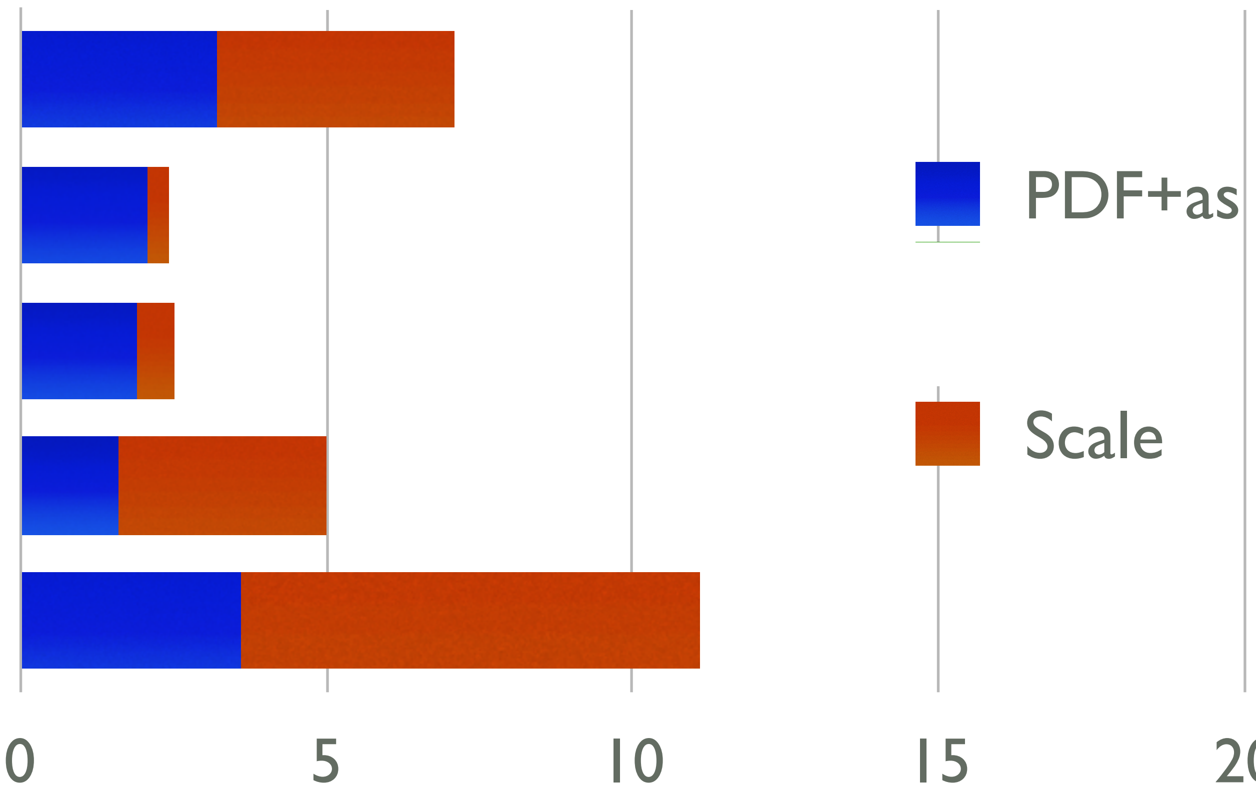
Higgs physics

PDF uncertainties limiting factor in the accuracy of theoretical predictions

PDF uncertainties

Yellow Report 4 (2016)



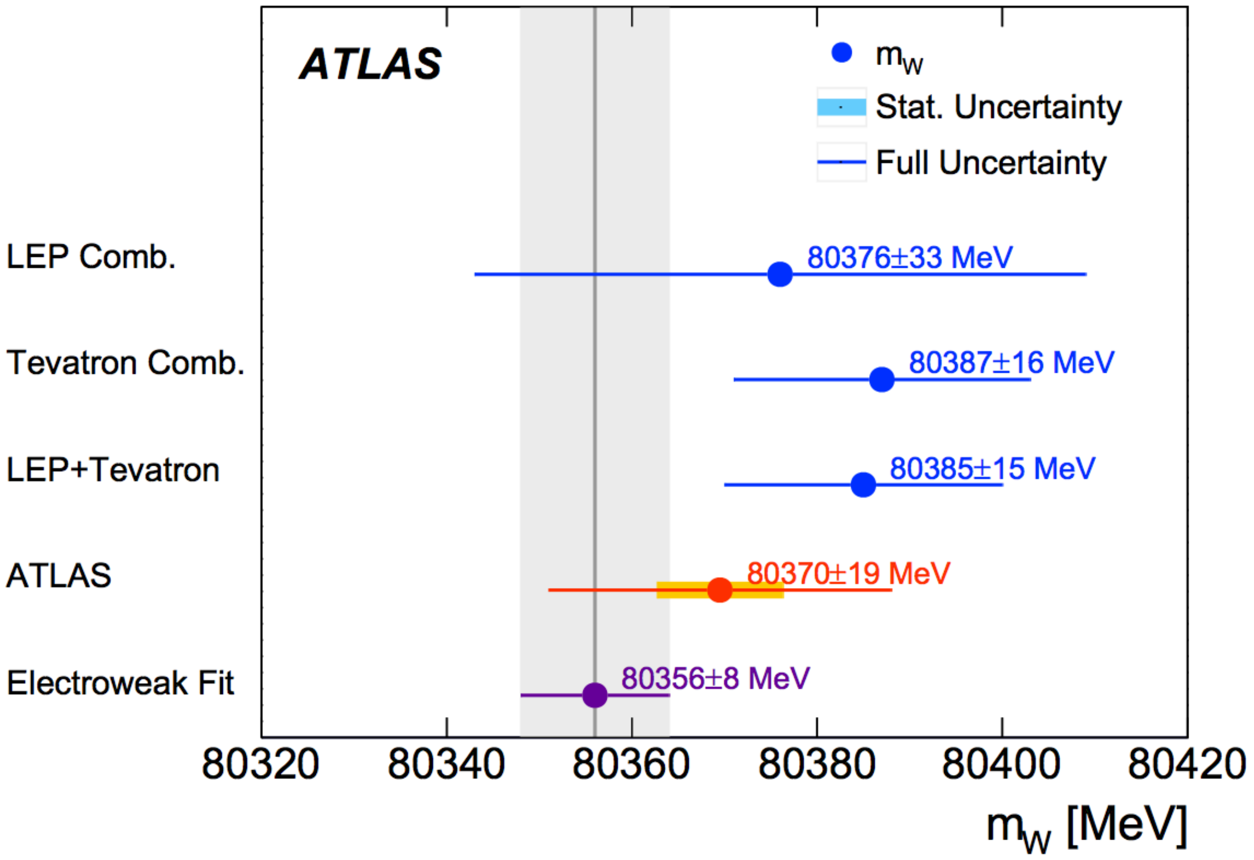


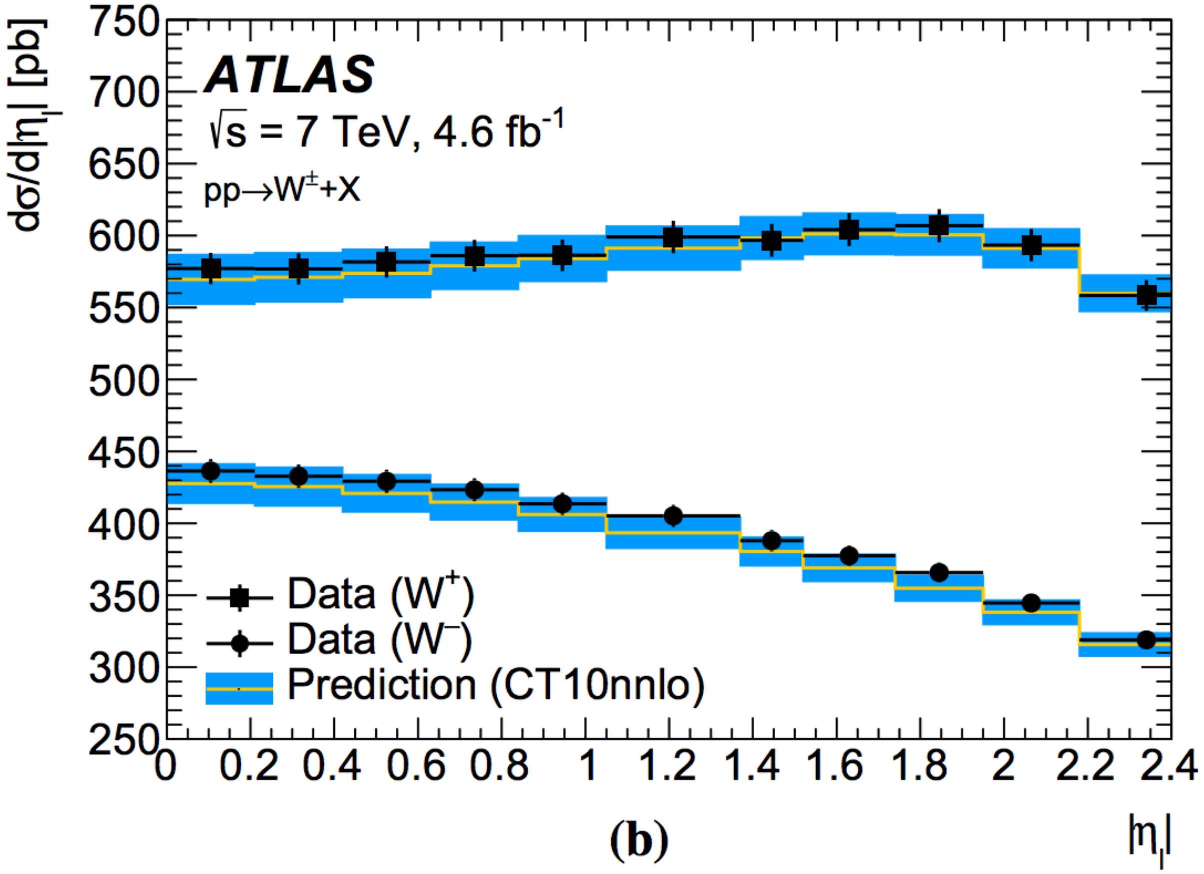
Higgs Production Channel	σ@13 TeV
ggF (N3LO)	**48.5 pb**
VBF (N2LO)	**3.78 pb**
WH (N2LO)	**1.37 pb**
ZH (N2LO)	**0.88 pb**
ttH (N1LO)	**0.51 pb**

Reduced (still often dominant) PDF uncertainties

PDF uncertainties

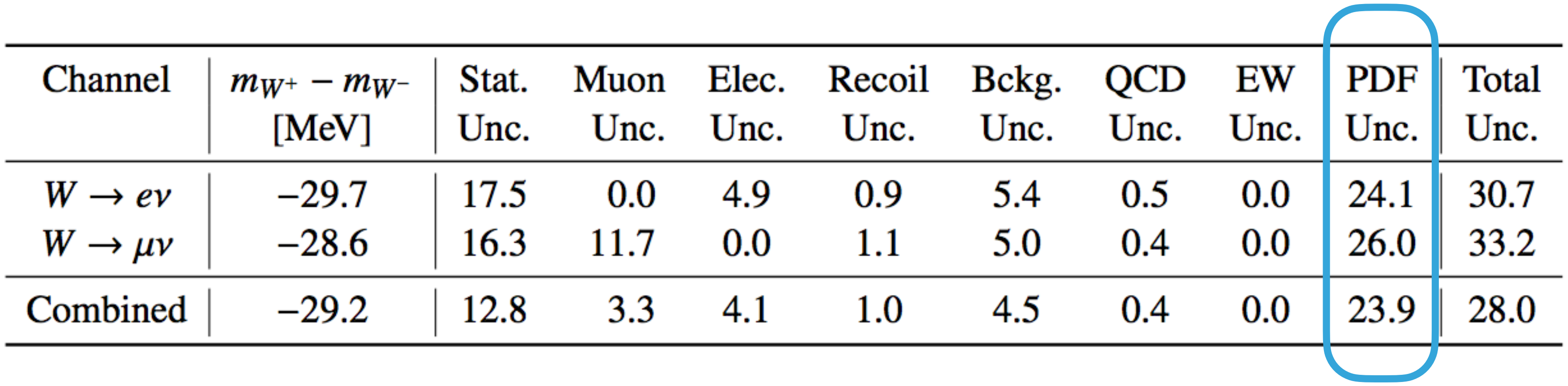
Determination of SM parameters





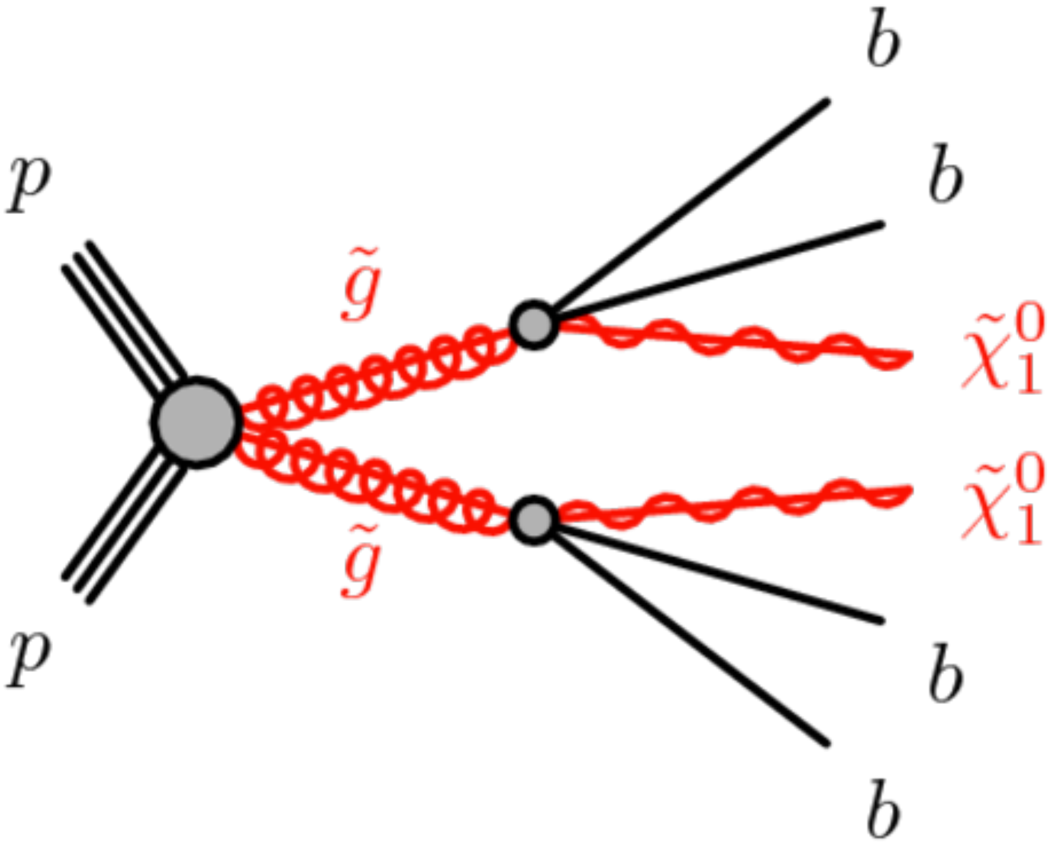
ATLAS collaboration, EPJC 78 (2018) 110

$$\eta = -\ln\tan(\theta/2)$$

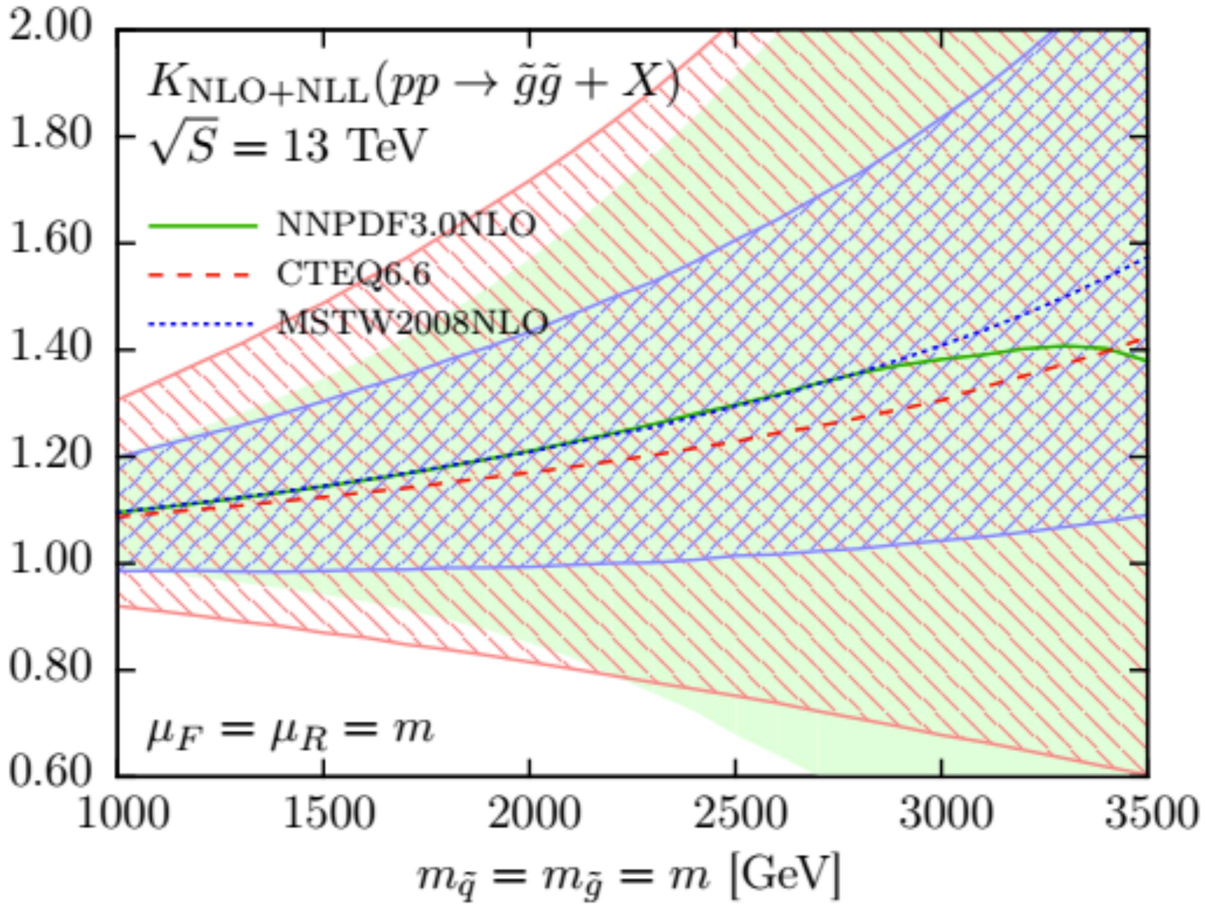


Channel	$m_{W^+} - m_{W^-}$ [MeV]	Stat. Unc.	Muon Unc.	Elec. Unc.	Recoil Unc.	Bckg. Unc.	QCD Unc.	EW Unc.	PDF Unc.	Total Unc.
$W \to e\nu$	−29.7	17.5	0.0	4.9	0.9	5.4	0.5	0.0	24.1	30.7
$W \to \mu\nu$	−28.6	16.3	11.7	0.0	1.1	5.0	0.4	0.0	26.0	33.2
Combined	−29.2	12.8	3.3	4.1	1.0	4.5	0.4	0.0	23.9	28.0

PDF uncertainties



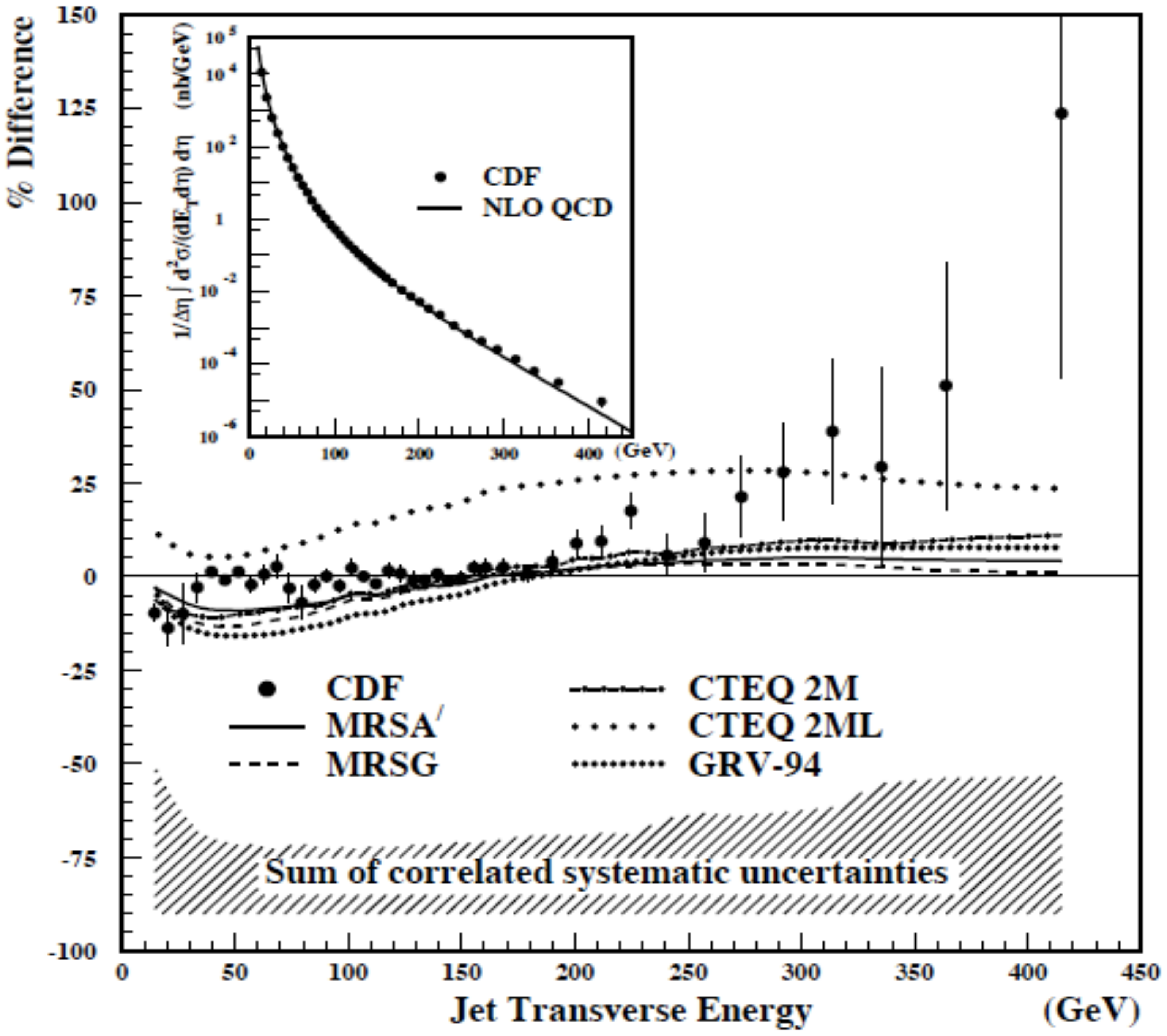
New Physics



Beenakker et al.
EPJC76 (2016)2, 53

PDF uncertainties are a limiting factor in the accuracy of theoretical predictions, both within **SM** and **beyond**

PDF uncertainties

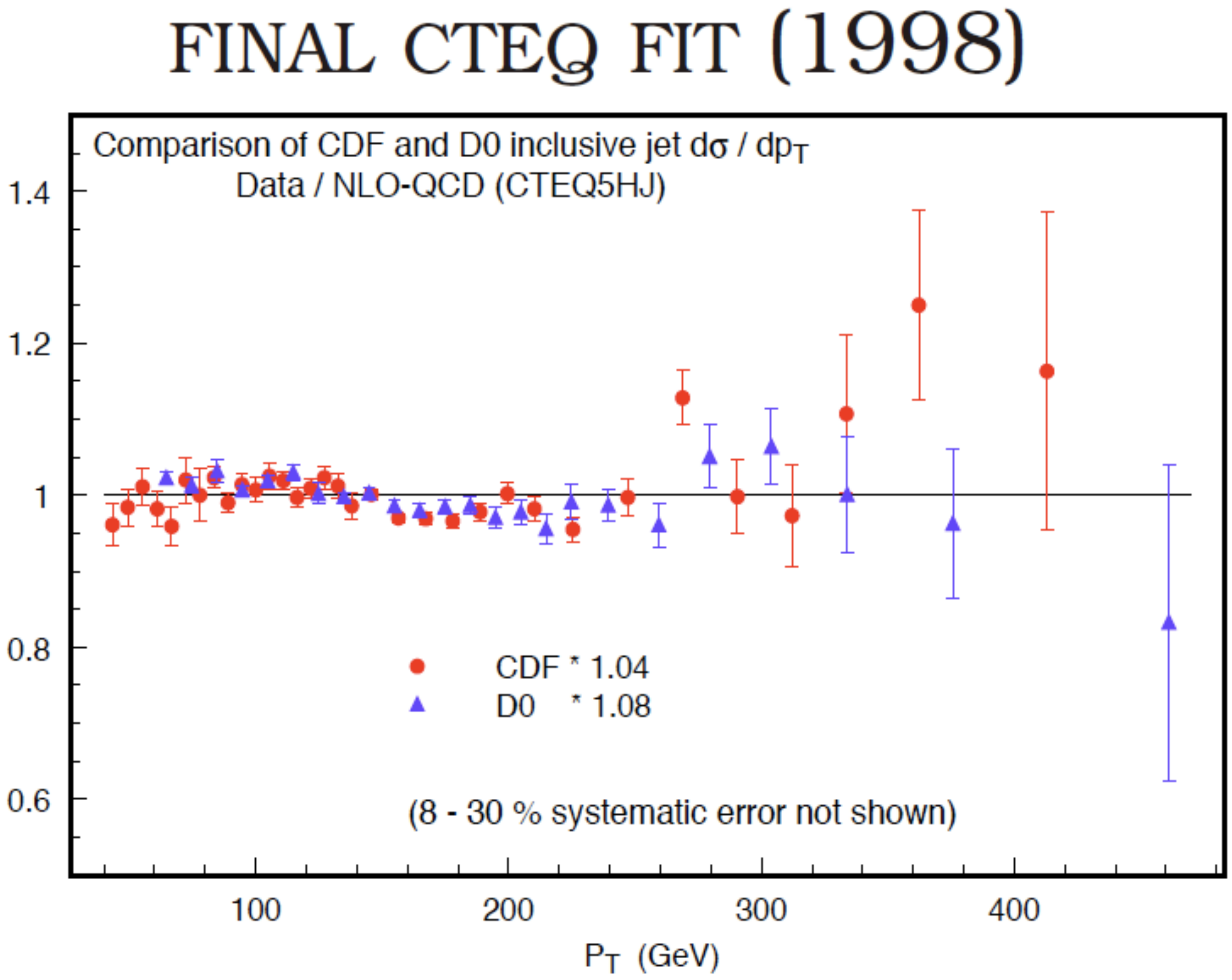


Historia magistra vitae est

Discrepancy between QCD calculations and CDF jet data (1995)

At that time there was no information on PDF uncertainties
and the theoretical prediction strongly depends on gluon shape at x>0.1

PDF uncertainties



Historia magistra vitae est

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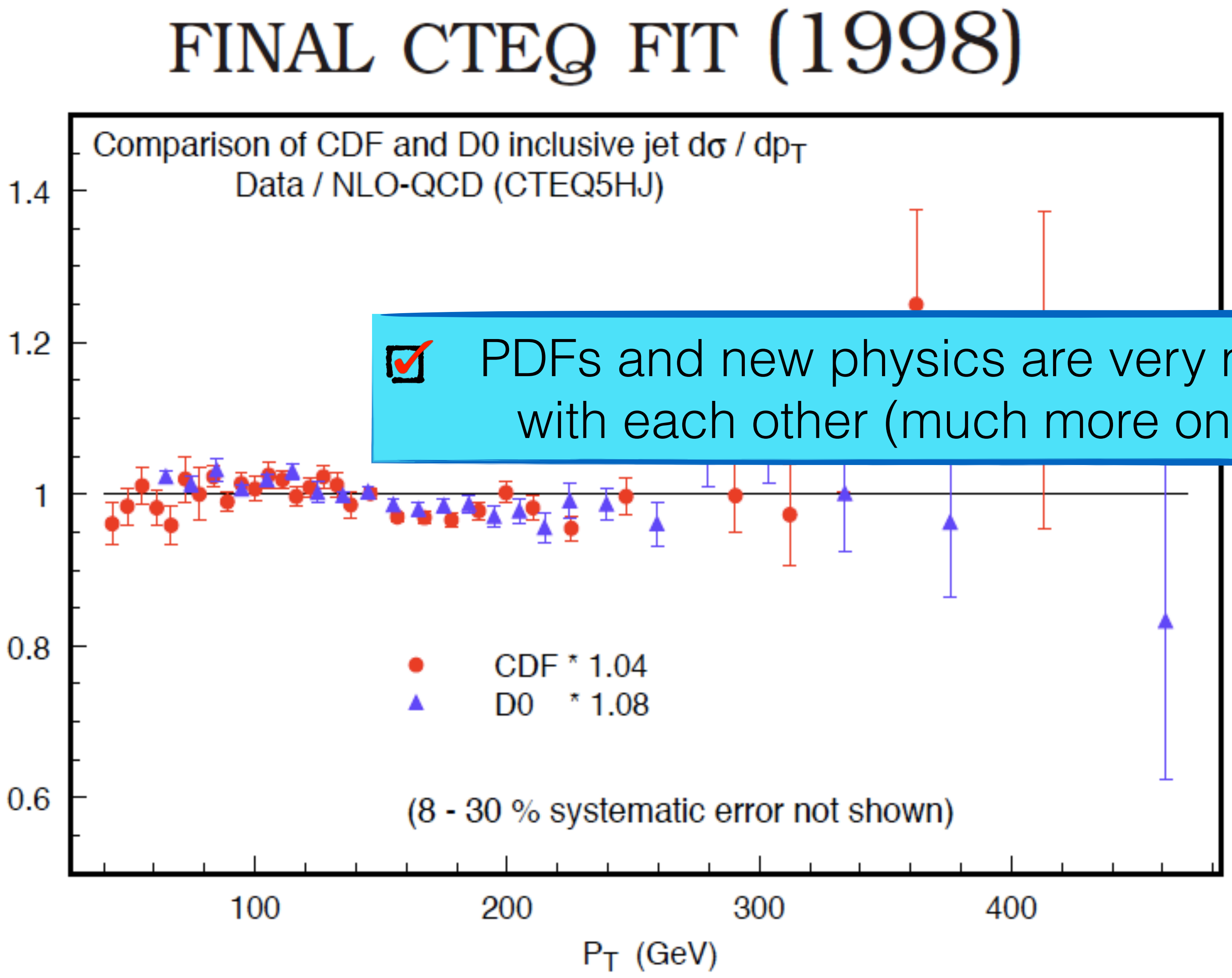
CTEQ re-performed the parton fit by including the jet data and the discrepancy was removed.

PDF uncertainties



☑ PDFs and new physics are very much related with each other (much more on lecture 5)

Historia magistra vitae est

Discrepancy between QCD calculations and CDF jet data

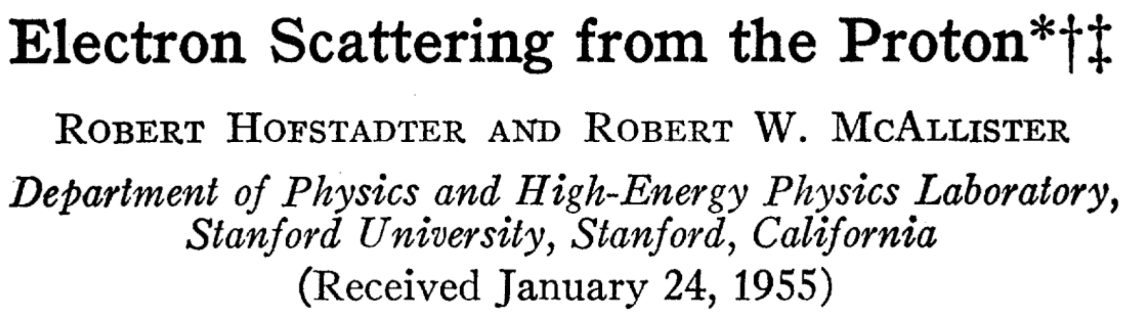
At that time there was no information on PDF uncertainties
and the theoretical prediction strongly depends on gluon shape at $x>0.1$

CTEQ re-performed the parton fit by including the jet data and the discrepancy was removed.

Parton model and QCD

Historic overview

- **1955**: Hofstadter et al observed first deviations in scattering of electron off proton from simple point-like Mott scattering ➜ Finite radius of proton ~ 0.7 fm

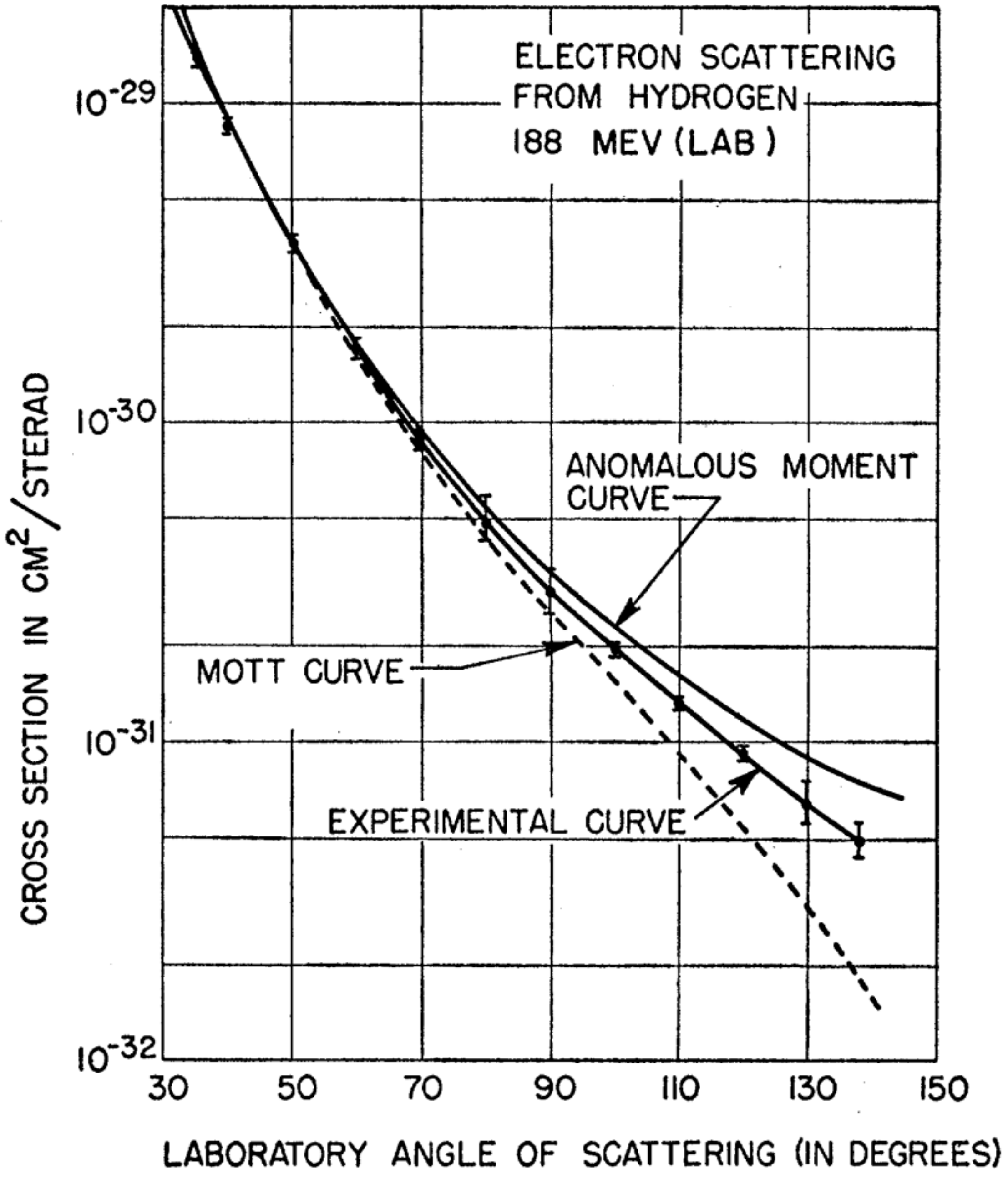


Electron Scattering from the Proton*†‡

Robert Hofstadter and Robert W. McAllister

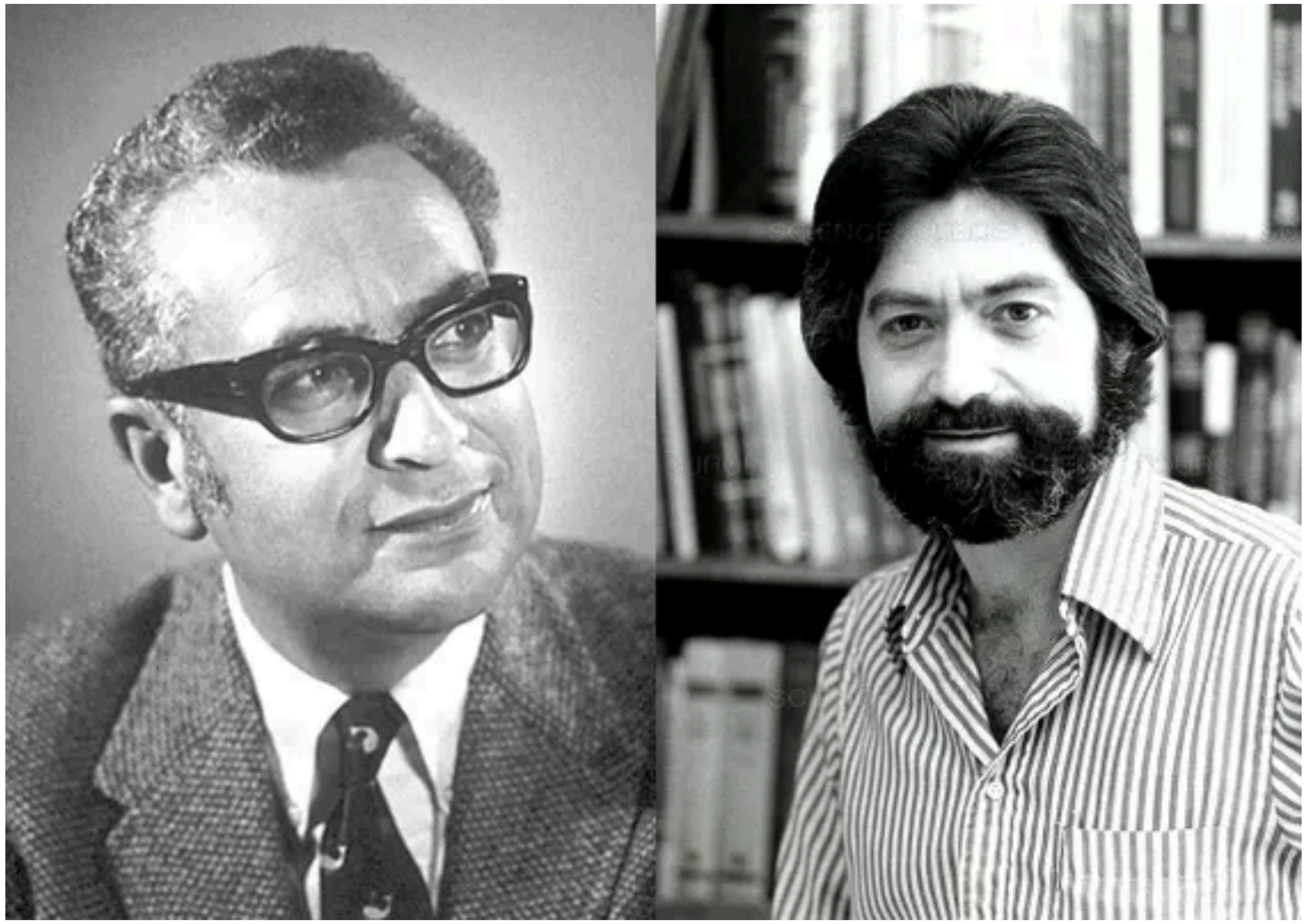
Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California

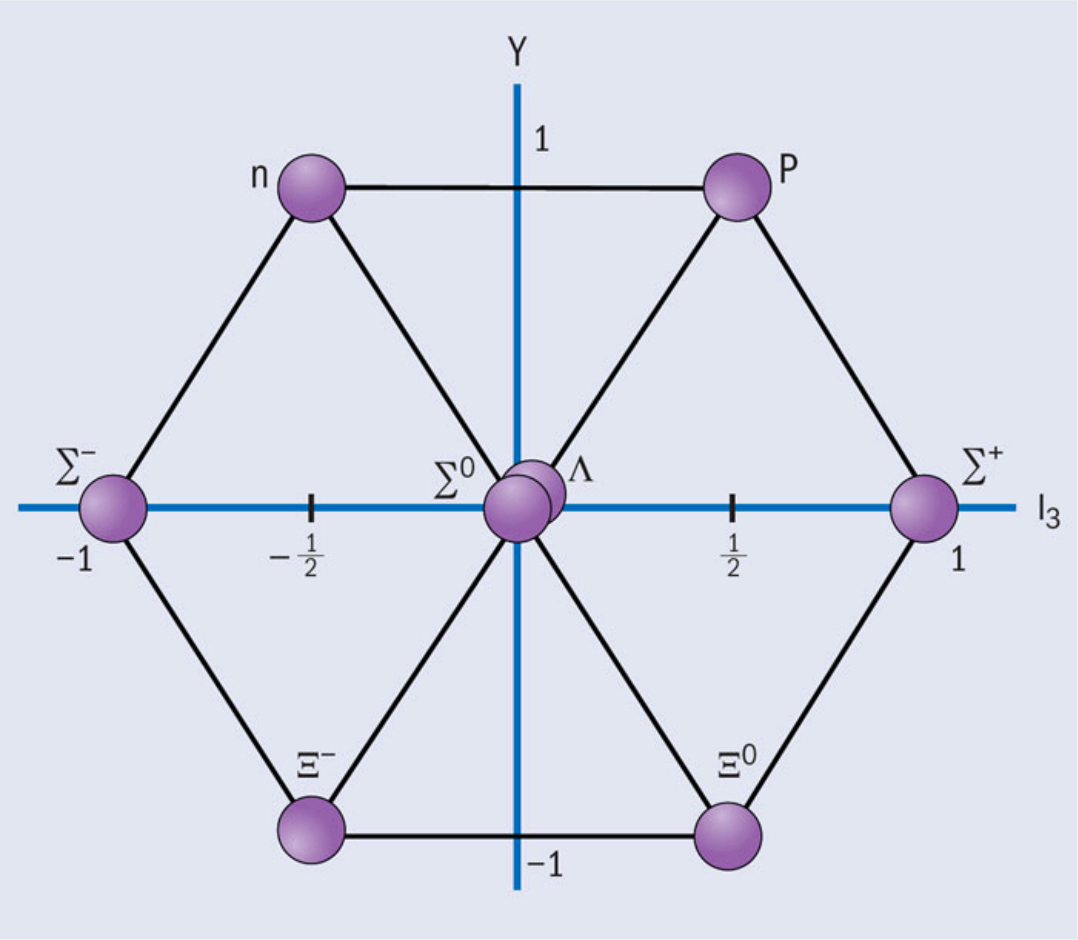
(Received January 24, 1955)



Historic overview

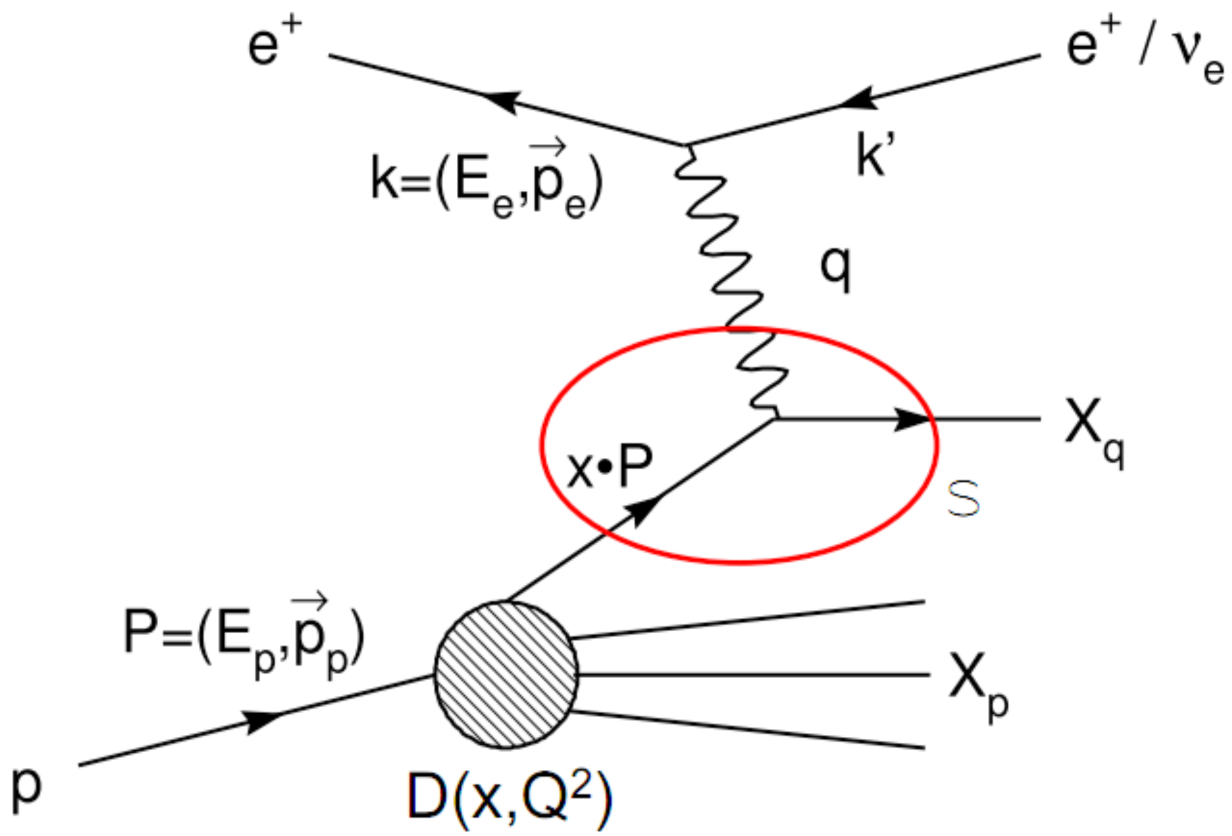
- **1964**: Zweig and Gell-Mann independently postulated existence of three aces (Zweig) or quarks (Gell-Mann) with fractional electric charge and spin–1/2 to explain proliferation of mesons and baryons in nucleon collision experiments. More of a mathematical model rather than particles! How could such objects be bound so tightly together?

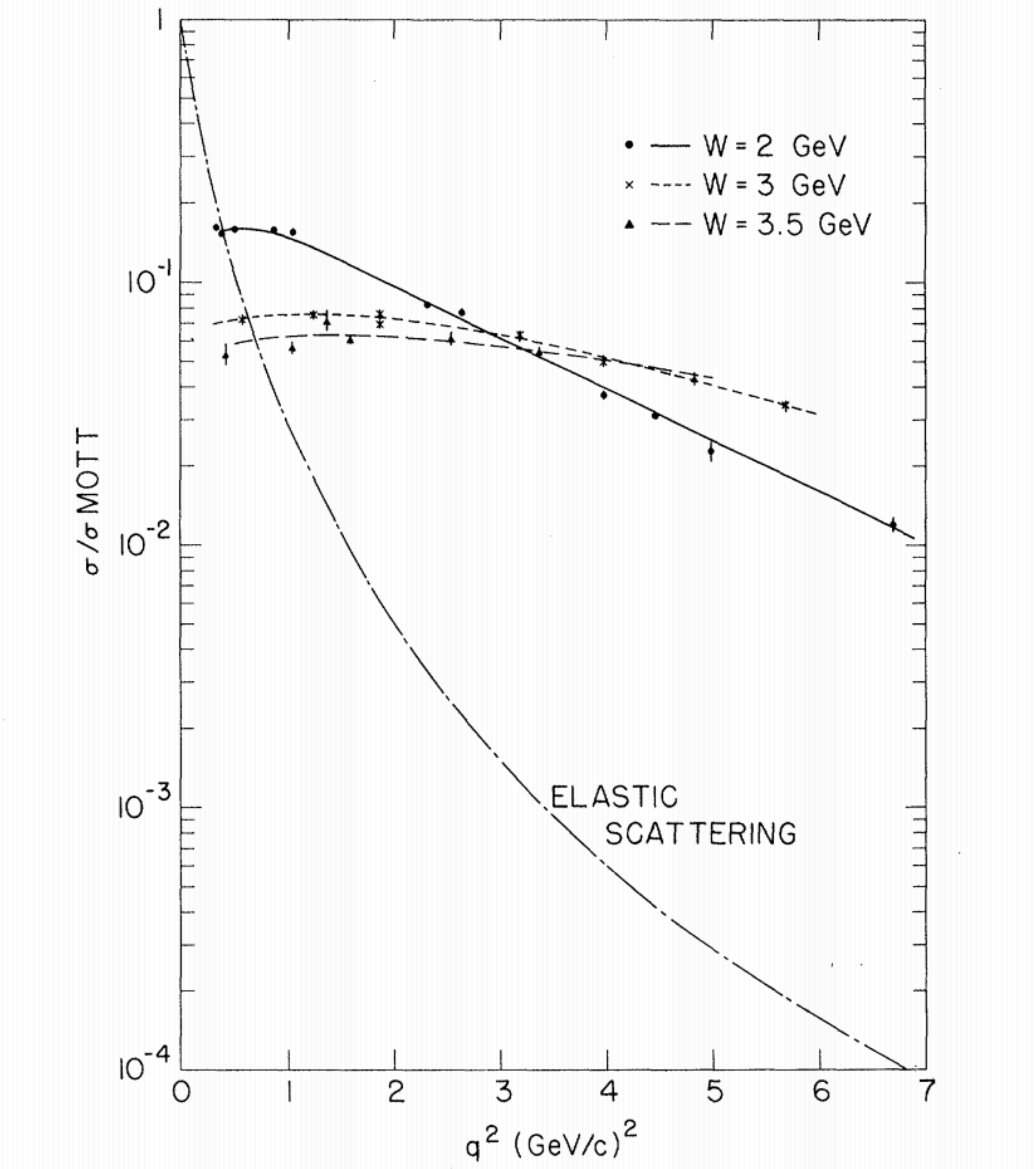




Historic overview

- **1967**: First deep-inelastic scattering experiments at SLAC 20 GeV linear collider gave first evidence of point-like elementary constituents which were later identified as quarks (Bjorken scaling)



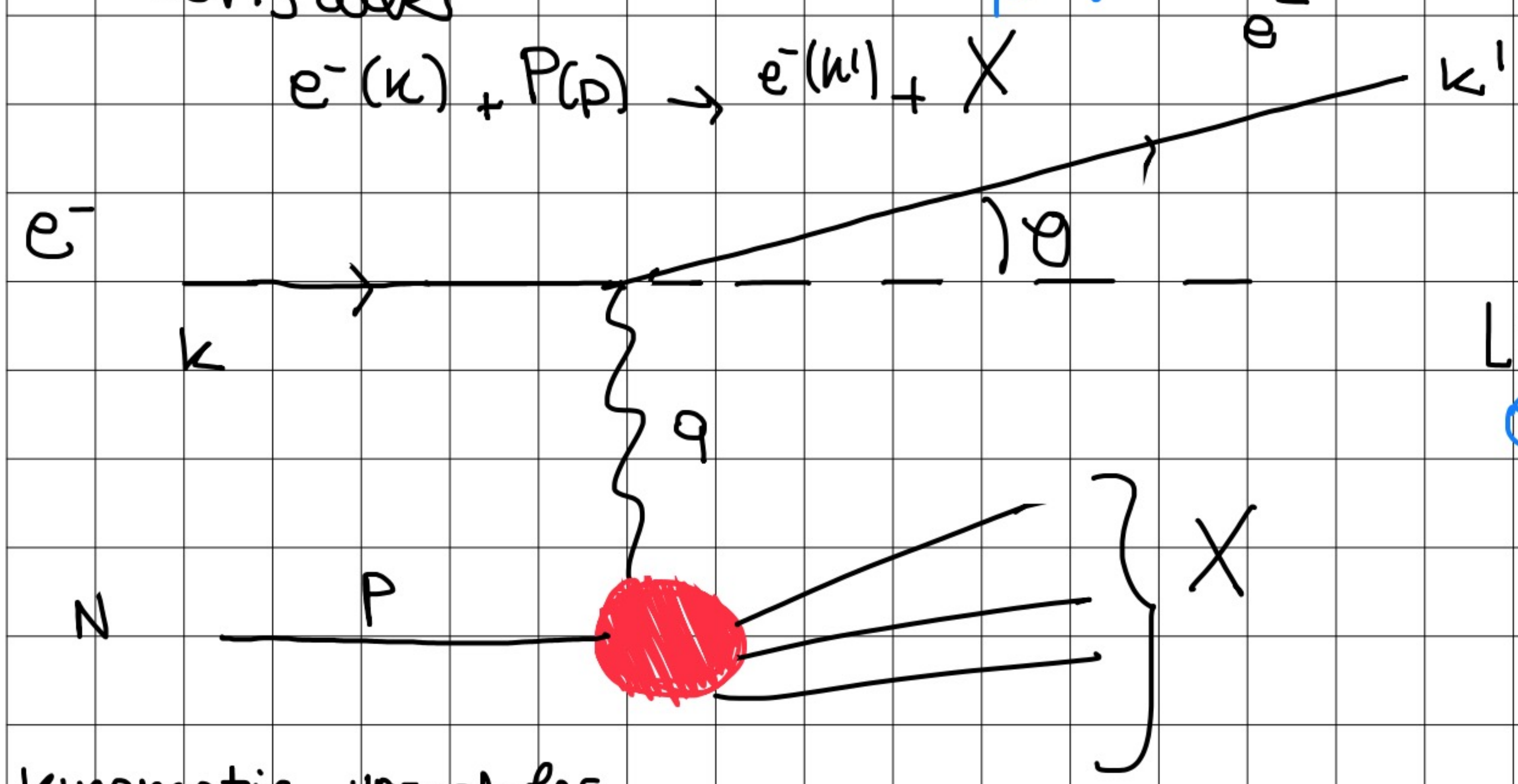


Deep Inelastic Scattering

Consider

$e^-(k) + P(p) \to e^-(k') + X$

↳ generic hadron system



LO diagram in QED

Kinematic variables

centre-of-mass energy

$$S = (P+K)^2$$

$$q = k - k'$$

q^2 γ virtuality

Deep Inelastic Scattering

Define Lorentz invariant quantities

in rest frame of p or c.o.m.

$$Q^2 = -q^2 = -(k-k')^2 = 2k\cdot k' = 2E_e E'_e (1-\cos\theta)$$

↓ ignore lepton mass

$$x_B = \frac{Q^2}{2p\cdot q}$$

Bjorken x

$x_B > 0$

$$y = \frac{p\cdot q}{p\cdot k} = \frac{E_e - E'_e}{E_e}$$

Inelasticity

$0 \leq y \leq 1$

$$M_X^2 = (p+q)^2 = \frac{Q^2(1-x)}{x}$$

Hadronic inv. mass

$x_B \leq 1$

Only two independent.

→ elastic limit $M_X^2 = M_p^2 \Leftrightarrow x_B = 1$ (only 1 indep.)

↓

$\underline{\text{deep inelastic limit}}$ $M_X^2 \gg M_p^2 \Leftrightarrow$ large Q^2 for fixed x

ignore proton mass $M_p \to 0$

Deep Inelastic Scattering

$$\mathcal{M}(e^- p \to e^- X) = \langle e^- | J^\mu | e^- \rangle \, g_{e\gamma} \, \frac{-g_{\mu\nu}}{q^2} \, g_{h\gamma} \, \langle X | J^\nu | h \rangle$$

↳ e

↳ coupling $h\gamma^*$

QED current

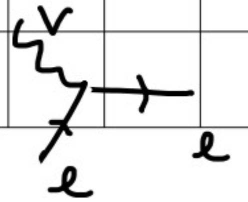
Let's generalise it to exchange of V boson γ^*, Z^*, W^* for any incoming lepton ℓ

$$\mathcal{M}(\ell p \to \ell' X) = \langle \ell | J^\mu | \ell' \rangle \, g_{\ell V} \, \frac{-g_{\mu\nu}}{q^2 - M_V^2} \, g_{hV} \, \langle X | J^\nu | h \rangle$$

M_V → mass of exchanged boson

$g_{\ell V}$, g_{hV} → couplings

$$g_{\ell V} = \kappa_V \sqrt{v_{\ell V}^2 + a_{\ell V}^2}$$



$$-ie\kappa_V \gamma_\mu \left(v_{\ell V} + \gamma_5 \, a_{\ell V} \right)$$

Deep Inelastic Scattering

$$d\sigma(ep \to e'X) = \frac{1}{2S} \frac{(g_{ev}\, g_{hv})^2}{(Q^2 + M_V^2)^2} L_{\mu\nu} W^{\mu\nu} (4\pi) \underbrace{\frac{d^3k'}{(2\pi)^3 2E'}}_{\downarrow}$$

② leptonic tensor

③ hadronic tensor

① final lepton phase space

① Phase space

$$\frac{d^3k'}{(2\pi)^3 2E'} = \frac{E' dE' d\cos\theta}{8\pi^2}$$

integrate over azimuthal angle φ

We can write it in terms of invariant

$$Q^2 = 2EE'(1 - \cos\theta) \Rightarrow dQ^2 = 2EE' d\cos\theta$$

$$2p\cdot q = \frac{s}{2E}(2E - E'(1+\cos\theta)) = \frac{s}{2E}\left[2(E-E') + \frac{Q^2}{2E}\right]$$

$\Rightarrow$ for fixed Q^2

$$dx_B = \frac{x_B^2}{Q^2}\frac{s}{E} dE'$$

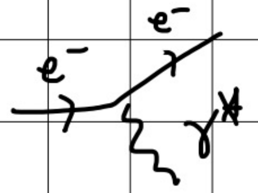
Deep Inelastic Scattering

$$\Rightarrow dx_B \, dQ^2 = \frac{2s x_B^2}{Q^2} E' dE' d\cos\theta \quad -$$

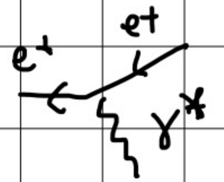
$$\frac{E'}{s} dE' d\cos\theta = \frac{Q^2}{2S^2 x_B^2} dx_B \, dQ^2 = \frac{y^2}{2Q^2} dx_B \, dQ^2$$

② Leptonic tensor

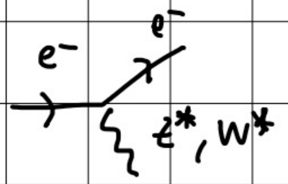
$$L_{\mu\nu} = \frac{1}{2} \langle l | J_\mu^+ | l' \rangle \langle l' | J_\nu | l \rangle$$



$$\bar{u}(k')(-ie\gamma_\mu) u(k)$$



$$\bar{v}(k)(-ie\gamma_\mu) v(k)$$



$$\bar{u}(k')\left[-iek_v \gamma_\mu (v_{ev} + a_{ev}\gamma_5)\right] u(k)$$

$$L_{\mu\nu} \propto \underset{e^- \text{ or } \nu}{\mathrm{Tr}[\not k \Gamma_\nu \not k' \Gamma_\mu]} \quad , \quad \underset{e^+ \text{ or } \bar{\nu}}{\mathrm{Tr}[\not k' \Gamma_\nu \not k \Gamma_\mu]}$$

Deep Inelastic Scattering

$$\Rightarrow \quad L_{\mu\nu} = 2\left[k_\mu k'_\nu + k_\nu k'_\mu - \frac{Q^2}{2} g_{\mu\nu} \pm i\, c_{\ell v}\, \varepsilon_{\mu\nu}{}^{\tau\sigma} k_\tau k'_\tau\right]$$

$$c_{\ell v} = \frac{2 a_{\ell v} v_{\ell v}}{v^2_{\ell v} + a^2_{\ell v}} \qquad \hookrightarrow \pm \begin{matrix} e^- & \nu \\ e^+ & \bar{\nu} \end{matrix}$$

③ Hadronic Tensor

$$W^{\mu\nu} = \frac{1}{2} \frac{1}{4\pi e^2} \sum_X \langle p | J^\mu | X \rangle \langle X | J^\nu | p \rangle \; \delta^{(4)}(p + q - P_X)$$

General form (using p^μ q^μ 4-vectors and $g^{\mu\nu}$ $\varepsilon^{\mu\nu\alpha\beta}$ isotropic tensor)

$$W^{\mu\nu} = -g_{\mu\nu} F_1 + \frac{1}{(p\cdot q)}\left[p^\mu p^\nu F_2 + i\,\varepsilon^{\mu\nu}{}_{\alpha\beta}\, p^\alpha p^\beta F_3 + (F_4 + iF_5)\, p^\mu q^\nu + (F_4 - iF_5)\, p^\nu q^\mu + F_6\, q^\mu q^\nu\right]$$

with F_i dimensionless functions of x, Q^2

- Time reversal $\Rightarrow$ $F_5 = 0$
- Conservation of EM current $q^\mu W^{\mu\nu} = W^{\mu\nu} q_\mu = 0$

$$\Rightarrow \quad W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \frac{\tilde{p}_\mu \tilde{p}_\nu}{(p\cdot q)} F_2(x, Q^2) + \frac{i\varepsilon_{\mu\nu\alpha\beta}\, p^\alpha q^\beta}{2(p\cdot q)} F_3(x, Q^2)$$

Deep Inelastic Scattering

$$\tilde{P}_\mu = P_\mu - \frac{(p\cdot q)}{q^2}\, q_\mu$$

$F_{1,2,3}$ → structure functions to be determined experimentally

If we require parity conservation (QED) $\Rightarrow F_3 = 0$

But if $\textcircled{Z}$ is exchanged $F_3 \neq 0$

▶ $$L_{\mu\nu} W^{\mu\nu} = 2Q^2 F_1 + \frac{4(p\cdot k)(p\cdot k')}{(p\cdot q)} F_2 \mp C_{e\nu} \frac{F_3}{(p\cdot q)}\, p\cdot(k+k')\, Q^2$$

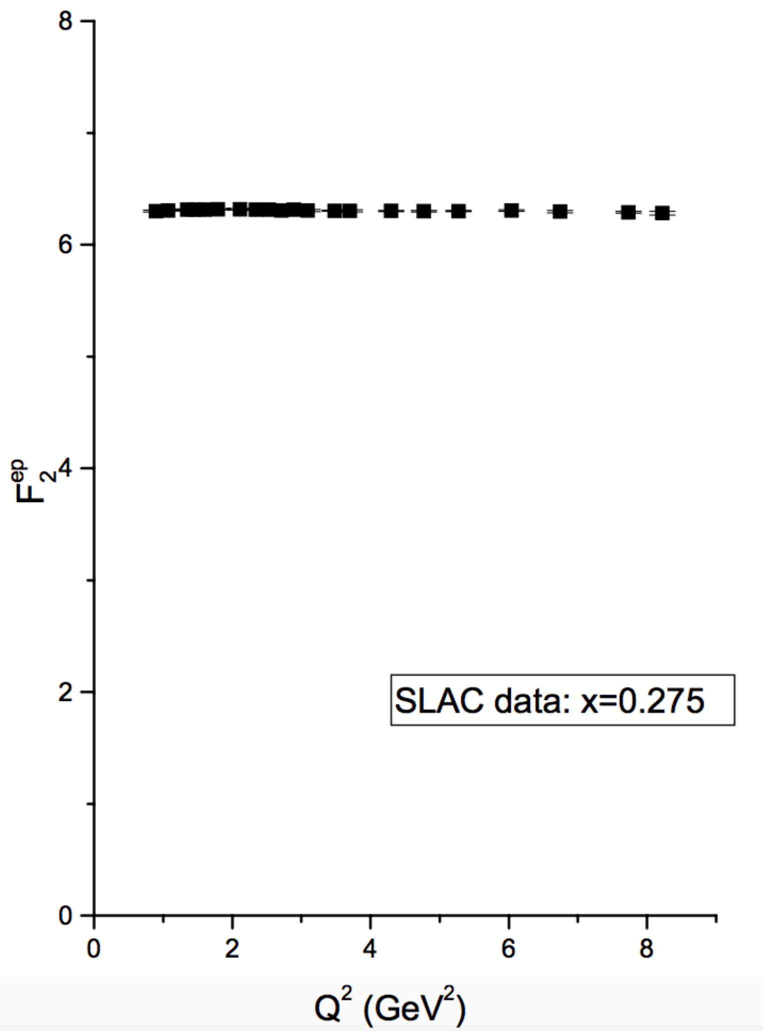
$$= \frac{2Q^2}{y^2}\left[y^2 F_1 + (1-y) F_2 \mp C_{e\nu}\, x \left(y - y^2/2\right) F_3\right]$$

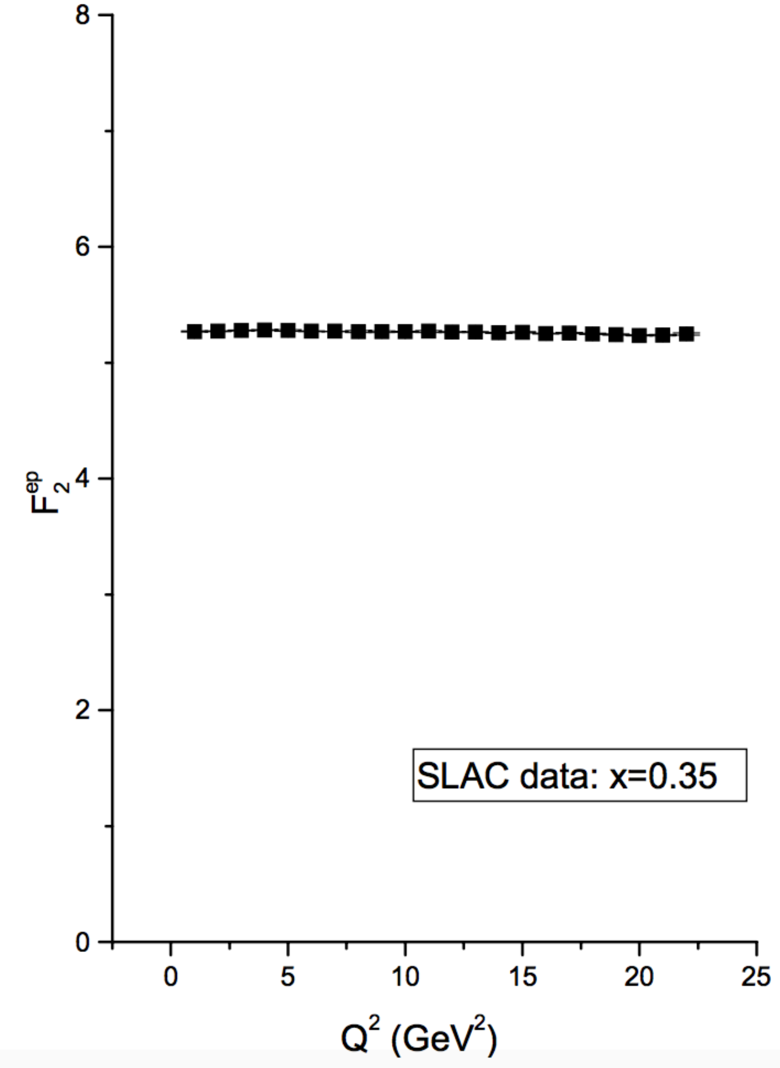
$$\frac{d\sigma}{dx\,dQ^2} = \frac{4\pi}{x}\frac{\alpha_{e\nu}\,\alpha_{h\nu}}{(Q^2+M_\nu^2)^2}\left[x y^2 \underset{\downarrow}{F_1} + (1-y) \underset{\downarrow}{F_2} \mp C_{e\nu}\, x\left(y - y^3/2\right) \underset{\downarrow}{F_3}\right]$$

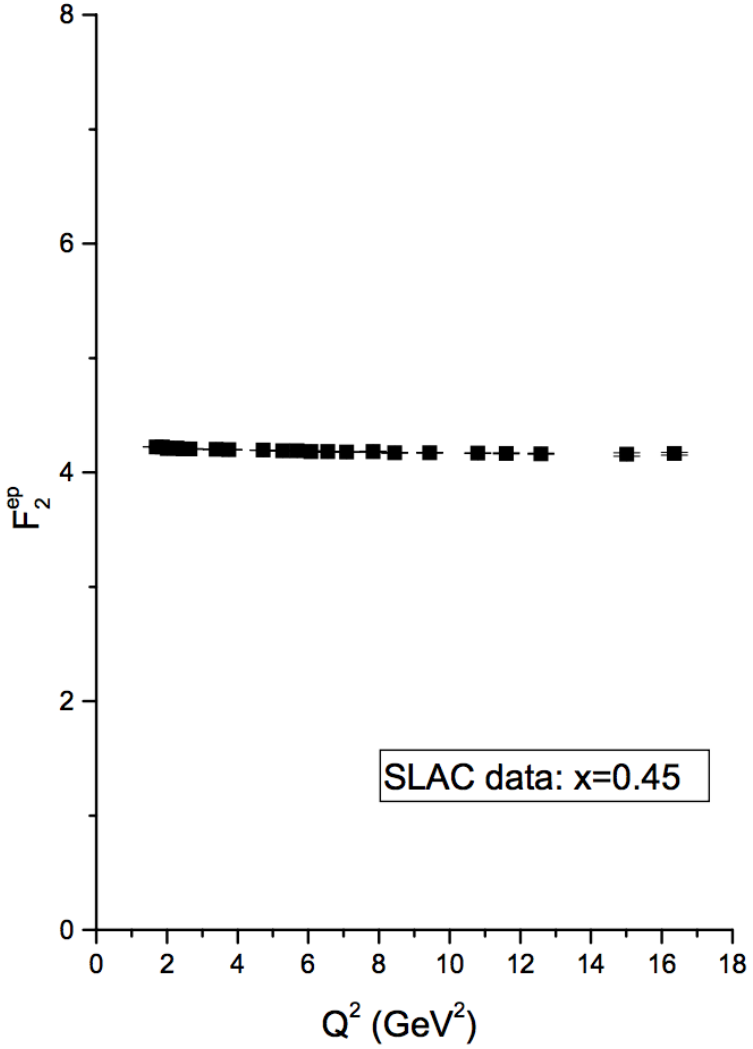
can be determined experimentally

Deep Inelastic Scattering

The surprising results at SLAC was that F1,2 did not vanish as Q2 increased, rather they remained finite and constant and depended only on xB - Bjorken scaling (1969)







Such scaling demonstrated that the exchanged vector boson (photon) scatters off point-like objects that have no mass or scale associated. The lepton scatters off charged spin 1/2 constituents (partons) that carry a fraction x of proton momentum

The Parton Model

Basic assumption

z: fraction of proton's momentum carried by the parton i

$$* \quad d\sigma(P) = \sum_{i \in \text{partons}} \int_0^1 dz \, \underbrace{f_i(z)} \, d\hat{\sigma}_i(zP)$$

Parton Distribution Functions

Scattering off massless spin 1/2 parton i with charge e_i (in units of proton charge) and momentum $\hat{p} = zP$

$$Q(\hat{p}) + \gamma^*(q) \rightarrow Q(\hat{p}')$$

γ exchange

$$\frac{1}{2}\sum |\mathcal{M}|^2 = \frac{1}{2} e^2 \underbrace{\mathrm{Tr}[\not{\hat{p}}\gamma^\nu \not{\hat{p}}' \gamma^\mu]}$$

$$4\left[\hat{p}^\nu \hat{p}'^\mu + \hat{p}^\mu \hat{p}'^\nu - g^{\mu\nu} \hat{p}\cdot\hat{p}'\right] \qquad \hat{p}' = \hat{p} + q$$

$$\Rightarrow \hat{W}^i_{\mu\nu}(\hat{p}, q) = \frac{e_i^2}{4\pi} \frac{1}{2} \int \frac{d^3\hat{p}'}{(2\pi)^3 2\hat{p}'_0} (2\pi)^4 \delta^{(4)}(\hat{p} + q - \hat{p}') \sum_{\text{spin}} \bar{u}(\hat{p}')\gamma_\mu u(\hat{p}) \bar{u}(\hat{p})\gamma_\nu u(\hat{p}')$$

The Parton Model

$$= e_i^2 \int d^4\hat{p}' \, \delta(\hat{p}'^2) \, \delta^{(4)}(\hat{p} + q - \hat{p}') \left(\hat{p}_\mu \hat{p}'_\nu + \hat{p}_\nu \hat{p}'_\mu - g_{\mu\nu} \, \hat{p}\cdot\hat{p}' \right)$$

$$= e_i^2 \, \delta\left((\hat{p} + q)^2\right) \left[\hat{p}_\mu (\hat{p} + q)_\nu + \hat{p}_\nu (\hat{p} + q)_\mu - g_{\mu\nu} \, \hat{p}\cdot(\hat{p} + q) \right]$$

$$= e_i^2 \, \delta(2\hat{p}\cdot q + q^2) \Big[2\hat{p}_\mu \hat{p}_\nu - 2\frac{\hat{p}\cdot q}{q^2} (\hat{p}_\mu q_\nu + \hat{p}_\nu q_\mu) + 2\left(\frac{\hat{p}\cdot q}{q^2}\right)^2 q_\mu q_\nu -$$

$$- 2\left(\frac{\hat{p}\cdot q}{q^2}\right)^2 q_\mu q_\nu - g_{\mu\nu} \, \hat{p}\cdot q \Big]$$

$$= \frac{e_i^2}{2\hat{p}\cdot q} \, \delta\left(\frac{2\hat{p}\cdot q}{2\hat{p}\cdot q} - \frac{Q^2}{2\hat{p}\cdot q} \right) \left[2\left(\hat{p}_\mu - \frac{\hat{p}\cdot q}{q^2} q_\mu\right)\left(\hat{p}_\nu - \frac{\hat{p}\cdot q}{q^2} q_\nu\right) + (\hat{p}\cdot q)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \right]$$

$$= e^2 \, \delta(1 - \hat{x}) \left[\frac{1}{2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{1}{(\hat{p}\cdot q)} \left(\hat{p}_\mu - \frac{\hat{p}\cdot q}{q^2} q_\mu\right)\left(\hat{p}_\nu - \frac{\hat{p}\cdot q}{q^2} q_\nu\right) \right]$$

$\downarrow$

with $\hat{x} = \dfrac{Q^2}{2\hat{p}\cdot q}$

The Parton Model

Applying $\ast$ to DIS cross section we get

$$\frac{4\pi\alpha^2}{Q^4} L_{\mu\nu} W^{\mu\nu}(p,q)\,\frac{y^2}{2Q^2}\,dQ^2\,dx = \frac{4\pi\alpha^2}{Q^4} L_{\mu\nu}\,\frac{y^2}{2Q^2}\,dQ^2 \sum_i \int_0^1 dz\, f_i(z)\, \hat{W}^{\mu\rho}(zp,q)\,d\hat{x}$$

Keeping into account that $\hat{x} = x_B/z$, we get

$$W_{\mu\nu}(p,q) = \sum_{i\in\text{partons}} \int_0^1 dz\, f_i(z)\,\frac{1}{z}\,\hat{W}^i_{\mu\nu}(zp,q)$$

Assuming $(zp+q)^2 \geq 0 \Rightarrow z \geq x_B$

$$\Rightarrow W_{\mu\nu}(p,q) = \sum_i \int_{x_B}^1 \frac{dz}{z}\, e_i^2 f_i \left[\frac{1}{2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{z}{p\cdot q}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)\right]$$

$$\delta(1 - x_B/z)$$

$$\frac{dz}{z} \to dz\,\delta(z - x_B)$$

$$\Rightarrow W_{\mu\nu}(p,q) = \sum_i e_i^2 f_i(x) \left[\frac{1}{2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{x}{p\cdot q}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)\right]$$

The Parton Model

Compared to general formula we get

$$\boxed{\begin{aligned} F_1(x,Q^2) &= \frac{1}{2} \sum_i e_i^2 f_i(x) \\ F_2(x,Q^2) &= \sum_i e_i^2 \, x \, f_i(x) \end{aligned}}$$

Explicit scaling! Also note that in PARTON MODEL

$$F_2(x) = 2x \, F_1(X)$$

Callan - Gross relation
typical of spin 1/2

Indeed F_1 → absorption of transversally polarized virtual photon

$F_L = F_2 - 2xF_1$ → " " longitudinally polarized " "

$F_L \ll F_1$ confirms spin 1/2 of partons

proof $\varepsilon^\mu \varepsilon^{*\nu} W_{\mu\nu}(p,q) \propto \sigma(p + \gamma^* \to X)$

Take longitudinally polarized photon

The Parton Model

- long. polarised γ

$$\varepsilon = \varepsilon_L = \lambda \left(p - \frac{p\cdot q}{q^2}\, q\right) \qquad |\lambda|^2 = \frac{q^2}{(p\cdot q)^2}$$

such that

$$q\cdot\varepsilon_L = 0$$
$$\varepsilon_L\cdot\varepsilon_L^* = -1$$

$$\sigma_L \propto \varepsilon_L^{\mu}\, \varepsilon_L^{*\nu}\, W_{\mu\nu}(p,q) = F_1(x,Q^2) - \frac{F_2(x,Q^2)}{2x}$$

- transv. polarized γ

$$p\cdot\varepsilon_T = 0$$
$$q\cdot\varepsilon_T = 0$$
$$\varepsilon_T\cdot\varepsilon_T^* = -1$$

$$\sigma_T \propto \varepsilon_T^{\mu}\, \varepsilon_T^{*\nu}\, W_{\mu\nu}(p\cdot q) = F_1(x,Q^2)$$

$$\Rightarrow \frac{\sigma_L}{\sigma_T} = 1 - \boxed{\frac{F_2(x,Q^2)}{2x F_1(x,Q^2)}} \rightarrow = 1 \text{ parton model} \Rightarrow \sigma_L = 0$$

Note that it would be $\neq 0$ if partons were spin 0.

Exercise I: Z contribution

- Show that, in the Parton model, considering also the contribution of a virtual Z boson and its interference with the photon one obtains:

$$F_2^{\gamma,Z}(x) = x\sum_{i=1}^{n_f} c_i[q_i(x) + \bar{q}_i(x)]$$

$$F_3^{\gamma,Z}(x) = \sum_{i=1}^{n_f} d_i[q_i(x) - \bar{q}_i(x)]$$

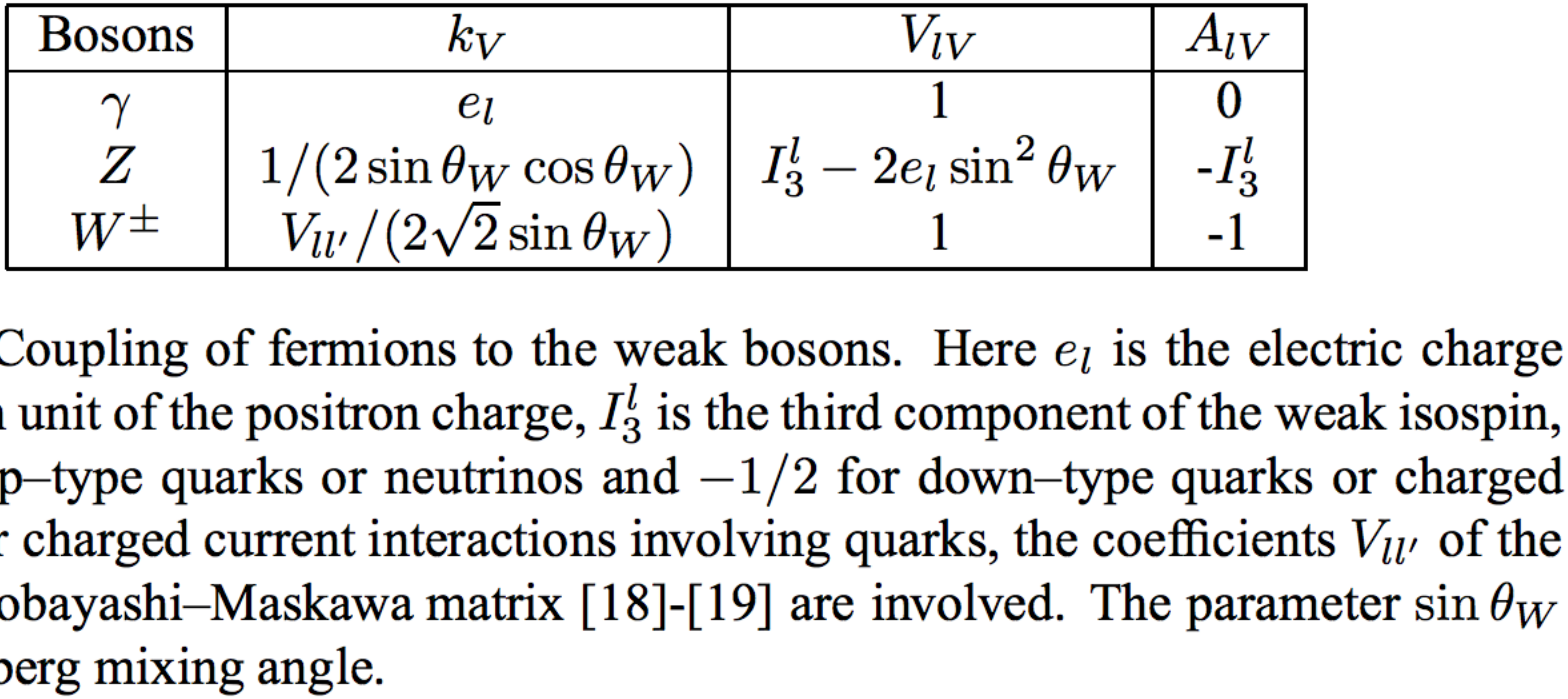
Where

$$c_i = e_i^2 - 2e_iV_{eZ}V_{iZ}P_Z + (V_{eZ}^2 + A_{eZ}^2)(V_{iZ}^2 + A_{iZ}^2)P_Z^2$$

$$d_i = -2e_iA_{eZ}A_{iZ}P_Z + 4V_{eZ}A_{eZ}V_{iZ}A_{iZ}P_Z^2$$

$$P_Z = \frac{Q^2}{(Q^2 + M_Z^2)(4s_w^2c_w^2)} \longrightarrow \begin{aligned} c_w &= \cos\theta_w \\ s_w &= \sin\theta_w \end{aligned}$$

Exercise I: Z contribution



Bosons	k_V	V_{lV}	A_{lV}
γ	e_l	1	0
Z	$1/(2\sin\theta_W\cos\theta_W)$	$I_3^l - 2e_l\sin^2\theta_W$	-I_3^l
$W^{\pm}$	$V_{ll'}/(2\sqrt{2}\sin\theta_W)$	1	-1

Table 1.1: Coupling of fermions to the weak bosons. Here e_l is the electric charge measured in unit of the positron charge, I_3^l is the third component of the weak isospin, $+1/2$ for up–type quarks or neutrinos and $-1/2$ for down–type quarks or charged leptons. For charged current interactions involving quarks, the coefficients $V_{ll'}$ of the Cabibbo–Kobayashi–Maskawa matrix [18]-[19] are involved. The parameter $\sin\theta_W$ is the Weinberg mixing angle.

Exercise II: Paschos-Wolfenstein relation

- Show that, in the Parton model, considering a (anti)neutrino-initiated DIS process on a deuteron target – assuming SU(2) isospin symmetry $u_n(x)=d_p(x)$ and $d_n(x) = u_p(x)$ – the ratio R

$$R = \frac{\sigma_{\mathrm{NC}}(\nu) - \sigma_{\mathrm{NC}}(\bar{\nu})}{\sigma_{\mathrm{CC}}(\nu) - \sigma_{\mathrm{CC}}(\bar{\nu})}$$

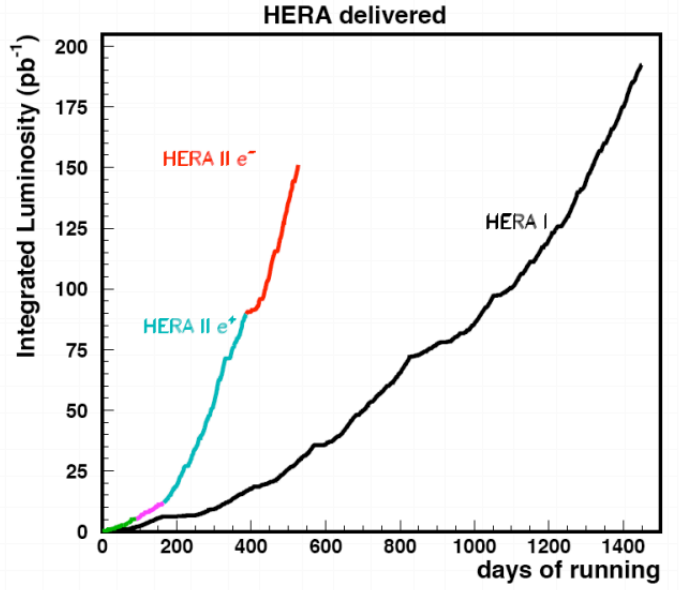
NC (mediated by Z) and CC (mediated by $W^{+/-}$), assuming strange and anti-strange to be equal in the target, is independent of Parton Distribution Functions and can be used to determined the Weinberg angle θ_W

$$R = \frac{1}{2}\left(\frac{1}{2} - \sin\theta_w^2\right)$$

You may use (without deriving it) the result (and set c, cbar = 0)

$$\begin{aligned}
F_2^{W^-} &= 2x\left(u + \bar{d} + \bar{s} + c\right), \\
F_3^{W^-} &= 2x\left(u - \bar{d} - \bar{s} + c\right), \\
F_2^{W^+} &= 2x\left(d + \bar{u} + \bar{c} + s\right), \\
F_3^{W^+} &= 2x\left(d - \bar{u} - \bar{c} + s\right),
\end{aligned}$$

The HERA collider



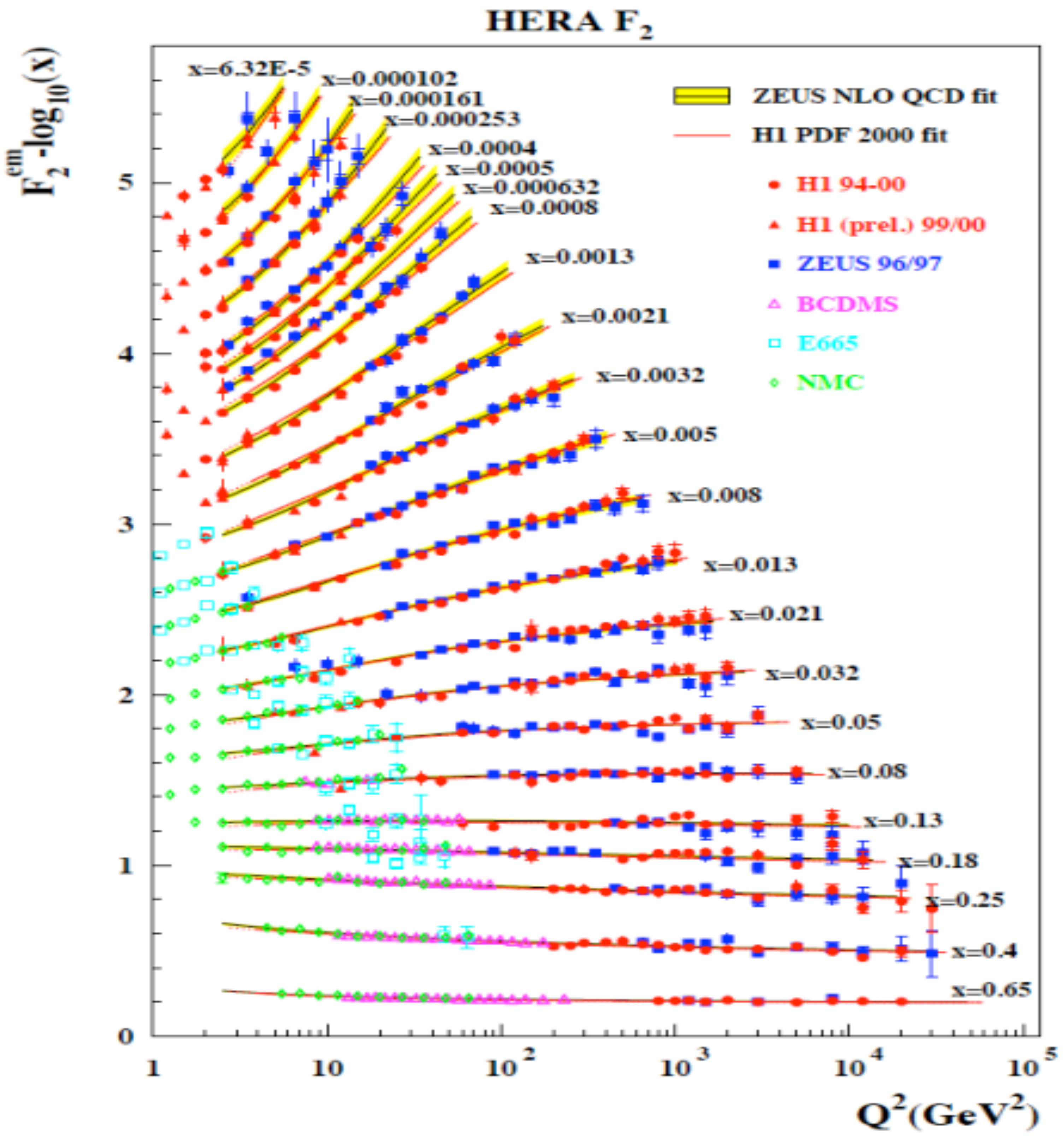
1992-2007

$$\sqrt{S} = 318\,\mathrm{GeV}$$

$$E_e = 27.5\,\mathrm{GeV}$$

$$E_p = 920\,\mathrm{GeV}$$

Scaling violation

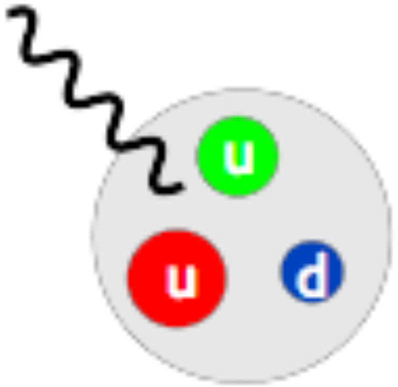


scaling violation

approx. scaling

QCD and improved parton model

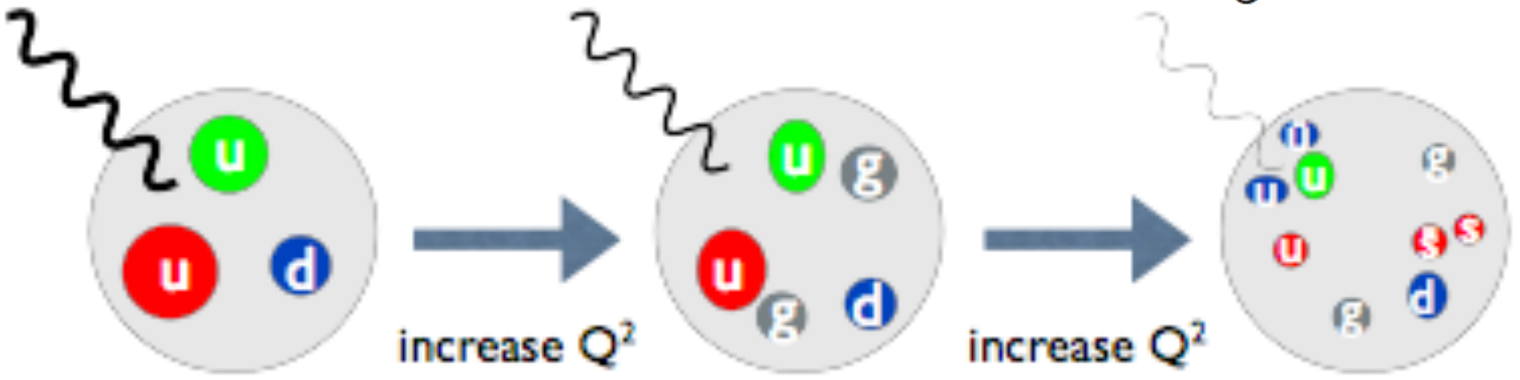
Parton model picture



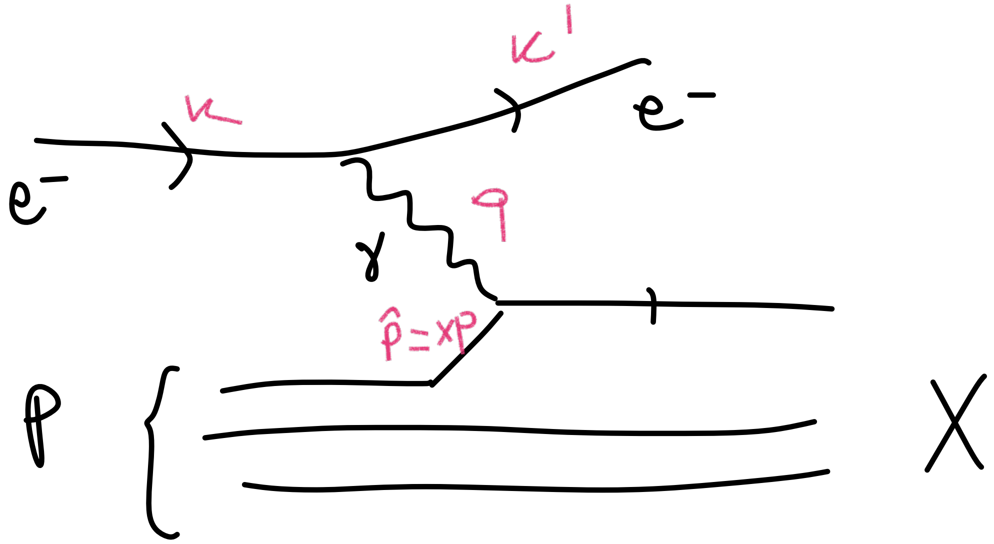
$$\sigma = \int dx\, f_i^{(p)}(x)\, \sigma^{(0)}(xp)$$

QCD-improved parton model

$$\sigma = \int dx\, f_i^{(p)}(x, \mu_F^2)\, \sigma(xp, \mu_F^2)$$



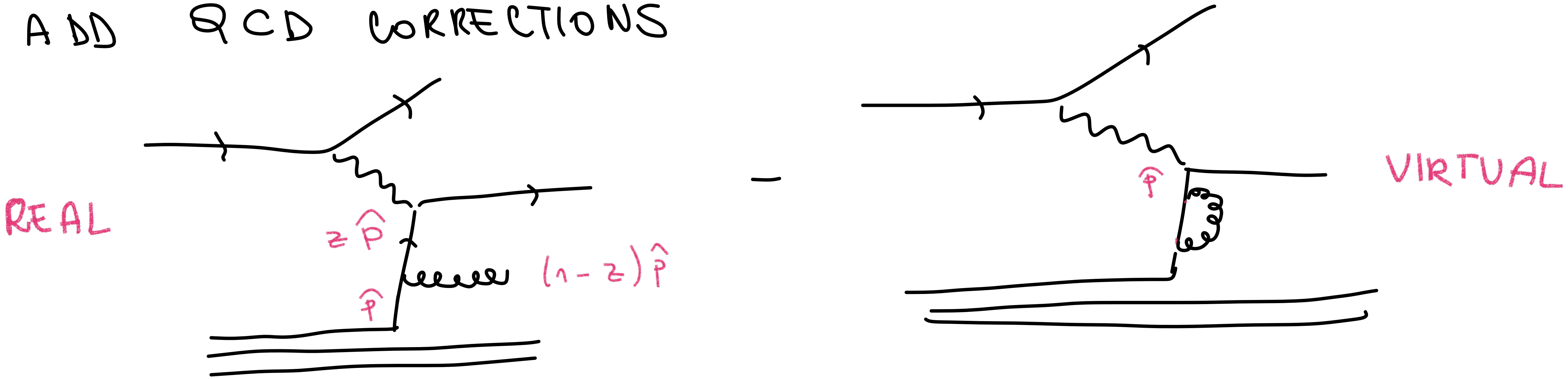
QCD and improved parton model



PARTON MODEL

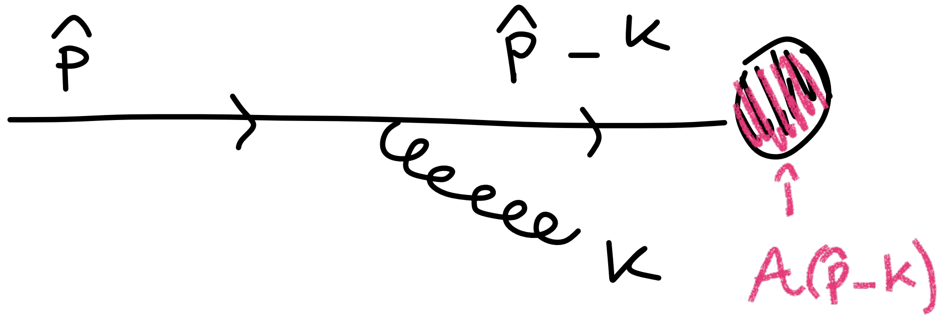
$$\sigma(e^-P \to e^-X) = \sum_i \int_0^1 dx\, f_i(x)\, \overset{\hat{\sigma}^{(0)}}{\hat{\sigma}(\underbrace{xP}_{\equiv\,\hat{p}})}$$

ADD QCD CORRECTIONS



QCD and improved parton model

real emission



sudakov parametrisation

$$k = (1-z)\hat{p} + k_T + \xi\eta$$

with η such that
4-vector

$\hat{p}\cdot k_T = 0$
$\eta\cdot k_T = 0$
$\eta^2 = 0$
$2\,p\cdot\eta = 1$

$$\hat{p} = \hat{p}_0\,(1, 0, 0, +1)$$
$$\eta = \frac{1}{4\hat{p}_0}\,(1, 0, 0, -1)$$
$$k_T = (0, k_{T,1}, k_{T,2}, 0)$$

From gluon on-shell condition, $k^2 = 0 \Rightarrow k_T^2 + 2\xi(1-z)\,\hat{p}\cdot\eta = 0 \Rightarrow \xi = -\frac{k_T^2}{1-z}$

From $(p-k)^2 < 0 \Rightarrow z < 1$

From $k_3 = 0 \Rightarrow |k_T^2| < 4\hat{p}_0^2(1-z)$

QCD and improved parton model

Parametrise phase space and integrate over azimuthal angle

$$\frac{d^3k}{(2\pi)^3 2k_0} = \frac{1}{16\pi^2}\frac{d|k_T|^2\,dz}{1-z}$$

Consider singular part of the amplitude

$$\mathcal{M}^{(1)}_{q,\text{sing}}(\hat{p},k) = g_s A_i(\hat{p}-k)\,\frac{\hat{\not{p}}-\not{k}}{(\hat{p}-k)^2}\,\not{\varepsilon}(k)\,t^a_{ij}\,u_j(\hat{p})$$

$$= g_s\,\frac{(1-z)}{k_T^2}\,A_i(\hat{p}-k)\,(z\hat{\not{p}}-\not{k}_T)\,\not{\varepsilon}(k)\,t^a_{ij}\,u_j(\hat{p})$$

$$= -\frac{g_s}{k_T^2}\,A_i(\hat{p}-k)\left[2z\,k_T\,\varepsilon(k)+(1-z)\not{k}_T\not{\varepsilon}(k)\right]t^a_{ij}\,u_j(\hat{p})$$

$$\Rightarrow \qquad |\mathcal{M}^{(1)}_{\text{q,sing}}|^2 = -\frac{2g_s^2}{|k_T|^2}C_F\,\frac{1+z^2}{z}\,|\mathcal{M}^{(0)}(z\hat{p})|^2$$

QCD and improved parton model

$$\hat{\sigma}^{(1)}_{q,R}(\hat{p}) = \frac{\alpha_s}{2\pi} C_F \int_0^1 \frac{dz}{1-z} \int_0^{|k_T^2|_{max}} d|k_T^2| \frac{1+z^2}{|k_T^2|} \hat{\sigma}_q^{(0)}(z\hat{p})$$

The partonic cross section at NLO in α_S (associated with real emission of a gluon off a quark displays two singularities:

- **SOFT** singularity (z → 1), which we regulate with parameter **ε** (→ 0)
- **COLLINEAR** singularity ($|k^2_T| \to 0$), which we regulate with parameter λ^2 (→ 0)

$$\hat{\sigma}^{(1)}_{q,R}(\hat{p}) = \frac{\alpha_s}{2\pi} C_F \int_0^{1-\varepsilon} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} d|k_T^2| \frac{1+z^2}{|k_T^2|} \hat{\sigma}_q^{(0)}(z\hat{p})$$

QCD and improved parton model

Add virtual corrections, which also have soft and collinear singularities

$$\hat{\sigma}_v^{(1)} = -\hat{\sigma}_q^{(0)}(\hat{p})\,\frac{\alpha_s}{2\pi}\,C_F \int_0^{1-\varepsilon} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|}\,(1+z^2)$$

$$\hat{\sigma}_{q,R+V}^{(1)}(\hat{p}) = \frac{\alpha_s}{2\pi}\,C_F \int_0^{1} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|}\,(1+z^2)\left[\hat{\sigma}_q^{(0)}(z\hat{p}) - \hat{\sigma}_q^{(0)}(\hat{p})\right]$$

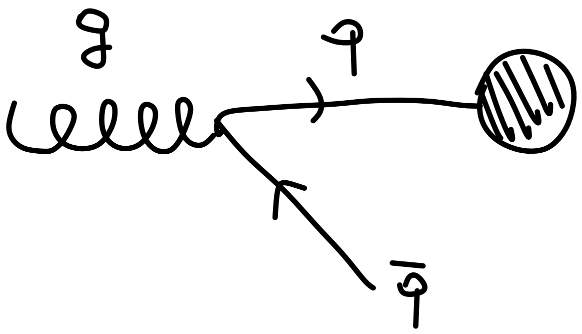
The soft singularity cancels between real and virtual contributions but the collinear singularity still there. Trick: split integration

$$\int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} \rightarrow \underbrace{\int_{\lambda^2}^{\mu_F^2} \frac{d|k_T^2|}{|k_T^2|}}_{\text{SINGULAR}} + \underbrace{\int_{\mu_F^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|}}_{\text{FINITE}}$$

QCD and improved parton model

$$\hat{\sigma}_q(\hat{p}) = \hat{\sigma}_q^{(0)}(\hat{p}) + \hat{\sigma}_q^{(1)}(\hat{p})$$

$$= \hat{\sigma}_q^{(0)}(\hat{p}) + \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 dz\, P_{qq}(z)\, \hat{\sigma}_q^{(0)}(z\hat{p}) \log\frac{M_F^2}{\lambda^2} + \text{NON-SINGULAR TERMS}$$



Add LO and NLO correction from initial quark + NLO contribution from gluon-initiated process with gluon splitting into quark-antiquark pair

$$\hat{\sigma}_g(\hat{p}) = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 dz\, P_{qg}(z)\, \hat{\sigma}_q^{(0)}(z\hat{p}) \log\frac{M_F^2}{\lambda^2} + \text{NON-SINGULAR TERMS}$$

$P_{qq}(z)$ and $P_{qg}(z)$ are universal functions associated to quark and gluon splittings, called splitting functions

QCD and improved parton model

Plug into the parton model equation:

$$\sigma(P) = \int_0^1 dx \, f_q(x) \, \hat{\sigma}_q \, (\underbrace{xP}_{\hat{p}}) + f_g(x) \, \hat{\sigma}_g \, (xP)$$

$$\Downarrow$$

$$\begin{aligned}\sigma(P) = \int_0^1 dx \Big[& f_q(x) \Big(\hat{\sigma}_q^{(0)}(xP) + \frac{\alpha_s}{2\pi} \int_0^1 dz \, P_{qq}(z) \, \hat{\sigma}_q^{(0)}(xzP) \log \frac{\mu_F^2}{\lambda^2} \Big) \\ & + f_g(x) \Big(\frac{\alpha_s}{2\pi} \int_0^1 dz \, P_{qg}(z) \, \hat{\sigma}_q^{(0)}(xzP) \log \frac{\mu_F^2}{\lambda^2} \Big) \Big] \\ + \int_0^1 dx \Big[& f_q(x) \, \hat{\sigma}_{q,\mathrm{Reg}}^{(1)}(xP, \mu_F^2) + f_g(x) \, \hat{\sigma}_{g,\mathrm{Reg}}^{(1)}(xP, \mu_F^2) \Big]\end{aligned}$$

QCD and improved parton model

Plug into the parton model equation:

$$\sigma(P) = \int_0^1 dx\, f_q(x)\, \hat{\sigma}_q\, (\underbrace{xP}_{\hat{p}}) + f_g(x)\, \hat{\sigma}_g\, (xP)$$

$$\Downarrow$$

$$\sigma(P) = \int_0^1 dx \Big[f_q(x) \Big(\hat{\sigma}_q^{(0)}(xP) + \frac{\alpha_s}{2\pi} \int_0^1 dz\, P_{qq}(z)\, \hat{\sigma}_q^{(0)}(xzP) \log\frac{\mu_F^2}{\lambda^2} \Big)$$

$$+ f_g(x) \Big(\frac{\alpha_s}{2\pi} \int_0^1 dz\, P_{qg}(z)\, \hat{\sigma}_q^{(0)}(xzP) \log\frac{\mu_F^2}{\lambda^2} \Big) \Big]$$

$$+ \int_0^1 dx \Big[f_q(x)\, \hat{\sigma}^{(1)}_{q,\mathrm{reg}}(xP, \mu_F^2) + f_g(x)\, \hat{\sigma}^{(1)}_{g,\mathrm{reg}}(xP, \mu_F^2) \Big]$$

Absorb collinear divergences into a redefinition of the parton distribution functions, which now depend on the factorisation scale

$$f_q(x, \mu_F^2) = \int_x^1 \frac{dy}{y} \Big\{ f_q(y) \Big[\delta\Big(1 - \frac{x}{y}\Big) + \frac{\alpha_s}{2\pi} P_{qq}\Big(\frac{x}{y}\Big) \log\frac{\mu_F^2}{\lambda^2} \Big]$$

$$+ f_g(y) \Big[\frac{\alpha_s}{2\pi} P_{qg}\Big(\frac{x}{y}\Big) \log\Big(\frac{\mu_F^2}{\lambda^2}\Big) \Big] \Big] \Big\}$$

QCD and improved parton model

With this redefinition of the PDFs, both PDFs and partonic cross section are finite and they both acquired dependence on arbitrary factorisation scale

$$\Rightarrow \quad \sigma(p) = \int_0^1 dx\, f_q(x,\mu_F^2)\,\hat{\sigma}_{q,res}(xP,\mu_F^2) + f_g(x,\mu_F^2)\,\hat{\sigma}_{g,res}(xP,\mu_F^2)$$

From PDF redefinition (similar to renormalisation) note that the dependence of PDFs on the scale is totally fixed by perturbation theory.
DGLAP evolution equation, similar to renormalisation group equations for α_S

$$\mu^2 \frac{\partial f_q(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\, P_{qq}\left(\frac{x}{y}\right) f_q(y,\mu^2) + P_{qg}\left(\frac{x}{y}\right) f_g(y,\mu^2)$$

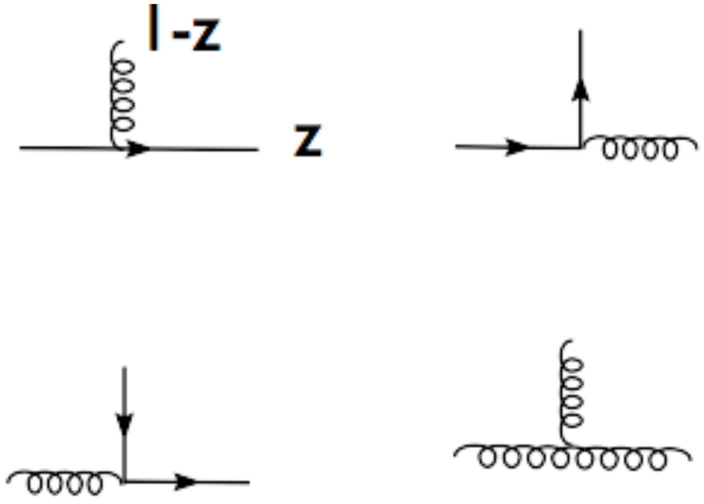
DGLAP evolution equations

When you put all flavours in, get 13 coupled integro-differential equations, which can be reduced to 11 decoupled and 2 coupled equation (with a change of basis in the space of PDFs)

Dokshitzer, Gribov, Lipatov, Altarelli, Parisi equations

$$\frac{d}{dt}\begin{pmatrix} q_i(x,t) \\ g(x,t)\end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \sum_{j=q,\bar{q}}\frac{d\xi}{\xi}\begin{pmatrix} P_{ij}\left(\frac{x}{\xi},\alpha_s(t)\right) & P_{ig}\left(\frac{x}{\xi},\alpha_s(t)\right) \\ P_{gj}\left(\frac{x}{\xi},\alpha_s(t)\right) & P_{gg}\left(\frac{x}{\xi},\alpha_s(t)\right)\end{pmatrix} \otimes \begin{pmatrix} q_j(\xi,t)\\ g(\xi,t)\end{pmatrix}$$

$$t = \log\frac{Q^2}{\mu_F^2}$$



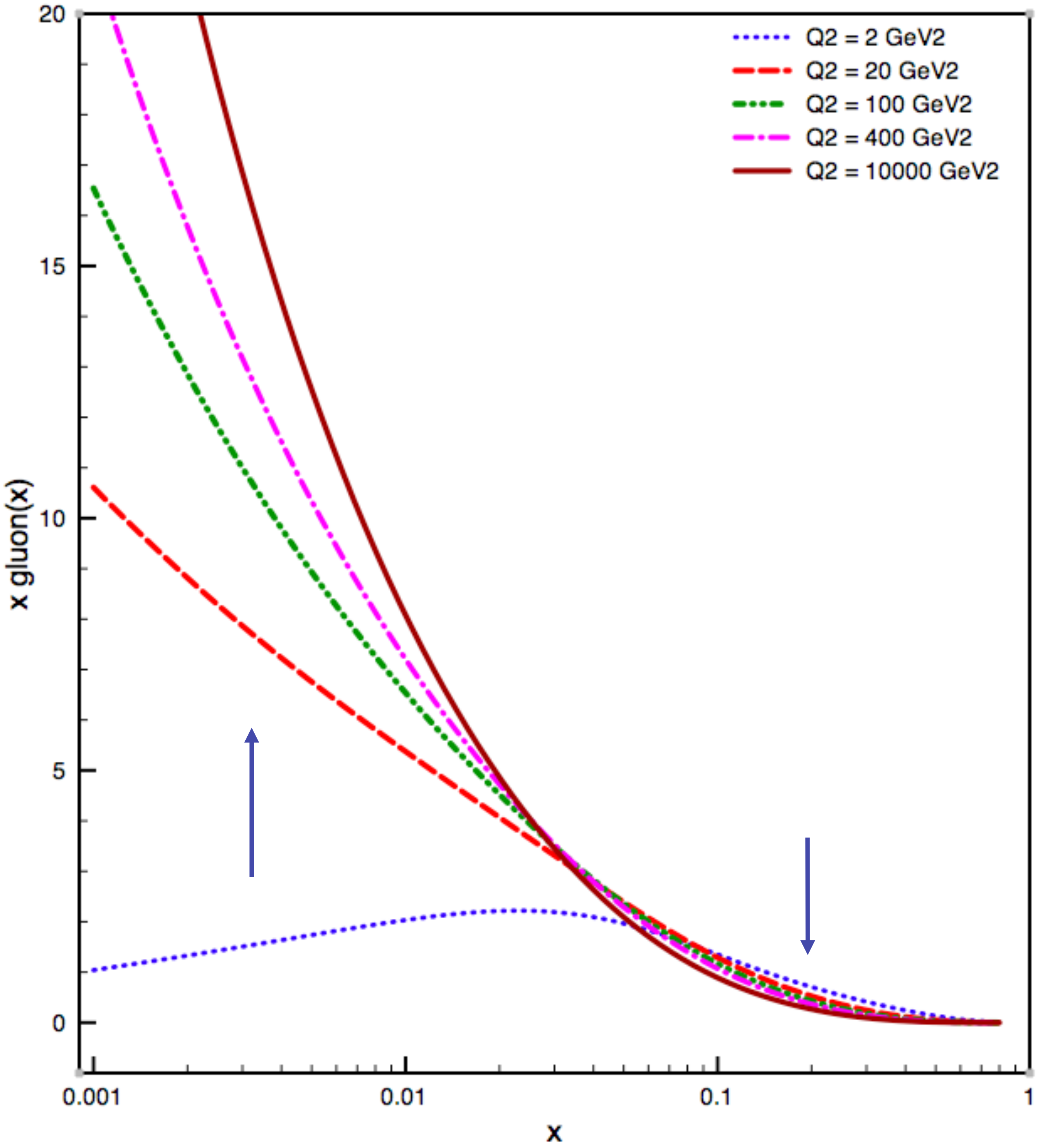
- Splitting functions known up to NNLO:

LO Dokshitzer; Gribov, Lipatov; Altarelli, Parisi (1977)

NLO Floratos,Ross,Sachrajda; Floratos, Lacaze, Kounnas, Gonzalez-Arroyo,Lopez,Yndurain; Curci,Furmanski Petronzio, (1981)

NNLO - Moch, Vermaseren, Vogt, 2004

DGLAP evolution equations



Gluon evolution

$$g(x,\mu^2) = \Gamma_{gq} \otimes \Sigma(x,\mu_0^2) + \Gamma_{gg} \otimes g(x,\mu_0^2)$$

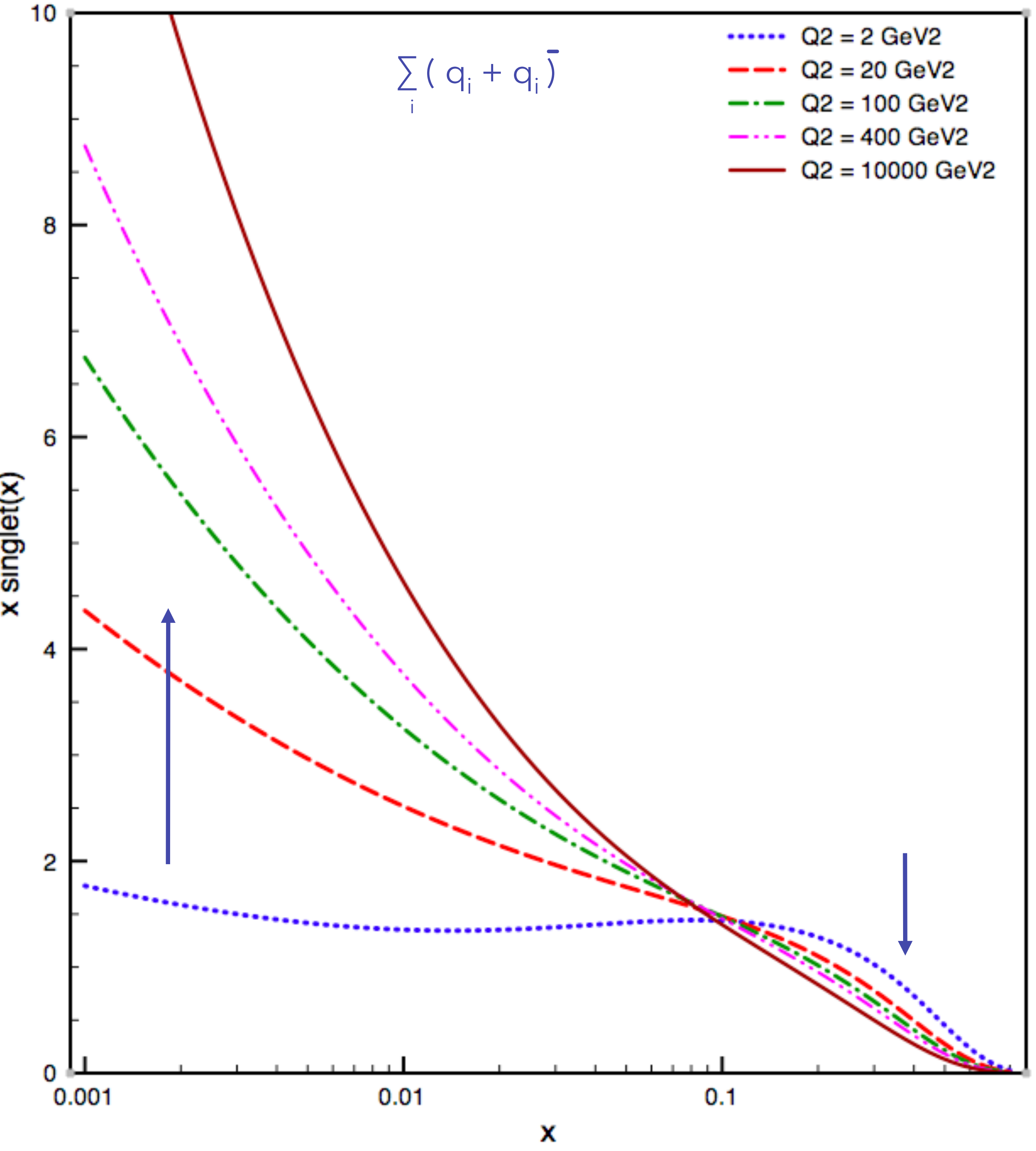
$$P_{gq}^{(0)}(x) = C_F \left[\frac{1+(1-x)^2}{x}\right]$$

$$P_{gg}^{(0)}(x) = 2N \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \delta(1-x)\frac{(11N - 4n_f T_R)}{6}$$

- Both P_{gg} and P_{gq} diverge for x → 0

- Gluon is depleted at large x

DGLAP evolution equations



Singlet evolution

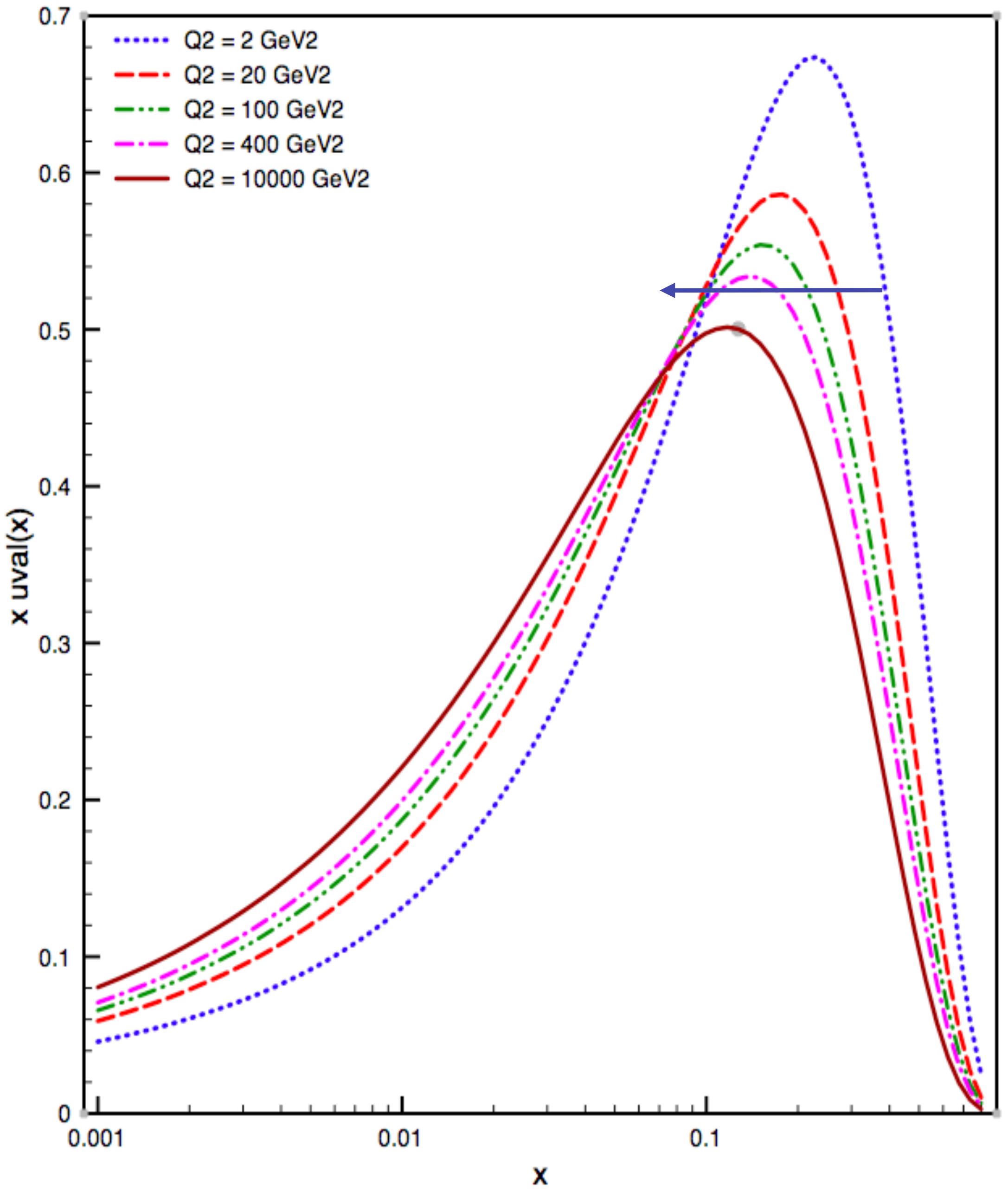
$$\Sigma(x,\mu^2) = \Gamma_{qq} \otimes \Sigma(x,\mu_0^2) + \Gamma_{qg} \otimes g(x,\mu_0^2)$$

$$\begin{aligned} P_{qq}^{(0)}(x) &= C_F\left[\frac{(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right] \\ P_{qg}^{(0)}(x) &= T_R\left[x^2 + (1-x)^2\right] \end{aligned}$$

- High-x gluon feeds growth of small-x gluon and quark

- Gluons can be seen because they help drive the quark evolution

DGLAP evolution equations



Non-singlet valence evolution

$$u_v(x, \mu^2) = \Gamma^v_{NS} \otimes u_v(x, \mu_0^2)$$

$$\boxed{P_{NS}^{(0),v} = P_{qq}^{(0)}(x) = C_F \left[\frac{(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right]}$$

- As Q^2 increases partons lose longitudinal momentum; distributions all shift to lower x

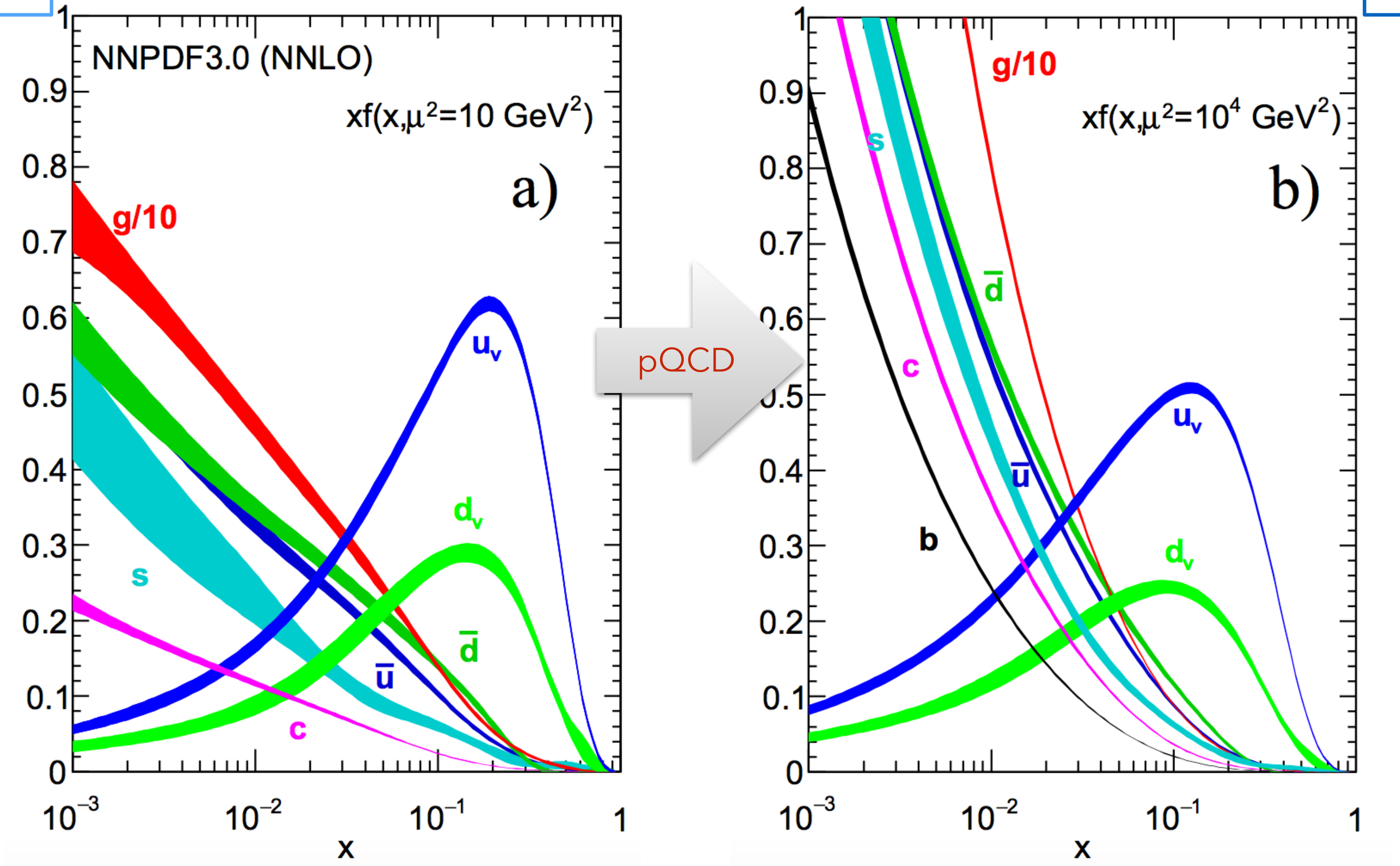
- Gluons can be seen because they help drive the quark evolution

DGLAP evolution equations

Functional dependence of PDFs on the scale is totally predicted up to NNLO accuracy by solving DGLAP evolution equations

Hadronic scale: global fit of PDFs

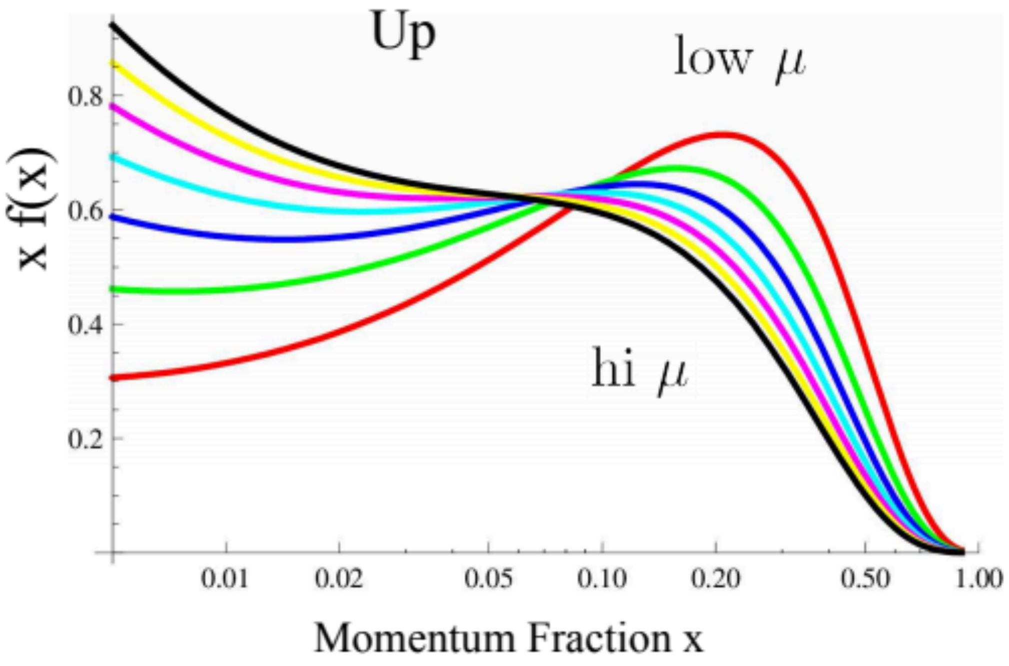
High scale: input to the LHC

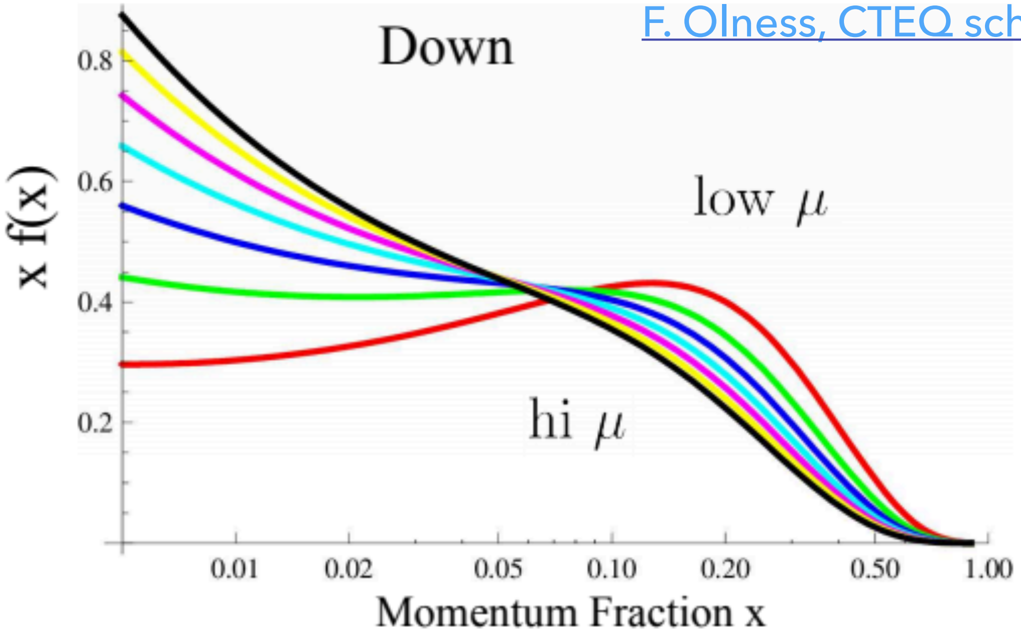


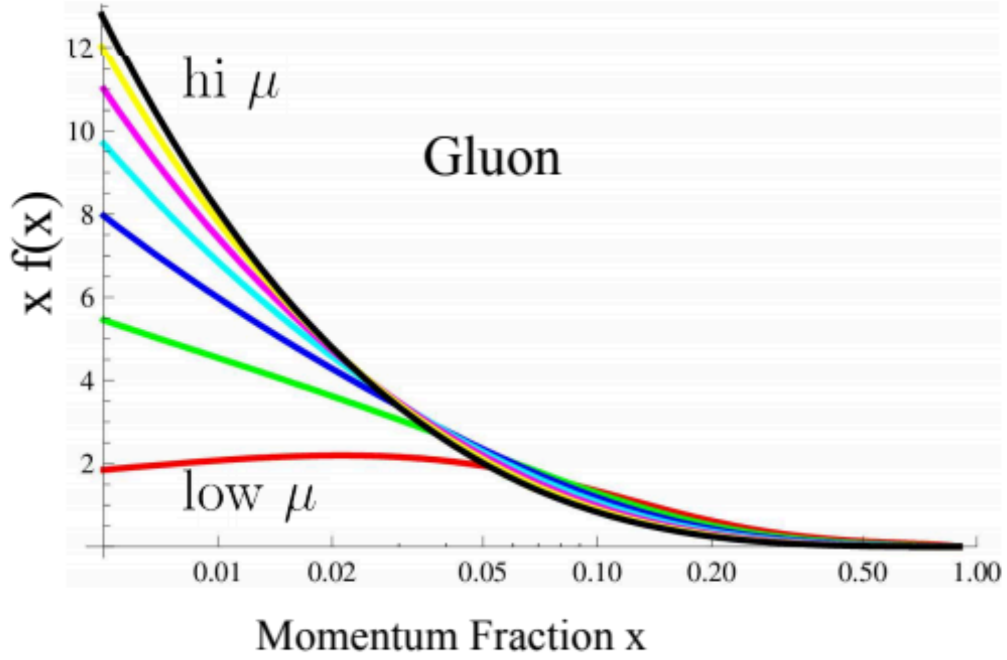
PDG 2016

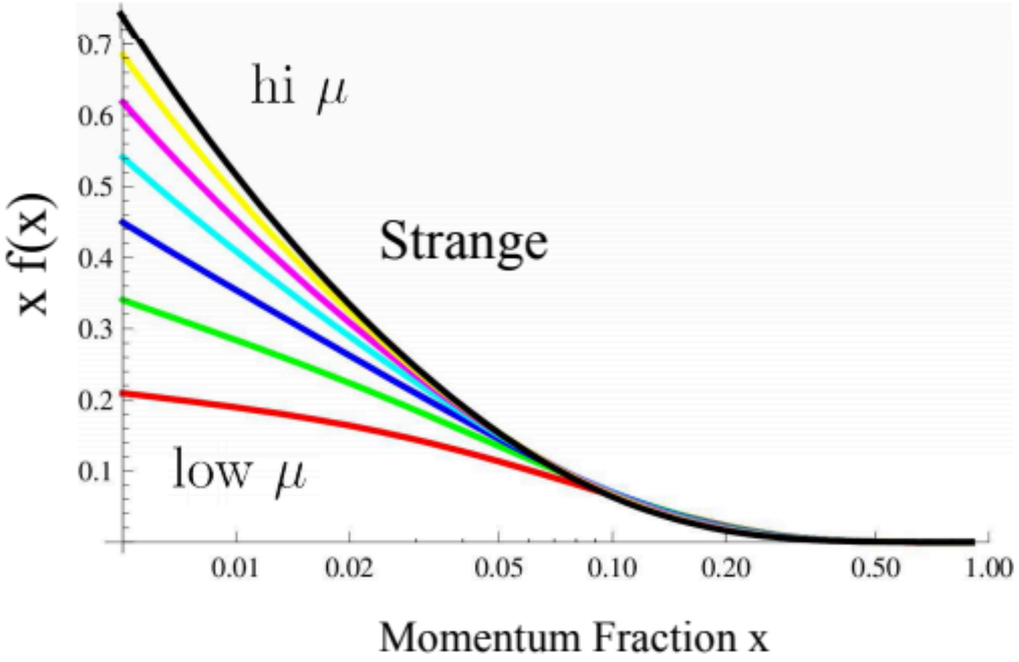
DGLAP evolution equations

F. Olness, CTEQ school 2017







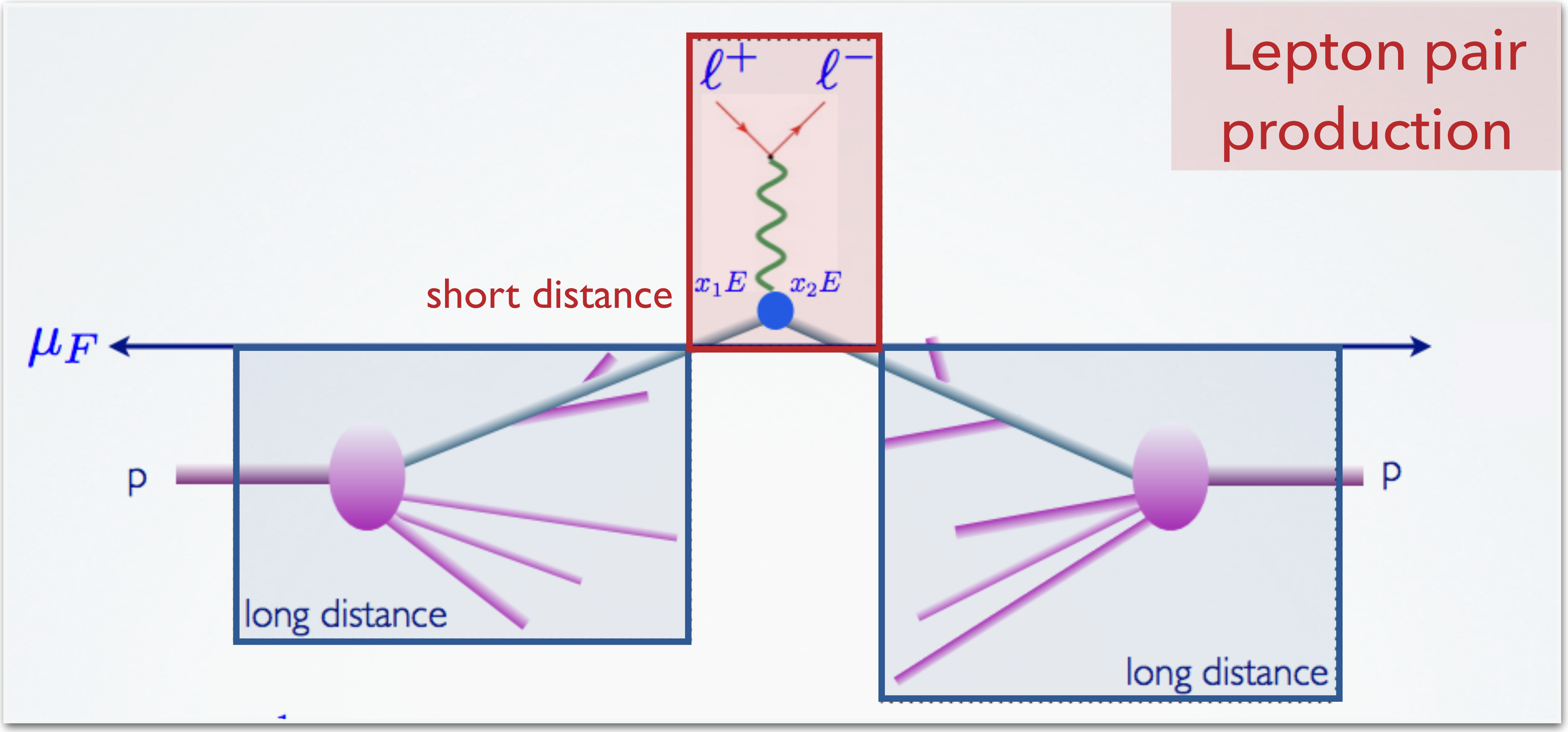


Collinear factorisation

Collinear Factorisation Theorem

$$\frac{d\sigma_H^{ep\to ab}}{dX} = \sum_{i=-n_f}^{+n_f} \int_{x_B}^{1} \frac{dz}{z}\, f_i(z,\mu_F)\, \frac{d\hat{\sigma}_i^{ei}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

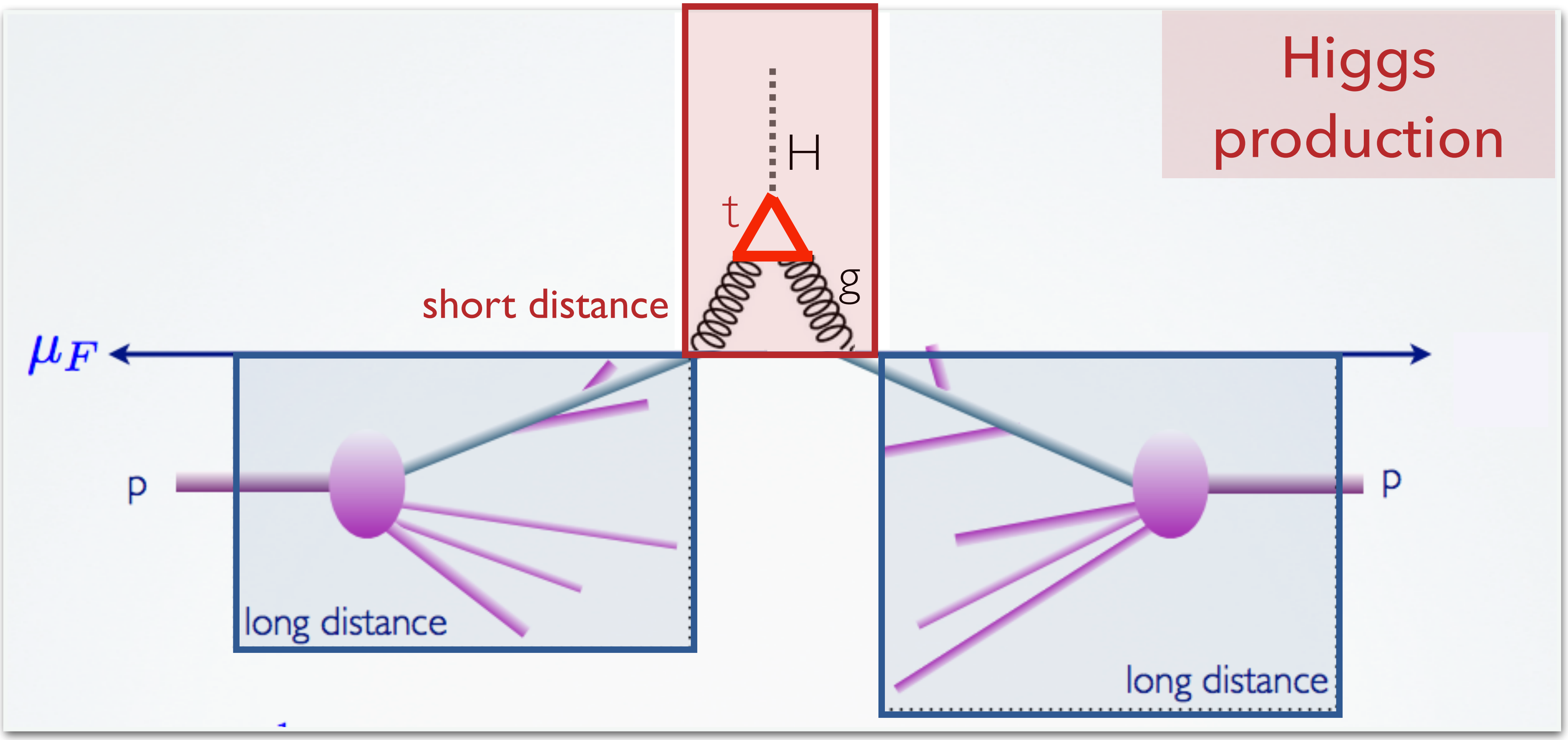
$$\frac{d\sigma_H^{pp\to ab}}{dX} = \sum_{i,j=-n_f}^{+n_f} \int_{\tau_0}^{1} \frac{dz_1}{z_1}\frac{dz_2}{z_2}\, f_i(z_1,\mu_F)\, f_j(z_2,\mu_F)\, \frac{d\hat{\sigma}_i^{ij}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$



Collinear Factorisation Theorem

$$\frac{d\sigma_H^{ep\to ab}}{dX} = \sum_{i=-n_f}^{+n_f} \int_{x_B}^{1} \frac{dz}{z}\, f_i(z,\mu_F)\, \frac{d\hat{\sigma}_i^{ei}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

$$\frac{d\sigma_H^{pp\to ab}}{dX} = \sum_{i,j=-n_f}^{+n_f} \int_{\tau_0}^{1} \frac{dz_1}{z_1}\, \frac{dz_2}{z_2}\, f_i(z_1,\mu_F)\, f_j(z_2,\mu_F)\, \frac{d\hat{\sigma}_i^{ij}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$



Wrap-up

- ➡ The structure of the proton has been a crucial ingredient to test and verify perturbative QCD and it is now key to the precision challenge that we are facing at the LHC

- ➡ Today's lecture
  - ✓ Parametrisation of the proton in terms of structure functions
  - ✓ Parton model picture
  - ✓ QCD - Improved parton model
  - ✓ DGLAP evolution equations
  - ✓ Collinear Factorisation Theorem

Extra material

Deep Inelastic Scattering

Slide from F Olness lectures
CTEQ school 2017

$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times F\left(\frac{Q^2}{\Lambda^2}\right)$$

Λ of order of the proton mass scale

$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times \sum_i e_i^2$$

Partonic cross sections

Slide from Gavin Salam lectures
Quy Nhon Vietnam 2018

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \qquad [\alpha_s \equiv \alpha_s(\sqrt{s_{e^+e^-}})]$$

$$= R_0 \left(1 + 0.32\alpha_s + 0.14\alpha_s^2 - 0.47\alpha_s^3 - 0.59316\alpha_s^4 + \cdots\right)$$

Baikov et al., 1206.1288
(numbers for γ-exchange only)

This is one of the few quantities calculated to N4LO

Good convergence of the series at every order
(at least for $\alpha_s(M_Z) = 0.118$)

Exercise I: Z contribution

- Show that, in the Parton model, considering also the contribution of a virtual Z boson and its interference with the photon one obtains:

$$F_2^{\gamma,Z}(x) = x\sum_{i=1}^{n_f} c_i[q_i(x) + \bar{q}_i(x)]$$

$$F_3^{\gamma,Z}(x) = \sum_{i=1}^{n_f} d_i[q_i(x) - \bar{q}_i(x)]$$

Where

$$c_i = e_i^2 - 2e_i V_{eZ} V_{iZ} P_Z + (V_{eZ}^2 + A_{eZ}^2)(V_{iZ}^2 + A_{iZ}^2)P_Z^2$$

$$d_i = -2e_i A_{eZ} A_{iZ} P_Z + 4V_{eZ} A_{eZ} V_{iZ} A_{iZ} P_Z^2$$

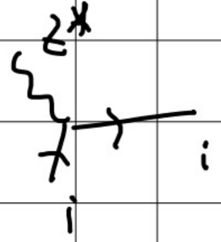
$$P_Z = \frac{Q^2}{(Q^2 + M_Z^2)(4s_w^2 c_w^2)} \longrightarrow \begin{aligned} c_w &= \cos\theta_w \\ s_w &= \sin\theta_w \end{aligned}$$

Solution I

① In the lectures we considered

$\frac{1}{2}\sum_{pol}\Big|$  $\Big|^2 = \frac{e_q^2}{2}\,\mathrm{Tr}\left[\hat{\not p}\,\gamma^\mu\,\hat{\not p}'\,\gamma^\mu\right]$

Add to hadronic tensor

$\frac{1}{2}\sum_{pol}\Big|$  $\Big|^2 = \frac{e^2 k_z^2}{2}\,\mathrm{Tr}\left[\hat{\not p}\,\gamma^\nu(v_{iz}+\gamma^5 a_{iz})\,\hat{\not p}'\gamma^\mu(v_{iz}+\gamma_5 a_{iz})\right]$

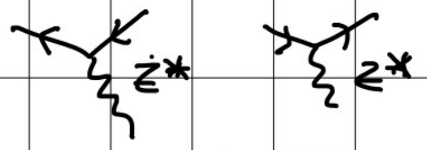
$$= \frac{e^2}{8 s_W^2 c_W^2}\left[(v_{iz}^2+a_{iz}^2)\,\mathrm{Tr}\left[\hat{\not p}\,\gamma^\nu\,\hat{\not p}'\,\gamma^\mu\right] + 2 v_{iz} a_{iz}\,\mathrm{Tr}\left[\hat{\not p}\,\gamma^\nu\,\hat{\not p}'\gamma^\mu\gamma^5\right]\right]$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$4\left[\hat{p}^\nu\hat{p}'^\mu + \hat{p}^\mu\hat{p}'^\nu - g^{\mu\nu}\,\hat{p}\cdot\hat{p}'\right] \qquad \underbrace{4i\epsilon^{\alpha\beta\nu\mu}\,\hat{p}_\alpha\hat{p}'_\beta}_{}$$

≠0 when contracted with anti-symm tensor

Similar contribution in leptonic tensor →



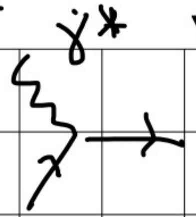
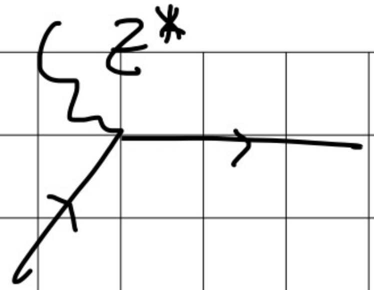
have different sign in front of anti symm. part

$v_{iz} \to v_{ez}$

$a_{iz} \to a_{ez}$

and propagator $\overbrace{\frac{Q^4}{(Q^2+M_Z^2)^2}}^{P_Z^2}\cdot\frac{1}{Q^4}$

Solution I

$\frac{1}{2} \sum_i \Bigg($  $\Bigg)^*$ $\Bigg($  $\Bigg)$

$$= -\frac{1}{2} e^2 k_\gamma k_Z \, \mathrm{Tr}\left[\hat{\not{p}} \gamma^\nu (v_{iZ} + a_{iZ} \gamma^5) \hat{\not{p}}' \gamma^\mu \right]$$

$$\simeq - \frac{e^2 e_J}{4 s_W c_W} \left[v_{iZ} \, \mathrm{Tr}\left[\hat{\not{p}} \gamma^\nu \hat{\not{p}}' \gamma^\mu \right] + a_{iZ} \, \mathrm{Tr}\left[\hat{\not{p}} \gamma^\nu \hat{\not{p}}' \gamma^\mu \gamma^5 \right] \right]$$

Similar contribution in leptonic tensor

$v_{iZ} \rightarrow v_{eZ}$

$a_{iZ} \rightarrow a_{eZ}$

and propagator

$$\frac{Q^{\cancel{4}\,2}}{\underbrace{(Q^2+M_Z^2)\,\cancel{Q^2}}_{P_Z}} \qquad \frac{1}{Q^4}$$

Exercise II: Paschos-Wolfenstein relation

- Show that, in the Parton model, considering a (anti)neutrino-initiated DIS process on a deuteron target – assuming SU(2) isospin symmetry $u_n(x)=d_p(x)$ and $d_n(x) = u_p(x)$ – the ratio R

$$R = \frac{\sigma_{\mathrm{NC}}(\nu) - \sigma_{\mathrm{NC}}(\bar{\nu})}{\sigma_{\mathrm{CC}}(\nu) - \sigma_{\mathrm{CC}}(\bar{\nu})}$$

NC (mediated by Z) and CC (mediated by $W^{+/-}$), assuming strange and anti-strange to be equal in the target, is independent of Parton Distribution Functions and can be used to determined the Weinberg angle θ_W

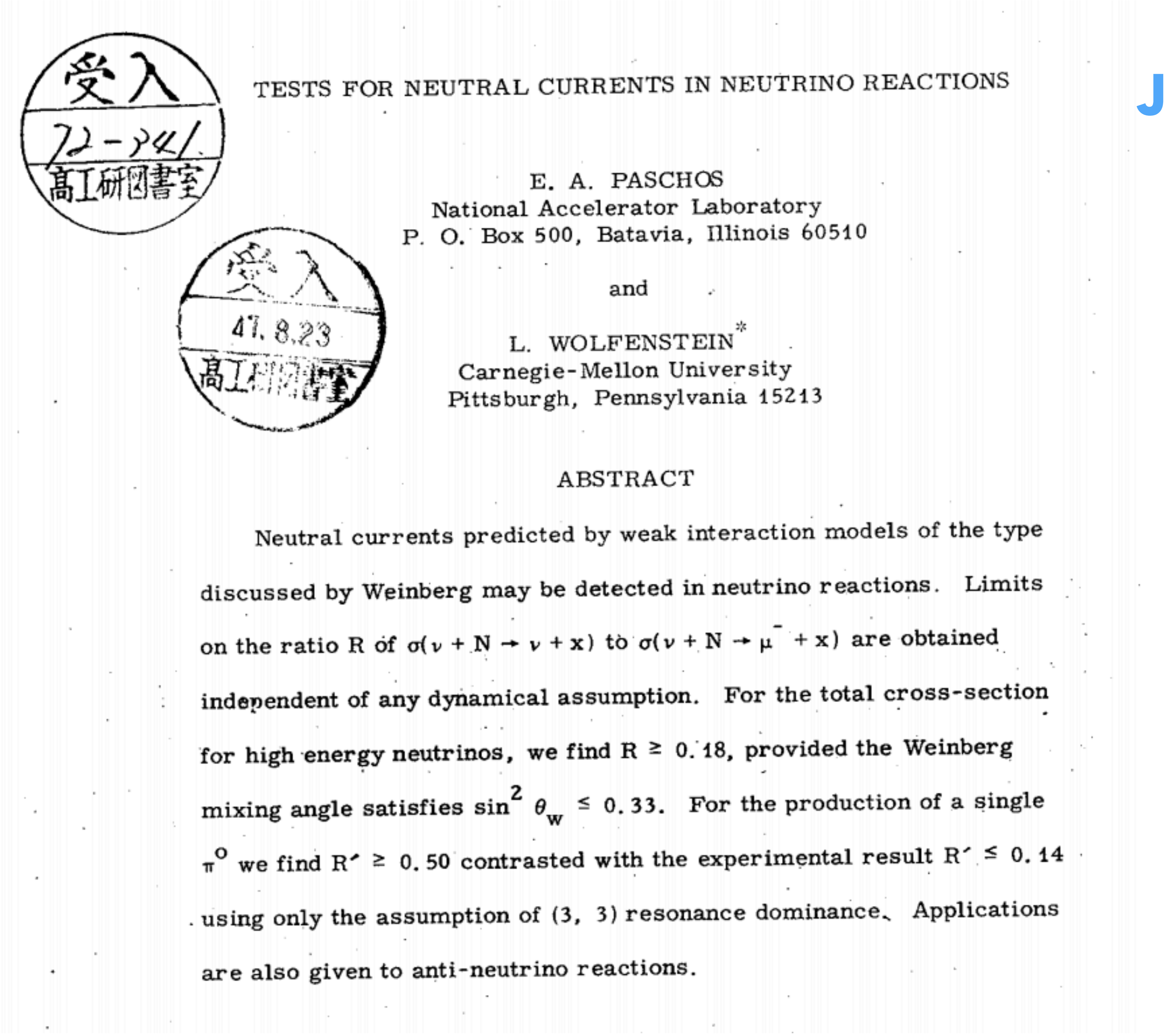
$$R = \frac{1}{2}\left(\frac{1}{2} - \sin\theta_w^2\right)$$

You may use (without deriving it) the result (and set c, cbar = 0)

$$\begin{aligned}
F_2^{W^-} &= 2x\left(u + \bar{d} + \bar{s} + c\right), \\
F_3^{W^-} &= 2x\left(u - \bar{d} - \bar{s} + c\right), \\
F_2^{W^+} &= 2x\left(d + \bar{u} + \bar{c} + s\right), \\
F_3^{W^+} &= 2x\left(d - \bar{u} - \bar{c} + s\right),
\end{aligned}$$

Exercise II: Paschos-Wolfenstein relation

July 1972



TESTS FOR NEUTRAL CURRENTS IN NEUTRINO REACTIONS

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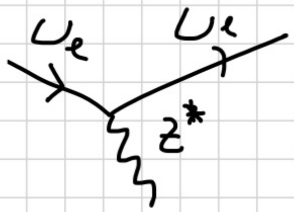
ABSTRACT

Neutral currents predicted by weak interaction models of the type discussed by Weinberg may be detected in neutrino reactions. Limits on the ratio R of $\sigma(\nu + N \rightarrow \nu + x)$ to $\sigma(\nu + N \rightarrow \mu^- + x)$ are obtained independent of any dynamical assumption. For the total cross-section for high energy neutrinos, we find $R \geq 0.18$, provided the Weinberg mixing angle satisfies $\sin^2 \theta_W \leq 0.33$. For the production of a single π^0 we find $R' \geq 0.50$ contrasted with the experimental result $R' \leq 0.14$ using only the assumption of (3, 3) resonance dominance. Applications are also given to anti-neutrino reactions.

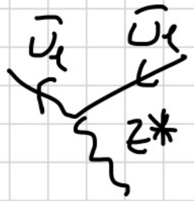
Solution II

II



$$= \bar{u}'(k') \frac{-ig}{4c_W} \gamma^\mu (1-\gamma_5) u(k)$$

$$L_{\mu\nu}(\nu) = \frac{1}{8c_W^2}\left(\mathrm{Tr}[\not k \gamma^\nu \not k' \gamma^\mu] - \mathrm{Tr}[\not k \gamma^\nu \not k' \gamma^\mu \gamma^5]\right)$$



$$= \bar{v}(k) \frac{-ig}{4c_W} \gamma^\mu (1-\gamma_5) v(k')$$

$$L_{\mu\nu}(\bar\nu) = \frac{1}{8c_W^2}\left(\mathrm{Tr}[\not k' \gamma^\nu \not k \gamma^\mu] - \mathrm{Tr}[\not k' \gamma^\nu \not k \gamma^\mu \gamma^5)\right)$$

Because

only difference in $\nu, \bar\nu$ incoming lepton

$$d\sigma = \frac{1}{2s} \frac{g_{\nu\nu}^2 \, g_{h\nu}^2}{(Q^2+M_V^2)^2} L_{\mu\nu} W^{\mu\nu} (4\pi) \frac{d^3k'}{(2\pi)^3 2E'}$$

↳ M_V^4 for $Q^2 \ll M_V^2$

$$d\sigma(\nu) - d\sigma(\bar\nu) \propto \Big[\underbrace{(\nu) - L_{\mu\nu}(\bar\nu)}\Big] \circ \frac{g_{\nu e}^2 \, g_{h\nu}^2}{M_V^4} W^{\mu\nu} (4\pi) \frac{d^3k'}{(2\pi)^3 2E'}$$

only antisymm. i.e. $\frac{i}{c_W^2} k_\alpha k'_\beta \, \epsilon^{\alpha\beta\mu\nu}$

Only non-zero contribution when contracted with asymm $W_{\mu\nu}$

Solution II

$$[L_{\mu\nu}(\nu) - L_{\mu\nu}(\bar{\nu})]\, W^{\mu\nu} = -\frac{1}{2(\tilde{p}\cdot q)c_W^2}\, k_\alpha k'_\beta\, \varepsilon^{\alpha\beta\mu\nu}\, \varepsilon_{\rho\sigma\mu\nu}\, \tilde{p}^\rho q^\sigma F_3(x)$$

$$= \frac{ME}{c_W^2}\,(2-y)\, x F_3^Z(x)$$

$$\Rightarrow \frac{d^2\sigma}{dx\,dy} = \frac{G_F^2 c_W^2 M_n E_\nu}{2\pi^2}\, x y\,(2-y)\, F_3^{NC}(x)$$

$$\int dy\,dx\,.. = \frac{G_F^2 c_W^2}{3\pi^2}\, E_\nu M_n \int dx\; x\, F_3(x)$$

with $F_3(x) \propto \frac{1}{2}\left(\frac{1}{2} - \frac{4}{3}s_W^2\right)(u - \bar{u} + c - \bar{c}) + \frac{1}{2}\left(\frac{2}{3}s_W^2 - \frac{1}{2}\right)(d - \bar{d} + s - \bar{s})$

using $u^P = d^N$, $d^P = u^N$ $\Rightarrow$ $u^P + d^P = u^N + d^N$ $\Rightarrow$ $u = d$, $\bar{u} = \bar{d}$

$s = \bar{s}$

$c = \bar{c} \cong 0$

$\rightarrow F_3^{NC}(x) \propto \left(\frac{1}{2} - s_W^2\right)[u(x) - \bar{u}(x)]$

Solution II

Charged current

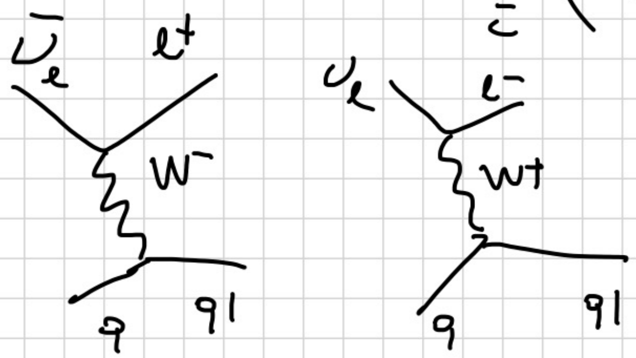
$$\underset{k}{\bar{u}}(p')\left[-\frac{ig}{2\sqrt{2}}\,\gamma^\mu(1-\gamma^5)\;U_{kj}\right]u_j(p$$

$\downarrow$

CKM Matrix (consider only 2 flavor)

$$U_{kj}=\begin{array}{c|cc} & d & s \\ \bar{u} & \cos\theta_c & \sin\theta_c \\ \bar{c} & -\sin\theta_c & \cos\theta_c \end{array} \overset{}{=} \begin{pmatrix} 1+\theta_c^2 & \theta_c \\ -\theta_c & 1+\theta_c^2 \end{pmatrix}$$

$\downarrow$ expand in θ_c and keep only $O(1)$



$$\Rightarrow F_3^{W^+} = 2x\,(d-\bar{u}+s-\bar{c})$$

$$F_3^{W^-} = 2x\,(u-\bar{d}-\bar{s}+c)$$

Analogously to NC

$$\sigma_{cc}(\nu)-\sigma_{cc}(\bar{\nu}) = \frac{2G_F^2}{3\pi^2}E_\nu M_N\int_0^1 x\,F_3^{cc}(x)\,dx$$

$$F_3^{cc}(x) = \frac{1}{2}(u+d+s+c-\bar{u}-\bar{d}-\bar{s}-\bar{c}) \longrightarrow (u-\bar{u})$$ same hypothesis

$$= \frac{2G_F^2}{3\pi^2}E_\nu M_N\int dx\,(u(x)-\bar{u}(x))$$

Solution II

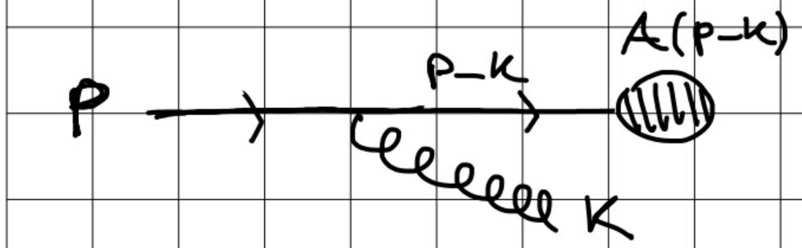
Taking ratio

$$R = \frac{\frac{1}{2}(1 - 2 s^2_w) \int_0^1 dx \left[u(x) - \bar{u}(x) \right] dx}{2 \int_0^1 dx \left[u(x) - \bar{u}(x) \right]}$$

$$= \frac{1}{2} \left(\frac{1}{2} - \sin^2 \theta_W \right)$$

QCD and improved parton model

REAL EMISSION



Sudakov parametrization

$$k = (1-z)\,p + k_T + \xi\,\eta$$

where η such that

$$p\cdot k_T = 0$$
$$\eta\cdot k_T = 0$$
$$\eta^2 = 0$$
$$2p\cdot\eta = 1$$

For example

$$p = p^0(1,0,0,1)$$
$$\eta = \frac{1}{4p^0}(1,0,0,-1)$$
$$k_T = (0, k_T^1, k_T^2, 0)$$

From $k^2=0$ (on-shell condition)

$$\Rightarrow\ k_T^2 + 2p\cdot\eta\,\xi(1-z) = 0 \quad \Rightarrow\ \xi = -\frac{k_T^2}{1-z}$$

From $(p-k)^2 < 0$

$$\Rightarrow\ z < 1$$

From $k_3 = 0$ $\Rightarrow$ $|k_T^2| < 4p_0^2(1-z)$

Detailed version 1/7

QCD and improved parton model

$$\frac{d^3k}{(2\pi)^3\,2k^0} = \frac{1}{16\pi^2}\,\frac{d|k_T^2|\,dz}{(1-z)}$$

(integrating over $d\varphi$)

Singular part of amplitude:

$$\mathcal{M}^{(1)}_{q,\text{sing}}(p,k) = g_s\,A_i(p-k)\,\frac{\overbrace{\not p-\not k}^{\to\, z\not p-\not k_T-\cancelto{O(k_T^2)}{\xi\not n}}}{\underbrace{(p-k)^2}_{\to\,\frac{1-z}{k_T^2}}}\,\not\epsilon(k)\,\underbrace{\boxed{t^A_{ij}}}_{\to\ \text{SU(3) gen fund. representation}}\,u_j(p)$$

$$= g_s\,\frac{(1-z)}{k_T^2}\,A_i(p-k)\,(z\not p-\not k_T)\,\not\epsilon(k)\,t^A_{ij}\,u_j(p)$$

But $\not p\,u(p)=0$

$k_T\cdot\epsilon(k)=0$

$$\Rightarrow\ (1-z)\,\not p\,\not\epsilon(k)\,u(p) = -\,2\epsilon_\mu(k)\,k_T^\mu\,u(p)$$

$$\Rightarrow\ \mathcal{M}^{(1)}_{q,\text{sing}}(p,k) = -\,\frac{g_s}{k_T^2}\,A_i(p-k)\,\left[2z\,k_T\,\epsilon(k) + (1-z)\,\not k_T\,\not\epsilon(k)\right]t^A_{ij}\,u_j(p)$$

Detailed version 2/7

QCD and improved parton model

$$|M^{(1)}_{qg^*q}|^2 = -\frac{2 g_s^2 C_F}{k_T^2}\;\frac{(1+z)^2}{z}\;|M^{(0)}(zp)|^2$$

where we have used $\sum \epsilon_\mu(k)\,\epsilon^*_\nu(k) = -g^T_{\mu\nu} = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{k\cdot n}$

$$t^A_{ij}\,t^A_{jk} = \delta_{ik}\,C_F \qquad C_F = \frac{N_c^2-1}{2N_c} = \frac{4}{3}$$

$$\Rightarrow\ \hat{\sigma}^{(1)}_q(p) = \frac{1}{16\pi^2}\int_0^1 \frac{dz}{1-z}\int_0^{|k_T^2|_{max}} d|k_T^2|\;\frac{1}{p\cdot p'}\;|M^{(1)}_q(p,k)|^2$$

$$\alpha_s = \frac{g_s^2}{4\pi} \qquad = \frac{\alpha_s C_F}{2\pi}\int_0^1 \frac{dz}{1-z}\int_0^{|k_T^2|_{max}} d|k_T^2|\;\frac{1+z^2}{|k_T^2|}\;\frac{1}{z(p\cdot p')}\;|M^{(0)}_q(zp)|^2 + \text{regular terms}$$

$$= \frac{\alpha_s C_F}{2\pi}\int_0^1 \frac{dz}{1-z}\int_0^{|k_T^2|_{max}} d|k_T^2|\;\frac{1+z^2}{|k_T^2|}\;\hat{\sigma}^{(0)}_q(zp)$$

divergences! SOFT $z \to 1$ Regulator $\varepsilon \to 0$

COLLINEAR $|k_T^2| \to 0$ regulator $\lambda \to 0$

Detailed version 3/7

QCD and improved parton model

$$\hat{\sigma}_q^{(1)}(p) = \frac{\alpha_s}{2\pi} C_F \int_0^{1-\varepsilon} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} (1+z^2)\, \hat{\sigma}_q^{(0)}(zp)$$

Adding virtual corrections $$-\hat{\sigma}_q^{(0)}(p) \frac{\alpha_s C_F}{2\pi} \int_0^{1-\varepsilon} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} (1+z^2)$$

the soft singularity cancels

$\Rightarrow \varepsilon \to 0$

$$\hat{\sigma}_q^{(1)}(p) = \frac{\alpha_s}{2\pi} C_F \int_0^{1} \frac{dz}{1-z} \int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} (1+z^2) \left[\hat{\sigma}_q^{(0)}(zp) - \hat{\sigma}_q^{(0)}(p) \right]$$

Still left with COLLINEAR divergence!

Introduce $\boxed{\mu_F}$ to split integration

$$\int_{\lambda^2}^{|k_T^2|_{max}} \frac{d|k_T^2|}{|k_T^2|} \to \underbrace{\int_{\lambda^2}^{\mu_F^2} \frac{d|k_T^2|}{|k_T^2|}}_{\text{singular}} + \underbrace{\int_{\mu_F^2}^{|k_T^2|_{MAX}} \frac{d|k_T^2|}{|k_T^2|}}_{\text{finite}}$$

Detailed version 4/7

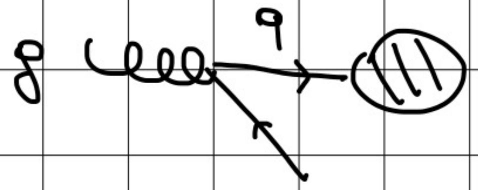
QCD and improved parton model

$$\Rightarrow \quad \hat{\sigma}_q(p) = \hat{\sigma}_q^{(0)}(p) + \hat{\sigma}_q^{(1)}(p)$$

universal function $P(q \to q)$

$$= \hat{\sigma}_q^{(0)}(p) + \frac{\alpha_s}{2\pi}\int_0^1 dz \; \overset{\nearrow}{P_{qq}(z)} \, \hat{\sigma}_q^{(0)}(zp) \log\frac{\mu_F^2}{\lambda^2} + \hat{\sigma}^{(1)}_{q,\mathrm{reg}}(p,\mu_F^2)$$

Of course quark can come from gluon



Doing the whole calculation, we get

$$\hat{\sigma}_g(p) = \hat{\sigma}_g^{(1)}(p)$$

$$= \frac{\alpha_s}{2\pi}\int_0^1 dz \; P_{qg}(z) \, \hat{\sigma}_q^{(0)}(zp) \log\frac{\mu_F^2}{\lambda^2} + \hat{\sigma}^{(1)}_{g,\mathrm{reg}}(p,\mu_F^2)$$

↳ In the parton model formula

$$\sigma(p) = \int_0^1 dy \left[f_q(y)\, \hat{\sigma}_q(yp) + f_g(y)\, \hat{\sigma}_g(yp) \right]$$

QCD and improved parton model

$$\sigma(p) = \int_0^1 dy \, \Big[f_q(y) \, \hat{\sigma}_q^{(0)}(yp)$$

$$+ \frac{\alpha_s}{2\pi} \int_0^1 dy \, f_q(y) \int_0^1 dz \, \hat{\sigma}_q^{(0)}(yz\,p) \, P_{qq}(z) \log \frac{\mu_F^2}{\lambda^2}$$

$$+ \frac{\alpha_s}{2\pi} \int_0^1 dy \, f_g(y) \int_0^1 dz \, \hat{\sigma}_g^{(0)}(yzp) \, P_{qg}(z) \log \frac{\mu_F^2}{\lambda^2}$$

$$+ \int_0^1 dy \, f_q(y) \, \hat{\sigma}^{(1)}_{q,reg}(yp, \mu_F^2) + \int_0^1 dy \, f_g(y) \, \hat{\sigma}^{(1)}_{g,reg}(yp, \mu_F^2)$$

terms $\propto \log \frac{\mu_F^2}{\lambda^2}$ can be reabsorbed into redefinition of f_q

$x = yz$

$$f_q(x, \mu_F^2) = \int_x^1 \frac{dy}{y} \Big\{ f_q(y) \Big[\delta\Big(1 - \frac{x}{y}\Big) + \frac{\alpha_s}{2\pi} P_{qq}\Big(\frac{x}{y}\Big) \log \frac{\mu_F^2}{\lambda^2} \Big]$$

$$+ f_g(y) \Big[\frac{\alpha_s}{2\pi} P_{qg}\Big(\frac{x}{y}\Big) \log \frac{\mu^2}{\lambda^2} \Big] \Big\}$$

Detailed version 6/7

QCD and improved parton model

So that

$$\sigma(P) = \int_0^1 dx \; f_q(x, \mu_F^2)\, \hat{\sigma}_q(xP, \mu_F^2) + f_g(x)\, \hat{\sigma}_g(xP, \mu_F^2)$$

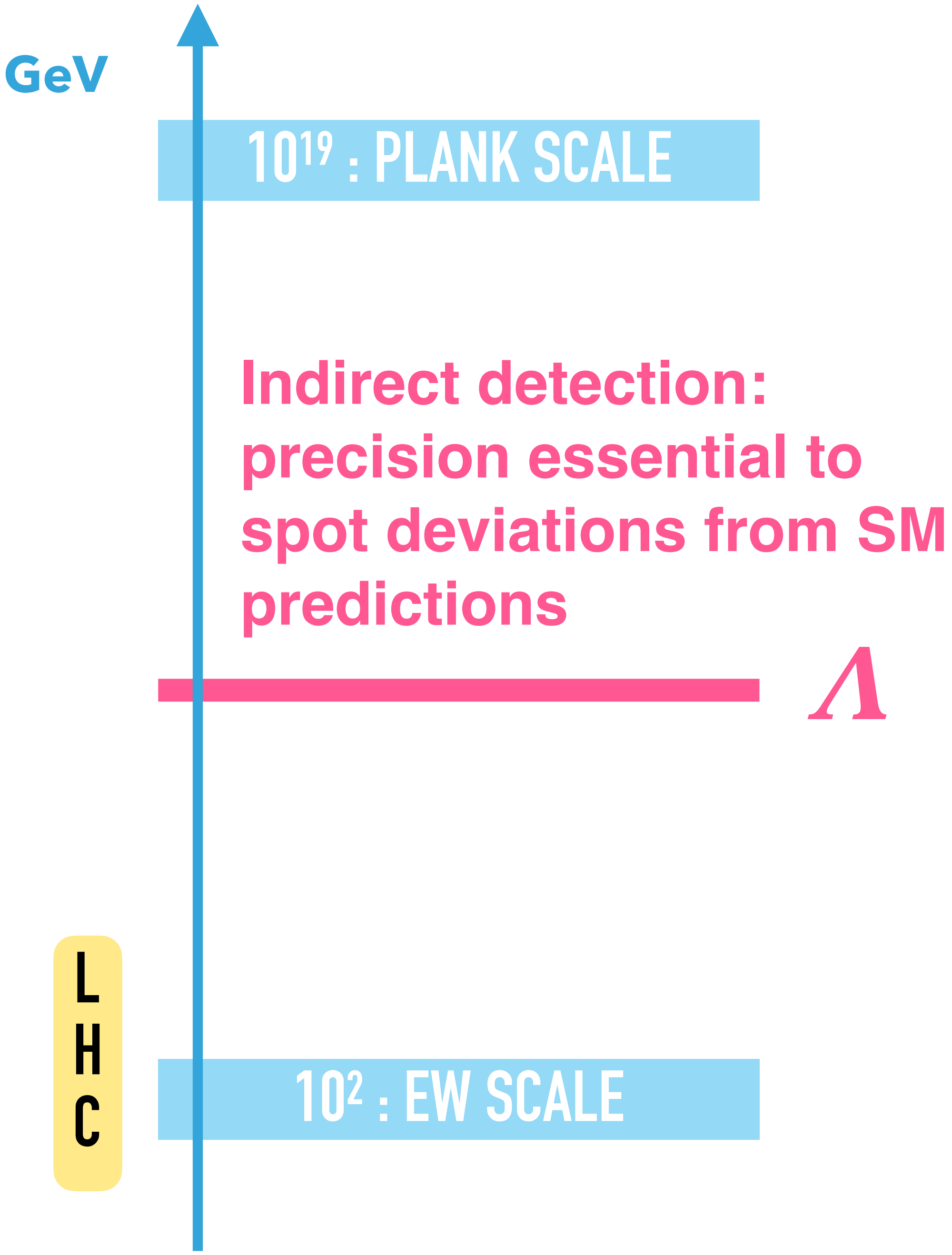
both depend on arbitrary FACTORIZATION scale

Note that however the dependence of $f_{q,g}(x, \mu_F^2)$ is totally ~~[illegible]~~ fixed by perturbation theory

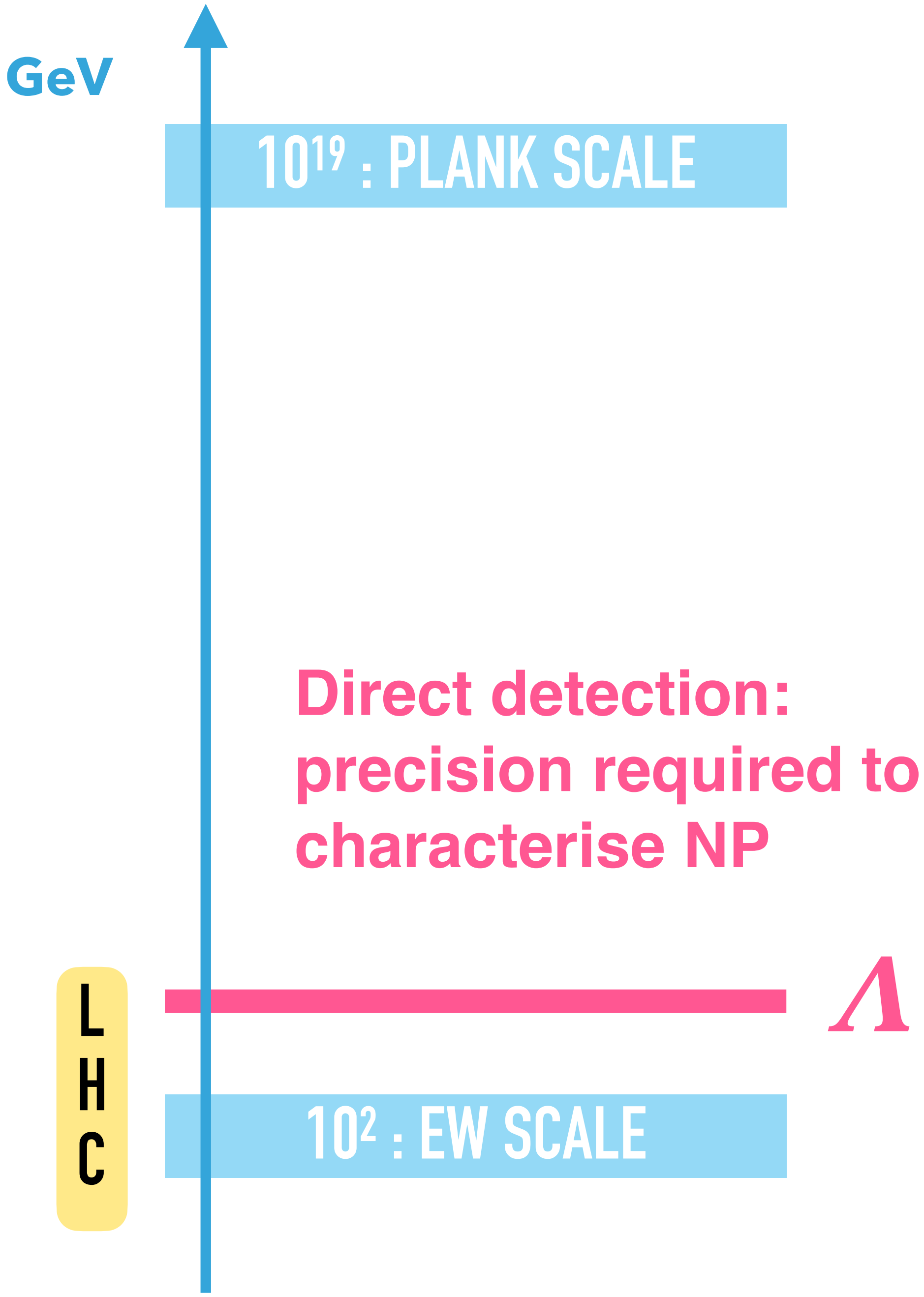
$$\mu^2 \frac{\partial f_q}{\partial \mu^2}(x, \mu^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{qq}\left(\frac{x}{y}\right) f_q(y, \mu^2) + P_{qg}\left(\frac{x}{y}\right) f_g(y, \mu^2) \right]$$

Detailed version 7/7

A precision era in particle physics



Indirect detection: precision essential to spot deviations from SM predictions



Direct detection: precision required to characterise NP

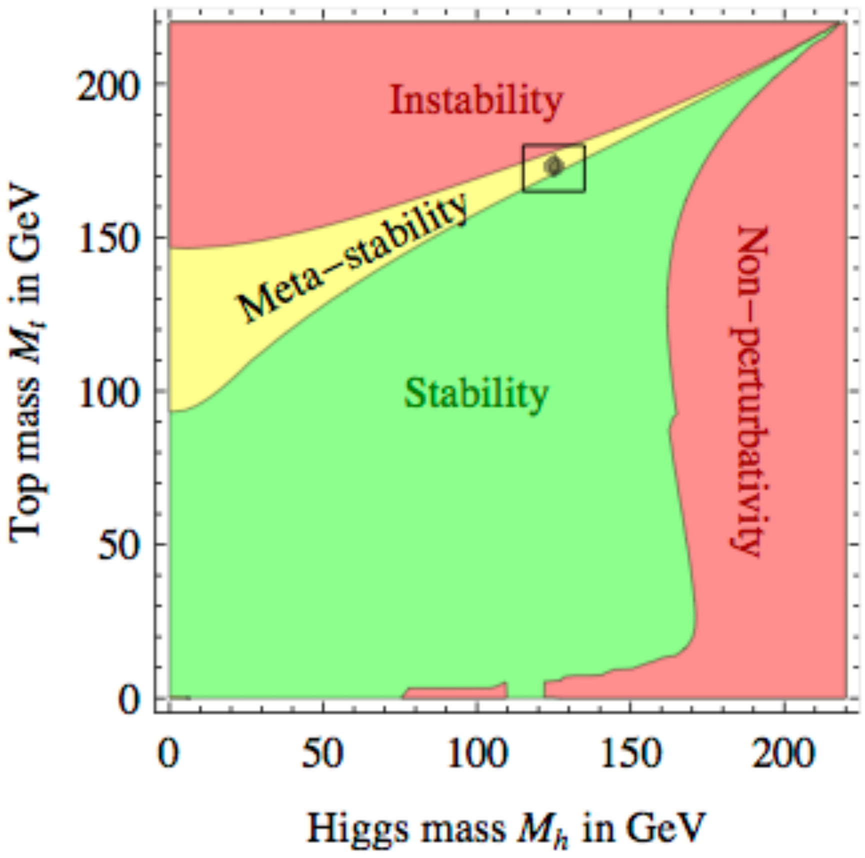
Some compelling questions

Higgs Boson

Hierarchy problem:
Huge gap between EW scale (10^2 GeV)
and Gravitational scale (10^{19} GeV)

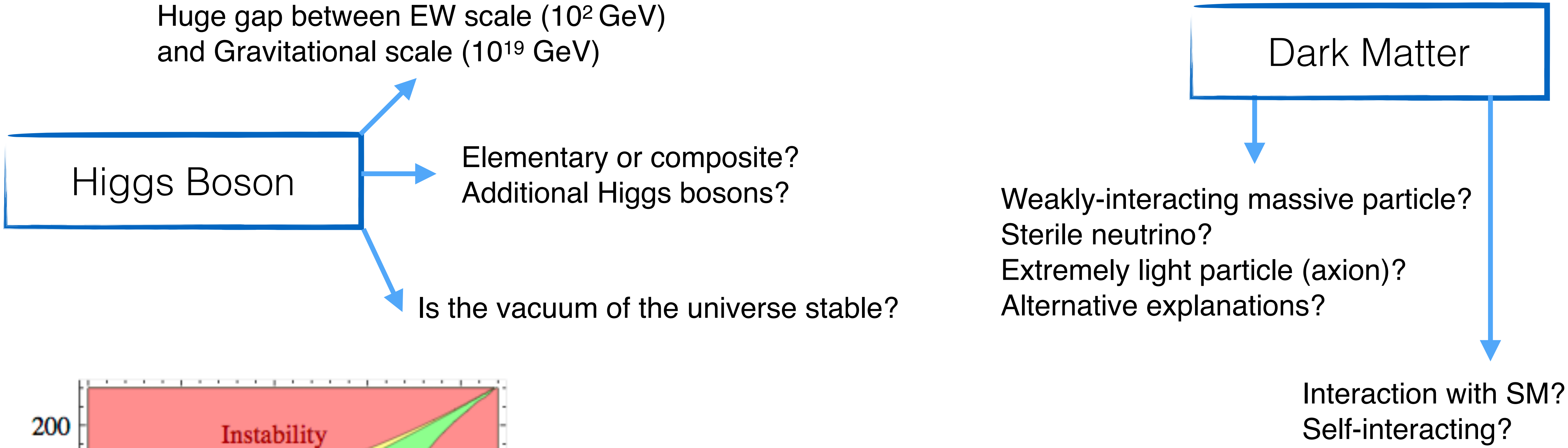
Elementary or composite?
Additional Higgs bosons?

Is the vacuum of the universe stable?



Degrassi et al, 2012

Some compelling questions



Higgs Boson

Hierarchy problem:
Huge gap between EW scale (10^2 GeV)
and Gravitational scale (10^{19} GeV)

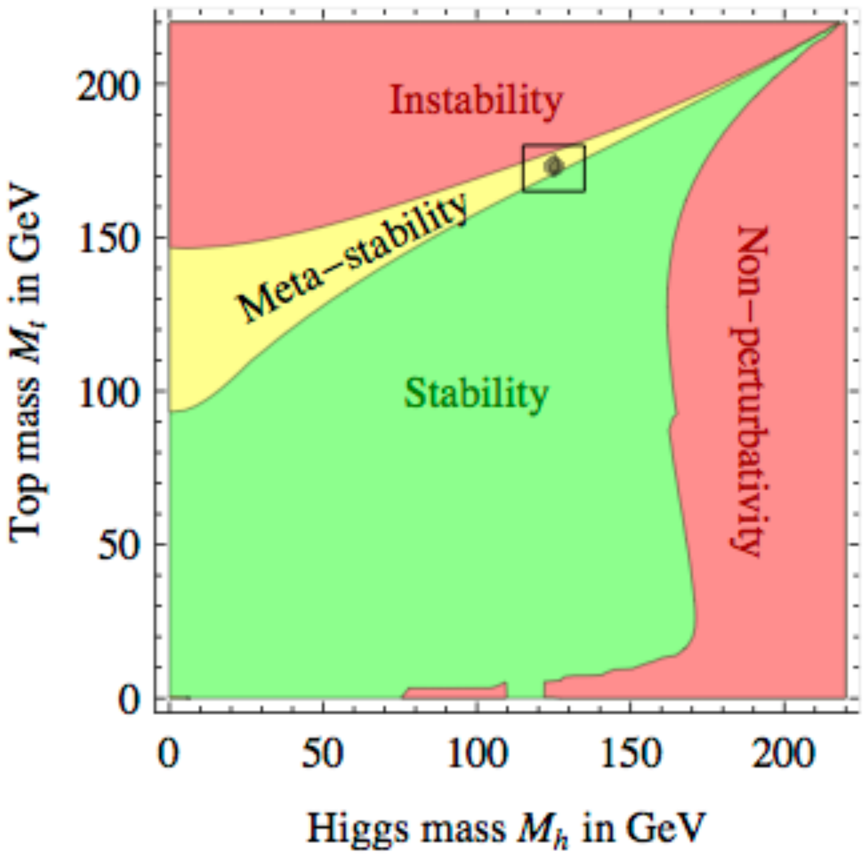
Elementary or composite?
Additional Higgs bosons?

Is the vacuum of the universe stable?

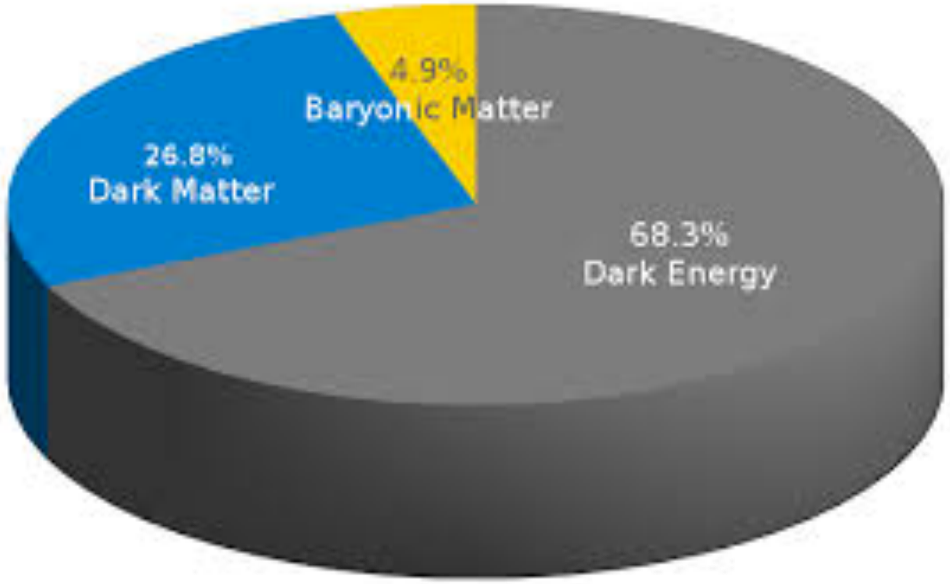
Dark Matter

Weakly-interacting massive particle?
Sterile neutrino?
Extremely light particle (axion)?
Alternative explanations?

Interaction with SM?
Self-interacting?



Degrassi et al, 2012



Some compelling questions

Higgs Boson

Hierarchy problem:
Huge gap between EW scale (10^2 GeV)
and Gravitational scale (10^{19} GeV)

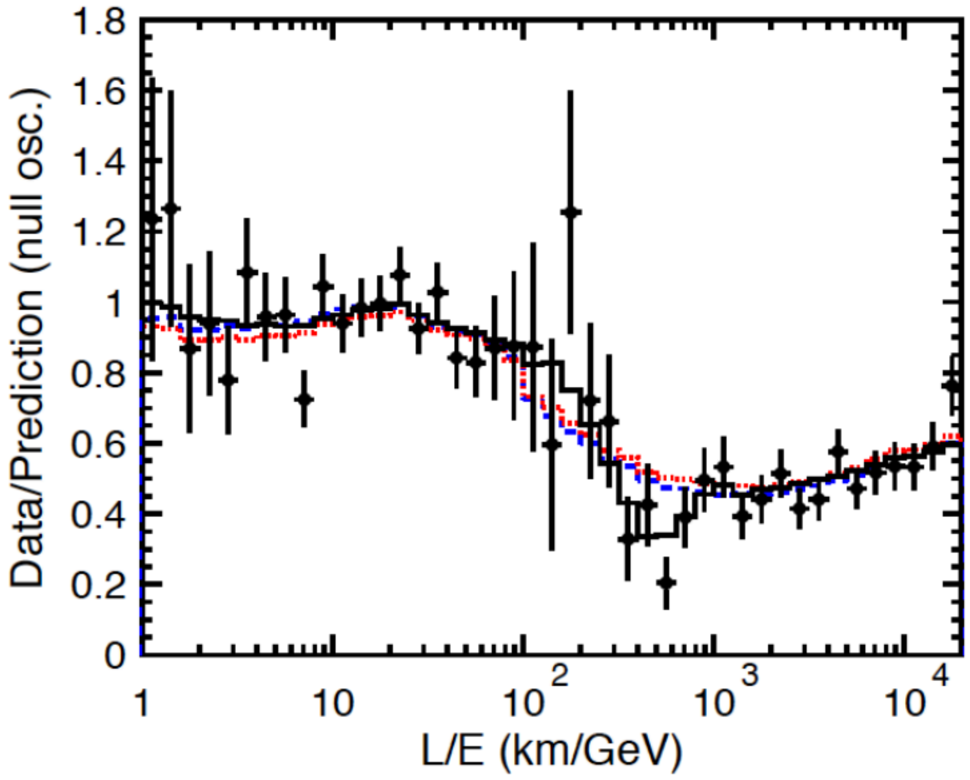
Elementary or composite?
Additional Higgs bosons?

Is the vacuum of the universe stable?

Dark Matter

Weakly-interacting massive particle?
Sterile neutrino?
Extremely light particle (axion)?
Alternative explanations?

Interaction with SM?
Self-interacting?



Super-Kamiokande observation of neutrino oscillations, 2004

Quarks and leptons

Why 3 families? Can we explain
mass & mixings?

Are neutrinos Majorana or Dirac
fermions?

Origin of matter-antimatter
asymmetry in the Universe?

DGLAP evolution equations

F. Olness, CTEQ school 2017