Centre of Excellence in Quark Matter
NNPDF
JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ
Research Council of Finland
HELSINKI INSTITUTE OF PHYSICS
Funded by the European Union
erc

On the positivity of $\overline{\mathrm{MS}}$ parton distributions

Felix Hekhorn

58th Rencontres de Moriond 2024

Overview

1. Introduction

2. From a physical scheme to $\overline{\mathrm{MS}}$

3. Conclusions

Introduction

History

Can $\overline{\mathrm{MS}}$ parton distributions be negative?

Candido, Forte, FH (2020) [JHEP11.129]

Positivity and renormalization of parton densities

Collins, Rogers, Sato (2022) [PRD105.076010]

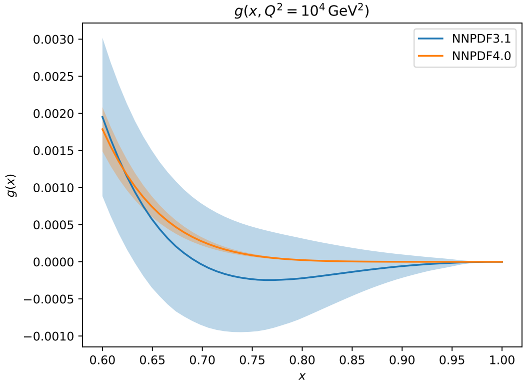
On the positivity of $\overline{\mathrm{MS}}$ parton distributions

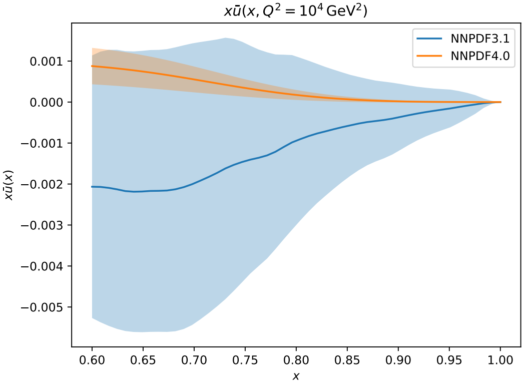
Candido, Forte, Giani, FH (2024) [EPJC84.335]

The small print

- Here I only consider parton distribution functions (PDFs)
- PDFs are, by definition, leading-twist objects
- only PDFs are universal
- “positive” = “non-negative”

Positivity can be a problem



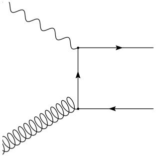
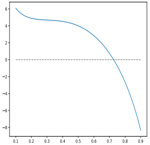


using NNPDF3.1 may yield $\sigma(pp \to Z') < 0$!

How come?

- leading order (LO) PDFs are positive ✓
- at next-to-leading order (NLO) collinear factorization is non-trivial and PDFs *could* turn negative

$$\sigma(Q^2) = C(\alpha_s(Q^2)) \otimes f(Q^2)$$

- example: gluon coefficient function in DIS  $+c.c. \xrightarrow{\overline{\mathrm{MS}}}$ 
- (mathematical) $\overline{\mathrm{MS}}$ leads to an over-subtraction
- a more physical scheme ($\mathtt{POS}$) leads to positive coefficient functions (→MCEG)

Study the scheme change!

From a physical scheme to $\overline{\mathrm{MS}}$

From a physical scheme to $\overline{\mathrm{MS}}$

In a physical scheme PDFs are identified with observables and hence positive by definition:

$$\sigma(Q^2) = \sigma_0 f^{\mathrm{PHYS}}(Q^2) + O\left(\frac{\Lambda^2}{Q^2}\right)$$

In $\overline{\mathrm{MS}}$ we compute perturbative coefficient functions:

$$\sigma(Q^2) = \sigma_0 C^{\overline{\mathrm{MS}}}(\alpha_s(Q^2)) \otimes f^{\overline{\mathrm{MS}}}(Q^2) + O\left(\frac{\Lambda^2}{Q^2}\right)$$

These coefficient functions connect the two schemes:

$$\Rightarrow f^{\overline{\mathrm{MS}}}(Q^2) = \left[C^{\overline{\mathrm{MS}}}(\alpha_s(Q^2))\right]^{-1} \otimes f^{\mathrm{PHYS}}(Q^2)$$

Inverting the coeffificient function

Coefficient functions are computed perturbatively:

$$C^{\overline{\mathrm{MS}}}(\alpha_s(Q^2), x) = \delta(1-x) + \frac{\alpha_s(Q^2)}{2\pi} C^{\overline{\mathrm{MS}},(1)}(x) + \mathcal{O}(\alpha_s^2)$$

from which we get the perturbative inverse:

$$\Rightarrow f^{\overline{\mathrm{MS}}}(Q^2) = \left[1 - \frac{\alpha_s(Q^2)}{2\pi} C^{\overline{\mathrm{MS}},(1)} \otimes \right] f^{\mathrm{PHYS}}(Q^2)$$

and hence the positivity condition

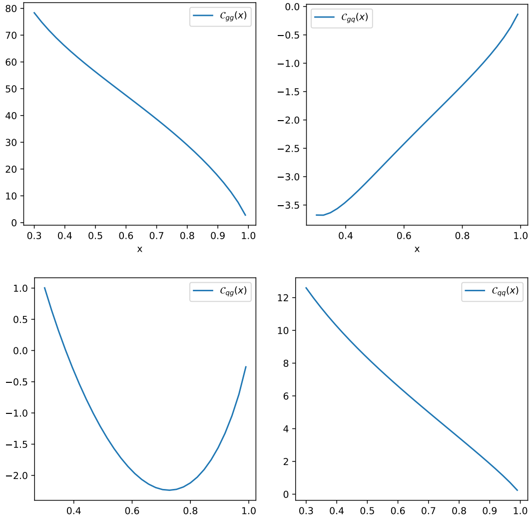
$$\frac{\alpha_s(Q^2)}{2\pi} \left| C_{ij}^{\overline{\mathrm{MS}},(1)} \otimes f_j^{\mathrm{PHYS}}(Q^2) \right| \leq \left| f_i^{\mathrm{PHYS}}(Q^2) \right|$$

Estimating the perturbative scale

positivity condition:

$$\frac{\alpha_s(Q^2)}{2\pi}\left|\sum_j \underbrace{\int_x^1 \frac{dy}{y} C_{F,ij}^{\overline{\mathrm{MS}},(1)}(y)}_{\mathcal{C}_{ij}(x)}\right| \leq 1$$

- use Higgs and DIS [NPB534.277]
- to impose positivity at $x = 0.8$ we get $\alpha_s(Q^2) \lesssim 0.2 \Rightarrow Q^2 \gtrsim 5\,\mathrm{GeV}^2$



Conclusions

Conclusions

- $\overline{\mathrm{MS}}$ PDF are positive in the perturbative regime
- positivity should be enforced at a large enough scale, e.g. $Q^2 = 5\,\mathrm{GeV}^2$
- this only holds for light PDFs

If the fitted PDF wants to go negative, it could mean

- that the perturbative regime breaks down, because e.g. higher-twist terms are present
- that the fitted object is far from the *true* PDF realized in nature

Conclusions

- $\overline{\mathrm{MS}}$ PDF are positive in the perturbative regime
- positivity should be enforced at a large enough scale, e.g. $Q^2 = 5\,\mathrm{GeV}^2$
- this only holds for light PDFs

If the fitted PDF wants to go negative, it could mean

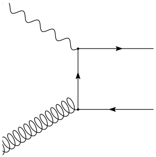
- that the perturbative regime breaks down, because e.g. higher-twist terms are present
- that the fitted object is far from the *true* PDF realized in nature

Danke! Thanks! Kiitos!

Backup slides

The gluon coefficient function in DIS

$$C_g^{(1)}(x, Q^2, \epsilon) = \left|\vphantom{\int}\right.$$



$$\left. + \, c.c. \right|^2 > 0 \tag{1}$$

$$= \left(\frac{Q^2(1-x)}{4\pi\mu^2 x}\right)^{-\epsilon} \frac{\Gamma(-\epsilon)}{2-2\epsilon} P_{qg}(x) + \ldots \tag{2}$$

$$\Rightarrow C_g^{\overline{\mathrm{MS}},(1)}(x) = P_{qg}(x)\left(\ln\left(\frac{1-x}{x}\right) - 4\right) + 3T_R \tag{3}$$

$$\Rightarrow C_g^{\mathrm{DPOS},(1)}(x) = 3(T_R - P_{qg}(x)) > 0 \tag{4}$$

Large x resummation

$$C^{\overline{\mathrm{MS}}}(x) = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[\delta(1-x)\Delta^{(1)} + C_F^{(1),\,\overline{\mathrm{MS}}}(x) + C_D^{(1),\,\overline{\mathrm{MS}}}(x)\right] + \mathcal{O}(\alpha_s^2) \tag{5}$$

$$\left[C_D^{(1),\,\overline{\mathrm{MS}}}\right]_{ij}(x) = c_i\delta_{ij}\left(2\left[\frac{\ln(1-x)}{1-x}\right]_+ - \frac{3}{2}\left[\frac{1}{1-x}\right]_+\right) = c_i\delta_{ij}\left[\frac{\ln(1-x)}{1-x}\right]_+ + \mathrm{NLL}(1-x), \tag{6}$$

$$\begin{aligned}\left[\delta(1-x) + \frac{\alpha_s}{2\pi}C_D^{(1),\,\overline{\mathrm{MS}}}(x)\right]_{ij}^{-1} &= \delta_{ij}\delta\left(1-x\right) \\ &- 2\delta_{ij}c_i\frac{\alpha_s}{2\pi}\left(\frac{\ln(1-x)}{\left(1+c_i\frac{\alpha_s}{2\pi}\ln^2(1-x)\right)^2}\frac{1}{1-x}\right)_+ + \mathrm{NLL}(1-x).\end{aligned} \tag{7}$$

Factorization theorem

$$F(x,Q^2) = \sum_j C_j(\epsilon, \alpha_s(\mu_r^2), \mu_r^2/Q^2) \otimes f_j(\epsilon, \mu_r^2) + O\left(\frac{\Lambda^2}{Q^2}\right) \quad (8)$$

with:

- $C_j(\epsilon)$ UV safe, IR safe, collinear divergent
- $f_j(\epsilon)$ UV safe, collinear divergent

$$F(x,Q^2) \sum_j C_j^R(\alpha_s(\mu_r^2), \mu_r^2/Q^2, \mu_f^2/Q^2) \otimes f_j^R(\mu_f^2) + O\left(\frac{\Lambda^2}{Q^2}\right) \quad (9)$$

with C_j^R and f_j^R finite