COLLINEAR STRUCTURE

LECTURE 1

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INFN
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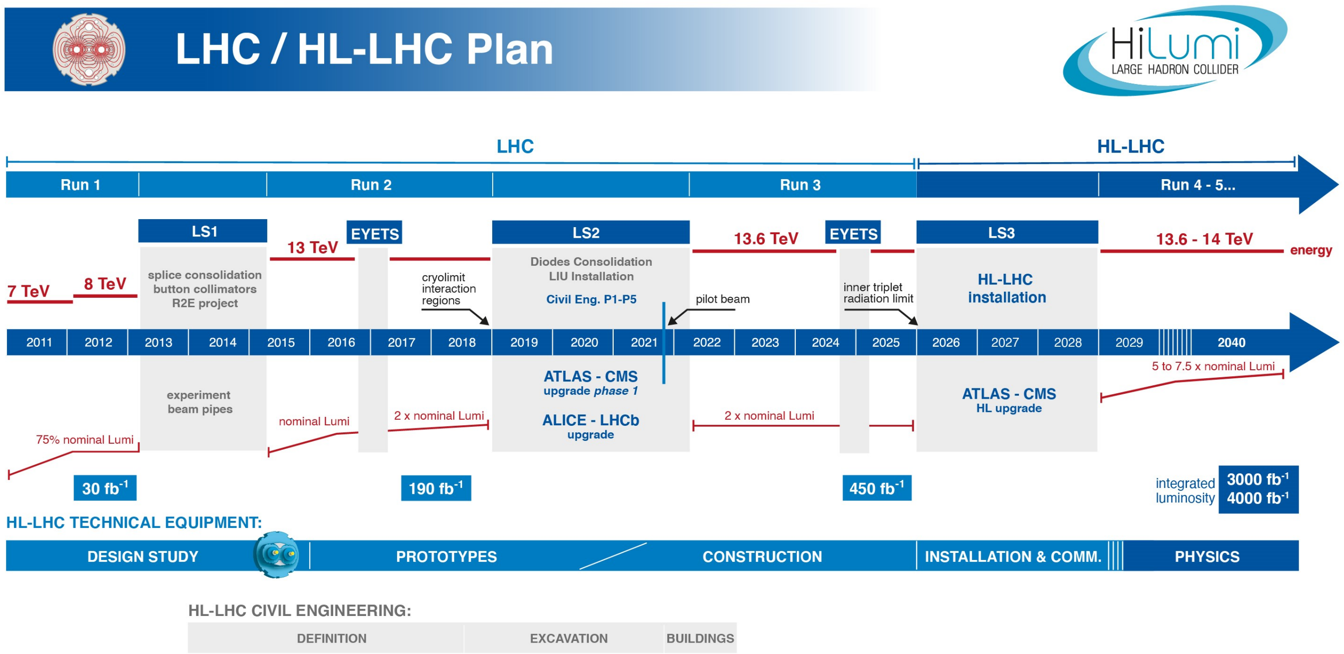
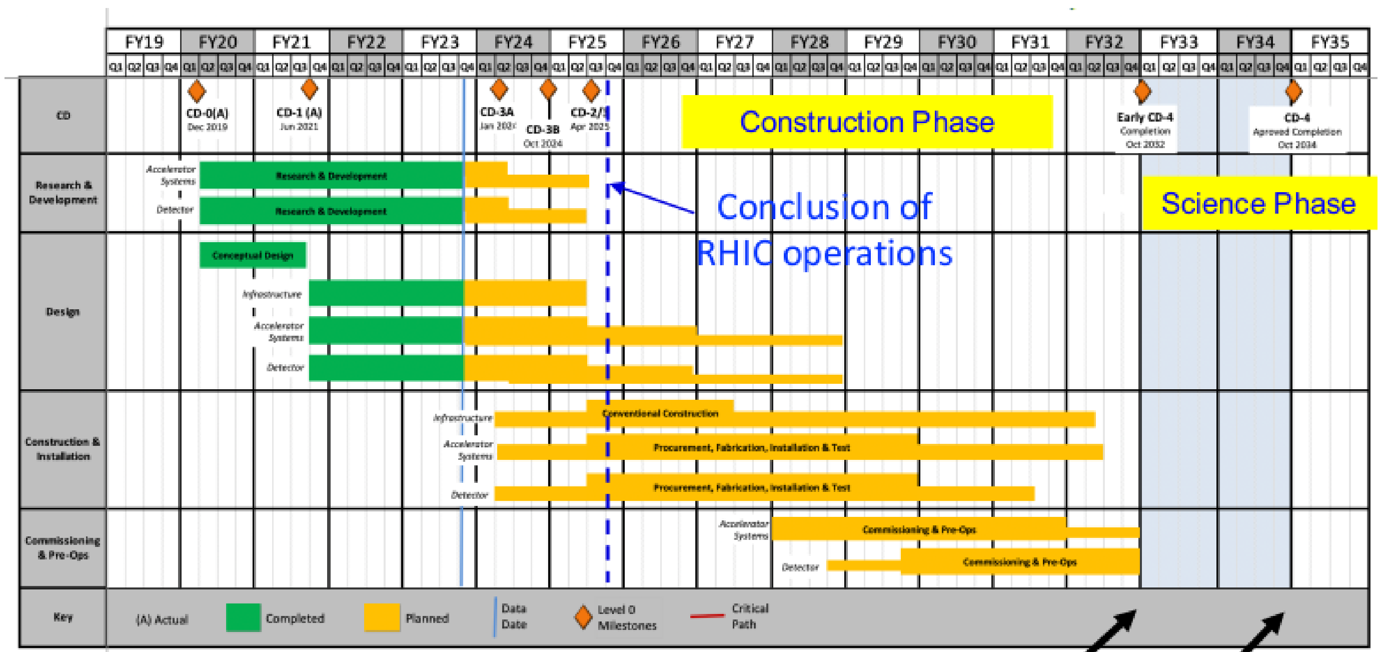
CFNS EIC school

Stony Brook, June 6 2024

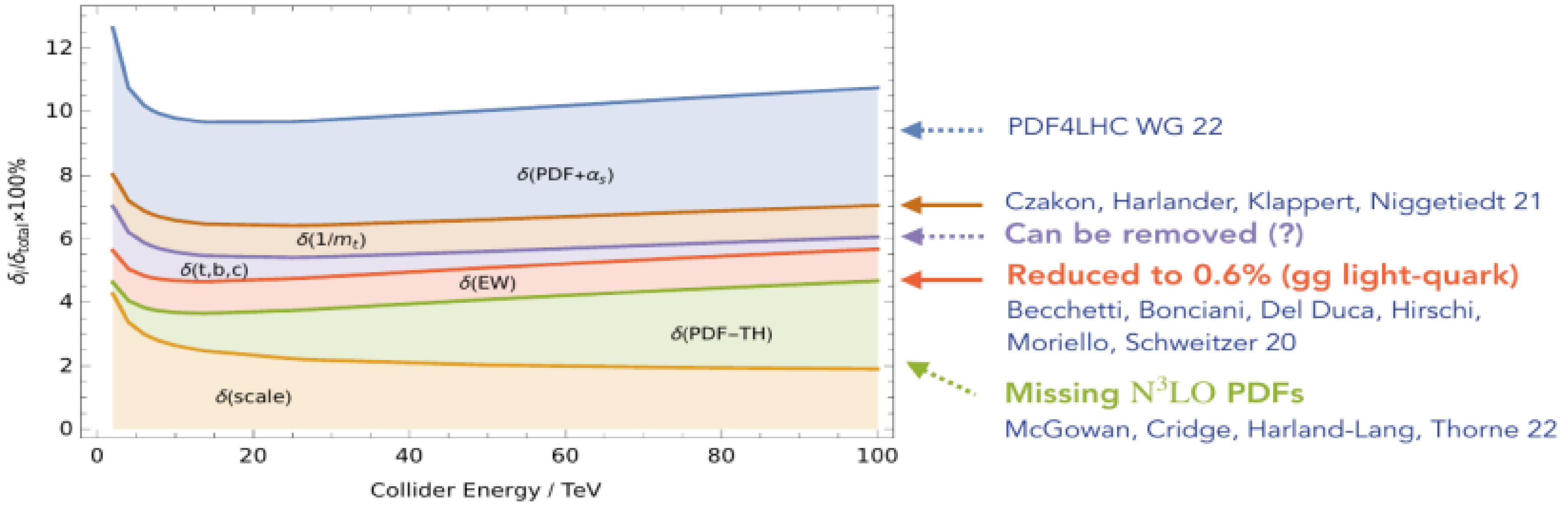
THE EIC-LHC SYNERGY





WHY IS IT INTERESTING?

UNCERTAINTIES:
HIGGS IN GLUON FUSION



(R. Röntsch, Les Houches 2023)

- (COLLINEAR) PARTON DISTRIBUTIONS $\Rightarrow$ DOMINANT UNCERTAINTY AT HADRON COLLIDERS
- DEEP-INELASTIC SCATTERING AT EIC $\Rightarrow$ DOMINANT SOURCE OF INFORMATION ON PDF IN THE HL-LHC ERA

QUESTIONS (AND ANSWERS)

- HOW DOES FACTORIZATION RELATE PHYSICAL OBSERVABLES TO PDFS?
- HOW DO PDFS LOOK LIKE, AND WHY?
- HOW ARE PHYSICAL OBSERVABLES AFFECTED BY PDFS, AND HOW CAN PDFS BE EXTRACTED FROM PHYSICAL OBSERVABLES?
- WHAT ARE PDF UNCERTAINTIES, AND WHERE DO THEY COME FROM?

THESE LECTURES

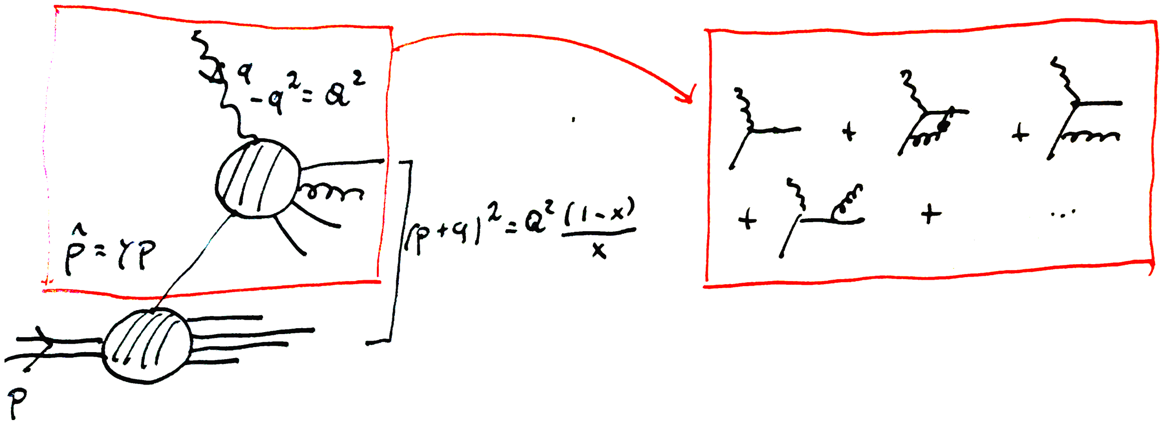
- A PRACTICAL APPROACH
- TRY TO ANSWER NAIVE QUESTIONS...
- HOPEFULLY RAISE DEEP QUESTIONS!

FACTORIZATION: REMINDER

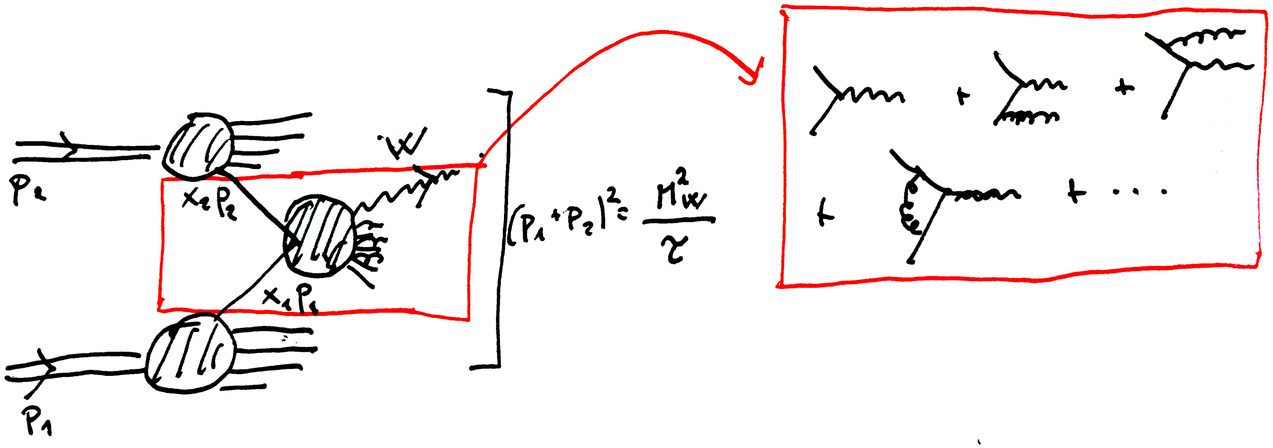
SIMPLEST CASE: TOTALLY INCLUSIVE CROSS-SECTION

LEPTON-HADRON

- SCALE $Q^2 = -q^2$
- SCALING VARIABLE $x = \frac{Q^2}{2p\cdot q}$
- FACTORIZATION $\Leftrightarrow$ ONLY ONE INCOMING PARTON

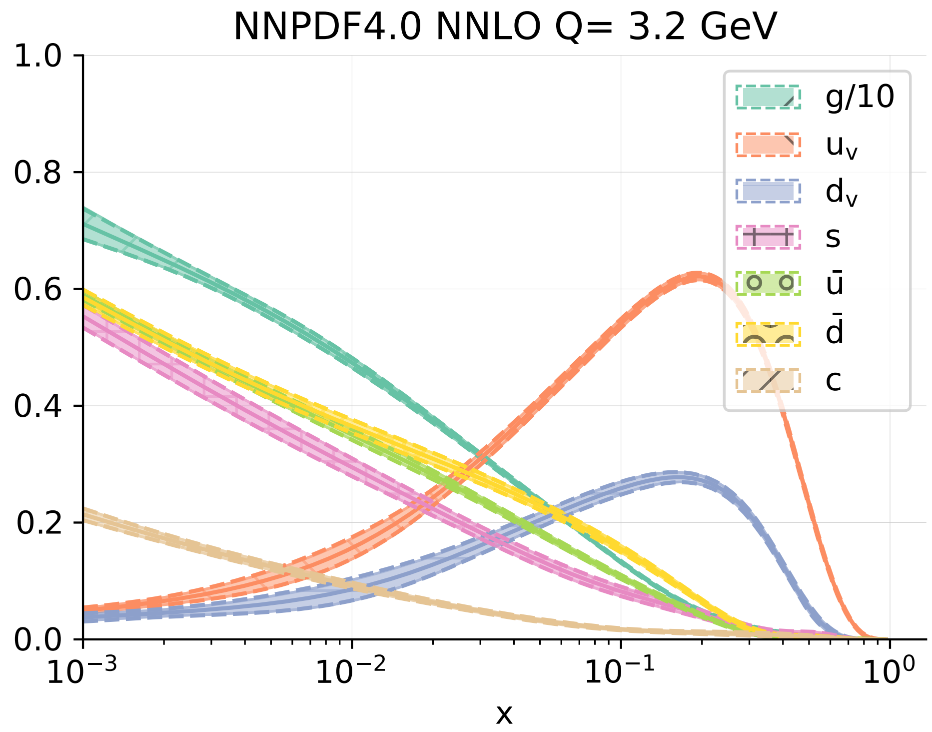


HADRON-HADRON



- SCALE M^2
- SCALING VARIABLE $\tau = \frac{M^2}{s}$
- FACTORIZATION $\Leftrightarrow$ ONE INCOMING PARTON FROM EACH HADRON

A FIRST LOOK AT PROTON PDFs (NNPDF4.0, 2021)



- A SET OF CORRELATED PROBABILITY DISTRIBUTIONS OF QUASI-PROBABILITY DISTRIBUTIONS
- $xf_i(x)$ PLOTTED VS. $\ln x \Rightarrow$ AREA = NUMBER
- "VALENCE" $u_v = u - \bar{u}$, $d_v = d - \bar{d} \Rightarrow$ PEAK (AROUND $x \sim 0.3$) $\Rightarrow$ FINITE NUMBER & MOM. FRACTION
- "SEA" $\bar{u}$, $\bar{d}$, $s + \bar{s}$, $c + \bar{c}$, $g \Rightarrow$ SMALL x GROWTH $\Rightarrow$ INFINITE NUMBER, FINITE MOMENTUM FRACTION

FACTORIZATION KINEMATICS: HADRONIC VS PARTONIC

ONE HADRON IN THE INITIAL STATE

$$\sigma(x) = \int_0^1 dz \int_x^1 dy \delta(x - yz) q(y) \hat{\sigma}(z) = \int_x^1 \frac{dy}{y} q(y) \hat{\sigma}\left(\frac{x}{y}\right) = \int_x^1 \frac{dy}{y} q\left(\frac{x}{y}\right) \hat{\sigma}(y) = [\sigma \otimes q](x)$$

- $x = x_B = \frac{Q^2}{2p \cdot q}$: SCALING VARIABLE FOR HADRONIC PROCESS $\Rightarrow$ MEASURED HADRON KINEMATICS: p PROTON MOMENTUM, q GAUGE BOSON MOMENTUM
- $z = \frac{Q^2}{2\hat{p} \cdot q}$: SCALING VARIABLE FOR PARTONIC PROCESS (COMPUTED PARTONIC FEYNMAN DIAGRAM) $\Rightarrow \hat{p}$ PARTON MOMENTUM
- y: INCOMING PARTON MOMENTUM FRACTION $\Rightarrow \hat{p} = yp$

TWO HADRONS IN THE INITIAL STATE

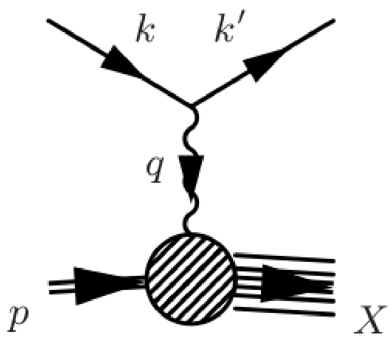
$$\sigma(\tau) = \int_0^1 dz \int_\tau^1 dx_1 dx_2 \delta(\tau - x_1 x_2 z) q_1(x_1) q_2(x_2) \hat{\sigma}(z) = \int_\tau^1 \frac{dx_1}{x_1} \int_\tau^1 \frac{dx_2}{x_2} q_1(x_1) q_2(x_2) \hat{\sigma}\left(\frac{\tau}{x_1 x_2}\right)$$

$$= \int_\tau^1 \frac{dy}{y} \mathcal{L}(y) \hat{\sigma}\left(\frac{\tau}{y}\right) = [\mathcal{L} \otimes \hat{\sigma}](\tau); \qquad \mathcal{L}(y) \equiv \int_y^1 \frac{dx_1}{x_1} q_1(x_1) q_2(y/x_1) = [q_1 \otimes q_2](y)$$

- $\tau = \frac{M^2}{s}$: SCALING VARIABLE FOR HADRONIC PROCESS $\Rightarrow s = (p_1 + p_2)^2$ COLLIDER C.M. ENERGY, M^2 FINAL STATE MASS
- $z = \frac{M^2}{\hat{s}}$: SCALING VARIABLE FOR PARTONIC PROCESS (COMPUTED PARTONIC FEYNMAN DIAGRAM) $\Rightarrow \hat{s} = (\hat{p}_1 + \hat{p}_2)^2$ C.M. ENERGY OF PARTON COLLISION
- x_1, x_2: INCOMING PARTON MOMENTUM FRACTIONS $\Rightarrow \hat{p}_i = x_i p_i$
- $z \leq 1 \Rightarrow x_i \geq \tau$
- $\mathcal{L}$: PARTON LUMINOSITY

DIS: A CLOSER LOOK

CROSS SECTION & STRUCTURE FUNCTIONS



- gauge boson virtuality: $q^2 = -Q^2$
- Bjorken x: $x = \frac{Q^2}{2p\cdot q}$
- Lepton fractional energy loss: $y = \frac{p\cdot q}{p\cdot k}$
- lepton-nucleon CM energy: $s = \frac{Q^2}{xy}$;
- virtual boson-nucleon CM energy $W^2 = Q^2\frac{1-x}{x}$;
- λ_l $\rightarrow$ lepton helicity; λ_p $\rightarrow$ proton helicity

$$\frac{d^2\sigma^{\lambda_p\lambda_\ell}(x,y,Q^2)}{dxdy} = \frac{G_F^2}{2\pi(1+Q^2/m_W^2)^2}\frac{Q^2}{xy}\left\{\left[-\lambda_\ell\, y\left(1-\frac{y}{2}\right)xF_3(x,Q^2) + (1-y)F_2(x,Q^2)\right.\right.$$

$$\left.\left.+y^2xF_1(x,Q^2)\right] - 2\lambda_p\left[-\lambda_\ell\, y(2-y)xg_1(x,Q^2) - (1-y)g_4(x,Q^2) - y^2xg_5(x,Q^2)\right]\right\}$$

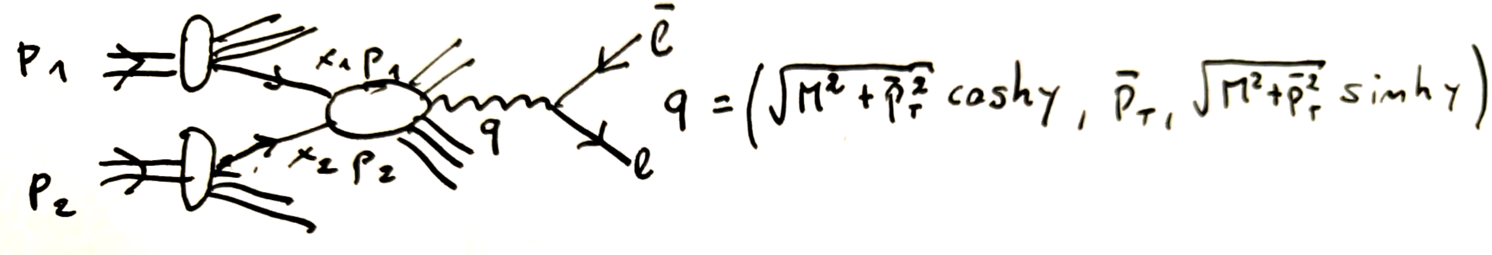
$$F^i(x,Q^2) = x\sum_j\int_x^1 \frac{dy}{y}C_j^i\left(\alpha_s(Q^2),\frac{x}{y}\right)f_j(y,Q^2)$$

$$g^i(x,Q^2) = x\sum_j\int_x^1 \frac{dy}{y}C_j^i\left(\alpha_s(Q^2),\frac{x}{y}\right)\Delta f_j(y,Q^2)$$

f_i, Δf_i $\Rightarrow$ UNPOL., POL., QUARK, ANTIQUARK & GLUON PDFs

	PARITY CONS.	PARITY VIOL.
UNPOL.	F_1, F_2	F_3
POL.	g_1	g_4, g_5

DRELL-YAN: A CLOSER LOOK
BEYOND THE TOTAL CROSS-SECTIONS



FULLY DIFFERENTIAL: $\frac{d\sigma}{dM^2 d\vec{p}_T dy}$ $\Rightarrow$ M^2 INVARIANT MASS; Y RAPIDITY; p_T TRANSVERSE MOMENTUM

RAPIDITY DISTRIBUTION

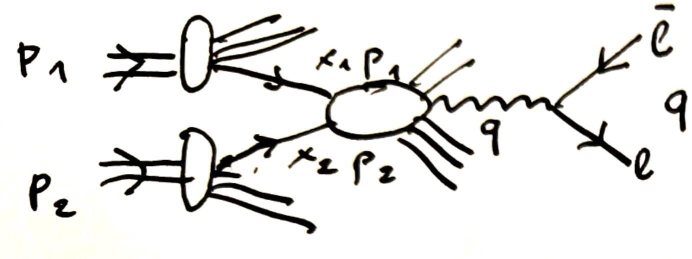
RAPIDITY: LONGITUDINAL BOOST OF FINAL STATE WR TO CM OF HADRONIC COLLISION;
$Y \Rightarrow$ HADRONIC CM; $y \Rightarrow$ PARTONIC CM

- $$\frac{d\sigma(s,M^2)}{dY\,dM^2} = \sum_{a,b}\int_\tau^1 dx_1\,dx_2\,dy\,f_{a/h_1}(x_1) f_{b/h_2}(x_2)\frac{d\hat\sigma_{ab\to X}}{dy\,dM^2}\left(x_1x_2s, M_X^2\right)\delta\left(Y - \frac{1}{2}\ln\frac{x_1}{x_2} - y\right)$$

- LEADING ORDER: $\frac{d\hat\sigma_{ab\to X}}{dy\,dM^2} = \sigma_0\delta(y)\tau\delta(1-\frac{\tau}{x_1x_2}) \Rightarrow \frac{d\sigma(s,M_X^2)}{dY\,dM^2} = \sigma_0\tau\sum_{ab} f_a(x_1)f_b(x_2)$;
  $x_i^{\rm LO} = \tau e^{\pm Y}$ $\tau = \frac{M^2}{s}$ $\Rightarrow$ BOOST DETERMINED BY IMBALANCE OF MOMENTUM FRACTIONS
  BEYOND LO $x_i^{\rm min} = x_i^{\rm LO}$

DRELL-YAN: A CLOSER LOOK

BEYOND TOTAL CROSS-SECTIONS



$$q = \left(\sqrt{M^2 + \vec{p}_T^{\,2}} \cosh y,\ \vec{p}_T,\ \sqrt{M^2 + \vec{p}_T^{\,2}} \sinh y\right)$$

TRANSVERSE MOMENTUM DISTRIBUTION

- $$\frac{d\sigma(s, M^2, p_t^2)}{dp_t^2 dM^2} = \sum_{a,b} \int_{x_i^{\min}}^1 dx_1\, dx_2\, dy\, f_{a/h_1}(x_1) f_{b/h_2}(x_2) \frac{d\hat{\sigma}_{ab\to X}}{dp_t^2 dM^2} \left(x_1 x_2 s, M^2, p_t^2\right)$$

- AT FIXED $|p_t|$, $\hat{s} = x_1 x_2 s \geq \hat{s}_{\min} = (\sqrt{M^2 + p_t^2} + \sqrt{p_t^2})^2$

  $$\Rightarrow x_1,\ x_2 \geq \frac{\hat{s}_{\min}}{s} = \tau \left(\sqrt{1 + \frac{p_t^2}{Q^2}} + \sqrt{\frac{p_t^2}{Q^2}}\right)^2 = \frac{\tau}{\left(\sqrt{1+\frac{p_t^2}{Q^2}} - \sqrt{\frac{p_t^2}{Q^2}}\right)^2}$$

  INTEGRAL OVER x_1, x_2 NOT A CONVOLUTION!

- NEW SCALE VARIABLE $Q^2 = \hat{s}_{\min} \Rightarrow$ NEW SCALING VARIABLE $\tau' = \frac{Q^2}{s} \Rightarrow$ CONVOLUTION!

  $$\frac{d\sigma}{dp_t^2 dM^2}(\tau', p_t^2, M^2) = \tau' \sum_{ij} \int_{\tau'}^1 \frac{dx}{x} \mathcal{L}_{ij}\left(\frac{\tau'}{x}\right) \frac{1}{x} \frac{d\hat{\sigma}_{ij}}{dp_t^2}(x, p_t^2, M^2)$$

MELLIN TRANSFORM

$$F(N) = \int_0^1 dx x^{N-1} f(x)$$

INVERSION FORMULA

$$f(x) = \int_{-i\infty}^{+i\infty} dN x^{-N} F(N)$$

$0 \leq x \leq 1$ COMPACT $\Leftrightarrow$ N COMPLEX

CONVOLUTIONS

$$h(x) = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y) = [f \otimes g](x) \Leftrightarrow H(N) = F(N)G(N)$$

N-SPACE FACTORIZED EXPRESSIONS:

- LEPTON-HADRON $F(N, Q^2) = \sum_i C_i \left(\alpha(Q^2), N\right) f_i(N, Q^2)$
- HADRON-HADRON $\sigma(x, N) = \sum_{ij} \hat{\sigma}_{ij} \left(\alpha(Q^2), N\right) \mathcal{L}_{ij}(N, Q^2)$
  $\mathcal{L}_{ij}(Q^2, N) = f_i(N) f_j(N)$

$x < 1 \Rightarrow$ LARGE/SMALL x $\Leftrightarrow$ LARGE/SMALL N

PARTON KINEMATICS vs. HADRON KINEMATICS

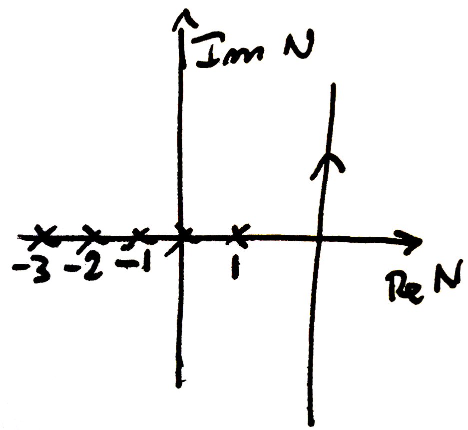
$$\sigma(\tau) = \sum_{ij} \int_\tau^1 \frac{dy}{y} \mathcal{L}_{ij}(y) \hat{\sigma}\left(\frac{\tau}{y}\right); \quad \mathcal{L}_{ij}(y) \equiv \int_y^1 \frac{dx_1}{x_1} f_i(y) f_j\left(\frac{y}{x_1}\right)$$

WHICH PARTON MOMENTUM FRACTION CONTRIBUTES TO A GIVEN HADRONIC PROCESS ?

INVERSION OF MELLIN TRANSFORMS

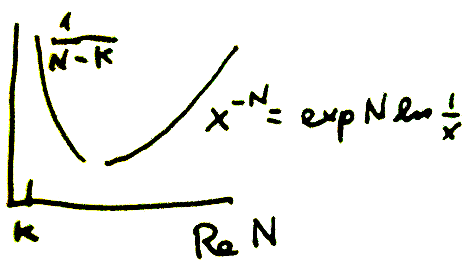
$$\Sigma(N) = \int_0^1 dx x^{N-1} \sigma(x) \Leftrightarrow \sigma(x) = \int_{-i\infty}^{+i\infty} dN x^{-N} \Sigma(N)$$

- INFINITE NUMBER OF SEA PARTONS $\Rightarrow N = 1$ DIVERGENCE
- INTEGRATE TO THE RIGHT OF CONVERGENCE ABSCISSA



SADDLE POINT INTEGRATION

$\int dx\, x^{N-1}\, x^{1-k} \sim \frac{1}{N-K}$



- SADDLE METHOD: $\int_{-i\infty}^{+i\infty} dN x^{-N} \Sigma(N) = \int_{-i\infty}^{+i\infty} dN e^{E(x,N)}$,
  TAYLOR EXPAND $E(x, N)$ ABOUT STATIONARY PT N_0 (SADDLE)

- SADDLE CONDITION $\frac{d}{dN}\left[-N \ln x + \ln \Sigma(N)\right]\Big|_{N=N_0} = 0$

  $\Rightarrow N_0(x, Q^2)$ DEP ON HADRONIC KINEMATICS

- $\Sigma(N) = \sum_{ij} \hat{\sigma}_{ij}(N) f_i(N) f_j(N)$

- PARTONIC $\hat{\sigma}_{ij}$: LO CONSTANT IN N SPACE

- PDF: $f_i(x) \sim x^{1-k} \Leftrightarrow f(N) \sim \frac{1}{N-k}$ DROPS

- x^{N-1}: $x < 1 \Rightarrow$ GROWS ON REAL N AXIS

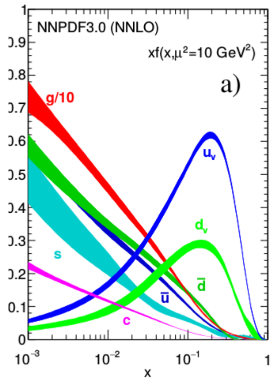
$$\sigma(x) = \frac{1}{\sqrt{2\pi E''(x, N_0)}} \Sigma(N_0) = \frac{1}{\sqrt{2\pi E''(x, N_0)}} \sum_{ij} f_i(N_0) f_j(N_0) \hat{\sigma}(N_0)$$

- HADRONIC CROSS-SECTION FACTORIZES WITH N FIXED BY HADRONIC KINEMATICS

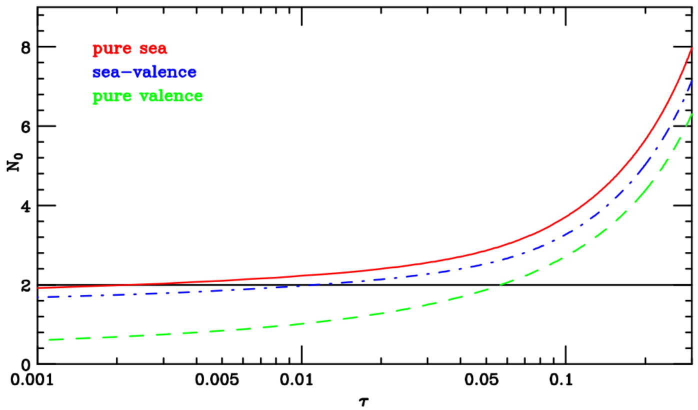
- SMALLER $x \Rightarrow x^{N-1}$ GROWS FASTER WITH $N \Rightarrow$ SADDLE MOVES TO THE LEFT
  $\Rightarrow \hat{\sigma}$ & PDF EVALUATED AT SMALLER N

HADRONIC VS PARTONIC
QUALITATIVE BEHAVIOUR

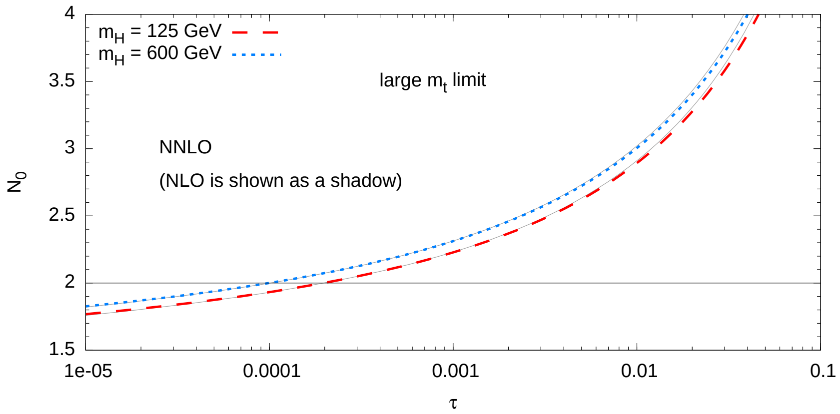
- THE EVALUATION: SEA, GLUON GROW FASTER THAN VALENCE
  $\Rightarrow$ N POLE MORE TO THE RIGHT
  $\Rightarrow$ SADDLE MOVES TO LARGER N
- THE INTERPRETATION: GLUON & ANTIQUARK SEA MORE LIKELY TO CARRY SMALL MOMENTUM FRACTION
  $\Rightarrow$ PARTONIC PROCESS CLOSER TO THRESHOLD
  $\Rightarrow$ LARGE x, LARGE N
- HIGGS $\Leftrightarrow$ GLUONS; DRELL-YAN $\Leftrightarrow$ QUARK-ANTIQUARK
  NOTE ANTIQUARK IN ANTIPROTON IS VALENCE!



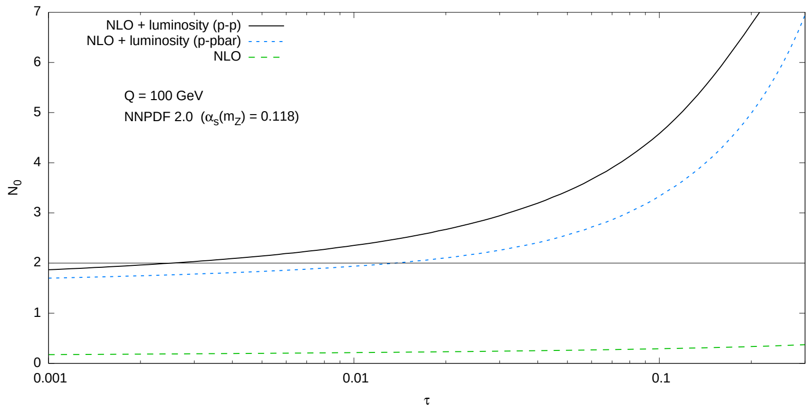
SADDLE VS $\tau = Q^2/s$



HIGGS



Z DRELL-YAN



FACTORIZATION & KINEMATICS SUMMARY

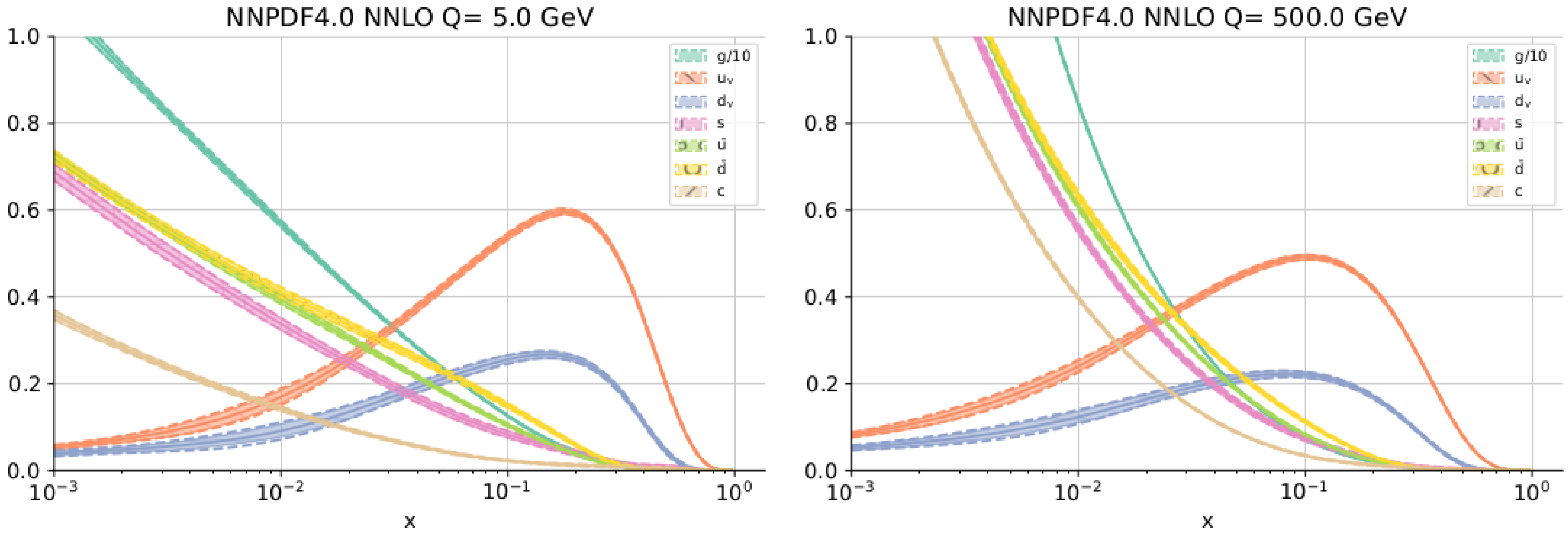
- LONGITUDINAL & TRANSVERSE KINEMATICS DECOUPLE
- LONGITUDINAL KINEMATICS FACTORIZES UPON CONVOLUTION

  $\Rightarrow$ (COLLINEAR) PDF (LEPTON-HADRON) OR LUMINOSITY (HADRON-HADRON)

  $\otimes$ PARTONIC CROSS SECTIONS
- CONVOLUTION UNDONE BY MELLIN
- SADDLE POINT MAPS HADRONIC KINEMATICS

  $\Rightarrow$ (REAL) N-MELLIN VALUE FOR PDF/LUMI & PARTONIC CROSS SECTION

PDFs: QUALITATIVE FEATURES



- NOTE $xf_i(x)$ (MOMENTUM DENSITY) SHOWN VS. $\ln x$ (AREA = NUMBER)
- NOTE GLUON DIVIDED BY 10
- VALENCE UP AND DOWN: PEAKED AT $x \sim 0.3$
- SEA ANTIQUARK AND GLUON GROW AT SMALL x
- LARGE x VALENCE SOMEWHAT SMALLER AT HIGH SCALE
- SMALL x SEA AND GLUON MUCH LARGER AT HIGH SCALE

SUM RULES

CONSTRUCT CONSERVED QUANTUM NUMBERS CARRIED BY PARTON DISTRIBUTIONS:

BARYON NUMBER

$$\int_0^1 dx\,(u^p - \bar{u}^p) = 2 = 2\int_0^1 dx\,\left(d^p - \bar{d}^p\right)$$

$$\int_0^1 dx\,(s^p - \bar{s}^p) = \int_0^1 dx\,(c^p - \bar{c}^p) = \int_0^1 dx\,\left(b^p - \bar{b}^p\right) = \int_0^1 dx\,(t^p - \bar{t}^p) = 0$$

MOMENTUM

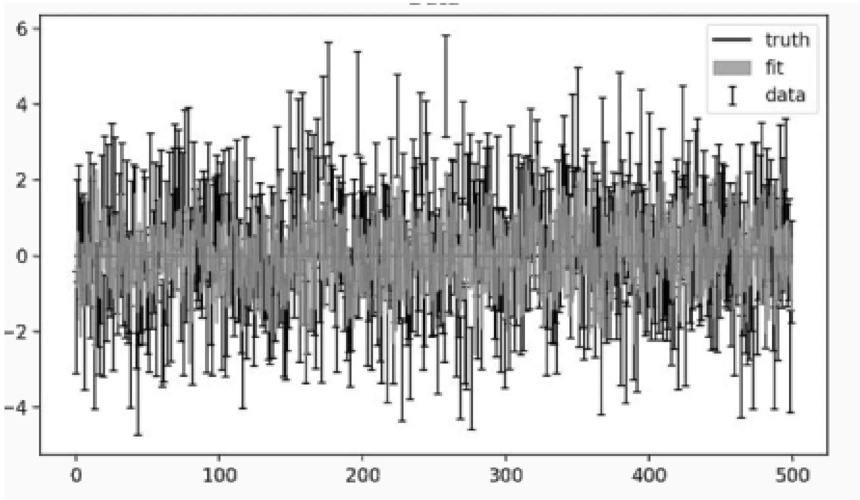
$$\int_0^1 dx x \left[\sum_{i=1}^{N_f} (q_i(x) + \bar{q}_i(x)) + g(x)\right] = 1$$

AT ALL SCALES

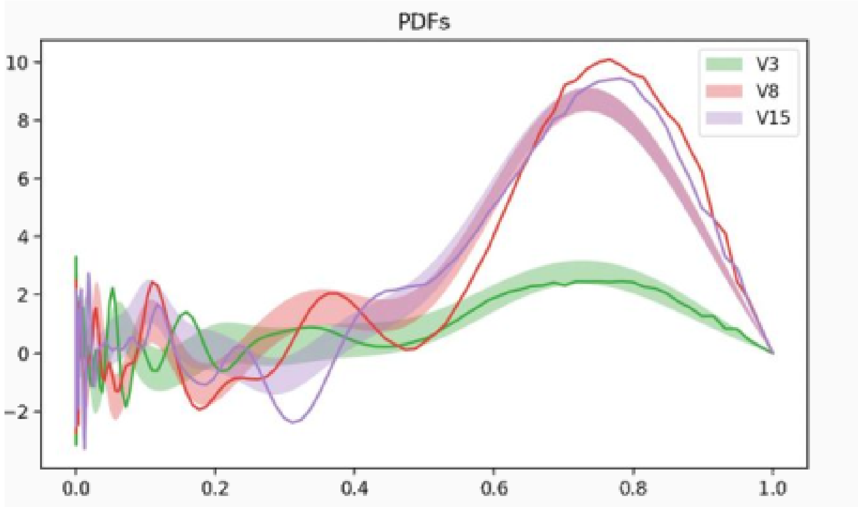
THE VALENCE BUMP

- IMPOSE SUM RULES, VANISHING AT $x = 1$
- FIT RANDOM DATA WITH A GAUSSIAN PROCESS
- QUALITATIVE SHAPE OF VALENCE PDFs REPRODUCED!

THE RANDOM DATA



THE VALENCE PDFs



THE SCALE DEPENDENCE OF PDFs
EVOLUTION EQUATIONS

$$\frac{d}{dt} q_{NS}(N,Q^2) = \frac{\alpha_s(t)}{2\pi} \gamma_{qq}^{NS}(N,\alpha_s(t)) q_{NS}(N,Q^2),$$

$$\frac{d}{dt} \begin{pmatrix} \Sigma(N,Q^2) \\ g(N,Q^2) \end{pmatrix} = \frac{\alpha_s(t)}{2\pi} \begin{pmatrix} \gamma_{qq}^{S}(N,\alpha_s(t)) & 2n_f \gamma_{qg}^{S}(N,\alpha_s(t)) \\ \gamma_{gq}^{S}(N,\alpha_s(t)) & \gamma_{gg}^{S}(N,\alpha_s(t)) \end{pmatrix} \begin{pmatrix} \Sigma(N,Q^2) \\ g(N,Q^2) \end{pmatrix}.$$

- LOG SCALE $t = \ln \frac{Q^2}{\Lambda^2}$:
- ANOMALOUS DIMENSIONS VS. SPLITTING FUNCTIONS
  $\gamma(N,\alpha_s(t)) \equiv \int_0^1 dx\, x^{N-1} P(x,\alpha_s(t))$
  INTEGRODIFFERENTIAL x SPACE VS. ODE N SPACE
- SINGLET $\Sigma(x,Q^2) = \sum_{i=1}^{n_f} \left(q_i(x,Q^2) + \bar{q}_i(x,Q^2) \right)$ VS.
  NONSINGLET $q^{NS}(x,Q^2) = q_i(x,Q^2) - q_j(x,Q^2)$ COMBINATIONS OF QUARK PDFs
- PERTURBATIVE EXPANSION OF ANOMALOUS DIMENSION
  $\gamma_i(N,\alpha_s(t)) = \gamma_i^{(0)}(N) + \alpha_s(t)\gamma_i^{(1)}(N) + \ldots$
  $\Rightarrow$ LOG RESUMMATION: LO $\Leftrightarrow$ LLQ^2; NLO $\Leftrightarrow$ LLQ^2, ...

THE SCALE DEPENDENCE OF PDFs
THE EVOLUTION BASIS

$$\frac{d}{dt} q_{NS}(N,Q^2) = \frac{\alpha_s(t)}{2\pi}\gamma_{qq}^{NS}(N,\alpha_s(t))q_{NS}(N,Q^2),$$

$$\frac{d}{dt}\begin{pmatrix}\Sigma(N,Q^2)\\ g(N,Q^2)\end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix}\gamma_{qq}^{S}(N,\alpha_s(t)) & 2n_f\gamma_{qg}^{S}(N,\alpha_s(t))\\ \gamma_{gq}^{S}(N,\alpha_s(t)) & \gamma_{gg}^{S}(N,\alpha_s(t))\end{pmatrix}\begin{pmatrix}\Sigma(N,Q^2)\\ g(N,Q^2)\end{pmatrix}.$$

HOW MANY DIFFERENT ANOMALOUS ARE THERE?

- LO: $\gamma_{qq}^{NS} = \gamma_{qq}^{S}$; $\Rightarrow$ 4 INDEP. ANOMALOUS DIMENSIONS
- NLO: $\gamma_{qq}^{S} \neq \gamma_{qq}^{NS,+} \neq \gamma_{qq}^{NS,-}$ $q^+ = q+\bar q$ VS $q^- = q-\bar q \Rightarrow$ 6 INDEP. AN. DIMS.
- NNLO: $\gamma_{qq}^{NS,-} \neq \gamma_{qq}^{NS,s}$ $q_i - \bar q_j$ $i=j$ VS. $i\neq j \Rightarrow$ 7 INDEP. ANOMALOUS DIMENSIONS

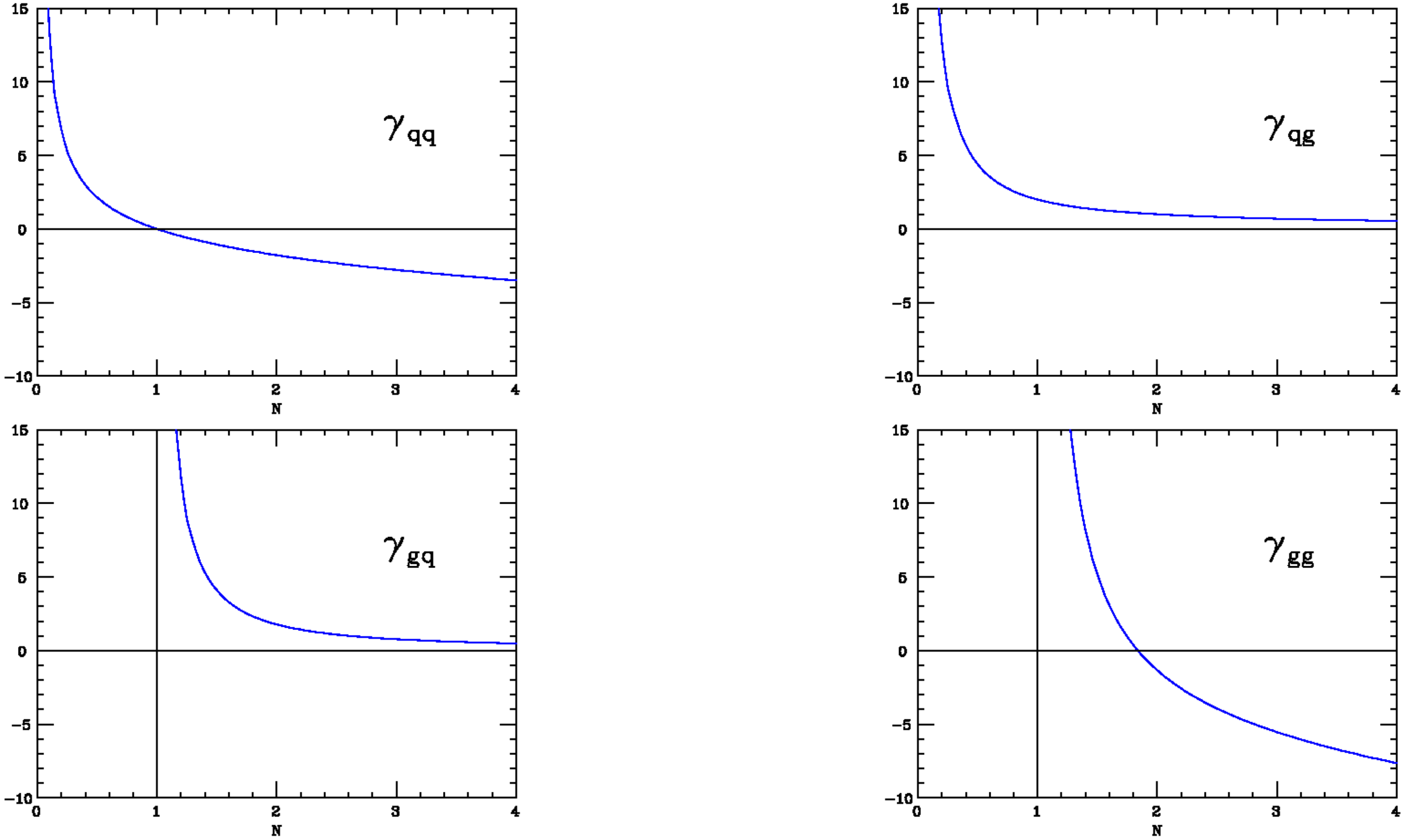
WHICH PDF COMBINATIONS EVOLVE INDEPENDENTLY?

- LO: g, $\Sigma = \sum_{i=1}^{n_f}\left(q_i(x,Q^2)+\bar q_i(x,Q^2)\right)$, ANY $2N_f-1$ LINEAR COMB. OF q_i, $\bar q_i$
- NLO: g, Σ, ANY N_f LINEAR COMB. OF $q_i-\bar q_i$, ANY N_f-1 LINEAR COMB. OF $q_i+\bar q_i$
- NNLO: g, Σ, VALENCE $V = \sum_{i=1}^{n_f}\left(q_i(x,Q^2)-\bar q_i(x,Q^2)\right)$, ANY N_f-1 LINEAR COMB. OF $q_i-\bar q_i$, ANY N_f-1 LINEAR COMB. OF $q_i+\bar q_i$

A COMMON CHOICE

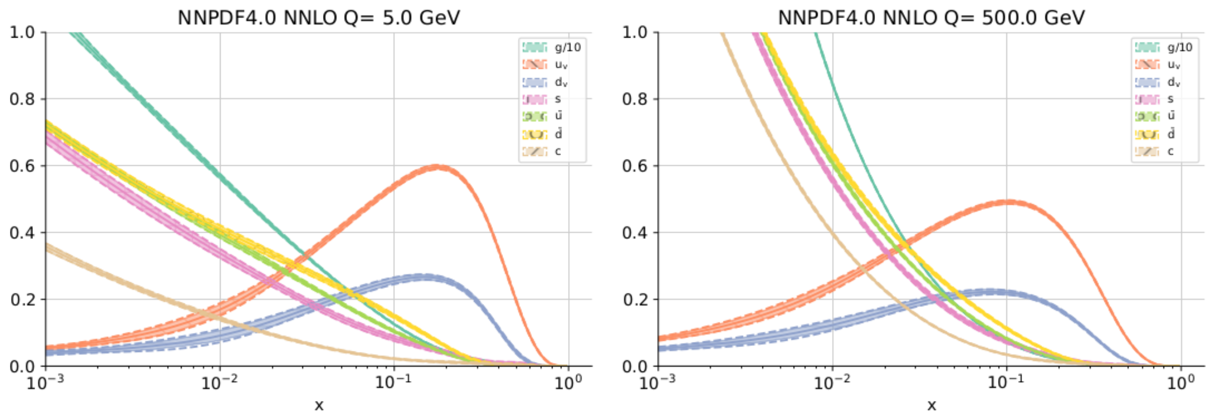
- GLUON g, SINGLET Σ, VALENCE V
- ITERATIVE NS COMBINATIONS OF $q_i^+ = q_i+\bar q_i$: $T_3 = u^+ - d^+$, $T_8 = u^+ + d^+ - 2s^+$, $T_{15} = u^+ + d^+ + s^+ - 3c^+ \ldots$ (labels $\to$ SU(N) gens.)
- ITERATIVE NS COMBINATIONS $q_i^- = q_i - \bar q_i$: $V_3 = u^- - d^-$, $V_8 = u^- + d^- - 2s^-$, $V_{15} = u^- + d^- + s^- - 3c^-, \ldots$

SCALE DEPENDENCE:
QUALITATIVE FEATURES (LEADING ORDER)



- AS Q^2 INCREASES, PDFS DECREASE AT LARGE x & INCREASE AT SMALL x DUE TO RADIATION
- GLUON SECTOR SINGULAR AT $N = 1 \Rightarrow$ GLUON GROWS MORE AT SMALL x
- $\gamma_{qq}(1) = 0 \Rightarrow$ BARYON NUMBER CONSERVATION (BEYOND LO $\Rightarrow \gamma_{qq}(1) = \gamma_{q\bar{q}}(1)$)
- $\gamma_{qq}(2) + \gamma_{qg}(2) = \gamma_{gq}(2) + \gamma_{gg}(2) = 0$ MOMENTUM CONSERVATION

PDFs: QUALITATIVE FEATURES



- SUM RULES ⇔ VALENCE BUMP
- GLUON ANOMALOUS DIMENSION SINGULAR AT $N = 1$ ⇔ SMALL x GROWTH OF GLUON
- SINGLET-GLUON MIXING ⇔ UNIVERSAL SMALL x BEHAVIOR OF ALL PDFs
- SIGN CHANGE OF ANOMALOUS DIMENSIONS (BARYON & MOMENTUM CONSERVATION) ⇔ DEPLETION OF LARGE x PDFs AT HIGH SCALE
- $\gamma_{qq}^{NS,-} \neq \gamma_{qq}^{NS,s} \Leftrightarrow q_i(x) - \bar{q}_i(x)$ DOES NOT EVOLVE MULTIPLICATIVELY $\Rightarrow$ CANNOT VANISH AT ALL SCALES

PDF FEATURES: SUMMARY

- VALENCE SUM RULE + KINEMATIC VANISHING $\Leftrightarrow$ VALENCE PEAK
- COLLINEAR FACTORIZATION $\Rightarrow$ ANOMALOUS DIMENSIONS & PDFs SHARE SINGULARITIES
- PERTURBATIVE EVOLUTION $\Rightarrow$ SMALL x RISE
- MOMENTUM SUM RULE $\Rightarrow$ STRONGER SMALL x RISE $\Leftrightarrow$ LARGE x DEPLETION

FOOD FOR THOUGHT

QUESTIONS

1. ARE PDFS PROBABILITIES? ARE THEY POSITIVE? IF NOT, WHY NOT?

2. CAN YOU UNDERSTAND WHY AT HIGHER ORDERS THERE ARE MORE INDEPENDENT ANOMALOUS DIMENSIONS?

   FOR INSTANCE WHY IS γ_{qq} FOR SINGLET AND NONSINGLET THE SAME AT LO AND NOT AT NLO?

   WHICH ARE THE FEYNMAN DIAGRAMS THAT DRIVE THE DIFFERENCE?

3. WHY DO SUM RULES HOLD AT ALL SCALES?

   CAN YOU EXPLAIN THIS IN TERMS OF ANOMALOUS DIMENSIONS OF OPERATORS?

4. DO YOU EXPECT PDFS TO VANISH WHEN $x \to 1$? WHY?