A determination of $\alpha_s(m_Z)$ at aN^{3}LO$_{\text{QCD}}\otimes$NLO$_{\text{QED}}$ from a global PDF analysis

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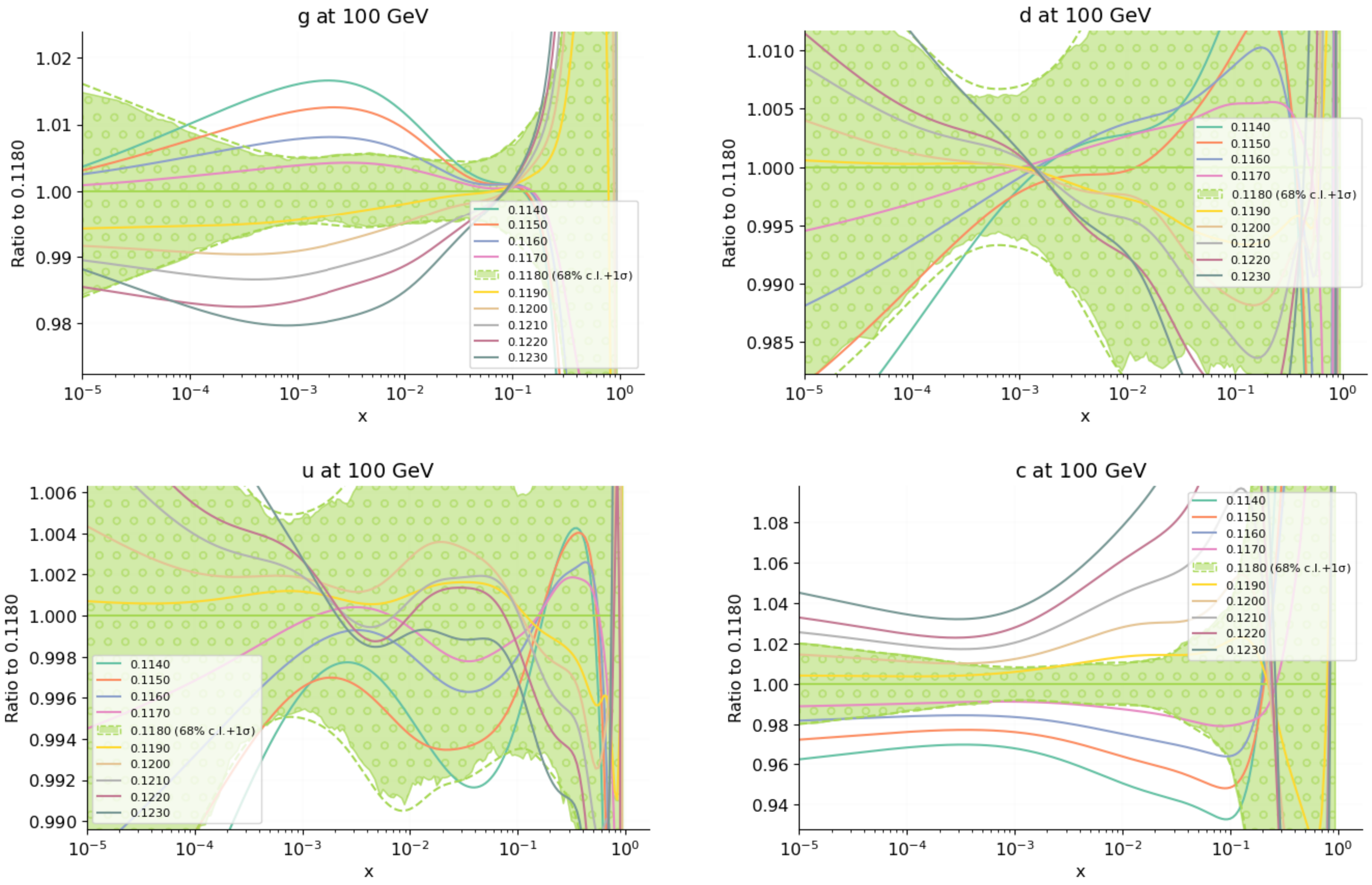
The University of Edinburgh

Cape Town, 25 March 2025

PDFs and α_s are correlated



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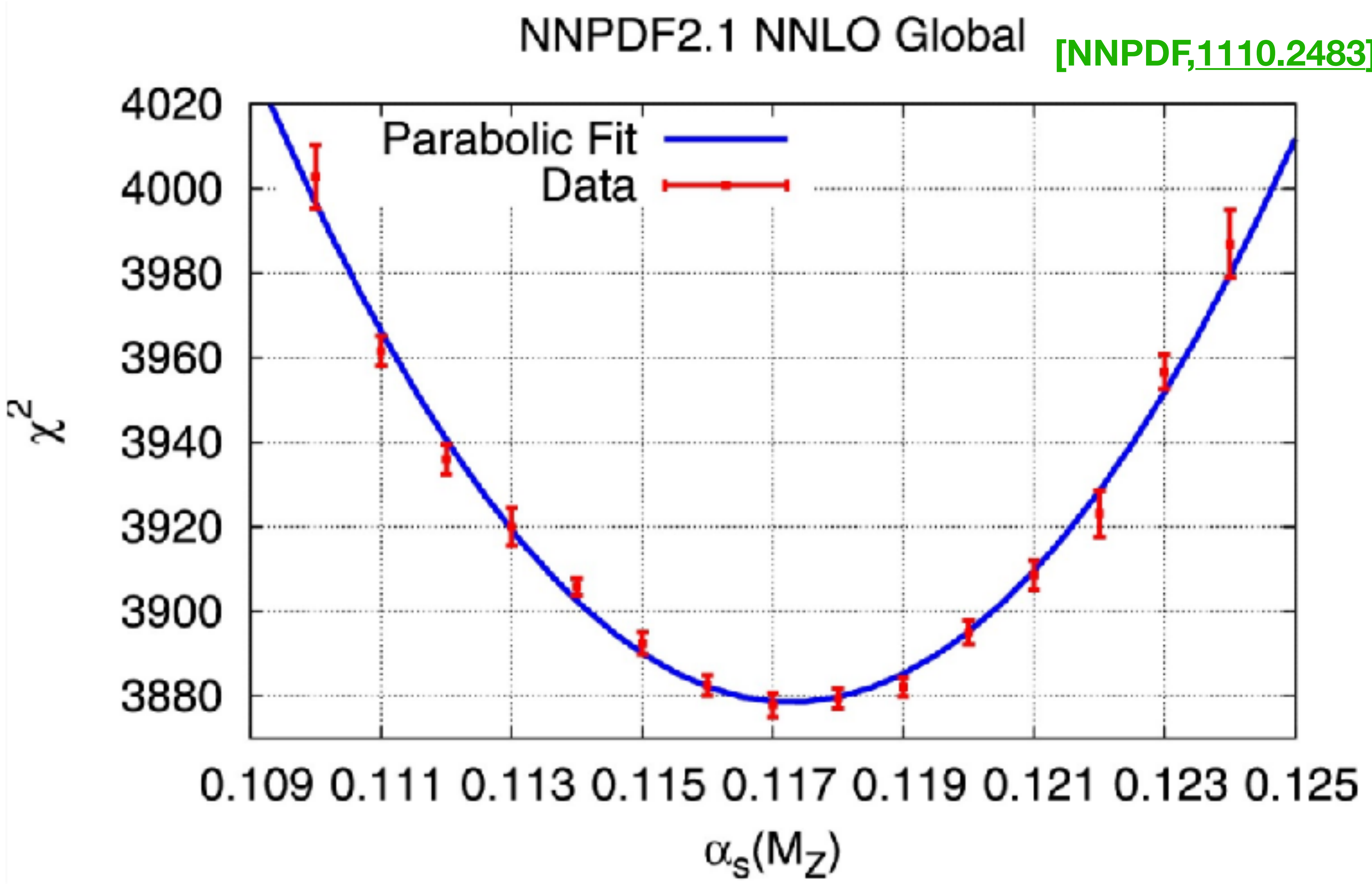

[NNPDF,1110.2483]

In many cases α_s is determined by extracting it from a parabolic fit to the χ^2 profile

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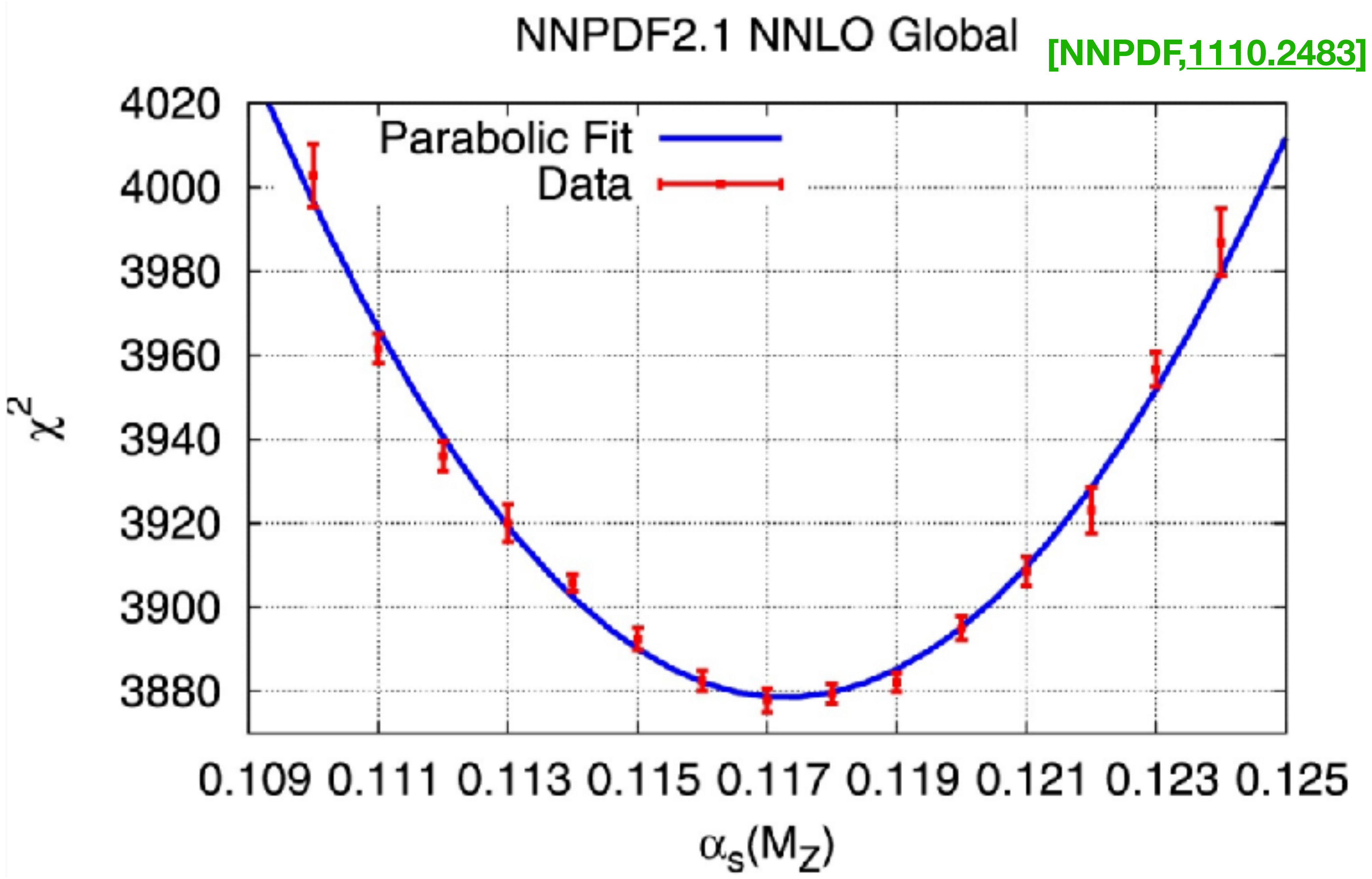

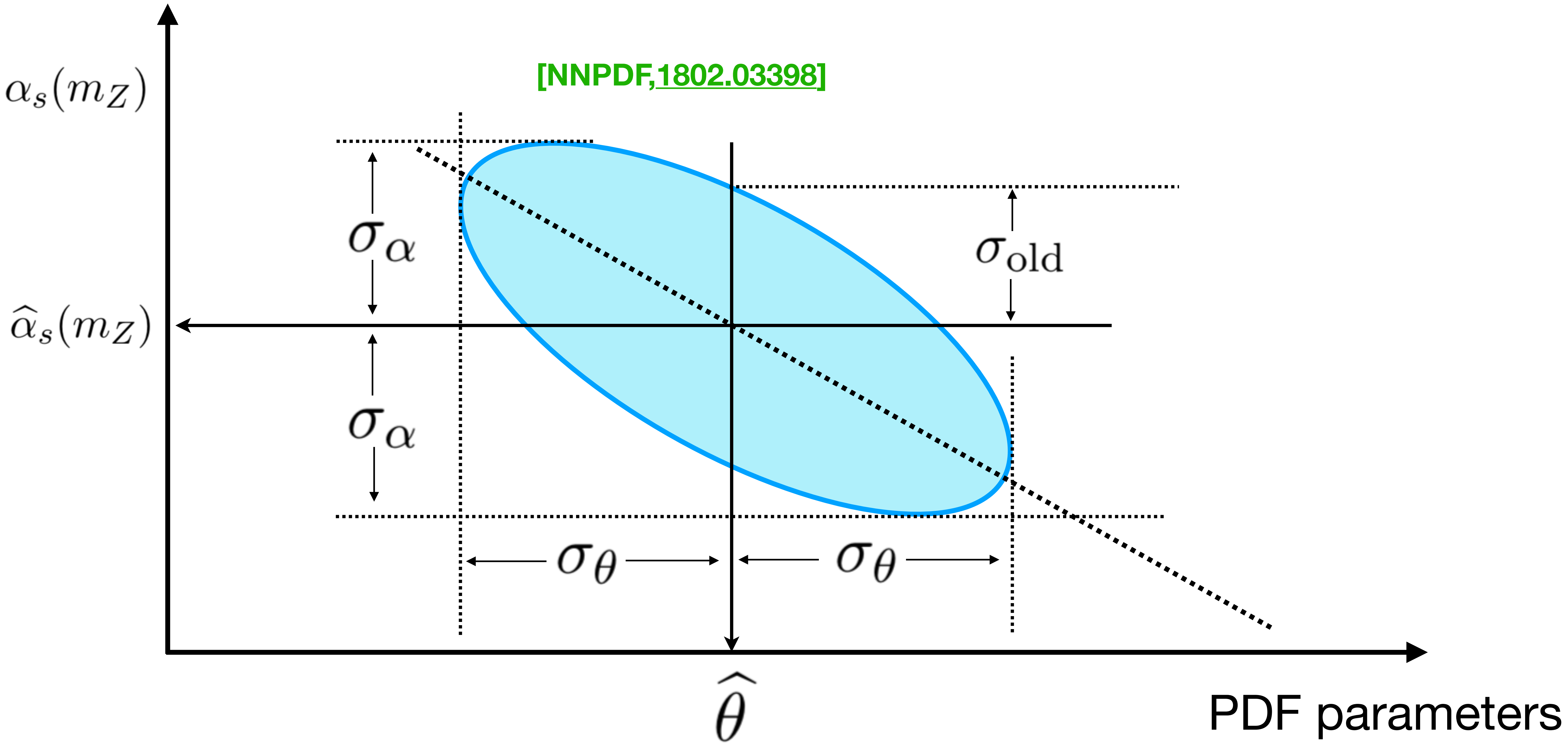

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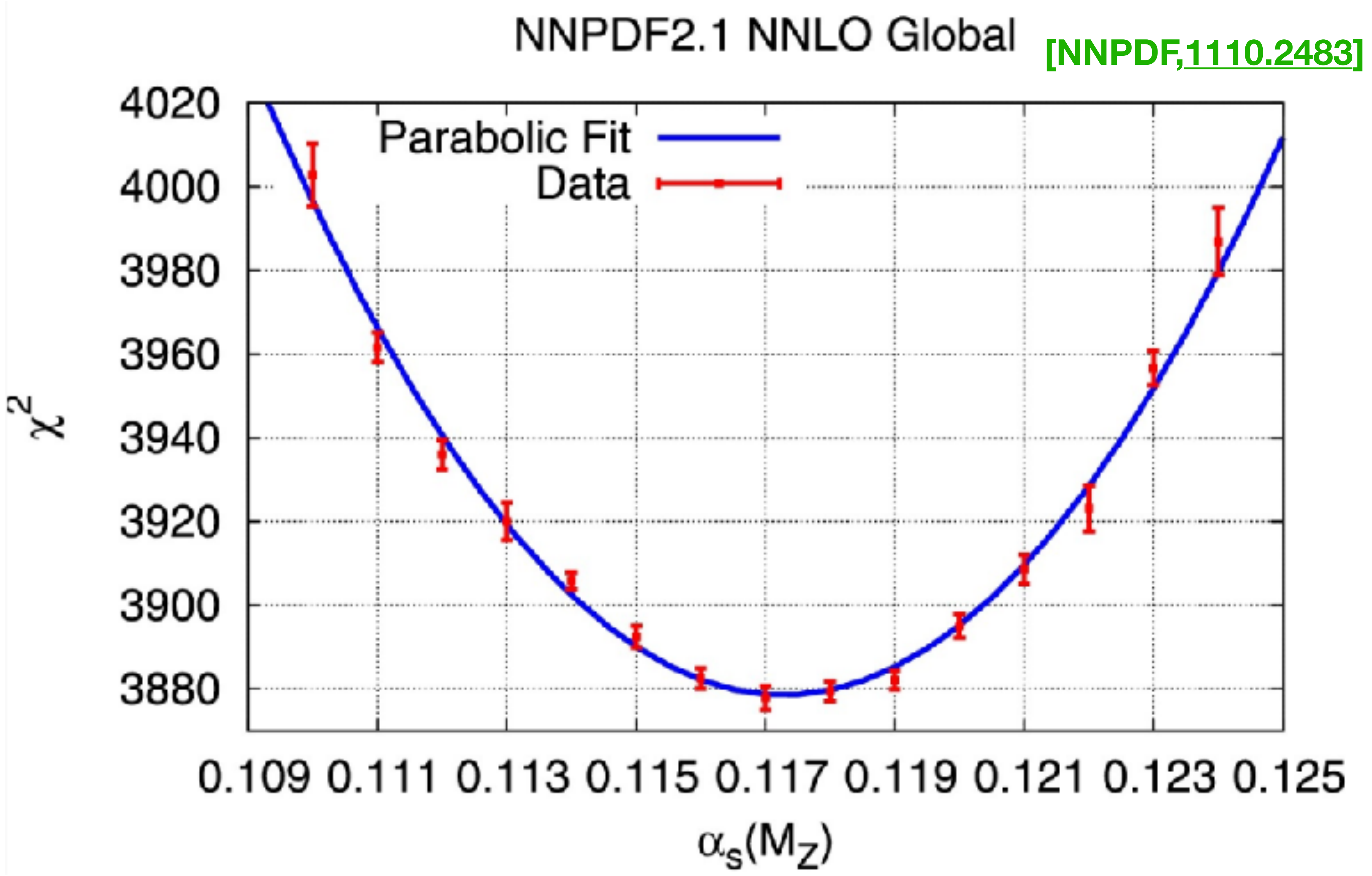

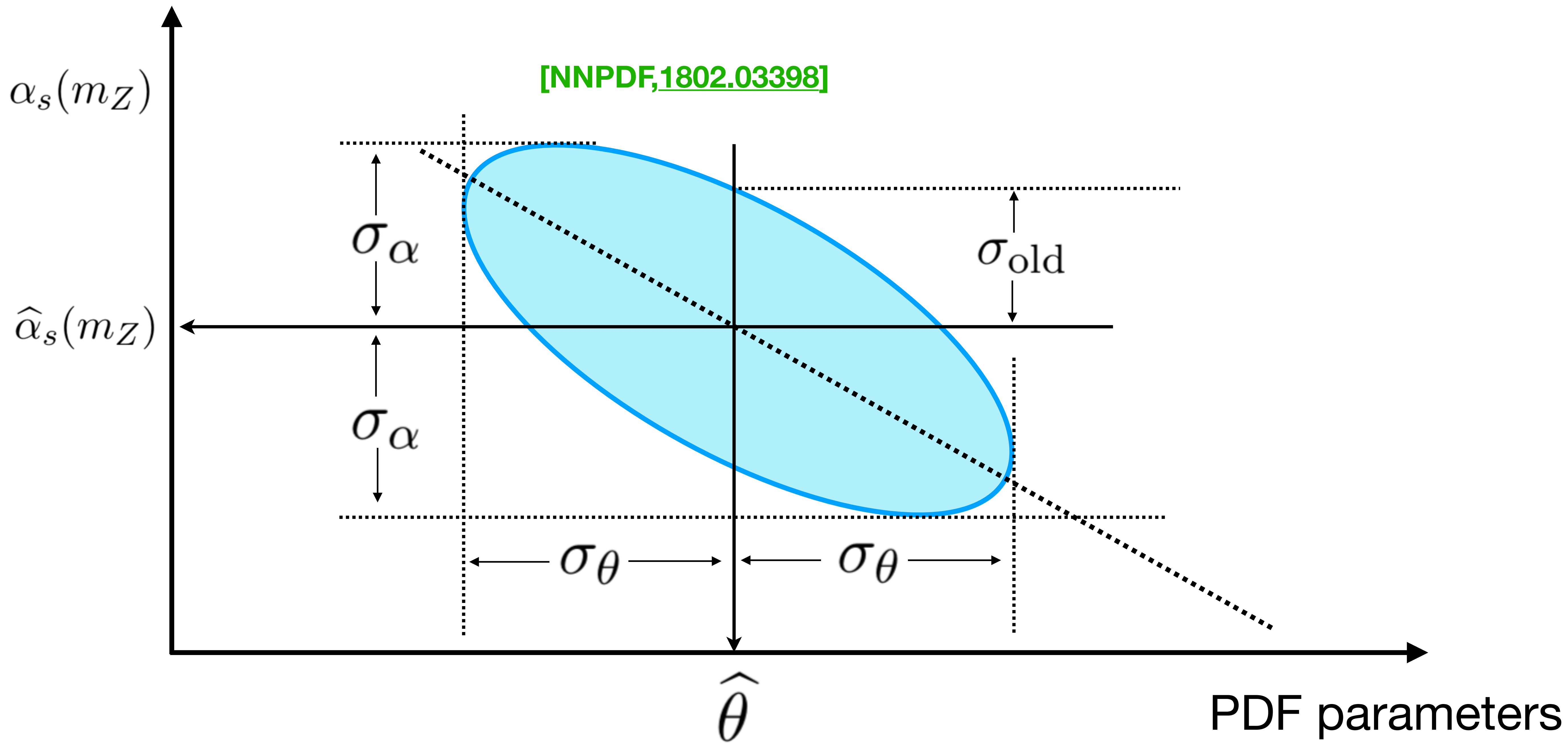

In many cases α_s is determined by extracting it from a parabolic fit to the χ^2 profile

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Ideally minimise α_s and PDF simultaneously

How to account for correlations between PDFs and α_s?

NNPDF can't (easily) treat α_s as another trainable parameter

Rerunning Monte Carlo generators and DGLAP evolution at every training step is not feasible, therefore predictions are stored in precomputed grids

Unlike partonic cross-sections, DGLAP is not a simple expansion in α_s

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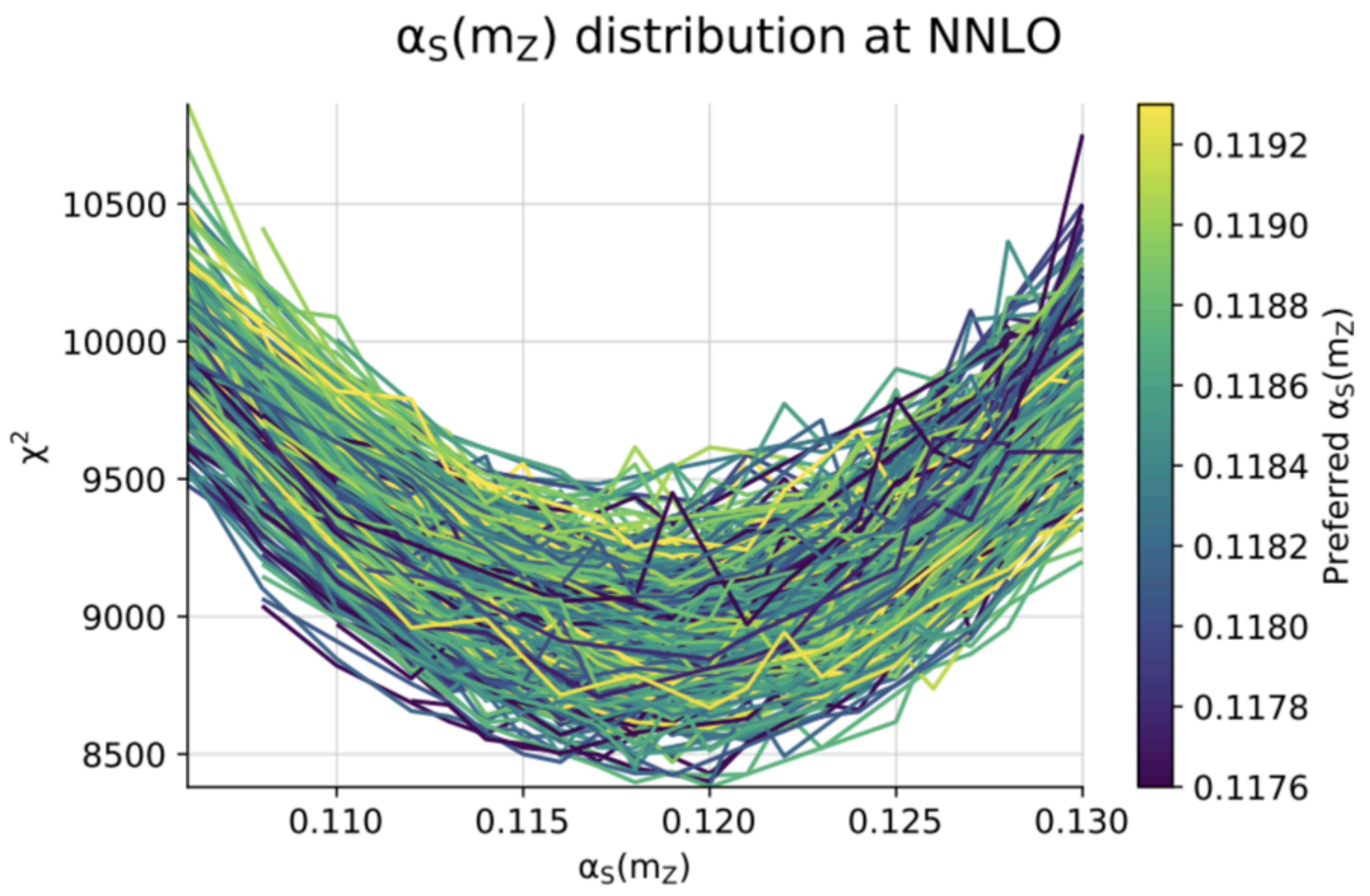
Unlike partonic cross-sections, DGLAP is not a simple expansion in α_s

Two methods have been developed to avoid this limitation:

1) **Multiple fits** of the same data replica, changing only the value of $\alpha_s(m_Z)$, thereby **correlating PDFs** at different $\alpha_s(m_Z)$
   [NNPDF, 1802.03398]
2) Based on **a single fit** with an $\alpha_s(m_Z)$ **theory covmat,** and computing the fit's preferred value for alphas a posteriori in a Bayesian framework
   [Ball, Pearson, 2105.05114]

The results shown in this talk correspond to the theory covmat method, but agreement is always within 1 per-mille



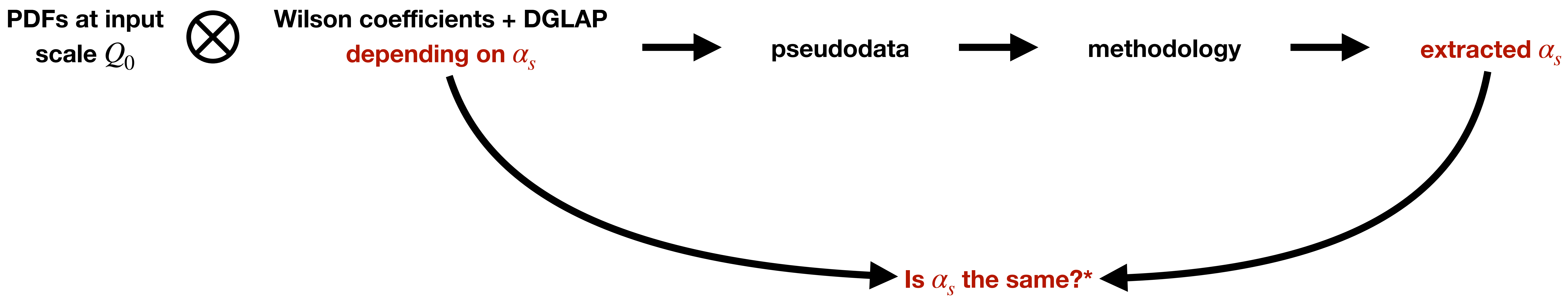
Correlated replicas fitted to the same data replica at different α_s

- Validation
- Results

Q: How to validate the methodologies?

A: Closure tests [Del Debio, Giani, Wilson, 2111.05787]

Basic idea: generate a global pseudo dataset from theory predictions and extract α_s from this

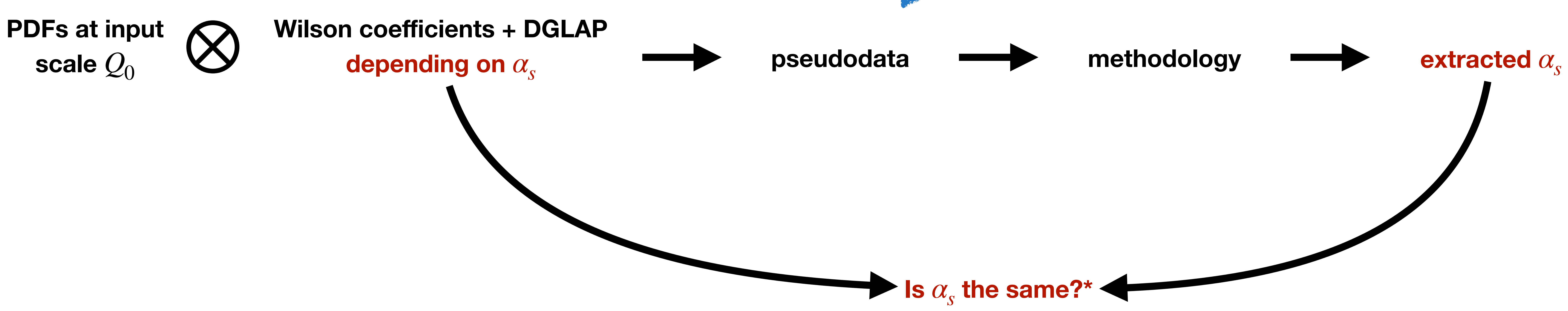


Q: How to validate the methodologies?

A: Closure tests [Del Debio, Giani, Wilson, 2111.05787]

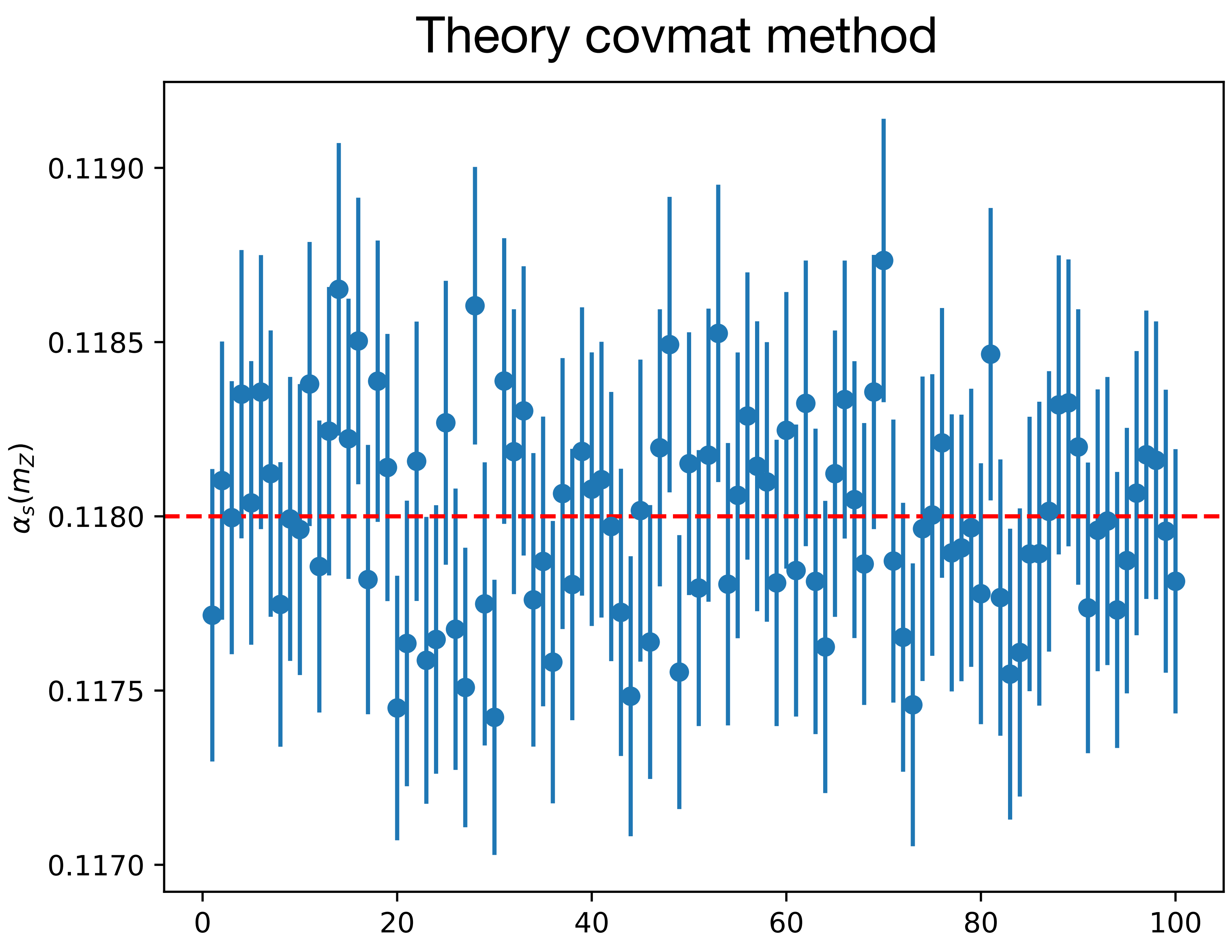
* Experimental data is sampled from a distribution, therefore pseudodata = prediction + noise

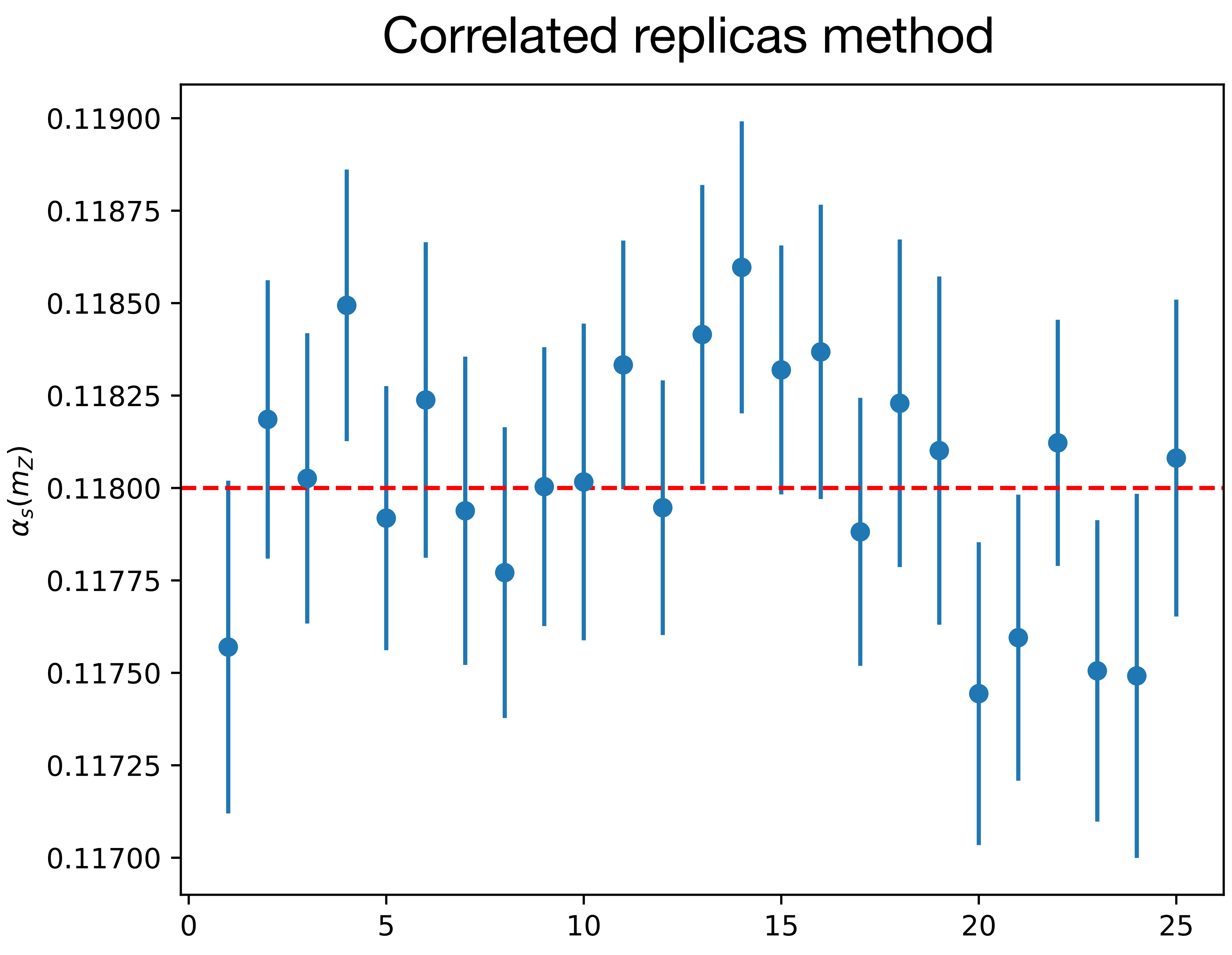
Basic idea: generate a global pseudo dataset from theory predictions and extract α_s from this



Validating the methodologies

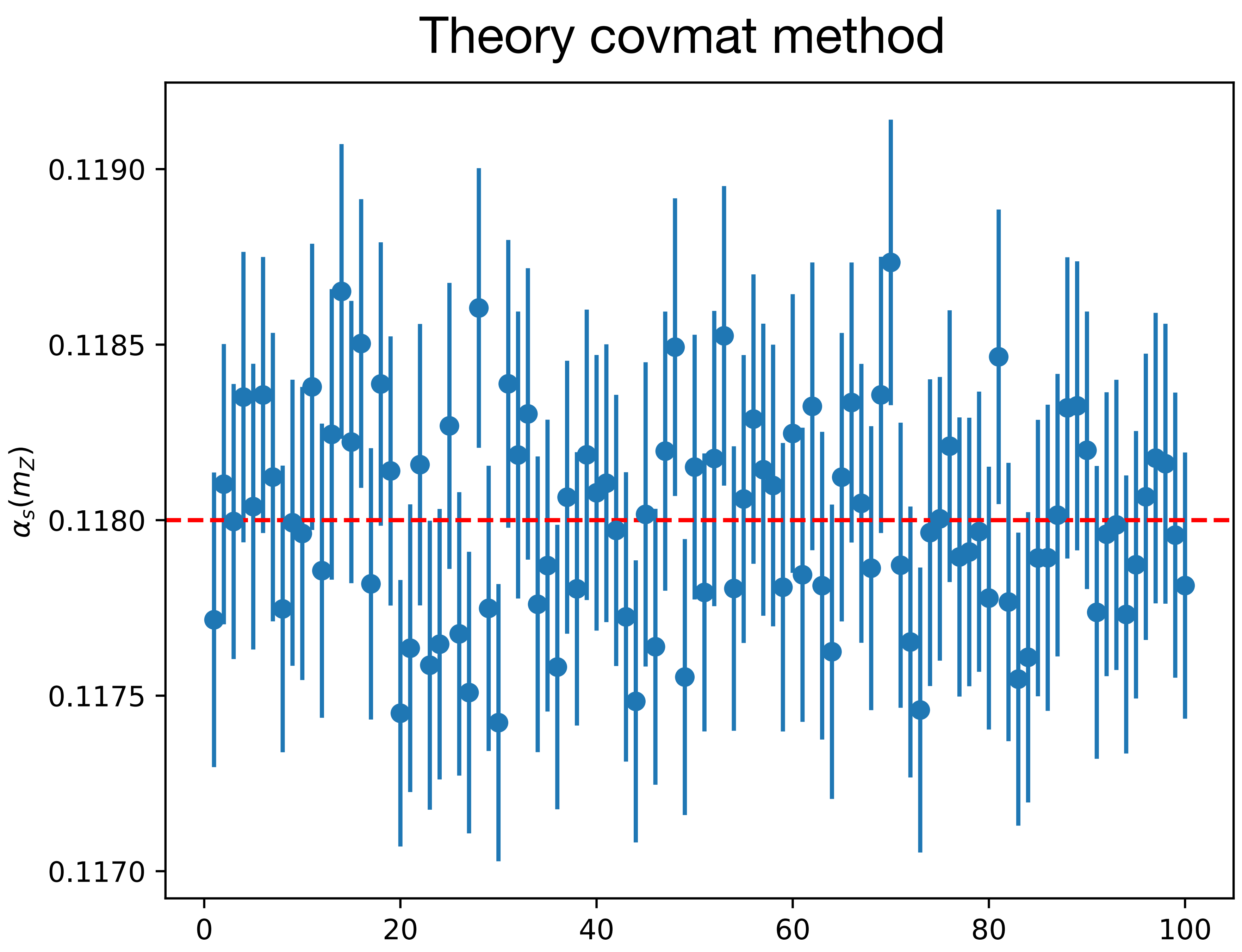
1) Generate pseudodata samples around $\alpha_s(m_Z) = 0.118$
2) Extract $\alpha_s(m_Z)$ for each pseudodata sample



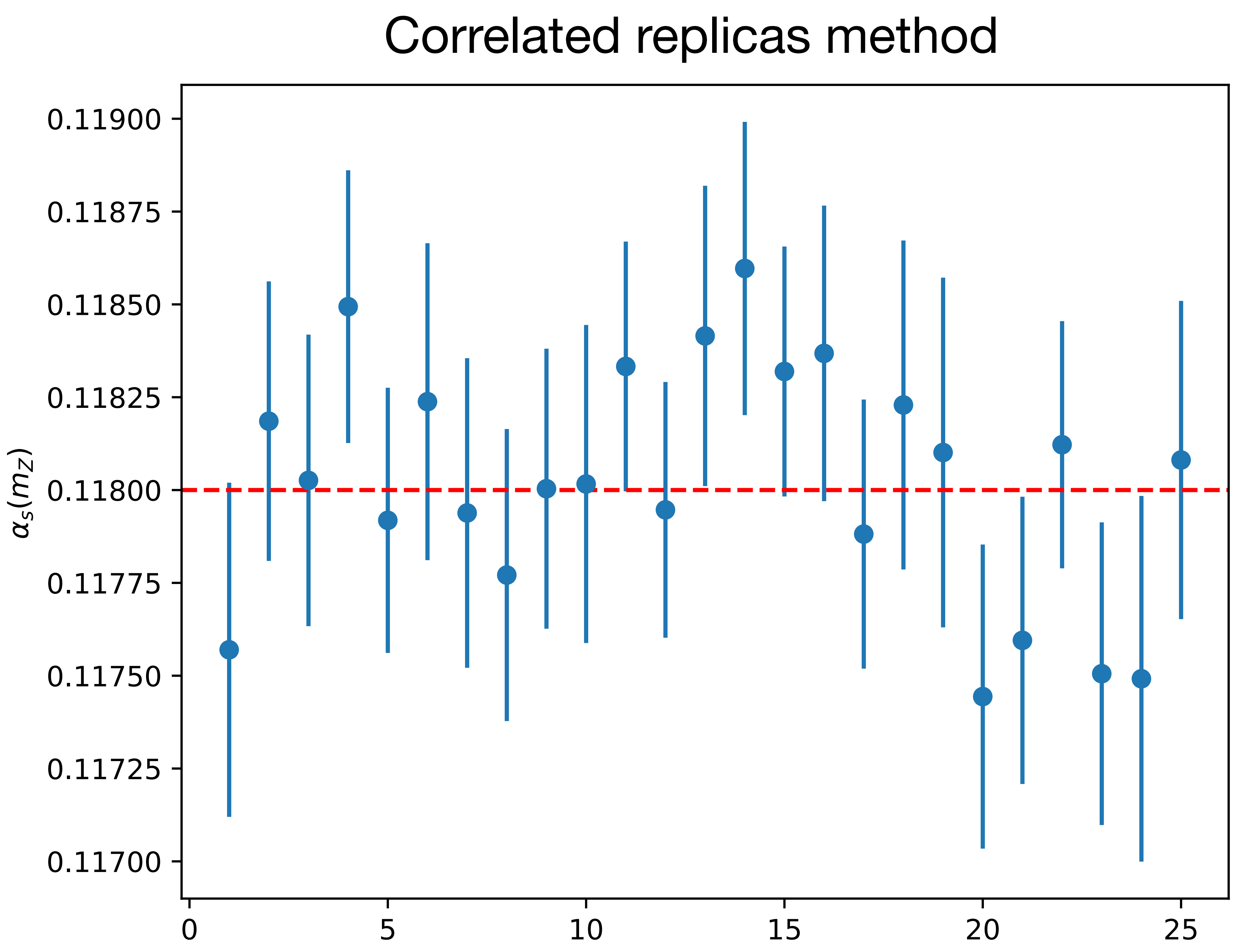


Validating the methodologies

1) Generate pseudodata samples around $\alpha_s(m_Z) = 0.118$
2) Extract $\alpha_s(m_Z)$ for each pseudodata sample
3) Check that our method returns the correct answer



$\alpha_s(m_Z) = 0.11800(4)$ ✓



$\alpha_s(m_Z) = 0.11804(8)$ ✓

Both methodologies pass the closure test!

Closure tests are a non-trivial check

Initially we were getting very large values for $\alpha_s(m_Z)$

Was this correct, or were we making a mistake?
How can we know?

Closure tests are a non-trivial check

Initially we were getting very large values for $\alpha_s(m_Z)$

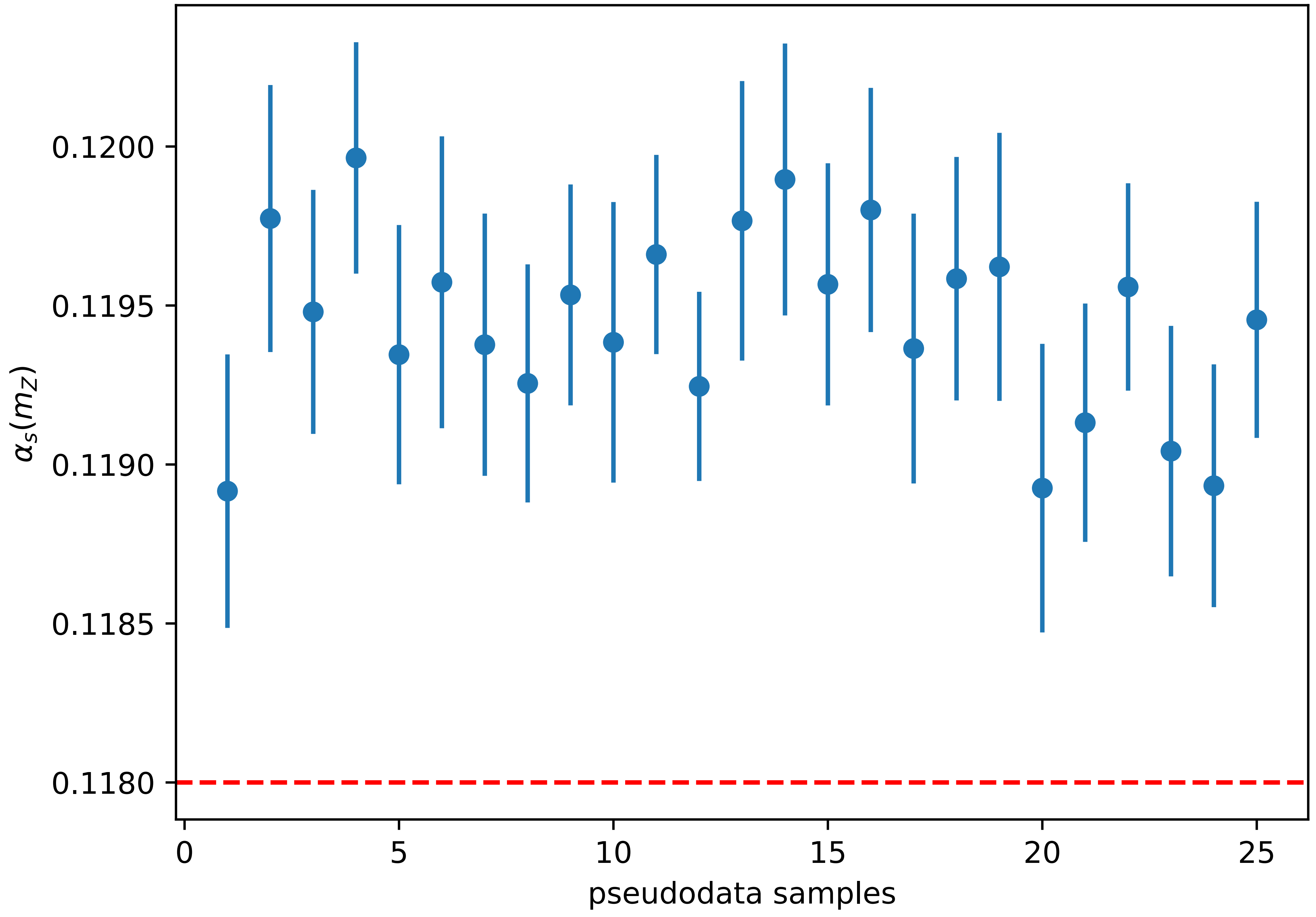
Was this correct, or were we making a mistake?
How can we know?

We can find out with a closure test!

Closure test pseudodata: $\alpha_s(m_Z) = 0.118$

Closure test result: $\alpha_s(m_Z) = 0.1197(6)$

This confirmed a problem with our methodology that we identified and fixed!



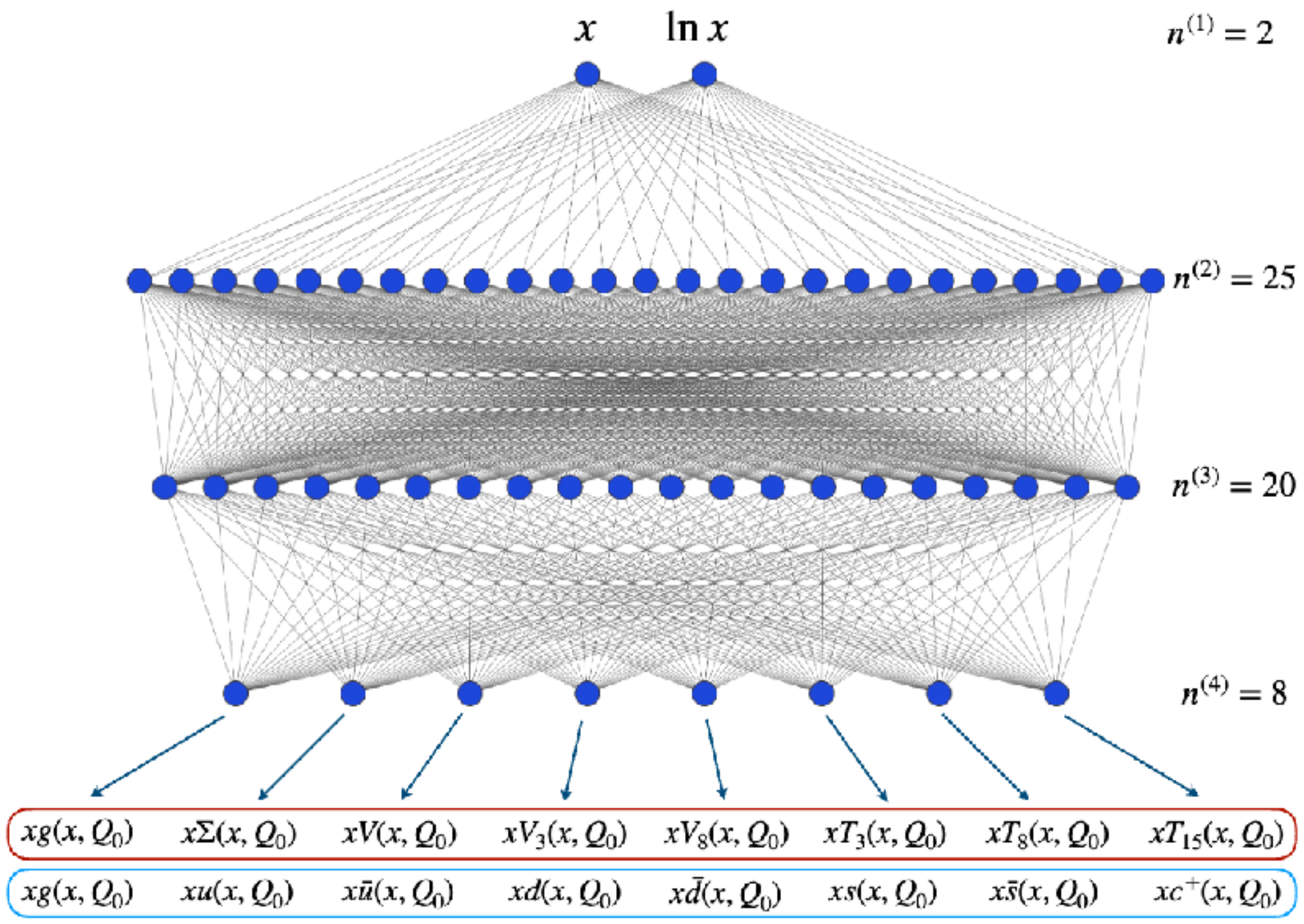
- Validation
- Results

Comparing to $\alpha_s(M_Z)$ based on NNPDF3.1 - methodology

NNPDF3.1: $\alpha_s(M_Z) = 0.1185(5)^{\mathrm{PDF}}$ **[NNPDF, 1802.03398]**

Changes in this determination based on NNPDF4.0:

- Fitting methodology (gradient descent, hyperoptimisation, single NN for all flavours…)
- Theory (MHOU, QED, aN^{3}LO)
- Data



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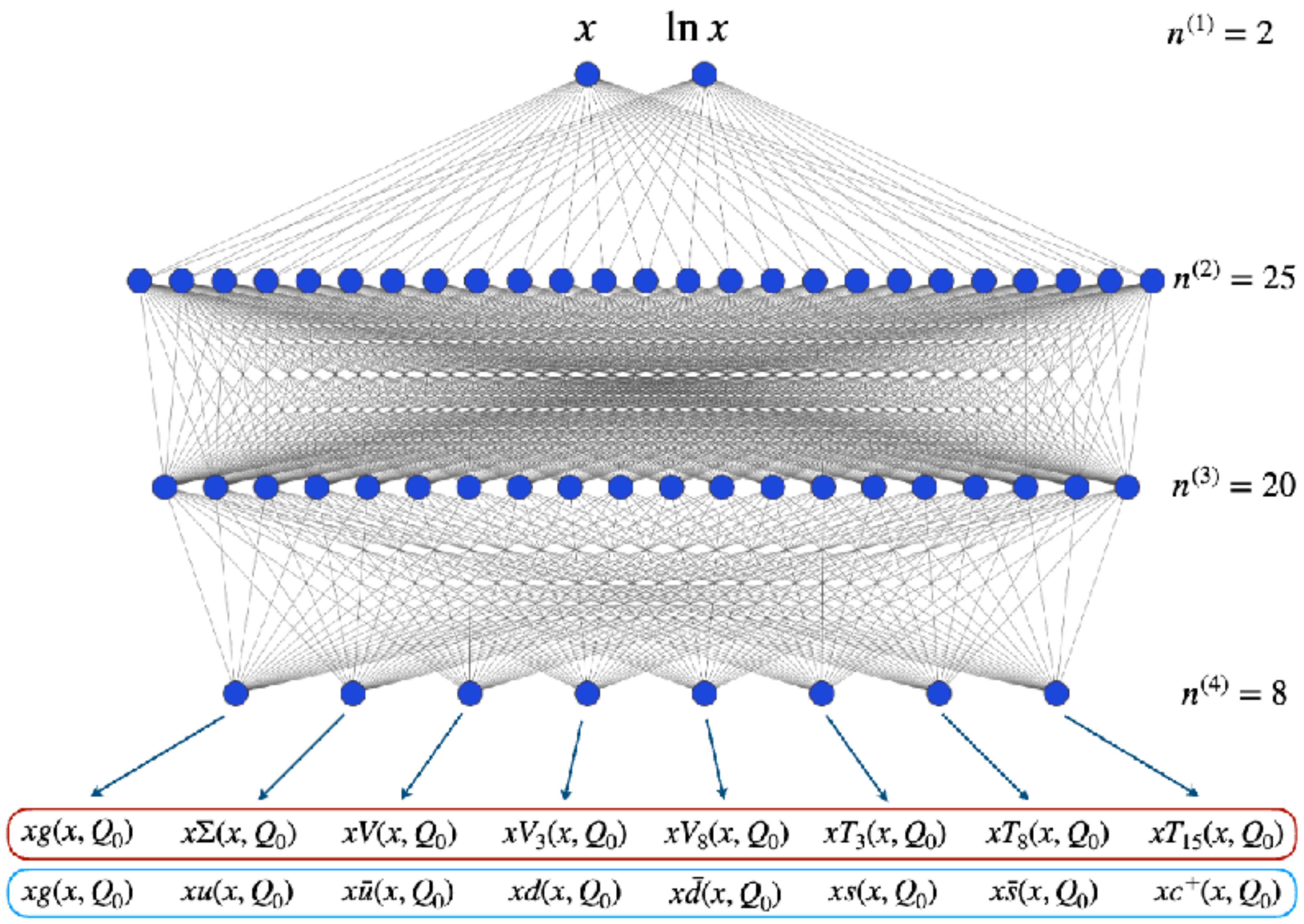
Let's first look at the **methodology**:

NNPDF4.0 methodology, NNPDF3.1-like dataset: $\alpha_s(M_Z) = 0.1188(5)^{\mathrm{PDF}}$

Consistent with the NNPDF3.1 result!



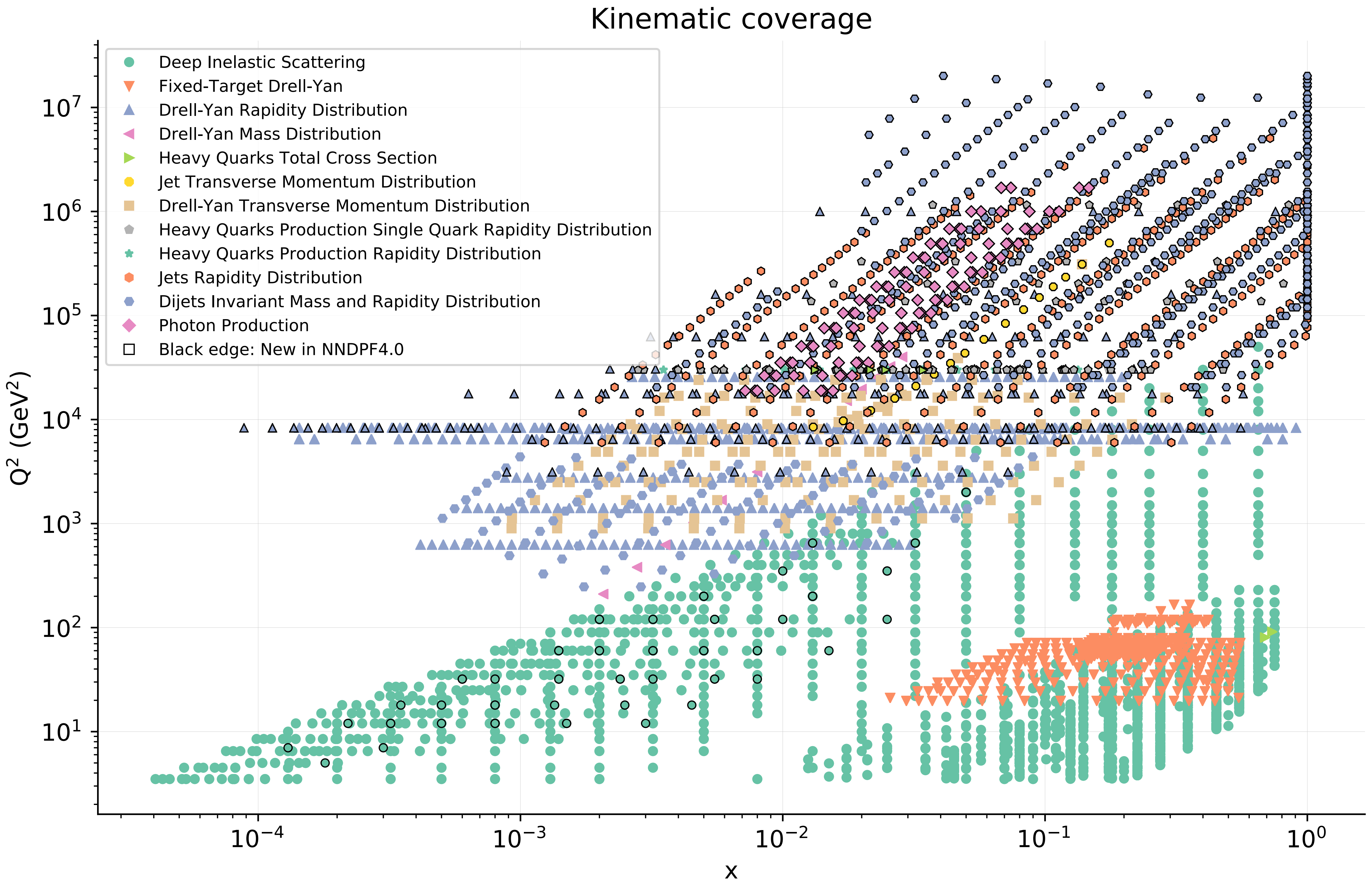
Comparing to $\alpha_s(M_Z)$ based on NNPDF3.1
missing higher order uncertainties

A big change is in the treatment of missing higher order uncertainties

MHOUs in NNPDF3.1 from $\alpha_s(m_Z)_{\rm NNLO} - \alpha_s(m_Z)_{\rm NLO}$

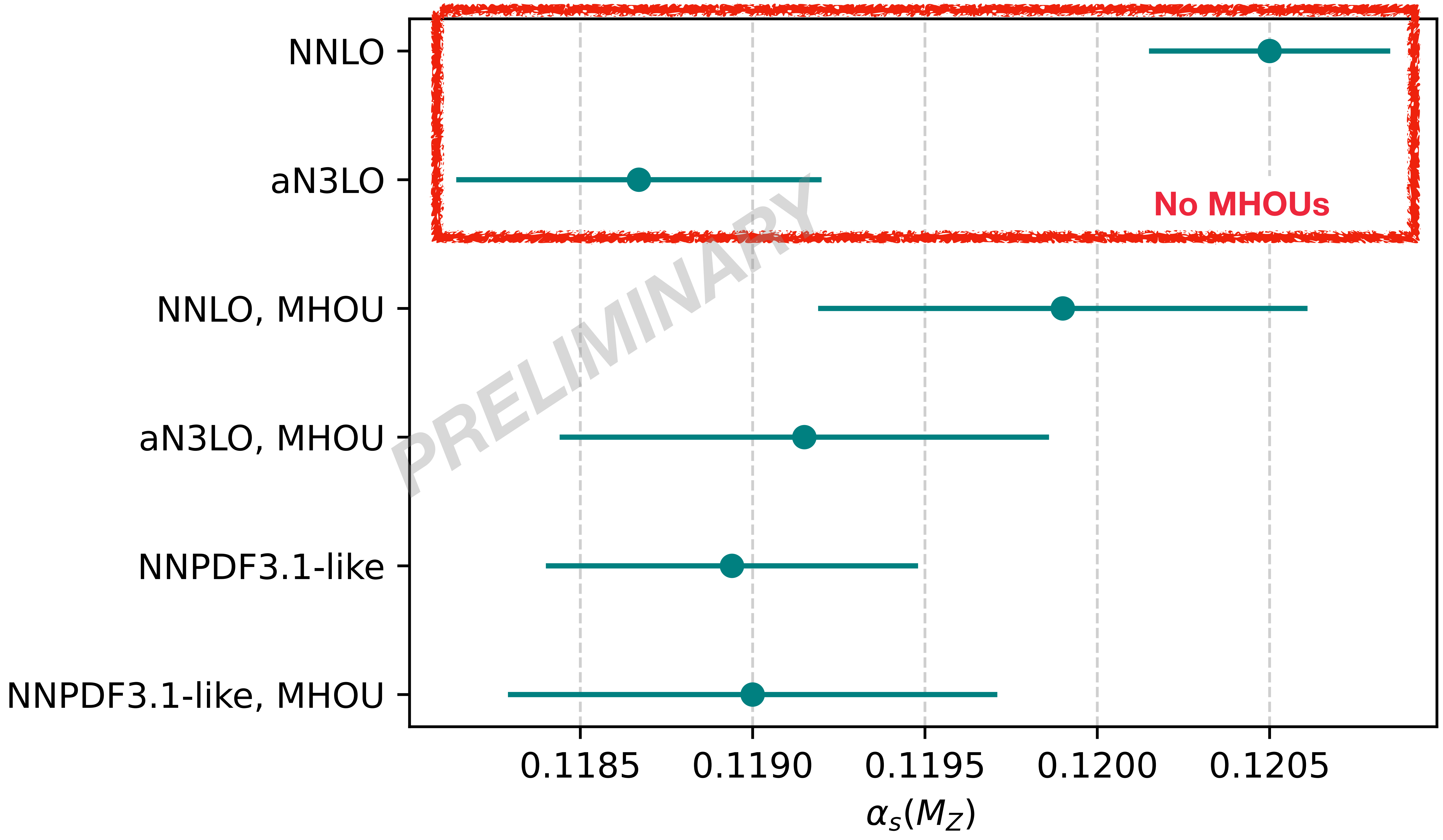
In this NNPDF4.0-based determination we include a **theory covariance matrix from scale variations** at the level of the fit **[NNPDF, 2401.10319]**

NNPDF3.1: $\alpha_s(m_Z) = 0.1185(5)^{\rm PDF}(1)^{\rm meth}(11)^{\rm MHOU} = 0.1185(12)$
NNPDF4.0, NNPDF3.1-like data: $\alpha_s(m_Z) = 0.1190(7)^{\rm PDF+MHOU}$



This determination also benefits from the full NNPDF4.0 dataset

Impact of missing higher order uncertainties (MHOUs) and aN^{3}LO [NNPDF, 2402.18635]



These and following results for the NNPDF4.0 dataset

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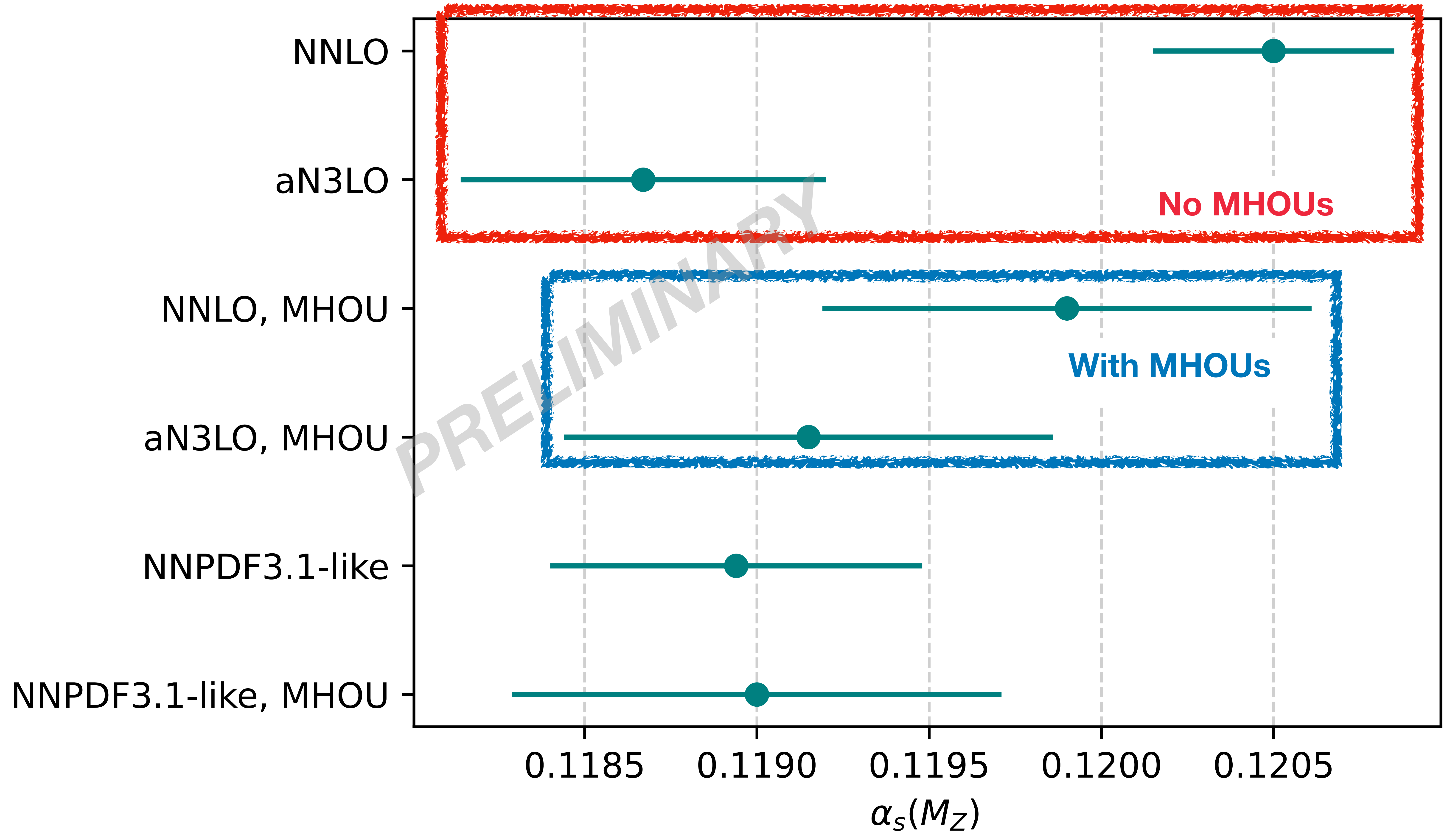

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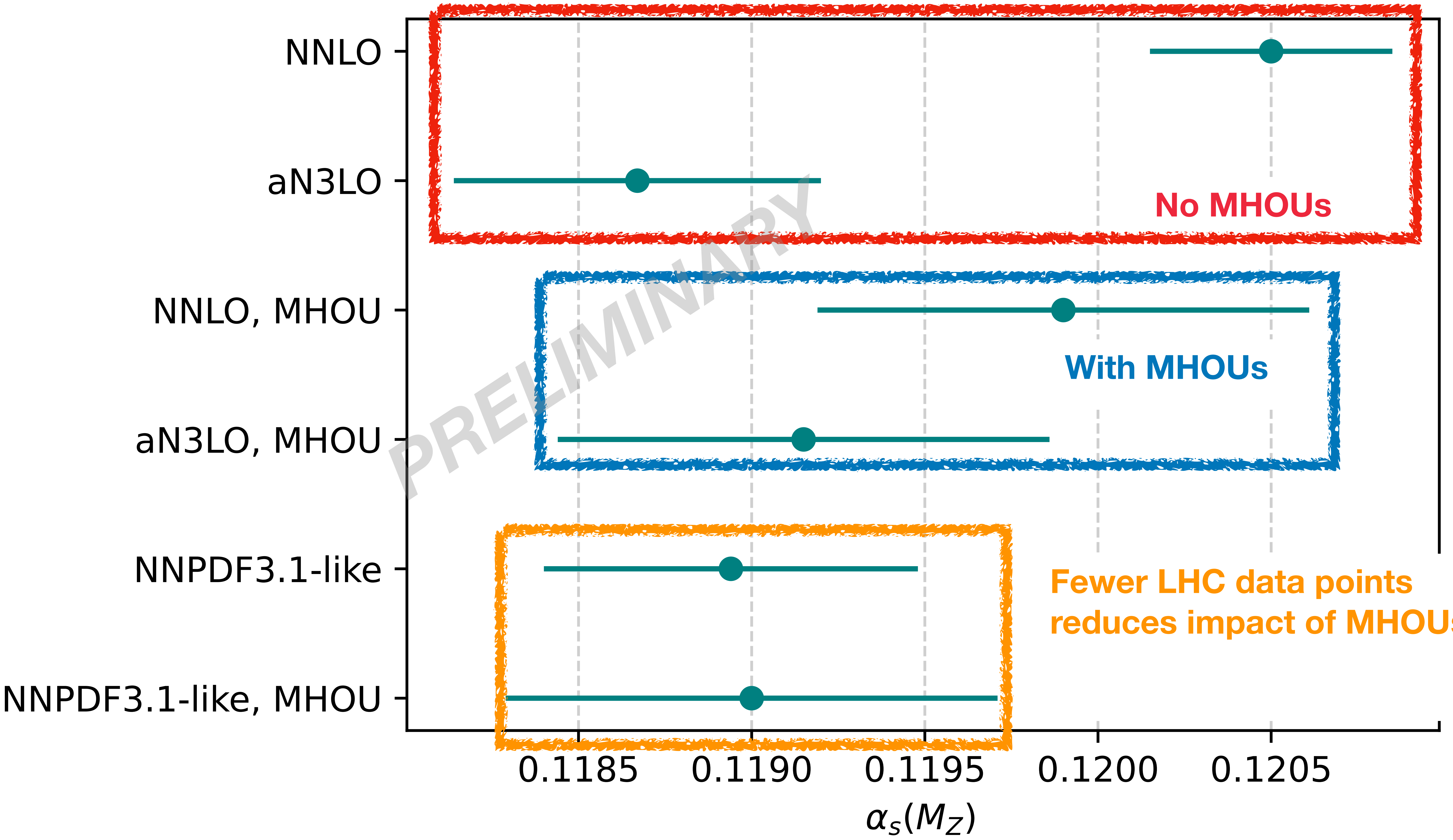
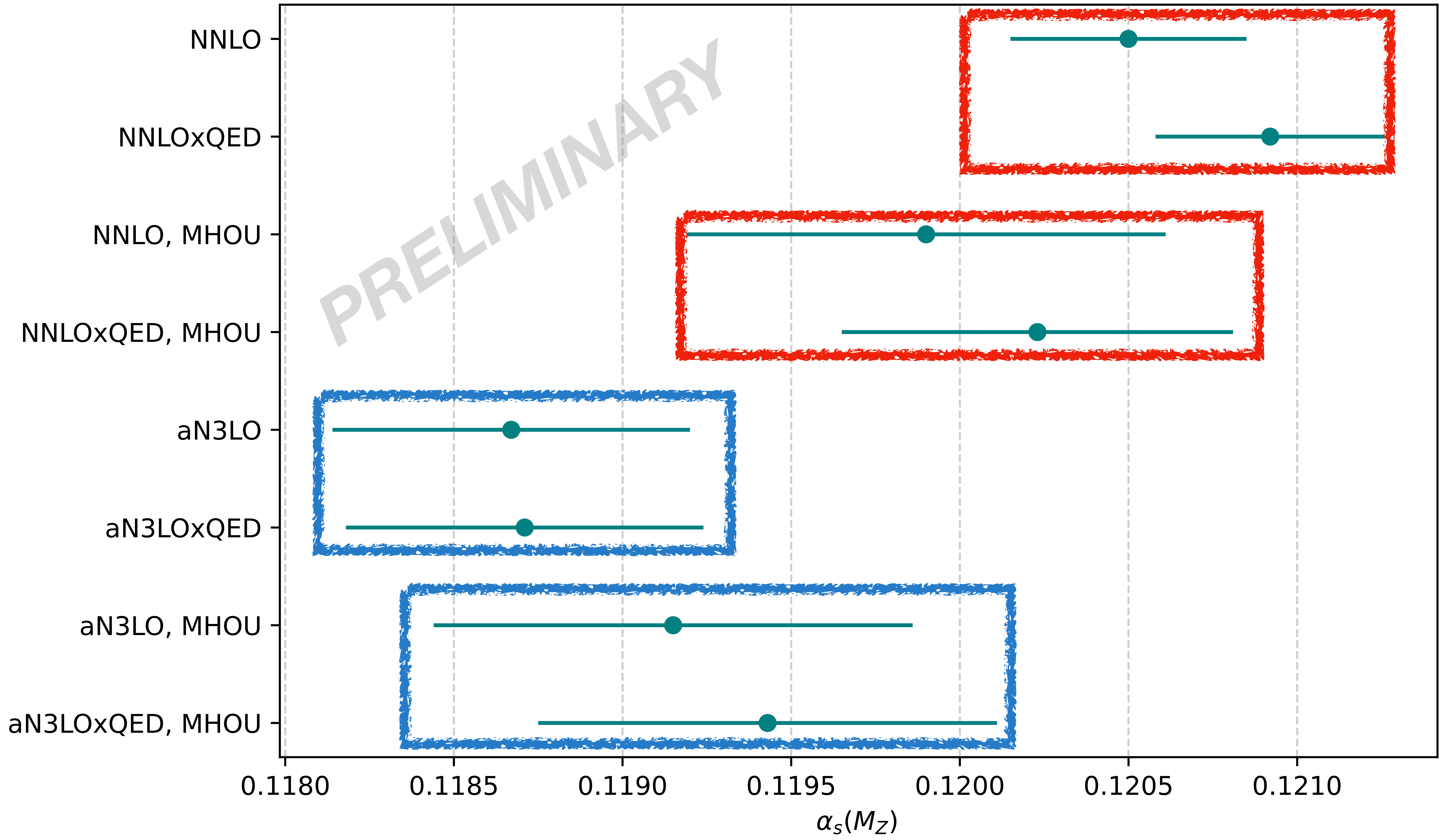


Fewer LHC data points reduces impact of MHOUs?

These and following results for the NNPDF4.0 dataset

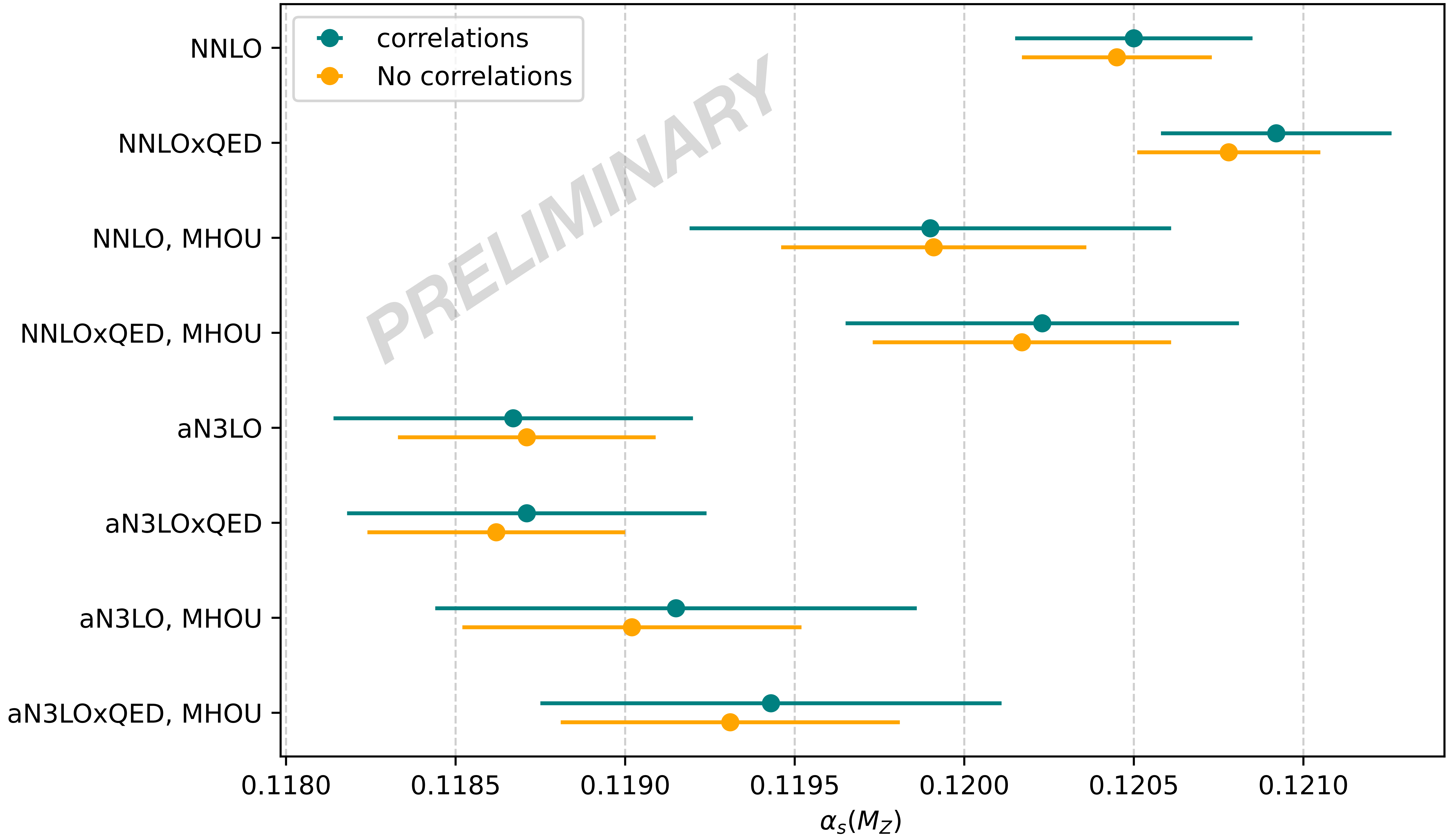
Impact of QED corrections and the photon PDF [NNPDF, 2401.08749]



- NLO QED corrections to DGLAP evolution
- Determine also the photon PDF

QED has a bigger impact at **NNLO** than at **aN3LO**

Impact of PDF-α_s correlations



Correlations increase the uncertainty by 25% to 60%

$\alpha_s(m_Z)$ at different values of m_t pole mass

- PDG value is $m_t = 172.4(7)$

How should we account for these uncertainties?

- Include m_t covmat requires computing grids for all datasets at the given m_t values (expensive)
- Add in quadrature, interpolating to the PDG uncertainties (negligible)

m_t	NNLO	NNLO, MHOU
175	0.1208(4)	0.1200(6)
172.5	0.1204(4)	0.1200(7)
170	0.1200(4)	0.1198(6)

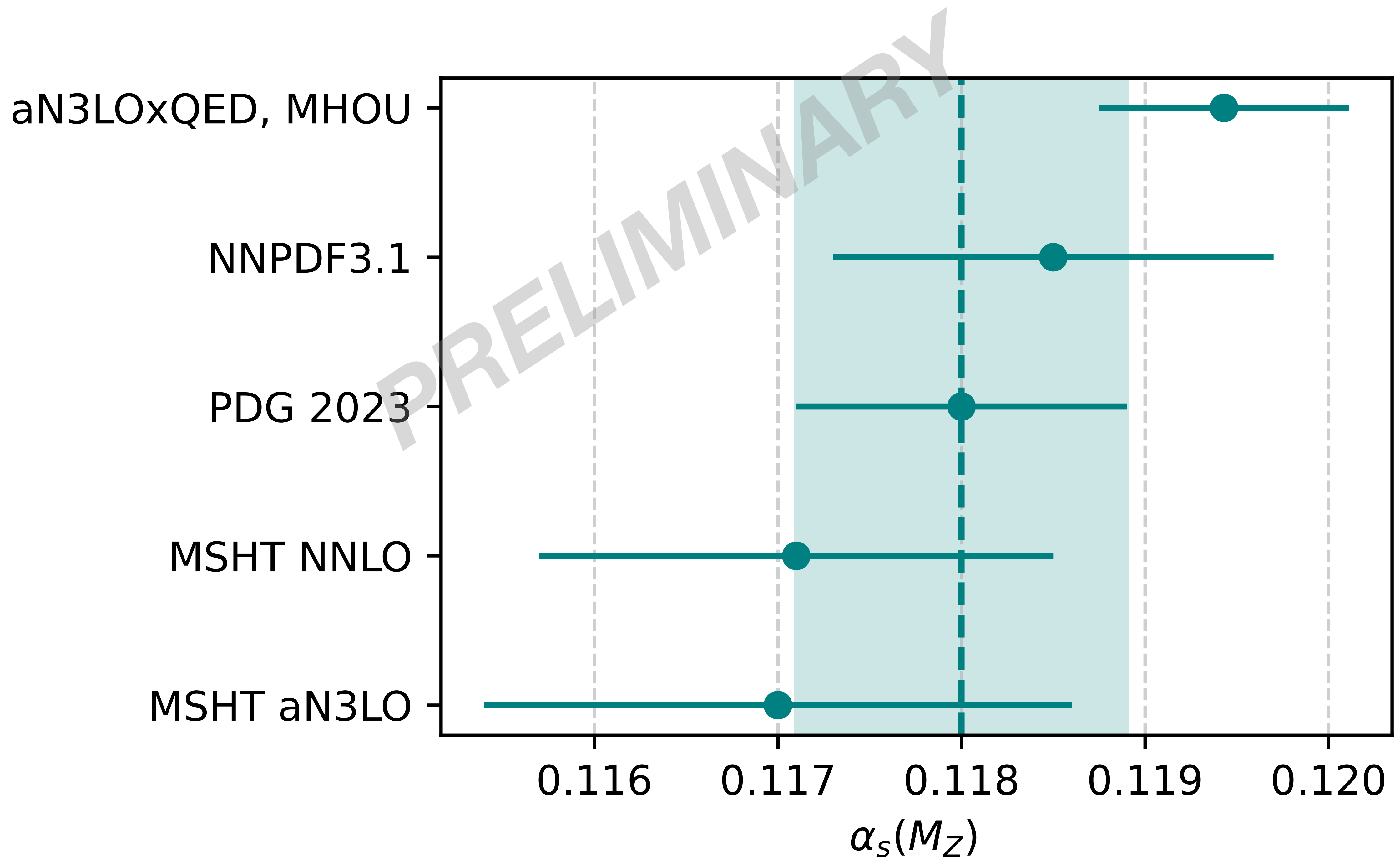
PRELIMINARY

Our most accurate results

$$\alpha_s(M_Z)^{\mathrm{aN3LO,QED,MHOU}} = 0.1194(7)$$

Summary and Outlook

- Strong correlations between the PDFs and α_s means that a simultaneous determination is needed
- Two methods agree within 1 per-mille for all cases
- Our methodologies have been validated by means of closure testing
- MHOUs improve the perturbative stability of α_s
- All effects (aN3LO, MHOU, QED) have to be considered simultaneously! Other methodological or theoretical effects to explore?
- $\alpha_s(M_Z)^{\text{aN3LO,QED,MHOU}} = 0.1194(7)$
- Next: simultaneous m_t, $\sin\theta_W$, … ?



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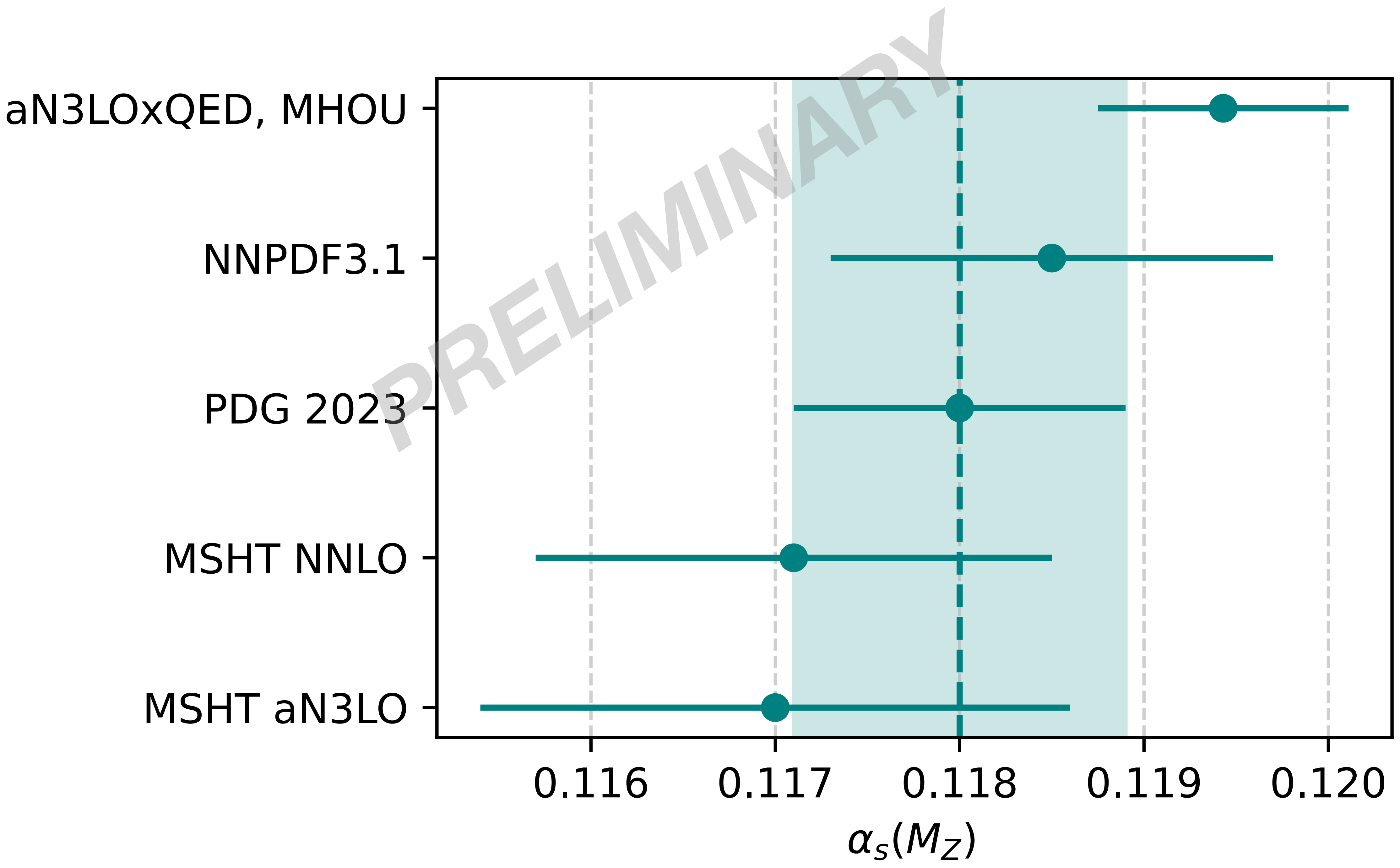


Thank you for your attention!

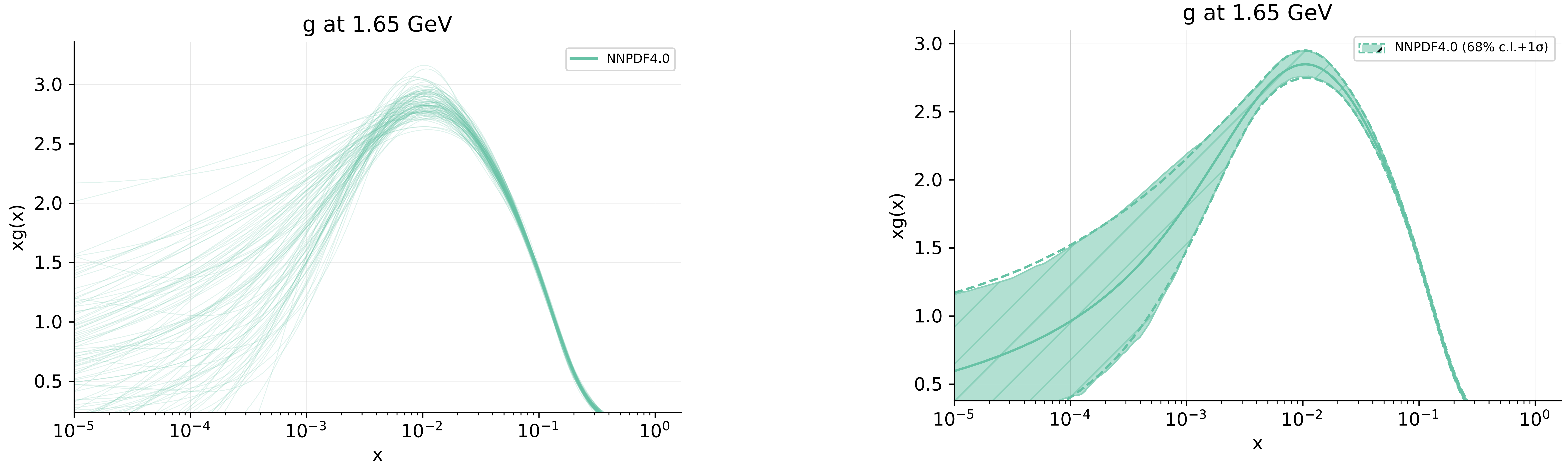
Backup slides

Propagating experimental uncertainty to PDFs

An NNPDF set (usually) consists of 100 PDF replicas produced as follows:

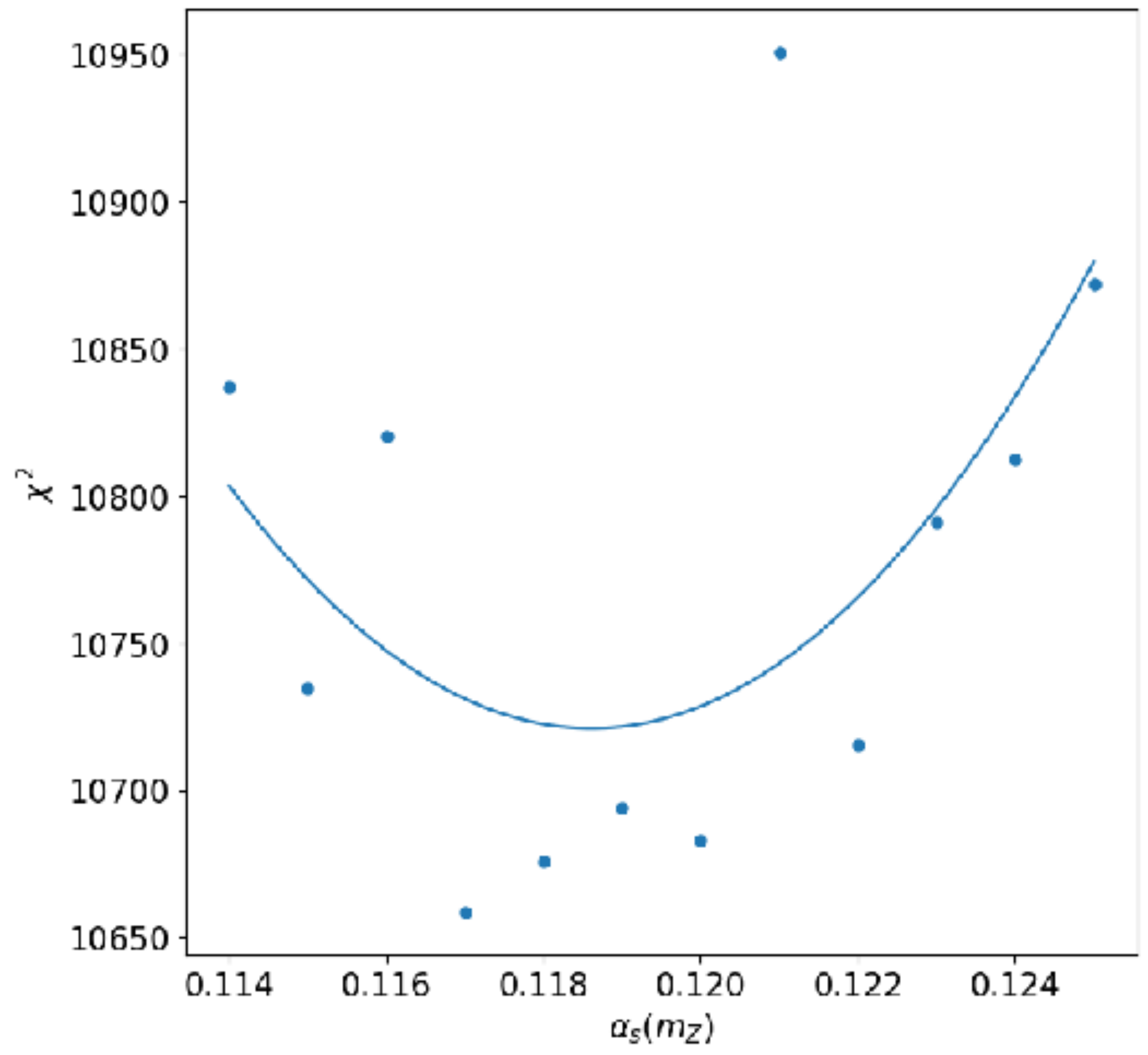
1. Assume experimental data is **defined** by a vector of central values and a covariance matrix
2. Sample this distribution to create 100 Monte Carlo replicas in data space
3. Perform a fit to each of the data replicas

➡A PDF set encoding experimental uncertainties



Simultaneous minimization of PDF and α_s

Correlated replicas method



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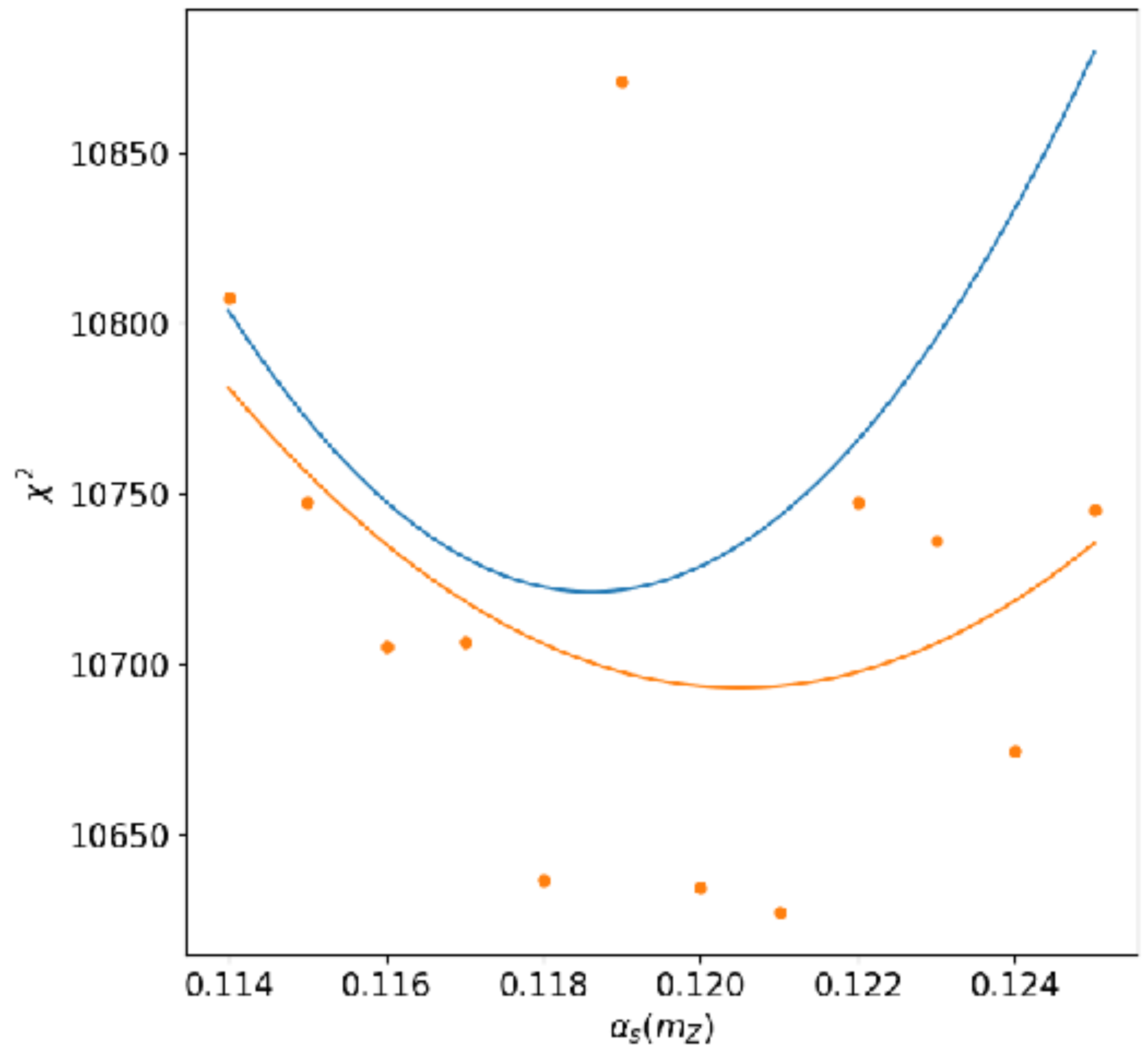

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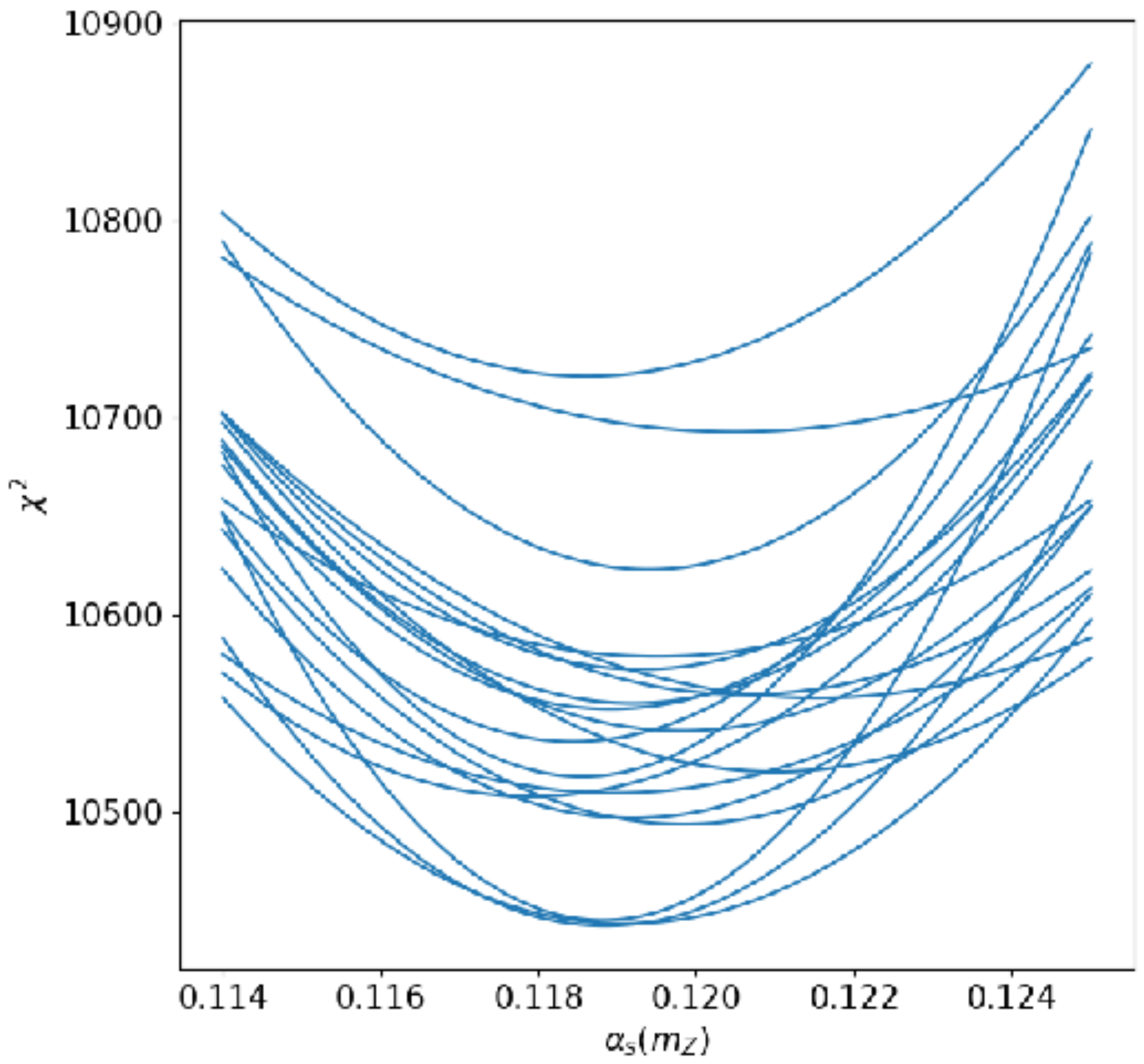

Fit the same data replica at different values of α_s and fit a parabola for each replica …

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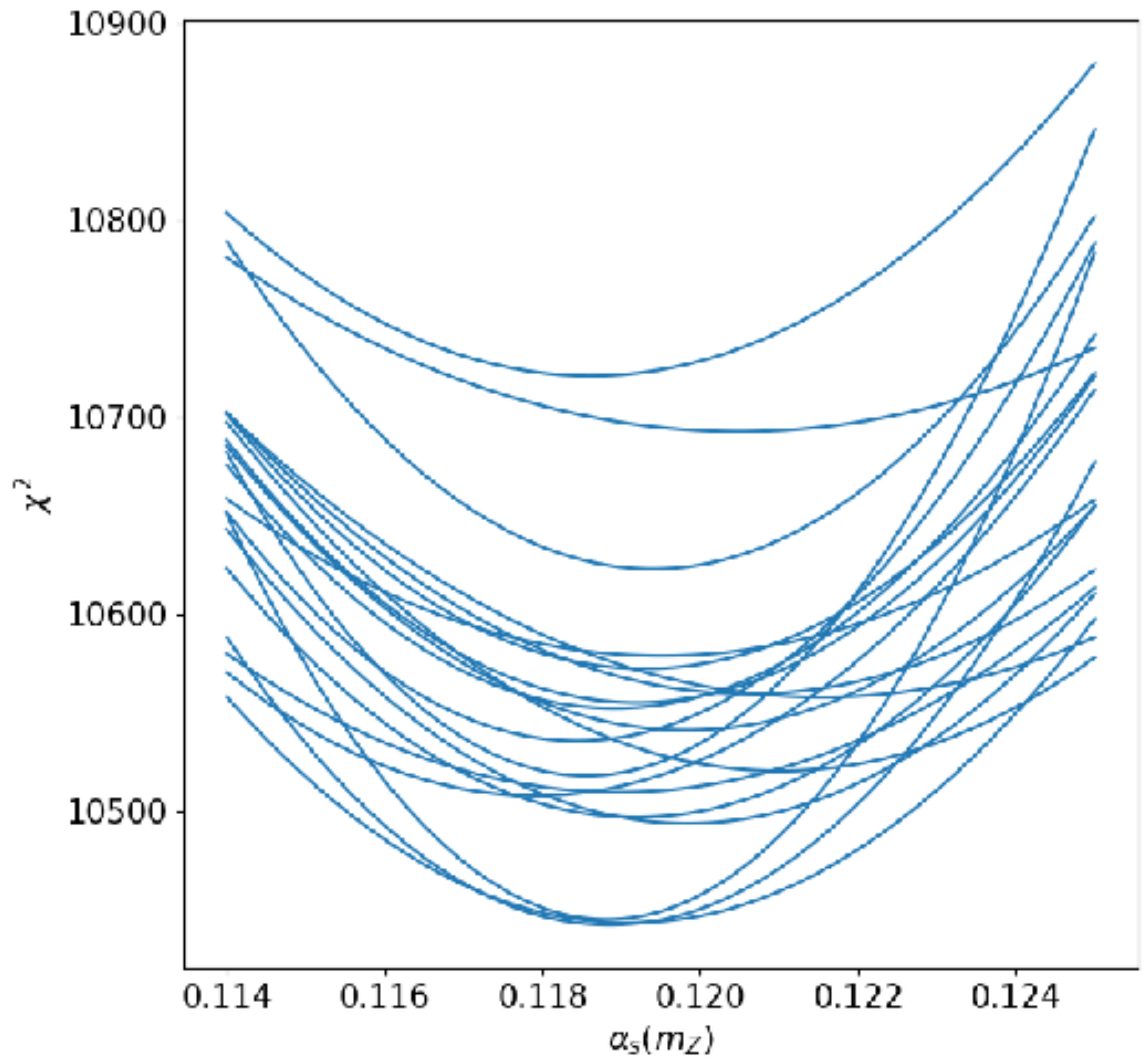
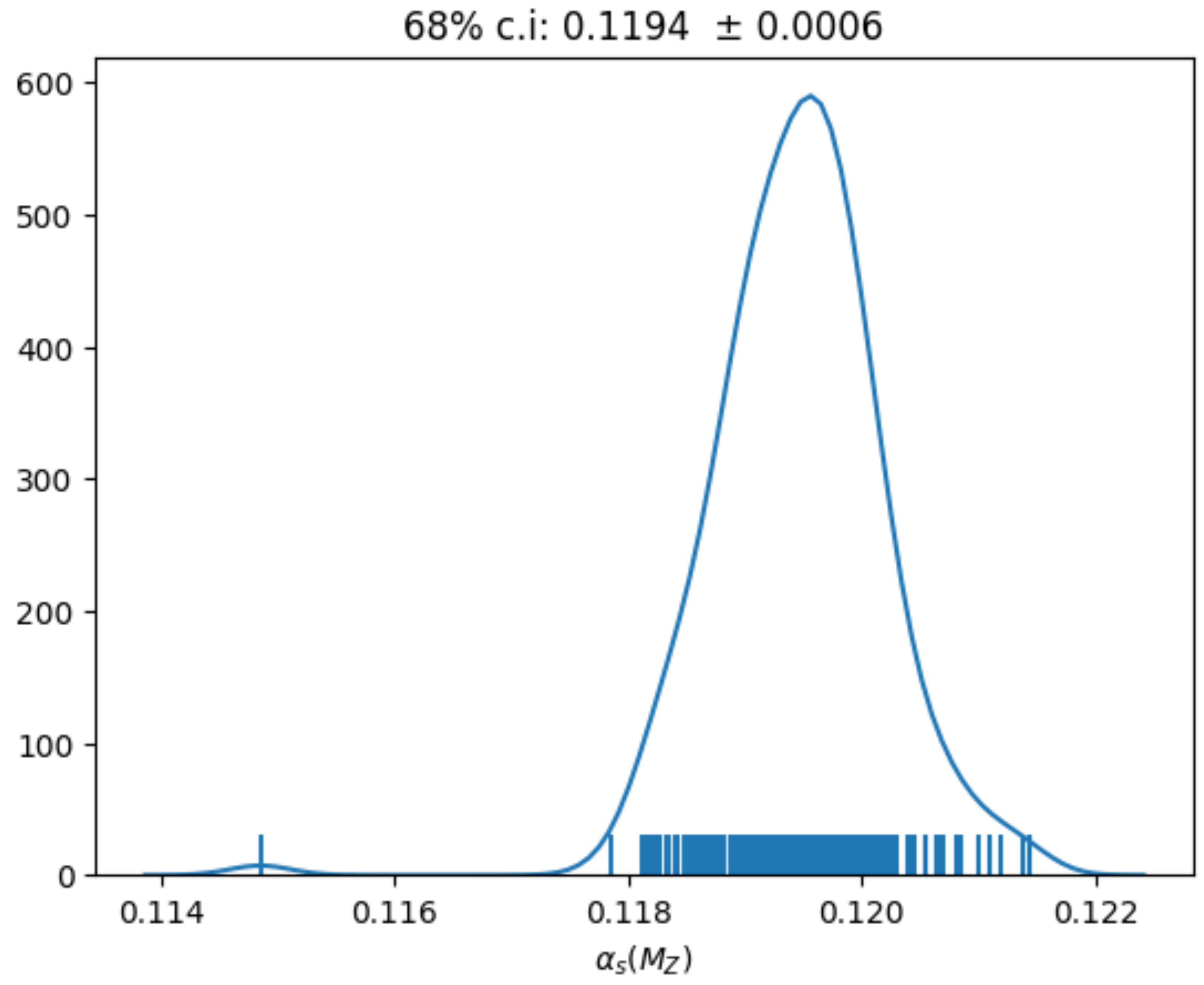




Fit the same data replica at different values of α_s and fit a parabola for each replica …

… then look at the distribution of minima of the parabolas

α_s from correlated theory uncertainties

Theory Covariance Method [arXiv:2105.05114]

The "correlated replicas" method is computationally costly because it involves fitting PDFs at many values of α_s

Alternatively, α_s can be determined in a **Bayesian framework** from nuisance parameters:

1. Model the theory uncertainty as a shift correlated for all datapoints
   $T \to T + \lambda \cdot \beta$, for $\beta \equiv T(\alpha_s^+) - T(\alpha_s^-)$
   $$P(T \mid D, \lambda) \propto \exp(\chi^2) = \exp\left(-\frac{1}{2}(T + \lambda \cdot \beta - D)^T C^{-1} (T + \lambda \cdot \beta - D)\right)$$
2. Choose a prior
   $$P(\Delta\alpha_s) \propto \exp\left(-\frac{1}{2}\lambda^2\right)$$
3. Marginalise over λ to get $P\left(T | D\right)$
4. Compute the posterior for λ
   $$P(\lambda \mid T, D) = \frac{P(T \mid D, \lambda) P(\lambda)}{P(T \mid D)} \propto \exp\left[-\frac{1}{2} Z^{-1} (\lambda - \bar{\lambda})\right]$$

$$Z = 1 - \beta^T (C + \beta\beta^T)^{-1} \beta \qquad \bar{\lambda}(T, D) = \beta^T (C + \beta\beta^T)^{-1} (D - T)$$

This idea can be extended to a real PDF fit [arXiv:2105.05114]

1) Perform fit with $C^{exp} \to C^{exp} + C^{\alpha_s}$, $C^{\alpha_s} = \beta\beta^T$
2) Once the fit has completed, compute α_s shift preferred by data as encoded in the fit

Prior dependence in the Theory Covariance Method

Prior dependence in the Theory Covariance Method

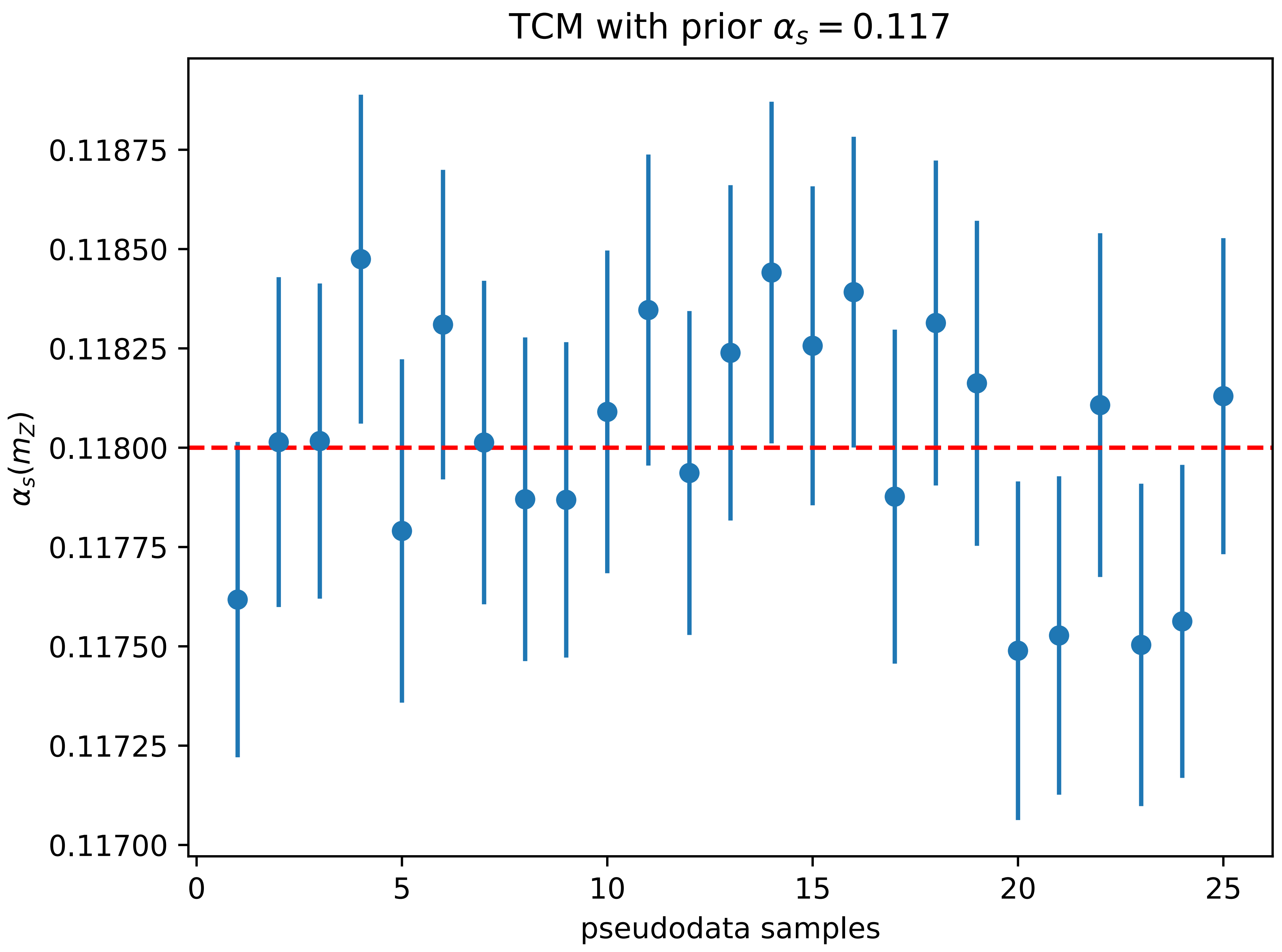
For some aspects of the fit we have to assume a value of $\alpha_s(m_Z)$, in reality we don't know the result so what if we choose "wrong"?

Consider the following

Pseudodata at $\alpha_s(m_Z) = 0.118$

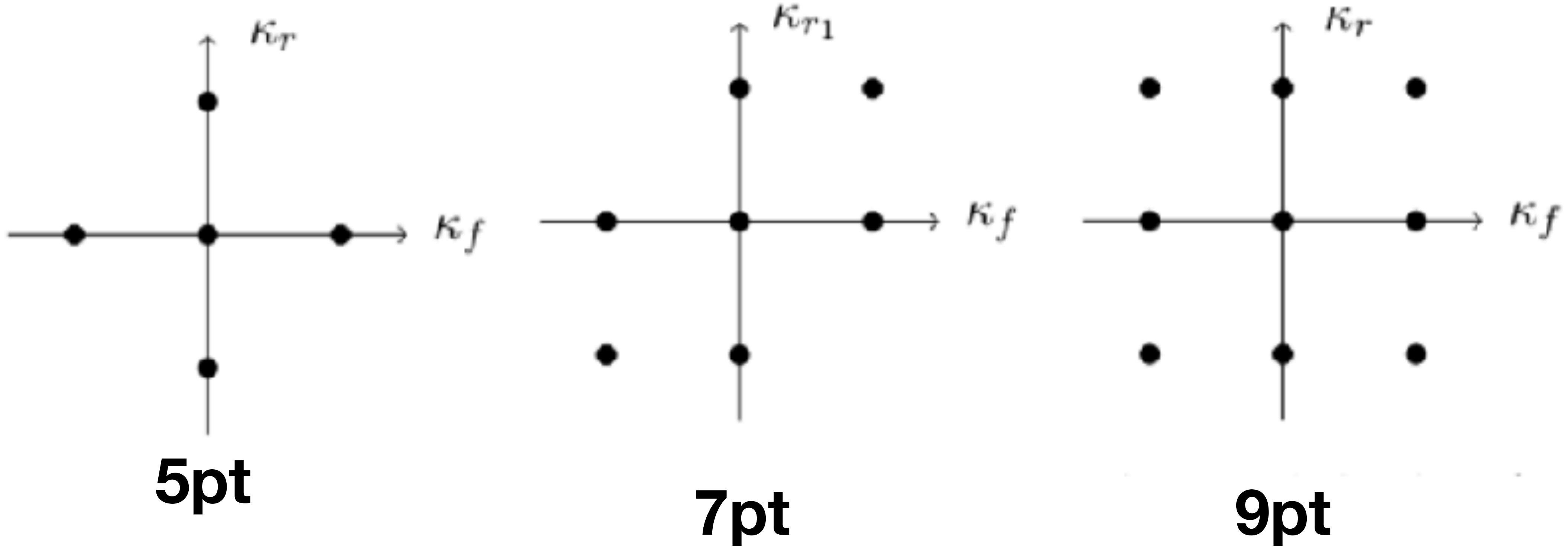
Prior assumption is $\alpha_s(M_Z) = 0.117$

Result moves towards the true result. We update assumption and iterate!



Theory uncertainties in PDFs

Missing higher order uncertainties (MHOUs) are estimated through 7 point scale variations



- In a fit we minimize the χ^2:

$$P(T \mid D\lambda) \propto \exp\left(-\frac{1}{2}(T-D)^T C^{-1}(T-D)\right) \equiv \exp\left(\chi^2\right)$$

- To account for MHOUs we treat the theory covmat on the same footing as the experimental covmat: $C = C_{\text{exp}} + C_{\text{MHOU}}$

$$C_{\text{MHOU},ij} = n_m \frac{1}{V_m} \sum \left(T_i(\kappa_f, \kappa_r) - T_i(0,0)\right)\left(T_j(\kappa_f, \kappa_r) - T_j(0,0)\right)$$

Validating the MHOU covmat

The MHOU covmat is validated by comparing the **shifts from scale variations at NLO** to the known **NNLO-NLO shifts**