NNPDFpol2.0: a first global determination of polarised PDFs at NNLO with theory uncertainties

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Outline of the Talk

Introduction & Motivations

- Helicity-/Spin-dependent PDFs are essentials to understand different physical processes (DIS, SIDIS, EDIS) $\Longleftrightarrow$ crucial inputs to **parton-to Fragmentation Function (FF)**
- Test QCD & Factorisation: provide more insights into QCD dynamics and test the validity of the **factorisation theorem**
- Precise knowledge of Polarised PDFs are crucial to decompose the **spin content of the proton**.

The proton spin is defined in terms of the Proton Sum Rules as R.L. Jaffe, A. Manohar [Nucl. Phys.B 337 (1990) 509-546], X.D. Ji [arXiv:9603249], W. Wakamatsu [arXiv:1004.0268], X. Ji, X. Xiong, F. Yuan, [arXiv:1202.2843]:

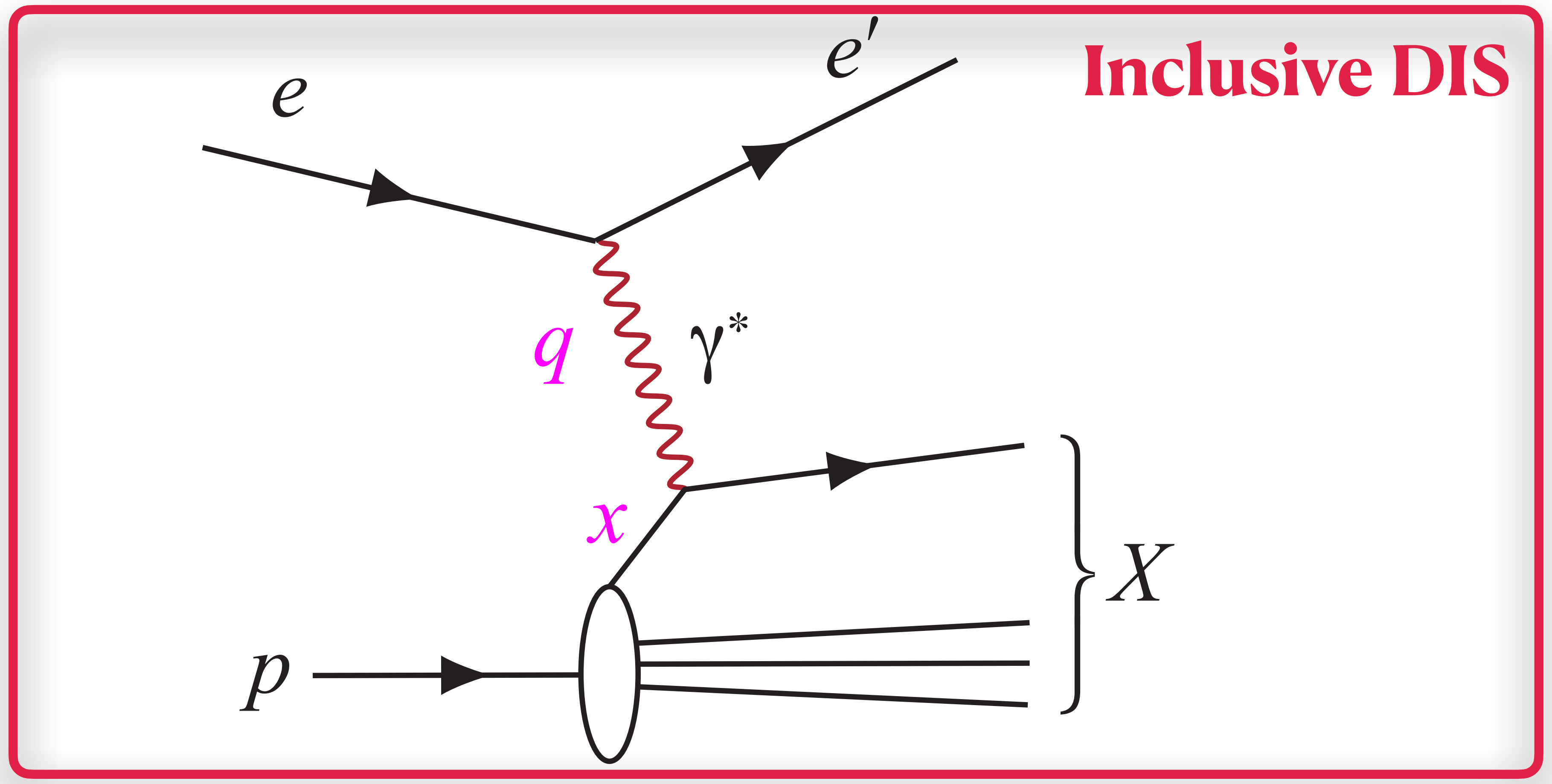
$$S_P \equiv \frac{1}{2} = \frac{1}{2} S_\Sigma(Q^2) + S_G(Q^2) + (\mathscr{L}_q + \mathscr{L}_g)(Q^2)$$

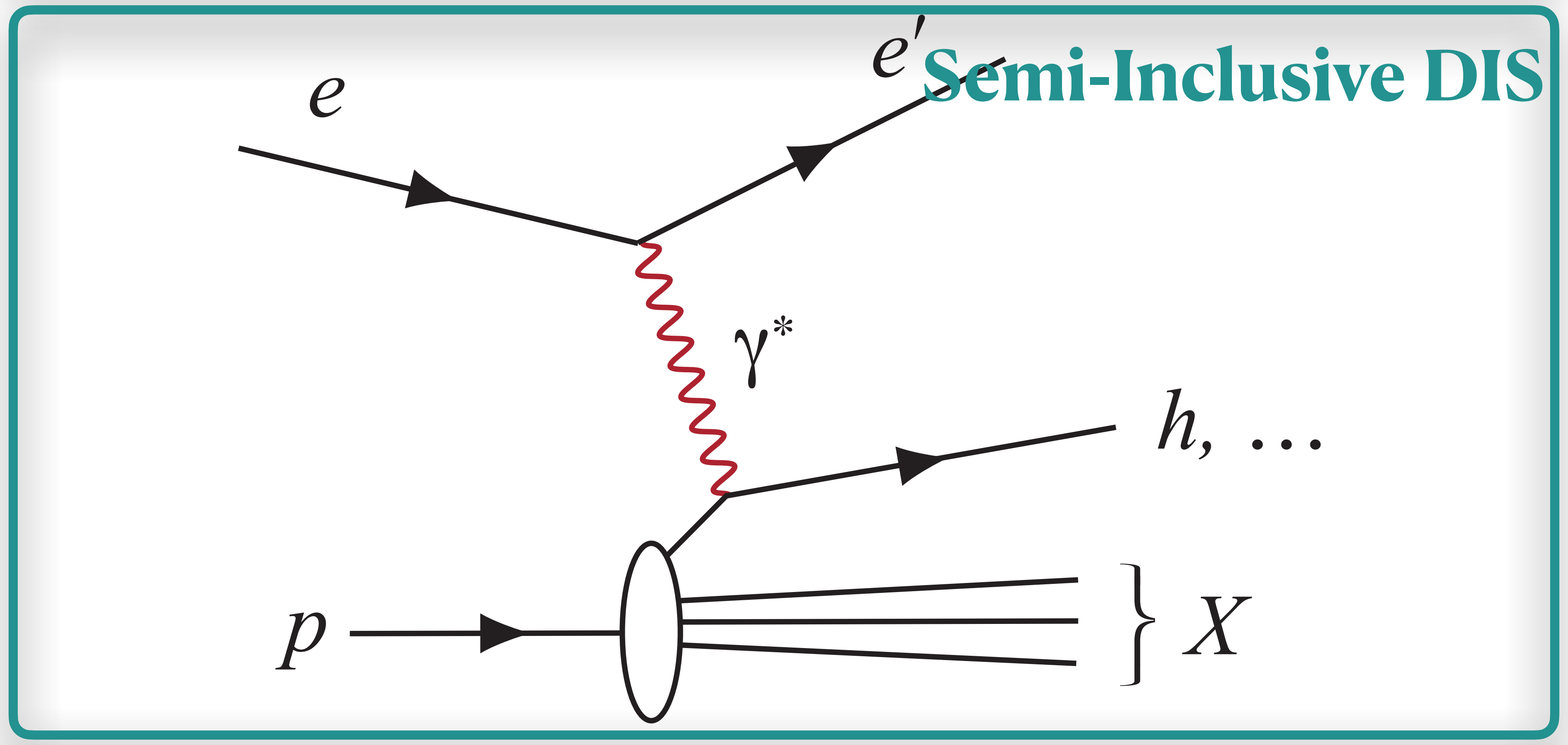
where the gluon and quark components are defined as:

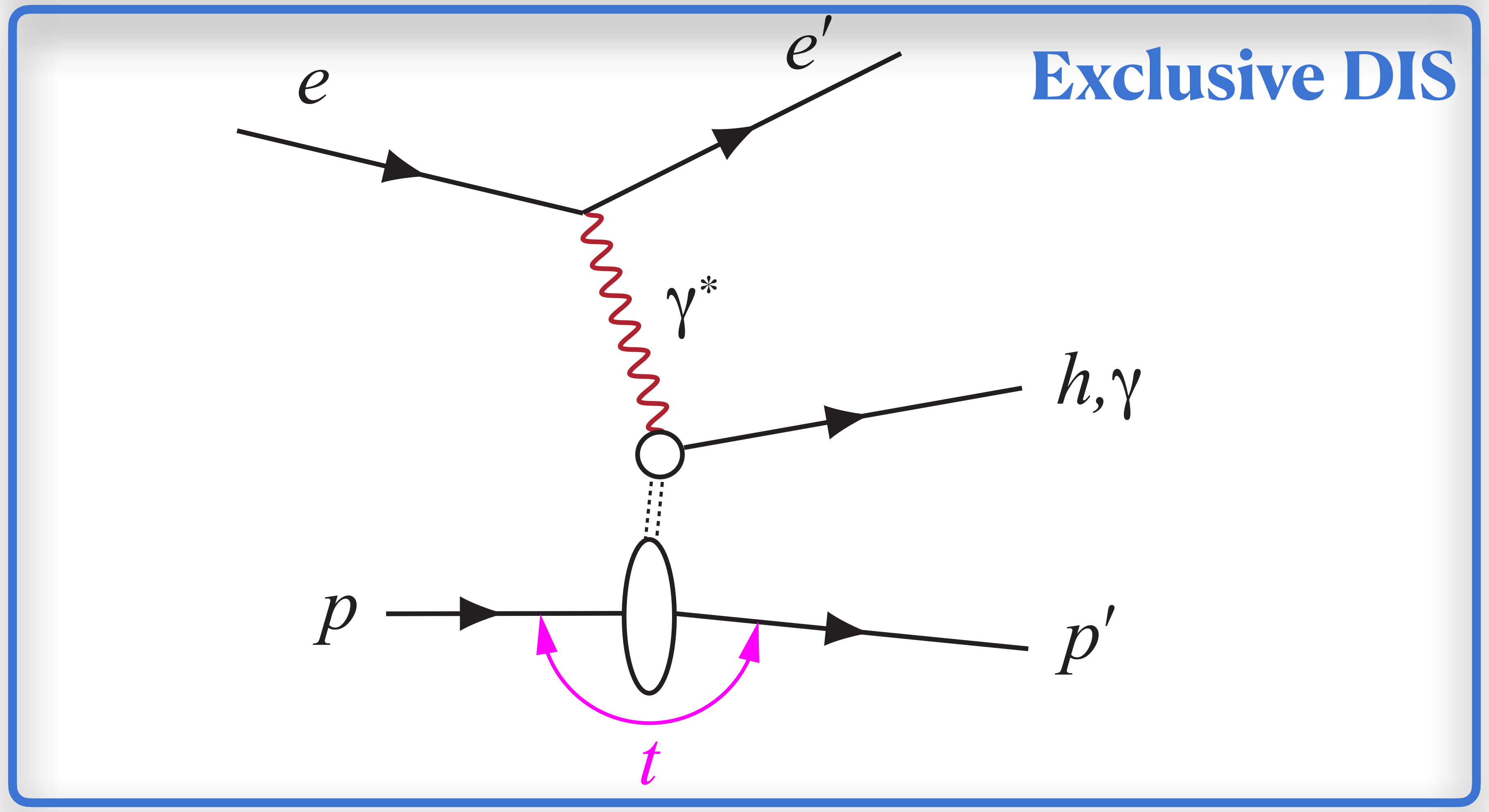
$$S_\Sigma\left(Q^2\right) = \int_0^1 (\Delta u^+ + \Delta d^+ + \Delta s^+)\left(x, Q^2\right) \mathrm{d}x$$

$$S_G\left(Q^2\right) = \int_0^1 \Delta g\left(x, Q^2\right) \mathrm{d}x$$

$\Delta\Sigma$ and Δg can be probed in **longitudinally polarised (SI)DIS** and **pp collisions** for $x \geq 10^{-3}$







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- Test QCD & Factorisation: provide more insights into QCD dynamics and test the validity of the factorisation theorem
- Precise knowledge of Polarised PDFs are crucial to decompose the spin content of the proton.

The p… s as R.L. Jaffe, … 9603249], W. Wa… 2843]:

These studies will be at the core of the Electron-Ion Collider (EIC) program

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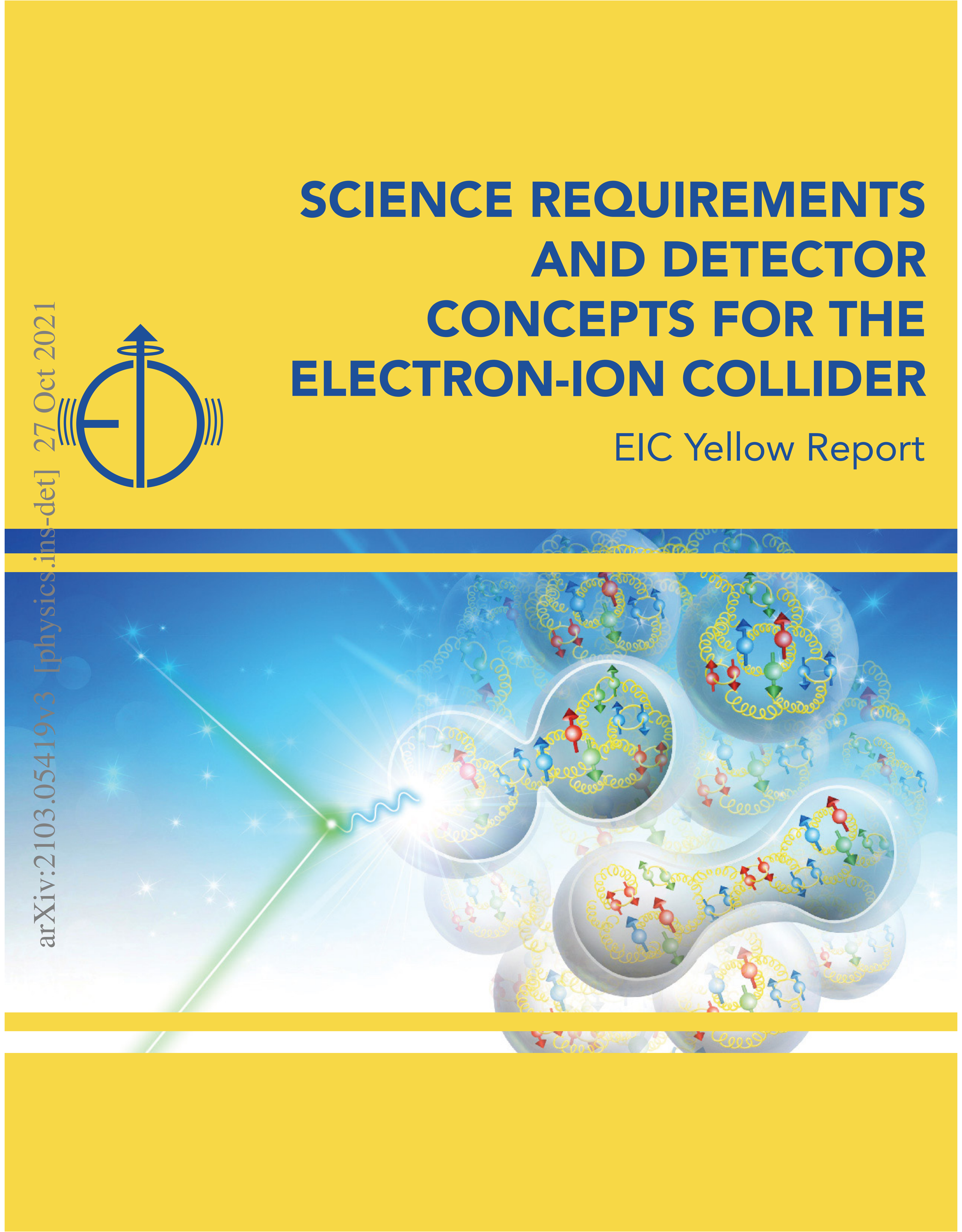


Outline of the Talk

Polarised DIS Observables

Polarised Differential Cross-Sections in *ep*-scatterings are expressed in terms of the polarised structure functions:

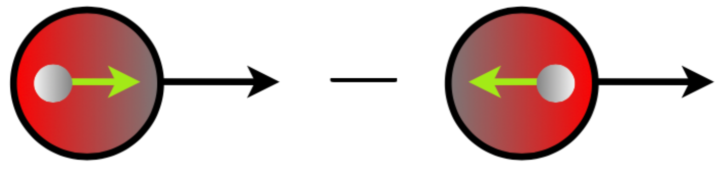
$$\frac{d^2\Delta\sigma^j}{dxdy} = \frac{4\pi\alpha^2}{xyQ^2}\xi^j\left[-\left[1+(1-y)^2\right]g_4^j + y^2 g_L^j + (-1)^p 2x\left[1-(1-y)^2\right]g_1^j\right]$$

$$g_1 = \left(\frac{1}{n_f}\sum_{k=1}^{n_f} e_{q_k}^2\right)\left(\sum_k C_k \otimes \Delta q_k\right)$$

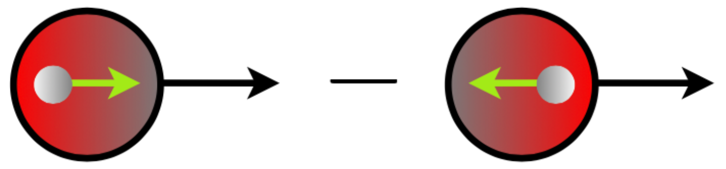
where the polarised (proton) parton densities are defined with respect to the spin of the parent nucleon as follows:

$$\Delta q_k(x) = \frac{1}{4\pi}\int dy^- e^{-ixP^+y^-}\left\langle h(P,S)\middle|\bar{\psi}\left(0,y^-,\mathbf{0}_\perp\right)\gamma^+\gamma^5\psi(0)\middle| h(P,S)\right\rangle + \cdots,$$

$$\Delta g(x) = \frac{1}{4\pi x P^+}\int dy^- e^{-ixP^+y^-}\left\langle h(P,S)\middle|G^{+\alpha}\left(0,y^-,\mathbf{0}_\perp\right)\tilde{G}_\alpha^+(0)\middle| h(P,S)\right\rangle + \cdots,$$

$\Delta q_k(x) \equiv$ 

$\equiv q_k^{\uparrow\uparrow}(x) - q_k^{\uparrow\downarrow}(x)$

$\Delta g(x) \equiv$ 

$\equiv g^{\uparrow\uparrow}(x) - g^{\uparrow\downarrow}(x)$

Following from the Operator definition, the Proton Spin receives contributions from the Orbital Angular Momentum:

$$S_P\left(Q^2\right) = \sum_f \left\langle P;S\middle|\hat{J}_f^z\left(Q^2\right)\middle|P;S\right\rangle \equiv \frac{1}{2} = \frac{1}{2}S_\Sigma\left(Q^2\right) + S_G\left(Q^2\right) + \mathcal{L}_q\left(Q^2\right) + \mathcal{L}_g\left(Q^2\right)$$

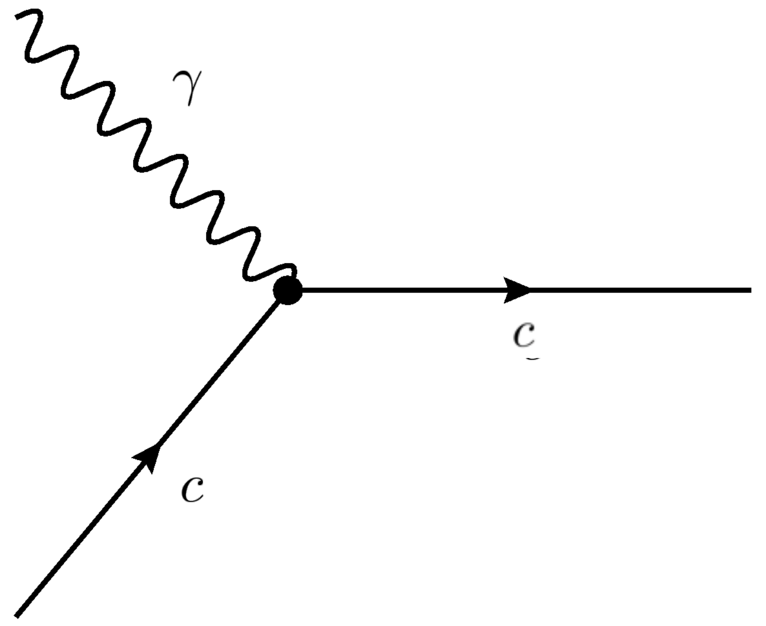
$$\left\langle P;S\middle|\hat{J}_\Sigma^z\left(Q^2\right)\middle|P;S\right\rangle \xrightarrow{\text{naive p.m.}} 2\left\langle S_\Sigma^{q+\bar{q}}\right\rangle,$$

$$\left\langle P;S\middle|\hat{J}_\Sigma^z\left(Q^2\right)\middle|P;S\right\rangle \xrightarrow{\overline{\text{MS}}} S_\Sigma\left(Q^2\right) - n_f\frac{\alpha_s\left(Q^2\right)}{2\pi}S_G\left(Q^2\right)$$

GM-VFNS for massive (charm) quarks

Diagrams	Definitions	Features

Zero-Mass



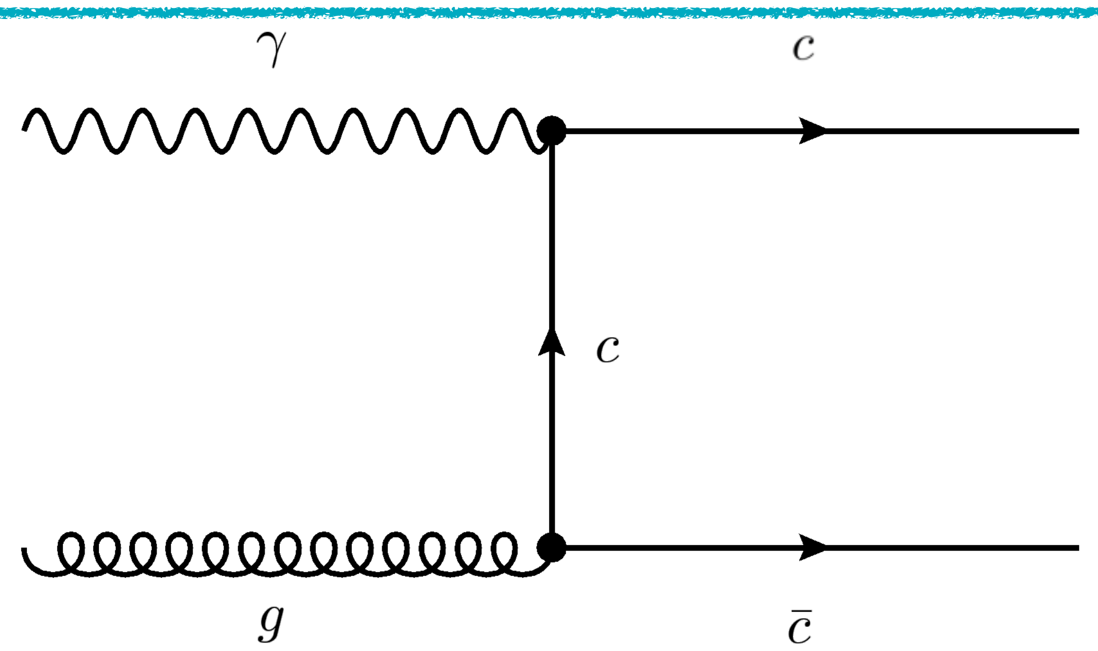
$$g_1^c(x,Q^2) \propto \sum_{k=g,u,d,s,c} \Delta C_k^{(n_f+1)}(x,Q^2) \otimes \Delta q_k^{(n_f+1)}(x,Q^2)$$

Charm treated in the same footing as LIGHT quarks

Appropriate at High Energies

Unreliable in the Threshold Region due to Mass Corrections

Fixed Flavour



$$g_1^c(x,Q^2) \propto \sum_{k=g,u,d,s} \Delta C_k^{(n_f)}(x,Q^2/m_c^2) \otimes \Delta q_k^{(n_f)}(x,Q^2)$$

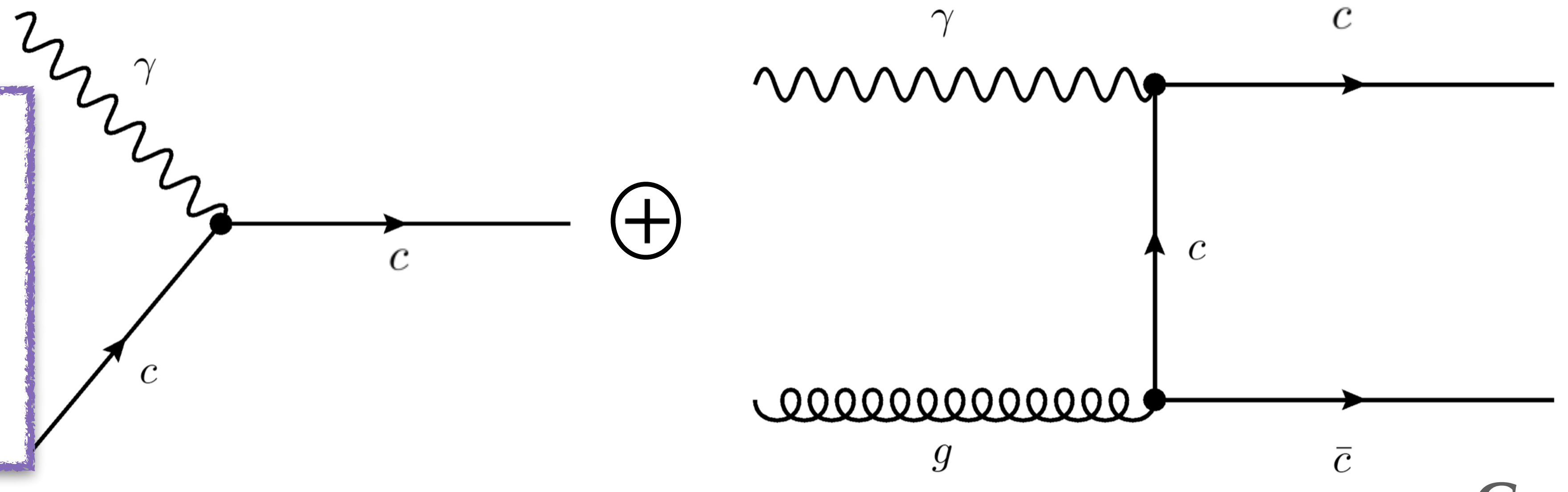
Charm mass effects accounted for exactly

No Charm in the PDF

Exact in the Threshold Region

Unreliable at mc2<<Q2 due to Large unresummed logarithms

GM-VFNS



$$g_1^c(x,Q^2) \propto \sum_{k=g,u,d,s,c} \left(\Delta C_j^{(n_f)} + \Delta C_j^{(n_f,0)}\right)\left(x,\frac{Q^2}{m_c^2}\right) \otimes \mathbf{A}_{jk}^{-1}(Q^2) \otimes \Delta q_k^{(n_f+1)}(x,Q^2) + \sum_{k=g,u,d,s,c} \Delta C_k^{(n_f+1)} \Delta q_k^{(n_f+1)}(x,Q^2)$$

Combines in a Consistent way Massless & Massive Calculations

Combines Best of Both Formalisms

Ingredients for constructing FONLL GM-VFNS

Anomalous Dimensions

Both Singlet and Non-Singlet Anomalous Dimensions are **analytically known** up to $\mathcal{O}(\alpha_s^3)$
Moch, Vermaseren, Vogt [arXiv:1409.5131] - [arXiv:1506.04517], Blümlein, Schneider, Schönwald [arXiv:2111.12401], Gluck, Reya, Stratmann, Vogelsgand [arXiv:9508347]

At **lowest-trivial order**, polarised qq Anomalous Dimensions is identical to its Unpolarised counterpart:

$$\Delta\gamma_{qq}^{(0)}(N, \alpha_s(Q^2)) = \gamma_{qq}^{(0)}(N, \alpha_s(Q^2))$$

Symmetry considerations imply that Non-Singlet polarised Anomalous Dimensions are directly related to the spin-averaged counterparts at ALL order via the following relation:

$$\Delta\gamma_{\mathrm{NS},\pm}^{(n)}(N, \alpha_s(Q^2)) = \gamma_{\mathrm{NS},\mp}^{(n)}(N, \alpha_s(Q^2))$$

Helicity conservation implies that the first Moment of the gluon-to-quark Splitting Function vanishes:

$$\int_1^0 dx\, x\Delta P_{qg}(x, \alpha_s(Q^2)) = 0$$

Matching Conditions

The components of the Matching Condition matrices $\Delta\mathrm{A}_{\alpha\beta}$ for all values of α and β are **known analytically** up to $\mathcal{O}(\alpha_s^2)$
Bierenbaum, Blümlein, Freitas, Goedicke, Klein, Schönwald [arXiv:2211.15337]

The expressions of the **Zeroth order** Matching Coefficients are trivial:

$$\mathrm{A}_{\alpha\beta}^{(0)}(x, Q^2/m_h^2) = \delta_{\alpha\beta}$$

At NLO, only $\Delta\mathrm{A}_{\alpha\beta}^{(1)}$ components with $\alpha = g, c, \bar{c}$ and j=g are non-zero while all other components with Quark lines contribute at NNLO

Coefficient Functions

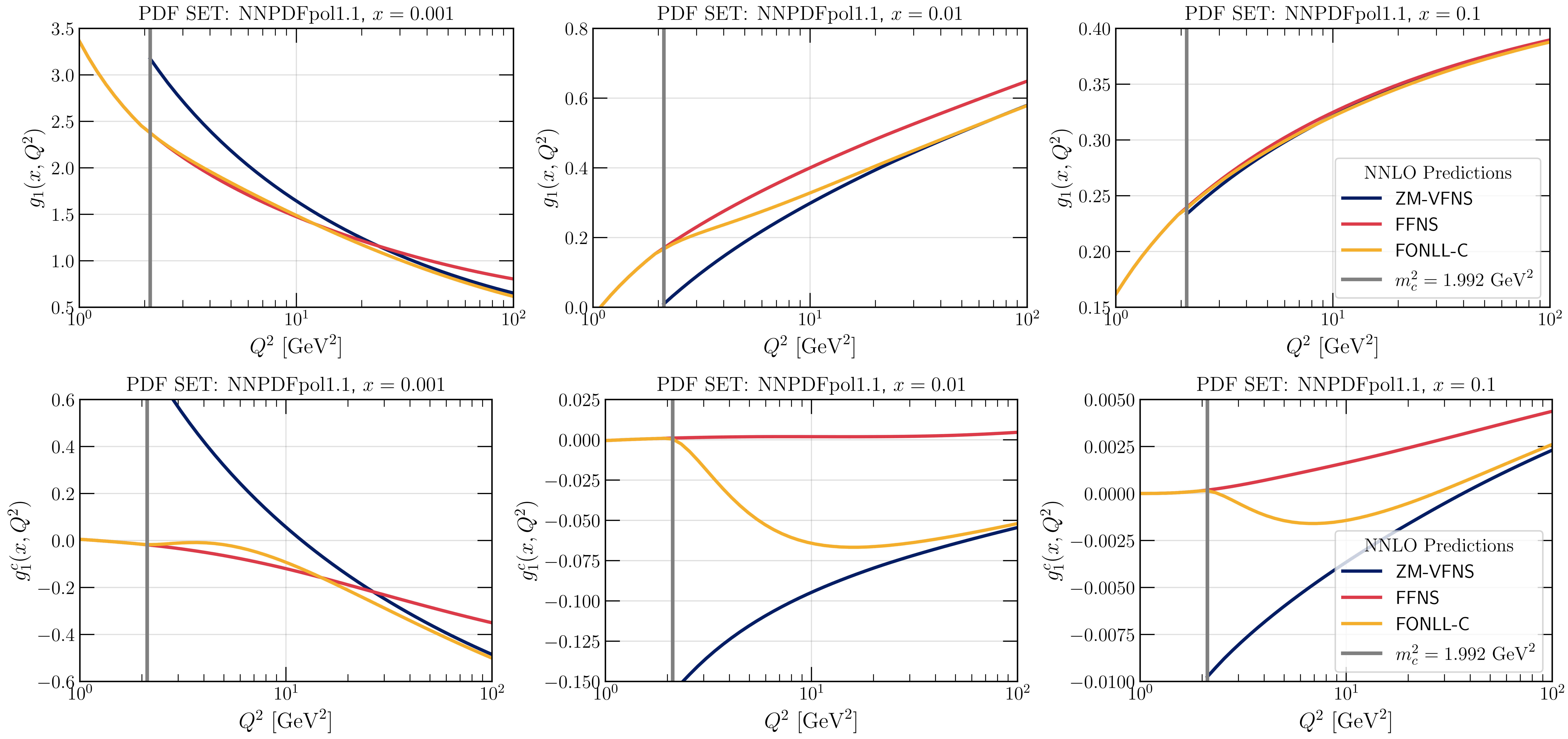
The pure massless coefficient functions entering the expression of g_1 are **known analytically up to NNLO** Ziljstra, Van Neerven [iNSPIRE:353973]

The massive coefficients are available up to NNLO with their asymptotic limits
Herkhorn, Stratmann [arXiv:1805.09026], Behring, Blümlein, Freitas, Von Matnteuffel [arXiv:1504.08217], Ablinger, Behring, Blümlein, Freitas, Von Matnteuffel, Schneider, Schönwald [arXiv:1912.02536] - [arXiv:2101.05733]

Heavy quark (charm) mass effects on polarised observables g_1

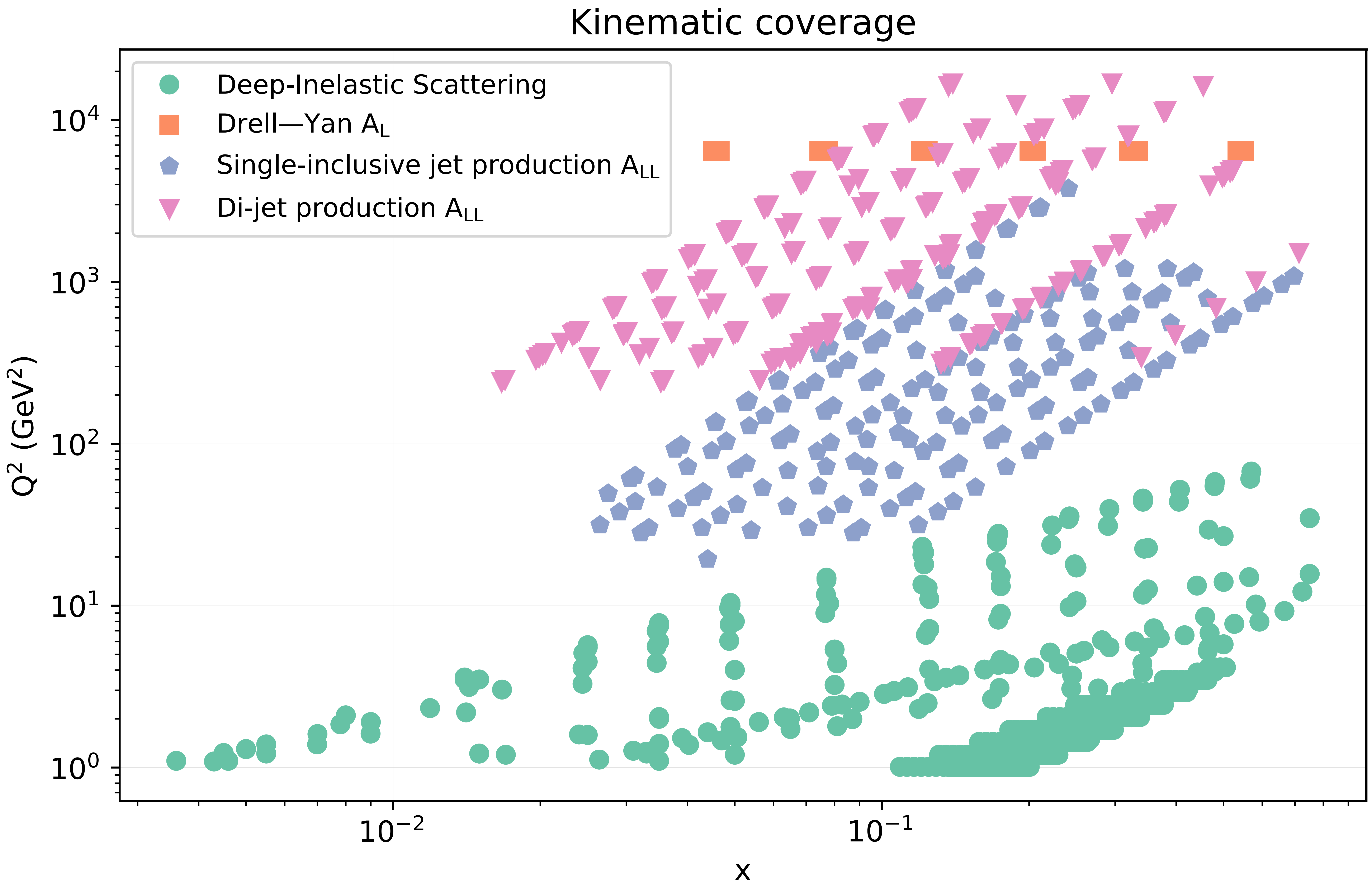
Mass effects are moderate for inclusive observables and **NNLO** effects are non-negligible with current DIS kinematics
$\Longleftrightarrow$ NLO QCD calculations might not be sufficiently accurate for EIC physics.

For tagged Charm final-state, **NNLO** effects are still non-negligible and **Charm Mass effects are sizeable**
$\Longleftrightarrow$ EIC physics require both higher-order corrections and mass effects



Outline of the Talk

Experimental Data



NNPDFpol2.0 includes measurements from various experiments and probes different processes:

- g_1 structure functions from COMPASS, SLAC, HERMES, JLAB, and SMC
- g_1/F_1 from SLAC and JLAB
- Longitudinal spin asymmetry at small-x from SMC
- W-boson longitudinal single spin asymmetry $A_L^{W^\pm}$ from STAR
- Single-inclusive jet and di-jet longitudinal double spin asymmetry $A_{LL}^{1/2-\mathrm{jet}}$ from STAR

Experimental measurements (after kinematic cuts: $Q^2 \geq Q^2_{\mathrm{min}} \equiv 1.0$ GeV^2 and $Q^2 \geq Q^2_{\mathrm{min}} \equiv 4.0$ GeV^2) do not sufficiently probe the **small-x region**

Estimate of Missing Higher Order Uncertainties (MHOUs)

For a given observable $\mathcal{O}$, **MHOUs** are commonly estimated by **varying the unphysical scales** in the **parton evolutions** and in the **partonic cross-sections**:

$$\mathcal{O}\left(\alpha_s\left(\mu^2\right), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2}\right) = \mathcal{L}\left(\alpha_s\left(\mu_F^2\right), \frac{Q^2}{\mu_F^2}\right) \mathcal{O}\left(\alpha_s\left(\mu_R^2\right), \frac{Q^2}{\mu_R^2}\right)$$

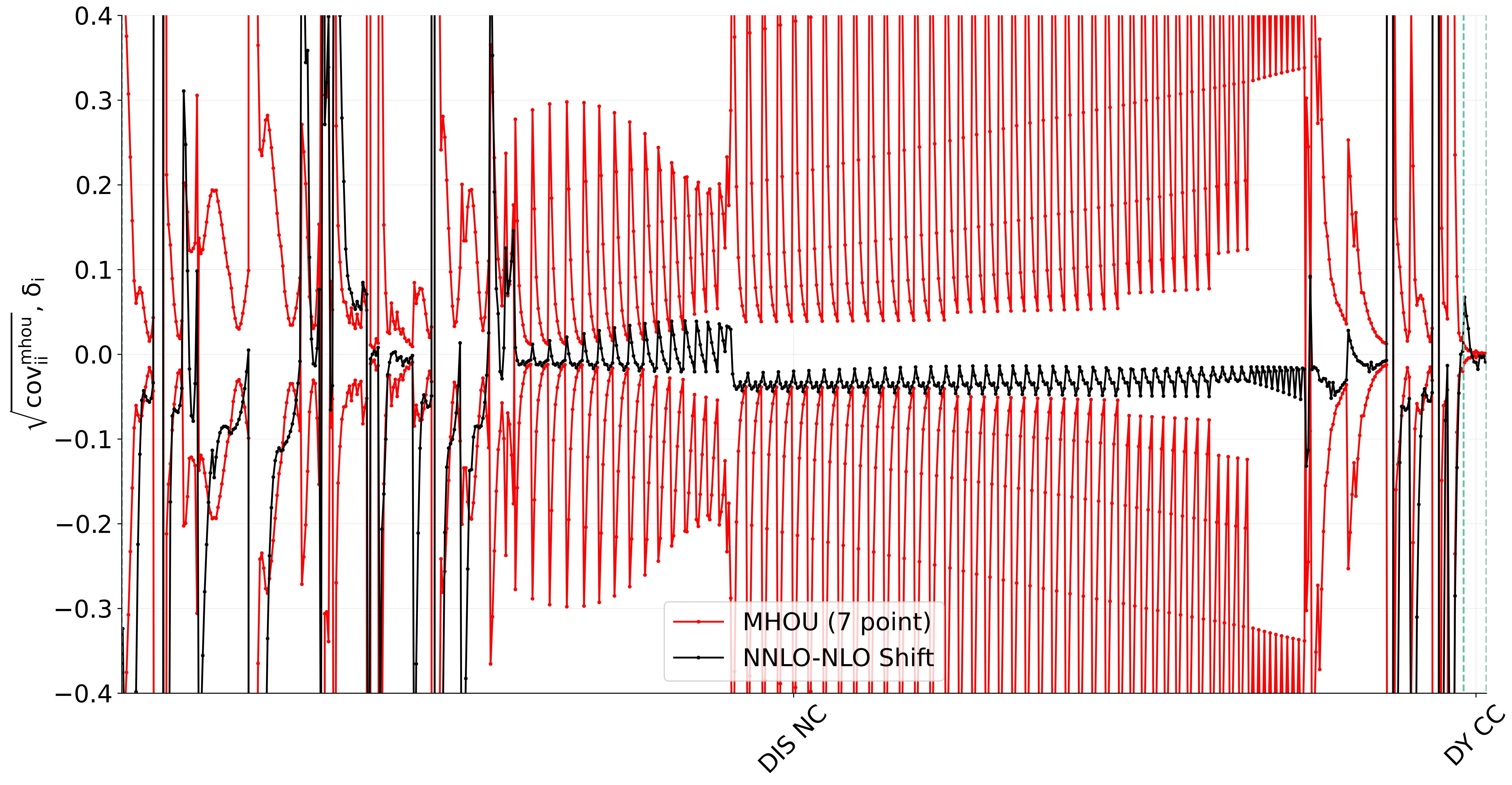
Variation of **Factorisation Scale** $\kappa_F = Q^2/\mu_F^2$ estimates MHOUs from Anomalous Dimensions in the evolution while variation of **Renormalisation Scale** $\kappa_R = Q^2/\mu_R^2$ estimates MHOUs from partonic cross-sections.

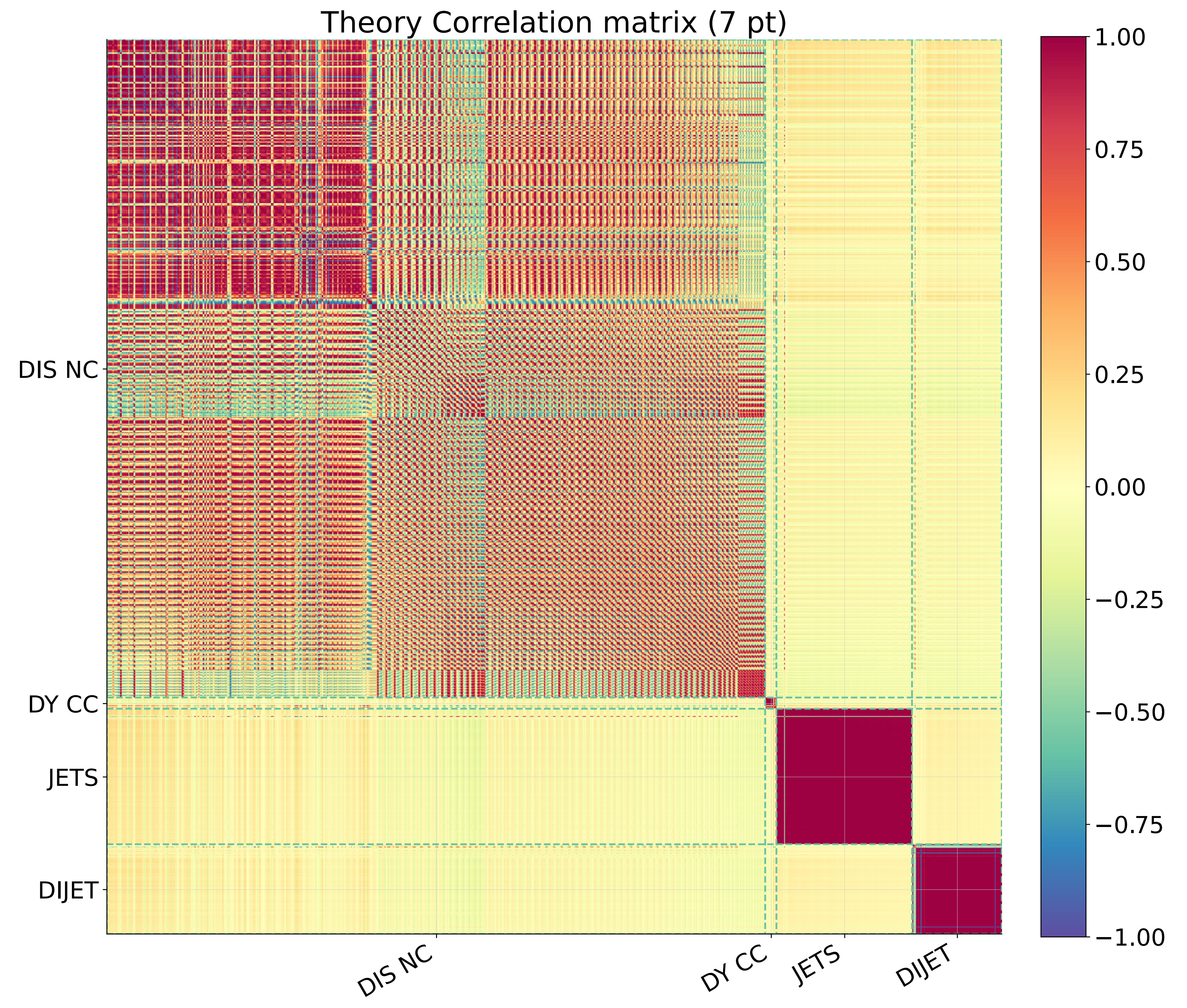
MHOUs can be added as a nuisance parameter to the Covariance Matrix [arxiv:1906.10698; arxiv:2105.05114]

$$\mathrm{cov}_{i,j} = \mathrm{cov}_{i,j}^{\mathrm{exp}} + \mathrm{cov}_{i,j}^{\mathrm{MHOU}},$$

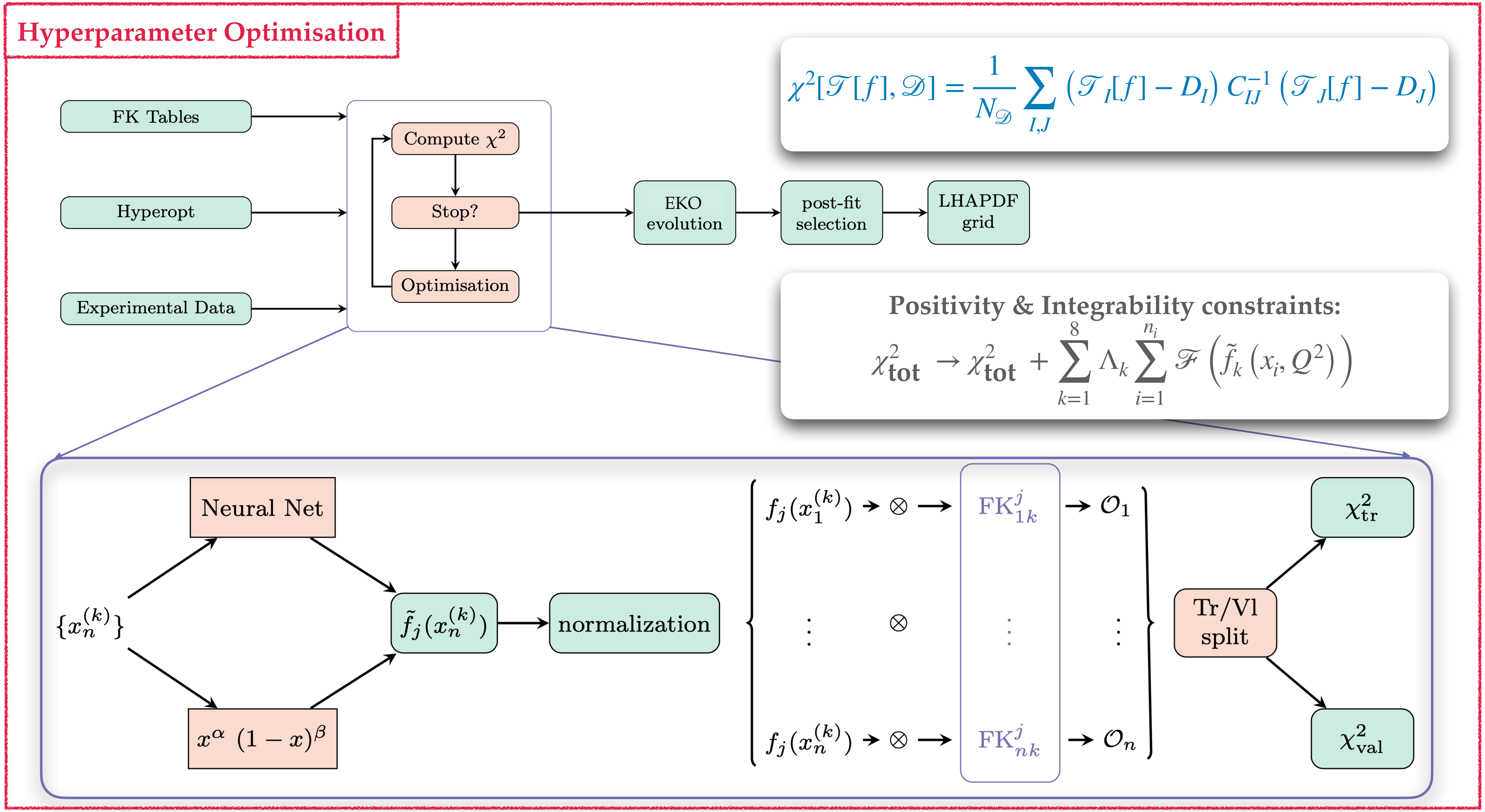
$$\mathrm{cov}_{i,j}^{\mathrm{MHOU}} = \frac{1}{N_{\mathrm{var}} - 1} \sum_{k=1}^{N_{\mathrm{var}}} \left(S_{i,k} - \bar{S}_i\right)\left(S_{j,k} - \bar{S}_j\right)$$

7-point scale variation prescription is used. Points belonging to the same process are CORRELATED by κ_R-variation while κ_F correlates all the points.





NNPDFpol2.0 methodology in a nutshell

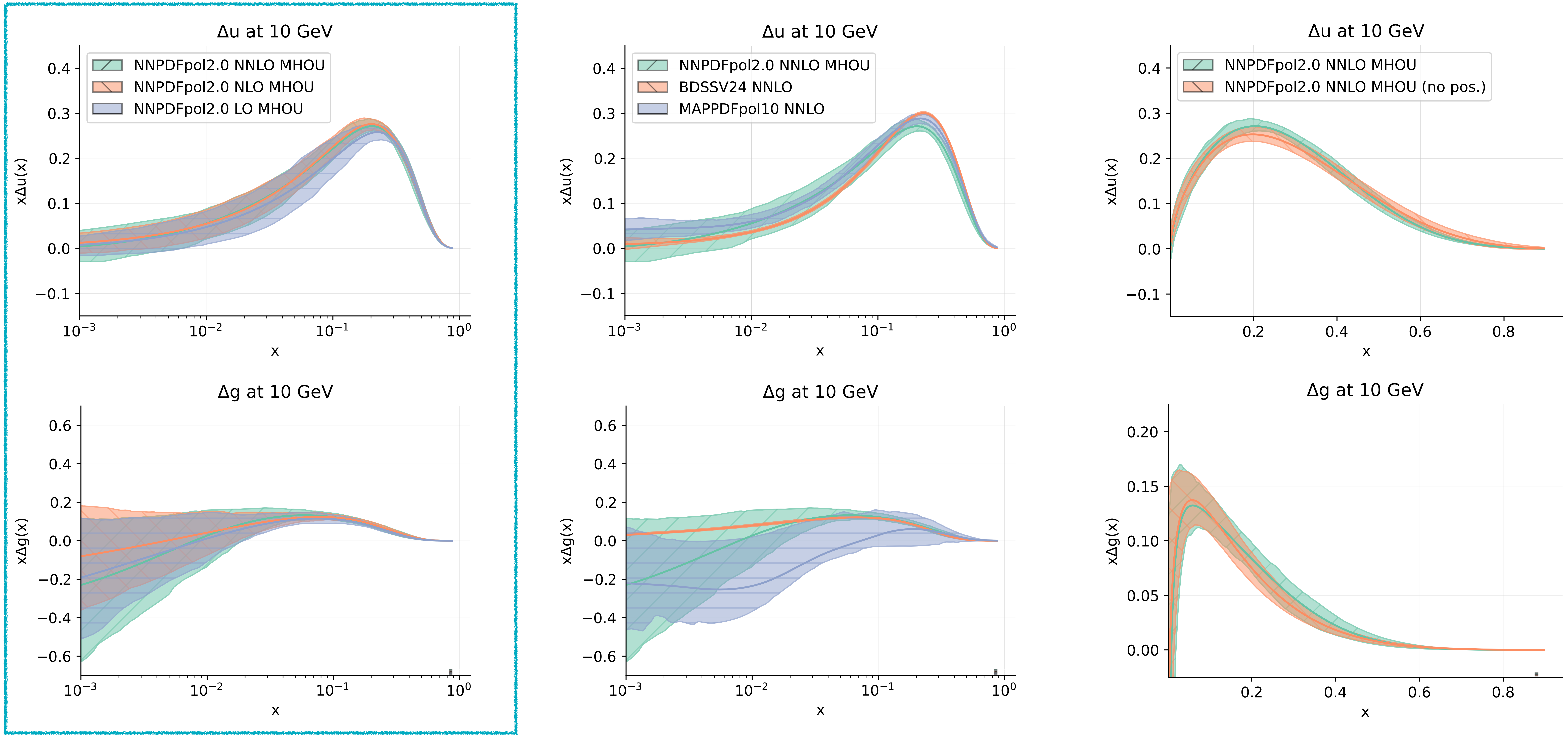


Outline of the Talk

NNPDFpol2.0 PDFs: Perturbative Convergence

NNLO corrections, **heavy quark** contributions, and **MHOUs** have **very limited effects**

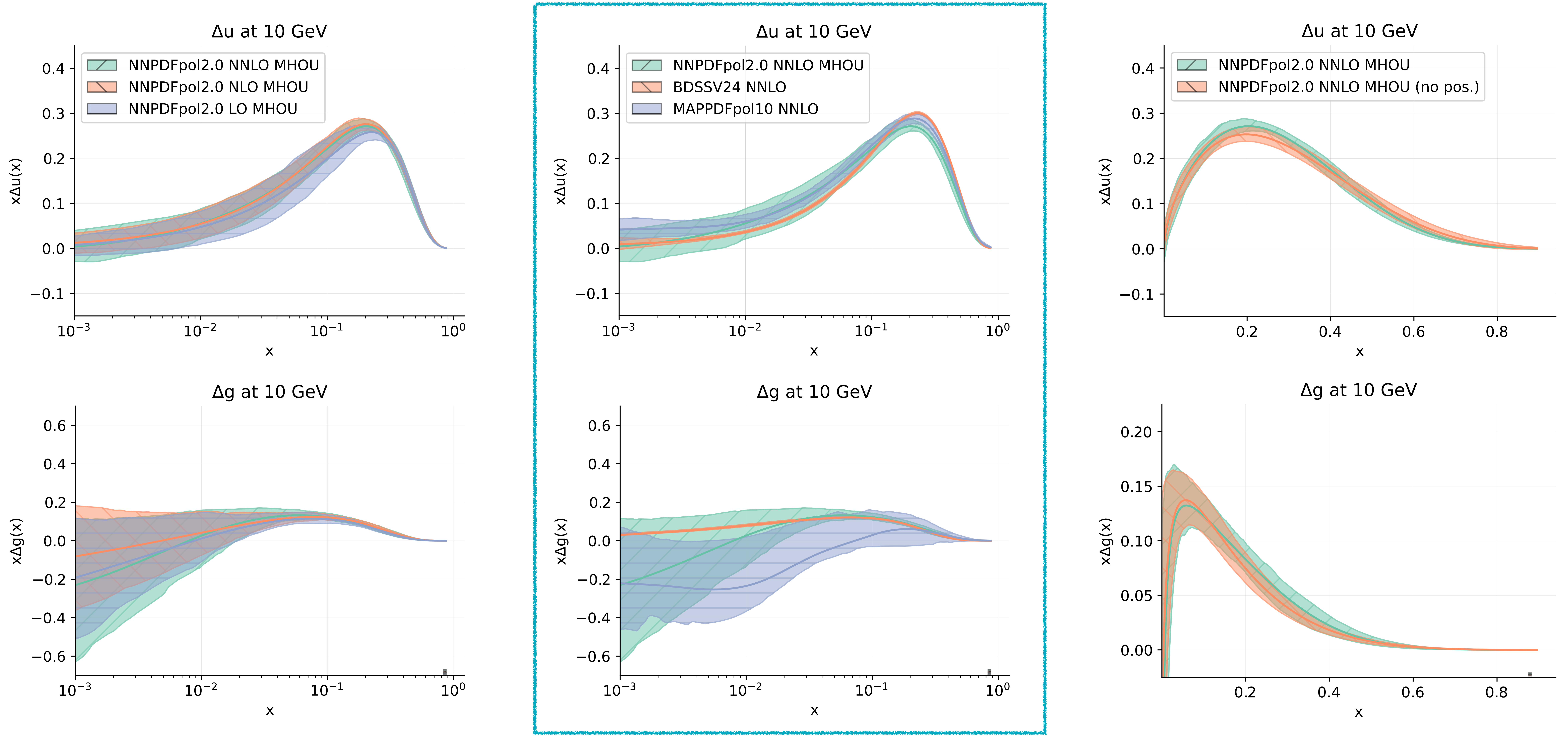
With current data, theoretical framework, and methodology, **perturbative expansion has converged**



NNPDFpol2.0 PDFs: Comparison to other sets

Very good agreement (in the central value) between NNPDFpol2.0 and BDSSV24 in the **data region ($x \geq 10^{-2}$)**

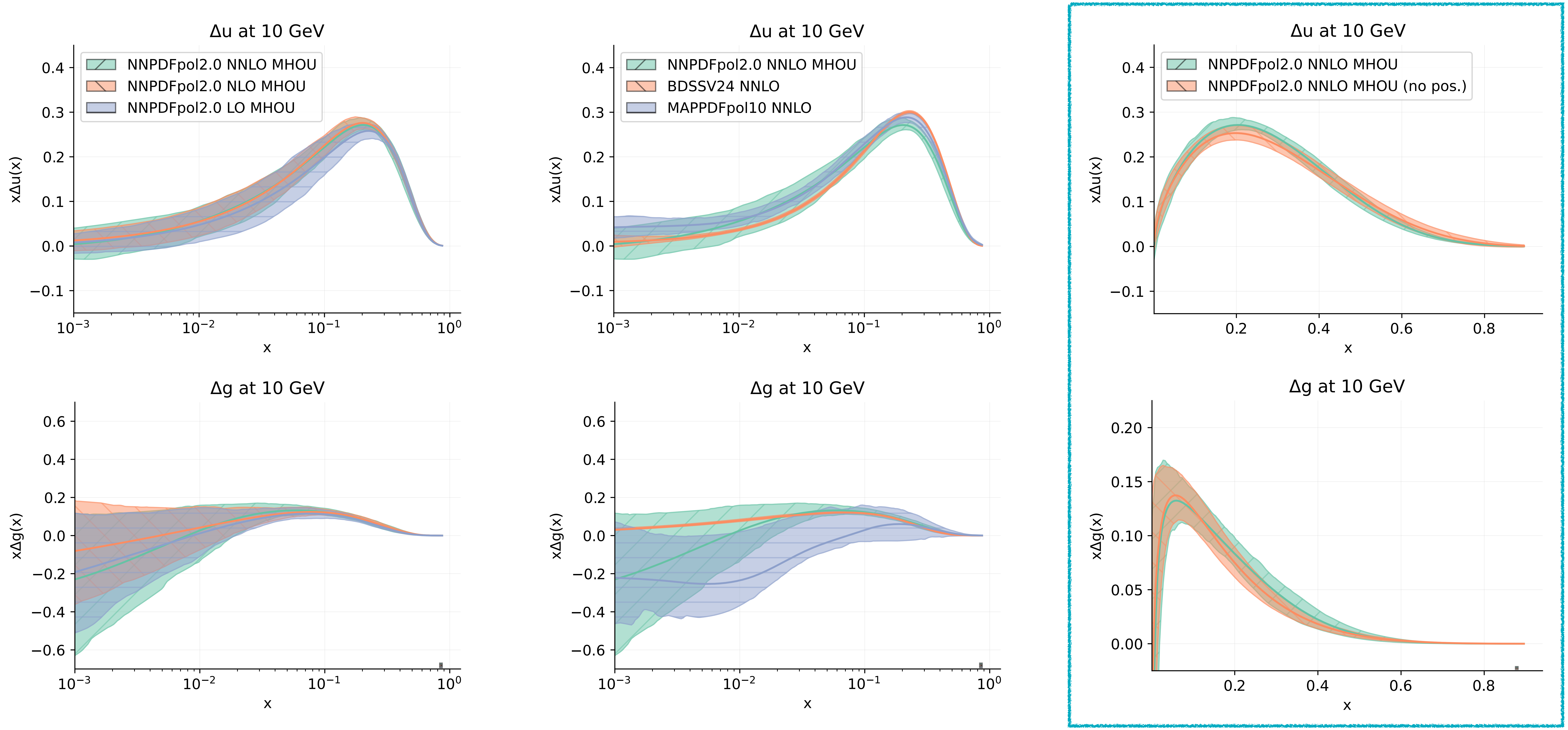
Differences (mostly attributed to data in NNPDFpol2.0 vs NNPDFpol1.1) are **within the one-sigma uncertainty bands**



NNPDFpol2.0 PDFs: Effect of Positivity

CLAIM: **W/o being enforced**, Gluon Positivity only satisfied by inclusions of **RHIC SI jet** and **JLab DIS** [arXiv:2201.0207]

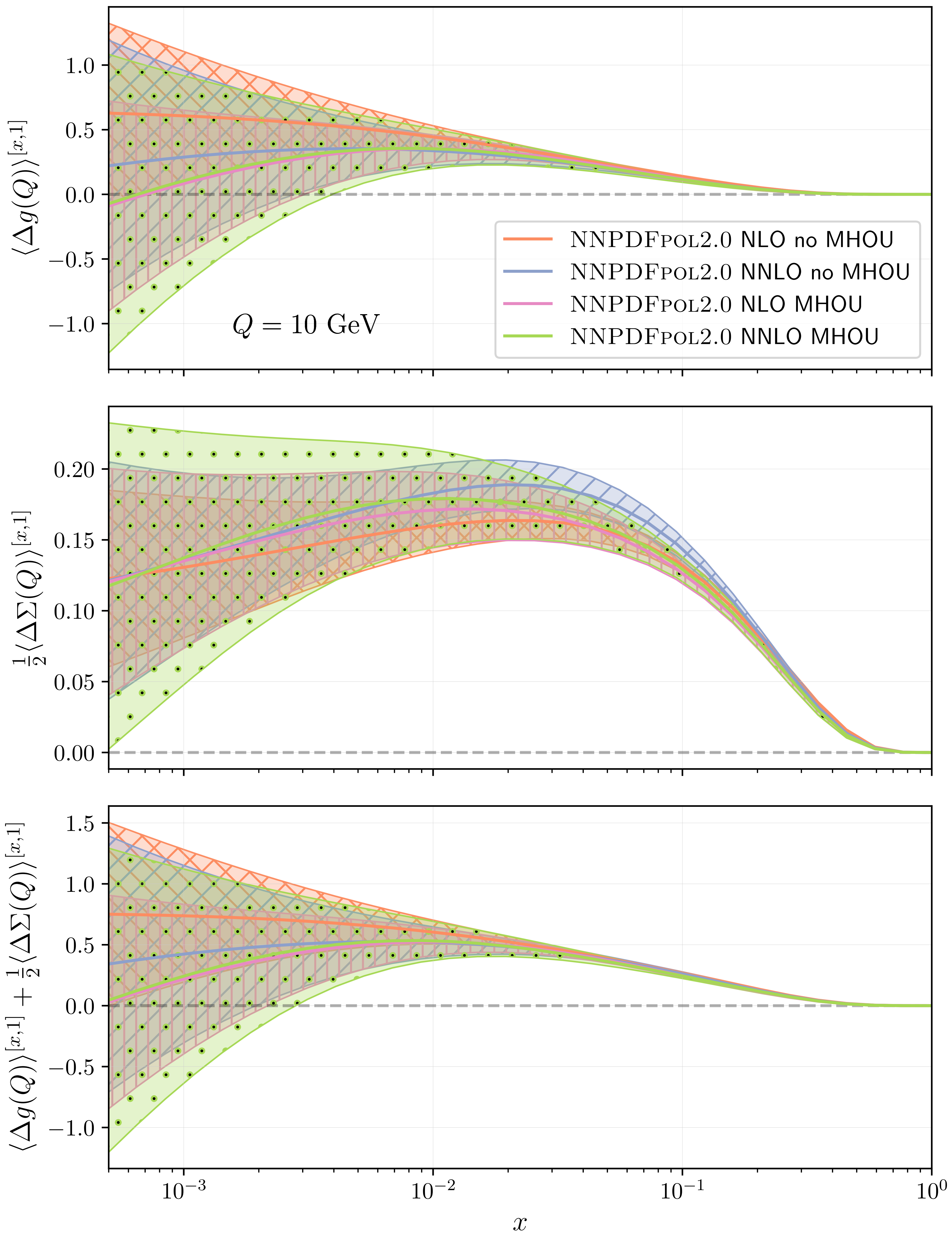
NOT QUITE TRUE: Positivity is immaterial to the sign of the Gluon $\Longleftrightarrow$ reduce large-x uncertainties



Outline of the Talk

	Introduction & Motivation	
Part I	GM-VFNS in Polarised DIS processes	[arXiv:2401.10127]
Part II	Data & Methodology	[arXiv:2410.16248]
Part III	NNPDFpol2.0 Helicity Dependent PDFs	[arXiv:2503.11814]
Part IV	**Phenomenology**	**[arXiv:2503.11814]**
	Conclusions & Outlook	

Phenomenological Implications

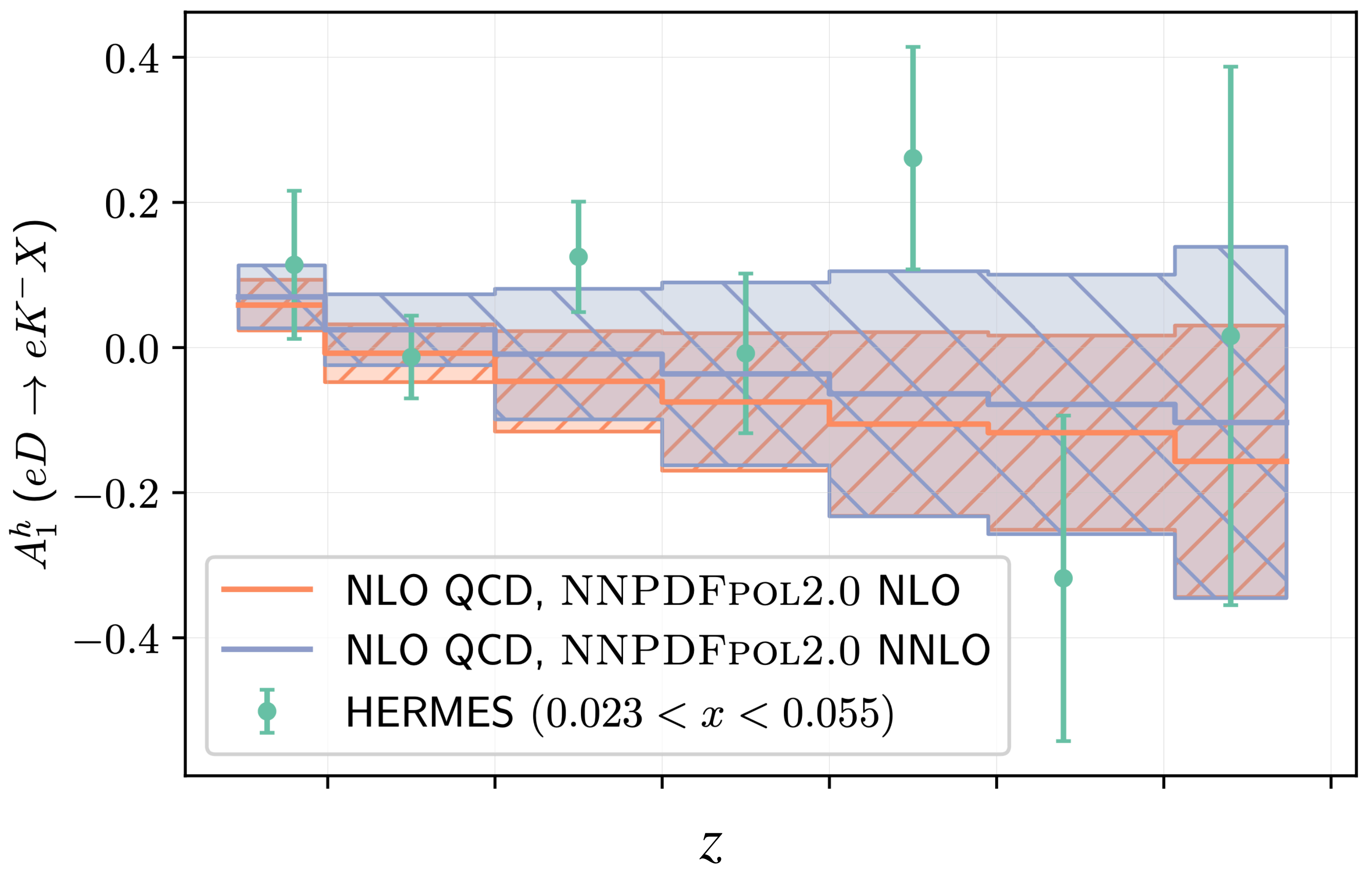


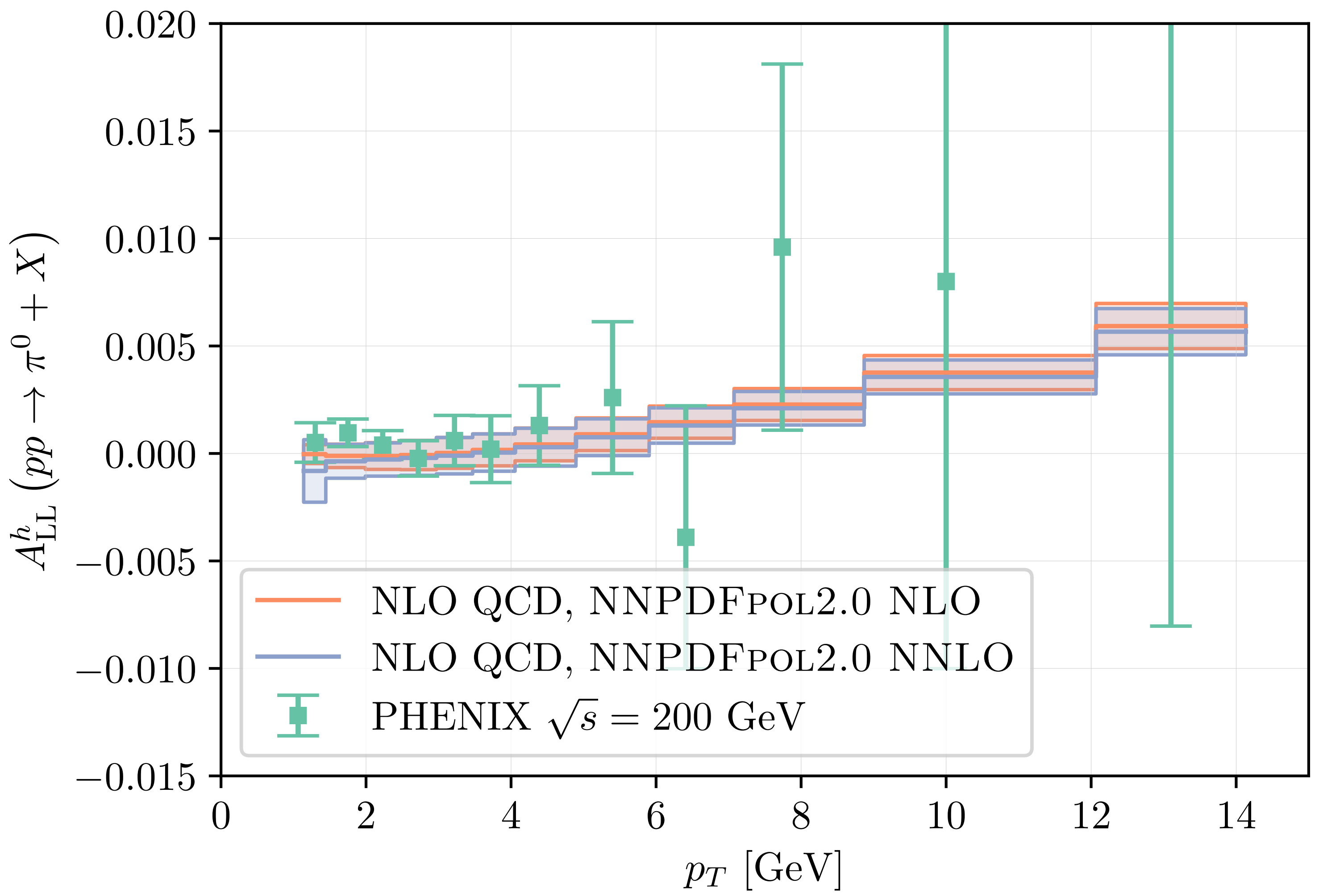
Spin content of the proton:

- ❖ Very small dependence on the perturbative accuracy
- ❖ Precision depends strongly on the small-x integration region
- ❖ Large uncertainties in small-x extrapolation consistent with PDF results

Single inclusive hadron production in SIDIS and pp collisions:

- ❖ NLO matrix elements are used in both predictions
- ❖ Predictions are in good agreement with data - within exp. uncertainties
- ❖ Exp. Uncertainties are very large $\Longleftrightarrow$ Constraints on polarised PDFs?





Outline of the Talk

PineAPPL: A Modern Fast Interpolation Grid

PineAPPLv1: "fast and flexible theory predictions for present and future colliders"

- ☑ support **ELECTROWEAK corrections**
- ☑ support **any arbitrary number of convolutions** $\Longrightarrow$ supports processes such as Single Inclusive Hadron Production in *pp* (**3 convolutions**), Exclusive Particle Production in *pp* and *pPb* (**4 convolutions**), …
- ☑ support **various combinations of initial- and final-states**:

$$\{\text{Unpolarized}, \text{Polarized}\} \otimes \{\text{PDF}, \text{FF}\}$$

- ☑ supports **additional variation of Fragmentation scale** μ_a and more…

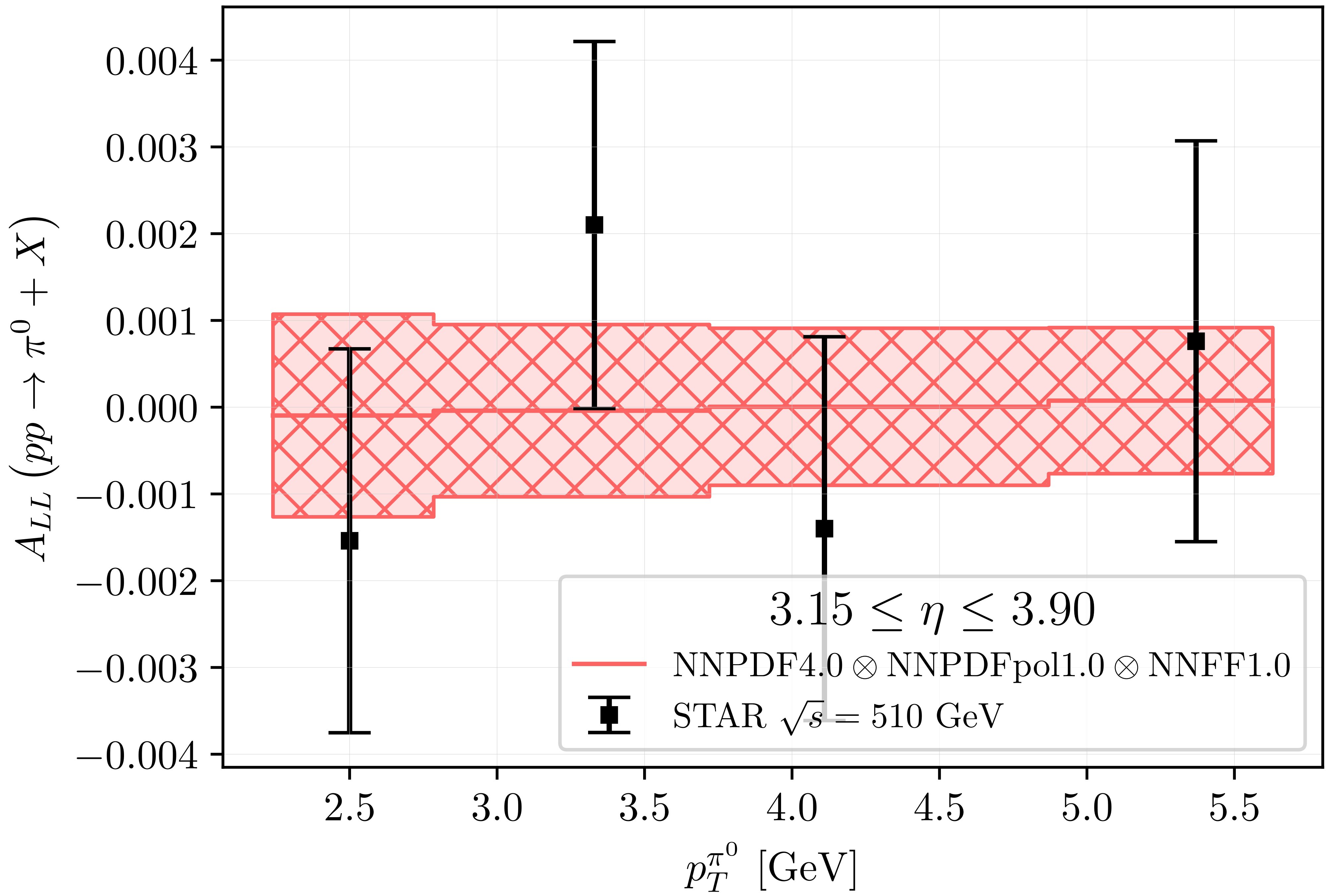
Rust passing | codecov 97% | docs passing | crates.io v1.0.0-alpha2 | Rust 1.80+

What is PineAPPL?

This repository contains programs, libraries and interfaces to read and write `PineAPPL` interpolation grids, which store theoretical predictions for high-energy collisions independently from their PDFs and the strong coupling.

PineAPPL grids are generated by Monte Carlo generators, and the grids in turn can be convolved with PDFs to produce tables and plots, such as the following one:

https://github.com/NNPDF/pineappl



Conclusions & Outlook

- Precise knowledge of Polarised PDFs is crucial to probe the spin content of the proton and will play an essential role in the EIC physics program
- NNPDFpol2.0 is based on **state-of-the-art theoretical** (mass effects, MHOUs, etc.) and **fitting frameworks** (new hyperparameter optimisation using ensemble method)
- RHIC data are relevant in constraining **key aspects of the polarised PDFs** (especially the Gluon)
- **Positivity constraints** ($|\Delta f_k| \leq f_k$) are **not crucial** in pinning down the **sign of the Gluon PDF**
- Formalism could be extend to extract the **parton-to Fragmentation Functions (FFs)**

THANKS FOR YOUR ATTENTION

“Wanderer above the Sea of Fog” by Caspar David Friedrich