aN^3LO PDF and Higgs cross sections

Felix Hekhorn

59^{th} Rencontres de Moriond

Outline

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What does aN^3LO mean?

Combining aN^3LO PDFs

What does aN^{3}LO mean?

Papers

- [2207.04739]: Approximate N^3LO parton distribution functions with theoretical uncertainties: MSHT20aN^3LO PDFs [EPJC83.185]
- [2402.18635]: The path to N^3LO parton distributions [EPJC84.659]
- [2406.16188]: A Benchmarking of QCD Evolution at Approximate N^3LO
- [2411.05373]: Combination of aN^3LO PDFs and implications for Higgs production cross-sections at the LHC

Note the publication dates on the arXiv!

How to compute theory predictions?

$$\sigma(Q^2) = \mathbf{C}(Q^2)\mathbf{E}^{(5)}(Q^2 \leftarrow m_b^2)\mathbf{A}^{(4)}(m_b^2)\mathbf{E}^{(4)}(m_b^2 \leftarrow Q_0^2)\mathbf{f}^{(4)}(Q_0^2)$$

with:

- **C**: coefficient function / partonic cross section
- **E**: evolution kernel operator $\Rightarrow$ splitting functions **P** / anomalous dimensions γ
- **A**: operator matrix elements

Operator Matrix Elements

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NNPDF

use exact expressions from Blümlein et al.

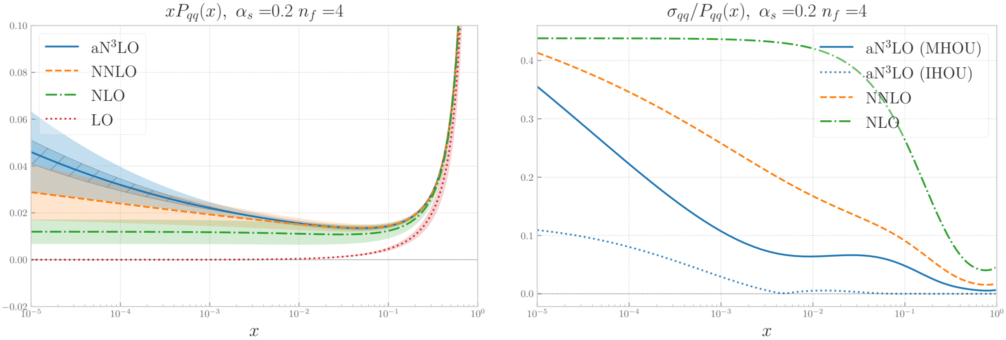
- [BBK09a]. Isabella Bierenbaum, Johannes Blümlein, and Sebastian Klein. Mellin Moments of the O(alpha**3(s)) Heavy Flavor Contributions to unpolarized Deep-Inelastic Scattering at Q**2 ensuremath >ensuremath > m**2 and Anomalous Dimensions. Nucl. Phys. B, 820:417-482, 2009. arXiv:0904.3563, doi:10.1016/j.nuclphysb.2009.06.005.
- [Blu00]. Johannes Blümlein. Analytic continuation of mellin transforms up to two-loop order. Computer Physics Communications, 133(1):76-104, Dec 2000. URL: http://dx.doi.org/10.1016/S0010-4655(00)00156-9, doi:10.1016/s0010-4655(00)00156-9.
- [BBK09b]. Isabella Bierenbaum, Johannes Blümlein, and Sebastian Klein. The Gluonic Operator Matrix Elements at O(alpha(s)**2) for DIS Heavy Flavor Production. Phys. Lett. B, 672:401-406, 2009. arXiv:0901.0669, doi:10.1016/j.physletb.2009.01.057.
- [ABK+11]. J. Ablinger, J. Blümlein, S. Klein, C. Schneider, and F. Wissbrock. The $O(alpha _s^3)$ Massive Operator Matrix Elements of $O(n_f)$ for the Structure Function $F_2(x,Q^2)$ and Transversity. Nucl. Phys. B, 844:26-54, 2011. arXiv:1008.3347, doi:10.1016/j.nuclphysb.2010.10.021.
- [ABB+14]. J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, C. Schneider, and F. Wißbrock. The 3-Loop Non-Singlet Heavy Flavor Contributions and Anomalous Dimensions for the Structure Function $F_2(x,Q^2)$ and Transversity. Nucl. Phys. B, 886:733-823, 2014. arXiv:1406.4654, doi:10.1016/j.nuclphysb.2014.07.010.
- [ABDF+14a]. J. Ablinger, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, and C. Schneider. The $O(alpha _s^3 T_F^2)$ Contributions to the Gluonic Operator Matrix Element. Nucl. Phys. B, 885:280-317, 2014. arXiv:1405.4259, doi:10.1016/j.nuclphysb.2014.05.028.
- [BBB+14]. A. Behring, I. Bierenbaum, J. Blümlein, A. De Freitas, S. Klein, and F. Wißbrock. The logarithmic contributions to the $O(alpha ^3_s)$ asymptotic massive Wilson coefficients and operator matrix elements in deeply inelastic scattering. Eur. Phys. J. C, 74(9):3033, 2014. arXiv:1403.6356, doi:10.1140/epjc/s10052-014-3033-x.
- [BAB+17]. Johannes Blümlein, Jakob Ablinger, Arnd Behring, Abilio De Freitas, Andreas von Manteuffel, Carsten Schneider, and C. Schneider. Heavy Flavor Wilson Coefficients in Deep-Inelastic Scattering: Recent Results. PoS, QCDEV2017:031, 2017. arXiv:1711.07957, doi:10.22323/1.308.0031.
- [ABDF+14b]. J. Ablinger, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, C. Schneider, and F. Wißbrock. The transition matrix element a_gq(n) of the variable flavor number scheme at o(α_s^3). Nuclear Physics B, 882:263-288, May 2014. URL: http://dx.doi.org/10.1016/j.nuclphysb.2014.02.007, doi:10.1016/j.nuclphysb.2014.02.007.
- [ABB+15]. J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. von Manteuffel, and C. Schneider. The 3-loop pure singlet heavy flavor contributions to the structure function f2(x,q2) and the anomalous dimension. Nuclear Physics B, 890:48-151, Jan 2015. URL: http://dx.doi.org/10.1016/j.nuclphysb.2014.10.008, doi:10.1016/j.nuclphysb.2014.10.008.
- [ABBlumlein+22]. J. Ablinger, J. and A. Behring, J. Blümlein, A. De Freitas, C. Schneider, A. Goedicke, C. von Manteuffel and K. Schonwald. The Unpolarized and Polarized Single-Mass Three-Loop Heavy Flavor Operator Matrix Elements $A_{gg,Q}$ and $Delta A_{gg,Q}$}. JHEP 12 (2022) 134, doi:10.1007/JHEP12(2022)134,
- [ABBlumlein+24]. J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. von Manteuffel, C. Schneider, K. Schönwald. The non-first-order-factorizable contributions to the three-loop single-mass operator matrix elements $A_{Qg}^{(3)}$ and $Delta A_{Qg}^{(3)}$.

MSHT

use approximations + nuisance parameters

Splitting Functions / Anomalous Dimensions

Strategy: combine available information and parametrize the remaining unknowns



Results by Moch et al. will be the new default! see talk by Andrea Pelloni

Partonic cross sections

- ✅ massless DIS, DY with $2 \to 1$ kinematics
- ⚠️ massive DIS: use available information to construct approximation
- ❗ everything else, e.g. differential DY, (di-)jets, top, …

but

- the situation for NNLO was/is just the same for, e.g., jets
- we account for missing higher order uncertainties: covariance matrix vs. nuisance parameters

Combining aN^{3}LO PDFs

Combining aN^{3}LO PDFs

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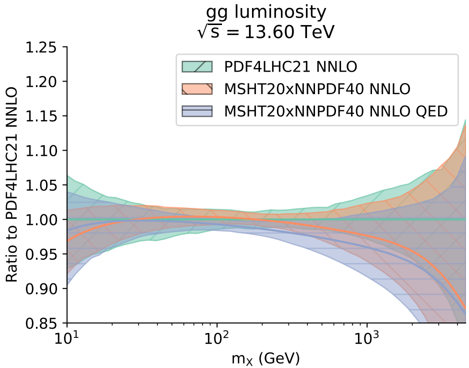
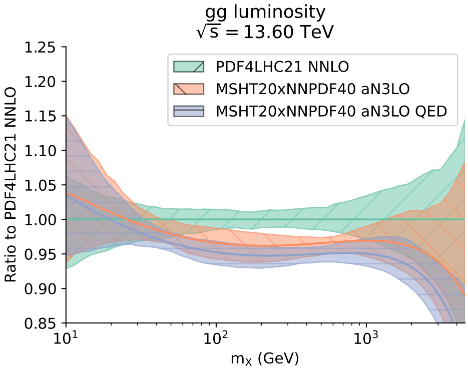
MSHT20xNNPDF40 = 100 replicas of MSHT20 + 100 replicas of NNPDF4.0

- usual differences in theory (e.g. heavy quark treatment), methodology (e.g. parametrization), or experiment setup remain
- this is a more conservative approach then PDF4LHC where parameters are (partly) harmonized

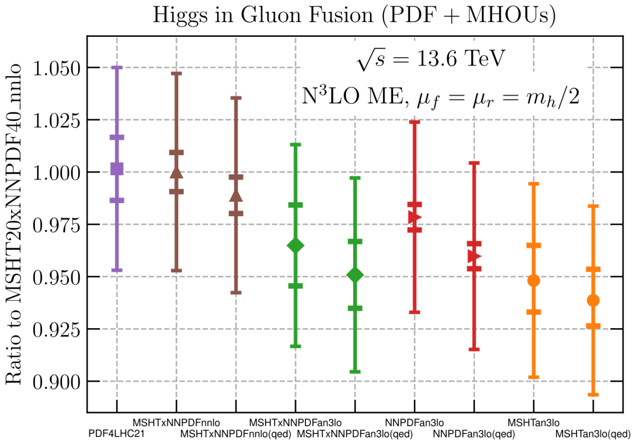
QED





- dataset selection is such that EW corrections are small
- QED only in evolution

$gg \to h$: PDF ratios



- PDF4LHC
- Combination@NNLO
  $\sigma = 46.49\,\text{pb}$
- Combination@N^{3}LO
- NNPDF4.0
- MSHT20

error bars:

- inner: PDF unc.
- outer: PDF unc. + MHOU

$gg \to h$: Estimating the PDF shift

Using the approximated shift due to mismatched PDF instead of the real one

$$\Delta^{\text{exact}} \equiv \left| \frac{\sigma^{\text{N}^3\text{LO}}_{\text{N}^3\text{LO}-\text{PDF}} - \sigma^{\text{N}^3\text{LO}}_{\text{NNLO}-\text{PDF}}}{\sigma^{\text{N}^3\text{LO}}_{\text{N}^3\text{LO}-\text{PDF}}} \right| \qquad \Delta^{\text{approx}} \equiv \frac{1}{2} \left| \frac{\sigma^{\text{NNLO}}_{\text{NNLO}-\text{PDF}} - \sigma^{\text{NNLO}}_{\text{NLO}-\text{PDF}}}{\sigma^{\text{NNLO}}_{\text{NNLO}-\text{PDF}}} \right|$$

yields a serious underestimation:

	NNPDF4.0	MSHT20	Combination
Δ^{exact}	2.2 %	5.3 %	3.3 %
Δ^{approx}	0.2 %	1.4 %	0.9 %

Summary

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$$\sigma_{\rm ggF} = 44.86 \pm 0.90_{\rm pdf}\ \text{pb} \quad \text{(pure QCD)}$$
$$\sigma_{\rm ggF} = 44.20 \pm 0.75_{\rm pdf}\ \text{pb} \quad \text{(QCD+QED).}$$

- N^3LO evolution is completed
- N^3LO cross sections are a long term project - meanwhile we use MHOU
- aN^3LO PDF have significant impact on $gg \to h$
- QED effects should be considered

Summary

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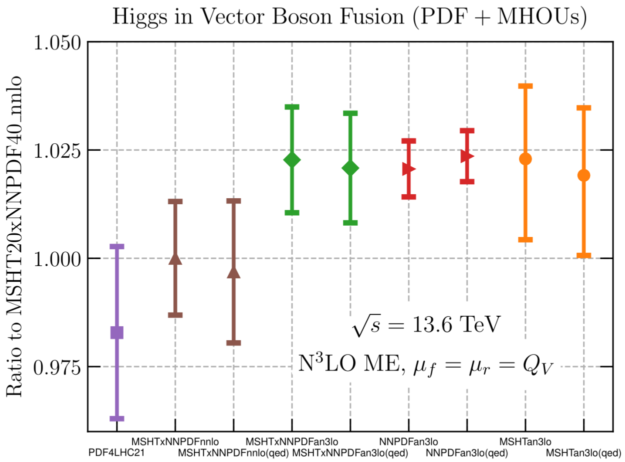
Danke! Thanks! Kiitos!

Backup slides

h VBF: PDF ratios



- PDF4LHC
- Combination@NNLO
  $\sigma = 4.35\,\mathrm{pb}$
- Combination@N^{3}LO
- NNPDF4.0
- MSHT20

error bars:

- inner: PDF unc.
- outer: PDF unc. + MHOU

h VBF: Estimating the PDF shift

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Using the approximated shift due to mismatched PDF instead of the real one yields a serious underestimation:

	NNPDF4.0	MSHT20	Combination
Δ^{exact}	1.3 %	2.3 %	2.3 %
Δ^{approx}	0.2 %	1.3 %	0.5 %

Higgs Cross Sections

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PDF set	pert. order (PDF)	$\sigma(gg \to h)$	$\sigma(h\ \mathrm{VBF})$
PDF4LHC21_mc	$\mathrm{NNLO_{QCD}}$	$46.56^{+1.5\%\ +4.4\%}_{-1.5\%\ -5.3\%}$	$4.27^{+2.0\%\ +2.0\%}_{-2.0\%\ -2.0\%}$
MSHT20xNNPDF40_nnlo	$\mathrm{NNLO_{QCD}}$	$46.49^{+0.9\%\ +4.2\%}_{-0.9\%\ -5.3\%}$	$4.35^{+1.3\%\ +1.3\%}_{-1.3\%\ -1.3\%}$
MSHT20xNNPDF40_nnlo_qed	$\mathrm{NNLO_{QCD}} \otimes \mathrm{NLO_{QED}}$	$45.97^{+0.9\%\ +4.3\%}_{-0.9\%\ -5.4\%}$	$4.34^{+1.6\%\ +1.6\%}_{-1.6\%\ -1.6\%}$
MSHT20xNNPDF40_an3lo	$\mathrm{aN^3LO_{QCD}}$	$44.86^{+2.0\%\ +4.4\%}_{-2.0\%\ -5.6\%}$	$4.45^{+1.2\%\ +1.2\%}_{-1.2\%\ -1.2\%}$
MSHT20xNNPDF40_an3lo_qed	$\mathrm{aN^3LO_{QCD}} \otimes \mathrm{NLO_{QED}}$	$44.20^{+1.7\%\ +4.3\%}_{-1.7\%\ -5.4\%}$	$4.44^{+1.2\%\ +1.2\%}_{-1.2\%\ -1.2\%}$
NNPDF40_an3lo_as_01180_mhou	$\mathrm{aN^3LO_{QCD}}$	$45.49^{+0.6\%\ +4.1\%}_{-0.6\%\ -5.2\%}$	$4.44^{+0.6\%\ +0.7\%}_{-0.6\%\ -0.6\%}$
NNPDF40_an3lo_as_01180_qed_mhou	$\mathrm{aN^3LO_{QCD}} \otimes \mathrm{NLO_{QED}}$	$44.62^{+0.6\%\ +4.1\%}_{-0.6\%\ -5.2\%}$	$4.45^{+0.6\%\ +0.6\%}_{-0.6\%\ -0.6\%}$
MSHT20an3lo_as118	$\mathrm{aN^3LO_{QCD}}$	$44.08^{+1.8\%\ +4.4\%}_{-1.6\%\ -5.6\%}$	$4.44^{+1.6\%\ +1.7\%}_{-1.8\%\ -1.8\%}$
MSHT20qed_an3lo	$\mathrm{aN^3LO_{QCD}} \otimes \mathrm{NLO_{QED}}$	$43.63^{+1.6\%\ +4.3\%}_{-1.3\%\ -5.3\%}$	$4.43^{+1.5\%\ +1.5\%}_{-1.8\%\ -1.8\%}$

Splitting Functions / Anomalous Dimensions

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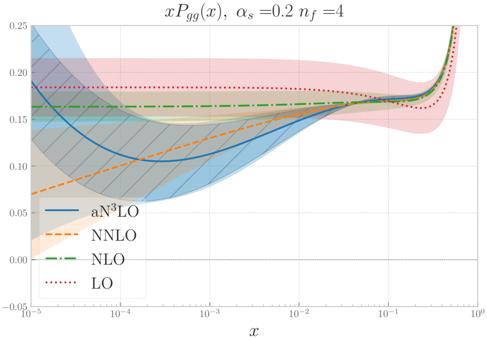
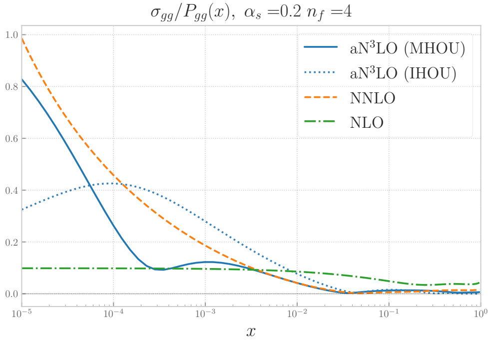
Strategy: combine available information and parametrize the remaining unknowns





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Splitting Functions / Anomalous Dimensions

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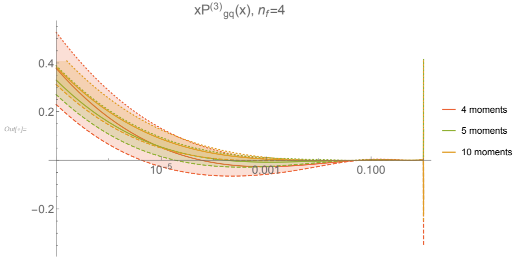

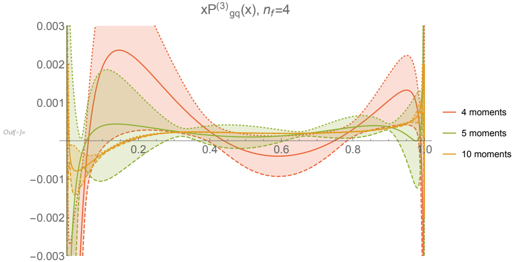

Splitting Functions / Anomalous Dimensions

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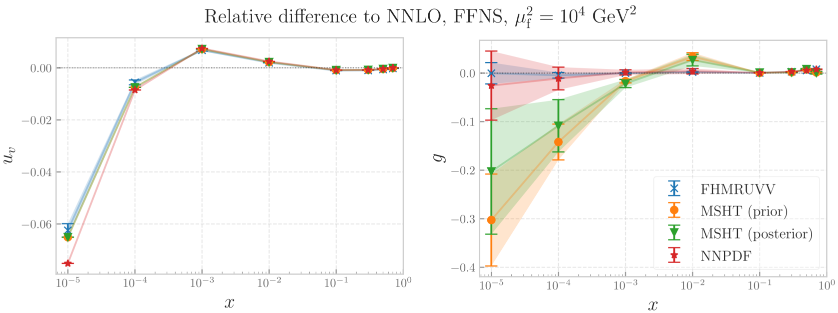

FHMRUVV = Falcioni, Herzog, Moch, Ruijl, Ueda, Vermaseren, Vogt will be the new default! see talk by Andrea Pelloni