

$N^3\text{LO}$ calculations and benchmarking of PDFs at $aN^3\text{LO}$ accuracy

LHC EW WG General Meeting

Emanuele R. Nocera

Università degli Studi di Torino and INFN, Torino

1 April 2026



UNIVERSITÀ
DI TORINO

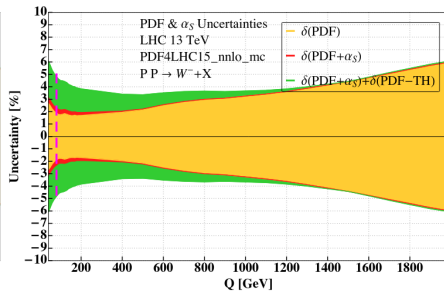
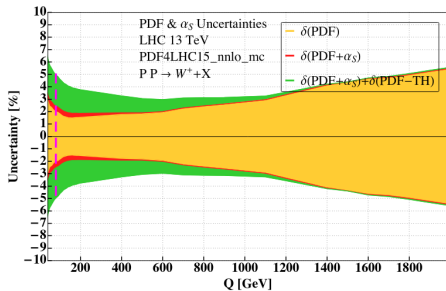
N³LO QCD corrections in PDF determination

NNLO is the precision frontier for PDF determination

N3LO is the precision frontier for partonic cross sections

Mismatch between perturbative order of partonic cross sections and accuracy of PDFs is becoming a significant source of uncertainty

$$\hat{\sigma} = \alpha_s^p \hat{\sigma}_0 + \alpha_s^{p+1} \hat{\sigma}_1 + \alpha_s^{p+2} \hat{\sigma}_2 + \mathcal{O}(\alpha_s^{p+3}) \quad \delta(\text{PDF} - \text{TH}) = \frac{1}{2} \left| \frac{\sigma_{\text{NNLO-PDFs}}^{(2)} - \sigma_{\text{NLO-PDFs}}^{(2)}}{\sigma_{\text{NNLO-PDFs}}^{(2)}} \right|$$



[JHEP 11 (2020) 143]

Perturbative input

Splitting Functions

Singlet ($P_{qq}, P_{gg}, P_{gq}, P_{qg}$)

- large- n_f limit [NPB 915 (2017) 335; JHEP 01 (2024) 029]
- small- x limit [JHEP 06 (2018) 145]
- large- x limit [NPB 832 (2010) 152; JHEP 04 (2020) 018; JHEP 09 (2022) 155]
- 10 lowest Mellin moments

[PLB 825 (2022) 136853; 842 (2023) 137944; 846 (2023) 138215; 848 (2024) 138351; 856 (2024) 138906; 860 (2025) 139194]

Non-singlet ($P_{NS,v}, P_{NS,+}, P_{NS,-}$)

- large- n_f limit [NPB 915 (2017) 335; JHEP 01 (2024) 029]
- small- x limit [JHEP 08 (2022) 135]
- large- x limit [JHEP 10 (2017) 041]
- 8 lowest Mellin moments [JHEP 06 (2018) 073]

DIS structure functions (F_L, F_2, F_3)

- DIS NC (massless) [NPB 492 (1997) 338; PLB 606 (2005) 123; NPB 724 (2005) 3]
- DIS CC (massless) [NPB 813 (2009) 220]
- massive from parametrisation combining known limits and damping functions [NPB 864 (2012) 399]

PDF matching conditions (transition elements)

– $A_{gg,H}, A_{Hg}, A_{qq,H}, A_{qg,H}$

[NPB 820 (2009) 417; 844 (2011) 26; 886 (2014) 733; 999 (2024) 116426; JHEP 12 (2022) 134; PLB 854 (2024) 138713]

Coefficient functions for other processes

- DY (inclusive) [JHEP 11 (2020) 143]; DY (y differential) [PRL 128 (2022) 052001]

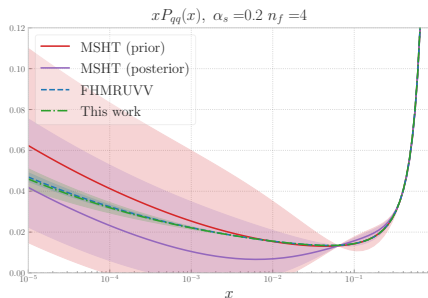
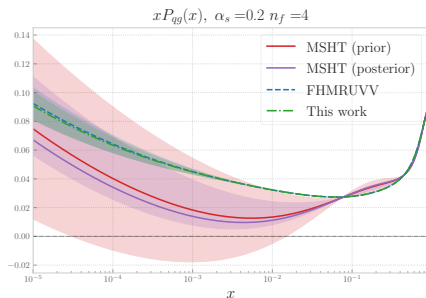
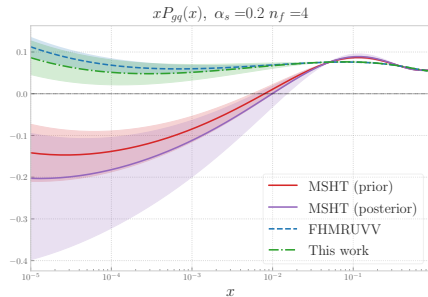
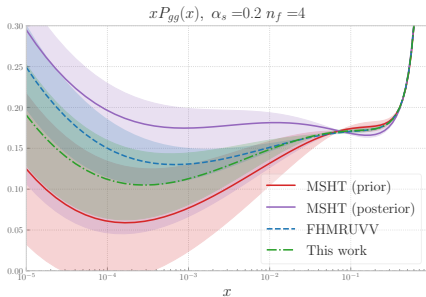
MSHT and NNPDF aN³LO PDF determinations

MSHT [EPJ C83 (2023) 185]

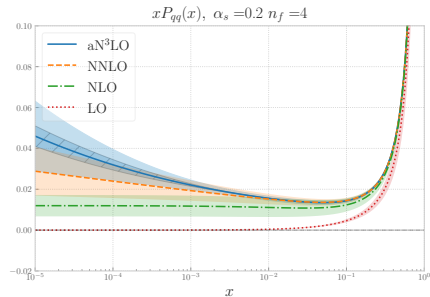
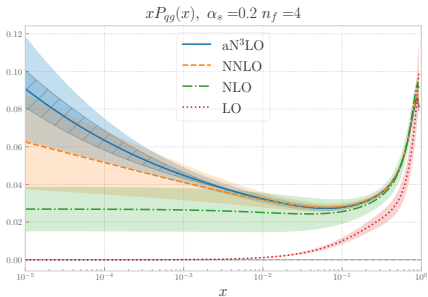
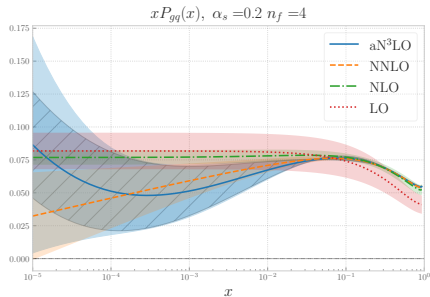
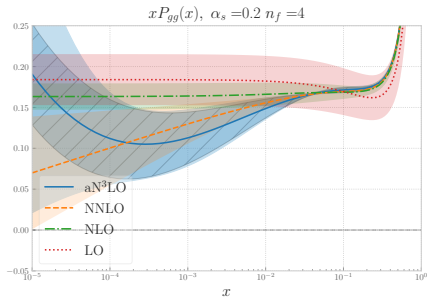
NNPDF [EPJ C84 (2024) 659]

splitting functions	linear combinations of as many interpolating functions, satisfying the known small- x , large- x , and large- n_f limits, as the number of known Mellin moments; coefficients fixed by known Mellin moments singlet: 4; non-singlet: 8	singlet: 5, 6 for $P_{ps}^{(3)}$; non-singlet: 8
IHOUs	nuisance parameters (1 per splitting function, for a total of 5) fitted to the data	estimated by varying the basis of interpolation functions and by constructing a corresponding theory covariance matrix
transition elements	approximations constructed similarly to splitting functions (5 nuisance parameters in total)	exact inclusion of all transition elements but $A_{gg,H}$ parametrised similarly to splitting functions
DIS heavy quarks	massive coefficient functions determined from parametrisations combining known limits and damping functions (continuous information)	
hadronic K -factors	parametrised as linear combinations of NNLO and NLO K -factors (2 nuisance parameters per process for a total of 10)	replaced by MHOUs
MHOUs	delegated to the 20 nuisance parameters (uncorrelated or correlated)	theory covariance matrix built from correlated 7-point scale variations

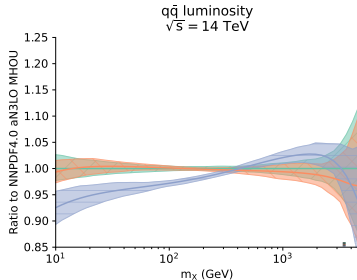
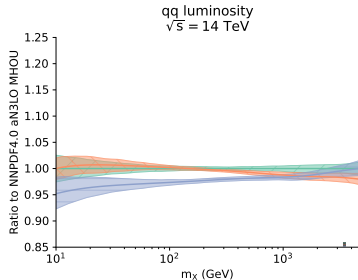
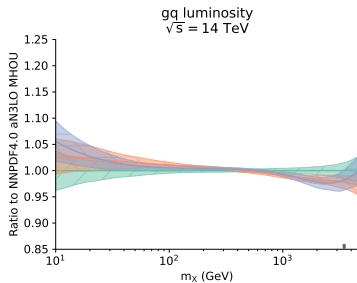
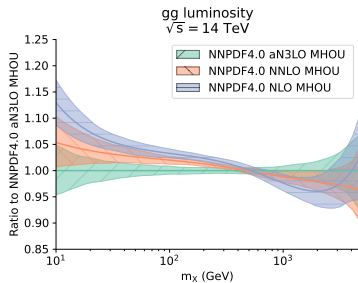
Singlet splitting functions: MSHT vs NNPDF



Singlet splitting functions: perturbative convergence

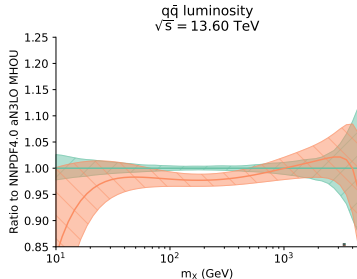
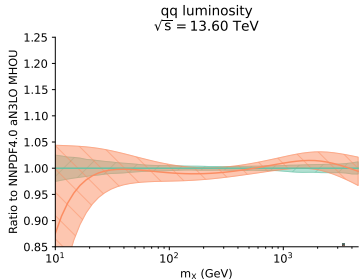
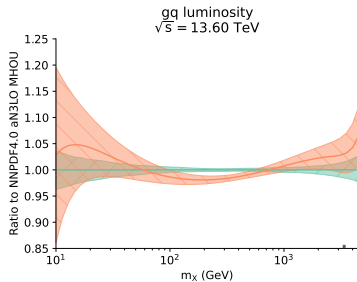
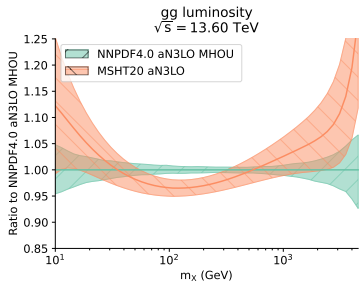


Impact of aN³LO corections on partonic luminosities



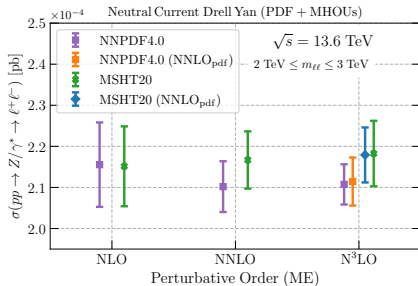
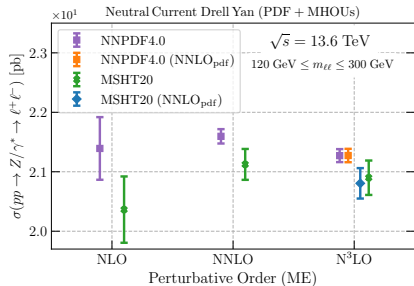
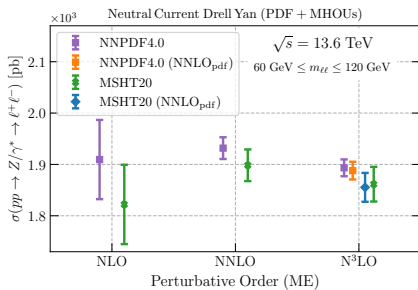
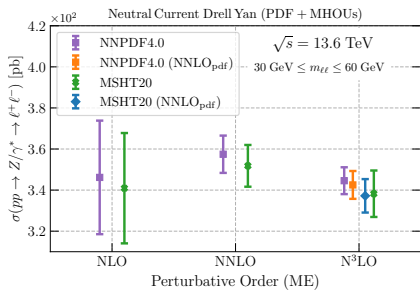
Trend observed for NNPDF4.0 [EPJ C84 (2024) 659] similar to MSHT20 [EPJ C83 (2023) 185]

aN³LO PDFs: MSHT20 vs NNPDF4.0

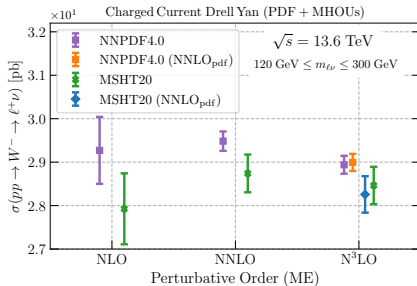
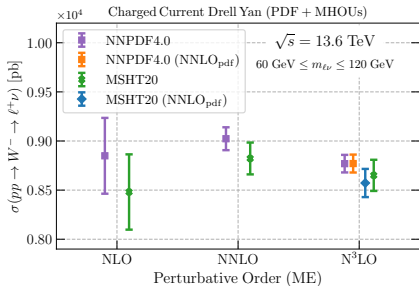
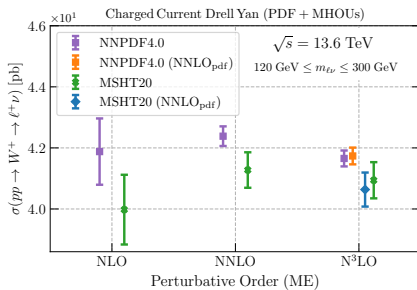
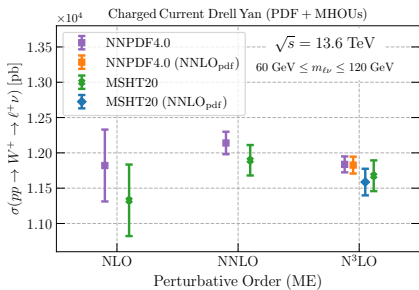


A 3% shift for $M_X \sim 100$ GeV, already at NNLO, partly sensitive to higher moments

Inclusive cross sections: NC Drell–Yan



Inclusive cross sections: CC Drell–Yan



Uncertainties

Process	σ (pb)	NNPDF4.0					MSHT20				
		δ_{th}	$\delta_{\text{PDF}}^{\text{noMHO}}$	$\delta_{\text{PDF}}^{\text{MHO}}$	$\Delta_{\text{NNLO}}^{\text{app}}$	$\Delta_{\text{NNLO}}^{\text{exact}}$	σ (pb)	$\delta_{\text{th}}\sigma$	δ_{PDF}	$\Delta_{\text{NNLO}}^{\text{app}}$	$\Delta_{\text{NNLO}}^{\text{exact}}$
W^+ (p)	1.2×10^4	1.0	0.5	0.5	1.1	0.1	1.2×10^4	1.9	1.7	2.3	0.8
W^- (p)	8.8×10^3	1.0	0.5	0.5	1.1	0.1	8.7×10^3	1.9	1.6	2.1	0.0
Z (p)	1.9×10^3	0.9	0.4	0.5	1.1	0.3	1.9×10^3	1.8	1.6	2.6	0.3
W^+ (hm)	4.7×10^{-4}	2.8	2.8	3.3	3.2	1.1	4.6×10^{-4}	4.0	3.9	2.0	1.3
W^- (hm)	1.4×10^{-4}	2.9	2.9	3.3	3.3	0.1	1.5×10^{-4}	4.2	4.2	2.0	0.6
Z (hm)	2.1×10^{-4}	2.3	2.3	2.5	3.4	0.3	2.2×10^{-4}	3.6	3.6	2.7	0.2

δ_{th} : PDF and 7-point scale variation uncertainties added in quadrature

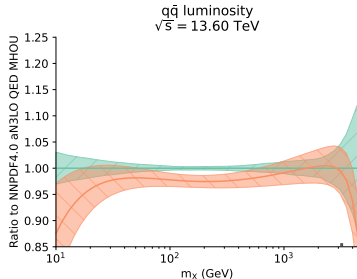
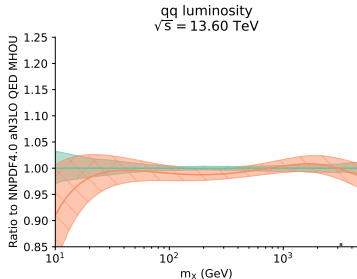
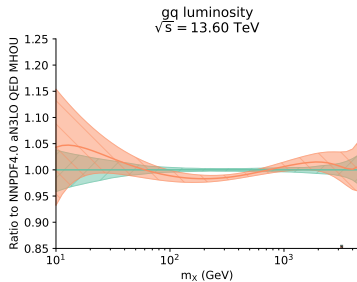
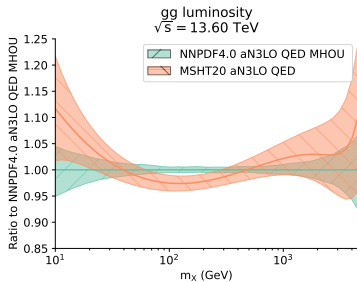
δ_{PDF} : PDF uncertainty

(NNPDF: MHO and noMHO indicate if MHOU were or were not included in the fit)

$$\Delta_{\text{NNLO}}^{\text{app}} \equiv \frac{1}{2} \left| \frac{\sigma_{\text{NNLO-PDF}}^{\text{NNLO}} - \sigma_{\text{NNLO-PDF}}^{\text{NNLO}}}{\sigma_{\text{NNLO-PDF}}^{\text{NNLO}}} \right|$$

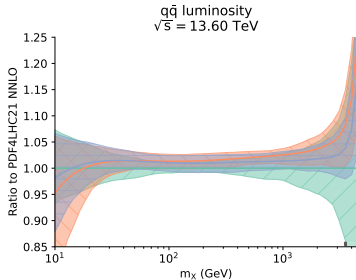
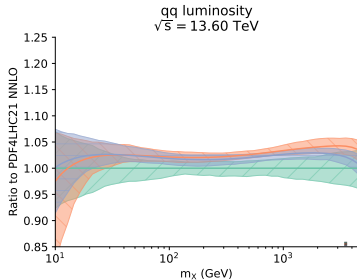
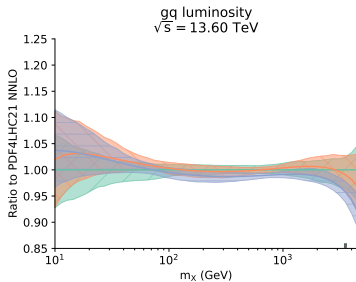
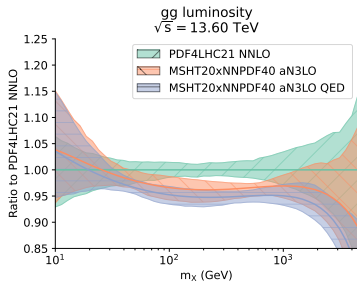
$$\Delta_{\text{NNLO}}^{\text{exact}} \equiv \left| \frac{\sigma_{\text{N}^3\text{LO-PDF}}^{\text{N}^3\text{LO}} - \sigma_{\text{NNLO-PDF}}^{\text{N}^3\text{LO}}}{\sigma_{\text{N}^3\text{LO-PDF}}^{\text{N}^3\text{LO}}} \right|$$

Combining QED and QCD higher order corrections



QED corrections slightly reduce discrepancies between the MSHT20 and NNPDF4.0

Combining MSHT and NNPDF aN³LO QED PDFs



Statistical combination performed as in PDF4LHC21 [JPG 52 (2025) 065002]

Updates from CT

Complete flavour decompositions of DIS structure functions at N³LO using approximate zero-mass Wilson coefficients with a rescaling variable [B. Wang and K. Xie PhD theses]

Ongoing implementation of massive N³LO heavy-quark coefficients to obtain N³LO DIS cross sections in the S-ACOT scheme

DGLAP evolution with APFEL++ [EPJ C84 (2024) 774], see also Candia [2512.22667]

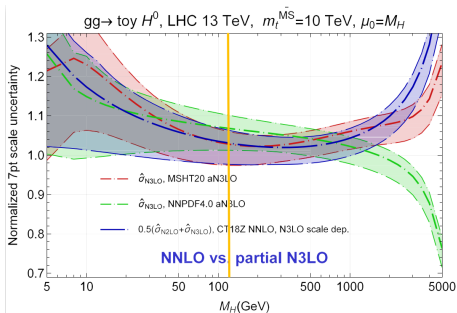
Inclusion of N³LO DY effects using NNLO ApplFast + N³LO/N²LO K -factor tables

First release of CT aN³LO PDFs expected at DIS2026 [J. Huston, HXSWG meeting December 2025]

Proposal for a CT18 NNLO+ prescription

- Use CTZ18 NNLO or CT18NNLO error sets
- Central predictions: average of predictions with $\hat{\sigma}_{\text{NNLO}}$ and $\hat{\sigma}_{\text{N}^3\text{LO}}$
- Scale uncertainty: compute using $\hat{\sigma}_{\text{N}^3\text{LO}}$

Is this prescription different from predictions with MSHT/NNPDF aN³LO PDFs?



[See also arXiv:2512.19779]

Are results stable upon inclusion of new results?

New in MSHT

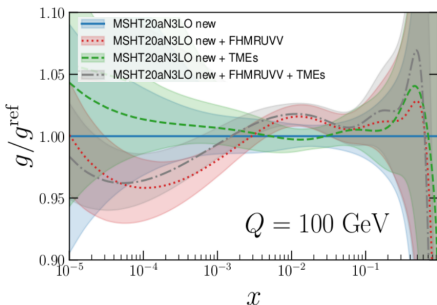
higher moments for singlet splitting funcs.

[PLB 842 (2023) 137944; 846 (2023) 138215; 848 (2024) 138351]

[PLB 856 (2024) 138906; 860 (2025) 139194]

computation of $A_{gg,H}$, $A_{H,g}$, $A_{qq,H}$, $A_{qg,H}$

[JHEP 12 (2022) 134; NPB 999 (2024) 116427]



[PoS DIS2025 (2025) 027]

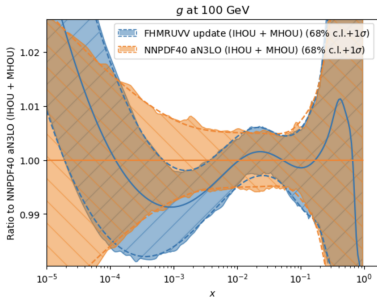
New in NNPDF

higher moments for singlet splitting funcs.

[PLB 856 (2024) 138906; 860 (2025) 139194]

improved parametrisation of $a_{H,g}^{(3)}$

[PLB 854 (2024) 138713]



[See G. Magni's talk PDF4LHC 2024]

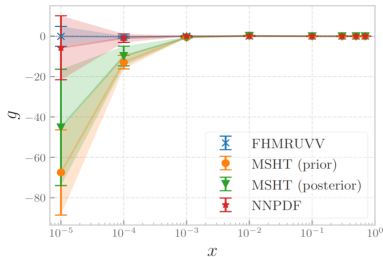
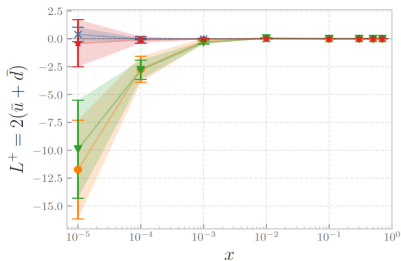
The gluon PDF moves up by 1.5% for MSHT, down by 1% for NNPDF near $x = 0.01$.

MSHT and NNPDF move closer together, which results in a 1.5% for $gg \rightarrow H$.

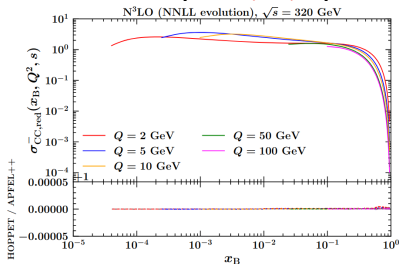
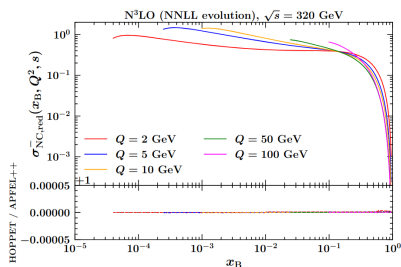
Main other features of N³LO comparison largely unaffected.

Do results depend on the numerical implementation?

Benchmark of aN³LO evolution [arXiv:2406.16188]



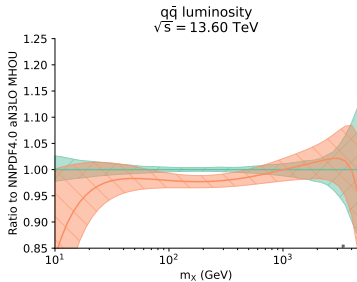
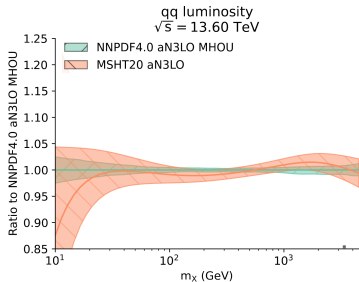
Benchmark of massless N³LO structure functions [EPJ C84 (2024) 774]



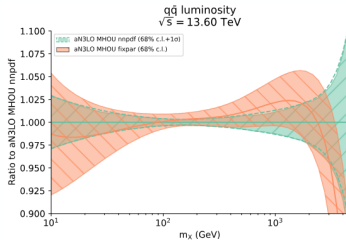
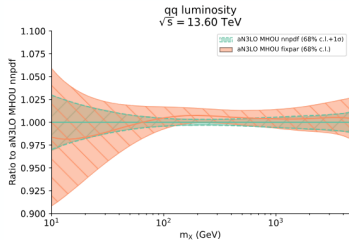
[See also the implementation in xFitter]

Are results stable upon the change of methodology?

Different theory and different methodology



Same theory, different methodology



[For details, see arXiv:2602.07118]

Summary

As experimental precision increases, theoretical accuracy should increase accordingly.

A precise and accurate determination of PDFs is key into this process.

A significant effort has been put to determine aN^3LO PDFs in recent years.

The exercise is not purely academic:

aN^3LO PDFs have a few percent impact on parton luminosities

PDF sets that combine aN^3LO and QED corrections exist

A MSHT-NNPDF combination of aN^3LO PDFs also exists

Should one be concerned by the fact that PDFs are aN^3LO and not N^3LO ?

Extensive investigations have been performed to benchmark numerical stability,
assess the impact of different perturbative input,
and disentangle effects due to theory from effects due to methodology.

Further investigations/benchmarks/combinations are foreseen
as part of the PDF4LHC activities also with the forthcoming aN^3LO CT PDF set.

Summary

As experimental precision increases, theoretical accuracy should increase accordingly.

A precise and accurate determination of PDFs is key into this process.

A significant effort has been put to determine aN^3LO PDFs in recent years.

The exercise is not purely academic:

aN^3LO PDFs have a few percent impact on parton luminosities

PDF sets that combine aN^3LO and QED corrections exist

A MSHT-NNPDF combination of aN^3LO PDFs also exists

Should one be concerned by the fact that PDFs are aN^3LO and not N^3LO ?

Extensive investigations have been performed to benchmark numerical stability,
assess the impact of different perturbative input,
and disentangle effects due to theory from effects due to methodology.

Further investigations/benchmarks/combinations are foreseen
as part of the PDF4LHC activities also with the forthcoming aN^3LO CT PDF set.

Thank you