

Modelling theory uncertainties in PDF determinations

Theoretical uncertainties in LHC electroweak precision measurements

Emanuele R. Nocera

Università degli Studi di Torino and INFN, Torino

11 September 2026



UNIVERSITÀ
DI TORINO

Which theory uncertainties?

$$F(x, Q^2) = \sum_i \int_x^1 \frac{dz}{z} f_i(x, Q^2) C_i \left(\frac{x}{z}, \alpha_s(Q^2) \right)$$

- Missing higher orders (MHO)
 - Unknown next terms in the perturbative series
 - Scale-variation directions and correlations
- Incomplete higher orders (IHO)
 - Unknown pieces of approximate order ($N^3\text{LO}$)
 - Ensemble of admissible approximations
- Nuclear effects
 - Relationship between bound- and free-nucleon predictions
 - Model or nuclear PDF ensemble, possibly iterated
- Higher twist/power corrections
 - Unknown kinematic correction functions
 - Parametrised corrections with priors
- SM parameters ($\alpha_s, m_c, M_W, \dots$)
 - Physical input parameters
 - Joint inference

How do theory uncertainties affect a PDF fit?

PDF determination is an inverse problem

Given a set of data D , determine $p(f|D)$ in the space of functions $f : [0, 1] \rightarrow \mathbb{R}$

The forward map (factorisation theorems) is complicated (convolution integral)

The set of data is heterogeneous (NC/CC DIS, NC/CC DY, jets, top ...)

The problem is typically solved by parametric regression

Approximate $p(f|D)$ with its projection in the space of parameters $p(\theta|D)$

The posterior $p(\theta|D) \propto p(D|\theta)p(\theta)$ is determined by optimising the likelihood

$$\chi^2[\theta] = \sum_{i,j}^{N_{\text{dat}}} (T_i[\theta] - D_i)(\text{cov}_{\text{exp}})_{ij}^{-1} (T_j[\theta] - D_j) = -2 \ln p(D|\theta) + \text{const.}$$

expected when experimental uncertainties are modelled as Gaussian

$$\epsilon^{\text{exp}} \sim \mathcal{N}(0, \text{cov}_{\text{exp}}) \quad D_i = T_i[\theta] + \epsilon^{\text{exp}}$$

With theory uncertainties, there is a shift δ^{th}

$$D_i = T_i[\theta] + \epsilon^{\text{exp}} + \delta^{\text{th}}$$

Modelling theory uncertainties

Validating theory uncertainties

1. Modelling theory uncertainties

Theory uncertainties as nuisance parameters

Parametrise the shift δ^{th} in terms of nuisance parameters λ

$$T_i[\theta] \rightarrow T_i[\theta] + \sum_a B_{ia} \lambda_a \quad \lambda_a \sim \mathcal{N}(0, 1)$$

with B_{ia} the matrix of theory uncertainty directions

$$\chi^2[\theta] \rightarrow \chi^2[\theta, \lambda] = \sum_{i,j}^{N_{\text{dat}}} (D_i - T_i[\theta] - \sum_a B_{ia} \lambda_a)^T (\text{cov}_{\text{exp}})_{ij}^{-1} (D_j - T_j[\theta] - \sum_a B_{aj} \lambda_a) + \lambda^T \lambda$$

Determine λ_a from the data so that $\partial\chi^2/\partial\lambda_a = 0$

The fit may move along physically motivated theory directions, paying a prior penalty

Advantages

- individual theory directions are identifiable
- posterior nuisance pulls can be inspected
- joint fits with model parameters are natural

Disadvantages

- increased dimensionality of the fit
- dependence on nuisance parameter basis
- theory directions may be degenerate

This approach is used by MSHT and CT for joint PDF/ $\alpha_s/m_c/m_t$ determinations, to estimate nuclear and HT effects, and by MSHT to estimate MHOUs and IHOUs

Theory uncertainties as a covariance matrix

Define a prior theoretical covariance matrix

$$(\text{cov}_{\text{th}})_{ij} = S_{ij} \quad S_{ij} = \sum_a B_{ia} B_{aj}$$

with B_{ia} the matrix of theory uncertainty directions

$$\chi^2[\boldsymbol{\theta}] \rightarrow \chi^2[\boldsymbol{\theta}] = \sum_{i,j}^{N_{\text{dat}}} (D_i - T_i[\boldsymbol{\theta}])^T (\text{cov}_{\text{exp}} + \text{cov}_{\text{th}})_{ij}^{-1} (D_j - T_j[\boldsymbol{\theta}])$$

Perform the fit as usual

The same theory directions appear as correlations between data points and processes

Advantages

- compact and computationally convenient
- compatible with usual PDF fitting
- correlations are treated globally

Disadvantages

- individual directions less transparent
- the guess on S must be educated enough
- parameter-dependent S requires care

This approach is used by NNPDF for joint PDF/ α_s / m_t determinations to estimate MHOUS, IHOUs, nuclear, and HT/PC effects

Are the two approaches the same?

The two approaches are two representations of the same linear Gaussian model

They are the same under the following assumptions:

- the theory response is linear in the nuisance parameters
 - the nuisance prior is Gaussian
- the covariance or nuisance directions are fixed with respect to the fitted parameters
 - nuisance parameters are consistently profiled or marginalised

The equivalence can be demonstrated analytically by profiling $\chi^2[\boldsymbol{\theta}, \boldsymbol{\lambda}]$ or equivalently by marginalising the the likelihood over the nuisance parameters

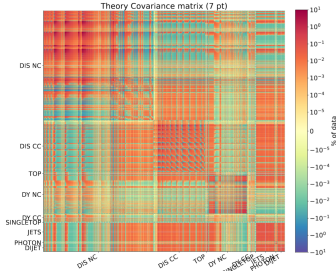
Are these assumptions compatible with the underlying physics?

How should theory directions be chosen?

To which extent is PDF determination affected by these assumptions?

Validation is crucial to answer these questions

MHOUs at NNLO: NNPDF



$$S_{ij} = \frac{1}{N} \sum_k \Delta_i^{(k)} \Delta_j^{(k)}$$

$$\Delta_i^{(k)} = T_i^{7\text{pt}}(\mu_R^{(k)}, \mu_F^{(k)}) - T_i(\mu_R^0, \mu_F^0)$$

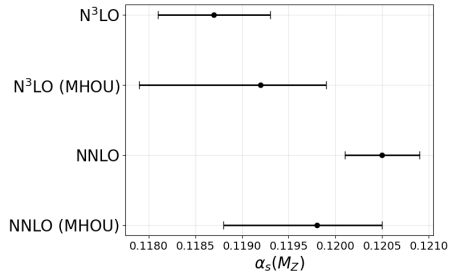
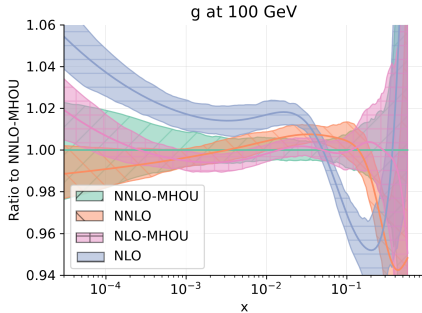
$$\chi_{\text{tot}}^2 = 1.23 \rightarrow \chi_{\text{tot}}^2 = 1.34 \text{ (no MHOUs, NLO)}$$

$$\chi_{\text{tot}}^2 = 1.13 \rightarrow \chi_{\text{tot}}^2 = 1.17 \text{ (no MHOUs, NNLO)}$$

Tensions relieved

Faster perturbative convergence

Small increase of PDF uncertainties



[EPJ C79 (2019) 282; EPJ C81 (2021) 37; EPJ C84 (2024) 517]

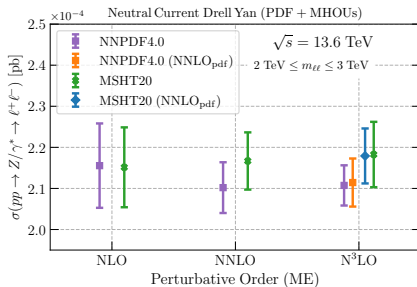
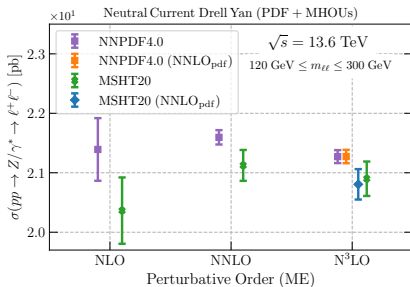
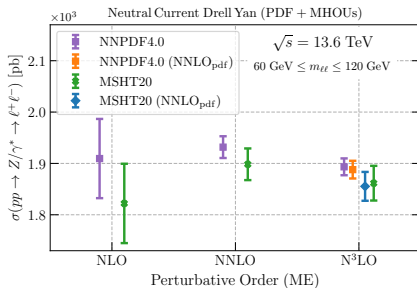
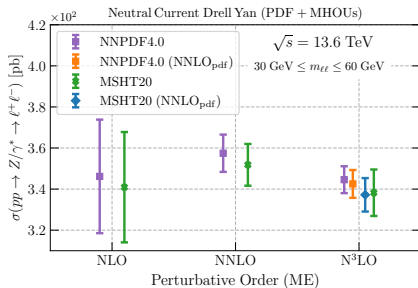
MHOUs and IHOUs at N³LO: MSHT and NNPDF

MSHT [EPJ C83 (2023) 185]

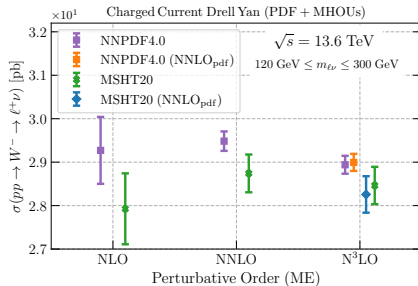
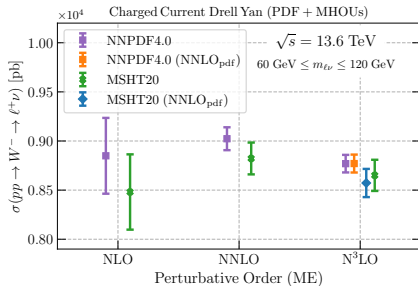
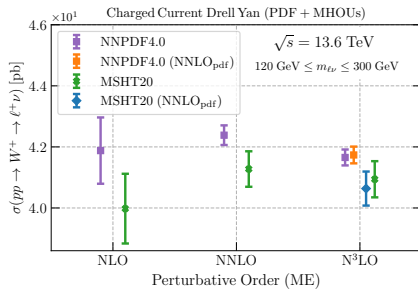
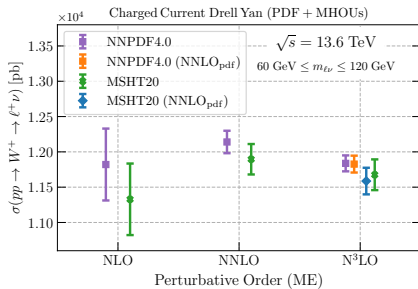
NNPDF [EPJ C84 (2024) 659]

splitting functions	linear combinations of as many interpolating functions, satisfying the known small- x , large- x , and large- n_f limits, as the number of known Mellin moments; coefficients fixed by known Mellin moments singlet: 4; non-singlet: 8	singlet: 5, 6 for $P_{ps}^{(3)}$; non-singlet: 8
IHOUs	nuisance parameters (1 per splitting function, for a total of 5) fitted to the data; varied basis of functions used and absorbed into uncertainty	estimated by varying the basis of interpolation functions and by constructing a corresponding theory covariance matrix
transition elements	approximations constructed similarly to splitting functions (5 nuisance parameters in total)	exact inclusion of all transition elements but $A_{gg,H}$ parametrised similarly to splitting functions
DIS heavy quarks	massive coefficient functions determined from parametrisations combining known limits and damping functions (continuous information)	
hadronic K -factors	parametrised as linear combinations of NNLO and NLO K -factors (2 nuisance parameters per process for a total of 10)	replaced by MHOUs
MHOUs	delegated to the 20 nuisance parameters (uncorrelated or correlated)	theory covariance matrix built from correlated 7-point scale variations

MHOUs and IHOUs at N³LO: NC Drell–Yan



MHOUs and IHOUs at N³LO: CC Drell–Yan



MHOUs and IHOUs at N³LO: uncertainties

Process	σ (pb)	δ_{th}	NNPDF4.0				σ (pb)	$\delta_{\text{th}}\sigma$	MSHT20		
			$\delta_{\text{PDF}}^{\text{noMHOu}}$	$\delta_{\text{PDF}}^{\text{MHOu}}$	$\Delta_{\text{NNLO}}^{\text{app}}$	$\Delta_{\text{NNLO}}^{\text{exact}}$			δ_{PDF}	$\Delta_{\text{NNLO}}^{\text{app}}$	$\Delta_{\text{NNLO}}^{\text{exact}}$
W^+ (p)	1.2×10^4	1.0	0.5	0.5	1.1	0.1	1.2×10^4	1.9	1.7	2.3	0.8
W^- (p)	8.8×10^3	1.0	0.5	0.5	1.1	0.1	8.7×10^3	1.9	1.6	2.1	0.0
Z (p)	1.9×10^3	0.9	0.4	0.5	1.1	0.3	1.9×10^3	1.8	1.6	2.6	0.3
W^+ (hm)	4.7×10^{-4}	2.8	2.8	3.3	3.2	1.1	4.6×10^{-4}	4.0	3.9	2.0	1.3
W^- (hm)	1.4×10^{-4}	2.9	2.9	3.3	3.3	0.1	1.5×10^{-4}	4.2	4.2	2.0	0.6
Z (hm)	2.1×10^{-4}	2.3	2.3	2.5	3.4	0.3	2.2×10^{-4}	3.6	3.6	2.7	0.2

δ_{th} : PDF and 7-point scale variation uncertainties added in quadrature

δ_{PDF} : PDF uncertainty

(NNPDF: MHOu and noMHOu indicate if MHOUs were or were not included in the fit)

$$\Delta_{\text{NNLO}}^{\text{app}} \equiv \frac{1}{2} \left| \frac{\sigma_{\text{NNLO-PDF}}^{\text{NNLO}} - \sigma_{\text{NLO-PDF}}^{\text{NNLO}}}{\sigma_{\text{NNLO-PDF}}^{\text{NNLO}}} \right|$$

$$\Delta_{\text{NNLO}}^{\text{exact}} \equiv \left| \frac{\sigma_{\text{N}^3\text{LO-PDF}}^{\text{N}^3\text{LO}} - \sigma_{\text{NNLO-PDF}}^{\text{N}^3\text{LO}}}{\sigma_{\text{N}^3\text{LO-PDF}}^{\text{N}^3\text{LO}}} \right|$$

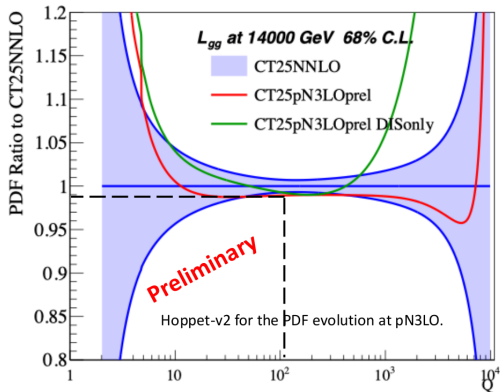
MHOUs and IHOUs at N³LO: CT

First pN³LO fits available based on new fitting code `Dragonwell`

DIS with heavy quarks at N³LO depends on missing mass terms and damping functions [B. Wang and K. Xie PhD theses]; validation of S -ACOT- χ for F_2 and F_2^c ongoing

Construction of approximate DIS $C_{2,g}^{(3)}$ from threshold and asymptotic limits depends on the choice of interpolation [arXiv:2606.25498]

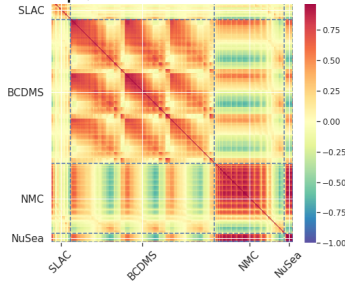
CT25pN3LOprel: DIS is pN³LO
All other CT25 data sets are NNLO
New LHC data sets in CT25 NNLO
reduce gg luminosity by $\sim 2.5\%$
w.r.t. CT18 NNLO, at m_H
Reduction of CT25pN3LOprel gluon
in the Higgs region is consistent
with MSHT/NNPDF aN³LO PDFs
The pN3LOprel fit has IHOUs
of order N³LO – not small
Increase of small- x gluon predicted by
DIS saturation scale in CT18X



HERA incl. DIS: $N_{\text{dat}} = 1120$; NNLO $\chi^2 = 1401$; N³LO $\chi^2 = 1399$

Nuclear uncertainties

Exp+Th correlation matrix



$$S_{ij} = \frac{1}{N} \sum_k \Delta_i^{(k)} \Delta_j^{(k)}$$

$$\Delta_i^{(k)} = T_i^N[f_N^{(k)}] - T_i^N[f_p]$$

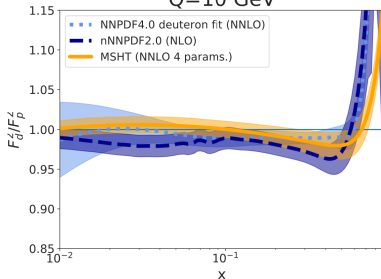
$$\chi_{\text{tot}}^2 = 1.17 \rightarrow \chi_{\text{tot}}^2 = 1.26 \text{ (no nucl. u.)}$$

$$\chi_{\text{LHCb}}^2 = 1.54 \rightarrow \chi_{\text{LHCb}}^2 = 1.76 \text{ (no nucl. u.)}$$

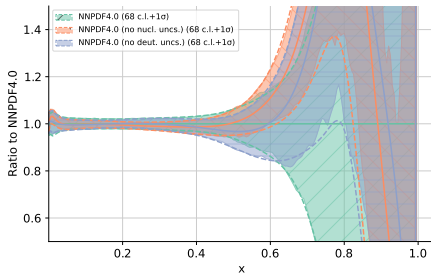
The bulk of the effect is due to nuclear uncertainties for heavy nuclei at large x

Reduced tension between DIS and LHC data

Q=10 GeV

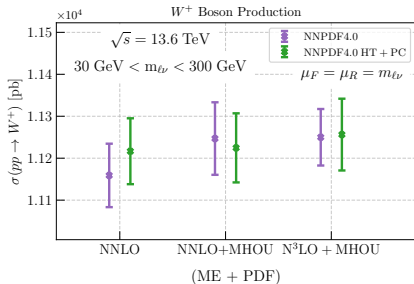
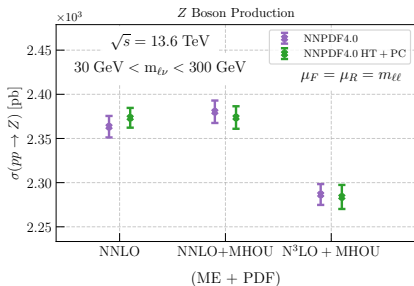


d at 1.7 GeV



[EPJ C79 (2019) 282; EPJ C81 (2021) 37; see also EPJ C81 (2021) 603; EPJ C63 (2009) 189]

Higher twist and power corrections



$$S_{ij} = \frac{1}{N} \sum_k \Delta_i^{(k)} \Delta_j^{(k)}$$

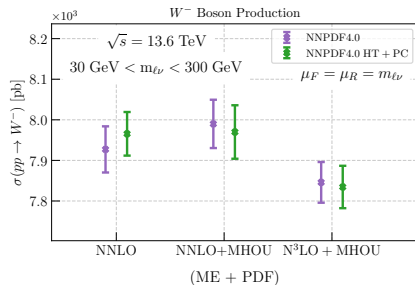
$$\Delta_i^{(k)} = T_i^{\text{HT,PC}(k)} - T_i^{\text{LT}}$$

$$T_i^{\text{HT}(k)} = T_i^{\text{LT}} \left(1 + \frac{H^{(k)}(x)}{Q^2} \right)$$

$$T_i^{\text{PC}(k)} = T_i^{\text{LT}} \left(1 + \frac{H^{(k)}(\eta)}{p_T} \right)$$

$$\chi_{\text{tot}}^2 = 1.11 \rightarrow \chi_{\text{tot}}^2 = 1.13 \text{ (no HT/PC NNLO)}$$

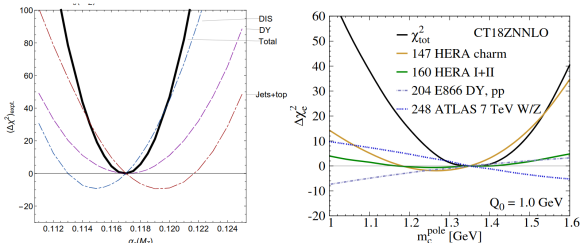
Tensions relieved, shift of central value



[EPJ C86 (2026) 281; ibid. 115]

Joint PDF-SM parameter determinations

Perform a scan of the best-fit minimum by varying α_s , m_c or m_b



[PRD 103 (2021) 014013; see also arXiv:2512.23792]

	σ	PDF unc.	m_c var.	m_b var.	α_s unc.
W^+ LHC (8 TeV)	7.075	+0.099 (+1.4%) -0.110 (-1.6%)	+0.008 (+0.12%) -0.014 (-0.19%)	+0.013 (+0.18%) -0.010 (-0.14%)	+0.064 (+0.91%) -0.060 (-0.85%)
W^- LHC (8 TeV)	4.955	+0.071 (+1.4%) -0.083 (-1.7%)	+0.005 (+0.09%) -0.009 (-0.19%)	+0.009 (+0.18%) -0.007 (-0.15%)	+0.044 (+0.88%) -0.042 (-0.84%)
Z LHC (8 TeV)	1.122	+0.014 (+1.3%) -0.017 (-1.4%)	+0.003 (+0.24%) -0.004 (-0.34%)	+0.0006 (+0.05%) -0.00004 (-0.003%)	+0.010 (+0.90%) -0.010 (-0.86%)
W^+ LHC (13 TeV)	11.53	+0.16 (+1.4%) -0.18 (-1.6%)	+0.024 (+0.21%) -0.028 (-0.24%)	+0.025 (+0.22%) -0.022 (-0.19%)	+0.12 (+1.0%) -0.11 (-0.94%)
W^- LHC (13 TeV)	8.512	+0.12 (+1.4%) -0.14 (-1.6%)	+0.013 (+0.15%) -0.019 (-0.23%)	+0.018 (+0.21%) -0.017 (-0.19%)	+0.080 (+0.94%) -0.078 (-0.91%)
Z LHC (13 TeV)	1.914	+0.024 (+1.3%) -0.029 (-1.5%)	+0.006 (+0.33%) -0.008 (-0.40%)	+0.0006 (+0.03%) +0.0004 (+0.02%)	+0.019 (+0.98%) -0.018 (-0.94%)

[EPJ C81 (2021) 744; for a N^3 LO determination of α_s , see EPJ C84 (2024) 1009]

N.B: m_c dependence largely reduced if charm is parametrised

Joint PDF- α_s determination: the CRM

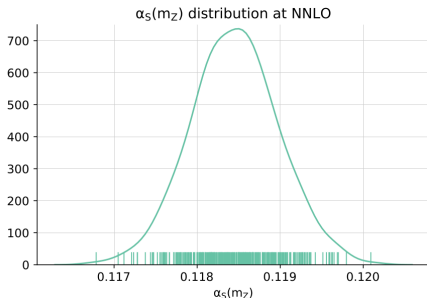
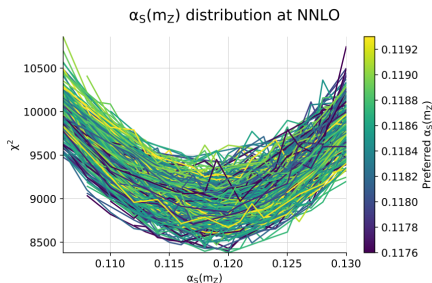
Frequentist parameter estimation by means of Monte Carlo resampling

For each data sample (replica), perform a scan in α_s

Each replica has a preferred value of α_s (the minimum of each parabola)

These preferred values form a Monte Carlo distribution

In this way, the PDF- α_s correlation is completely taken into account



Determine α_s from the central value and variance of the distribution

Joint PDF- α_s determination: the TCM

Bayesian parameter estimation by means of a theoretical covariance matrix

$$S_{ij} = \beta_i \beta_j$$

correlated uncertainty covariance matrix

$$T \rightarrow T + \lambda \beta$$

nuisance parameter λ on prediction T

$$P(T|D, \lambda) \propto \exp \left[-\frac{1}{2} (T + \lambda \beta - D)^T C^{-1} (T + \lambda \beta - D) \right]$$

probability of prediction T given the data D and the nuisance parameter λ

$$P(\lambda) \propto \exp \left(-\frac{\lambda^2}{2} \right)$$

uncertainty on nuisance parameter

$$P(T|D) \propto \exp \left[-\frac{1}{2} (T - D)^T (C + S)^{-1} (T - D) \right]$$

probability of prediction T given the data

$$P(\lambda|T, D) \propto \exp \left[-\frac{1}{2} Z^{-1} (\lambda - \bar{\lambda}(T, D))^2 \right]$$

posterior distribution of the nuisance parameter λ

with $\bar{\lambda}(T, D) = \beta^T (C + S)^{-1} (D - T)$, $Z = 1 - \beta^T (C + S)^{-1} \beta$, and C exp. cov. mat.

Treat $\alpha_s - \alpha_s^0$ as nuisance parameter with prior $P(\alpha_s)$ centered about α_s^0

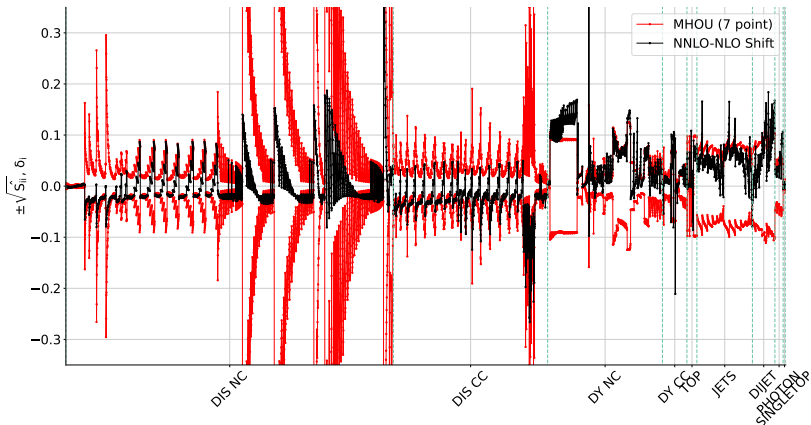
2. Validating theory uncertainties

MHOUs: can the model anticipate the next order?

Compare the NLO MHOU covariance matrix against the NNLO–NLO difference

$$\delta_i = \frac{T_i^{\text{NNLO}} - T_i^{\text{NLO}}}{T_i^{\text{NLO}}} \quad \hat{S}_{ij}^{\text{NLO}} = \frac{S_{ij}^{\text{NLO}}}{T_i^{\text{NLO}} T_j^{\text{NLO}}}$$

Compare δ_i to $\pm\sqrt{\hat{S}_{ii}}$ and check if $\delta_i \sim \pm\sqrt{\hat{S}_{ii}}$



DIS: very conservative; DY: some underestimation; other process: good estimate

MHOUs: what about correlations?

Compare the eigenvalues of the MHOUs covariance matrix to the projection of the shift along its eigenvectors

$i = 1, \dots, N_{\text{dat}}$ data space

$\alpha = 1, \dots, N_{\text{sub}}$ eigenvector subspace

$N_{\text{dat}} = 4616 \gg N_{\text{sub}} = 22$

$e_i^\alpha; \lambda^\alpha = (s^\alpha)^2$ eigensystem

$\delta^\alpha = \sum_{i=1}^{N_{\text{dat}}} \delta_i e_i^\alpha$ projection of δ_i on e_i^α

$\delta_i^S = \sum_{\alpha=1}^{N_{\text{sub}}} \delta^\alpha e_i^\alpha$ component of δ_i in S

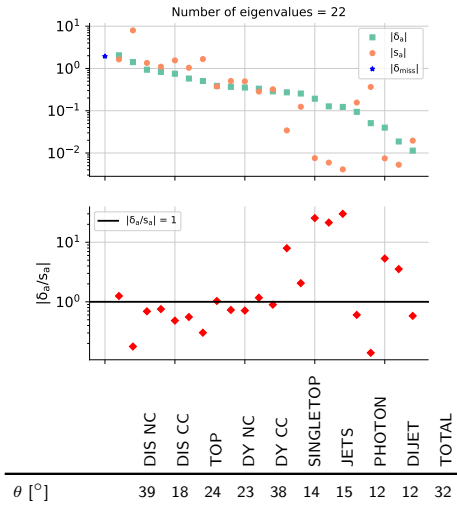
$\delta_{\text{miss}} = \delta_i - \delta_i^S$ shift missed by MHOUs

$\theta = \arccos(|\delta^S|/|\delta|)$ δ - δ^S angle

Check that $\delta^\alpha \leq s^\alpha$ 68% of the times

Check that $|\delta_{\text{miss}}|$, θ small and $|\delta^S| \sim |\delta|$

Scale variations to estimate MHOUs are OK

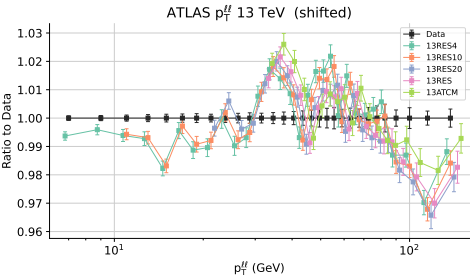
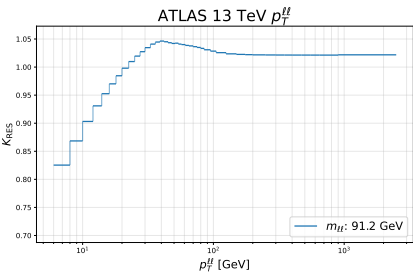


Unlike an envelope, an oversized covariance eigenvalue does not by itself shift the fit: it only downweights residuals along that direction. If the data residual has little projection onto that eigenvector, the effect is largely innocuous

MHOUs: the Z -boson transverse momentum distribution

There may be one case of caution, though: Zp_T production at small p_T

$$\theta_{Zp_T} \sim 60^\circ, \text{ largely driven by ATLAS 13 TeV}$$



Data set

N_{dat}

correlation

envelope prescription

ATLAS $p_T^{\ell\ell}$ 8 TeV ($p_T^{\ell\ell}, y^{\ell\ell}$)	44	1.21	1.27	1.23	1.21	0.53	0.55	0.54	0.53
ATLAS $p_T^{\ell\ell}$ 8 TeV ($p_T^{\ell\ell}, m^{\ell\ell}$)	48	1.62	3.65	3.56	4.31	0.28	0.32	0.29	0.26
CMS $p_T^{\ell\ell}$ 8 TeV ($p_T^{\ell\ell}, y^{\ell\ell}$)	28	1.69	1.64	4.64	4.53	0.18	0.24	0.39	0.47
ATLAS $p_T^{\ell\ell}$ 13 TeV	19	3.94	8.42	13.5	17.6	0.29	0.32	0.35	0.33
CMS $p_T^{\ell\ell}$ 13 TeV	16	0.74	0.80	1.03	1.63	0.52	0.62	0.63	0.70

[arXiv:2607.19183]

Joint PDF- α_s determination: closure tests

Assume true underlying PDFs and $\alpha_s(M_Z)$ value: $\alpha_s^0(M_Z) = 0.118$

Generate data distributed according to the experimental covariance matrix

Run the correlated replica method and the theoretical covariance method on these data

Do statistics on N_r “runs of the universe” turning Bayesian into frequentist

$$\langle \alpha_s \rangle = \frac{\sum_{j=1}^{N_r} \frac{\alpha_s^{(j)}}{(\sigma_\alpha^{(j)})^2}}{\sum_{j=1}^{N_r} \frac{1}{(\sigma_\alpha^{(j)})^2}} \quad \text{weighted mean over } N_r \text{ runs}$$

$$\langle \sigma_\alpha \rangle = \left[\frac{1}{N_r} \sum_{j=1}^{N_r} \frac{1}{(\sigma_\alpha^{(j)})^2} \right]^{-1/2} \quad \text{weighted uncertainty}$$

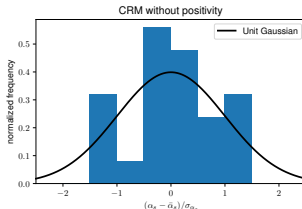
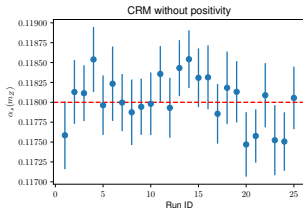
$$P = \frac{\frac{1}{N_r} \sum_{j=1}^{N_r} (\alpha_s^{(j)} - \alpha_s^0)}{\langle \sigma_\alpha \rangle / N_r} \quad \text{pull}$$

$$R_{bv} = \sqrt{\frac{1}{N_r} \sum_{j=1}^{N_r} \left(\frac{\alpha_s^{(j)} - \alpha_s^0}{\sigma_\alpha^{(j)}} \right)^2} \quad \text{Bias/variance: mean square deviation vs truth vs unc.}$$

[EPJ C85 (2025) 1001; see also JHEP 08 (2026) 159 for a simultaneous determination of α_s and m_t]

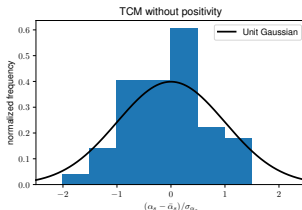
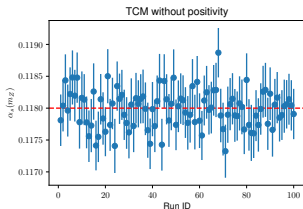
Joint PDF- α_s determination: CRM vs TCM

CRM: 250 MC replicas $\times$ 12 values of α_s , $0.114 \leq \alpha_s \leq 0.123$; 25 runs of the universe
 $\langle \alpha_s \rangle = 0.118029$ $\langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000077$ $P = 0.38$ $R_{bv} = 0.80 \pm 0.09$



TCM: 550 MC replicas; 100 runs of the universe

$\langle \alpha_s \rangle = 0.117984$ $\langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000041$ $P = 0.39$ $R_{bv} = 0.71 \pm 0.05$



CRM and TCM in excellent agreement; uncertainties validated

Detecting bias: multiplicative uncertainties

Collider data: multiplicative uncertainties in experimental covariance matrix

D'Agostini bias: using experimental covariance matrix in maximum likelihood estimate leads to an underestimate of the true value [Nucl.Instrum.Meth. A346 (1994) 306]

NNPDF solution: t_0 method [JHEP 05 (2010) 075]

Replace the experimental covariance matrix with the t_0 covariance matrix and iterate

$$\text{cov}_{ij,\text{exp}} \rightarrow \text{cov}_{ij,t_0} = \delta_{ij}(\sigma^u D)^2 + \sum_c^{N_{\text{add}}} (\sigma^c D)_i (\sigma^c D)_j + \sum_c^{N_{\text{mult}}} (\sigma^c T_0)_i (\sigma^c T_0)_j$$

New problem: theory predictions depend on α_s . Should we vary T_0 with α_s or not?

CRM varying T_0 with α_s : 25 runs of the universe

$$\langle \alpha_s \rangle = 0.119450 \quad \langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000077 \quad P = 19 \quad R_{bv} = 3.80 \pm 0.16$$

CRM with fixed T_0 : 25 runs of the universe

$$\langle \alpha_s \rangle = 0.118152 \quad \langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000070 \quad P = 2.2 \quad R_{bv} = 0.97 \pm 0.11$$

If $\text{cov} = \text{cov}_{\text{exp}} + \text{cov}_{\text{th}}$ depends on a parameter, the complete Gaussian log-likelihood is

$$\chi^2[\boldsymbol{\theta}, \alpha_s] = -2 \ln(p(D|\boldsymbol{\theta}, \alpha_s)) - \ln \det(\text{cov}) + \text{const.}$$

Minimising only the quadratic term rewards α_s values that produce larger uncertainties: cov^{-1} becomes smaller without paying the compensating penalty $\ln \det \text{cov}$

3. Conclusions

Summary

Nuisance parameters and covariance matrices encode the same linear Gaussian model

Various sources of theory uncertainties have been modelled in PDF determination: MHOUs, IHOUs, nuclear, higher-twist/power corrections, and standard model parameter

The difficult and physically meaningful choice
is the set of uncertainty directions and their correlations

Validation is what turns a plausible uncertainty model into a quantitatively credible one

For the sake of PDF determination
scale variations are by and large a good uncertainty model to represent MHOUs

Summary

Nuisance parameters and covariance matrices encode the same linear Gaussian model

Various sources of theory uncertainties have been modelled in PDF determination:
MHOUs, IHOUs, nuclear, higher-twist/power corrections, and standard model parameter

The difficult and physically meaningful choice
is the set of uncertainty directions and their correlations

Validation is what turns a plausible uncertainty model into a quantitatively credible one

For the sake of PDF determination
scale variations are by and large a good uncertainty model to represent MHOUs

Thank you

4. Backup

Perturbative input (circa 2024)

Splitting Functions

Singlet ($P_{qq}, P_{gg}, P_{gq}, P_{qg}$)

- large- n_f limit [NPB 915 (2017) 335; JHEP 01 (2024) 029]
- small- x limit [JHEP 06 (2018) 145]
- large- x limit [NPB 832 (2010) 152; JHEP 04 (2020) 018; JHEP 09 (2022) 155]
- 10 lowest Mellin moments

[PLB 825 (2022) 136853; 842 (2023) 137944; 846 (2023) 138215; 848 (2024) 138351; 856 (2024) 138906; 860 (2025) 139194]

Non-singlet ($P_{NS,v}, P_{NS,+}, P_{NS,-}$)

- large- n_f limit [NPB 915 (2017) 335; JHEP 01 (2024) 029]
- small- x limit [JHEP 08 (2022) 135]
- large- x limit [JHEP 10 (2017) 041]
- 8 lowest Mellin moments [JHEP 06 (2018) 073]

DIS structure functions (F_L, F_2, F_3)

- DIS NC (massless) [NPB 492 (1997) 338; PLB 606 (2005) 123; NPB 724 (2005) 3]
- DIS CC (massless) [NPB 813 (2009) 220]
- massive from parametrisation combining known limits and damping functions [NPB 864 (2012) 399]

PDF matching conditions (transition elements)

– $A_{gg,H}, A_{Hg}, A_{qq,H}, A_{qg,H}$

[NPB 820 (2009) 417; 844 (2011) 26; 886 (2014) 733; 999 (2024) 116426; JHEP 12 (2022) 134; PLB 854 (2024) 138713]

Coefficient functions for other processes

- DY (inclusive) [JHEP 11 (2020) 143]; DY (y differential) [PRL 128 (2022) 052001]

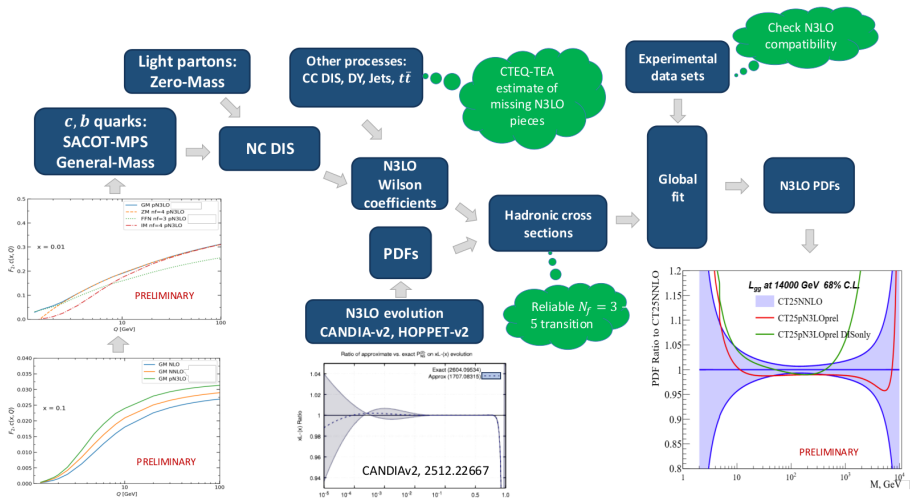
Perturbative input (circa 2026)

N3LO Ingredients		MSHT20aN3LO 2207.04739	Evolution Benchmarking 2406.16188	DIS2025 proceedings 2510.09321	DIS2026 updates	Future (Ongoing)
Splitting Functions	Singlet $qq_{PS}, qg,$ gq, gg	4 moments for all (own approximations)	10 moments: $qq_{PS}, qg,$ 5 moments: gq, gg	10 moments FHMURVV central	10 moments for all FHMURVV central + error	11 moments for all (FHMURVV 2512.10783). Further work on uncertainty in nuisance parameter approach
	Non-Singlet $qq_{NS+}, -, S$	8 moments (own qq_{NS+})	8 moments (Moch et al qq_{NS+})	8 moments (own qq_{NS+})	8 moments (own qq_{NS+})	Exact - see later (Gehrmann et al)
Transition Matrix Elements		$A_{Hq}^{PS}, A_{gq,H}$ known; $A_{gg,H}, A_{Hg}, A_{qq,H}^{NS}$ approximated	-	All known (included via grids as in 2411.05373*)	All known (included via grids as in 2411.05373*)	Include Exact expressions (Ablinger et al 2510.02175) (consistency checks already performed)
DIS Coefficient Functions	Light Mass	Known ZM-VFNS				
	Heavy Mass	FFNS known LL small- x , unknown NLL and x -dependence. Known mass threshold info $f(\ln(W^2/4M_H^2 - 1))$, unknown Q^2 interpolation to ZM-VFNS - interpolated based on lower orders				Improve estimate of best value and uncertainty for $\mathcal{O}(\alpha_S^3)$ FFNS coefficient functions at low Q^2 (more important now no uncertainty in TMEs)

*2411.05373: aN3LO PDF Combination, MSHT + NNPDF

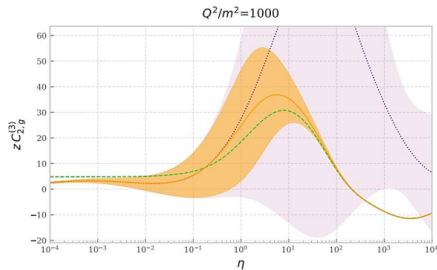
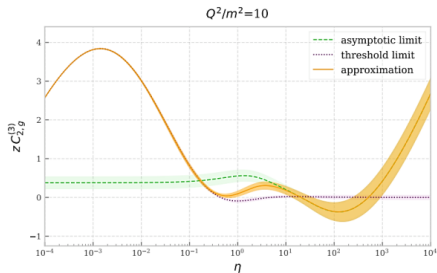
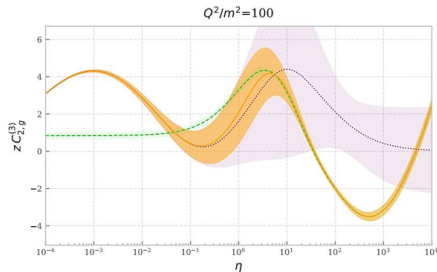
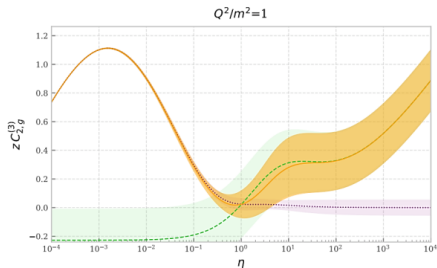
The effect of this updated input is to bring the MSHT and NNPDF4.0 gluon PDFs closer

CT roadmap to N³LO PDFs

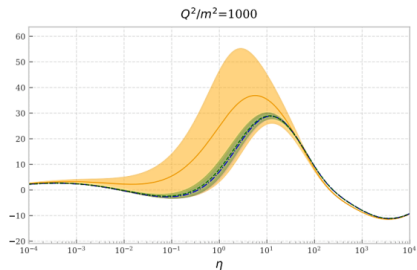
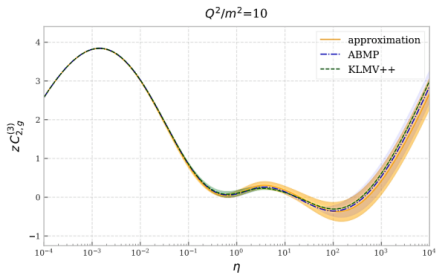
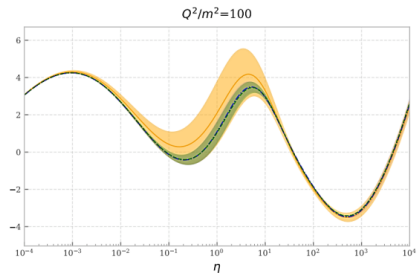
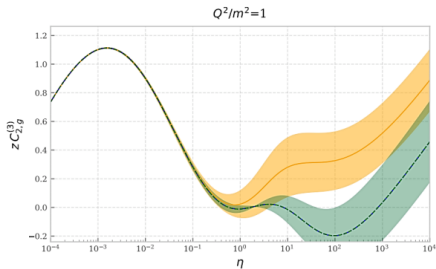


[Slide by courtesy of Marco Guzzi, Jeey Huston, Pavel Nadolsky]

Reconstructing $C_{2,g}^{(3)}$



Reconstructing $C_{2,g}^{(3)}$



Validating S-ACOT

