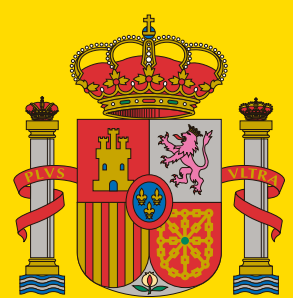


Status and challenges of global analyses of PDF at N3LO

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**13th Resummation, Evolution and Factorization
(REF 2026)
Santiago de Compostela - October 2026**



MINISTERIO
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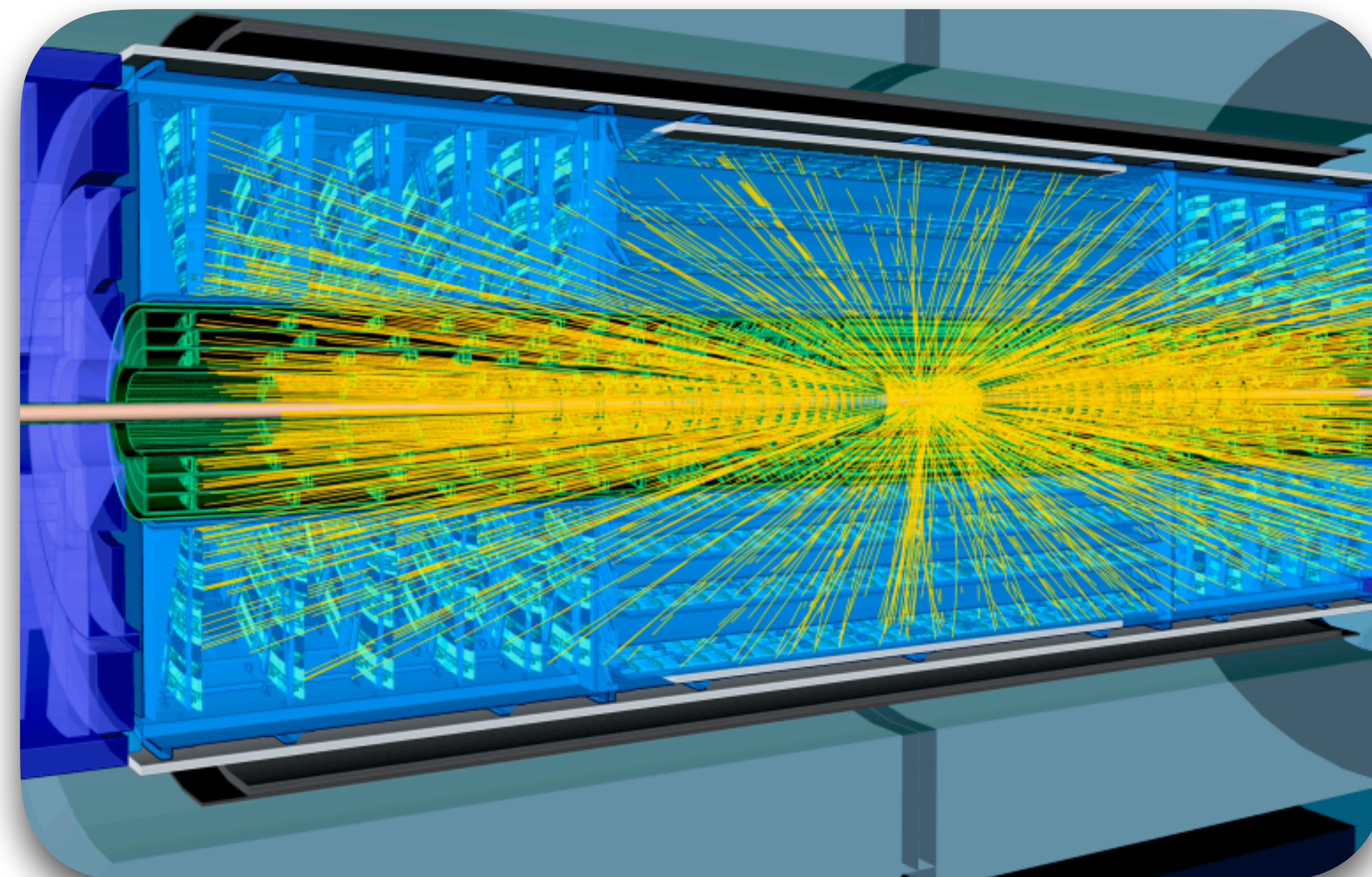
Introduction and motivation



The N3LO revolution?

Approximated N3LO PDFs

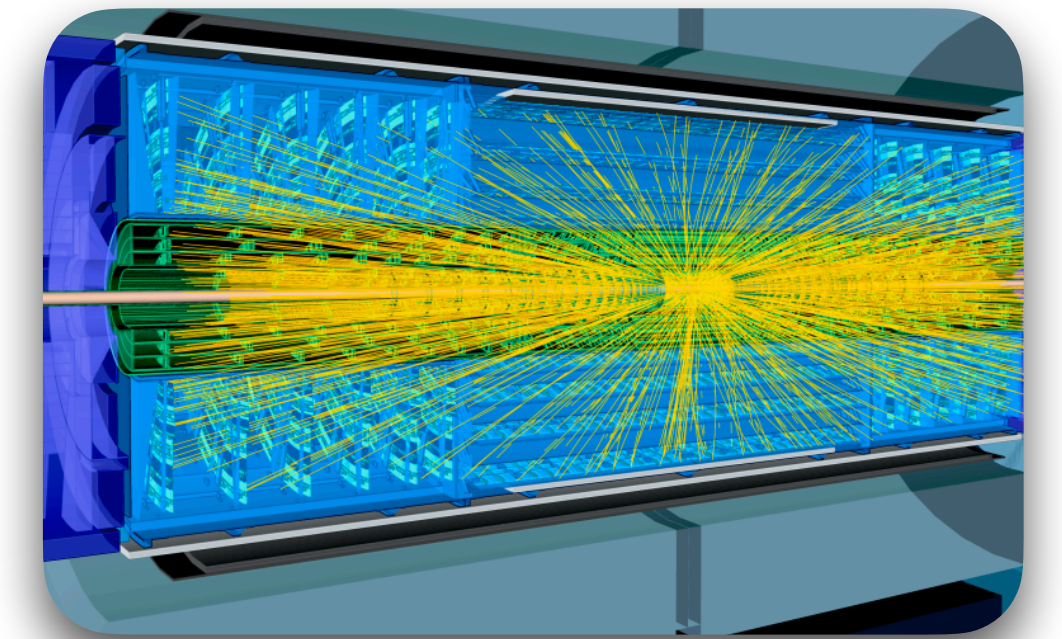
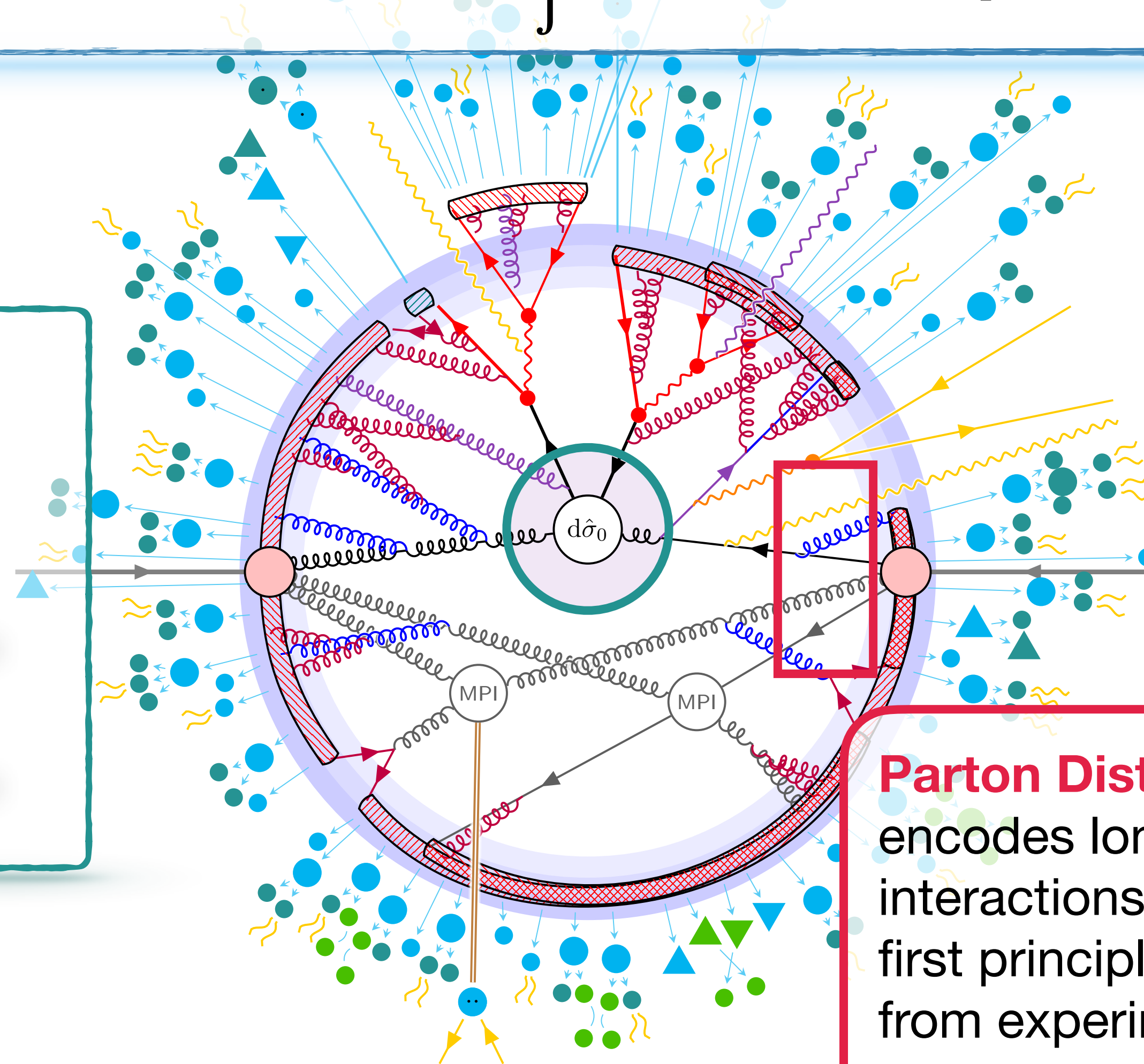
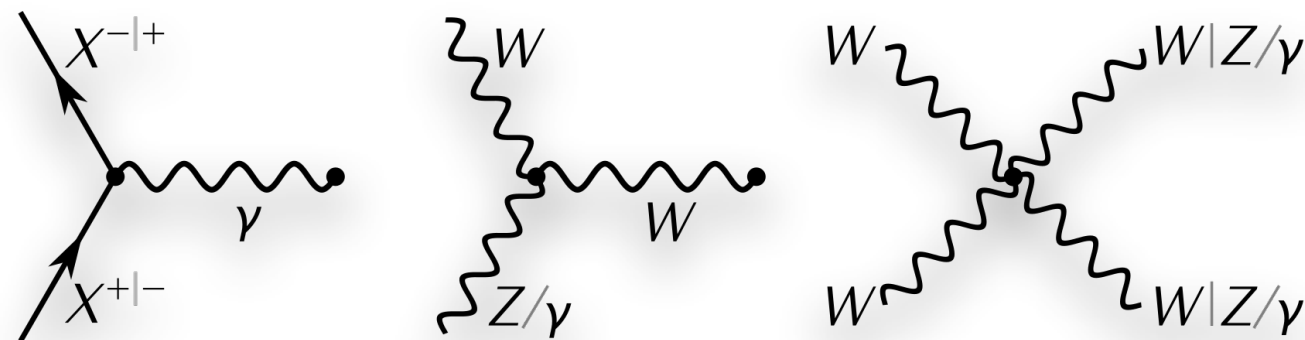
Conclusions



Ingredients for collider predictions in High Energy Physics

Experiments measure an Observable: $\mathcal{O} = \int d\Phi_n dx_1 dx_2 f_1(x_1, \mu_F^2) f_2(x_2, \mu_F^2) |M(\{p_n\}, \mu_R)|^2$

Hard scattering $\hat{\sigma}_0$: encodes short-range interactions, computed perturbatively



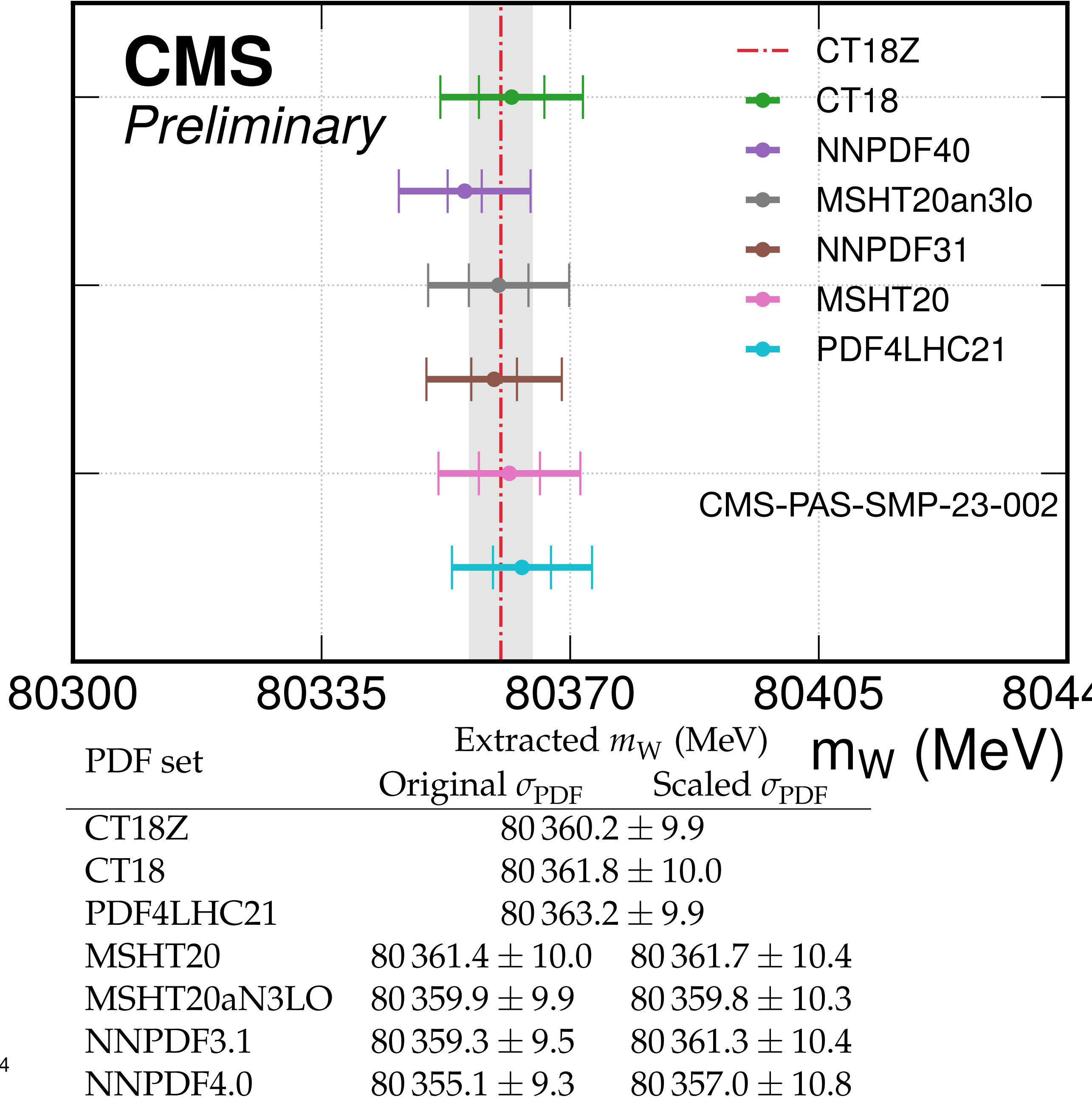
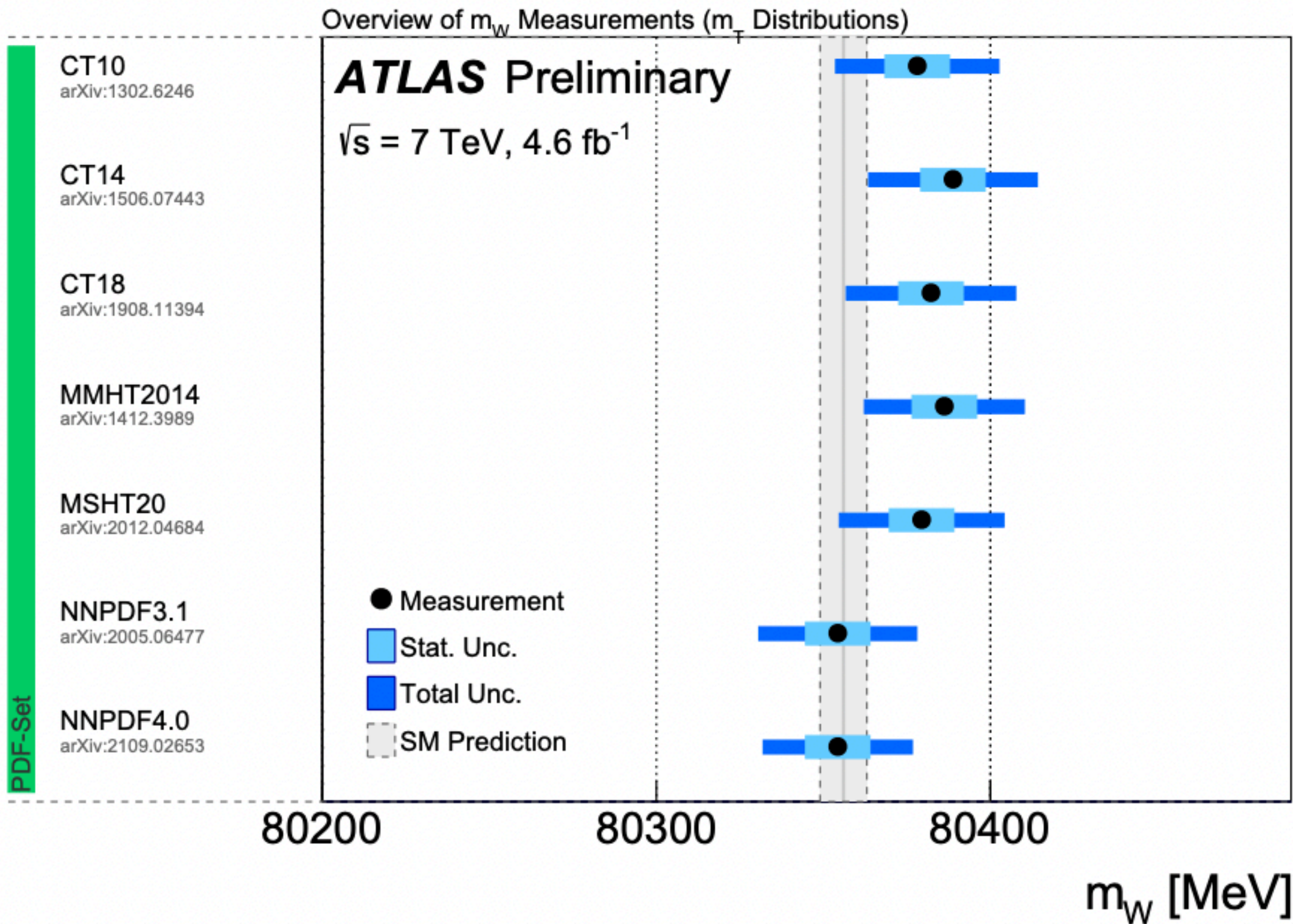
Parton Distribution Functions (PDFs): encodes long-range non-perturbative interactions. They cannot be computed from first principles and have to be determined from experimental Data.

Important: PDFs are Universal

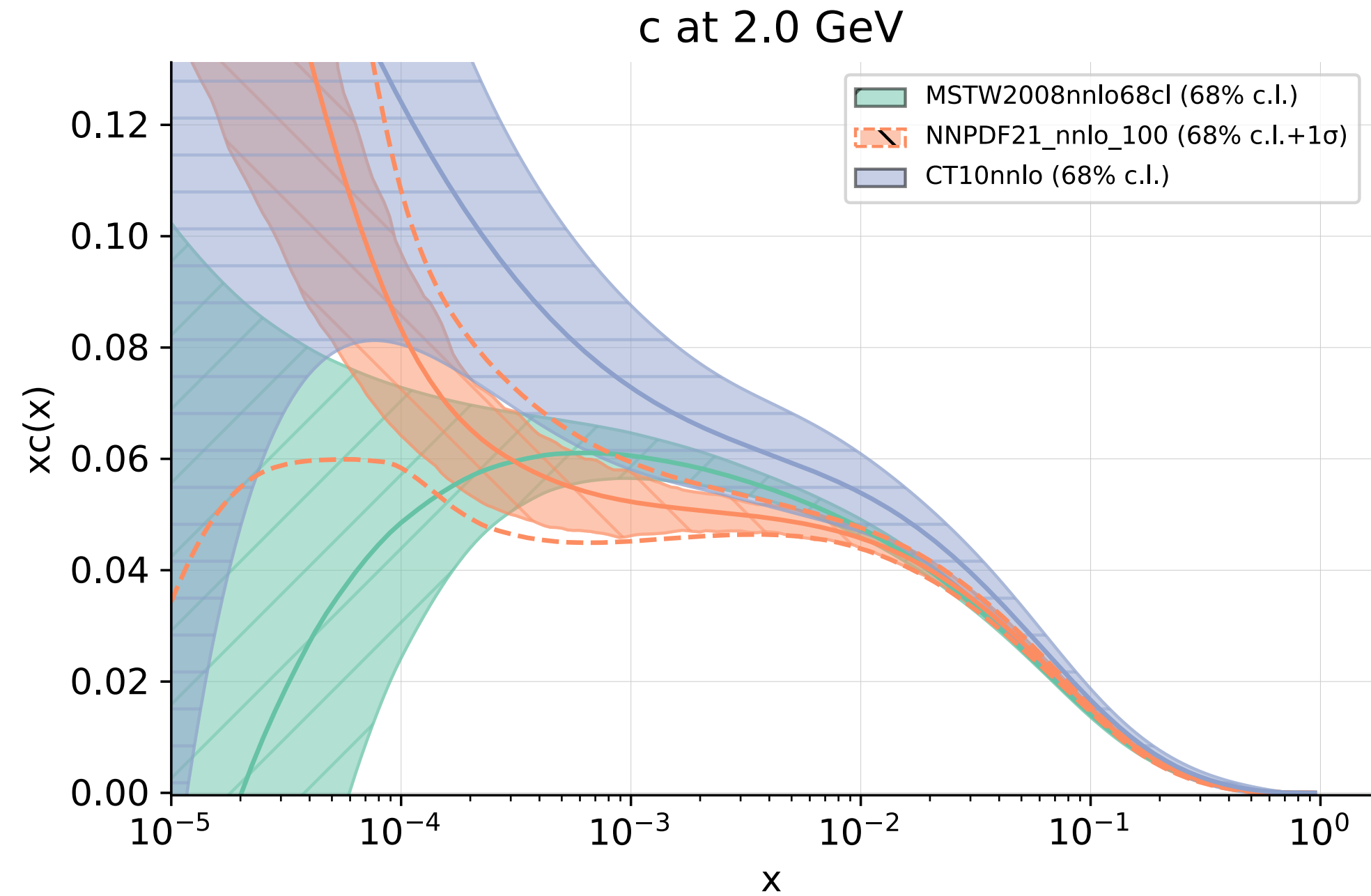
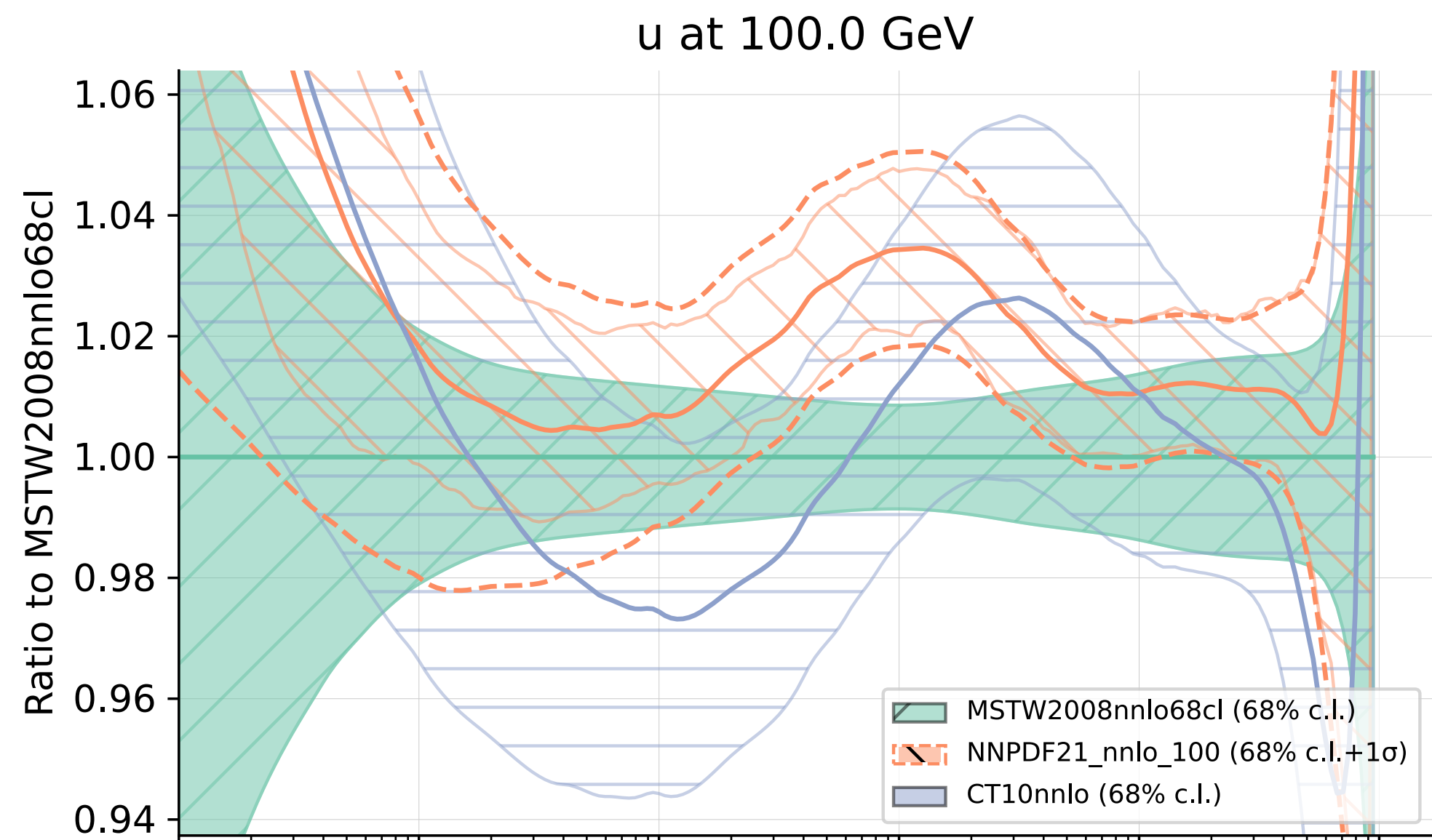
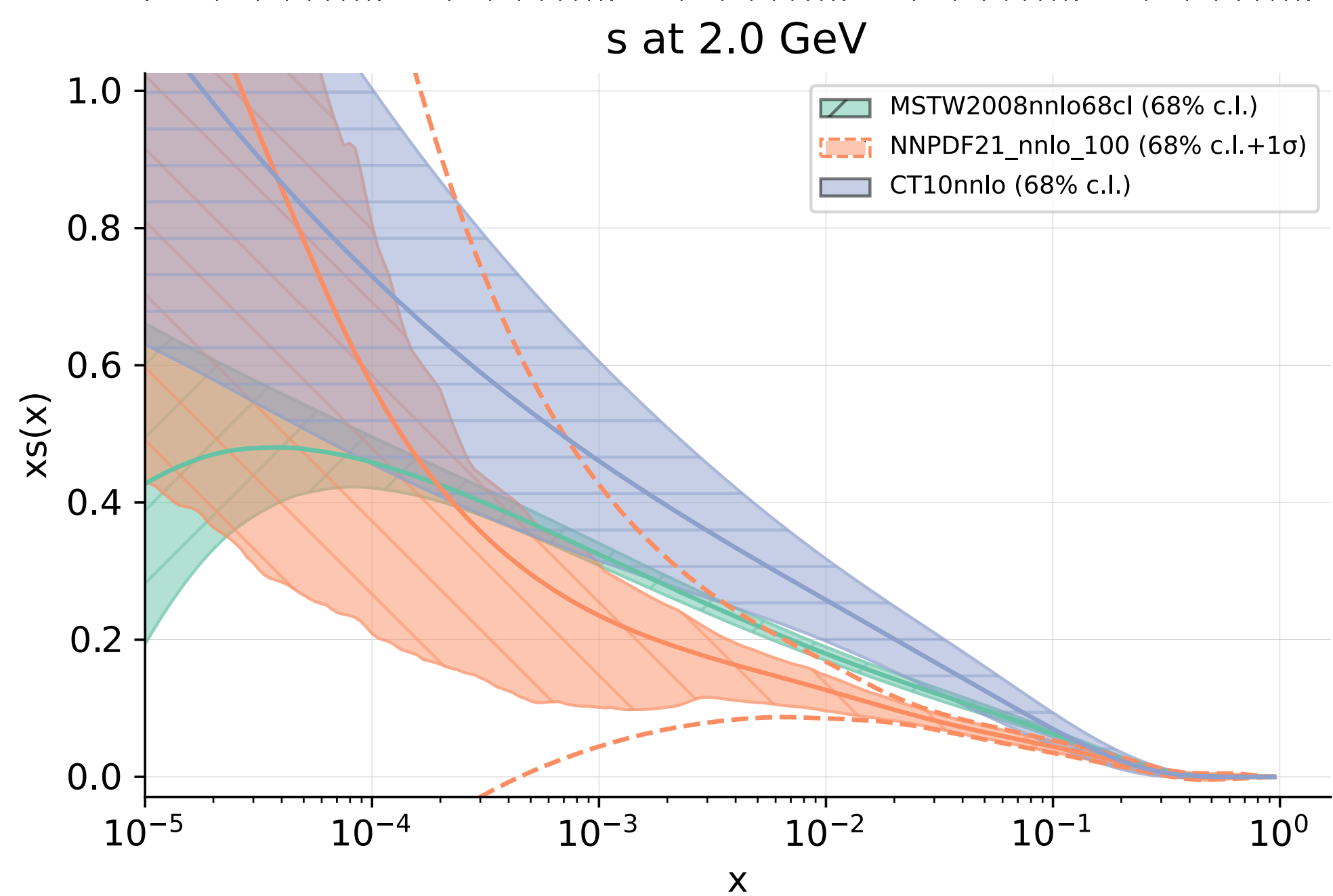
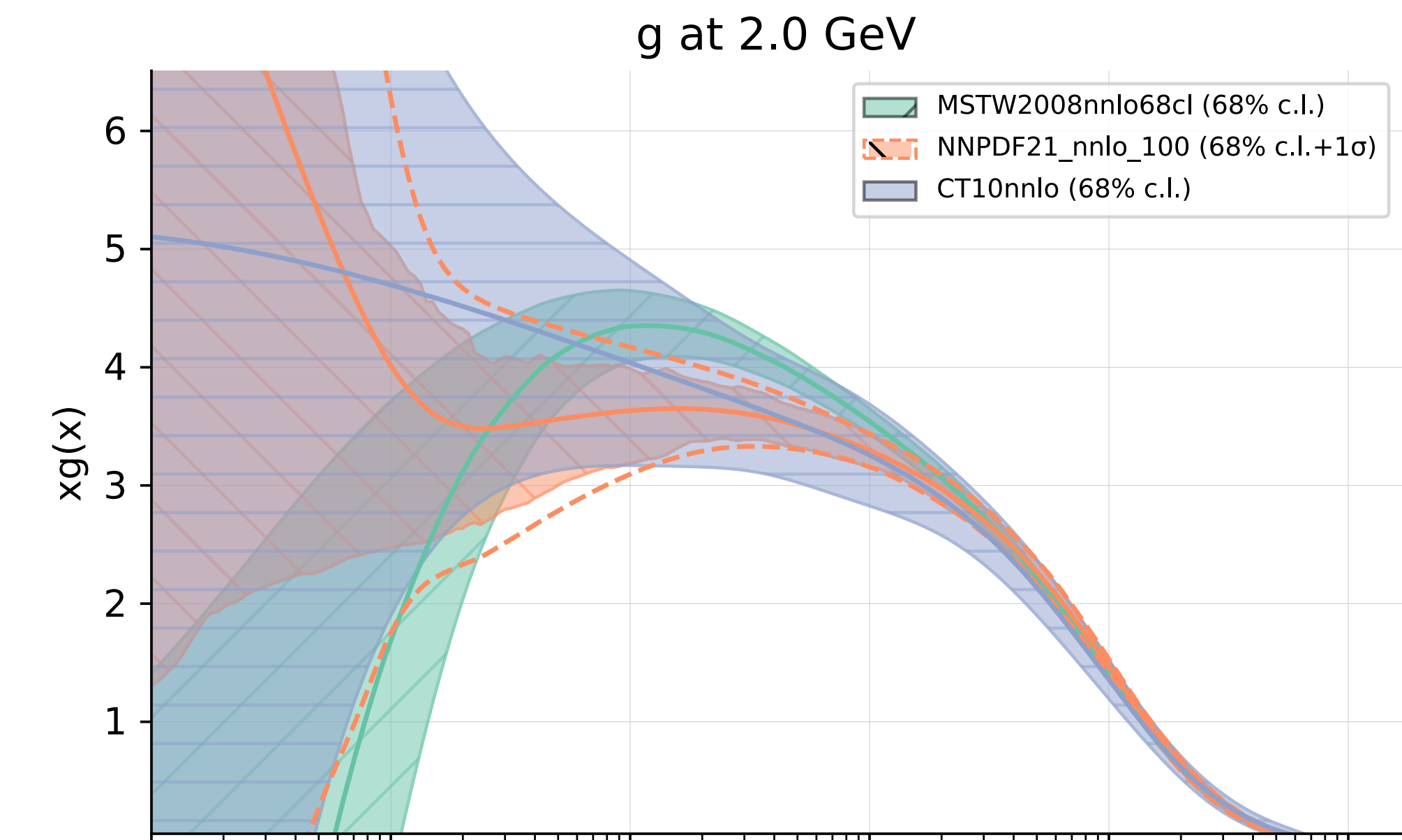
*terms & conditions apply

What is the impact of the PDF determination?

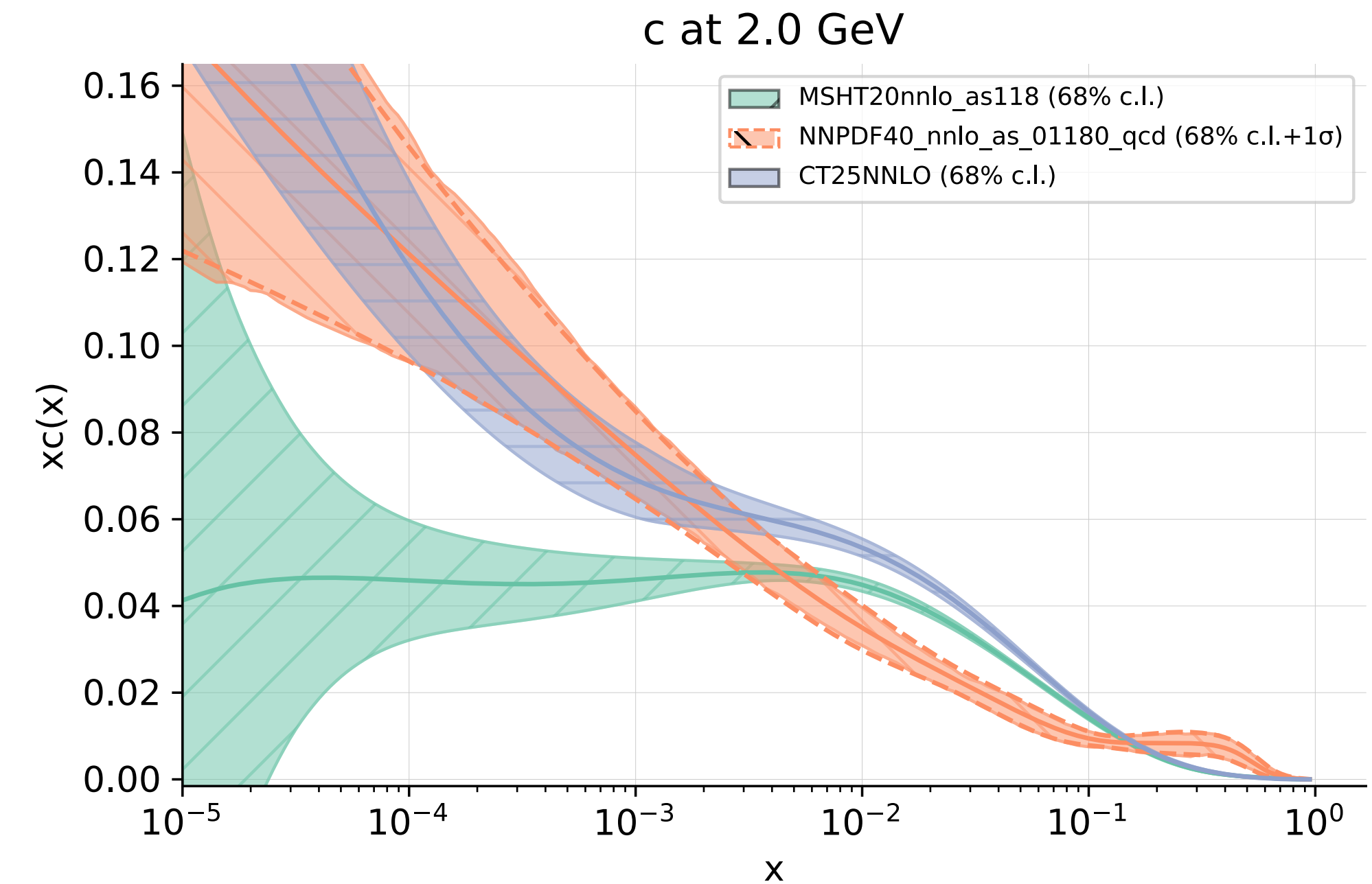
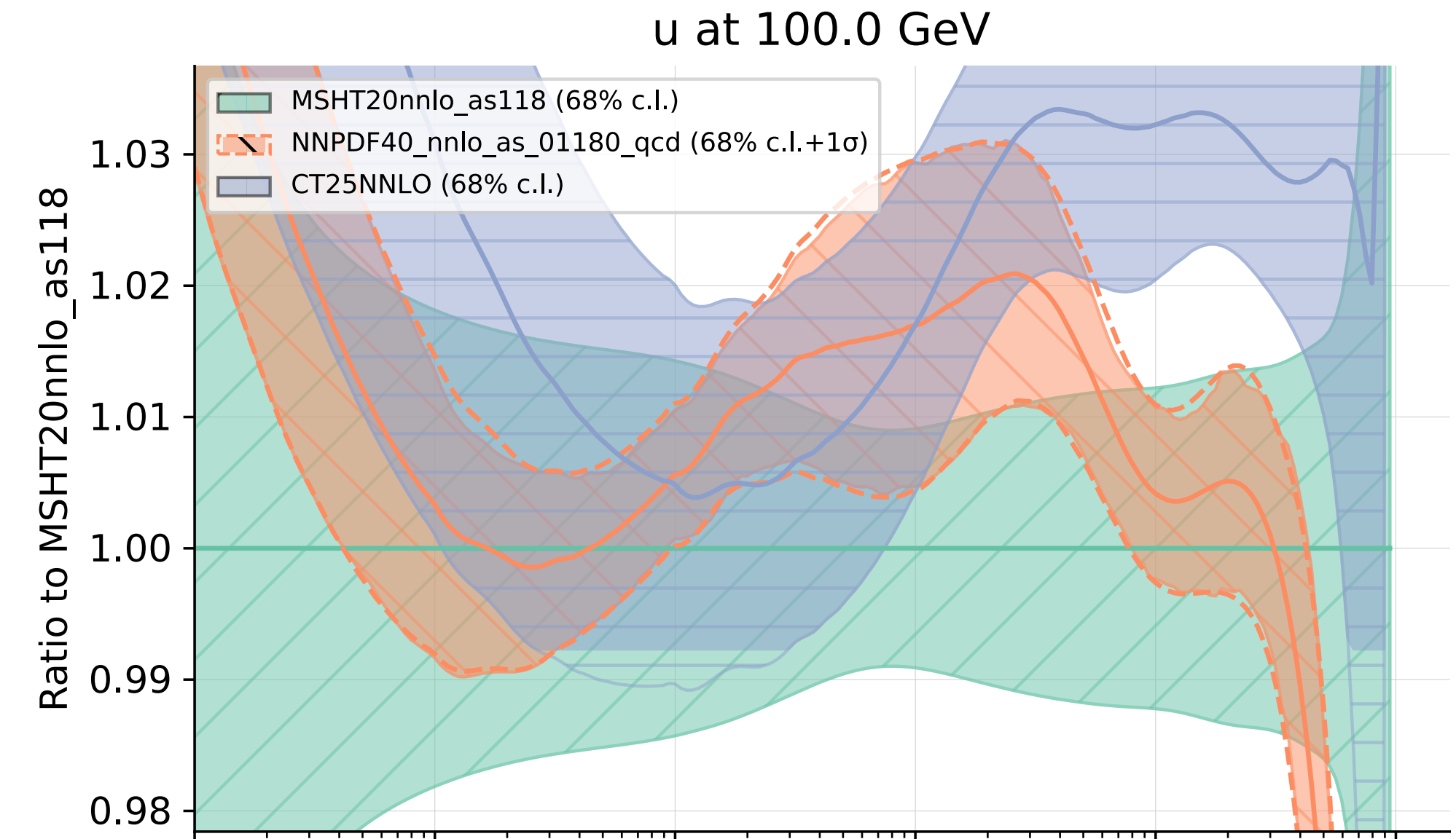
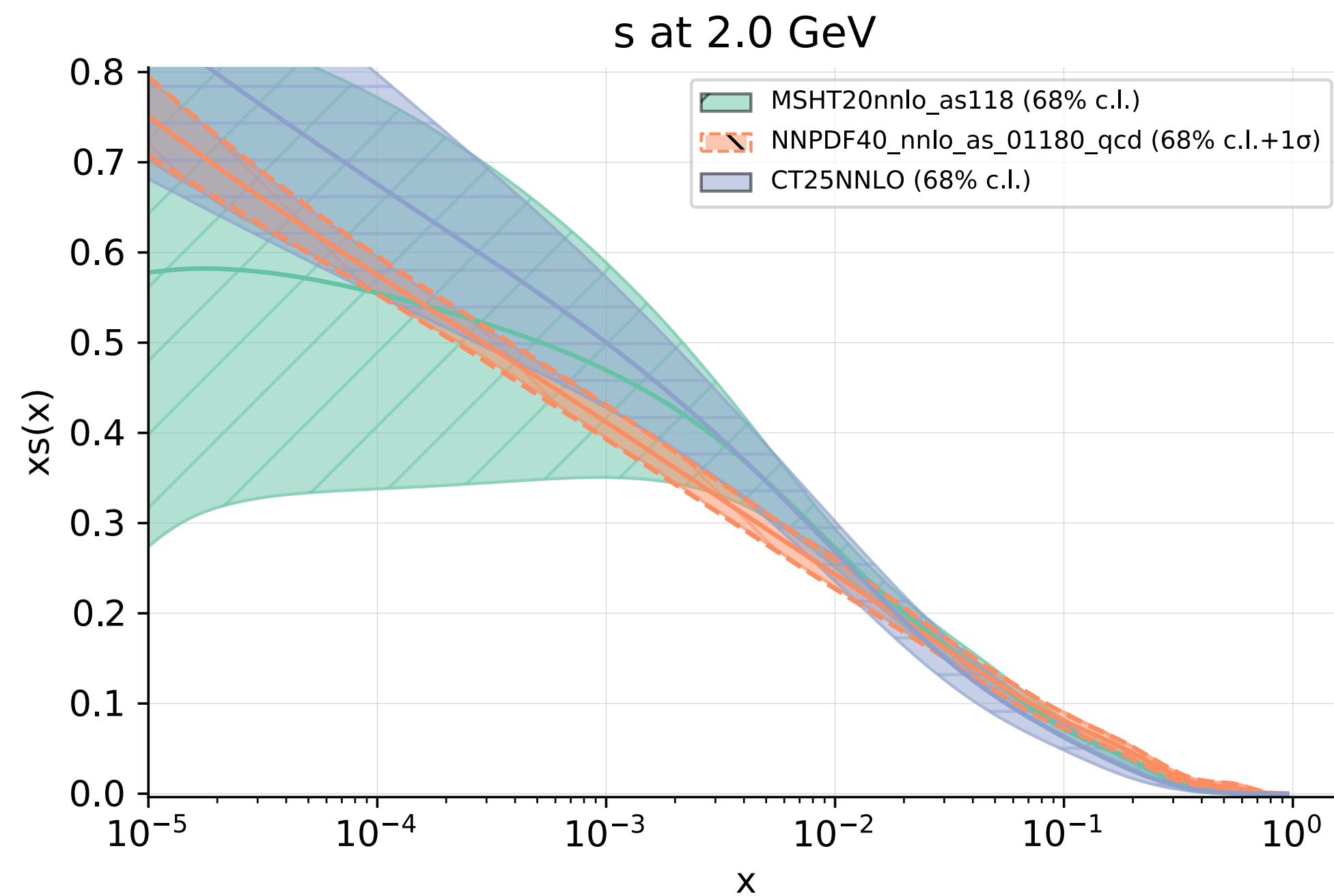
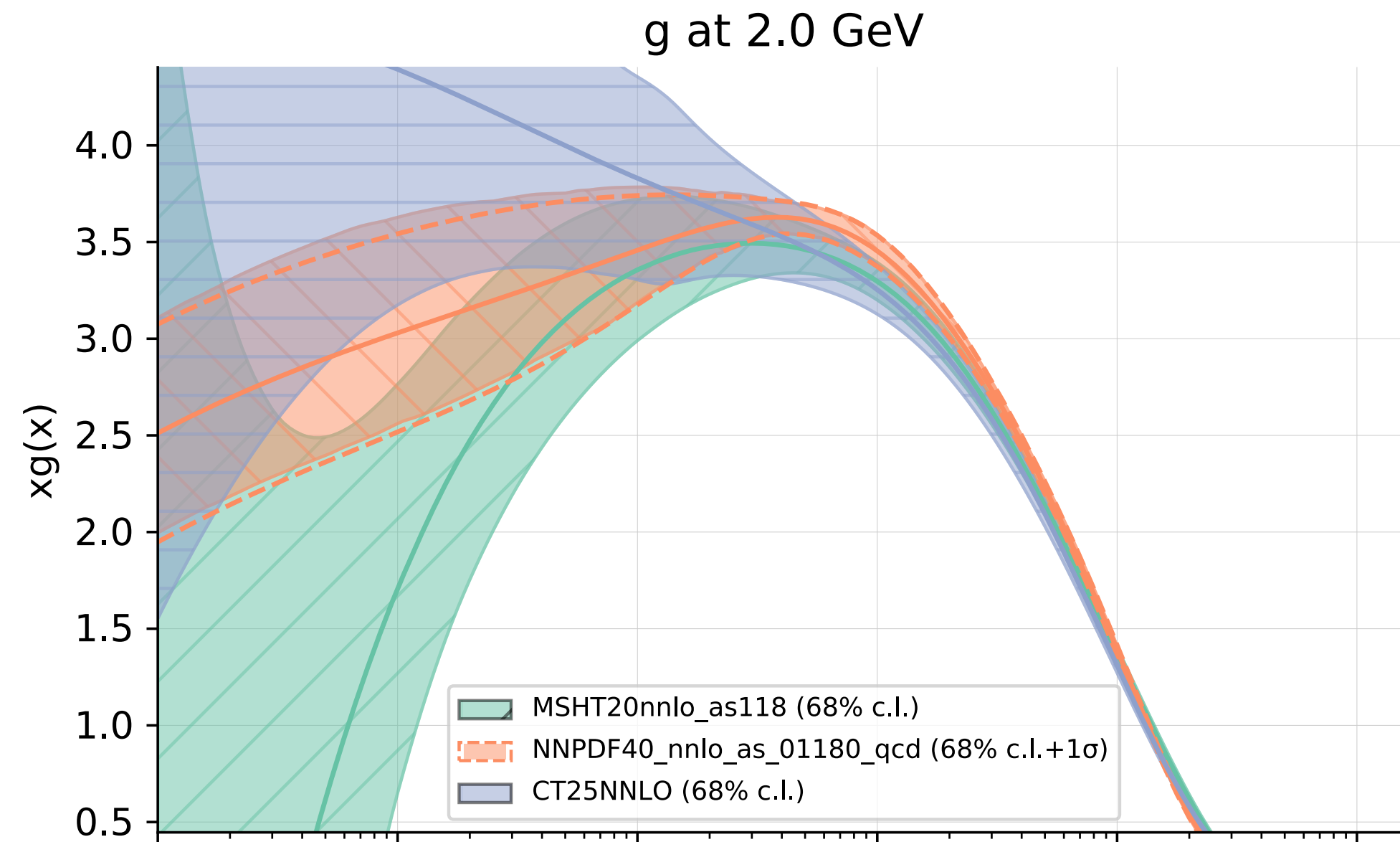
The PDF determination is at the core of any prediction for hadron colliders, therefore being able to compute it accurately will affect all measurements and predictions.



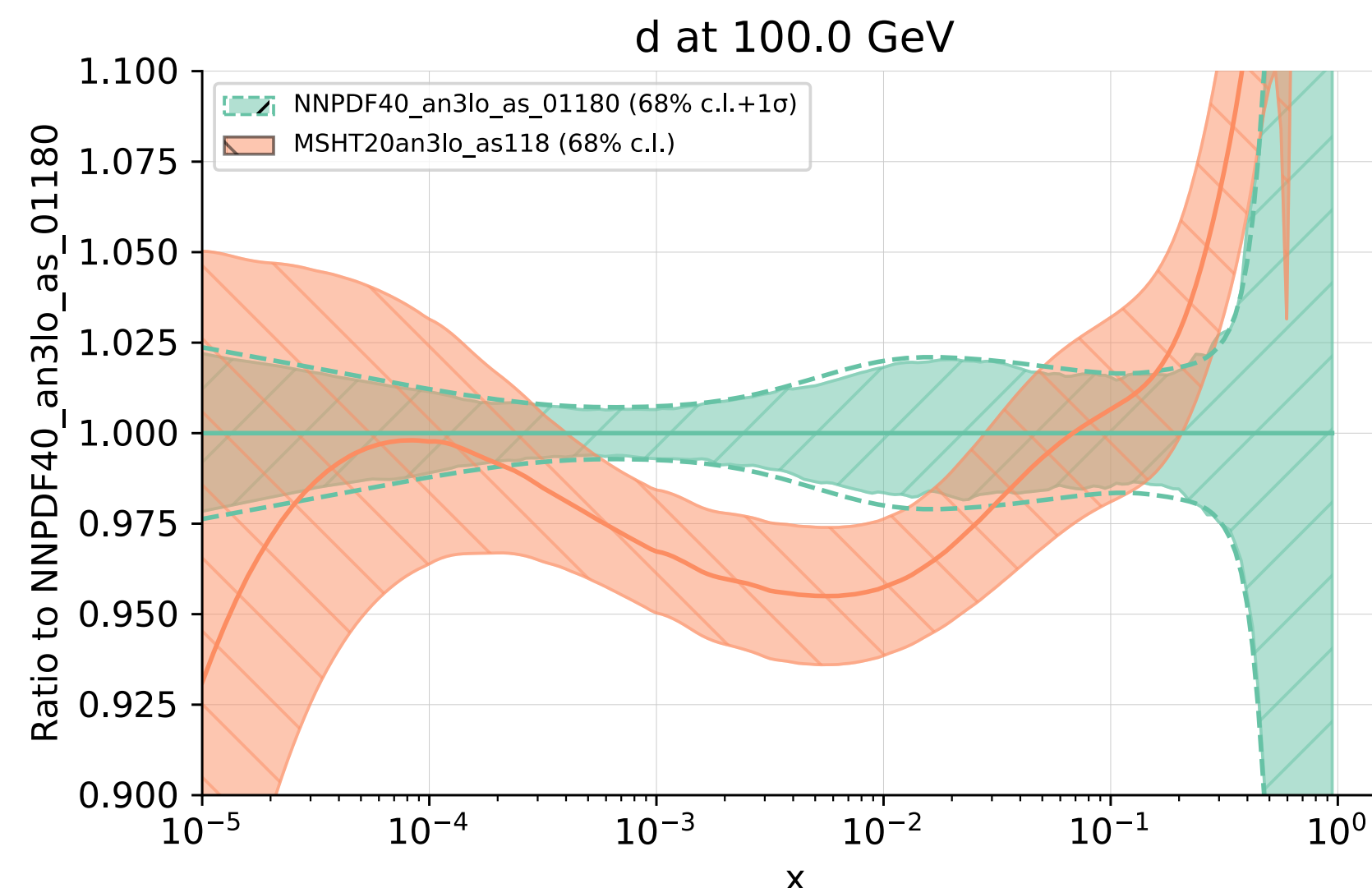
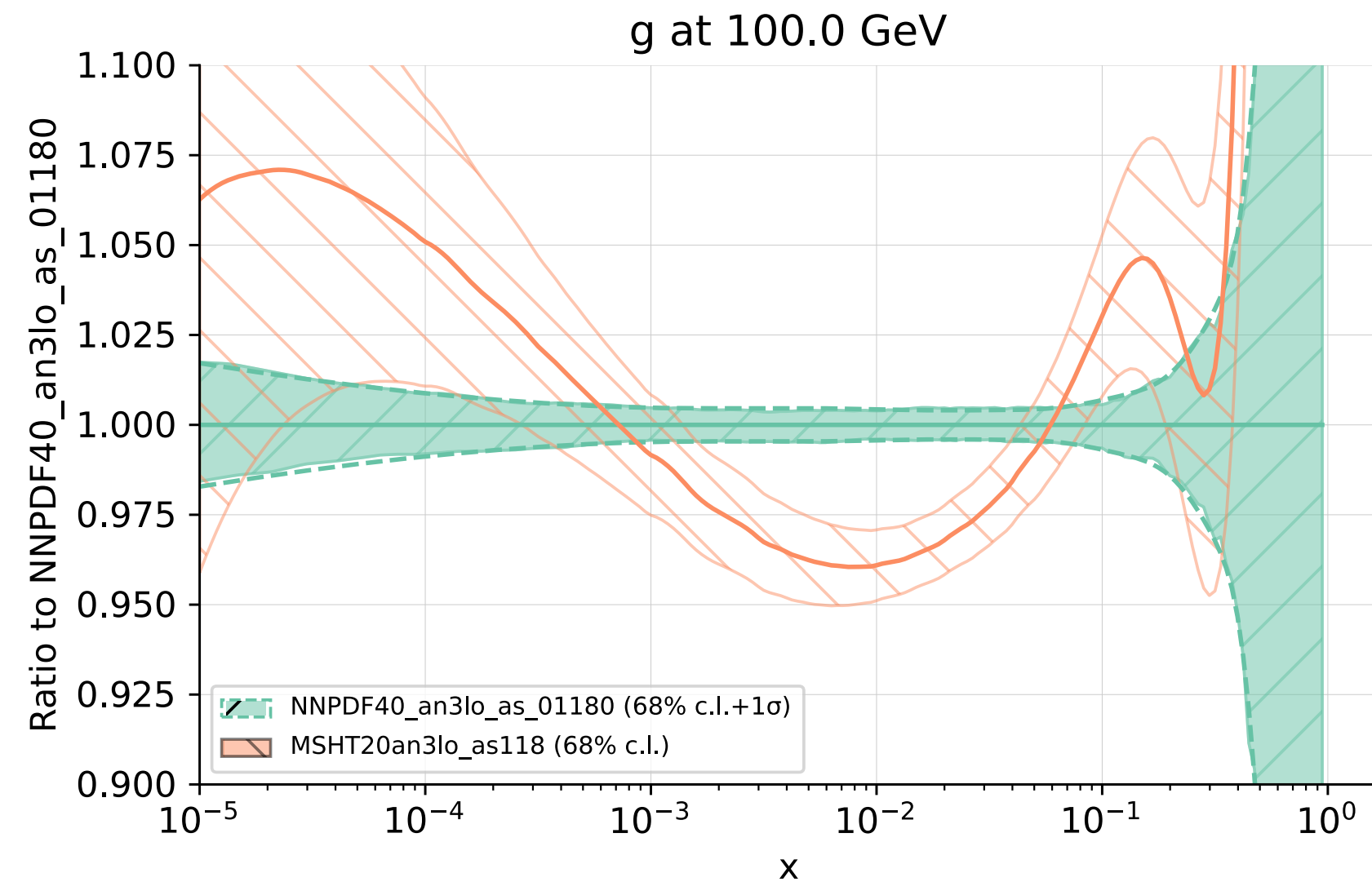
Parton Distribution Functions... 15 years ago



Parton Distribution Functions... today: NNLO



Parton Distribution Functions... today: aN3LO



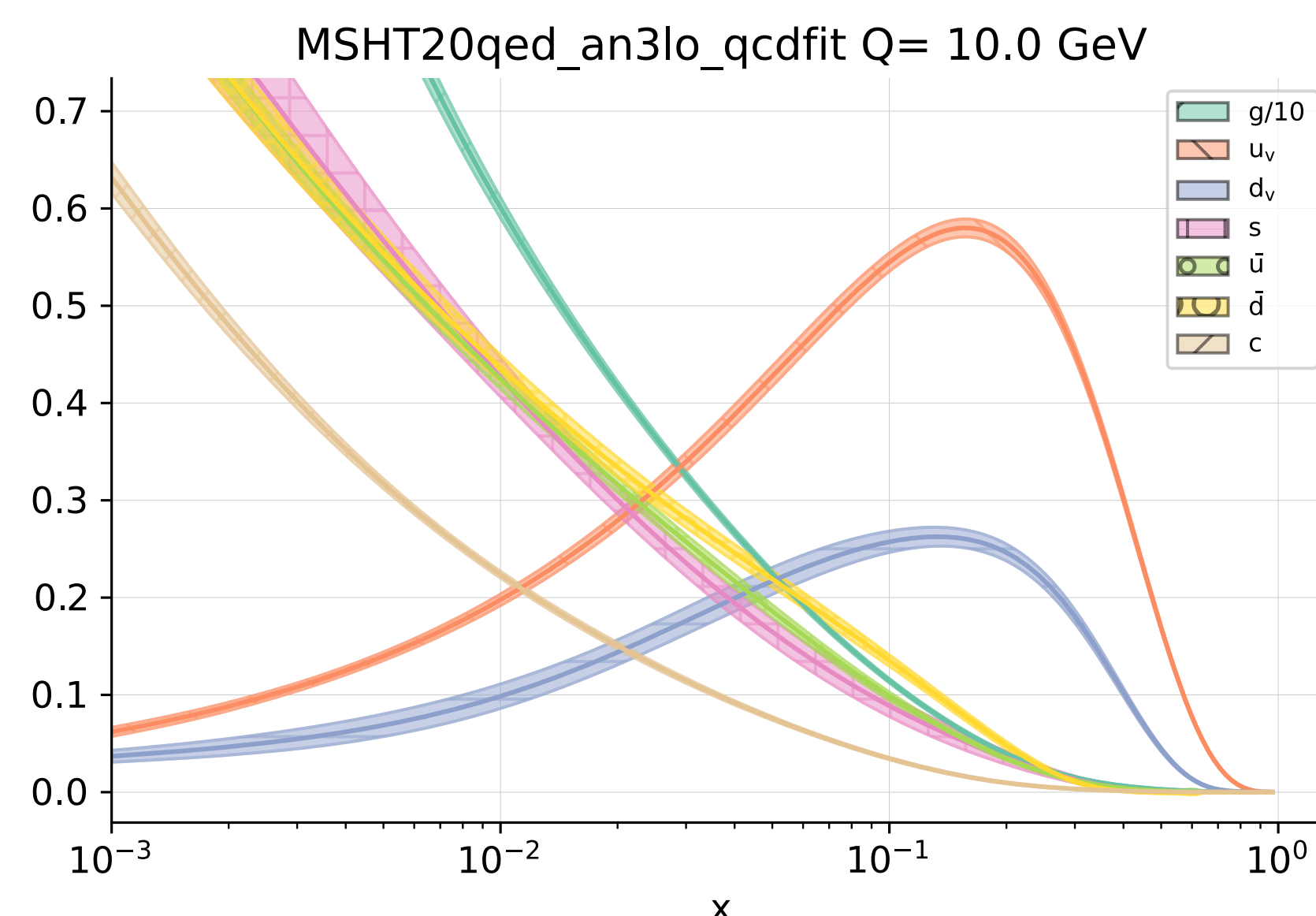
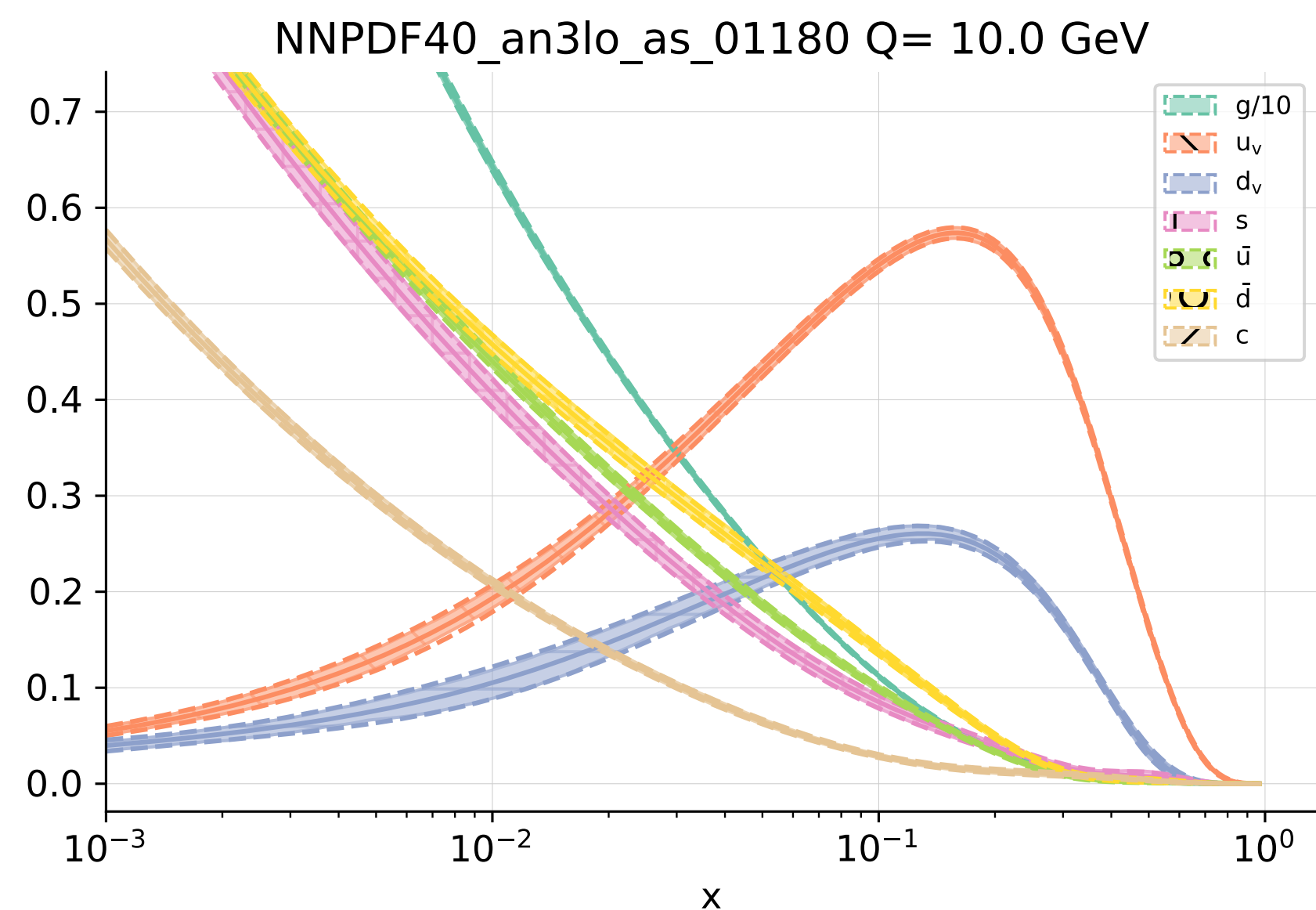
“a” because they are approximated:
the full splitting functions are still not available and N3LO partonic cross sections are only included for DIS data, hadronic is still NNLO

Only MSHT and NNPDF available
at the moment, a first combination
has been provided in:

Combination of aN3LO PDFs and
implications for Higgs production
cross-sections at the LHC

MSHT & NNPDF collaborations

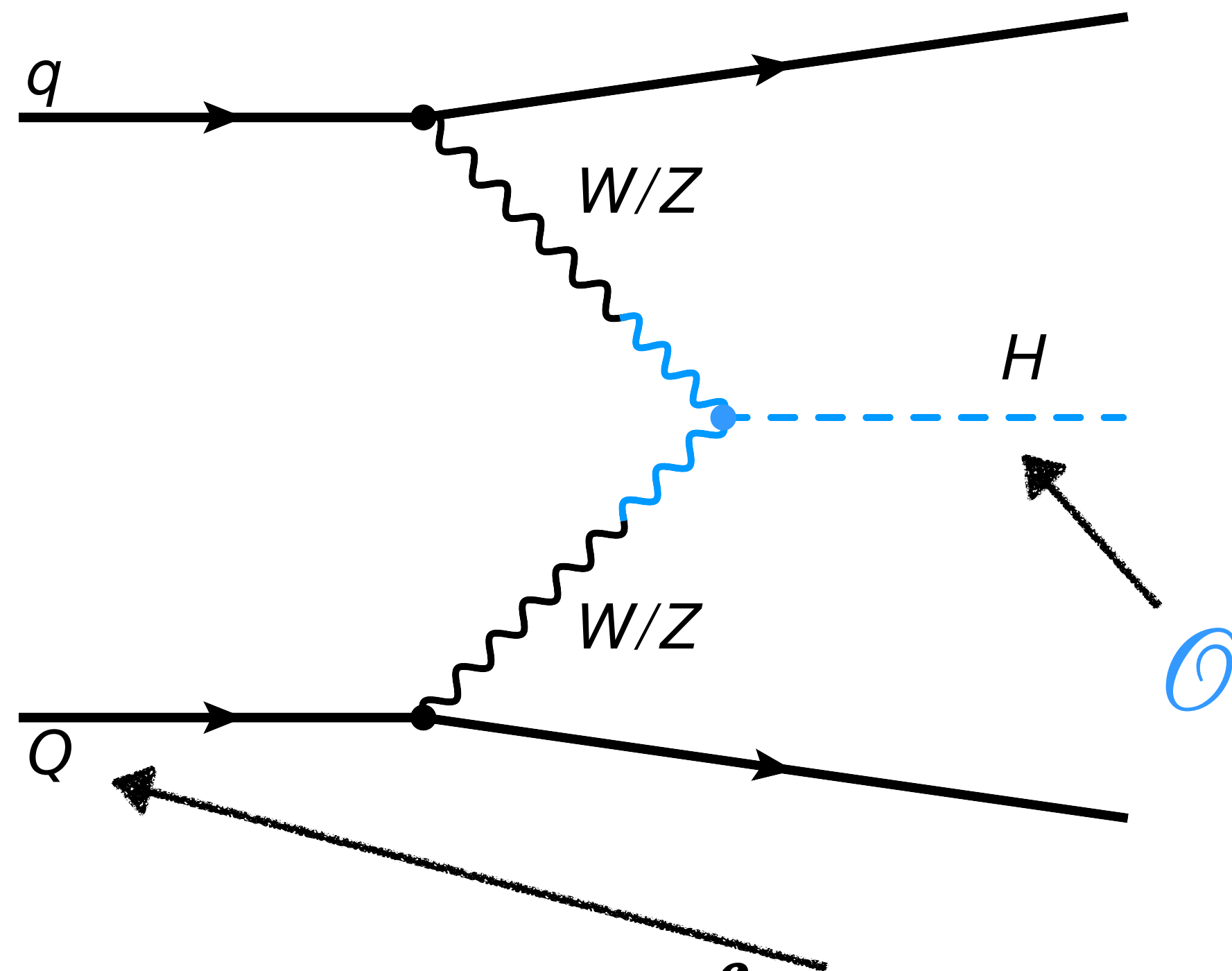
[hep-ph/2411.05373](https://arxiv.org/abs/hep-ph/2411.05373)



An update by CTEQ + updates from all
groups with the latest information
available (making that “a” smaller with
each passing day!) and a full
benchmark will come in the near future!

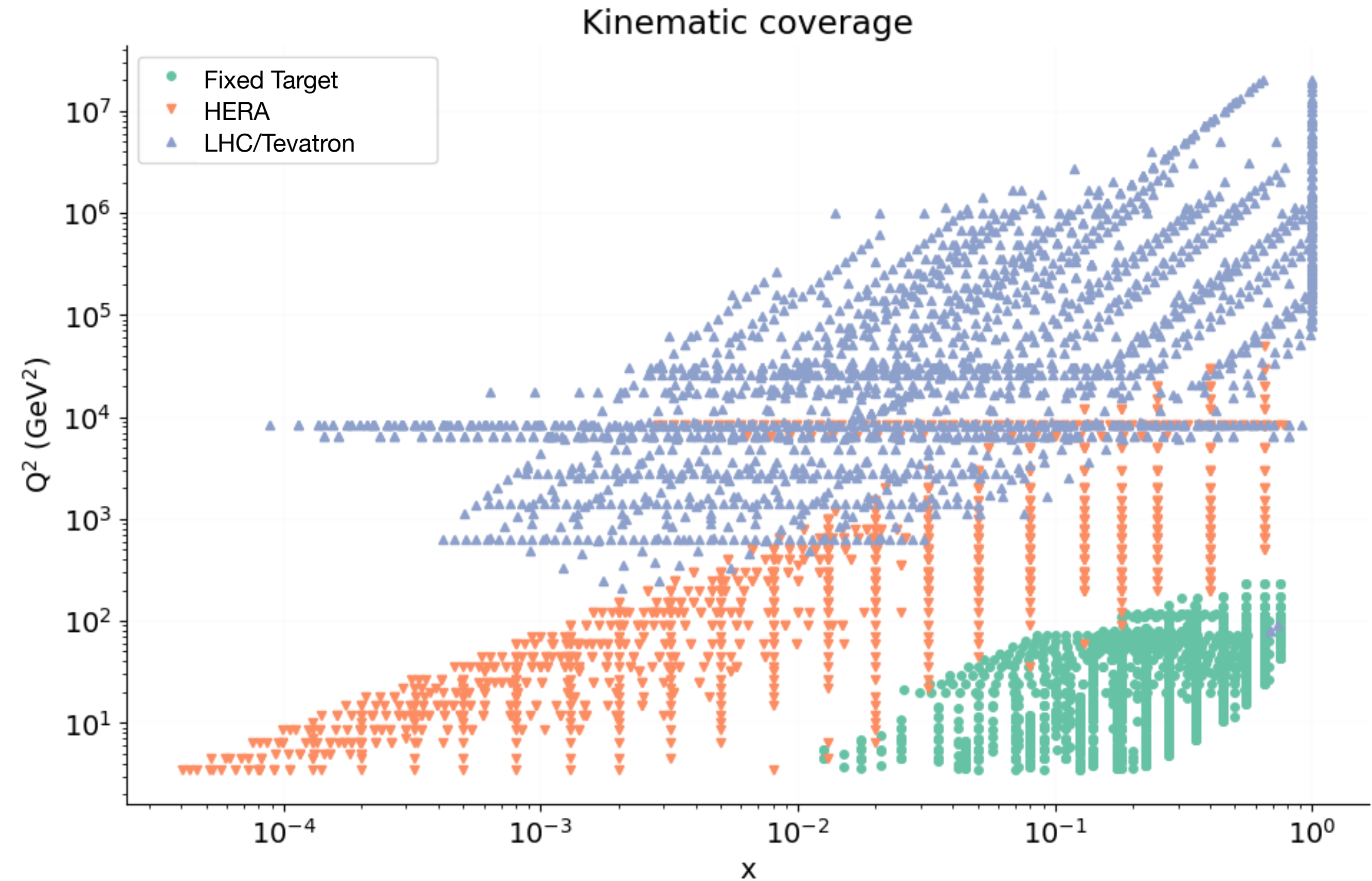
PDF determination ingredients

Theory + Data + Methodology (+ thoughts and prayers)

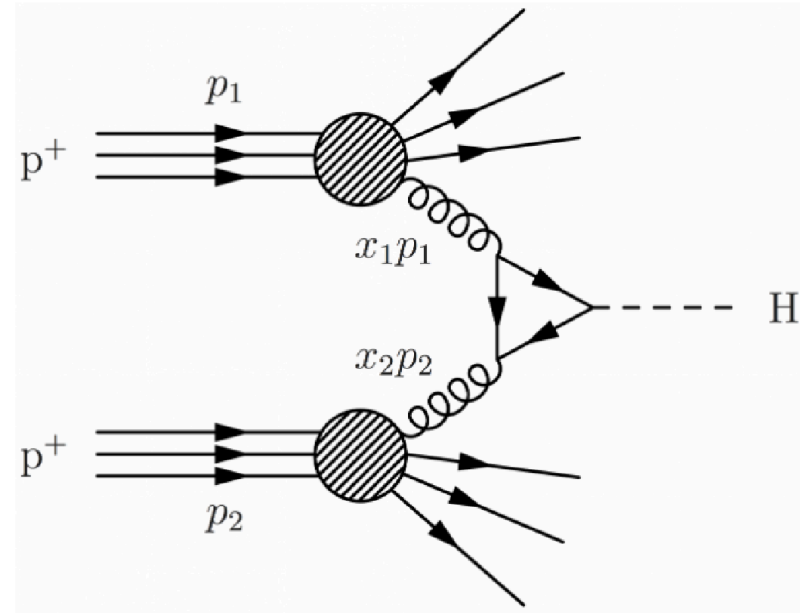


$$\mathcal{O} = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \hat{\sigma}_{ij}(x_1, x_2, \mu_R, \mu_F)$$

Input: quark type and fraction of the energy participating in the collision



Accuracy in theory translates to accuracy on PDFs



$$\mathcal{O} = \int d\Phi_n dx_1 dx_2 f_1(x_1, \mu_F^2) f_2(x_2, \mu_F^2) |M(\{p_n\}, \mu_R)|^2$$

Parton Distribution Functions

Hard-scattering amplitude

In perturbation theory we describe observable $\mathcal{O}$ at a fixed order in terms of the coupling constant of QCD, α_s :

$$\mathcal{O} = \alpha_s^k (\mathcal{O}_{\text{LO}} + \alpha_s \mathcal{O}_{\text{NLO}} + \alpha_s^2 \mathcal{O}_{\text{NNLO}} + \dots)$$

With a typical reduction of theoretical uncertainties at each higher order, to exemplify:

- LO (100 % error)
- NLO (10-20%)
- NNLO (a few %)
- N3LO (1% or better)

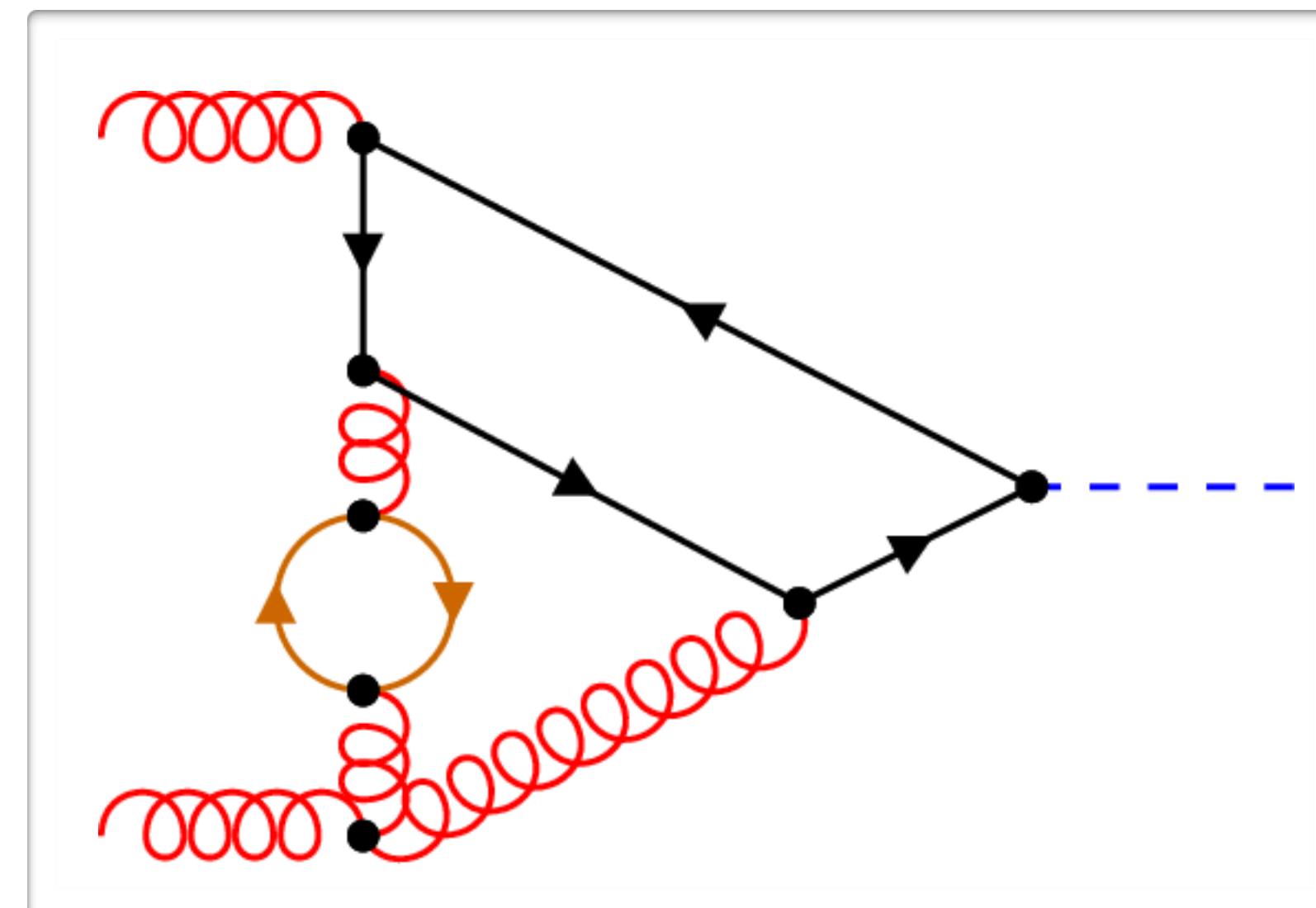
NB: this talk focuses on the fixed order accuracy of the PDF determination. Many other effects of theoretical origin should also be considered for a complete picture: QED, EW, higher twists, resummation, interpolation accuracy, etc.

Introduction and motivation

The N3LO revolution?

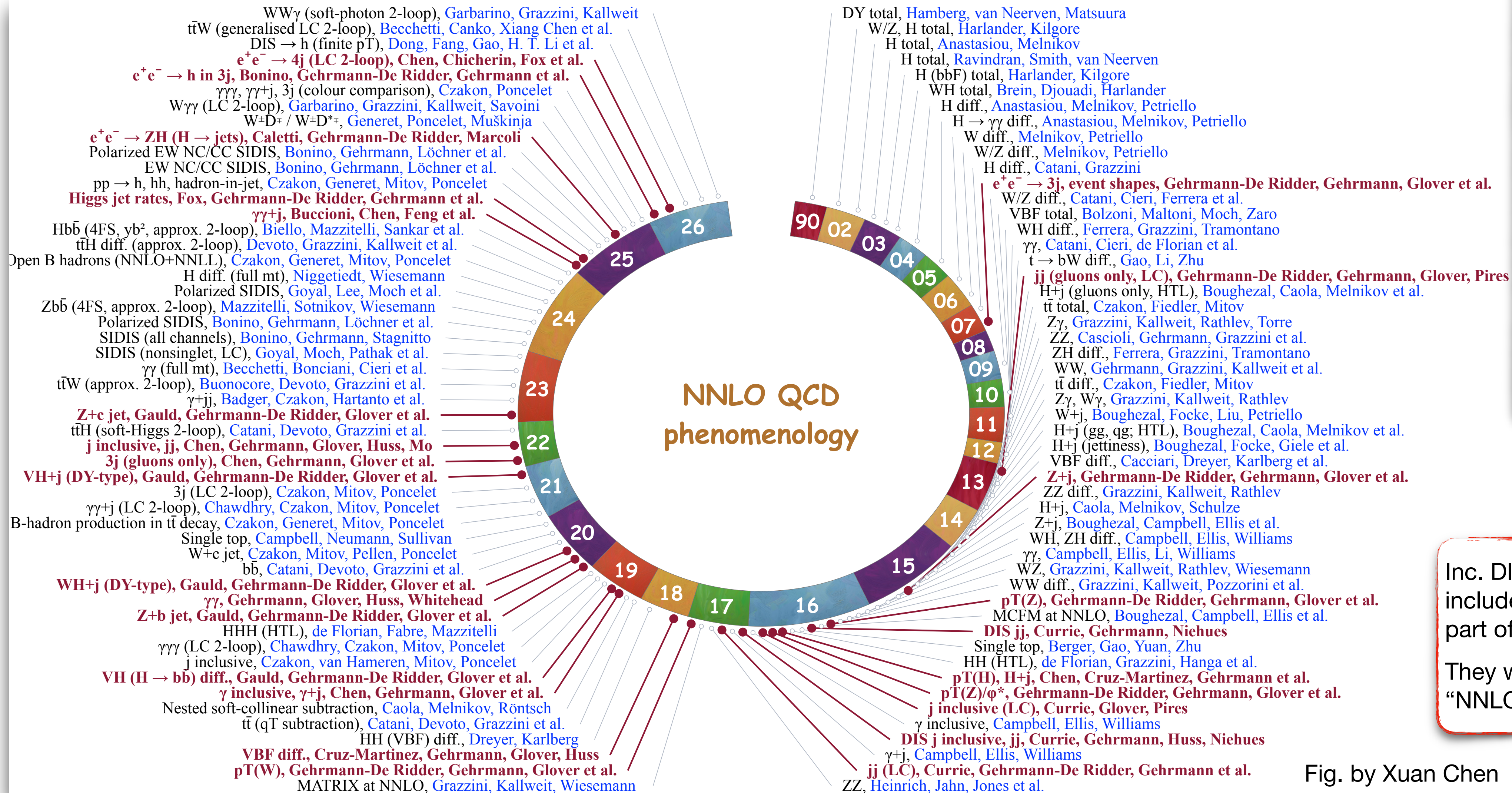
Approximated N3LO PDFs

Conclusions



The NNLO revolution standard

NLO is a solved (automated) problem. NNLO not so much, but many calculations are already available



Many of these are nowadays included in NNLO PDF fits for:

- DY (NC & CC)
- V + jet
- TTBAR
- Single Top
- Jets and dijets
- Prompt photon

Inc. DIS and DGLAP evolution not included in this list, but known (and part of PDF fits) since a long time.

They were the true driver of the "NNLO" label in PDFs.

Fig. by Xuan Chen

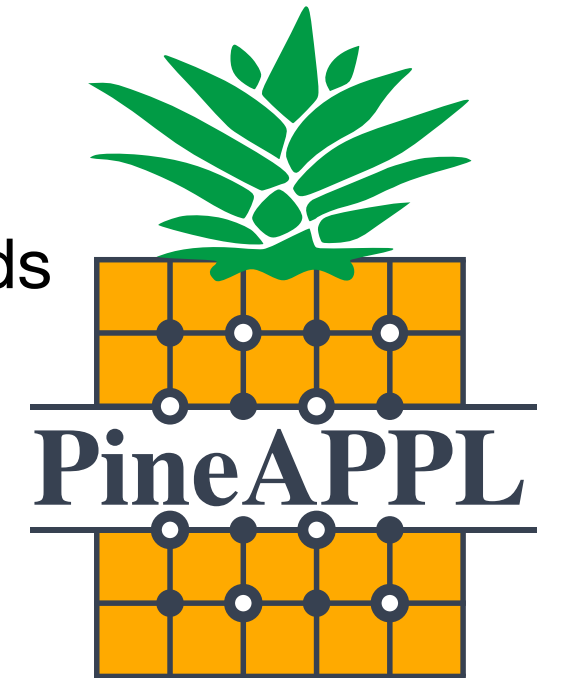
From now on, expect exact NNLO PDFs

While PDFs have been “NNLO” for a while, exact inclusion of NNLO calculations is very challenging and hadronic NNLO calculations are included by means of constant factors. Only recently, NNLO accurate interpolation grids have made *true* NNLO PDFs possible.

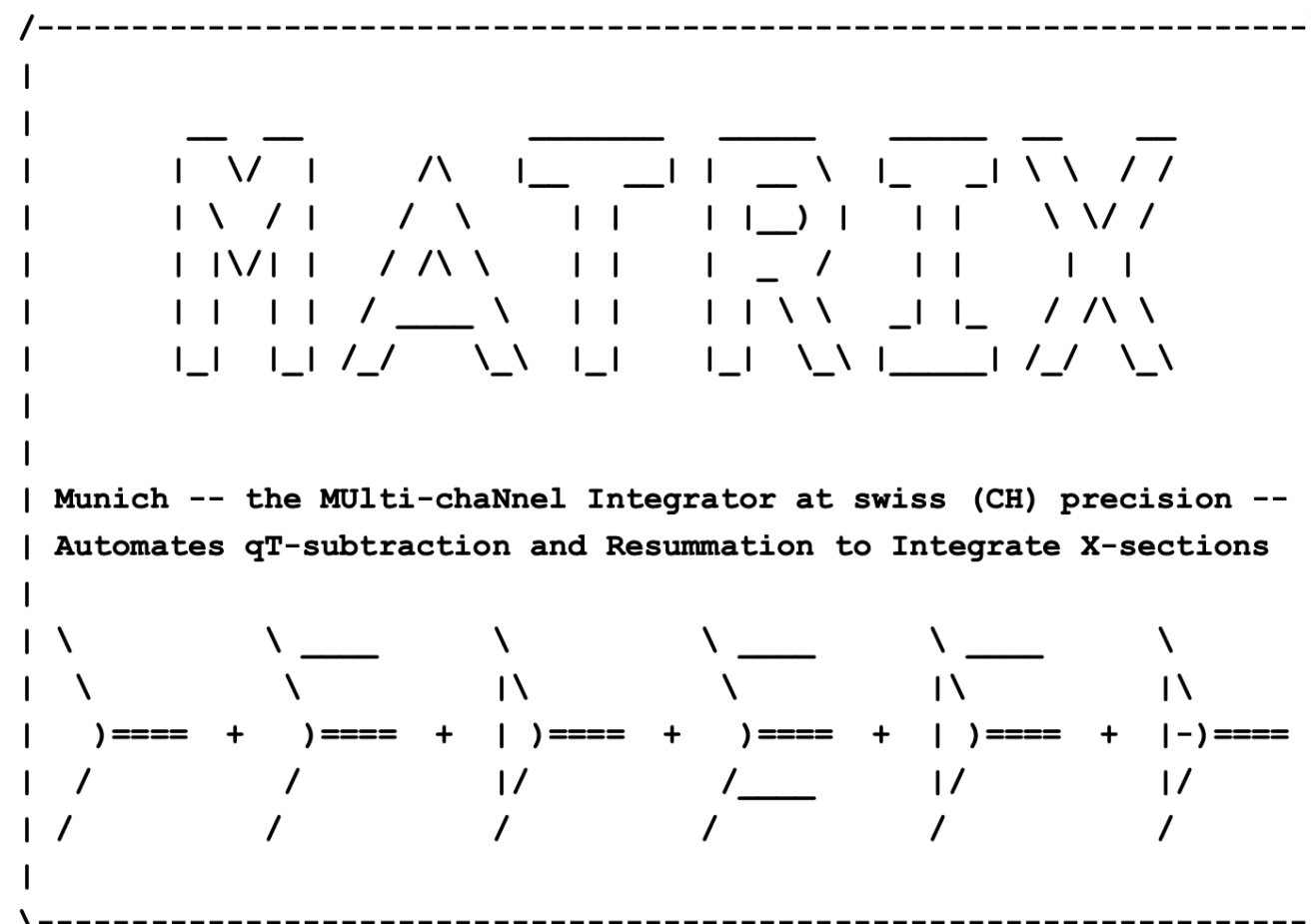
$$\frac{d^4\hat{\sigma}_{ij}}{d\mu_F d\mu_R dx_1 dx_2}$$

We can bin the cross section in the variables relevant for the PDF determination using interpolation grids

We can further optimize the grids by considering the PDFs at an initial scale, but that’s just a technical point.



For the purposes of PDF fitting the following public code cover almost all of our necessities for hadronic collisions by generating public grids.



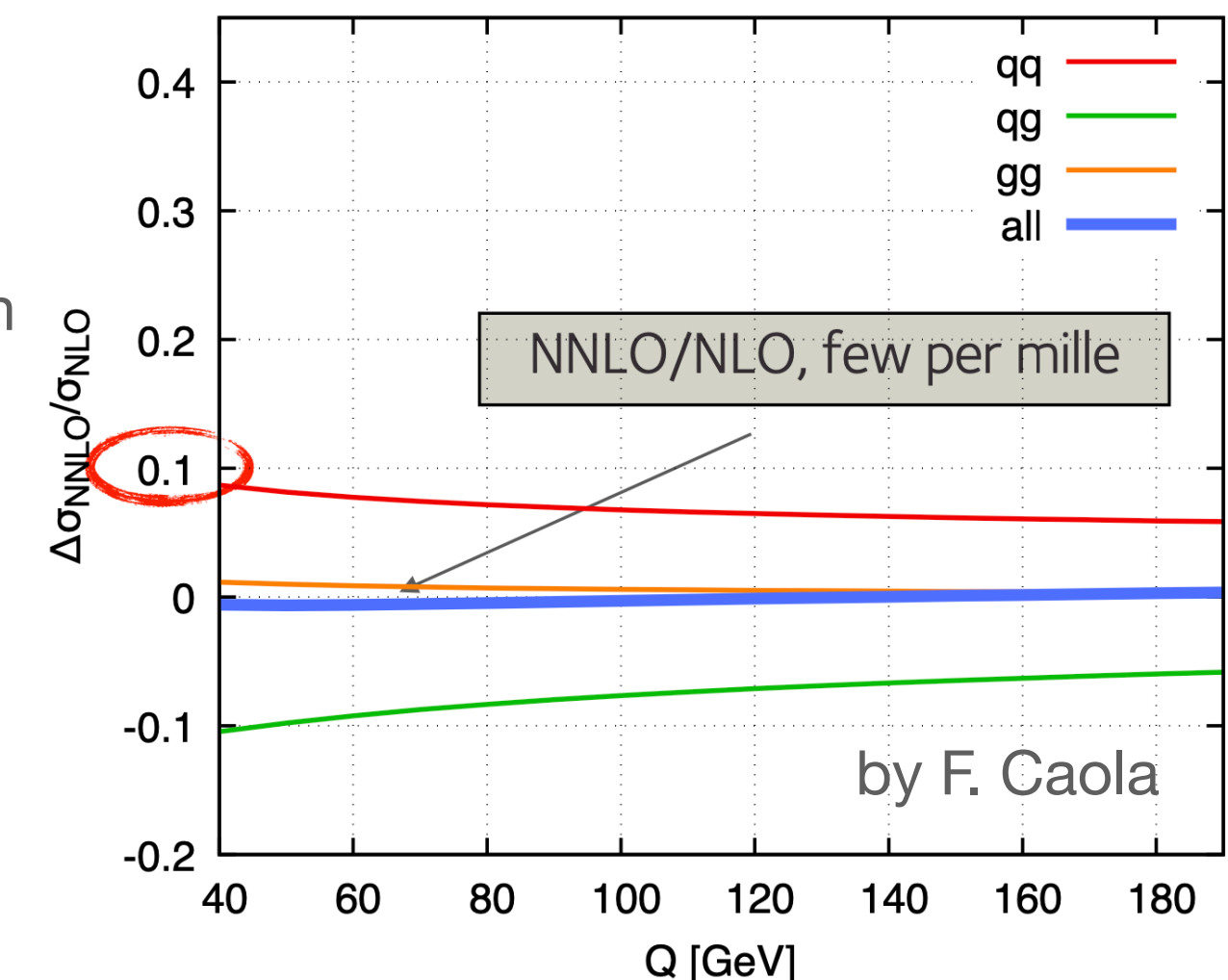
M. Grazzini, S. Kallweit, M. Wiesemann, et al
hep-ph/1711.06631



A. Huss et al - hep-ph/2503.22804

A constant factor can potentially hide away important physics through accidental cancellations.

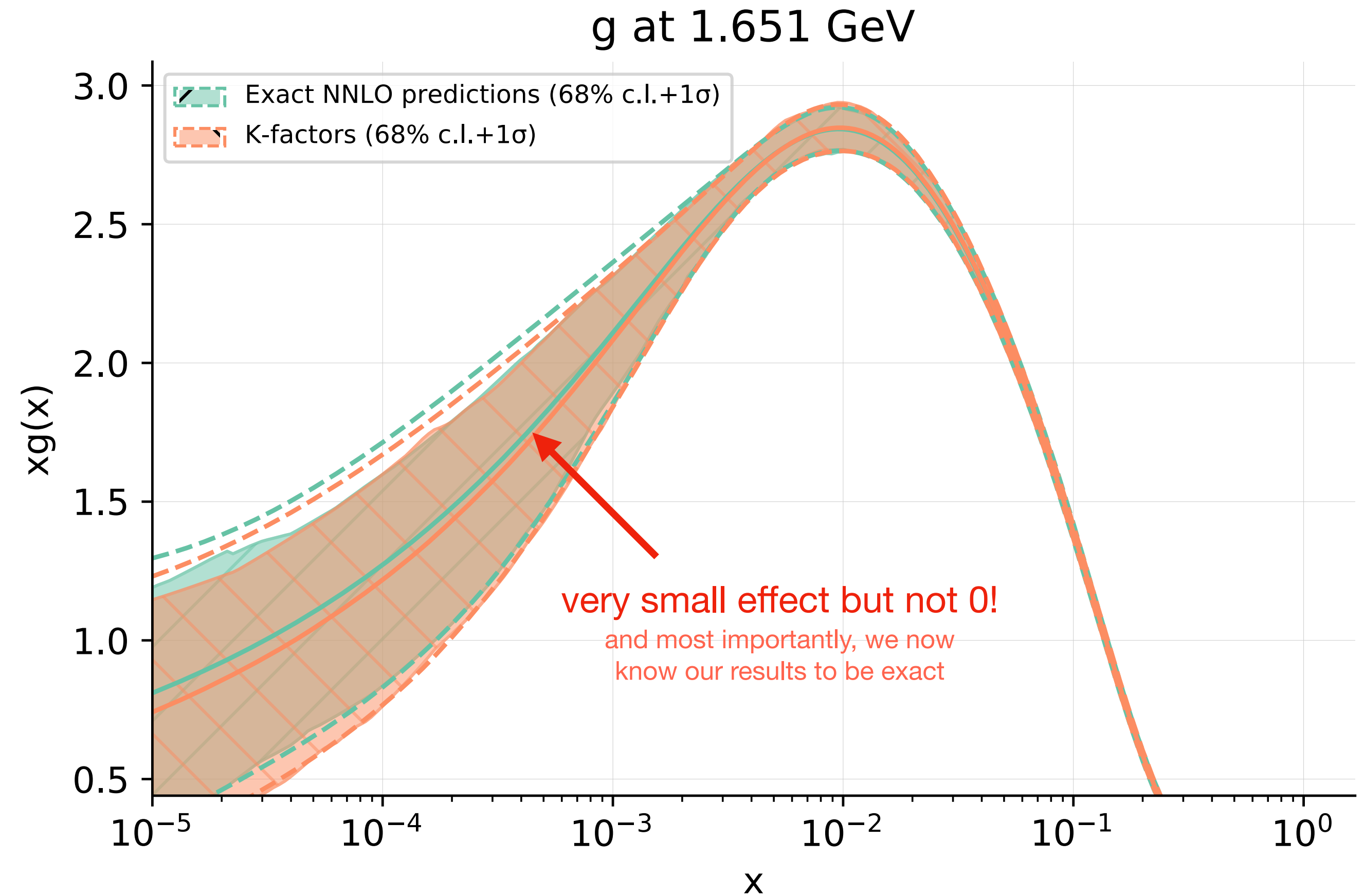
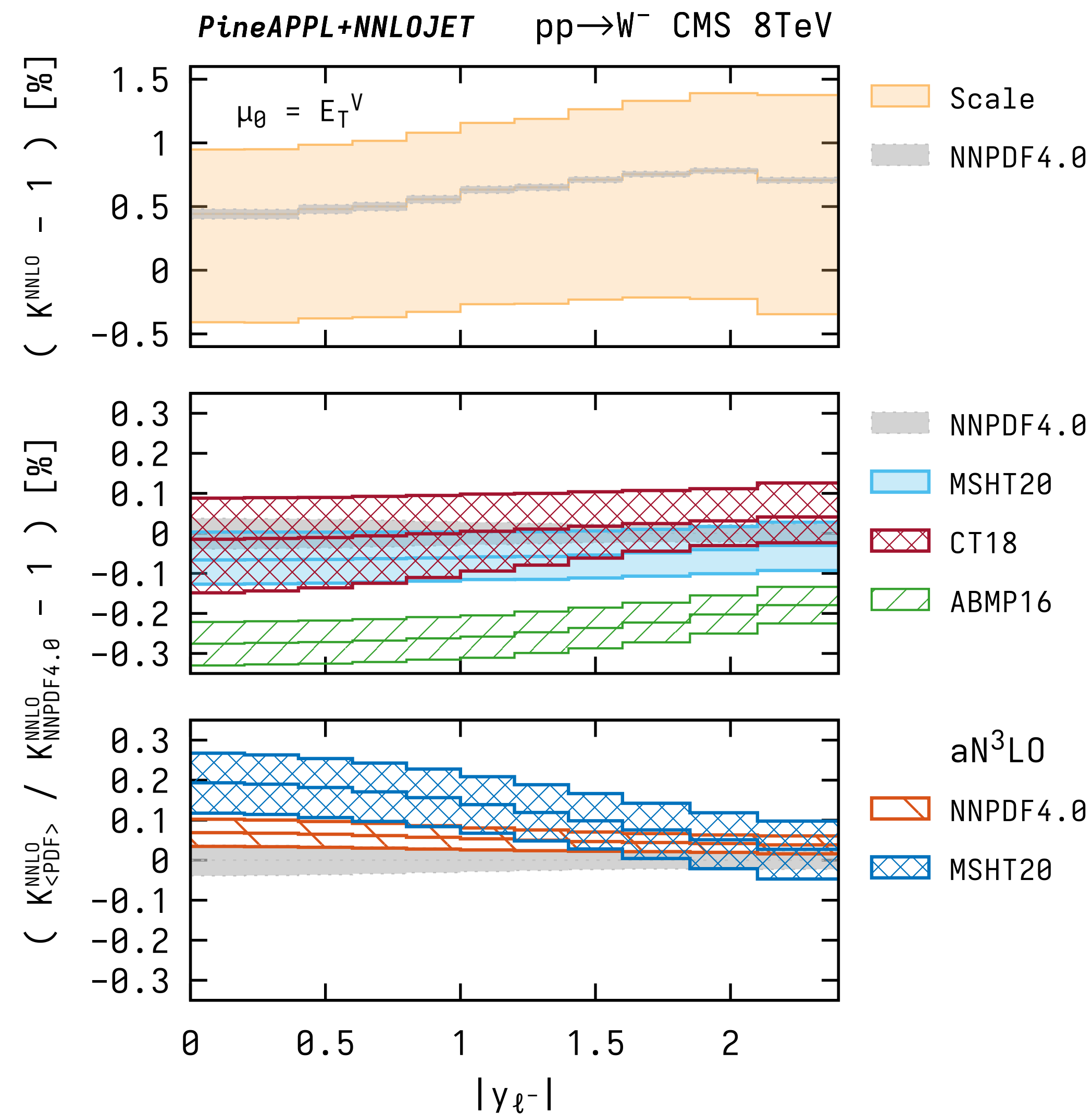
See for instance the case of Drell—Yan.



Other codes have also been interfaced with grid interpolation tools for NNLO results: vrap, apfel, MCFM or top++

A case study: DY @ NNLO

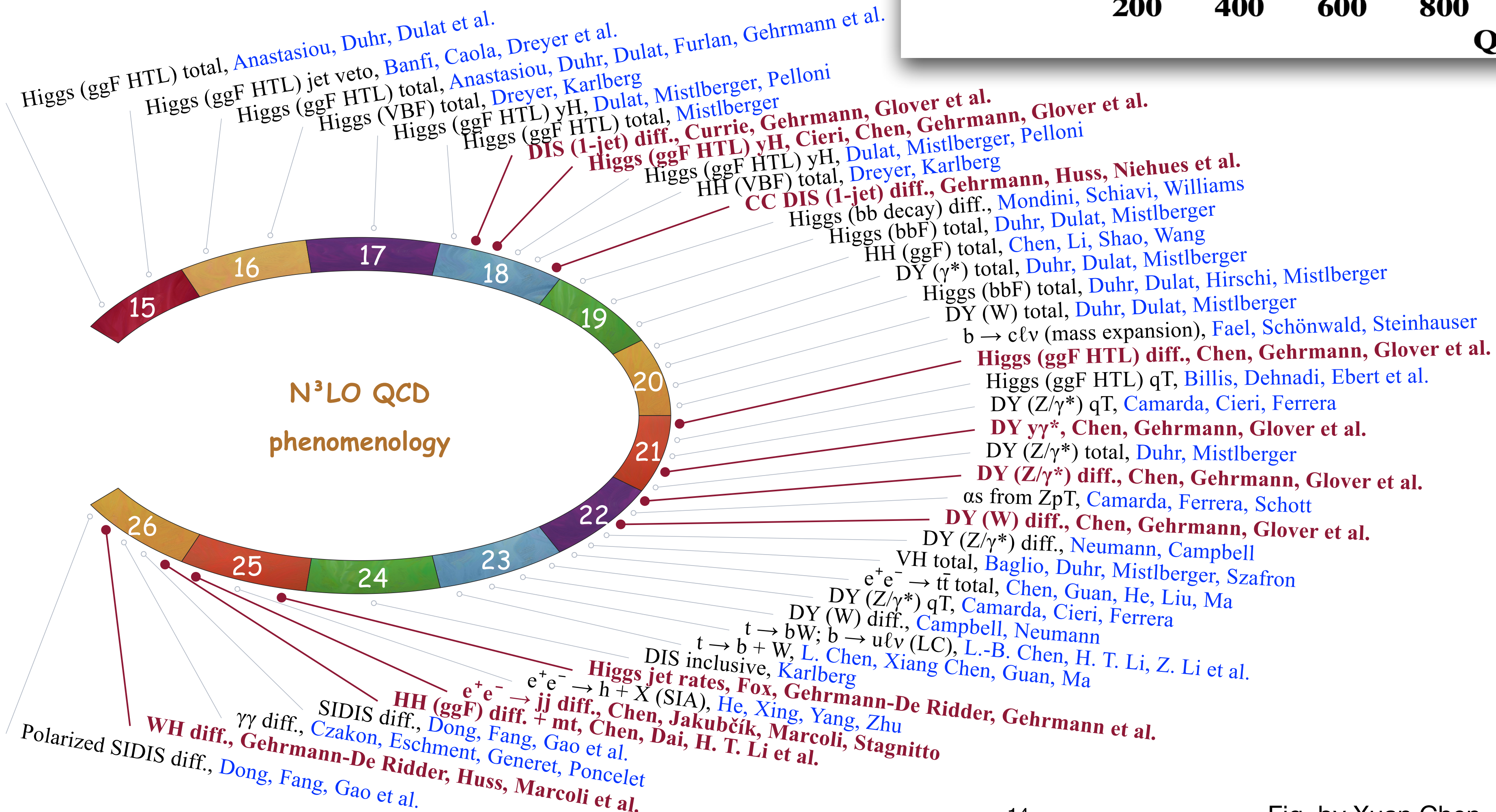
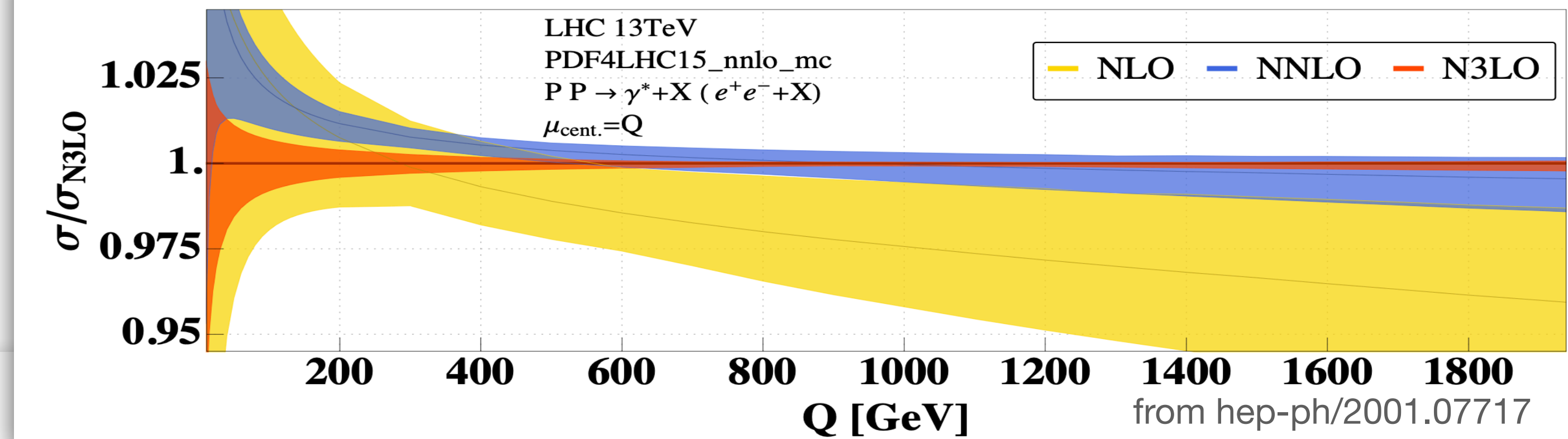
Fast interpolation grids for the Drell–Yan process
JCM, A. Huss, C. Schwan - hep-ph/2501.13167



Thanks to the enormous effort by the theory community, we have now access to interpolation grids for all* processes and data included in PDF fits.

We are now ready to go confidently for the next step!

The N3LO revolution



And what about DIS?

✓ Massless coefficients

Available at N3LO, also in the form of interpolation grids and public codes.

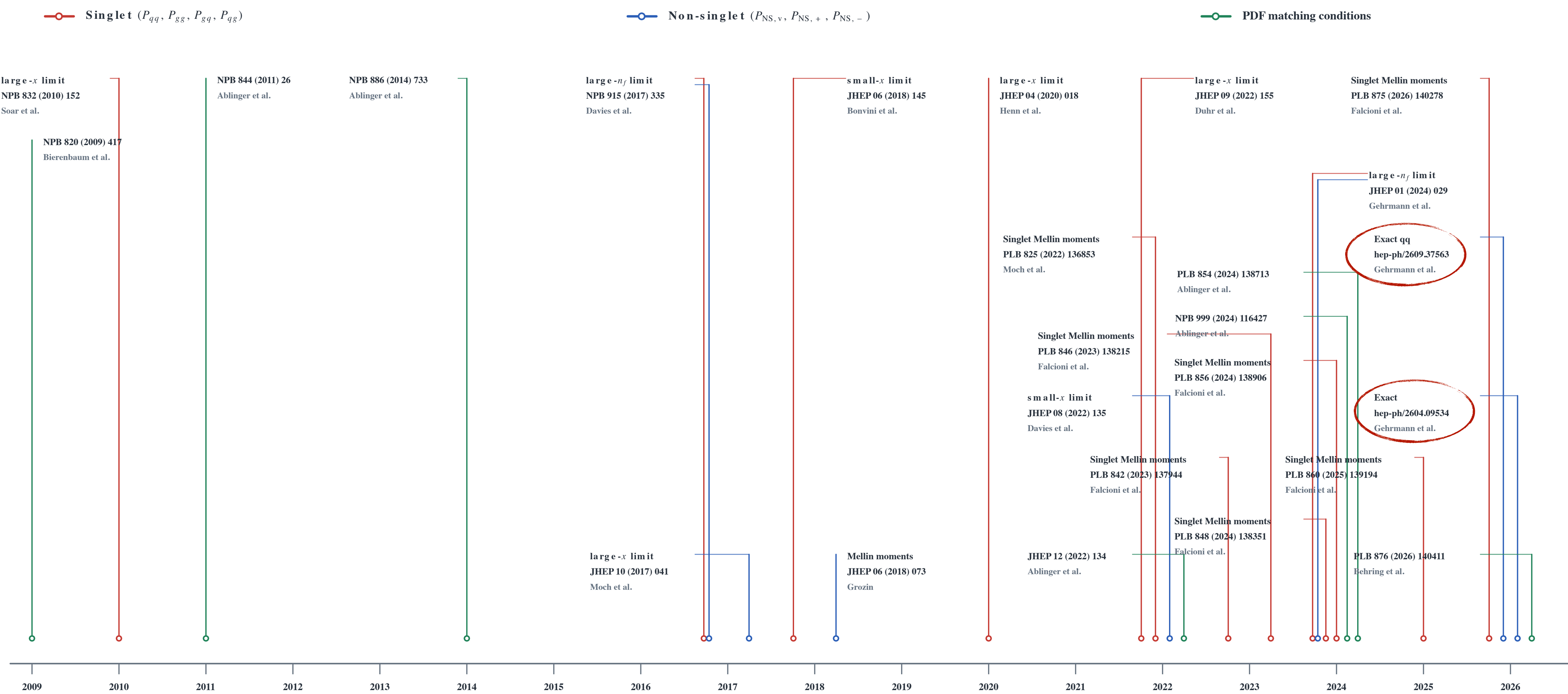
✗ Massive coefficients

Available at $\mathcal{O}(\alpha_s^3)$ with some approximations.

In practice, the approximation has a small effect on the PDF determination (see e.g., [hep-ph/2606.25498](#))

N3LO revolution

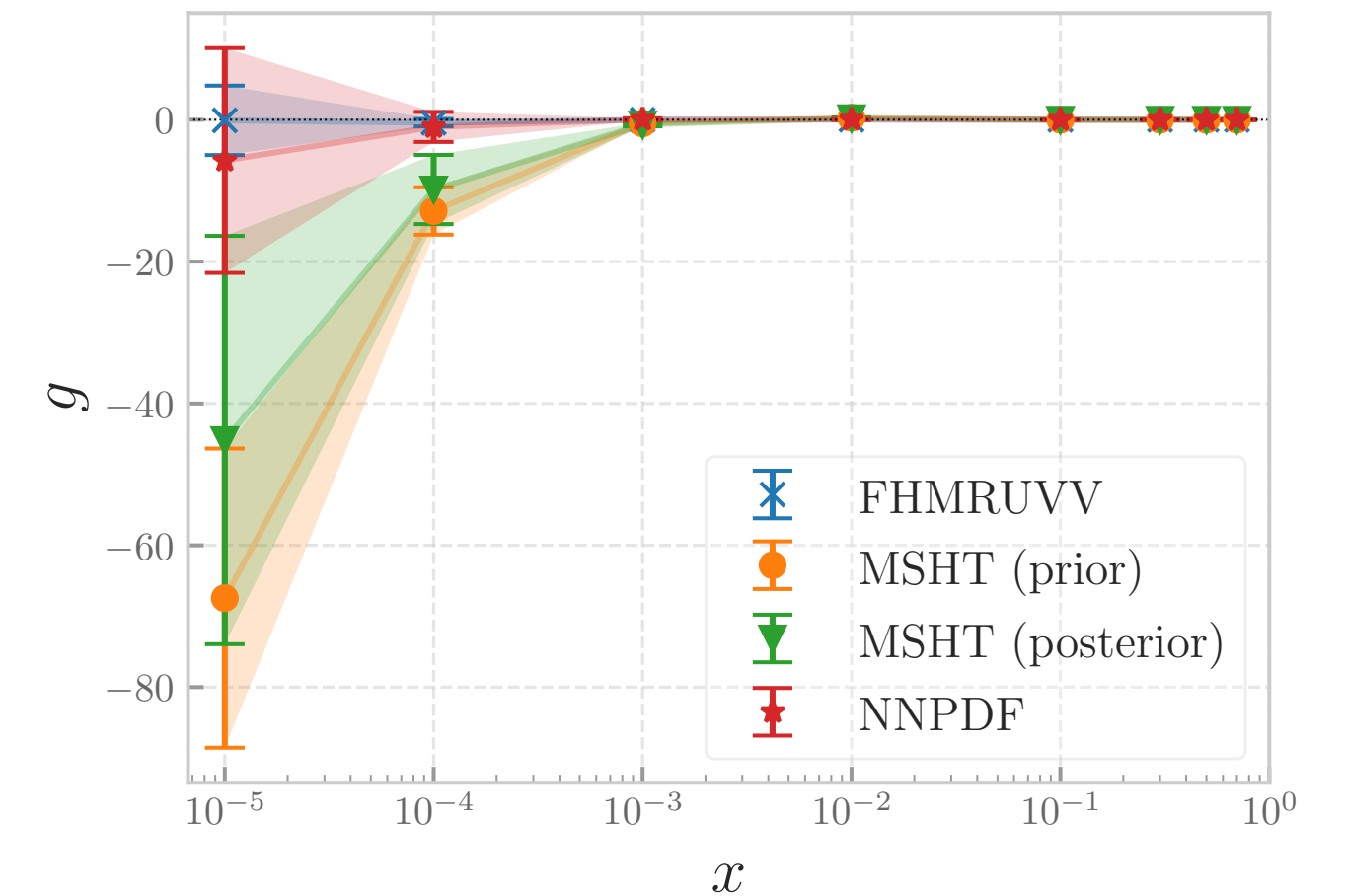
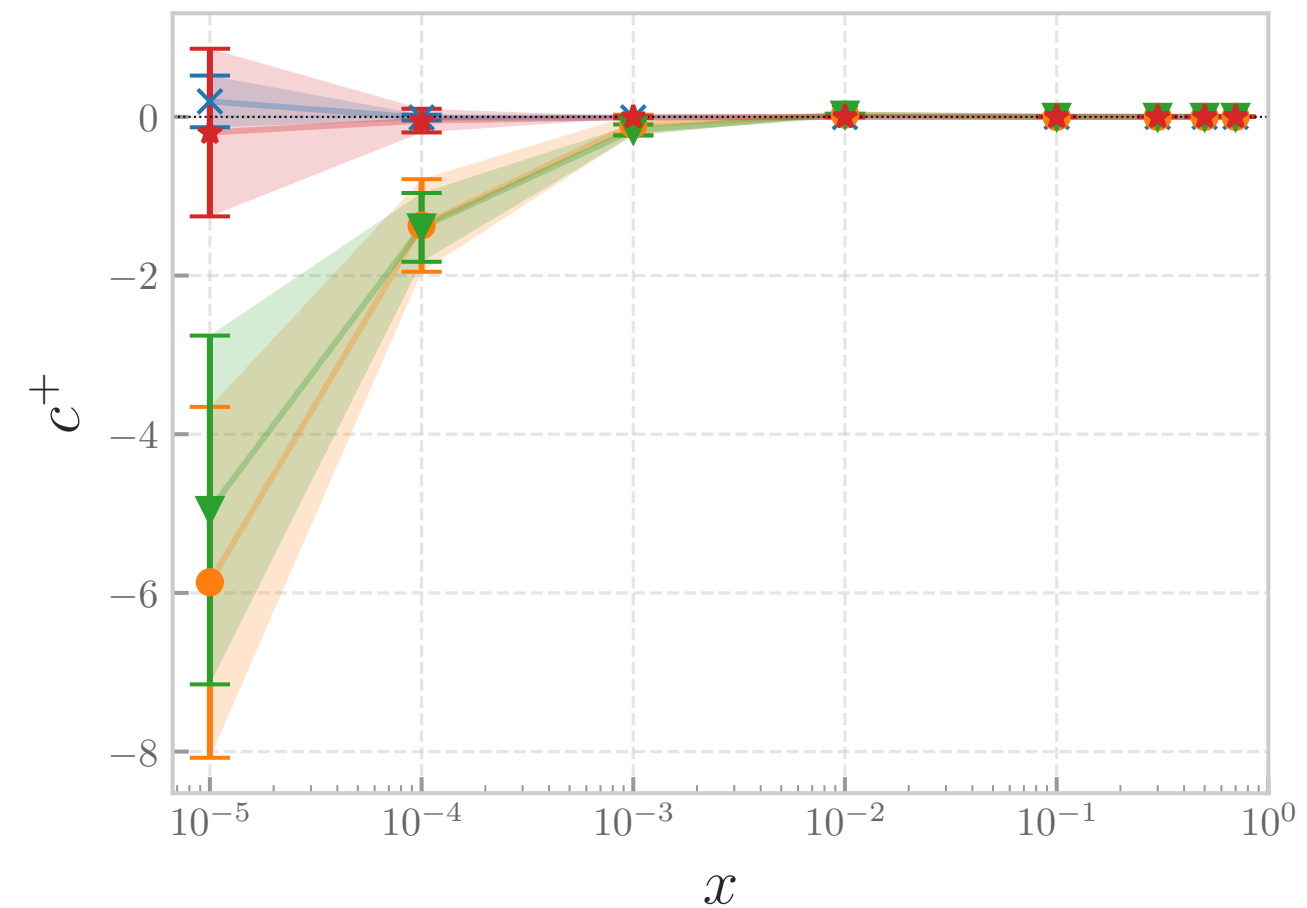
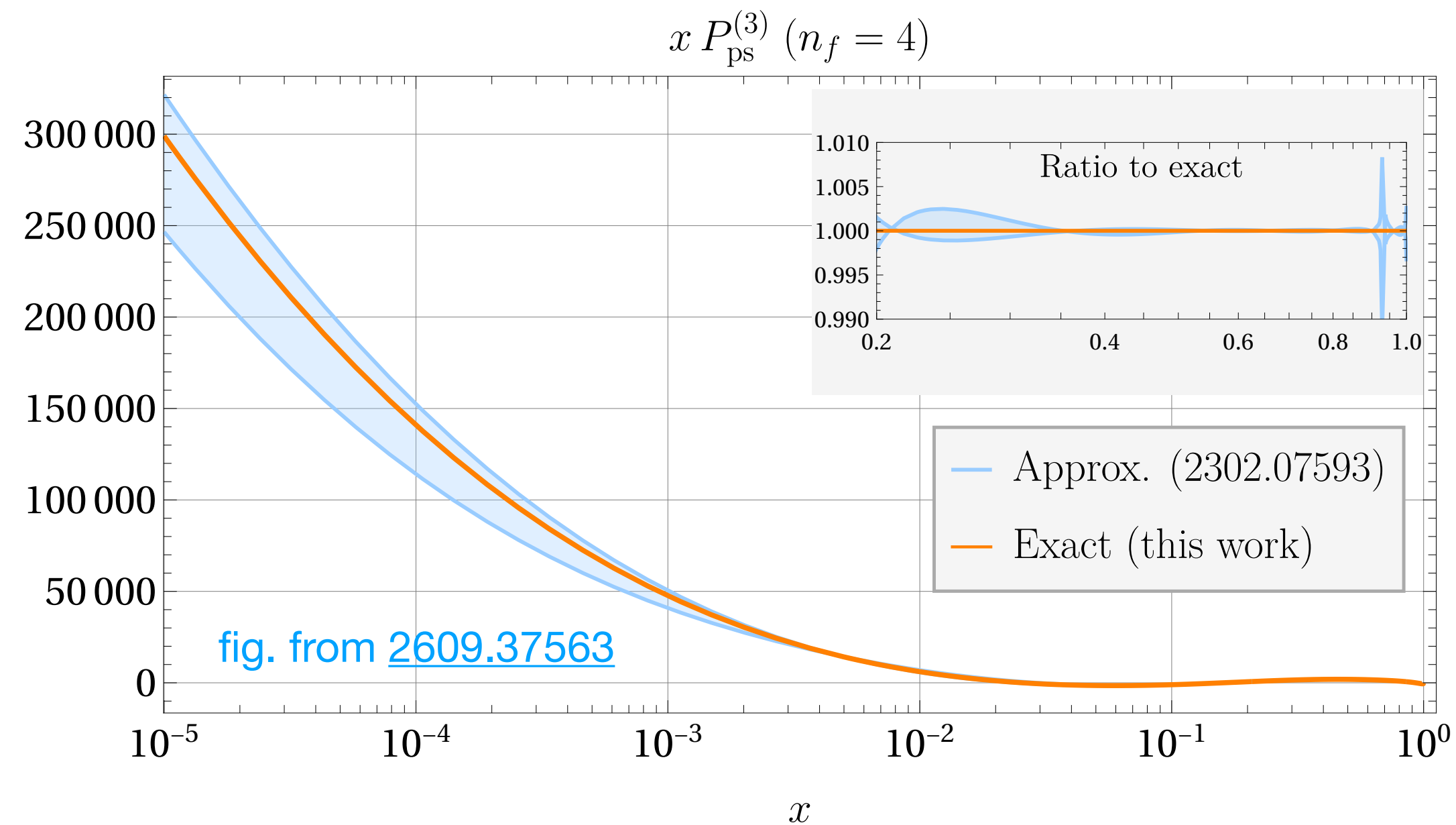
For PDF determination we also need N3LO DGLAP equations:



For many years we only had access to approximated results by considering certain limits

Recently the results are so complete (and even exact in some cases!) we will soon stop talking about “approximated N3LO”

PDFs at (approximated) N3LO



figs. from [2406.16188](#)

NB: Even after all theoretical ingredients are available, some delay is to be expected until PDFs can truly exploit them.

The PDFs consist of many different moving pieces and until they are not all in place (and well validated). The progress is, however, quite fast thanks to the existence of public tools (actively maintained!) that give us (fitters) access to these new calculations.

APFEL++

V. Bertone - [hep-ph/1708.00911](#)



A. Karlberg, P. Nason,
G. Salam, G. Zanderighi,
F. Dreyer
- [hep-ph/2510.09310](#)

A. Candido, F. Hekhorn, G. Magni - [hep-ph/2202.02338](#)

EKO
Evolution Kernel Operators

Candia-v2

C. Hampson, M. Guzzi
- [hep-ph/2512.22667](#)

What can we truly use in PDF fits?

✓ Approximated (and exact) DGLAP evolution

Incorporated already in the evolution codes used by the various collaborations. Benchmarks have already been performed (e.g., 2406.16188). New benchmarks with more recent results are in progress.

✓ DIS coefficients (both for massive and massless structure functions)

Public codes are available. Benchmarks for massless coefficients are available (see e.g. 2404.15711), and between the approximations for massive coefficients used by the various collaborations are currently underway.

~ Inclusive Drell—Yan exact N3LO results

While there are available results for Drell—Yan inclusive cross sections at N3LO, PDF fits need to be compared to data and therefore fiducial cross sections must also be computed. While they exist, the computing cost is at the moment excessive to include them systematically in PDF fits.

✗ N3LO 2 → 2 hadronic coefficients

Other hadronic coefficients (TTBAR production, jets, V+j, etc) are not expected to be available at N3LO in the near future. And surely not in a computing-practical way.

For the time being, (a)N3LO PDFs will be limited to the DGLAP equations and DIS data. In the *best* case scenario, Drell—Yan fiducial corrections might be incorporated in the future. Anything else won't be available in the coming generations of PDFs... but we will keep our eyes peeled for new results 

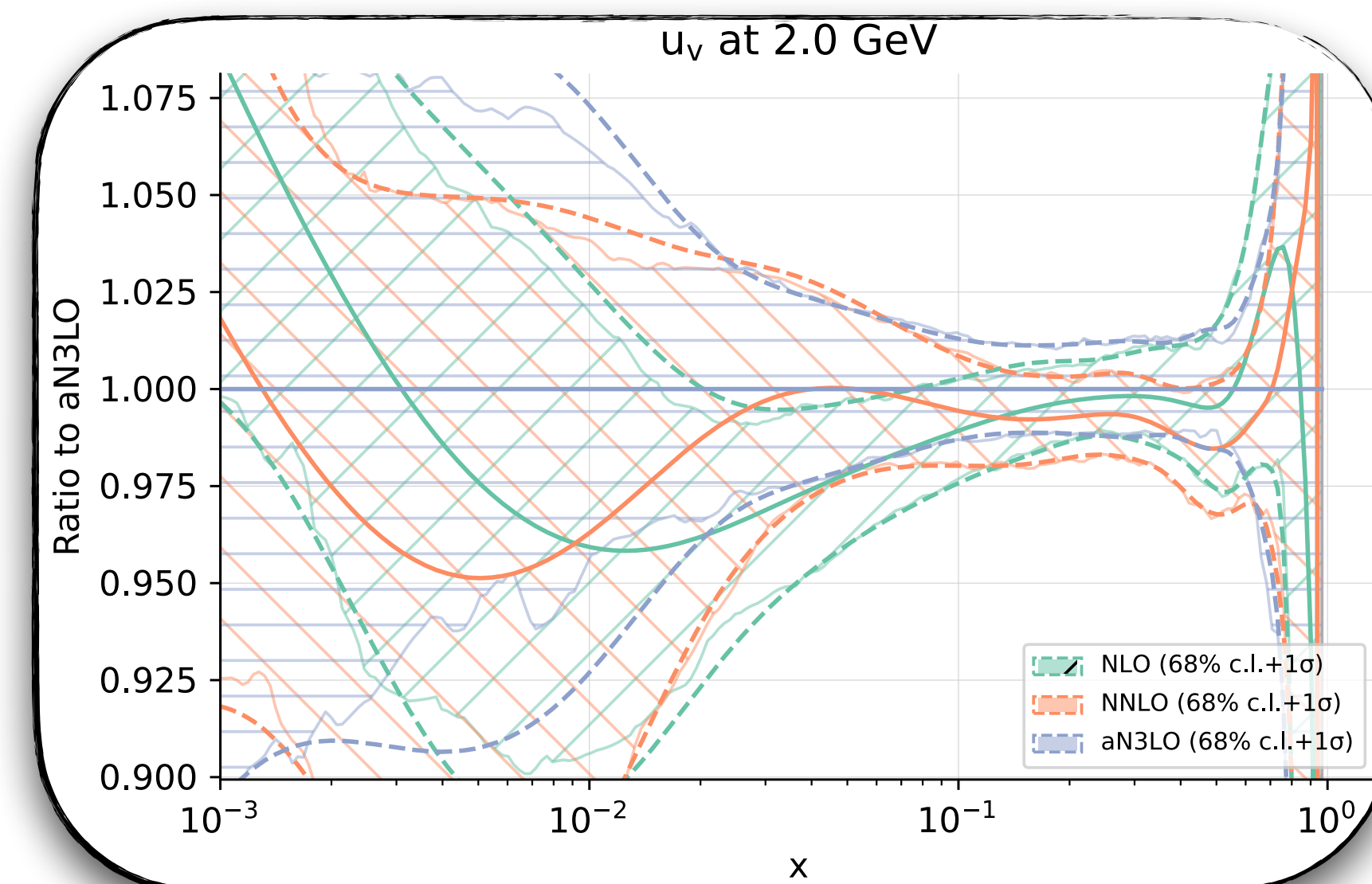
Introduction and motivation

The N3LO revolution?

Approximated N3LO PDFs



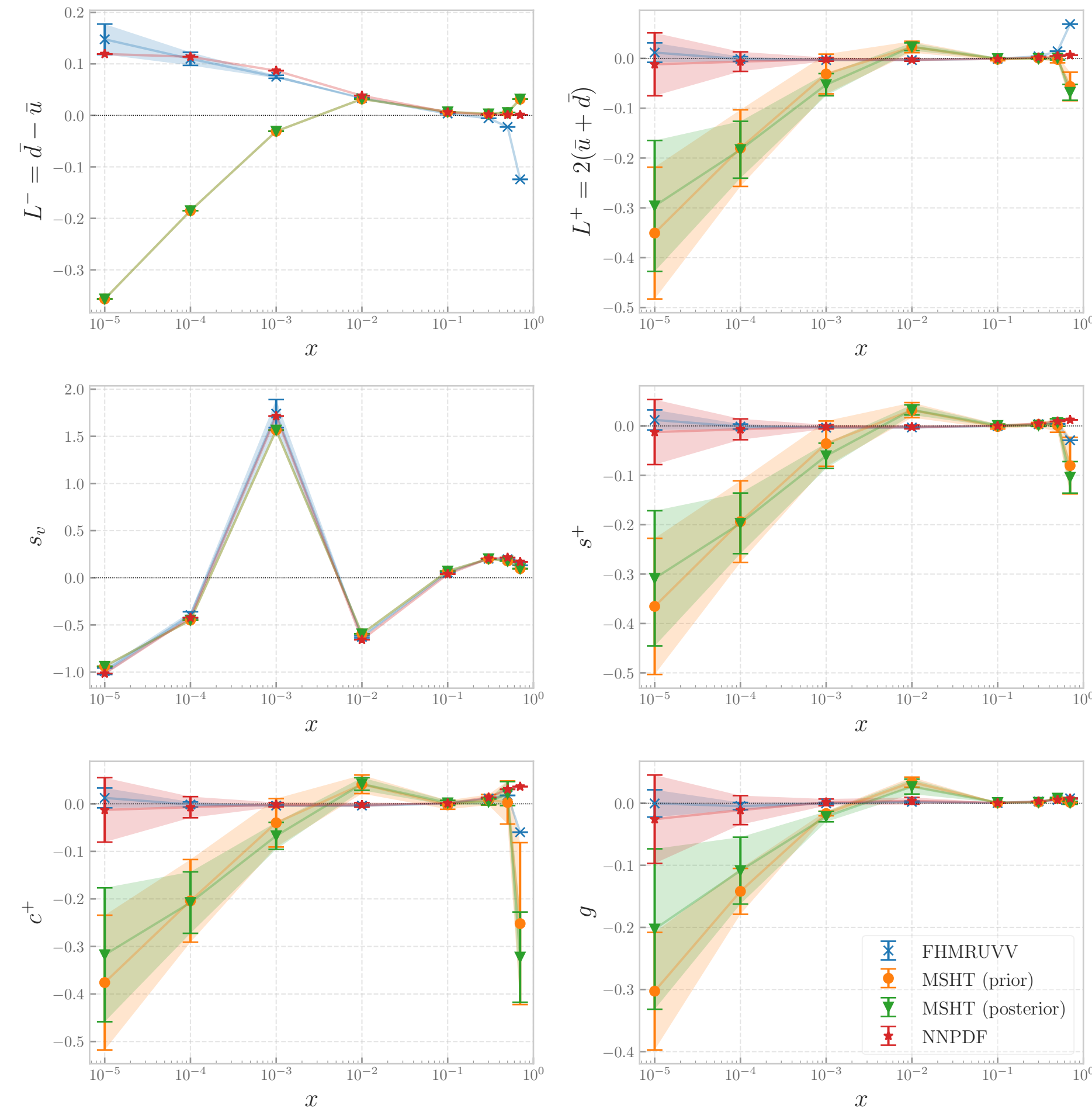
Conclusions



Approximated N3LO PDFs

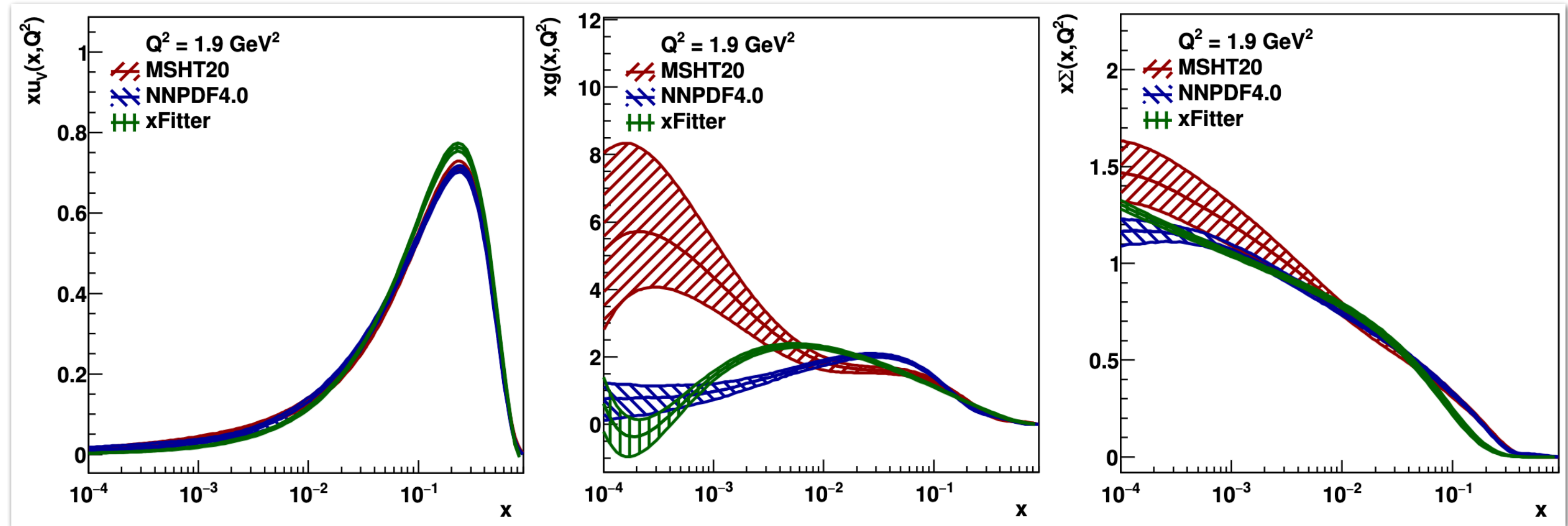
At the moment both the MSHT and NNPDF collaboration have performed global N3LO fits, although both of them are already “outdated”. MSHT’s aN3LO was presented in 2022 and NNPDF’s at the beginning of 2024. The CTEQ collaboration is expected to present their aN3LO PDF in the near future, NNPDF will provide a N3LO PDF together with NNPDF4.1 (expected 2027), the xFitter collaboration is also including interfaces to N3LO evolution/coefficients.

In other words, the results in the following slides are a frozen instant of time which do not exploit all information available *now*. On the other hand (the good news is) new and more accurate information seems to be largely compatible with previous approximated results.



Relative difference of aN3LO wrt NNLO for the different approximations of the evolution for a fixed PDF. Compares MSHT and NNPDF with the theoretical FHMURVV*. Plot from hep-ph/2406.16188

Comparison between xFitter (preliminary, DIS only), NNPDF4.0 and MSHT20 at aN3LO. Slide shown by F. Giuli at DIS Bologna 2026.

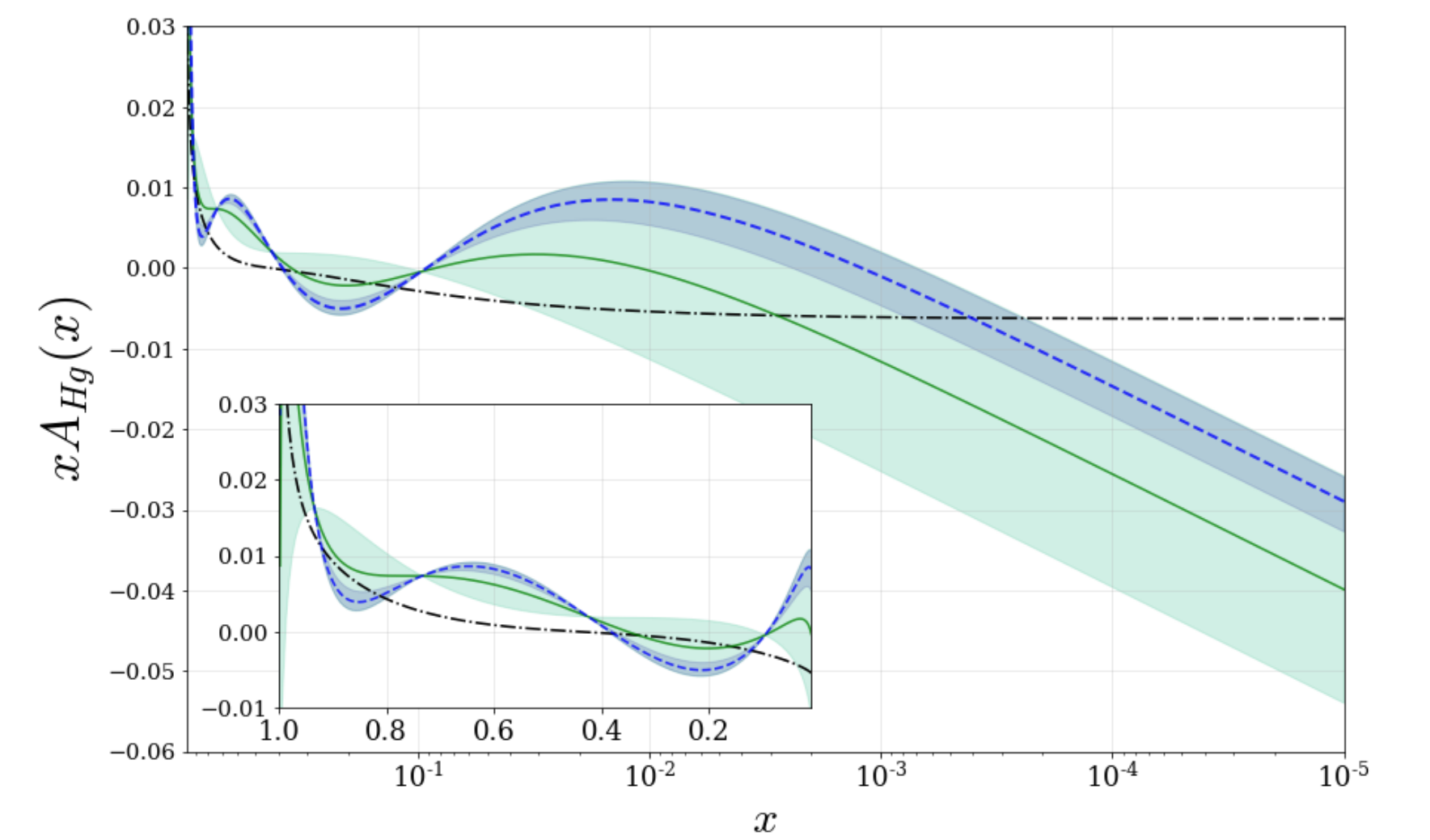
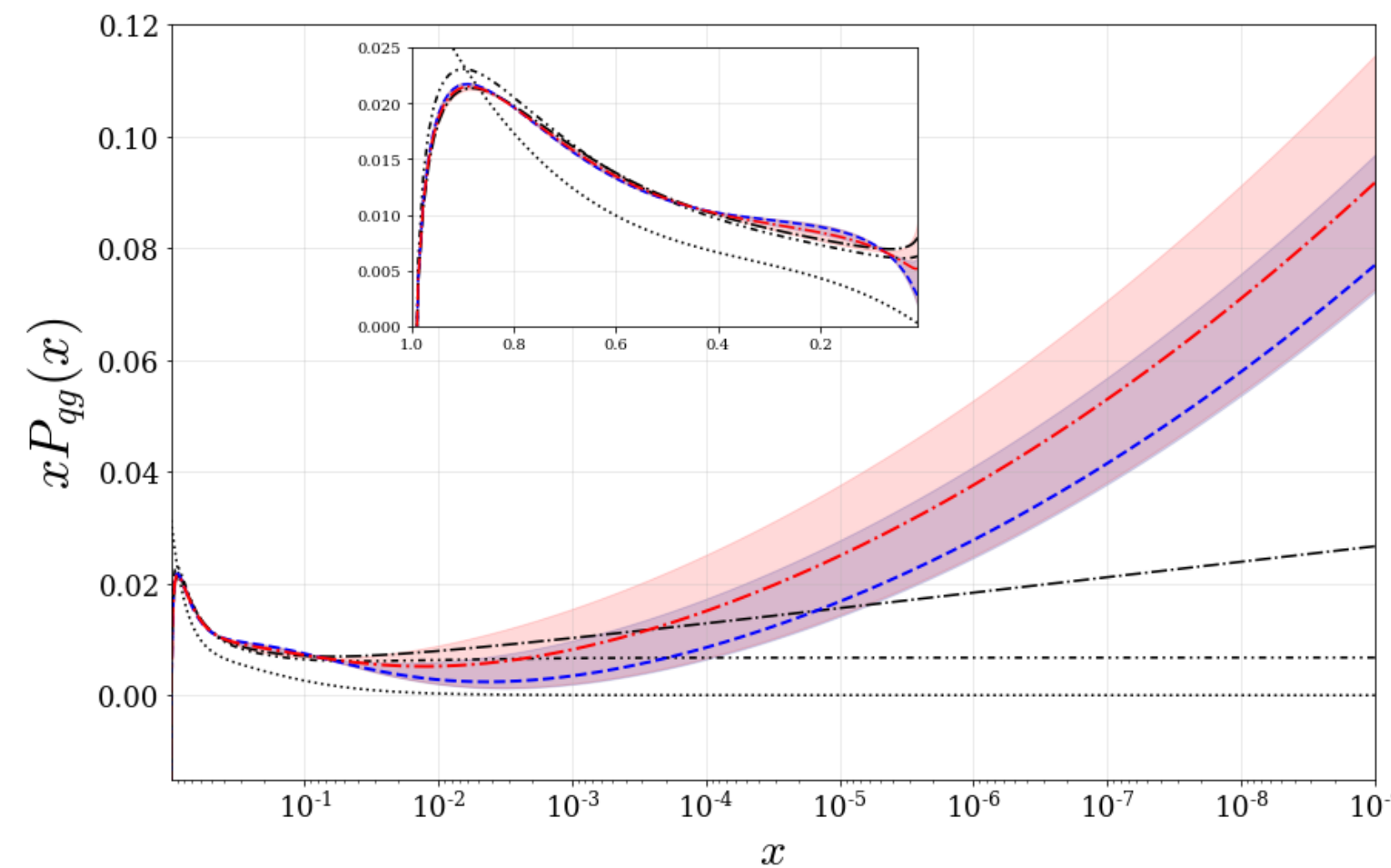
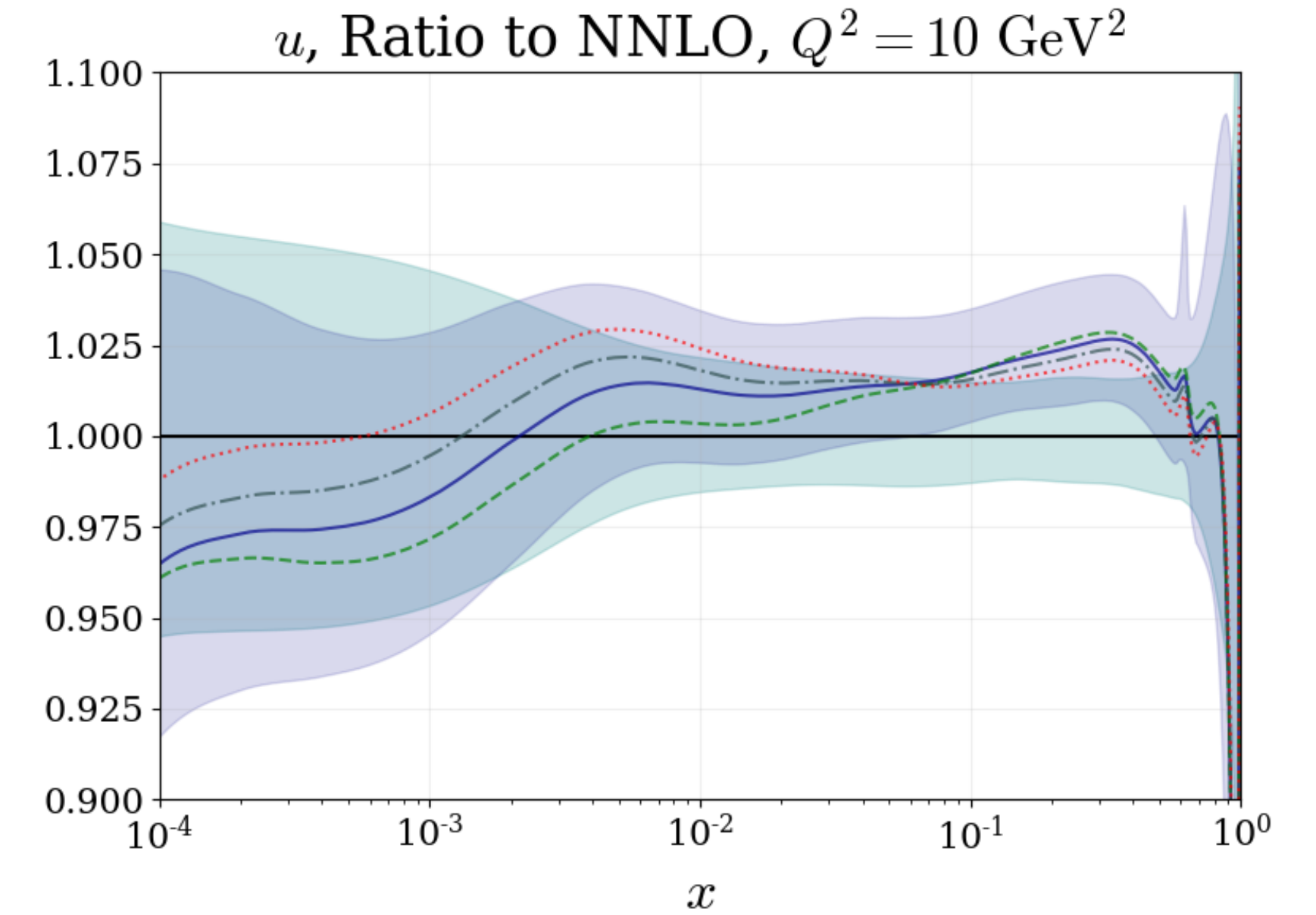
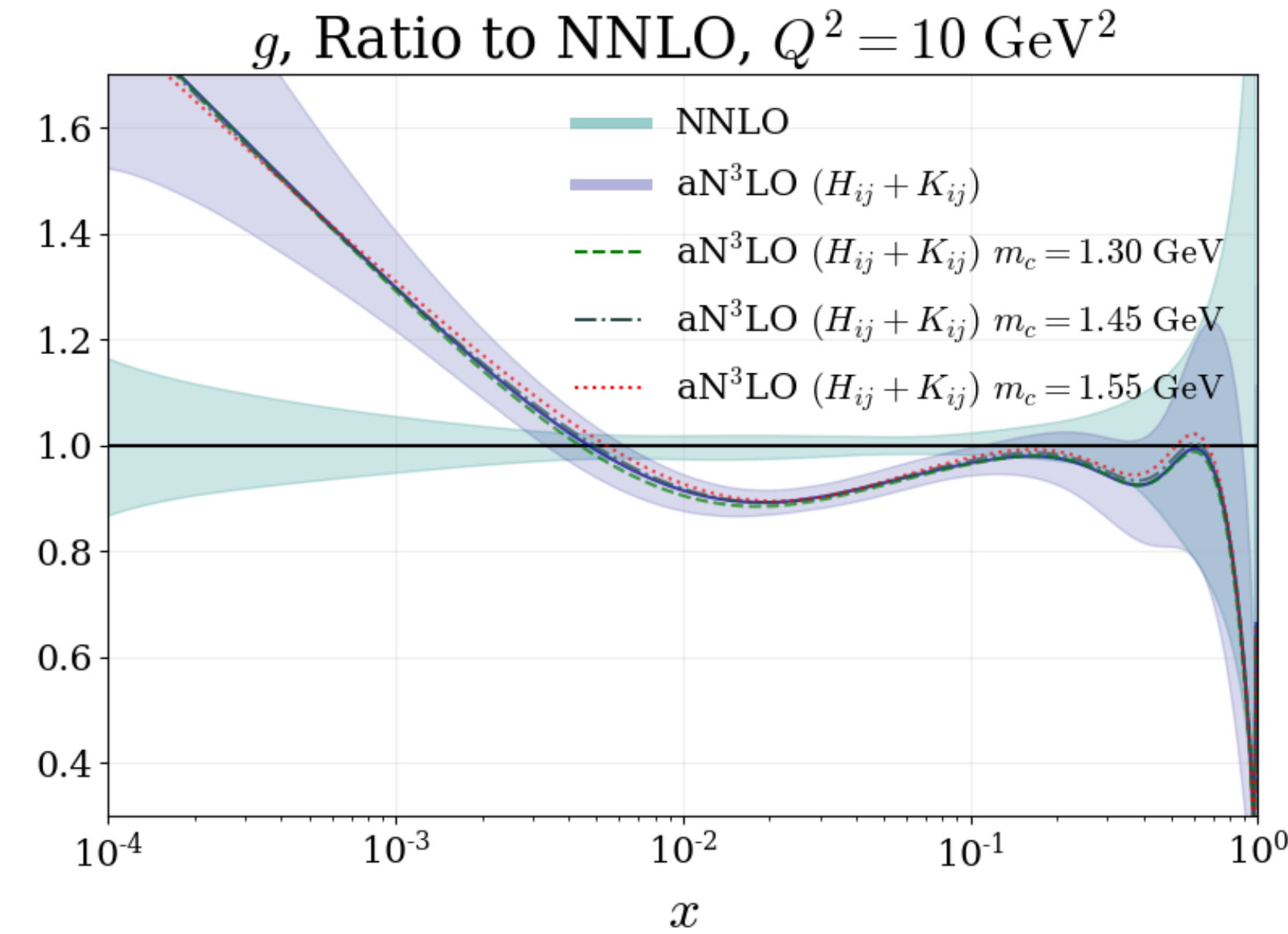


*Released aN3LO from NNPDF used an in-house implementation and the differences here reflect that, but current evolution code and future releases use the FHMURVV splitting functions.

MSHT20 aN3LO

McGowan, Cridge, Harland-Lang, Thorne - hep-ph/2207.04739

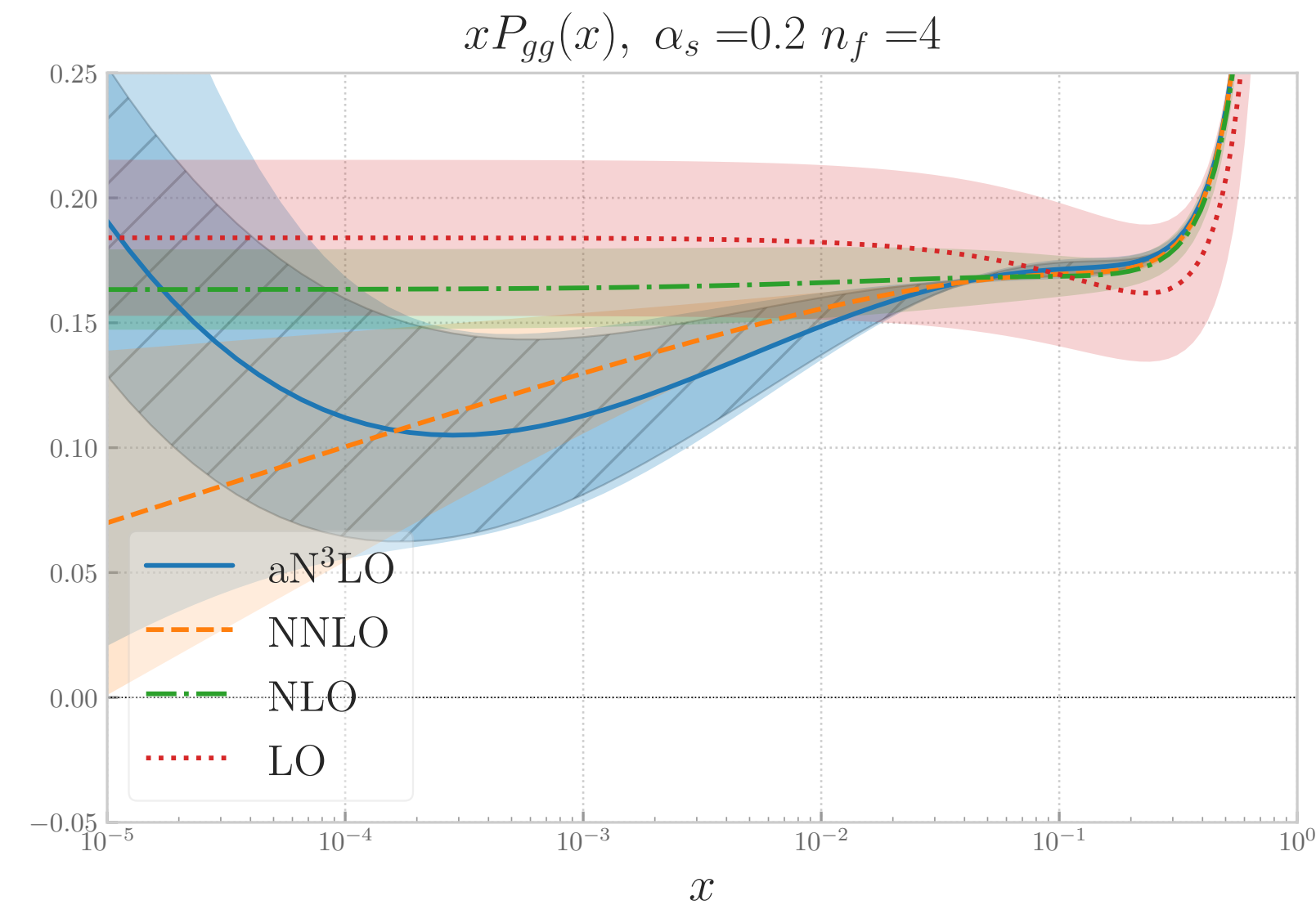
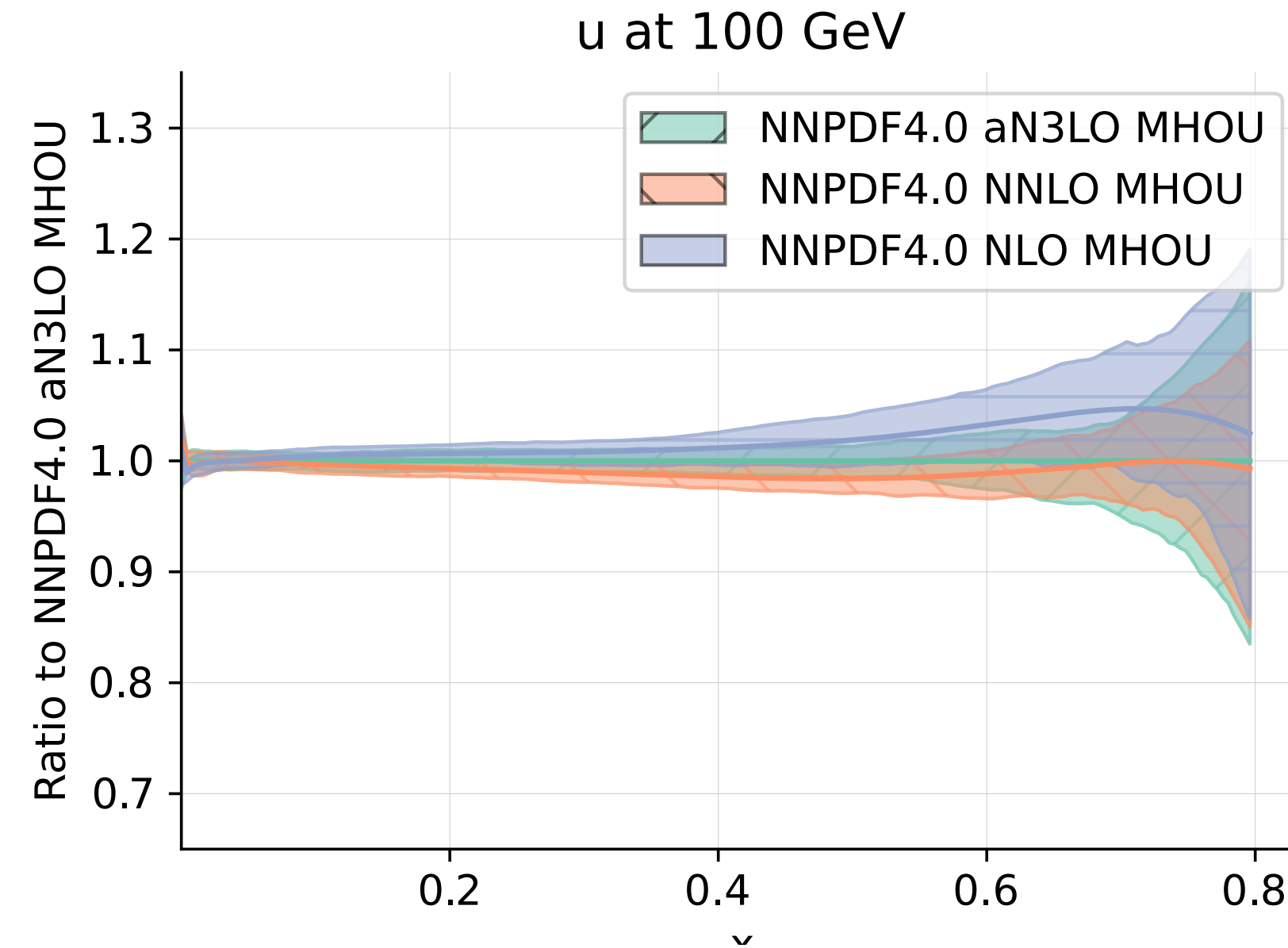
- **MHOU and IHOU:** nuisance parameters fitted to the data
- **Splitting functions and OMEs:** approximated from the small/large- x limits, large- n_f and known Mellin moments
- **Known mellin moments:** at the time, singlet: 4; non-singlet: 8
- **Massive DIS coefficients:** approximated (see hep-ph/1205.5727)
- **Hadronic contributions at N3LO:** parametrized as linear combinations of NNLO and NLO k-factors (also with nuisance parameters)



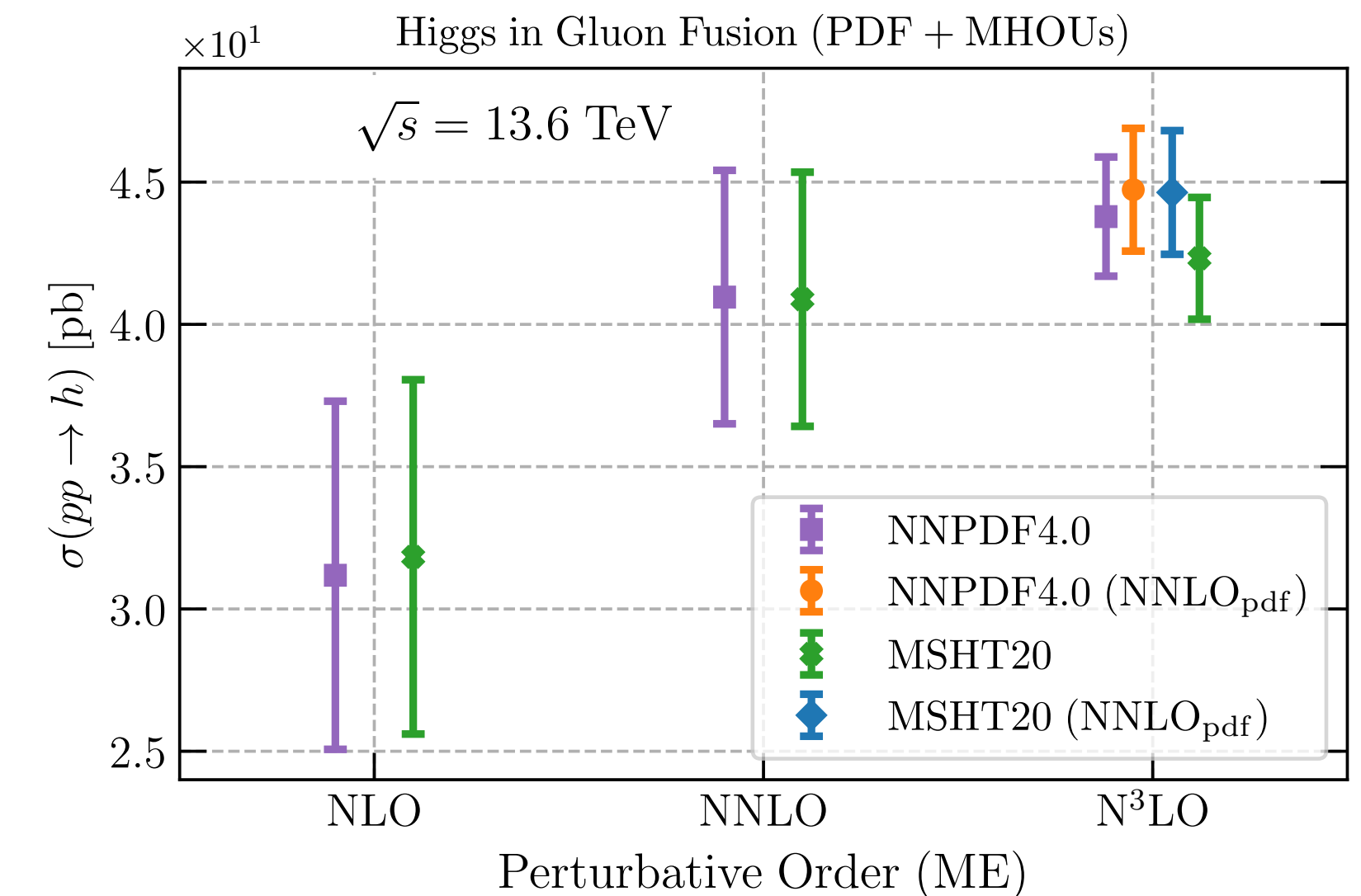
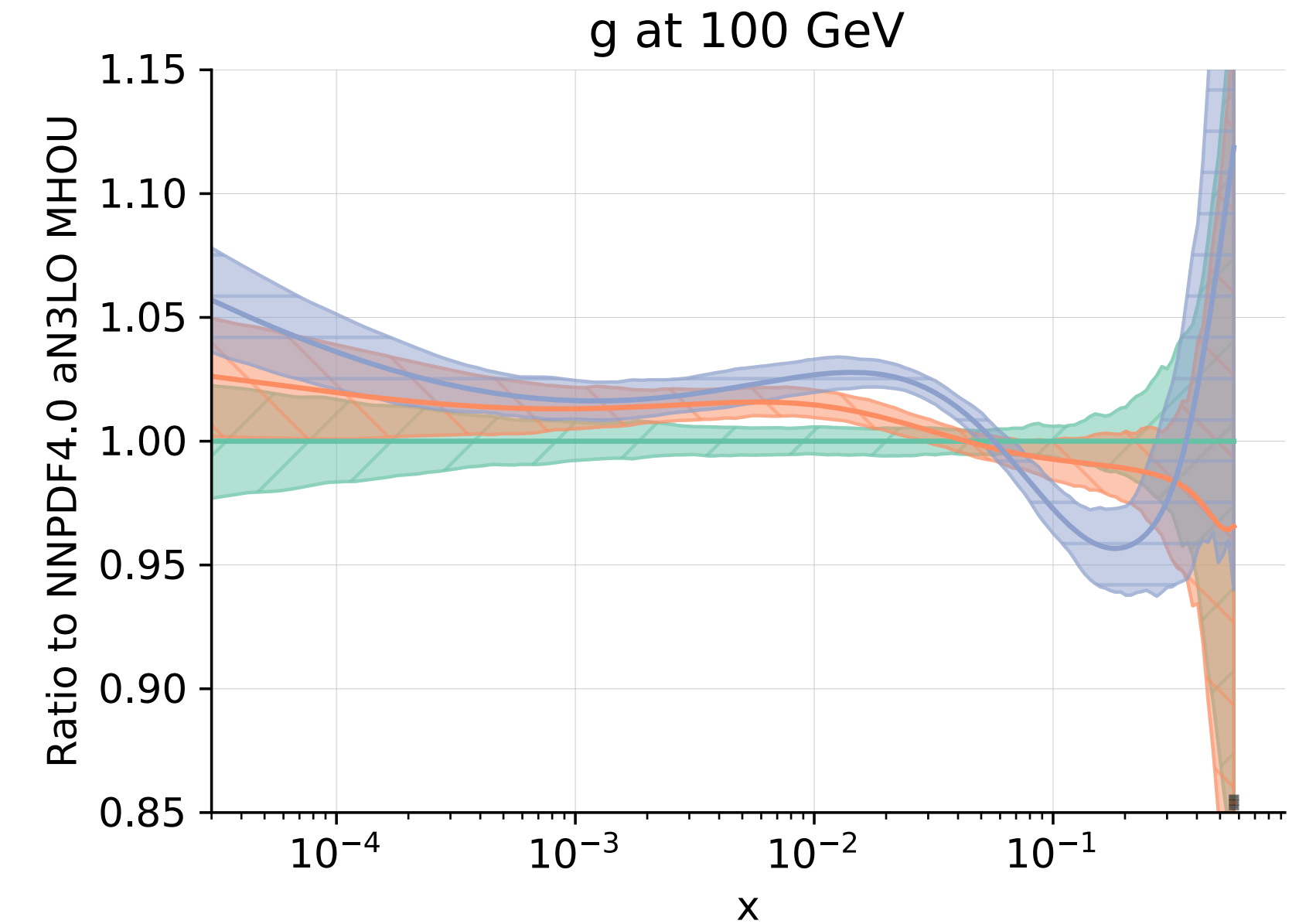
Blue bands: central value and uncertainties of splitting functions and OMEs after the nuisance parameters are varied upon the global fit

NNPDF4.0 aN3LO

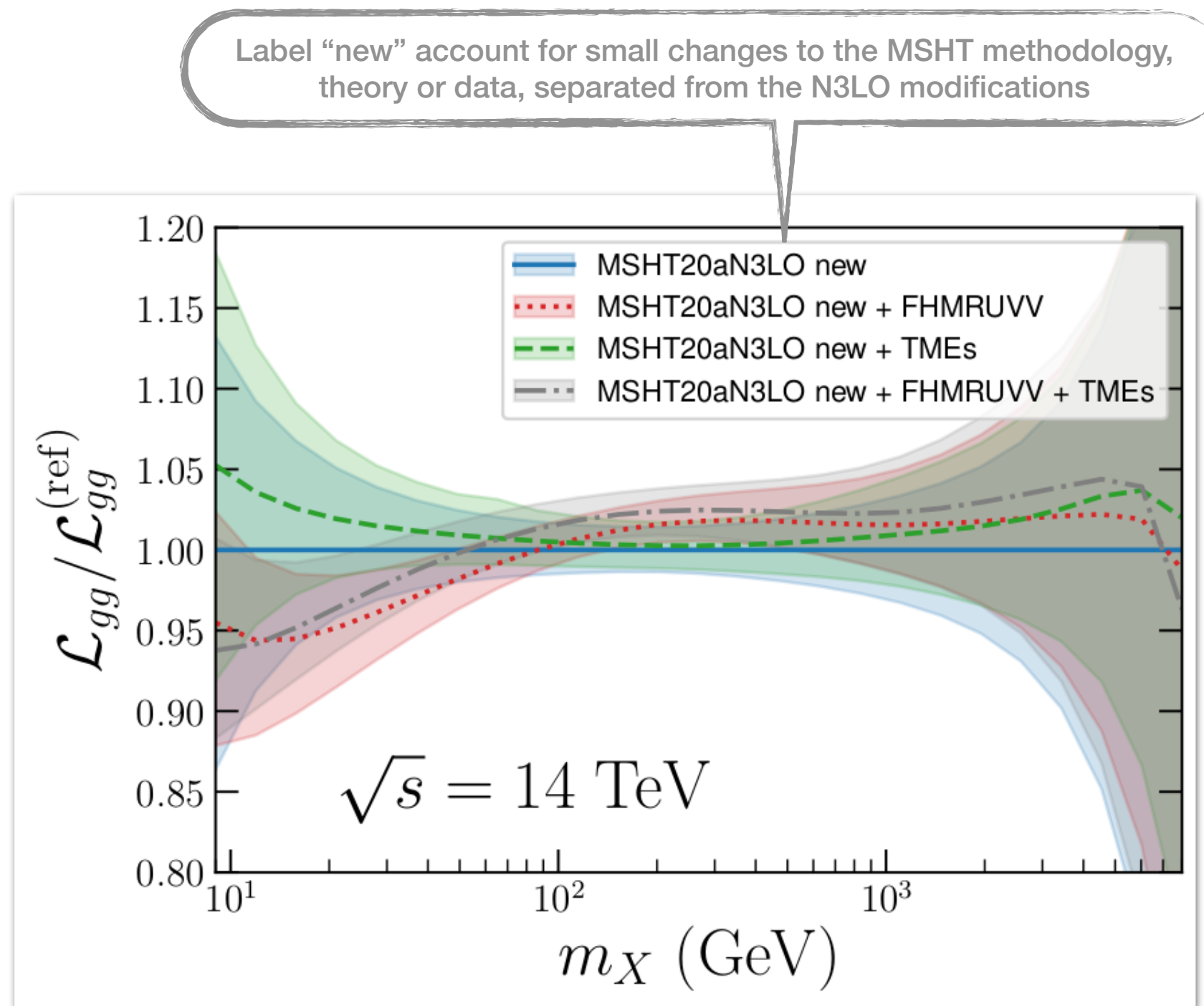
- ▶ **MHOU and IHOU**: theory covariance matrix formalism with scale variations for MHOU and parametrisation variations for IHOU
- ▶ **Splitting functions and OMEs**: approximated from the small/large- x limits, large- n_f and known Mellin moments. OMEs included exactly (for all but AggH)
- ▶ **Known mellin moments**: at the time, singlet: 6; non-singlet: 8
- ▶ **Massive DIS coefficients**: approximated (see hep-ph/1205.5727)
- ▶ **Hadronic contributions at N3LO**: estimated via MHOU



uncertainty bands < N3LO include scale variations

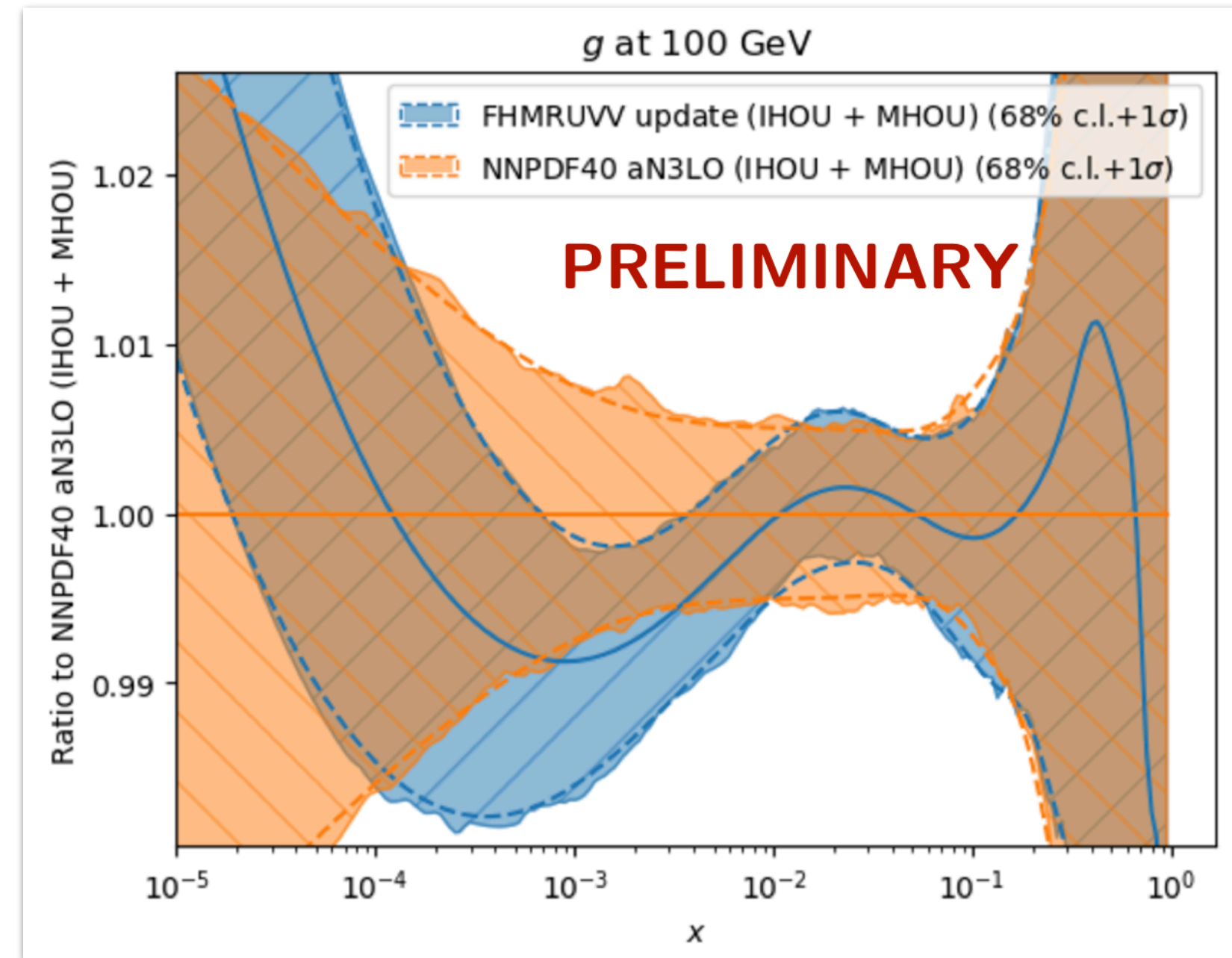


Recent updates



Talk by T. Cridge at PDF4LHC June 2026

MSHT has implemented recently available Mellin moments + exact computation of OMEs

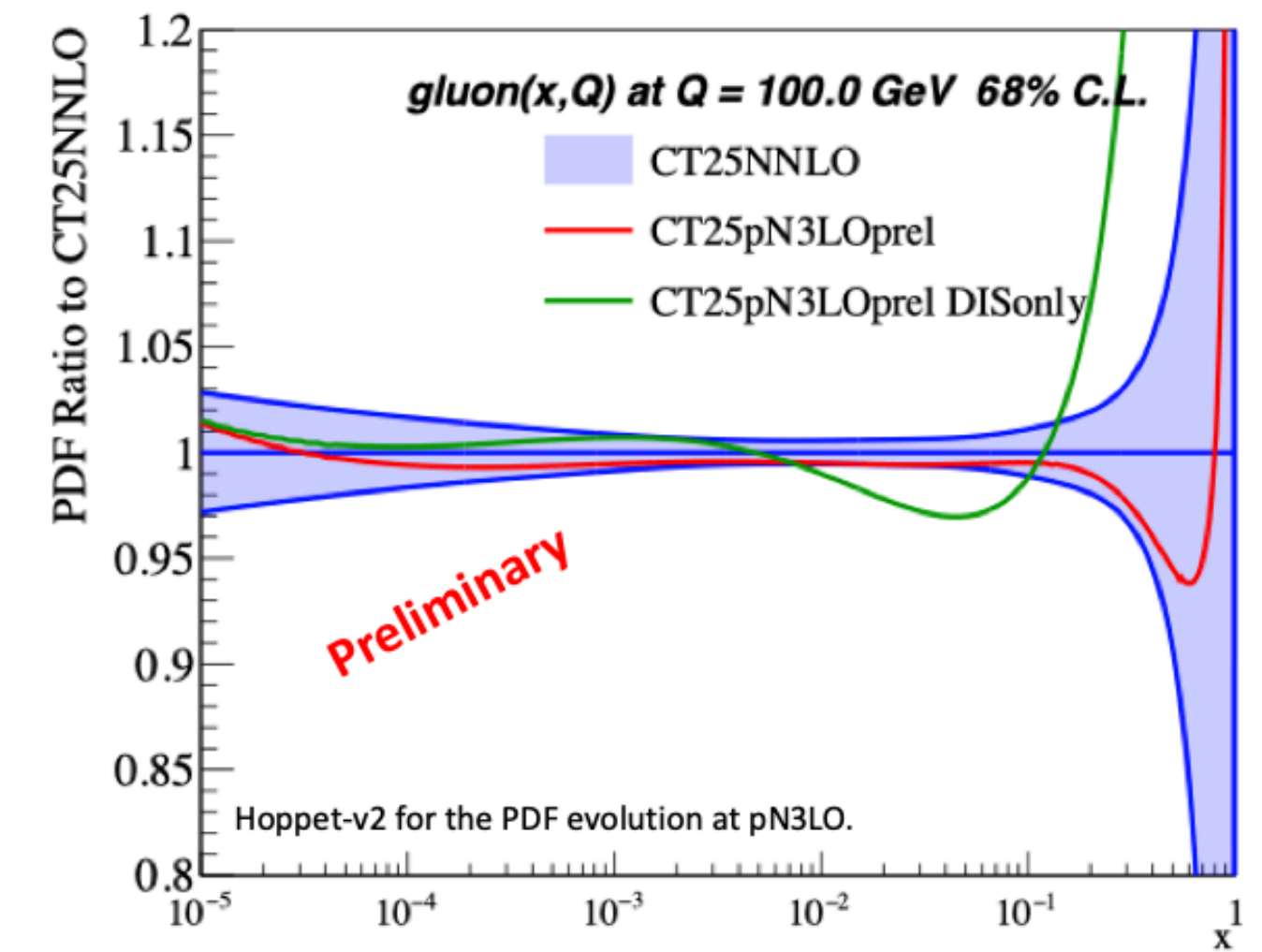


Talk by G. Magni at PDF4LHC '24

NNPDF has included new Mellin moments and moved to use directly the splitting functions by FHMRUVV

PDF4LHC 2026 will be held at CERN, November 23rd and 24th. More updates expected there.

Talk by M. Guzzi at PDF4LHC June 2026



Preliminary CT25 results recently shown at a PDF4LHC working meeting

Combination and benchmark

NNPDF and MSHT collaborations
hep-ph/2411.05373

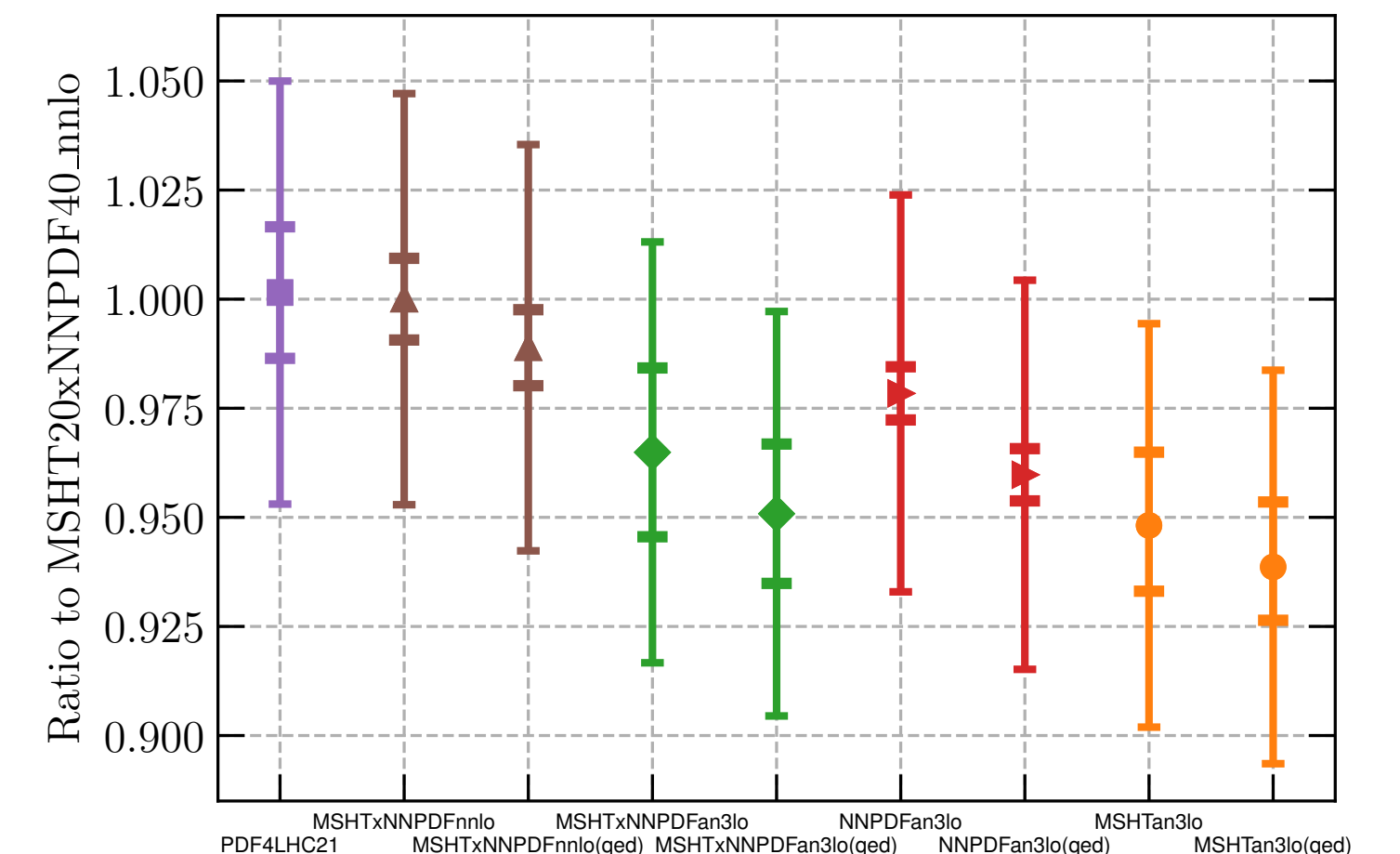
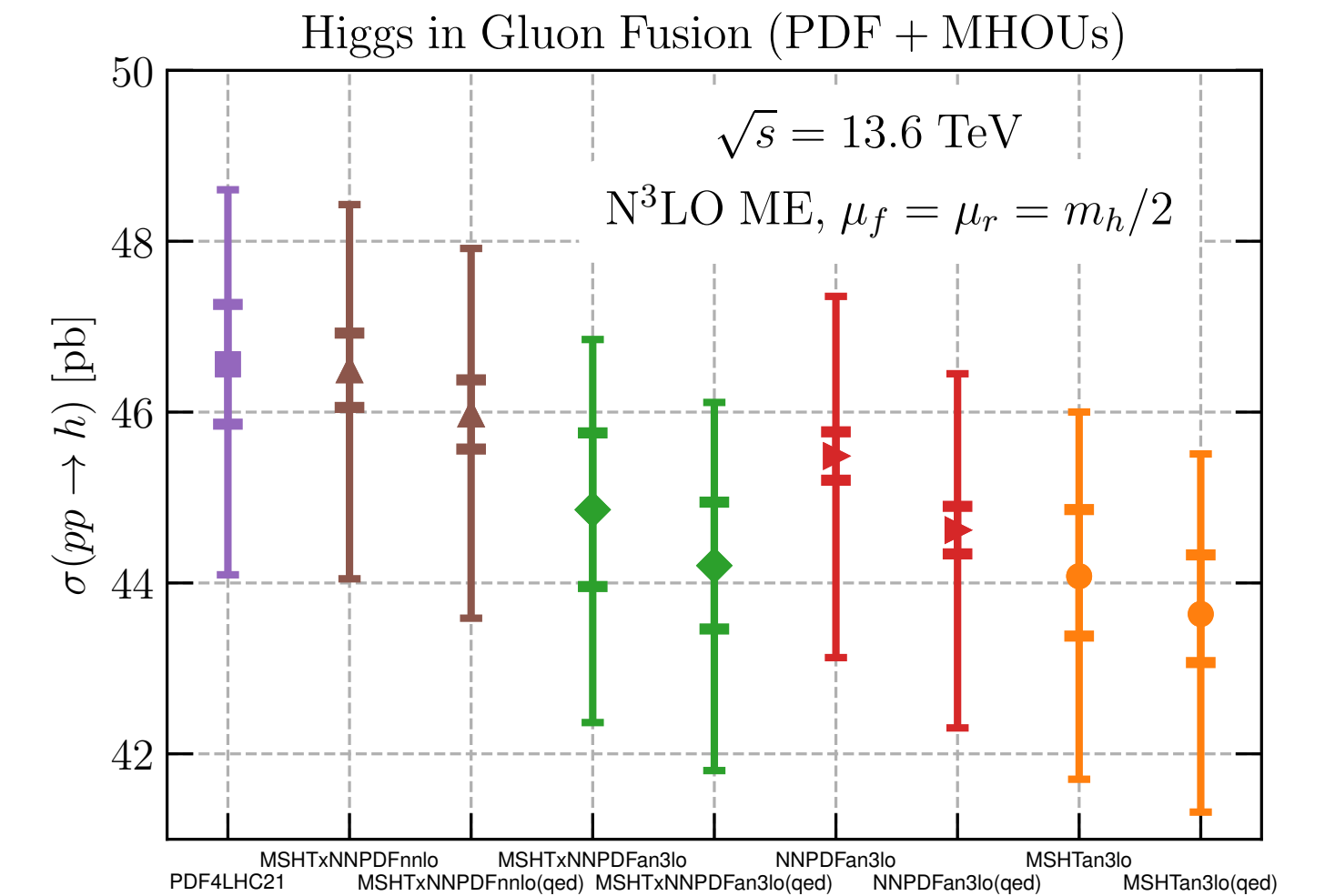
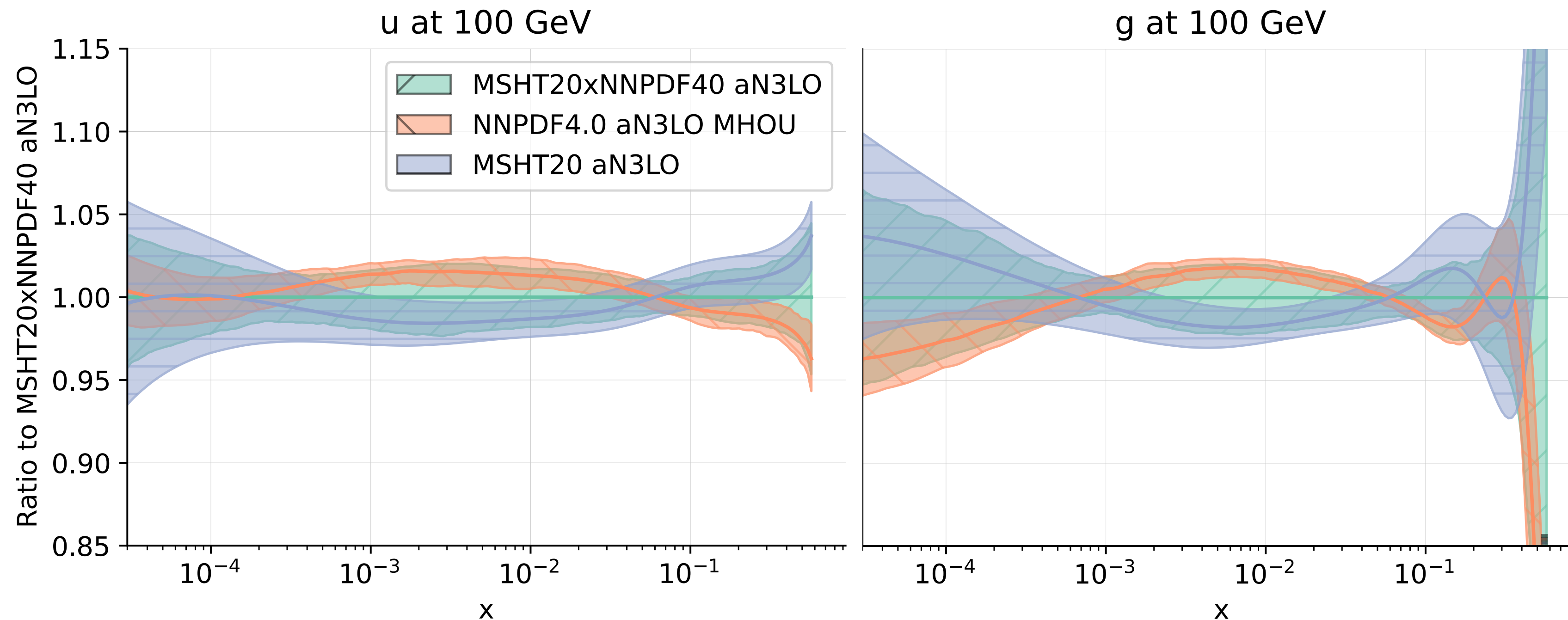
The MSHT and NNPDF collaborations have produced a PDF4LHC-like combination of replicas of both aN3LO results.

The explicit goal is to combine the existing results, which means 1) inconsistencies due to different evolution approximation being used 2) safe only at higher scales, relevant for LHC phenomenology.



Central value: unweighted average

Uncertainty: accounts for both uncertainties and central value differences



Comparison between aN3LO results with different methodologies, data and theory

E.g., the recent combination of MSHT and NNPDF: [hep-ph/2411.05373](https://arxiv.org/abs/hep-ph/2411.05373)

In the combination of MSHT and NNPDF aN3LO PDFs many different effects are confounded:

1. Different methodologies
2. Slightly different choices of datasets
3. Different theory settings (most notably charm fitted or not)

Can we separate the methodological effects from the physics choices?

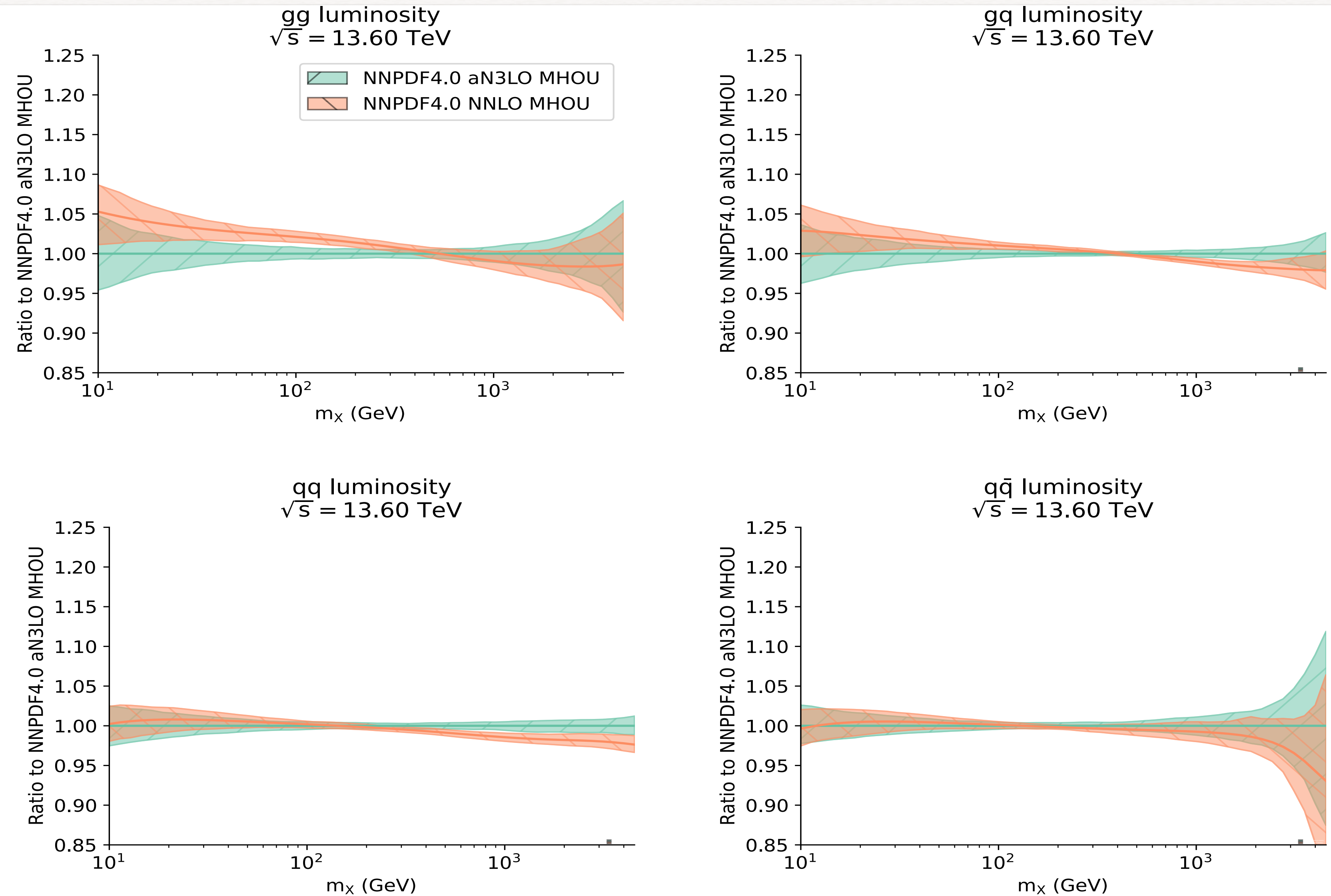


Figure 4 from [hep-ph/2411.05373](https://arxiv.org/abs/hep-ph/2411.05373)

Comparison between aN3LO results: same data and theory, isolating the effect of the methodology

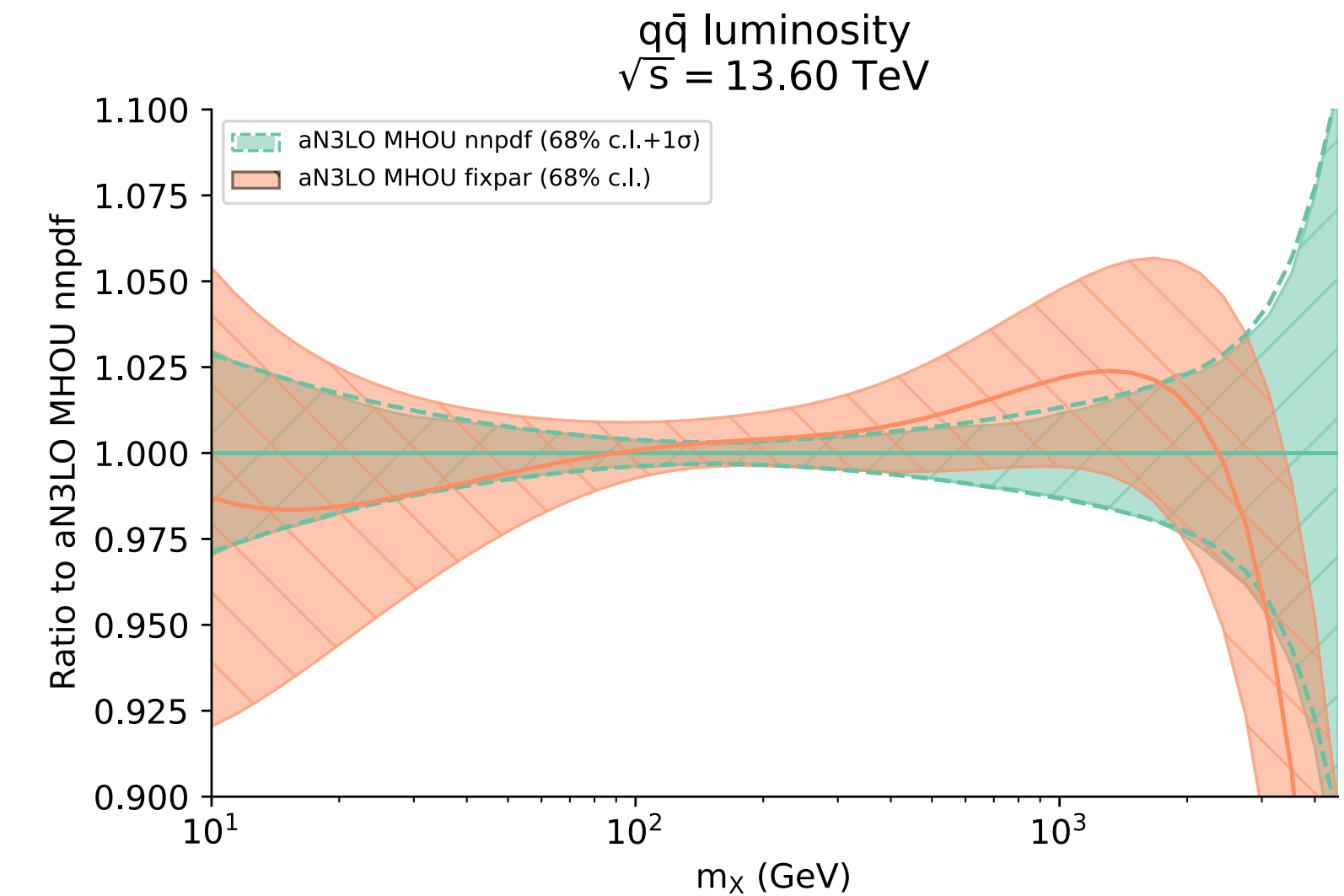
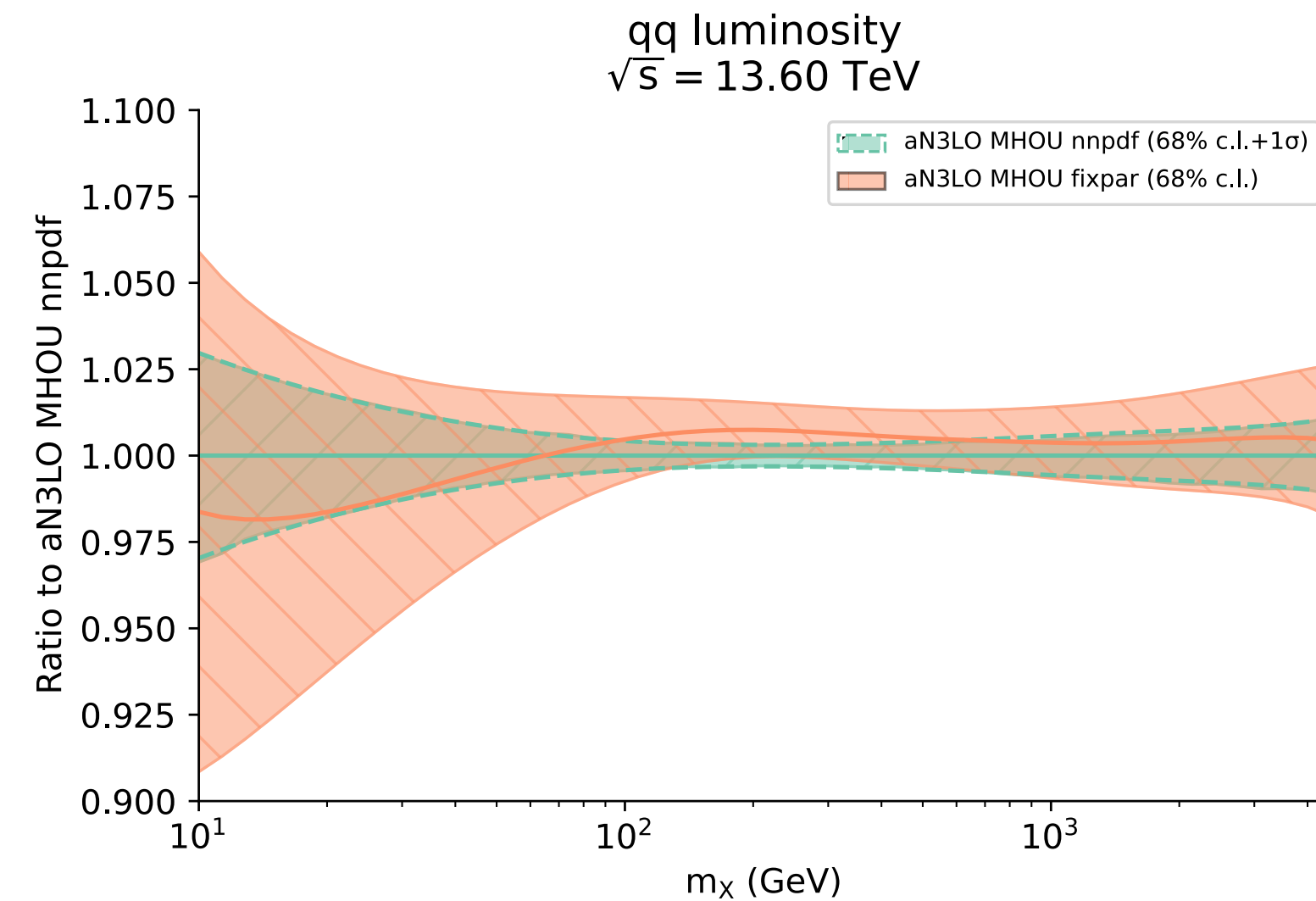
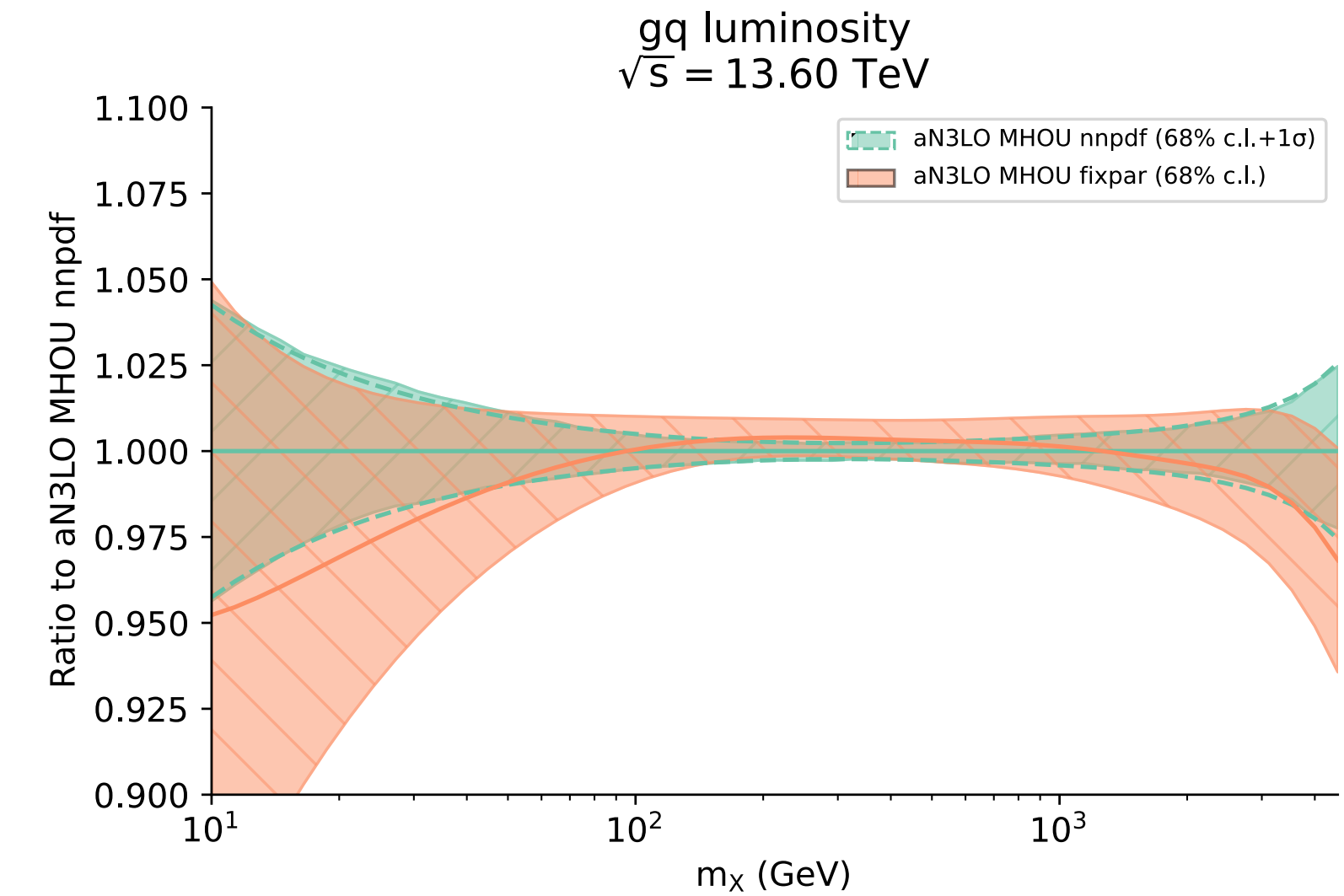
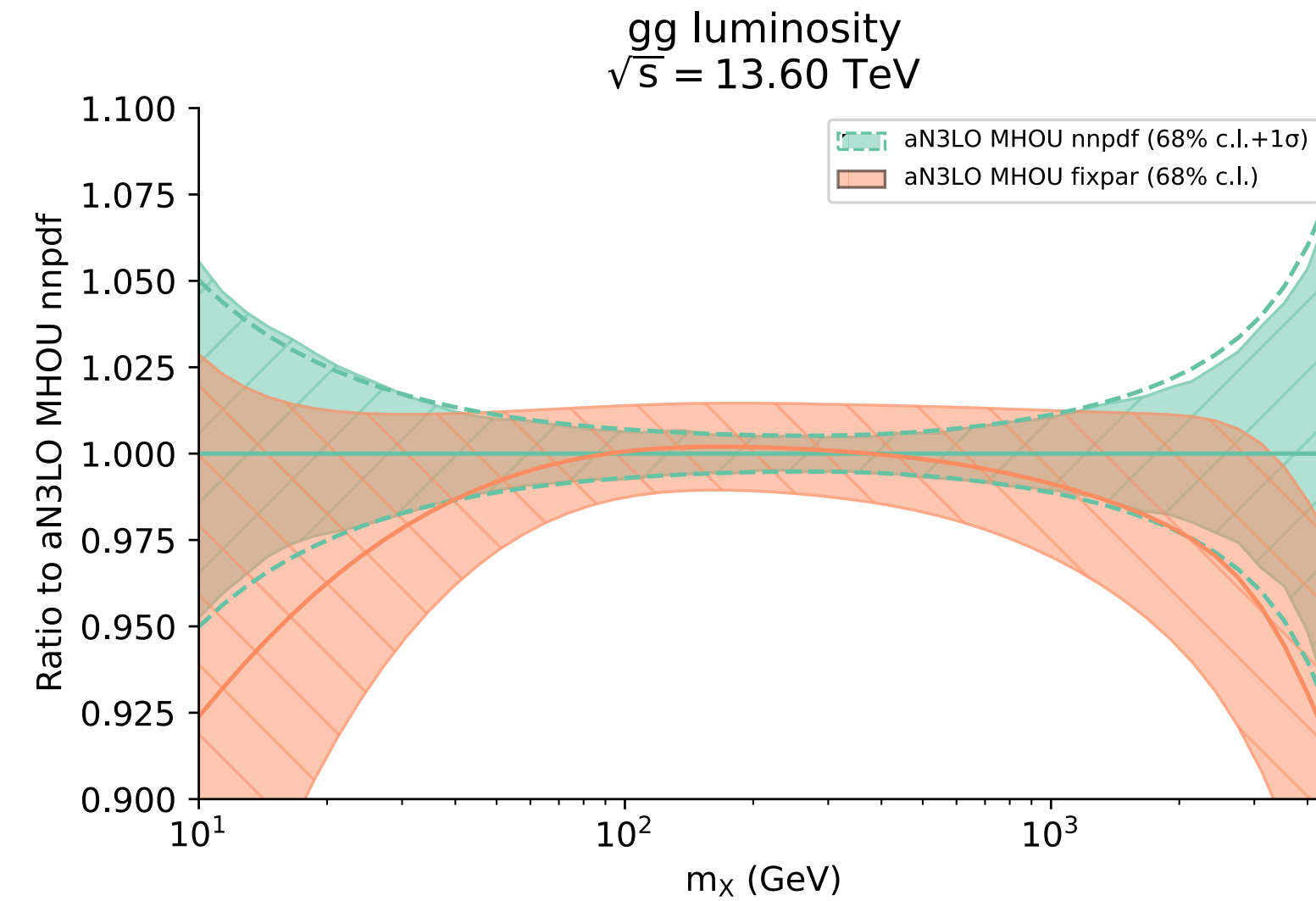


Assessing the impact of fitting methodology at aN3LO with FPDF: an open source tool for extracting Parton distribution functions in the Hessian approach

JCM, T. Giani, L. Harland-Lang - hep-ph/2602.07118

FPDF allows us to incorporate the same data and theory for two completely different methodologies.

Within this framework, it is possible to see that the **bulk of the differences** is driven by data and theory, and **not methodologies**.

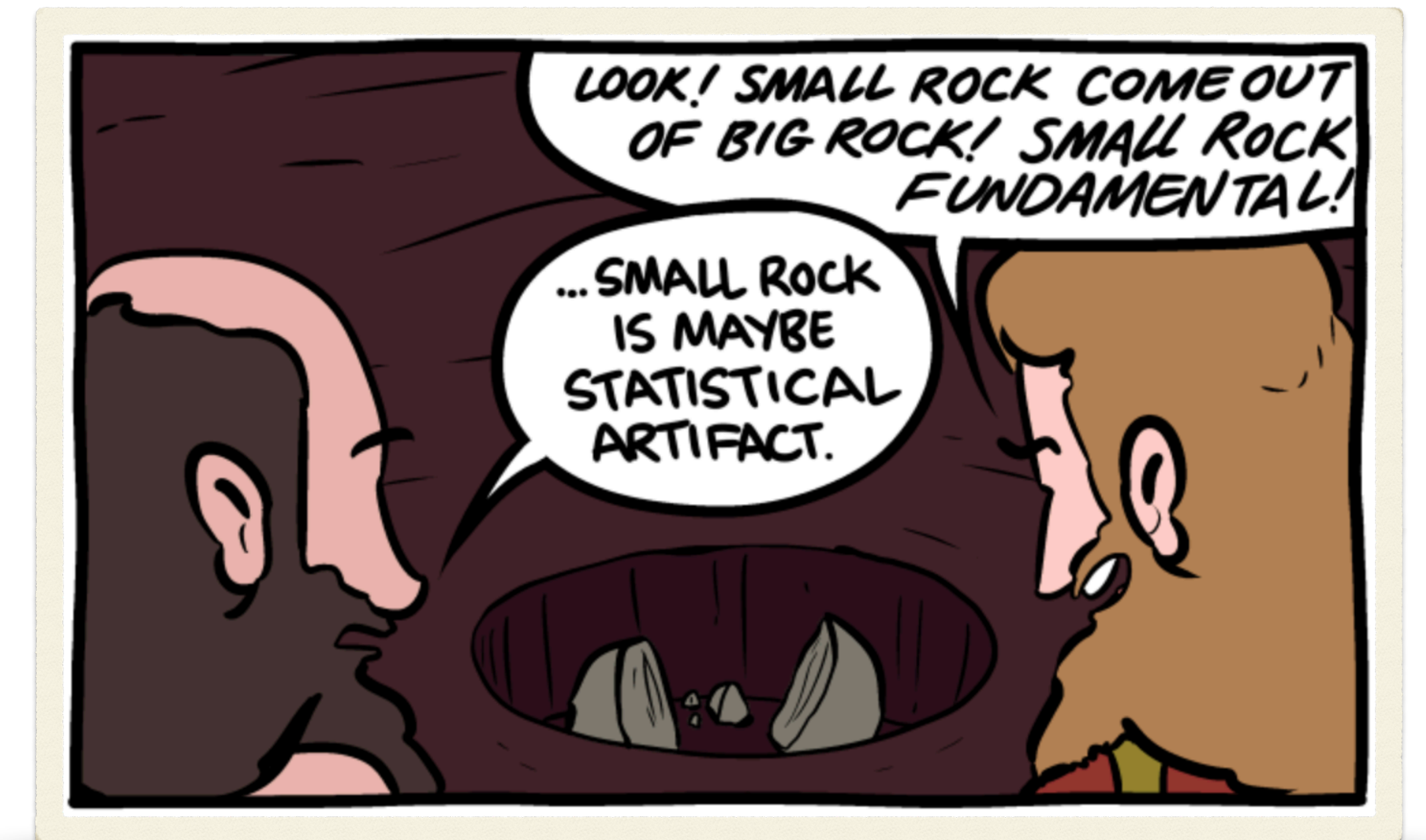


Introduction and motivation

The N3LO revolution?

Approximated N3LO PDFs

Conclusions



Conclusions

Thanks to the latest developments theoretical calculations, we are truly in the N3LO era of PDFs.

- The determination of Parton Distribution Function is one of the most important ingredients of predictions for highly energetic hadron collisions.
- The NNLO label is now truly justified, with no approximations left in the PDF, the work is now to fully justify removing the “a” from aN3LO.
- Theoretical advances in splitting function and massive coefficients for DIS make the leftover uncertainty very small.
- N3LO predictions for hard coefficients are right now the biggest bottleneck towards the exact N3LO label!
- Benchmarks ongoing between collaborations.

Backup

The NNPDF ongoing implementation

slides by F. Hekhorn at PDF4LHC meeting

New FONLL features [JHEP10.004]



- ▶ we need $c^{(3c)}(Q^2)$, $c^{(3c\cap 4)}(Q^2)$, $c^{(4)}(Q^2)$ anyway and EKO can compute $f^{(3)}(Q^2)$
 - ▶ the connection between $f^{(3)}(Q^2)$ and $f^{(4)}(Q^2)$ becomes trivial
 - ▶ can be implemented at any order
 - ▶ can combine N^jLO with N^kLL with any j, k
 - ▶ matching point between schemes can be arbitrary
 - ▶ intrinsic charm falls natural into place
 - ▶ consider simultaneous charm and bottom mass effects
-
- ▶ exact results from Blümlein et al. are implemented
 - ▶ “official” implementation (**libome**) is not available in Mellin space (yet) for EKO
 - ▶ not yet implemented:
 - ▶ [2512.13508]: $A_{Qq,ps,sea}$; expected impact: negligible; “It turns out that the distributions $(\Delta)f_Q(x, Q^2) - (\Delta)f_{\bar{Q}}(x, Q^2)$ are non-vanishing but very small and require huge luminosities to be measured”

FONLL with coexisting flavor number PDFs [JHEP10.004]

Combine fixed-flavor and resummed predictions:

$$\sigma^{\text{FONLL}}(Q^2) = \sigma^{(3c)}(Q^2) + \sigma^{(4)}(Q^2) - \sigma^{(3c\cap 4)}(Q^2)$$

with

$$\sigma^{(3c)}(Q^2) = c^{(3c)}(Q^2) \otimes f^{(3)}(Q^2)$$

$$\sigma^{(4)}(Q^2) = c^{(4)}(Q^2) \otimes f^{(4)}(Q^2)$$

$$\sigma^{(3c\cap 4)}(Q^2) = c^{(3c\cap 4)}(Q^2) \otimes f^{(3)}(Q^2)$$

old construction [NPB834.116] uses a single PDF:

$$\sigma^{\text{FONLL}}(Q^2) = c^{\text{FONLL}}(Q^2) \otimes f^{(4)}(Q^2)$$

FONLL is consistent with other GM-VFNS at NLO ✓

- ▶ FHMPRUVV splitting functions are implemented and default (in-house parametrization [EPJC84.659] is deprecated)
- ▶ not yet implemented:
 - ▶ [2512.10783]: small update for P_{gq} ; expected impact: negligible; “Our new results are well inside the error bands, reinforcing that these approximations can be safely employed in analyses of parton distributions and hard-scattering processes.”
 - ▶ [2604.09534]: exact NS; expected impact: small; “With the complete results obtained here, we show that these approximations are reliable within their quoted uncertainty band.”

The NNPDF ongoing implementation

slides by F. Hekhorn at PDF4LHC meeting

NNPDF is open source! [EPJC81.958]

N3LO ingredients of NNPDF are based on the implementations in EKO and Yadism of the splitting functions, and massive and massless DIS coefficients.



<https://github.com/NNPDF/pineappl>

Fast interpolation grid library



NNPDF
Neural Network Parton Distribution Functions

18 followers <http://nnpdf.science/> nnpdf@mi.infn.it

Overview Repositories 24 Projects Packages

Popular repositories

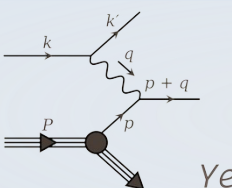
<https://github.com/NNPDF/>



<https://github.com/NNPDF/eko>

DGLAP solver

Evolution Kernel Operators



Yadism
Yet Another DIS Module

<https://github.com/NNPDF/yadism>

DIS observables



All tests NNPDF passing Documentation passing Test CommonData passing

DOI: [10.5281/zenodo.15691478](https://doi.org/10.5281/zenodo.15691478)

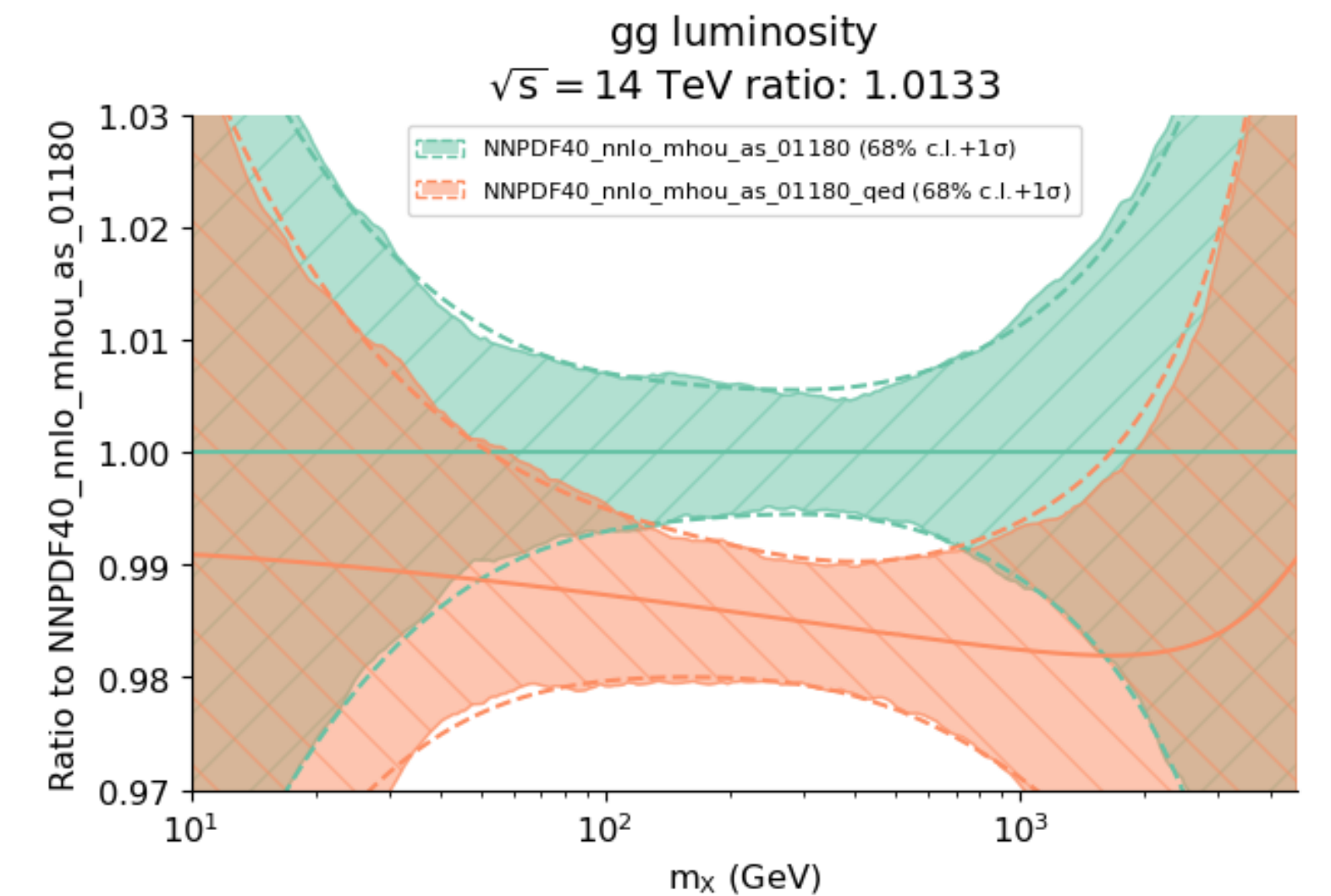
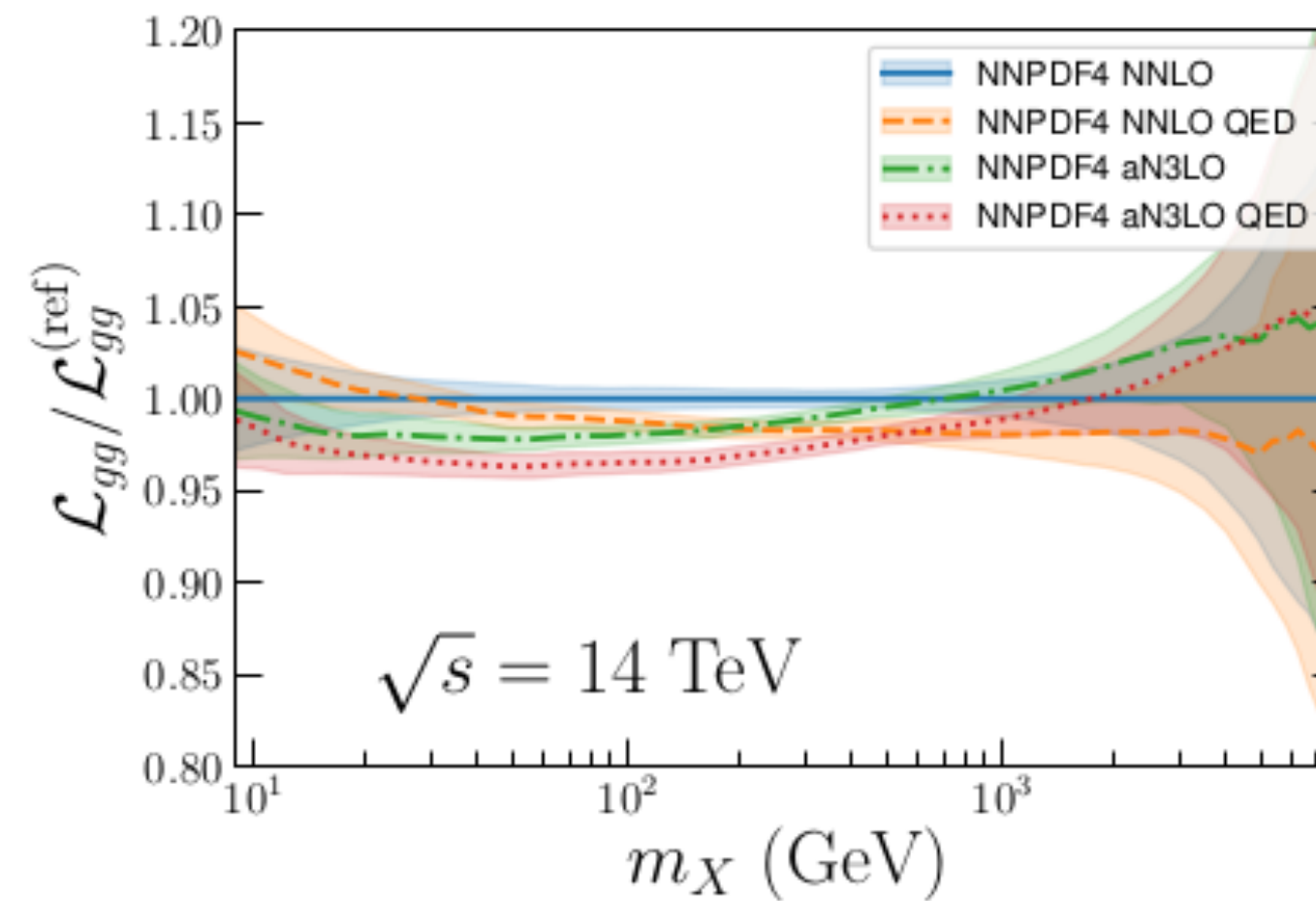
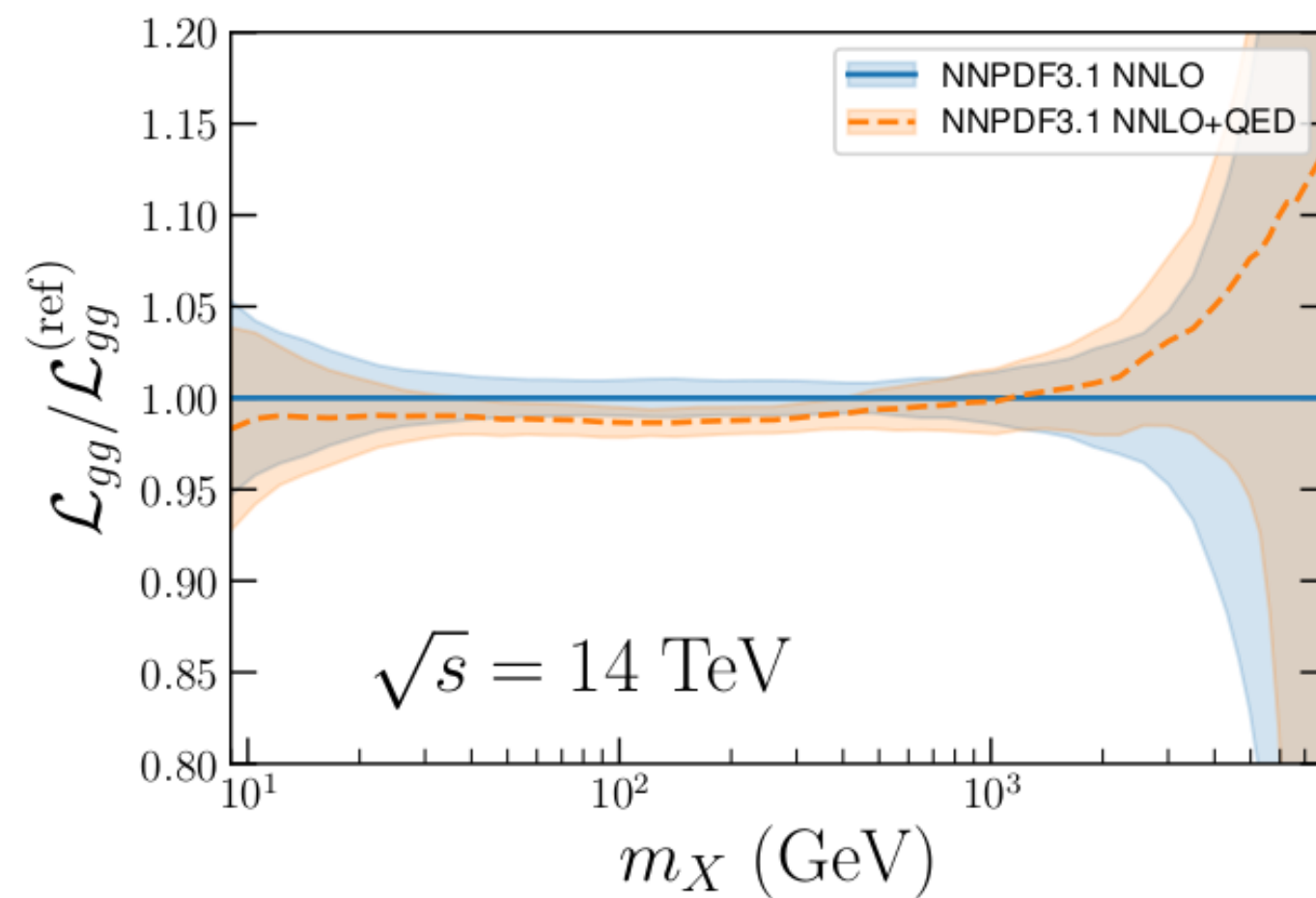
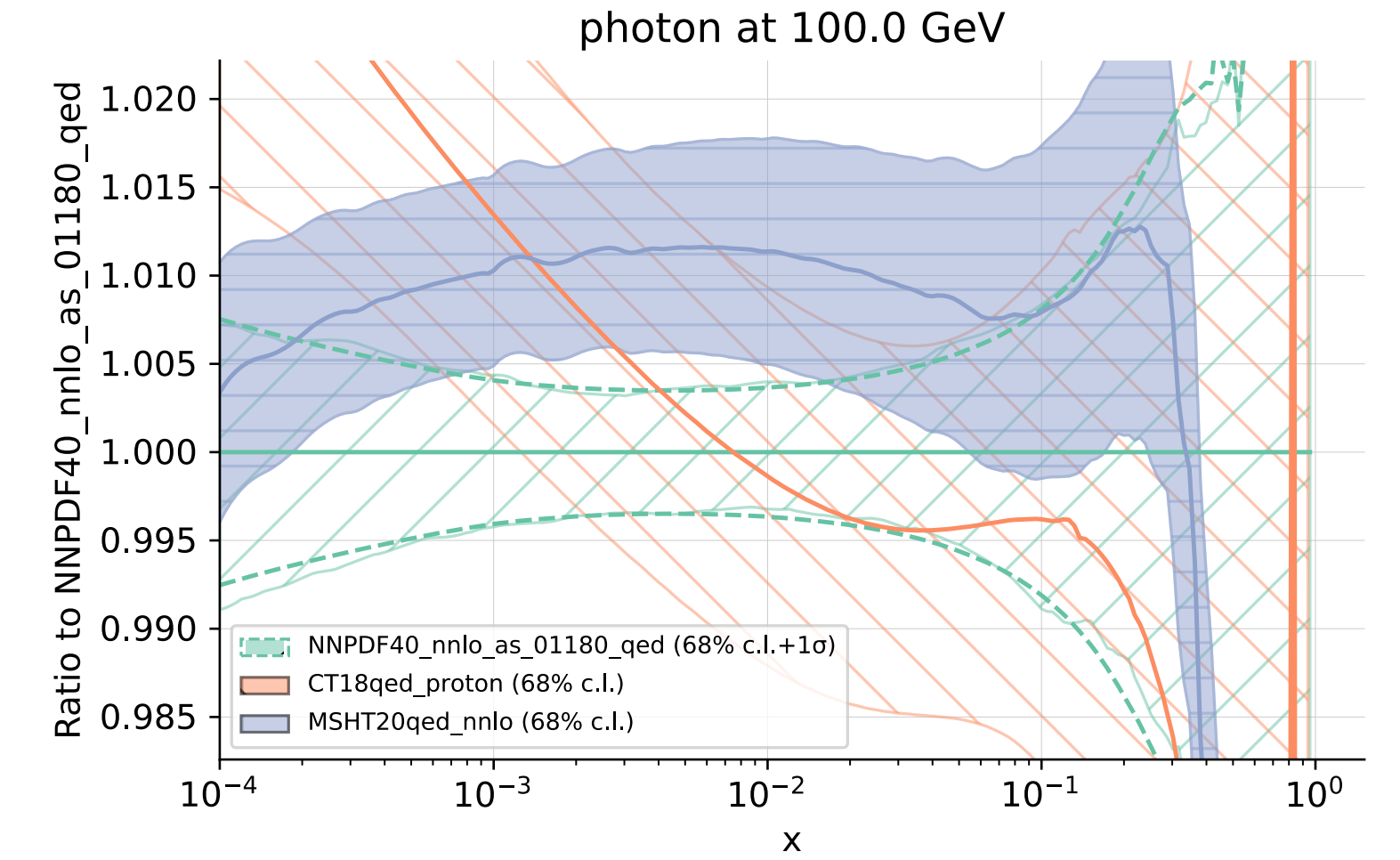
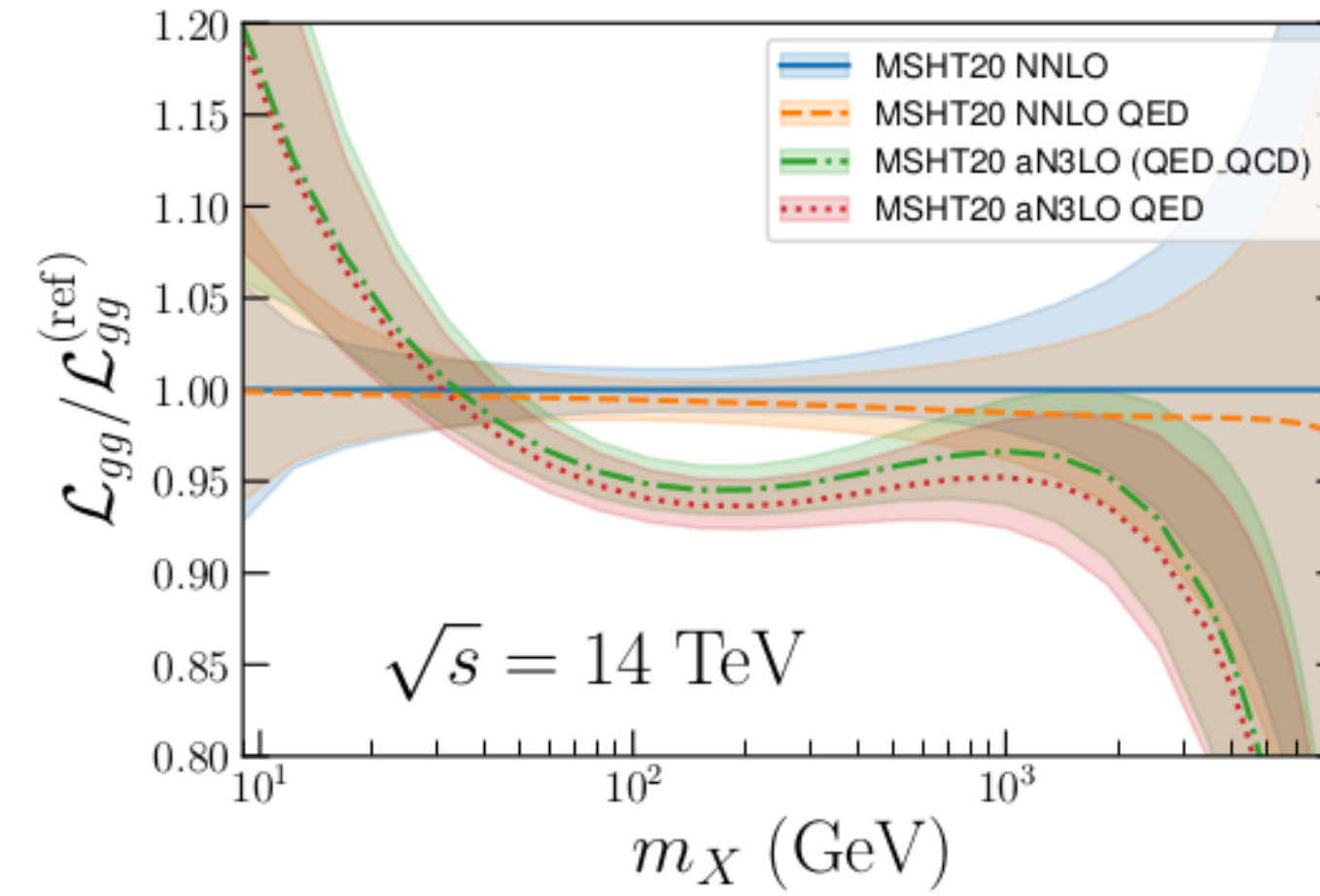
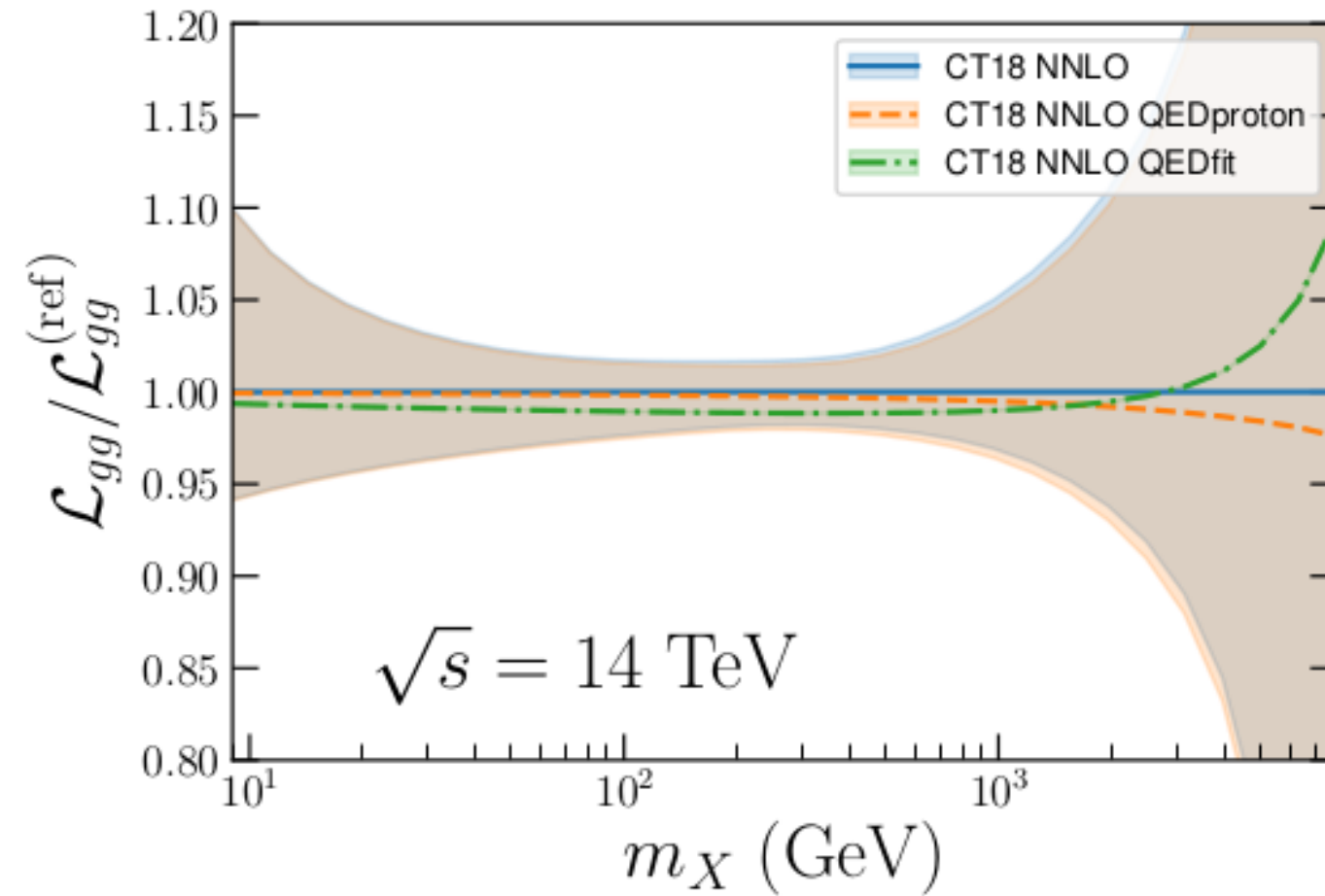
NNPDF: An open-source machine learning framework for global analyses of parton distributions

<https://github.com/NNPDF/nnpdf>

/nnpdf_data: all experimental data
/n3fit: core fitting methodology
/validphys2: (overarching) methodology

QED effects in Parton Distribution Functions

No longer so negligible when extra orders are incorporated!



A. Cooper-Sarkar, T. Cridge, T. Hobbs, J. Huston, P. Nadolsky, M. Ponce-Chavez, K. Xie - hep-ph/2508.06603

T. Cridge, **JCM**, J. Huston - hep-ph/2602.06908